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Risk Analysis for Reservoir Operation

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The planning of reservoir operation presents decision makers with a trade-off between competing functions, which are energy production and flood control in this study. To optimally resolve the trade-off between maximization of energy revenues and minimization of downstream losses, the interaction between the expected value and variance of revenues (accruing from the reservoir operation) is included in a stochastic daily reservoir operation planning model. By parametrically varying the expected value and variance of the objective function, the risk-averse nature of decision makers is incorporated, resulting in a range of feasible alternative policies that reflect the decision maker's attitude toward revenue maximization and poor performance of the reservoir operation.

1. INTRODUCTION

A major difficulty in the development of reservoir planning models is to derive a suitable objective function. This is primarily due to (1) the stochastic nature of several variables present in any planning model (e.g., streamflows), (2) the inadequacy of expected or average performance criteria to reflect the typical decision maker's aversion to poor (or in the extreme case, catastrophic) outcome of an adopted set of release policies, and (3) the multiobjective nature of reservoir operation [*Cohon and Marks*, 1975]. Items 1 and 2 above are closely related (item 3 is discussed in section 2). Since the objective function is a function of random variables, it is customary to average it by means of an expectation operator to make it mathematically tractable. This leads to the computation of release policies that maximize the average performance of the reservoir but ignore serious negative effects that unusual, yet probable, events may have on those served by the reservoir functions.

A critique of the average performance criterion is given by *Eckstein* [1958], who pointed out that initiatives for structural measures (e.g., reservoirs) with flood control purposes arise largely from public reaction to catastrophic floods. Nevertheless, justifying the economic feasibility of a flood control project is usually based on the concept of expected annual damages (EAD). This type of analysis shows that the contribution of rare (i.e., with a long return period) floods to the EAD is relatively small. Even though the EAD is usually used to justify a flood control project, the actual size of the flood control structure is actually larger than the size of the reservoir that would minimize long-term, average damages. This is so because a structure that minimizes the EAD would protect against frequent (i.e., short to medium return period) floods that contribute most to the EAD but would not protect against the rare events, the ones that usually create the political pressure to implement such protection measures.

The critique of the expected performance approach for sizing flood control reservoirs is also applicable to the use of release policies that maximize expected or average performance as the basis for the operation of a multipurpose reservoir. As an alternative to the expected performance criterion, *Dantzig* [1956] proposed the use of a utility function to reflect the risk-averting attitudes of decision makers. *Davis* [1975] presented a method based on the average performance criterion but introduced a penalty factor to account for departures from expected targets. *Datta and Burges* [1984] and *Orlovski et al.* [1984] discussed the shortcomings of the expected value criterion, and in the latter paper, a minimax approach was introduced to derive release policies that are more representative of the actual attitudes of decision makers toward risk arising from the probabilistic performance of a reservoir system. *Mariño and Loaiciga* [1985*a*] analyzed the effect of conservative constraints on the flood control pool imposed by the Army Corps of Engineers on the operation schedules of a multireservoir system. They pointed out that risk aversion to poor performance of release schedules is reflected in practice by a stringent set of constraints on the flood control pool.

This paper presents a stochastic daily model for short-term (i.e., 1-month ahead planning horizon) reservoir operation. The model produces a set of alternative feasible release policies that represent various combinations of expected values and variances (a measure of risk, as shown later) of the objective function. The daily model is imbedded within a monthly model (with a 1-year planning horizon) previously developed by the authors [*Mariño and Loaiciga*, 1985*b*]. The relationship of our approach to a general optimization model based on the concept of expected utility maximization [*Freund*, 1956] is discussed, and an application to the planning of operations of Shasta reservoir, located in northern California, is presented. The remainder of the paper is organized as follows: Section 2 presents the theoretical background and the development of the planning model. Section 3 contains an application of the model, as well as numerical data and details on computational aspects. Section 4 closes this paper with a summary and conclusions of the findings.

2. METHODOLOGY

2.1. *Objective Function*

The reservoir used as a test case, i.e., the Shasta reservoir, is a multipurpose dam located in northern California. The Shasta reservoir impounds the Sacramento River about 200 miles north of the city of Sacramento. It operates to satisfy three functions: Flood control, energy production, and water supply for agricultural activities during the crop-growing season in California (June through September). During the rainy season (November through April), water supply demands are at negligible levels and flood control and hydroelectric energy production are the two main functions to be met. During the growing season, flood control is of no concern (at least in a short-term, daily basis) and hydroelectric energy production and water supply become the dominant functions. *Mariño and Loaiciga* [1983] provided an extensive description

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of the physical and contractual constraints imposed on the operation of Shasta reservoir, as well as the objectives of its operation. The authors also derived optimal monthly schedules that maximize annual benefits accruing from the operation of Shasta [*Mariño and Loaiciga*, 1985*b*]. The computed monthly storages are specified as probabilistic constraints to the daily model as discussed below.

The stochastic daily model developed in this study considers two operational functions: hydroelectric energy production and flood control. The model is applied to regulate daily storages during one month of the rainy season when water supply for agricultural use is of marginal importance. The objective is then given by

$$\max_{u_t, \forall t} E\{U[R - \text{FCL}]\} \tag{1}$$

in which R denotes the revenues from energy sales, FCL represents the monetary losses from flood control damages, u_t specifies the daily (nonstochastic) penstock releases during the tth day, t denotes a time index for the entire planning horizon (i.e., 30 days, as explained later), and $E\{U[\ \ ]\}$ denotes the expected utility of the monetary payoff $R - \text{FCL}$. Utility refers to the preferences of the decision makers with regard to the monetary revenues accruing from the reservoir operation. Since $R - \text{FCL}$ is a stochastic function, so is the utility of $R - \text{FCL}$, hence the need for the expectation operator E. It must be emphasized that expected utility of monetary benefits is not the same as the expected monetary benefits (or average or expected performance, as termed earlier); in particular, the former fully accounts for the risk-averse nature of system managers, whereas the latter does not. Furthermore, the choice of a utility function is necessary in this study only insofar as it provides a good theoretical background for the final methodology to develop a set of alternative feasible release policies, as shown in subsection 2.3. *Bechard et al.* [1981] proposed a similar linear combination of functions as that given in equation (1) as a basis for the objective function in a reservoir operation model. They showed the suitability of such an objective function when a deterministic performance criterion is used with a sequential update of streamflow forecasts and storages.

To express equation (1) in terms of releases u_t, the continuity equation for the reservoir, i.e.,

$$x_t = x_{t-1} - r_t - u_t + w_t \tag{2}$$

(where x_t, r_t, and w_t denote the beginning of period storage, spillage, and the streamflows for the tth day, respectively) must be written in terms of the initial, known storage x_1. Since we consider a winter month in this application, evaporation losses are minimal and neglected in equation (2). Spillages are expressed as function of storages,

$$r_t = c + d(x_t + x_{t+1}) \tag{3}$$

in which c and d are coefficients of the linear approximation to spillages. Equation (3) was originally developed by *Mariño and Loaiciga* [1985*b*]. By substituting (3) into (2) and expanding the right-hand side of (2) to express it as a function of the known, initial storage x_1, one obtains

$$x_t = a_{t-1}x_1 - \sum_{l=1}^{t-1} b_l u_{t-l} + \sum_{l=1}^{t-1} b_l w_{t-l} - \sum_{l=1}^{t-1} C_l \tag{4}$$

in which

$$a_{t-1} = \left(\frac{1-d}{1+d}\right)^{t-1} \tag{5}$$

$$b_l = \frac{(1-d)^{l-1}}{(1+d)^l} \tag{6}$$

$$C_l = c\,\frac{(1-d)^{l-1}}{(1+d)^l} \tag{7}$$

and $t = 2, 3, \cdots, N, N+1$, where N is the number of days in the planning horizon.

The revenues, R, accruing from hydroelectric energy sales equal the sum of daily revenues, i.e.,

$$R = \psi \sum_{t=1}^{N} [a + b(x_t + x_{t+1})]u_t \tag{8}$$

in which ψ denotes the unit price of energy (\$/MWh), $a + b(x_t + x_{t+1})$ is an energy production rate (MWh/volume of release), and a and b are coefficients developed previously by *Mariño and Loaiciga* [1985*b*].

The flood losses component of the objective, FCL, is expressed as

$$\text{FCL} = \sum_{t=1}^{N} [a' + b'(u_t + r_t)] \tag{9}$$

in which a' and b' are coefficients. Equation (9) was derived by approximating damage-release relationships linearly within the feasible range of total release $u_t + r_t$ (see subsection 2.2). By substituting (4) into (8) and (9) and by substituting the resulting expressions for (8) and (9) into (1), one obtains, after lengthy algebraic operations, the linearized expression

$$\max_{u_t, \forall t} E\left\{U\left[\sum_{t=1}^{N} q_t u_t + K\right]\right\} \tag{10}$$

in which

$$q_t = -\sum_{l=1}^{N-t} (d_{t+l} b_l) - b'(1 - 2dm_t) \qquad t = 1, 2, \cdots, N-1 \tag{11}$$

$$q_t = -b' \qquad t = N \tag{12}$$

$$d_t = b(w_t + w_{t-1}) - 2\Psi(ad + bc) \qquad t = 2, 3, \cdots, N \tag{13}$$

$$m_t = \sum_{l=1}^{N-t} b_l \qquad t = 1, 2, \cdots, N-1 \tag{14}$$

$$K = b'\left[-2d \sum_{t=1}^{N-1} m_t w_t + Nc + 2dx_1 \sum_{t=2}^{N} a_{t-1} - 2d \sum_{l=1}^{N-1} (N-l)C_l + 2dx_1\right] \tag{15}$$

Coefficients a, b, and Ψ are defined after equation (8); b_l and b' are defined after equations (6) and (9), respectively; coefficients c and d are given in equation (3); K is a random term, independent of releases, in which a_{t-1}, C_l, and m_t are given by equations (5), (7), and (14), respectively. The coefficients q_t, $t = 1, 2, \cdots, N-1$, in equation (10) are random since they are a linear function of the random streamflows (see equation (13)).

Without any loss of generality, the following utility function is proposed to model the preferences of the decision makers,

$$U(R - \text{FCL}) = 1 - e^{-\phi(R - \text{FCL})} \tag{16}$$

in which

$$R - \text{FCL} = \sum_{t=1}^{N} q_t u_t + K$$

The exponential utility function in (16) has the concavity property typical of conservative (i.e., risk-averse) decision makers. The decision maker's risk aversion is determined by the coefficient of risk aversion ϕ in (16). The larger the ϕ, the more conservative the decision maker is. The exponential utility function in (16) is a subcase of the general HARA/LRT family of utility functions [*Cass and Stiglitz*, 1970], in which the coefficient of risk aversion is constant. Such utility functions are widely accepted to model risk aversion [e.g., *Dantzig*, 1956; *Freund*, 1956; *Wiens*, 1976; *Paris*, 1979].

If one assumes that linear combinations of streamflows (i.e., $\sum a_l w_l$, as the one appearing in (13)) are normally distributed (a mild assumption due to the central limit theorem), then, after substituting (16) into (10) and taking expectations, one obtains the following objective function:

$$\max_{u_t, \forall t} \sum_{t=1}^{N} u_t E(q_t) - \frac{\phi}{2}(\mathbf{u}'\Lambda\mathbf{u} + 2\mathbf{u}'\boldsymbol{\sigma}_{12}) \tag{17}$$

in which Λ is the covariance of the vector $\mathbf{q}$ whose elements are $q_1, q_2, \cdots, q_N$; $\boldsymbol{\sigma}_{12}$ is the vector covariance between the vector $\mathbf{q}$ and K; and $\mathbf{u}'$ is a row vector whose elements are $u_1, u_2, \cdots, u_N$. In equation (17), constant terms that have no influence on the maximization (i.e., $E(K)$ and σ_{22}, the variance of K) are left out. Covariances Λ and $\boldsymbol{\sigma}_{12}$ must be estimated, as explained below.

2.2. *Constraints*

The ending storage x_{N+1} is random and unknown. From the optimal monthly storage policies developed by *Mariño and Loaiciga* [1985*b*], a target value $x_{N+1}{}^T$ is used as a reference value in the following probabilistic (or chance) constraint:

$$P(x_{N+1}{}^T - \delta_l \le x_{N+1} \le x_{N+1}{}^T + \delta_u) \ge \gamma \tag{18}$$

which specifies that the ending storage be within the range $(x_{N+1}{}^T - \delta_l, x_{N+1}{}^T + \delta_u)$ with probability γ, where δ_l and δ_u are deviations about the target $x_{N+1}{}^T$. In addition, there are probabilistic constraints on the minimum and maximum values of the storages,

$$P(x_t \ge X_{\min}) \ge \alpha \qquad t = 2, 3, \cdots, N+1 \tag{19}$$

$$P(x_t \le X_{\max}) \ge \beta \qquad t = 2, 3, \cdots, N+1 \tag{20}$$

and there are constraints on penstock releases u_t, i.e.,

$$u_t \ge U_{\min} \qquad t = 1, 2, \cdots, N \tag{21}$$

$$u_t \le U_{\max} \qquad t = 1, 2, \cdots, N \tag{22}$$

in which $X_{\min}$ and $X_{\max}$ denote minimum and maximum storages, respectively; α and β are the probabilities with which constraints (19)–(20) are satisfied; and $U_{\min}$ and $U_{\max}$ are minimum and maximum penstock releases, respectively. Notice that the continuity equation (4) has already been substituted into the objective function and that spillages are implicitly constrained since they are expressed as a function of storages (equation (3)). The deterministic equivalents of (18)–(20) are given in the next subsection where the planning model is summarized.

2.3. *Summary and Discussion of Planning Model*

After substituting (4) into (18)–(20) and converting the chance constraints (18)–(20) into their deterministic equivalents, the planning model can be stated as

$$\max_{u_t, t=1,2,\cdots,N} \sum_{t=1}^{N} u_t E(q_t) - \frac{\phi}{2}(\mathbf{u}'\Lambda\mathbf{u} + 2\mathbf{u}'\boldsymbol{\sigma}_{12}) \tag{23}$$

subject to

$$k^{(\gamma)} - x_{N+1}{}^T - \delta_u + a_N x_1 - \sum_{l=1}^{N} C_l \le \sum_{l=1}^{N} b_l u_{t-l}$$
$$\le g^{(\gamma)} - x_{N+1} + \delta_l + a_N x_1 - \sum_{l=1}^{N} C_l \tag{24}$$

$$\sum_{l=1}^{t-1} u_{t-l} b_l \le a_t{}^{(\alpha)} + a_{t-1} x_1 - X_{\min} - \sum_{l=1}^{t-1} C_l \tag{25}$$
$$t = 2, 3, \cdots, N+1$$

$$\sum_{l=1}^{t-1} u_{t-l} b_l \ge b_t{}^{(\beta)} + a_{t-1} x_1 - X_{\max} - \sum_{l=1}^{t-1} C_l \tag{26}$$
$$t = 2, 3, \cdots, N+1$$

$$u_t \ge U_{\min} \qquad t = 1, 2, \cdots, N \tag{27}$$

$$u_t \le U_{\max} \qquad t = 1, 2, \cdots, N \tag{28}$$

Equations (24)–(26) are the deterministic equivalents of the chance constraints (18)–(20), respectively. In equation (24), $k^{(\gamma)}$ and $g^{(\gamma)}$ are the values for which

$$P\left(\sum_{l=1}^{N} b_l w_{t-l} \le k^{(\gamma)}\right) \ge \gamma$$

$$P\left(\sum_{l=1}^{N} b_l w_{t-l} \ge g^{(\gamma)}\right) \ge \gamma$$

respectively. In equations (25) and (26), $a_t{}^{(\alpha)}$ and $b_t{}^{(\beta)}$ are the values for which

$$P\left(\sum_{l=1}^{t-1} b_l w_{t-l} \ge a_t{}^{(\alpha)}\right) \ge \alpha$$

$$P\left(\sum_{l=1}^{t-1} b_l w_{t-l} \le b_t{}^{(\beta)}\right) \ge \beta$$

respectively. The distribution of the convolution

$$\sum_{l=1}^{t-1} b_l w_{t-l}$$

is derived in section 3.

The solution to the model given by (23)–(28) requires the specification of γ, α, and β in (18)–(20), respectively; furthermore, the risk aversion coefficient ϕ in (23) must be known. *Paris* [1979] has shown that ϕ can be determined from the values of γ, α, and β in some kinds of economic models; for our problem, the required information in Paris' approach is not available. The approach followed herein is then to specify γ, α, and β based on information available from previous studies on suitable reliability levels (i.e., γ, α, and β) for reservoirs in northern California [*Mariño and Mohammadi*, 1983; *Mariño and Loaiciga*, 1983]. The lack of information about ϕ is adequately overcome by noticing that the stochastic revenue function

$$R - \text{FCL} = \sum_{t=1}^{N} q_t u_t + K$$

has a variance equal to $\mathbf{u}'\Lambda\mathbf{u} + 2\mathbf{u}'\boldsymbol{\sigma}_{12} + \sigma_{22}$ (all terms previously defined). Then, the problem of maximizing expected utility (i.e., equation (23)) is paired to that of minimizing the variability or dispersion of revenues $R - \text{FCL}$ (such variability introduces the uncertainty in the outcome of any release policy and is the source of risk) subject to a parametric constraint on the level of expected revenues, i.e.,

$$\sum_{t=1}^{N} u_t E(q_t) + E(K)$$

see equation (16). The parametric constraint specifies that the level of expected revenue be at least at some minimum monetary threshold. Minimization of the variance of revenues subject to constraints (24)–(28) plus a constraint on expected revenues that is varied parametrically yields a trade-off curve between the variance of the revenue (i.e., the risk) and its expected value. The idea is then that the decision maker chooses any variance-expected value combination on such a curve and that will imply the choice of a specific release policy, for each point in the variance-expected value curve has an associated feasible release policy. The decision maker is then certain that any release policy entails a given level of risk and expected level of revenues. The adopted release policy would be the one that the decision maker would choose if problem (23)–(28) was solved with the correct value for the risk aversion coefficient ϕ. The trade-off curve between variance and expected value is appealing, for it shows the set of feasible risk–monetary payoff combinations available to the decision maker. To summarize, the problem to be solved is

$$\min_{u_t, t=1,2,\cdots,N} \quad \mathbf{u}'\Lambda\mathbf{u} + 2\mathbf{u}'\boldsymbol{\sigma}_{12} \tag{29}$$

subject to

$$\sum_{t=1}^{N} u_t E(q_t) + E(K) \geq M^{(k)} \tag{30}$$

and constraints (24)–(28). The parametric constraint (30) specifies that the expected revenue is to exceed some level $M^{(k)}$, where k denotes that the minimization in (29) will be done for different values of $M^{(k)}$ to develop the feasible variance–expected revenue curve.

3. Model Application

3.1. *Basic Data*

The model developed in the preceding section (equations (29)–(30) and (24)–(28)) was applied to daily flow regulation of Shasta reservoir during the month of April of the water year (October–September) 1979–1980. Table 1 contains the values of key input variables. Table 2 contains the values of the coefficients (c, d), (a, b), and (a', b') appearing in (3), (8), and (9), respectively. Those coefficients are the basis for many other expressions in the model (e.g., (5)–(7) and (11)–(15)).

3.2. *Estimation of Expected Values and Covariances*

The model implementation requires the expected values $E(q_t)$ and $E(K)$ (see equations (11) and (15), respectively), $t = 1, 2, \cdots, N - 1$ (q_N is constant, as indicated in equation (12)). Moment estimators for q_t, $t = 1, 2, \cdots, N - 1$, and K were computed from daily data for the period 1921–1980. The estimators are

$$E(q_t) = \bar{q}_t = -\frac{1}{NY}\sum_{j=1}^{NY}\sum_{l=1}^{N-t} d_{t+l}{}^{(j)} b_l - b'(1 - 2dm_t) \tag{31}$$

$$t = 1, 2, \cdots, N-1$$

in which $d_{t+l}{}^{(j)}$ is given by equation (13) and the superscript j is an index for the year number (i.e., $j = 1$ for 1921, $j = 2$ for 1922, $\cdots$, $NY = 60$ for 1980),

$$E(K) = \bar{K} = \frac{1}{NY}\sum_{j=1}^{NY}\left(-2b'd\sum_{t=1}^{N-1} m_t w_t{}^{(j)}\right) + b'\left[Nc + 2dx_1\sum_{t=2}^{N} a_{t-1} - 2d\sum_{l=1}^{N-1}(N - l)C_l + 2dx_1\right] \tag{32}$$

in which $w_t{}^{(j)}$ denotes the tth day in April of the jth year.

The covariance of the vector $\mathbf{q}$ whose elements are q_1, q_2, $\cdots$, q_N, denoted by Λ, is estimated by means of the expression

$$\hat{\sigma}_{ij} = \frac{1}{(NY - 1)}\left[\sum_{j=1}^{NY} q_i{}^{(j)} q_j{}^{(j)} - NY\bar{q}_i\bar{q}_j\right] \tag{33}$$

for $i, j = 1, 2, \cdots, N - 1$; $\hat{\sigma}_{ij}$ denotes the estimator of the ijth element of Λ (a symmetric matrix); and $q_i{}^{(j)}$ denotes the observed value for q_i during the jth year. Since q_N is a constant, the $(N - 1)$th row and column of Λ are zero. The estimator of $\boldsymbol{\sigma}_{12}$, the covariance between $\mathbf{q}$ and K, is given by

$$\hat{s}_i = \frac{1}{(NY - 1)}\left[\sum_{j=1}^{NY} q_i{}^{(j)} K^{(j)} - NY\bar{K}\bar{q}_i\right] \tag{34}$$

$$i = 1, 2, \cdots, N - 1$$

in which $\hat{s}_i$ denotes the estimator for the ith element of $\boldsymbol{\sigma}_{12}$ ($\hat{s}_N = 0$) and $K^{(j)}$ denotes the observed value for K during the jth year.

The distribution of the convolution

$$\sum_{l=1}^{t-1} b_l w_{t-l}$$

required to specify (24)–(26) is simply that of a linear combination of random normal variables; therefore, such convolution is also normally distributed with expected value

$$\sum_{l=1}^{t-1} b_l \bar{w}_{t-l}$$

where $\bar{w}_{t-l}$ is the expected value of w_{t-l}, which is estimated by

$$\bar{w}_{t-l} = \frac{1}{NY}\sum_{j=1}^{NY} w_{t-l}{}^{(j)} \tag{35}$$

TABLE 1. Basic Data for the Optimization Model

	Value
Reliabilities	
α	0.90
β	0.99
γ	0.99
Storages, m^3	
x_1	4.6626×10^9
$x_{N+1}{}^T$	4.8106×10^9
δ_l	0.3700×10^9
δ_u	0.1234×10^9
Bounds	
Storages, m^3	
$X_{\min}$	0.7401×10^9
$X_{\max}$	5.0573×10^9
Releases, m^3/d	
$U_{\min}$	1.2335×10^7
$U_{\max}$	6.1674×10^7

TABLE 2. Values of Coefficients

Equation	Coefficient	Value
(3)	c	−274
	d	0.0360
(8)	a	234
	b	0.0231
	Ψ	50
(9)	a'	−187,500
	b'	18,750

Use of equations (3), (8), and (9) requires that storages, releases, and spillages be expressed in units of volume that are measured in thousands of acre-feet (kaf); 1 kaf = 1.23×10^6 m^3.

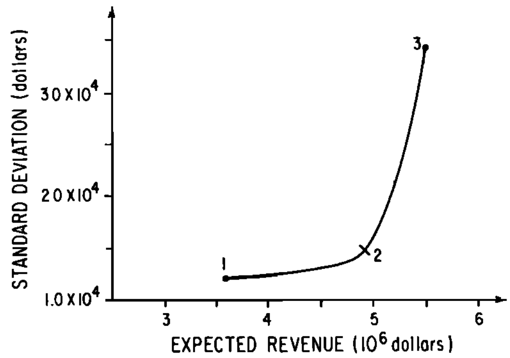

Fig. 1. Standard deviation versus expected value of revenues.

in which $w_{t-l}^{(j)}$ is the observed streamflow during the $(t-l)$th day of the jth year. The covariance of the convolution

$$\sum_{l=1}^{t-1} b_l w_{t-l}$$

is given by

$$\sum_{l=1}^{t-1} b_l^2 V(w_{t-l}) + 2 \sum_{k<m} \sum_{m} b_k b_m \operatorname{cov}(w_k, w_m) \qquad (36)$$

$$1 \le k \le t-2$$
$$2 \le m \le t-1$$

in which $V(w_{t-l})$ and cov (w_k, w_m) denote the variance of w_{t-l} and the covariance of w_k and w_m, respectively, and are estimated by

$$\hat{V}(w_{t-l}) = \frac{1}{(NY-1)}\left\{\sum_{j=1}^{NY} [w_{t-l}^{(j)}]^2 - NY(\bar{w}_{t-l}^{(j)})^2\right\} \qquad (37)$$

$$\hat{\operatorname{cov}}(w_k, w_m) = \frac{1}{(NY-1)}\left\{\sum_{j=1}^{NY} w_k^{(j)} w_m^{(j)} - NY\, \bar{w}_k^{(j)} \bar{w}_m^{(j)}\right\} \qquad (38)$$

in which $\hat{V}(\ )$ and $\hat{\operatorname{cov}}(\ ,\ )$ denote estimators; $\bar{w}_s^{(j)}$ denotes the expected value of streamflow during the sth day of the jth year (where s is a suitable subindex, i.e., $s = t - l$, k or m) and is estimated by (35).

3.3. *Computational Aspects and Analysis of Results*

The solution of (29)–(30) plus constraints (24)–(28) was obtained by means of a reduced-gradient method [*Murtaugh and Saunders*, 1978]. Computations were performed in a DEC VAX 11/780 computer with CPU processing time of about 30 s in all runs (each run for a different level of expected revenue; see equation (30)).

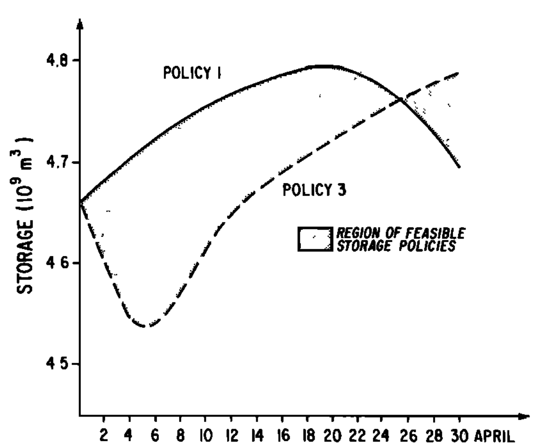

Fig. 2. Range of feasible storage policies.

TABLE 3. Penstock Releases (u_t) and Spillages (r_t) for Policies 1 and 3 (in 10^7 m^3/d)

Day t	u_t Policy 1	u_t Policy 3	r_t Policy 1	r_t Policy 3
1	1.2	6.2		
2	1.2	6.2		
3	1.2	6.2		
4	1.2	5.3		
5	1.2	4.2	0.1	
6	1.2	2.9	0.2	
7	1.2	1.4	0.3	
8	1.2	1.2	0.3	
9	1.2	1.2	0.4	
10	1.2	1.2	0.4	
11	1.2	1.2	0.5	
12	1.2	1.2	0.5	
13	1.2	1.2	0.6	
14	1.2	1.2	0.6	
15	1.2	1.2	0.6	
16	1.2	1.2	0.7	
17	1.2	1.2	0.7	
18	1.2	1.2	0.7	0.1
19	1.2	1.2	0.7	0.2
20	1.7	1.2	0.8	0.3
21	1.9	1.2	0.7	0.3
22	1.7	1.2	0.7	0.4
23	1.8	1.2	0.7	0.4
24	1.7	1.2	0.7	0.5
25	1.8	1.2	0.7	0.5
26	1.7	1.2	0.7	0.6
27	3.3	1.2	0.6	0.6
28	3.8	1.2	0.5	0.6
29	6.2	1.2	0.3	0.7
30	6.2	1.2		0.7

Figure 1 shows the curve depicting the locii of expected revenue versus the square root of the variance (i.e., the standard deviation) of revenue. Notice that the standard deviation of revenues remains relatively constant up to an expected value of about $5,000,000. For larger expected revenues, the corresponding standard deviation increases rapidly. An interesting feature of Figure 1 is that any increase in revenues is at the expense of an increase in the variance and, consequently, in the risk associated with a given release policy. Such one-to-one relationship explains why reservoir managers do not follow release policies that would maximize expected profits, i.e., do not choose a release policy associated with point 3 in Figure 1. Instead, release policies associated with less risky outcomes, predictably between points 1 and 2, are adopted. For the planning of operations, the decision maker would enter Figure 1 at a point which is consistent with his or her preferences about the level of expected revenue and corresponding risk. This point uniquely defines an optimal release policy. The curve depicted in Figure 1 is restricted to the regions between points 1 and 3 by the constraint set. The storage policies corresponding to points 1 and 3 define a range of feasible storages that can be chosen and that correspond to points in the curve of Figure 1. Figure 2 shows such a range of feasible storage policies (i.e., the region comprehended between the curves labeled policy 1 and policy 3, which correspond to the points 1 and 3 of Figure 1). Penstock releases and spillages corresponding to any storage policy are readily computed by means of equations (2) and (3). Table 3 displays the releases u_t and spillages r_t corresponding to storages policies 1 and 3 in Figure 2. The model developed in subsection 2.3 yields u_t, $\forall t$, from which spillages and storages were derived from equations (3) and (4), respectively. By parametrically varying equation (30), different operation strategies are readily obtained.

4. SUMMARY AND CONCLUSIONS

Decision making under uncertainty commonly involves having to choose between a set of feasible strategies. Some strategies offer a relatively high expected benefit or revenue but at the expense of an increased variability (or risk) on the probable outcome associated with a given strategy or policy. The problem of trading expected revenue by variability of the outcome has been analyzed in relation to reservoir operation. The drawback of using the expected performance criterion for planning reservoir operation has been discussed. As an alternative approach, a model that incorporates the risk-averse nature of reservoir managers has been developed. This model is based on a general class of utility functions and, from a computational standpoint, has been expressed as a parametric, quadratic, mean-variance model.

The set of feasible storage policies has been derived. Also, the corresponding trade-off curve between the expected revenue and the standard deviation of such revenues is given for the daily operation of Shasta reservoir. The decision maker can set a specific level of expected revenues accruing from a two-purpose (i.e., hydropower production and flood control) operation and the proposed model yields the corresponding releases, spillages, storages, and a level of revenue variability.

Computationally, the model has been found quite efficient; from a conceptual standpoint, the model yields not only information about alternative operation policies, but also on the basic elements that characterize the decision making process with which reservoir managers are faced, namely, expected performance and uncertainty on the outcome of any operational schedule.

NOTATION

a coefficient in the energy revenue equation (8).
a' coefficient in the flood loss equation (9).
a_{t-1} initial storage term in the continuity equation (4).
b coefficient in the energy revenue equation (8).
b' coefficient in the flood loss equation (9).
b_t coefficient in the continuity equation specified in equation (6).
C_t coefficient in the continuity equation specified in equation (7).
c coefficient in the spillage equation (3).
d coefficient in the spillage equation (3).
d_t coefficient in the objective function specified by equation (13).
FCL flood loss damage equation (9).
K random revenue in equation (10).
$\bar{K}$ expected value of random revenue.
$M^{(k)}$ parametric value of expected revenue in equation (30).
m_t coefficient of the objective function given by equation (14).
N number of periods (days) in the planning horizon.
NY number of years of daily records.
q_t random coefficient of penstock releases in objective function.
$\bar{q}_t$ expected value of q_t.
R energy revenues equation (8).
r_t releases (spillages) during tth day.
$\hat{s}_i$ estimator of the ith element of $\boldsymbol{\sigma}_{12}$.
$U_{\max}$ maximum penstock release.
$U_{\min}$ minimum penstock release.
u_t penstock releases during tth day.
$\mathbf{u}$ vector of penstock releases for entire planning horizon.
w_t streamflow during the tth day.
$\bar{w}_t$ expected value of streamflow during the tth day.
$X_{\max}$ maximum storage.
$X_{\min}$ minimum storage.
x_t beginning storage during tth day.
α reliability level for exceeding minimum storage.
β reliability level for being below maximum storage.
γ reliability level for chance constraint on final storage.
δ_u deviation above target storage.
δ_l deviation below target storage.
Λ covariance matrix of random coefficients q_t.
$\hat{\sigma}_{ij}$ estimator of the ijth element of Λ.
$\boldsymbol{\sigma}_{12}$ covariance between matrix coefficients q_t and random term K.
σ_{22} variance of K.
ϕ risk aversion coefficient.
ψ unit price per MWh energy.

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