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A Wave-Induced Transport Process in Marine Sediments

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We show how surface wave action can increase the rate of transport of solutes into a sandy seabed by orders of magnitude via a mechanism known as mechanical dispersion. It is most effective for large sediment permeability and thickness, high surface wave amplitude, and shallow water. A method for setting up the appropriate transport equation, valid when dispersion is well developed, is given. Its dispersion term contains two mechanical dispersion parameters that can be estimated roughly from existing data when the sediments are well sorted. The dispersion can be inhomogeneous and anisotropic in homogeneous, isotropic sediments. The effect of surface wave action on transport into sediments on the eastern U.S. shelf is shown to be significant under certain conditions. The effect on thawing of subsea permafrost beneath Prudhoe Bay, Alaska, seems negligible.

INTRODUCTION

The mechanisms by which nutrients, pollutants, or solutes are transported into marine sediments are of interest for several reasons, the most obvious of which is their control on rates and depths of chemical or biological processes. The best understood and probably most common of the transport mechanisms is molecular diffusion along chemical gradients, and many chemical processes have been successfully analyzed in its context (*Berner* [1980] is a convenient reference). Bio-turbation is also known to be important in some situations. There is also a less studied class of physical rather than chemical or biological transport mechanisms that involves bulk motion of the pore water in the sediments. When these mechanisms operate, they may be important because they are potentially so much faster than the others. In this paper one of these physical mechanisms, the enhancement of transport rates by wave-driven motion of the water in the sediments, is examined.

The wave-driven mechanism is but one member of a family of possible convective processes, and it is well to consider it in that context. For example, there are convective processes that may be loosely termed flushing, stirring, and floating, as well as others. In flushing, the pore water is continuously replaced by a current with time average unequal to zero, which is driven by motion of the overlying free water. This seems to occur in an intertidal and narrow subtidal strip [*Riedl et al.*, 1972]. In the stirring process addressed by this paper, the pore water is agitated by the motion in the overlying free water caused by surface waves, but the motion is periodic and the time average current is zero. In the floating process the pore water is floated out of the sediments due to larger density in the overlying free water. Examples in which floating seems to be important are given by *Thorstenson and Mackenzie* [1974], who found seasonal sediment water composition changes in Harrington Sound, Bermuda, that were too large to be explained by diffusion; *Hesslein* [1980], who found the entry of tritiated water into lake sediments to be much faster than diffusive; *Harrison and Osterkamp* [1978], who found the rates

of thawing of subsea permafrost and the salinity profiles in the thawed layer to be characteristic of convective processes, and who used simple theory to show that a diffusive regime is often unstable to convection; and *Musgrave and Reeburgh* [1982], who find time-varying temperature fields in lake sediments that cannot be described by a diffusive model.

The wave-stirring processes considered here (not to be confused with the stirring of the sediment particles that occurs if they are resuspended) has already received some attention. *Riedl et al.* [1972] refer to it as subtidal pumping and discuss evidence for it based on the position of the redox boundary in marine sediments. They point out, by way of Darcy's law and in situ measurements of the pore water velocity field, that a great deal of mass can be transported into sandy sediments by wave-induced motion. Of course, a great deal of mass may also be transported out again because the motion is periodic, so their calculation, while suggestive, is not really a transport theory. Nevertheless, theory explored here supports their basic idea.

THEORY

Background

When groundwater moves through an aquifer, it is known that any chemical concentration gradients in it are smeared out at a rate which greatly exceeds that predicted on the basis of molecular diffusion. Experiment and theory indicate that the effect is strongly velocity dependent. The basic reasons for the phenomenon are fairly well understood. The most important is that fluctuations in the directions of the streamlines, with respect to the direction of mean flow, occur on the scale of the sediment grain size. A similar dispersive phenomenon should be caused in subsea sediments by pore water motion that is driven by surface waves.

In this section we shall adapt conventional theory, normally used in ground water or reservoir applications, to the wave problem, in which the pore water motion is periodic. Near shore the wave-induced motion may have a component which is not periodic and which will also contribute to transport and dispersion, but that effect is not treated here. We shall assume the simplest possible version of the conventional theory (see *Fried and Combarous* [1971] for a review), although the

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question of the best formulation is still open (Smith and Schwartz [1980], for example). A more important point discussed later is the limit of applicability of the conventional theory to periodic motion.

It is intuitively reasonable, and it is borne out by laboratory experiment, that at low pore water velocity any possible effects of mechanical dispersion will be overwhelmed by molecular diffusion. But there exists a reasonably well defined velocity above which mechanical dispersion begins to become important. This is the velocity at which the efficiencies of convective and diffusive transport are equal, on a distance scale equal to the pore size, which can be crudely represented by the grain size d of the sediments. Stated more formally, dispersion becomes significant for velocity v , such that the Peclet number Pe , defined by

$$Pe = \frac{vd}{\kappa_f} \quad (1)$$

becomes greater than unity. Here κ_f is the molecular diffusivity of the solute of interest in free water. When $Pe > 1$ in a field situation, dispersion can be estimated from suitable parameters measured as functions of v in the laboratory.

It is unlikely that a large Peclet number is a sufficient criterion for the importance of mechanical dispersion in our particular problem of periodic motion, in which we take v in (1) to be the velocity amplitude. If the scale of motion of a pore water parcel during a wave period does not exceed the pore (or grain) size, it is unlikely that much dispersion will occur regardless of the Peclet number [see Raats and Scooter, 1968]. A related, but less physically significant problem from the point of view of dispersion, is that Darcy's law, which we will use to calculate the pore water velocity field, may be questioned under this circumstance. As the scale of motion increases, and finally greatly exceeds the pore (or grain) size, it seems likely that the conditions of the conventional theory are approached, and that presently available experimentally measured values of the dispersion parameters can be applied to the periodic motion problem to give rough results.

Transport Equation

As the basis of the simplest theory, we adopt the following transport equation for the concentration S of the material of interest in the pore water:

$$\nabla \cdot (\boldsymbol{\kappa} \cdot \nabla S) - \mathbf{v} \cdot \nabla S - qS - \frac{\partial S}{\partial t} = 0 \quad (2)$$

t is time, \mathbf{v} is the wave-induced pore water velocity field, q is a constant sink strength factor, and $\boldsymbol{\kappa}$ is the mechanical dispersion tensor; $\mathbf{v} \cdot \nabla S$ is the convective term, and qS is a reaction term appropriate for simple first-order processes. This particular form of the reaction term is adopted as a convenient example; most of the subsequent discussion of the nature of the mechanical dispersion term is independent of it.

Even in homogeneous isotropic sediments, the mechanical dispersion in (2) must be represented by a tensor $\boldsymbol{\kappa}$ because of anisotropy due to a particular direction of the pore water velocity vector \mathbf{v} . Experiment shows that in isotropic sediments the dispersion tensor can be represented by

$$\boldsymbol{\kappa}' = \begin{pmatrix} \kappa_L & 0 & 0 \\ 0 & \kappa_T & 0 \\ 0 & 0 & \kappa_T \end{pmatrix} \quad (3)$$

which holds only in a coordinate system oriented with its first axis parallel to the pore water velocity vector \mathbf{v} . Along this axis the dispersion coefficient has the value κ_L (L stands for longitudinal). Dispersion also occurs perpendicular to \mathbf{v} ; it is isotropic in this plane in the sense that it can be represented by the single coefficient κ_T (T stands for transverse). κ_L and κ_T are found to depend upon velocity magnitude v , and both approach the familiar molecular diffusivity κ when the Peclet number $Pe \ll 1$ (equation (1)). For $Pe \gg 1$, $\kappa_L > \kappa_T$, usually by an order of magnitude, so that the dispersion is indeed anisotropic.

In the present problem of periodic wave-induced motion, the direction of the velocity vector \mathbf{v} changes in space and time, so equation (3) does not hold in a fixed coordinate system. $\boldsymbol{\kappa}$ in a fixed system can be found by using the usual rules for tensor transformation under a specified coordinate system rotation angle, which is the angle θ made by \mathbf{v} with respect to the sea bed; θ is a function of space and time. The fixed Cartesian coordinates are taken to be (x, y, z) , where x and z are in the seabed surface, respectively parallel and perpendicular to the direction of wave propagation, and y is down. The rotation occurs about the z axis, and θ is taken to be positive in the clockwise direction. After $\boldsymbol{\kappa}$ in the fixed system is worked out, the first (dispersive) term in (2) can be evaluated

$$\begin{aligned} \nabla \cdot (\boldsymbol{\kappa} \cdot \nabla S) &= \frac{\partial}{\partial x} \left(\alpha \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha \frac{\partial S}{\partial y} \right) + \kappa_T \frac{\partial^2 S}{\partial z^2} \\ &+ \frac{\partial}{\partial x} \left(\beta \cos 2\theta \frac{\partial S}{\partial x} - \beta \sin 2\theta \frac{\partial S}{\partial y} \right) \\ &+ \frac{\partial}{\partial y} \left(-\beta \sin 2\theta \frac{\partial S}{\partial x} - \beta \cos 2\theta \frac{\partial S}{\partial y} \right) \end{aligned} \quad (4)$$

where α is the mean dispersion coefficient, or effective diffusivity, given by

$$\alpha = \frac{\kappa_L + \kappa_T}{2} \quad (5a)$$

and

$$\beta = \frac{\kappa_L - \kappa_T}{2} \quad (5b)$$

Note that when $Pe \ll 1$, $\alpha \rightarrow \kappa$ and $\beta \rightarrow 0$ because both κ_L and $\kappa_T \rightarrow \kappa$, the molecular diffusivity.

Pore Water Velocity Field

We now consider the pore water velocity field \mathbf{v} , which can be calculated from Darcy's law and incompressibility, using the pressure variations induced by surface waves as the seabed boundary condition. The special case of homogeneous isotropic sediments of infinite depth (depth greater than the surface wavelength) and of a single driving wavelength is considered first; how the transport behaves under different conditions is discussed later. By using the simplest version of Darcy's law and the coordinate system of the last section, the velocity field is

$$v_x = ue^{-hy} \cos(hx - \omega t) \quad (6a)$$

$$v_y = -ue^{-hy} \sin(hx - \omega t) \quad (6b)$$

$$v_z = 0 \quad (6c)$$

Here t is time, h and ω are, respectively, the wave number and angular frequency of the surface wave, and u is a velocity

amplitude given by

$$u = \frac{kha}{\cosh hD} \quad (7)$$

where a is the wave amplitude, D is the water depth, and k is the quotient of hydraulic conductivity and porosity, a more convenient parameter in our applications than the hydraulic conductivity itself. The factor $a/\cosh hD$ is the amplitude (expressed as a head) of the seabed pressure variation, estimated from linear wave theory [Lamb, 1945]. By the same theory ω and h are related by

$$\omega = (gh \tanh hD)^{1/2} \quad (8)$$

where g is the gravitational acceleration. Pore water motion is significant only if the wavelength exceeds the water depth, because only then is the sea bed pressure variation significant.

Equation (6) is basically the velocity field as first calculated by Putnam [1949]. It does not take into account the deformability of the sediments, compressibility of the pore water, perturbation of the overlying free water by the pore water motion, or inertial effects. Starting from (6) and a complete statement of Darcy's law including inertial terms, one can easily show that these terms are negligible as long as $(\omega k)/g$ and $(\alpha(hk)^2)/g \ll 1$, which is true in all of the problems discussed here. The unimportance of inertial effects was noted by Reid and Kajiura [1957], who also showed that the perturbation of the overlying free water by the pore water motion is small. The other effects are discussed by Yamamoto *et al.* [1978]. Since none of the effects appears to add anything essential in our particular application, we have used the simplest treatment.

Three properties of the velocity field defined by (6) are of particular interest:

1. The scale of motion of a water parcel: under the good assumption that the scale of motion is much less than $1/h$ (the wavelength divided by 2π), it is easily shown that a pore water parcel moves in a circle with diameter given by

$$\text{dia} = \frac{2u}{\omega} e^{-hy} \quad (9)$$

The ratio of this to the grain size d , which we call the orbit number (Or), is

$$\text{Or} = \frac{2u}{\omega d} e^{-hy} \quad (10)$$

2. Velocity magnitude: the magnitude v of \mathbf{v} is given by

$$v = ue^{-hy} \quad (11)$$

which depends on depth below seabed y only.

3. Velocity direction: the velocity vector \mathbf{v} makes an angle θ with the x axis given by

$$\theta = hx - \omega t \quad (12)$$

if θ is measured in the clockwise direction. This direction for θ is consistent with that used in equation (4).

The convective term in (2) can be written in terms of this velocity field from (6). It becomes

$$\mathbf{v} \cdot \nabla S = ue^{-hy} \left(\cos \theta \frac{\partial S}{\partial x} - \sin \theta \frac{\partial S}{\partial y} \right) \quad (13)$$

Simplification of the Transport Equation

Substitution of (4) and (13) for $\nabla \cdot (\boldsymbol{\kappa} \cdot \nabla S)$ and $\mathbf{v} \cdot \nabla S$ in (2) results in a transport equation that is too complicated to be of

much use, especially since by equation (12) θ is a function of both x and t . However, surface waves are a relatively high frequency phenomenon, while the response of the concentration S is relatively sluggish. This suggests that the transport equation should be averaged over a wave period. If the change in S during the wave period is indeed negligible, this gets rid of all the θ -dependent terms, at least in the case under consideration in which the pressure variation at the seabed is dominated by a single wavelength. The transport equation would then take the simple form

$$\frac{\partial}{\partial x} \left(\alpha \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left(\alpha \frac{\partial S}{\partial y} \right) + \kappa_r \frac{\partial^2 S}{\partial z^2} - qS - \frac{\partial S}{\partial t} \simeq 0 \quad (14)$$

A more rigorous justification for dropping the θ -dependent terms is given in the appendix, where an example of special interest is considered: the steady state case applying for long wavelength, and S constant at the seabed. It is described by

$$\alpha (d^2 S / dy^2) - qS = 0 \quad (15)$$

which has the solution

$$S = S_0 \exp \left(- \left(\frac{q}{\alpha} \right)^{1/2} y \right) \quad (16)$$

where S_0 is the concentration at the seabed. In this solution a measure of how far the seabed concentration penetrates the sediments before it is consumed by the sinks is the "penetration depth," defined by

$$\text{penetration depth} = \left(\frac{\alpha}{q} \right)^{1/2} \quad (17)$$

Equations (15) and (16) apply when this depth is much less than the wavelength. The flux of S into the seabed (per area of pore space) is

$$-\alpha \frac{dS}{dy} \Big|_{y=0} = (q\alpha)^{1/2} S_0$$

The ratio of the penetration depth to that of the purely molecular case ($\alpha = \kappa$), and the similar ratio for the seabed flux of S , are the same and are given by

$$\text{penetration and flux ratios} = \left(\frac{\alpha}{\kappa} \right)^{1/2} \quad (18)$$

This quantity is of interest because it is a useful indicator of the enhancement of the reaction rate over the purely molecular situation.

Dependence of the Transport Equation on the Velocity Field

The equation (14) version of the transport contains the assumption of the particular velocity field defined by (6), which is appropriate for thick, homogeneous isotropic sediment and a single driving wavelength. It is easy to see what would happen if the requirement of thick sediment were relaxed. Consider the opposite case in which the sediment thickness is much less than the wavelength, and the sediment is underlain by an impermeable boundary. Then it is straightforward to show from Darcy's law that the pore water parcel orbits are not circles but straight lines oriented parallel to the seabed, along which oscillatory motion takes place. The angle θ defining the direction of the motion is therefore constant and zero,

which allows (4) to be simplified to

$$\nabla \cdot (\kappa \cdot \nabla S) = \frac{\partial}{\partial x} \left(\kappa_L \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa_T \frac{\partial S}{\partial y} \right) + \left(\kappa_T \frac{\partial^2 S}{\partial z^2} \right)$$

Since $\theta = 0$, no rotation is necessary, and this form of the dispersive term is obvious from (3). If, as before, the convective terms are neglected, the transport equation (2) becomes

$$\frac{\partial}{\partial x} \left(\kappa_L \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa_T \frac{\partial S}{\partial y} \right) + \kappa_T \frac{\partial^2 S}{\partial z^2} - qS - \frac{\partial S}{\partial t} = 0$$

This thin sediment case implies a considerably slower vertical (y direction) transport into the seabed than does the thick sediment case described by equation (14), because for $Pe \gg 1$, κ_T is about an order of magnitude less than κ_L , or roughly 5 times less than α by equation (5a). However, the applicability of the values of κ_L and κ_T as usually measured is doubtful, just as in the case $Or \ll 1$ for circular motion. In the general finite sediment thickness case the orbits are ellipses and both diameters should be much greater than grain size for the measured values to apply.

This discussion of the shallow sediment limit points the way to the more general case. Only for thick sediments and a single driving wavelength will the orbits be circular, and the dispersive isotropy in the vertical (x - y) plane evident in equation (14) exist. Assuming, as discussed earlier, that it is valid to average over a wave period, the more general form of dispersive term (equation (4)) can be written

$$\nabla \cdot (\kappa \cdot \nabla S) = \frac{\partial}{\partial x} \left(\kappa_x \frac{\partial S}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa_y \frac{\partial S}{\partial y} \right) + \kappa_z \left(\frac{\partial^2 S}{\partial z^2} \right) - \frac{\partial}{\partial x} \left(\kappa_{xy} \frac{\partial S}{\partial y} \right) - \frac{\partial}{\partial y} \left(\kappa_{xy} \frac{\partial S}{\partial x} \right)$$

where the four effective diffusivities are defined by the wave period averages

$$\kappa_x = \langle \alpha + \beta \cos 2\theta \rangle$$

$$\kappa_y = \langle \alpha - \beta \cos 2\theta \rangle$$

$$\kappa_z = \langle \kappa_T \rangle$$

$$\kappa_{xy} = \langle \beta \sin 2\theta \rangle$$

APPLICATION

Behavior of the Peclet and Orbit Numbers and the Dispersion Parameters

As a first step in applying the theory to a problem of interest, the Peclet number Pe , and orbit-to-grain-size ratio Or should be investigated. As defined by equations (1) and (10), both of these quantities involve the grain size, which implicitly assumes that the sediments are well sorted. This is not necessary for mechanical dispersion to occur nor for most of the previous theory to hold, but it is a useful limit to consider because the limitations on what can be predicted with available dispersion parameter data are more obvious. To express Pe and Or explicitly in terms of grain size, an expression relating the hydraulic conductivity to porosity ratio k to grain size d is needed. We use

$$k = cd^2 \quad (19)$$

where c is a constant equal to 1.84×10^4 (m s)⁻¹ for a porosity of 0.4 [Krumbein and Monk, 1942]. This value for c holds

at 20°C; it is about half as large at 0°C. In two of the following examples, k is specified but sorting is not complete. Then an effective grain size, designated by d_{ef} , defined by

$$d_{ef} = \left(\frac{k}{c} \right)^{1/2} \quad (20)$$

is used in the theory to obtain order of magnitude results. With the help of equations (7), (8), (11), and (19), equations (1) and (10) give

$$Pe = \frac{cd^3}{\kappa_f} \cdot e^{-hy} \frac{ha}{\cosh hD} \quad (21)$$

$$Or = 2cd \cdot e^{-hy} \frac{ha}{(gh \tanh hD)^{1/2} \cosh hD} \quad (22)$$

which apply for thick sediments, the simplest situation. These are written as a product of two factors, the first representing the sediments, and the second representing the water depth D and sea state, as defined by the wave number h and amplitude a . Note the different dependence on grain size d . For a given wave period, the square root factor in (22) is given, being the angular frequency ω . Pe and Or then have the same dependence on h and D . Near the seabed ($y = 0$) their maxima occur at $D = 0$ if the wave amplitude is also given. Therefore mechanical dispersion should be most strongly developed near shore, because it should vary monotonically with both Pe and Or .

When applicable to periodic motion, numerical values of the dispersion parameters can be obtained from graphical representations of experimental data; those summarized by *Fried and Combarous* [1971] are used in the following examples. When $Pe \gg 1$, the longitudinal dispersion coefficient (κ_L) is given approximately by $1.8vd$. For $Pe \gg 1$, the transverse coefficient κ_T is typically 10 times smaller. The mean value of the two, α , is then

$$\alpha \approx vd \quad Pe \gg 1 \quad (23)$$

which is a rough but simple and useful result. As a short cut to evaluating α in this limit if Pe has already been evaluated from equation (21), notice from equation (1) that α is the same as $\kappa_f Pe$, where κ_f is the free water molecular diffusivity. However, κ_f cancels (see equation (21)), so α is independent of it when $Pe \gg 1$. In the following examples we are more interested in the ratio α/κ , where κ is the porous medium molecular diffusivity; κ does depend on κ_f . For the experimental data used, $\kappa = 0.67 \kappa_f$; the numerical factor depends on the structure or 'formation factor' of the porous medium. In the following examples we use a value for κ_f of 1.5×10^{-9} m² s⁻¹, which applies to NaCl at 25°C. An easily shown consequence of (23) is that $\alpha \propto d^3$ when $Pe \gg 1$, showing the sensitivity of effective diffusivity to grain size. But we expect this relationship (as well as equation (23)) to hold only if $Or \gg 1$ as well.

Example 1: Coarse Sand and Shallow Rough Water

We first consider an extreme case which should favor wave-induced transport: thick sediments relative to the wavelength, large grain size d , shallow water depth D and high wave amplitude a . Suppose $d = 2$ mm, $D = 5$ m, wavelength = 50 m, and $a = 0.5$ m. By equations (21) and (22), $Pe = 5.1 \times 10^3$ and $Or = 9$. Or is probably large enough that the large dispersion implied by the large Pe indeed occurs, and that the experimental value of α given by equation (23) is at least roughly applicable. Then $\alpha/\kappa = 7.6 \times 10^3$ and $(\alpha/\kappa)^{1/2} = 87$. In this ex-

ample the effective diffusivity α is increased almost 4 orders of magnitude over its molecular value κ by the wave-induced motion. The penetration depth and seabed flux of S , $(\alpha/\kappa)^{1/2}$ by equation (18), are increased almost 2 orders of magnitude.

Example 2: Shelf Area of the Eastern United States

We next consider the shelf area of the eastern U.S., which is of interest because *Riedl et al.* [1972] considered the effect of wave-induced processes there. A basic problem, as we have seen, is the sensitivity of mechanical dispersion to sediment permeability distribution from the seabed to sediment depths on the order of a wavelength. Since the permeability profile is unknown, we use the seabed value, 20 darcies typically [*Riedl et al.*, 1972], and assume that it applies at depth. After conversion to the hydraulic conductivity to porosity ratio k (assuming a porosity of 0.4), 20 darcies gives an effective grain size d of 0.16 mm by equation (20). A wave period of 7 s and amplitude of 0.75 m seem fairly representative for the shelf. We calculate the dispersion for a water depth of 10 m, which gives a wavelength of 60 m from equation (8). Equations (21) and (22) give values for Pe and Or of 2.5 and 0.32. The mechanical dispersion enhancement factor α/κ for the diffusivity, and the factor $(\alpha/\kappa)^{1/2}$ for the penetration depth and flux as estimated from available dispersion data, are 2.3 and 1.5, respectively.

These factors are likely to be upper limits because the permeability may decrease with depth, and because $Or < 1$. It therefore seems that mechanical dispersion is usually not well developed on the eastern U.S. shelf. However, we note that a doubling of grain size and of wave amplitude would make $Or > 1$ and $Pe \approx 40$, suggesting that mechanical dispersion can be well developed under favorable conditions.

Example 3: Prudhoe Bay, Alaska

We next consider the case of Prudhoe Bay, Alaska, which was the origin of our interest in wave-induced transport processes. Much of the shelf of arctic Alaska is underlain by subsea permafrost, which is thawing at a rate that is probably controlled to a large extent by the rate of salt transport into the seabed. Gravity-driven convection of the pore water in the thawed layer underlying the seabed is an obvious candidate for the principal salt transport mechanism occurring in near-shore areas at Prudhoe Bay [*Harrison and Osterkamp*, 1978], and it is of interest to see whether wave-induced transport could operate as well. The measured values of k are usually in the range 0.8×10^{-7} to 8×10^{-7} m s⁻¹. By using the upper limit, the effective grain size d_{ef} is 9.3×10^{-3} mm by equation (20). Taking depth D as 3 m, wavelength as 50 m (about the same as the thawed sediment thickness), and amplitude a as 0.5 m, one finds that $Pe = 3 \times 10^{-4}$, and $Or = 2 \times 10^{-2}$. Therefore wave-induced salt transport does not seem to be a factor in permafrost thawing at Prudhoe Bay.

DISCUSSION

The following factors favor wave-driven mechanical dispersion: large grain size or high permeability, large sediment thickness, large wave amplitude, and shallow water. Dispersion should occur for insoluble suspended material as well as for solutes. In homogeneous isotropic sediments, mechanical dispersion is inhomogeneous in the sense that the dispersion coefficients are usually spatially variable, and anisotropic in

the sense that a single scalar dispersion coefficient can be defined only in special cases. A necessary condition for significant mechanical dispersion is that the Peclet number on the scale of the grain size be greater than 1. But the pore water motion is periodic, and the Peclet number defined here in terms of the velocity amplitude may exceed 1 even when the scale of water motion is less than the grain size; in this case we expect mechanical dispersion to be small. When the scale of motion in both horizontal and vertical directions considerably exceeds the grain size, the approach used here is probably fairly good. Basically the approach has been to parameterize the dispersion term in the transport equation by two parameters, the longitudinal and transverse coefficients of dispersion, which may be estimated roughly from existing data. As the scale of motion decreases, not only the existing data, but also this simple parameterization may be inappropriate. Ultimately, the best parameterization of the transport equation, and the numerical values in it, are matters to be settled by experiment. Our simple approach has at least indicated some of the basic properties of wave-driven dispersion, such as its anisotropy and dependence on sea state, and allowed rough numerical estimates of its effect in limiting cases.

It is possible to imagine situations in which wave-driven dispersion is not important, even when implied by these considerations. A condition that favors it, large sediment permeability, also favors Rayleigh instability to gravity-driven convection. For example, in the case of heavy, salty free water overlying lighter, fresher pore water, the transport of salt into the sediments may take place, not primarily by wave action, but by a larger scale gravity-driven sinking of the heavy water into, and floating of the lighter water out of, the sediments. When the density difference is reversed, or when it does not exist and the transport of a trace material is of interest, wave action may be important.

There is another aspect of the relationship between gravity and wave-driven pore water motion that is interesting. One might think that the agitation by wave action would help to destabilize the pore water in situations tending to gravitational instability, and to favor the onset of gravity-driven convection. But the opposite may be true, because the wave action may give rise to a large effective diffusivity, which tends to stabilize a system against convection.

APPENDIX

A complete form for the transport equation, obtained by substituting equations (4) and (13) in (2), is easily simplified for an illustrative special case. Suppose the wavelength is much larger than all the other length scales, including the penetration depth, so that $h \rightarrow 0$. Then θ becomes dependent on t only (equation (12)), and α and β become constant. Then none of the coefficients in equations (4) and (13) or in the resulting transport equation, depends upon x . If there is no x or z dependence in the initial or boundary conditions, all the derivatives with respect to x and z vanish, and we are left with

$$\alpha \frac{\partial^2 S}{\partial y^2} - qS - \frac{\partial S}{\partial t} - \beta \cos 2\omega t \frac{\partial^2 S}{\partial y^2} - u \sin \omega t \frac{\partial S}{\partial y} = 0$$

A solution is

$$S = S_0 \exp \left(- \left(\frac{q}{\alpha} \right)^{1/2} y - \Gamma_1 \sin 2\omega t - \Gamma_2 \cos \omega t \right) \quad (24)$$

where

$$\Gamma_1 = \frac{q\beta}{2\omega\kappa}$$

$$\Gamma_2 = \left(\frac{q}{\alpha}\right)^{1/2} \frac{u}{\omega}$$

Sufficient conditions for this to reduce to the solution of equation (15) are $\Gamma_1 \ll 1$ and $\Gamma_2 \ll 1$.

The origin of the $\Gamma_1 \sin 2\omega t$ term in equation (24) is mechanical dispersion, and the condition $\Gamma_1 \ll 1$ has a simple meaning. For $Pe \ll 1$, $\beta = 0$ (see the discussion of equations (5a) and (5b) and Γ_1 is automatically zero, as it should be because of its mechanical dispersion origin. For larger Pe , $\beta/\alpha \lesssim 1$, so the condition $\Gamma_1 \ll 1$ implies that $q/(2\omega) \ll 1$. Since q^{-1} is a time constant reflecting the time required for the sinks to consume the concentration S if, for example, the seabed supply were turned off, this condition is the statement that this time constant be much larger than the wave period. This condition is probably always satisfied in practice. The origin of the $\Gamma_2 \cos \omega t$ term in equation (24) is convection, and the condition $\Gamma_2 \ll 1$ also has a simple meaning. Since u/ω is the pore water orbit radius (equation (9)), and $(\alpha/q)^{1/2}$ is the penetration depth (equation (17)), this condition is the statement that the former be much smaller than the latter. This seems reasonable, because of the convection origin of the Γ_2 term. This condition is also probably always satisfied in practice.

There are two other ways of looking at the conditions $\Gamma_1 \ll 1$ and $\Gamma_2 \ll 1$. First, since the angular frequency ω occurs in the denominators of both conditions, they are satisfied for high frequency. Second, since the sink strength q occurs in the numerators of both, they are satisfied for small sink strength. Evidently, the sinks must not be too greedy if their effect on concentration is to be described by a simple differential equation.

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