Title
E.E. REVIEW COURSE - LECTURE VIII.

Authors
Martinelli, E.
Leppard, J.
Perl, H.

Publication Date
1952-04-21

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

UCRL 1888

ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE VIII
April 21, 1952
E. Martinelli

(Notes by: J. Leppard, H. Perl)

I. MAGNETIC FIELDS

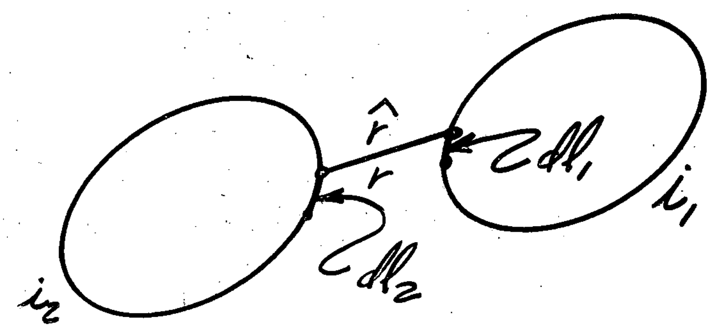
A. Electrostatic Case

Coulomb's Law which is given for the electrostatic case can be stated thus: $\vec{F} = f\, e_1\, e_2 / r^2$ in which $\vec{F}$ is in dynes, r in centimeters,

f = 1 (dimensionless), gives the value of the charge e, in electrostatic units (ESU).

B. Magnetic Case

The magnetic case has an equivalent which was first determined experimentally by Ampere. Given two closed loops of wire of length ℓ.



then $d\vec{F} = \dfrac{C\, i_1\, i_2}{r^2} \left[d\ell_1 \times (d\ell_2 \times \hat{r}) \right]$

Using electromagnetic units (EMU)

i is measured in ab amperes = 10 amperes.

r is measured in centimeters.

C = 1 (dimensionless).

DISCLAIMER

$\vec{dF}$ is in dynes

Using MKS units

$\vec{F}$ is measured in Newtons

i is measured in amperes

ℓ is measured in meters

$C = \frac{\mu}{4\pi}$ (μ = permeability of free space)

$= 4\pi \times 10^{-7}$ Henries/meter = volt sec/amp.

The relationship of these two systems of units is:

1 EMU of current = 3×10^{10} ESU charge/sec.

An expression for the magnetic field due to an infinitesimal current is:

$\vec{dB} = i_2\, d\ell_2 \times r/r^2$ then $\vec{dF} = i_1\, d\ell_1 \times \vec{dB}$

Integrating yields an integral formula for calculating $\vec{B}$.

$$\vec{B} = \int \frac{i\, d\ell \times \hat{r}}{r^2}$$

As shown, this formula gives an answer in EMU units. For MKS units insert the constant

$$\frac{\mu_o}{4\pi}$$

The units of magnetic field are:

MKS = Webers/meter^2

EMU = Gauss

and 1 Weber = volt second

1 Gauss = 10^{-4} Webers/m^2

II. MAGNET DESIGN

In the design of a small magnet with a small gap, analogous cases can be formulated using a more familiar group of functions.

Consider a loop of copper wire about a dielectric (Fig. 1) with an impressed voltage potential V_1 and V_2 as compared with a small electro-magnet (Fig. 2).

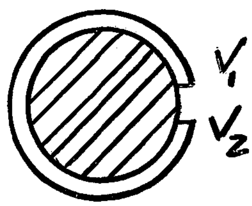


FIG. 1

CURRENT

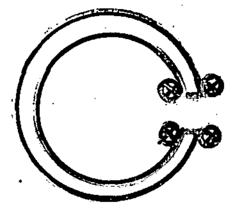


FIG. 2

MAGNETIC INDUCTION

(assuming the divergence of the current in the conductor = 0)

Expressed thus:

$\nabla \cdot i = 0$

then

$$\oint \frac{\hat{i} \cdot d\ell}{\sigma} = \text{EMF}$$

(differential form of Ohm's Law)

$$I = \int \hat{i}\,\hat{n}\,ds \text{ (total current through conductor)}$$

$R = E/I$

$\nabla \cdot \vec{B} = 0$

then

$$\int \frac{\vec{B} \cdot d\ell}{\mu} = 4\pi I \text{ (total current)}$$

assuming

$\mu \vec{H} = \vec{B}$

$4\pi I = \text{MMF}$

(μ is equivalent to conductivity)

$$N = \int \vec{B} \cdot \hat{n}\,ds$$

$$R' = \frac{\Omega}{N} \text{ (} R' = \text{reluctance)}$$

$$R' = \frac{L}{\mu A}$$

Having defined the term reluctance (R'), consider an electro-magnet and its air gap of length L with cross sectional area A (Fig. 3)

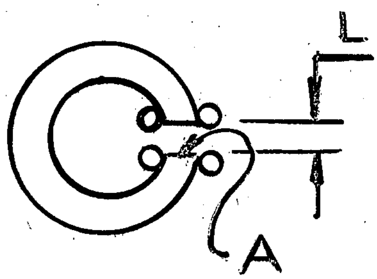


FIG. 3

$$R' \text{ Gap} = \frac{L \text{ Gap}}{A \text{ Gap}} \quad \text{(Neglect Fringe Effects)}$$

$$R' \text{ Iron} = \frac{L \text{ Iron}}{A \text{ Iron}\,\mu}$$

therefore, for an efficient magnet design:

$$R'_{iron} \ll R'_{gap}$$

considering the flux (N) of the gap:

$$N_{gap} = B_{gap} A_{gap} \text{ (in Gauss).}$$

$$= \frac{\Omega}{R'}$$

$$= \frac{4\pi \; n \; I \; A_{gap}}{L_{gap}}, \text{ when n I is ab ampere turns}$$

$$\text{or } n \; I = \frac{B_{gap} \; L_{gap}}{.4\pi}$$

$$\approx 2 \; B_{gap} \; L_{gap} \text{ (L in inches),}$$

B in Gauss

These formulae assume no field outside the magnet and that $\vec{B}$ does not saturate the iron.

III. MAGNETIC SHIELDING

The problem of magnetic shielding can be considered as follows:

Assume a uniform magnetic field (Fig. 5)

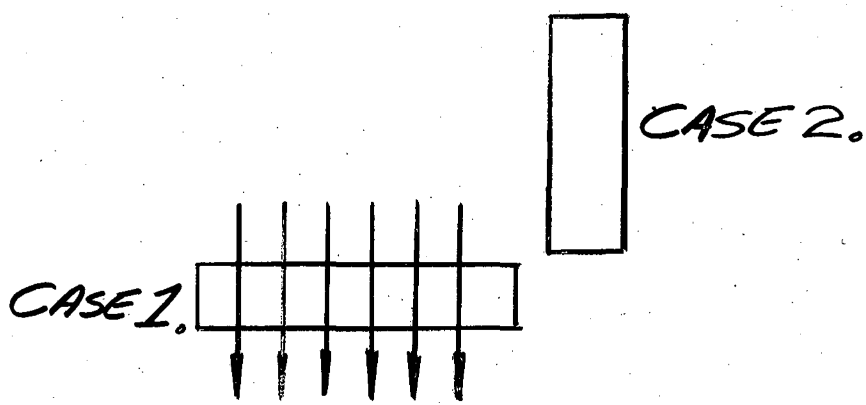


FIG. 5

CASE 1. ($\vec{B}$ $\perp$ to the iron. $\vec{B}$ is continous).

$B_{iron} = B_{air}$, $H_{iron} = \frac{B_{iron}}{\mu}$

CASE 2. ($\vec{B}$ $||$ to the iron).

$H_{iron} = H_{air}$, $B_{iron} = \mu H_{air}$

$B_{air} = H_{air}$

From the above cases it is seen that the vertical shielding in Case 2 is unsatisfactory in that the iron becomes saturated.

Consider again the iron in Cast 1. (Fig. 6)

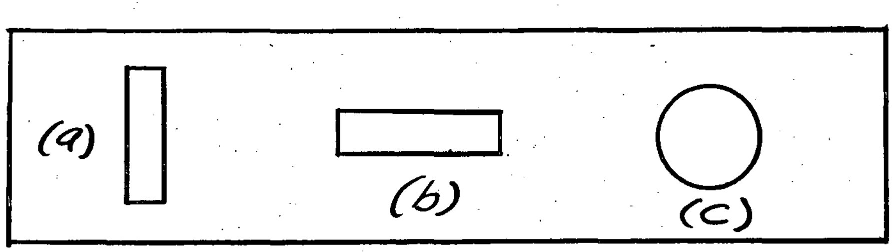


FIG. 6

(a) $\vec{H}_{iron} = \vec{H}_{hole}$ when hole is $\parallel$ to $\vec{B}$,

$$= B_{air}/\mu$$

$$= B_{hole}$$

(b) $\vec{B} = \vec{B}_{hole}$ when hole is $\perp$ to $\vec{B}$

(c) $$B_{inside\ hole} = 2\vec{B}_{iron}\frac{\frac{1}{\mu}}{\frac{1}{\mu}+1} \sim \frac{2\vec{B}}{\mu}$$

Therefore, magnetic shielding design criteria should include the following:

1. Iron shielding mass large with respect to shielded mass.

2. An aperture in the iron shielding whose greater dimension is parallel to the magnetic field. (However, practical geometry dictates circular or elliptical openings).

3. A liner of additional shielding material can be included within these apertures.

SYMBOLS

$\vec{H}$ = magnetic field intensity

$\vec{B}$ = magnetic flux density

i or I = current

ℓ or L = length

C = constant

f = constant

μ = permeability

R = resistance

R' = reluctance

Ω = magnetomotive force

A = area

σ = conductivity

N = flux (total)

n = turns

MMF = magnetomotive force