Title
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Publication Date
2016-06-01

DOI
10.1016/j.jebo.2016.02.007

Peer reviewed

Group Size and Cooperation among Strangers*

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April 7, 2015

Abstract

We study how group size affects cooperation in an infinitely repeated n-player Prisoner's Dilemma (PD) game. In each repetition of the game, groups of size $n \leq M$ are randomly and anonymously matched from a fixed population of size M to play the n-player PD stage game. We provide conditions for which the contagious strategy (Kandori, 1992) sustains a social norm of cooperation among all M players. Our main finding is that if agents are sufficiently patient, a social norm of society-wide cooperation becomes easier to sustain under the contagious strategy as $n \to M$. In an experiment where the population size M is fixed and the group size n varies, we find strong evidence that cooperation rates in the treatment with a large group size are significantly higher than in the treatment with a small group size.

JEL Classification Nos: C72, C73, C78, Z13.

Keywords: Cooperation, Social Norms, Group Size, Repeated Games, Random Matching, Prisoner's Dilemma, Experiment.

*We have benefited from comments by Bram Cadsby, Gabriele Camera, Erik Kimbrough, Ming Li, Quang Nguyen and seminar participants at Nanyang Technological University, Shanghai University of Finance and Economics, 2012 China International Conference on Game Theory and Applications, the 4th Annual Xiamen University International Workshop on Experimental Economics. Duffy acknowledges research support from the UCI School of Social Sciences. Xie gratefully acknowledges the hospitality of the Economic Growth Center at the Division of Economics in Nanyang Technological University and funding support from FQRSC (2010-NP-133118).

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1 Introduction

What choice of group size maximizes (or minimizes) the possibility of achieving a social norm of cooperation in a finite population of self-interested strangers? This question would seem to be of considerable relevance to a wide variety of different settings involving the matching of strategic but essentially anonymous players, for example, the number of students assigned to each class, passenger seating configurations on airplanes or the number of jurors in a legal proceeding. In this paper we offer an answer to this question. Specifically, we consider a population of players of fixed size M. In every period, $t = 1,2,...\infty$, players in this population are randomly matched to form groups of size n and play an n-person Prisoner's Dilemma game with the members of their group. The total number of groups, M/n, is assumed to be an integer (i.e., M is a multiple of n).

The $n = 2$ person version of this environment has been previously studied by Kandori (1992), who shows that a social norm of cooperation among anonymous, randomly matched players is sustainable under certain conditions on the game. Kandori further shows that a social norm of cooperation among strangers in the $n = 2$ case becomes more difficult to sustain as M gets large and the possibility vanishes in the limit as $M \to \infty$. By contrast, in this paper we fix M and ask: for what value(s) of $n \geq 2$ is a social norm of cooperation among strangers easiest to achieve? In other words, is there an optimal group size for maximizing the likelihood of cooperative outcomes?

Our answer is that under certain conditions–specifically if agents are sufficiently patient– a social norm of cooperation among strangers, which is sustained by universal play of a "contagious" trigger strategy, becomes steadily easier to achieve as n gets larger, and becomes easiest to achieve when $n = M$. That is, we find that cooperation can be easiest to sustain when the group size is as large as possible. This seemingly counterintuitive finding readily follows from the logic of the contagious trigger strategy that is used to support cooperation among randomly matched, non-communicative and anonymous "strangers." Intuitively, if agents are sufficiently patient, then the costs of igniting a contagion toward mutual defection are greatest when the matching group size, n, equals the population size, M. On the other hand, once a defection has started in the community, the benefits to slowing down the contagious process are also minimized in this same case where $n = M$. Therefore, the players' incentives to follow the contagious strategy are easiest to satisfy when the group size is as large as possible. However, we also find that if agents are insufficiently patient, then the relationship between the group size, n, and the ease with which a social norm of universal cooperation among strangers is sustained can be non-monotonic as $n \to M$.

Our findings serve to generalize Kandori's (1992) extension of the folk theorem for repeated games with random, anonymous matchings to the multiple-player ($n > 2$) Prisoner's Dilemma game. The n-player version of the Prisoner's Dilemma game is widely used to model a variety of *social* dilemmas including, e.g., the tragedy of the commons (Hardin (1968)). In addition, we show that our monotonicity result holds more broadly in two different settings. In the first setting, players in each group of size n are randomly paired to play the traditional 2-person Prisoner's Dilemma game but are able to observe information on the outcome of play by other pairs of players in their n-player matching group. In the second setting, the payoff matrix of the n-player game is changed to reflect the incentives provided in a binary public good game.

We also provide an empirical test of our main theoretical results by designing and implemented an n-player Prisoners' Dilemma game experiment. In this experiment, we fix the population at size $M = 12$ and we study the indefinitely repeated game in which players from the population are randomly and anonymously matched in each repetition to play an $n = 2$ or $n = 6$-player version of the Prisoner's Dilemma stage game. We find strong evidence that cooperation rates are higher in the $n = 6$ matching group treatment as compared with the $n = 2$ matching group treatment as subjects learn, with experience, the more immediate consequences of triggering an infectious wave of defection when the group size is larger. We show further how these differences in cooperation rates between the two different group sizes are fully consistent with our main theoretical findings.

This paper contributes to the theoretical and experimental literature on sustaining cooperation among anonymous, randomly matched players. While this is an admittedly stark environment, it is an important benchmark case in both the theoretical and experimental literature and one that naturally characterizes many types of socio-economic interactions.[1] In addition to the original seminal paper by Kandori (1992), Ellison (1994) and Dal Bo (2007) provide further generalizations of how a social norm of cooperation may be sustained among anonymous, randomly matched players in 2-player Prisoner's Dilemma games. Xie and Lee (2012) extend Kandori's result to 2-player "trust" games under anonymous random matchings. Camera and Gioffre (2014) offers a tractable analysis of the contagious equilibria by characterizing a key statistic of contagious punishment processes and deriving closed-form expressions for continuation payoffs off the equilibrium path. Experimentally, Duffy and Ochs (2009) report on an experiment that examines play in an indefinitely repeated, two-player Prisoner's Dilemma game and find that a cooperative norm does not emerge in the treatments with anonymous random matching but does emerge under fixed pairings as players gain more experience. Camera and Casari (2009) examine cooperation under random matching by focusing on the role of private or public monitoring of the anonymous (or non-anonymous) players' choices. They find that such monitoring can lead to a significant increase in the frequency of cooperation relative to the case of no monitoring. Duffy et al. (2013) test the contagious equilibrium in the lab using trust games and find that information on past play significantly increases the level of trust and reciprocity under random matchings. Camera et al. (2013) report wide heterogeneity in strategies employed at the individual level in an experiment in which anonymous randomly matched subjects play the Prisoner's Dilemma game in sequences of indefinite duration. Compared with this previous literature, our paper is the first to theoretically and experimentally extend the analysis of the contagious equilibrium from a 2-player stage game to an n-player stage game. On the other hand, our main theoretical and experimental finding, that a cooperative social norm is easier to sustain with a larger rather than a smaller group size, finds some support in the previous literature

[1] There is also an experimental literature that studies cooperation in repeated Prisoner's Dilemma games of indefinite duration among fixed pairs of players (partners) e.g., Dal Bó (2007), Aoyagi and Fréchette (2009), Dal Bó and Fréchette (2011), Fudenberg et al. (2012). Engle-Warnick and Slonim (2006) examines a trust game of indefinite duration with fixed pairs.

if one considers a large group size to be a partial substitute for public monitoring or fixed matching.[2]

There are also several experimental papers that study the consequences of group size for contributions to a public good, e.g., Isaac and Walker (1988), Isaac et al. (1994) and Xu et al. (2013). There are, however, important differences between our setup and that of Issac and Walker that prevent a direct comparison: 1) the strategy space is continuous in Issac and Walker's public good game and not binary as in the n-player game that we study; 2) Issac and Walker's players are in fixed matches of size n for all repetitions of the public good game whereas in our setup players are randomly matched into groups of size n in each repetition of the game, and finally 3) Issac and Walker study a finitely repeated game whereas we study an infinitely repeated game.[3]

The rest of the paper is organized as follows. Section 2 presents our model and section 3 presents our main theoretical results on the consequences of group size for the sustainability of social norms of cooperation among anonymous and randomly matched strangers. Section 4 shows how our results for the n-player game extend to the more traditional 2-player game when information on play of the game is available to all n players in the matching group used to form pairs of players. Section 5 shows how our framework maps into the classic public good game of Isaac and Walker (1988). Section 6 reports on the findings of an experiment testing our main theoretical results. Finally, section 7 concludes with a brief summary and some suggestions for future research.

2 The Model

Consider a finite population of M players. Time is discrete, the horizon is infinite and all players have a common period discount factor, $\delta \in [0, 1]$. In each period, the M players are randomly and anonymously matched into m groups of size $n \leq M$, with all matchings being equally likely, that is, we assume that M is a multiple of n with multiplier m. The randomly matched group members then simultaneously and without communication play an n-player Prisoner's Dilemma game where each player chooses a strategy from the set $\{C, D\}$, with C representing cooperation and D representing defection. Let i denote the number of members of the group choosing to cooperate (i.e., the number

[2]We note that when the group size, n, is set equal to the largest possible value, the population size M, then our model converges to one of perfect public monitoring and fixed matching. Thus for group sizes less than M, one can view larger group sizes as being closer approximations to perfect public monitoring and fixed matchings.

[3]Isaac and Walker (1988) and Isaac et al. (1994) examine how groups of size 4, 10, 40 and 100 play a repeated public good game. One of their main findings is that, holding the marginal per capita return (MPCR) to the public good constant, an increase in the number of players, n, leads to no change or an increase (depending on the MPCR) in the mean percentage of each player's fixed and common endowment that is contributed toward the public good, and this effect is strongest with group sizes of 40 and 100 in comparison with group sizes of 4 and 10. Xu et al. (2013) examine the effectiveness of an individual-punishment mechanism in larger groups of 40 participants compared with groups of smaller groups of size four. They find that the individual punishment mechanism is effective when the MPCR is constant but not effective when the marginal group return (MGR) is held constant (in which case MPCR is decreasing). Therefore, in this public goods literature, the contribution rate to the public goods is increasing with the group size only when the MPCR is constant and the group return is increasing with the group size. In this sense, our monotonicity result is stronger since we normalize the payoff so that it is comparable to the case with fixed group return and decreasing MPCR with the group size.

of "cooperators") other than the representative player himself so that $0 \leq i \leq n-1$. Let C_i and D_i denote the payoffs to cooperation and defection, respectively, when there are i cooperators. An n-player Prisoner's Dilemma game is defined by the following three assumptions regarding these payoffs:

A1: $D_i > C_i$ for $0 \leq i \leq n-1$.

A2: $C_{i+1} > C_i$ and $D_{i+1} > D_i$ for $0 \leq i < n-1$.

A3: $C_{n-1} > D_0$.

Assumption A1 says that defection is always a dominant strategy. Assumption A2 says that payoffs are increasing with the number of cooperators. Finally, assumption A3 says that if all participants adopt the dominant strategy, the outcome is sub-optimal relative to the mutual cooperation outcome. These conditions are standard in the literature on n-person Prisoner's Dilemma games (See, e.g., Okda (1991, Assumption 2.1)). We further suppose that the payoff matrix is symmetric for each player in the group and is as given in Table 1.

number of cooperators in the group	0	1	2	...	$n-1$
C	C_0	C_1	C_2	...	C_{n-1}
D	D_0	D_1	D_2	...	D_{n-1}

Table 1: The Payoff Matrix of the n-Player Prisoner's Dilemma Game

We next define the "contagious strategy" following Kandori (1992) and show that a social norm of cooperation can be sustained as a sequential equilibrium if all players adopt this strategy. Define a player as a "c-type" if in all previous repetitions of the game this player and all of the other $n-1$ group members with whom he has interacted in all prior periods have never chosen D, i.e., the outcome of the stage game played in every prior period has been cooperation, C, by every group member the player has encountered. Otherwise, the player is a "d-type" player. (Note that the presence of c-type players in any period does not preclude the presence of d-type players in the same period among the population (or "community") of players of size $M \geq n$). The "contagious strategy" can now be defined as follows: A player chooses C if he is c-type and chooses D if he is d-type.

We next provide a set of sufficient conditions that sustains the contagious strategy as a sequential equilibrium when the group size is n. We first introduce some notation. Let X_t be the number of d-type players at time t. Define $A_n = (a_{ij}^n)$ to be an $M \times M$ transition probability matrix where $a_{ij}^n = \Pr(X_{t+1} = j | X_t = i$ and all players follow the contagious strategy) given group size n. Define $B_n = (b_{ij}^n)$ as an $M \times M$ transition probability matrix where $b_{ij}^n = \Pr(X_{t+1} = j | X_t = i$ and one d-type player deviates to playing C while all other players follow the contagious strategy) given group size n. Let $H_n = B_n - A_n$, which indicates how the diffusion of defection is delayed by the unilateral deviation of one of the d-type players. Define $Z_n = (\rho_0^n \ \rho_1^n \ \ldots \ \rho_{n-1}^n)$, where $\rho_0^n, \rho_1^n, \ldots, \rho_{n-1}^n$ are $M \times 1$ vectors such that the ith element of ρ_j^n is the conditional probability that a d-type player meets j c-type players in the group when there are i d-type players in the community given that the group size is n (i.e., $Z_n = (z^n_{ij})$ is an $M \times n$ matrix where $z^n_{ij} = \Pr$(a d-type player meets $j-1$ c-type players in his group in period $t|X_t = i$) given a group size of n). Define e_i as a $1 \times M$ vector whose ith element is 1 and with zeros everywhere else. Finally, define column vectors $v_n = (D_0, D_1, \ldots, D_{n-1})^T$ and $u_n = (C_0, C_1, \ldots, C_{n-1})^T$, whose ith element is the payoff for a player from choosing D and C respectively, given that there are $i-1$ other players in the group who choose C.

Next we show that a one-shot deviation from the contagious strategy is unprofitable after any history. On the equilibrium path, a one-shot deviation is unprofitable if

$$\frac{C_{n-1}}{1-\delta} \geq \sum_{t=0}^{\infty} \delta^t e_1 A_n^t Z_n v_n. \tag{1}$$

The left hand side of (1) is the payoff from cooperating forever and the right-hand side of (1) is the payoff that the player earns if the player initiates a defection and defects forever afterward. Off the equilibrium path, following Kandori (1992), we identify a sufficient condition for a one-shot deviation to be unprofitable under any consistent beliefs. Suppose there are k d-type players, where $k = n, n+1, \ldots, M$.[4] Then a one-shot deviation off the equilibrium path is unprofitable if

$$\sum_{t=0}^{\infty} \delta^t e_k A_n^t Z_n v_n \geq e_k Z_n u_n + \delta \sum_{t=0}^{\infty} \delta^t e_k B_n A_n^t Z_n v_n. \tag{2}$$

The left hand side of (2) is the payoff that a d-type player earns from playing D forever when there are k d-type players including the player himself, while the right hand side of (2) is what a d-type player receives when he deviates from the contagious strategy, playing C today and then reverting back to playing D forever after. Inequalities (1) and (2) can be manipulated into equilibrium conditions 1 and 2 in the following lemma.

Lemma 1 *The contagious strategy constitutes a sequential equilibrium if the following two conditions are satisfied:*

$$\begin{aligned} &\textit{Equilibrium Condition 1:} \quad && C_{n-1} \geq (1-\delta) e_1 (I - \delta A_n)^{-1} Z_n v_n, \\ &\textit{Equilibrium Condition 2:} \quad && e_k Z_n (v_n - u_n) \geq \delta e_k H_n (I - \delta A_n)^{-1} Z_n v_n. \end{aligned}$$

The intuition behind equilibrium conditions 1 and 2 is similar to that for the $n = 2$ case studied by Kandori (1992). When a player is on the equilibrium path, he has no incentive to deviate from cooperation when δ is sufficiently large. When a player is off the equilibrium path, he has no incentive to deviate from continued play of the contagious strategy if the extra payoff from defection in the current period, $v_n - u_n$, is large enough. Using Lemma 1 we can prove the following theorem.

Theorem 1 *Under uniformly random matching, the contagious strategy described above constitutes a sequential equilibrium strategy for any finite population size, M, if δ, $C_{n-1} - D_o$, and $v_n - u_n$ are sufficiently large.*

Proof: See Appendix A.

[4] Since the player under consideration is a d-type, there must be at least n d-type players in the community.

3 Main Results

In this section we ask the following question: Fixing the population size M, which group size n maximizes the possibility of achieving a social norm of cooperation among strangers?[5] Although we can characterize the equilibrium conditions for the contagious strategy, we cannot derive closed-form solutions since the formulas for the elements of the transition matrix A and B become too complicated to derive for group sizes $n > 2$.[6] Therefore, in this section we switch to the use of numerical methods.[7]

Furthermore, for greater tractability we focus on a simple symmetric specification for the payoff parameters that satisfy assumptions A1-A3. Specifically, we normalize $C_0 = 0$ and set $D_i - C_i = \alpha$ for $0 \leq i \leq n-1$ and $C_{i+1} - C_i = D_{i+1} - D_i = \beta$ for $0 \leq i < n-1$. Under these assumptions, the payoff matrix (Table 1) now takes on the specific form shown in Table 2. We will consider the robustness of our findings to a slightly different parameterization of the n-player PD game payoff matrix later in section 5.

number of cooperators in the group	0	1	2	...	$n-1$
C	0	β	2β	...	$(n-1)\beta$
D	α	$\alpha+\beta$	$\alpha+2\beta$	...	$\alpha+(n-1)\beta$

Table 2: The Simpler Payoff Matrix for the n-Player Prisoner's Dilemma Game

Finally, we note that under our parameterization it may be easier to achieve full cooperation with a larger group size since the payoff from cooperation, $(n-1)\beta$, grows with the group size, n. To properly correct for this dependency, we also normalize the payoff matrix in such a way that the payoff from full cooperation is fixed and constant. Specifically, we always set $(n-1)\beta = 1$ for any n (i.e., we set $\beta = 1/(n-1)$). Note that under this normalization, to satisfy assumption A3, we must have $\alpha < C_{n-1} = 1$ for all $n \geq 2$.

In order to examine the question raised above, we first fix $M = 12$ and examine changes in the two equilibrium conditions as the group size takes on the values $n = 2, 3, 4, 6, 12$. We find that a common pattern emerges in this setting as $n \to M$. We begin by presenting numerical results for a fixed value of $\delta = 0.9$. We will later consider cases where the value of δ is varied.

3.1 Equilibrium Condition 1

We first examine the effect of increases in the group size, n, on equilibrium condition 1. Although we are mainly interested in the case where payoffs are normalized to eliminate the dependency on n, for the moment we keep payoffs for equilibrium condition 1 in their original unnormalized form (i.e., $C_{n-1} = (n-1)\beta$), so that we can derive some intuition as to how the discounted summation of

[5]In Appendix B, we also ask how the answer to this question changes if instead of fixing M, we vary both M and n but in such a way that the number of groups, m, is held constant.

[6]Kandori (1989) provides transition matrix formulas for the $n = 2$ case only.

[7]The Mathematica program used for the numerical results is available upon request.

the probability of earning each payoff outcome changes with the group size. Equilibrium condition 1 (as stated in Lemma 1) for various values of n is given as follows:

$n = 2$: $\beta \geq 0.70492\alpha + 0.29508(\alpha + \beta) \Rightarrow \alpha \leq 0.705\beta$;

$n = 3$: $2\beta \geq 0.75032\alpha + 0.07575(\alpha + \beta) + 0.17393(\alpha + 2\beta) \Rightarrow \alpha \leq 1.58\beta$;

$n = 4$: $3\beta \geq 0.78347\alpha + 0.03034(\alpha + \beta) + 0.05466(\alpha + 2\beta) + 0.13153(\alpha + 3\beta) \Rightarrow \alpha \leq 2.47\beta$;

$n = 6$: $5\beta \geq 0.81002\alpha + 0.00586(\alpha + \beta) + 0.02928(\alpha + 2\beta) + 0.03904(\alpha + 3\beta) + 0.01464(\alpha + 4\beta) + 0.10117(\alpha + 5\beta) \Rightarrow \alpha \leq 4.25\beta$;

$n = 12$: $11\beta \geq 0.9\alpha + 0.1(\alpha + 11\beta) \Rightarrow \alpha \leq 9.9\beta$.

Several patterns regarding the discounted summation of probabilities for each payoff should be noticed. First, the summation of the probabilities assigned to each payoff on the right hand side of these inequalities is always 1, which is proved in Lemma 2 in the appendix. Second, the probability of earning $D_0 = \alpha$, the first term on the right hand side (i.e., the discounted summation of the probability of meeting no cooperators in the group once defection has started) is increasing with the group size n. Third, the probability of earning $D_{n-1} = \alpha + (n-1)\beta$, the last term on the right hand side (i.e., the discounted summation of the probability of meeting $n-1$ cooperators in the group if the player chooses defection in the current period) is decreasing with the group size n. Finally, the discounted summation of the probability of earning $D_1, D_2, \ldots, D_{n-2}$ converges to 0 as n gets larger. These patterns are intuitive if we consider the extreme case where the group size is equal to the population size, i.e., $n = M = 12$. In that case, if a player chooses to defect his defection spreads to the entire population so that in the next period, he will never meet any cooperators in his group. Thus, only in the current period will the defecting player meet $n-1$ cooperators and gain D_{n-1}.

Now we impose the parametric normalization discussed above that eliminates the dependency of the equilibrium condition on n. In this case, equilibrium condition 1 for various values of n (and $\beta = (n-1)^{-1}$) is simplified as follows:[8]

$n = 2$: $\alpha \leq 0.705\beta = 0.705$, for $\beta = 1$;

$n = 3$: $\alpha \leq 1.58\beta = 0.788$, for $\beta = 1/2$;

$n = 4$: $\alpha \leq 2.47\beta = 0.823$, for $\beta = 1/3$;

$n = 6$: $\alpha \leq 4.25\beta = 0.85$, for $\beta = 1/5$;

$n = 12$: $\alpha \leq 9.9\beta = 0.9$, for $\beta = 1/11$.

We observe that equilibrium condition 1 becomes monotonically less restrictive as the group size n becomes larger. Intuitively, with a larger group size, an initial defection spreads to more "innocent" (c-type) players. Furthermore, via the random re-matching each period, defection spreads to the entire population of M players much faster since there are fewer groups given the fixed population size, M, and a larger group size, n. These two effects together imply that the contagious process is faster with a larger group size n and thus the payoff from starting a defection is reduced, making the condition on the equilibrium path easier to satisfy. Note that a slightly different normalization, for instance, $D_0 = 0$, $D_i = i\beta$, $C_i = D_i - \alpha$, $C_{n-1} = 1$, gives similar results.

[8]Notice that the regulative condition $\alpha < 1$ is always satisfied for $n \geq 2$.

More generally, given a group of size n, we can write equilibrium condition 1 as:

$$(n-1)\beta \quad \geq \quad p_0^n\alpha + p_1^n(\alpha+\beta) + \ldots + p_{n-1}^n(\alpha+(n-1)\beta).$$

where $p_j^n \equiv (1-\delta)e_1(I-\delta A_n)^{-1}\rho_j^n$ denotes the discounted summation of the probability of meeting j cooperators (c-types) in a group of size n once a player has initiated a defection. From this condition we can derive a more sufficient condition:

$$(n-1)\beta \geq \sum_{j=0}^{n-1} p_j^n\alpha + \sum_{j=1}^{n-1} p_j^n(n-1)\beta. \tag{3}$$

Since we have already shown that $\sum_{j=0}^{n-1} p_j^n = 1$ in Lemma 2, inequality (3) above can be simplified to:

$$\alpha \leq p_0^n(n-1)\beta.$$

Finally, imposing the normalization that $\beta = (n-1)^{-1}$, we have

$$\alpha \leq p_0^n. \tag{4}$$

Proposition 1 *If p_0^n is increasing in n, then Condition (4) (Equilibrium condition 1) is monotonically less restrictive as the group size n increases.*

3.2 Equilibrium Condition 2

We next examine the effects of increases in the group size, n, on equilibrium condition 2. Given our payoff specification that $D_i - C_i = \alpha$ for $i = 0, 1, \ldots, n-1$, the left hand side of equilibrium condition 2, the extra payoff from defection, is equal to α. The right hand side of equilibrium condition 2, the payoff for a d-type player from slowing down the contagious process, achieves its highest value when the number of d-type players are at a minimum, i.e., when $k = n$. Thus it is sufficient to compare equilibrium condition 2 at $k = n$ for different group sizes, $n = 2, 3, 4, 6, 12$. Similar to equilibrium condition 1, we first present equilibrium condition 2 with the original payoff parameters and then we impose our normalization later. Equilibrium condition 2 (as stated in Lemma 1) for various values of n is given as follows:

$k = n = 2$: $\alpha \geq -0.29077\alpha + 0.29077(\alpha+\beta) \Rightarrow \alpha \geq 0.291\beta$;

$k = n = 3$: $\alpha \geq -0.19856\alpha + 0.09489(\alpha+\beta) + 0.10367(\alpha+2\beta) \Rightarrow \alpha \geq 0.302\beta$;

$k = n = 4$: $\alpha \geq -0.18757\alpha + 0.12632(\alpha+\beta) + 0.05126(\alpha+2\beta) + 0.00998(\alpha+3\beta) \Rightarrow \alpha \geq 0.259\beta$;

$k = n = 6$: $\alpha \geq -0.01154\alpha + 0.00190(\alpha+\beta) + 0.00506(\alpha+2\beta) + 0.00379(\alpha+3\beta) + 0.00076(\alpha+4\beta) + 0.00003(\alpha+5\beta) \Rightarrow \alpha \geq 0.023\beta$;

$k = n = 12$: $\alpha \geq 0$.

The right hand side of equilibrium condition 2, the net payoff for a d-type player from slowing down the contagious process when he reverts to cooperation rather than continuing defection, is decomposed into the change in receiving each D_i in the future. Several patterns are again noticeable. First, the summation of the change in the probabilities associated with each payoff D_i is always 0, which is proved in Lemma 2 in the appendix. Second, the change in the probability of earning

$D_0 = \alpha$ (i.e., the change in the discounted summation of the probability of meeting no cooperators in the group) is negative (0 in the 12-person PD), and the change in the probabilities of earning $D_i = \alpha + i\beta$ (i.e., the change in the discounted summation of the probability of meeting i cooperators in the group) are positive (0 in the 12-person PD). This shows that the d-type player's deviation from the contagious strategy in the current period decreases this d-type player's probability of meeting no c-type players in the group in future periods, and increases the probabilities of meeting any positive number of c-type players in the group in the future. Furthermore, the decrease in the probability of meeting no c-type players is the summation of the increase in the probabilities of meeting any positive number of c-type players. Finally, the extent of the change in the probabilities is decreasing as the group size, n, becomes larger. It is sufficient to see this point from the decrease in the probability of meeting no c-type players, that is, the absolute value of the first probability on the right hand side of earning $D_0 = \alpha$ is decreasing as n gets larger. In the case of $k = n = M = 12$, when the player under consideration is a d-type player, all the other players in the population are also d-type players. Therefore, there is no effect working to slow down the contagious process if this d-type player chooses to deviate from the contagious strategy by choosing cooperation.

Now again we impose the normalization condition that $\beta = (n-1)^{-1}$. Doing so yields the following versions of equilibrium condition 2 for the various group sizes n:

$n = 2$: $\alpha \geq 0.291\beta = 0.291$ for $\beta = 1$

$n = 3$: $\alpha \geq 0.302\beta = 0.151$ for $\beta = 1/2$

$n = 4$: $\alpha \geq 0.259\beta = 0.086$ for $\beta = 1/3$

$n = 6$: $\alpha \geq 0.023\beta = 0.005$ for $\beta = 1/5$

$n = 12$: $\alpha \geq 0$

From the above results, equilibrium condition 2 becomes less restrictive with a larger group size, n. Intuitively, it is also due to the faster contagious process associated with a larger group size. When the speed of contagion is faster, the effect for a single d-type player to slow down the contagious process becomes smaller. So the d-type player has less of an incentive to deviate from the contagious strategy off the equilibrium path by reverting back to playing cooperation again.

More generally, given a group of size n, we can write equilibrium condition 2 as:

$$\alpha \quad \geq \quad q_0^n \alpha + q_1^n (\alpha + \beta) + \ldots + q_{n-1}^n (\alpha + (n-1)\beta).$$

where $q_j^n \equiv \delta e_n H_n (I - \delta A_n)^{-1} \rho_j^n$ denotes the change in the discounted summation of the probability of meeting j c-type players in the group when the d-type player reverts back to playing cooperation instead of defection given that the group size is n and there are $k = n$ d-type players in the population. If $q_j^n > 0$ for $j = 1, \ldots, n-1$, then we can derive a more sufficient condition:

$$\alpha \geq \sum_{j=0}^{n-1} q_j^n \alpha + \sum_{j=1}^{n-1} q_j^n (n-1)\beta. \tag{5}$$

Given that $\sum_{j=0}^{n-1} q_j^n = 0$ as shown in Lemma 2, inequality (5) can be simplified to:

$$\alpha \geq -q_0^n (n-1)\beta.$$

Imposing the normalization $(n-1)\beta = 1$, we have

$$\alpha \geq -q_0^n. \tag{6}$$

Proposition 2 *If $q_0^n < 0$, $q_j^n > 0$ for $j = 1, \ldots, n-1$, and $|q_0^n|$ is decreasing in n, then Condition (6) (Equilibrium condition 2) is monotonically less restrictive as the group size n increases.*

3.3 Numerical Findings for Different Values of δ

Propositions 1-2 require restrictions on p_0^n and $|q_0^n|$ so that a set of more sufficient equilibrium conditions for the contagious strategy to sustain a social norm of cooperation among strangers becomes monotonically less restrictive as the group size n increases. We next ask whether these conditions hold, i.e., whether p_0^n is increasing in n and whether $|q_0^n|$ is decreasing in n.

Notice first that $p_0^n = (1-\delta)e_1(I - \delta A_n)^{-1}\rho_0^n$ and $q_0^n = \delta e_n H_n (I - \delta A_n)^{-1}\rho_0^n$ are both functions of δ and n. (Implicitly they are also functions of M since the transition probability matrix A_n and B_n also depend on M). Therefore, we compute p_0^n and q_0^n for different group sizes, n, and for different discount factors, δ, all under a fixed $M = 12$. The results of these numerical calculations are shown in Table 3. The cells shown in boldface are those that guarantee the existence of the contagious equilibrium.

p_0^n for given n and δ							
δ	0.01	0.1	0.3	0.5	0.7	0.9	0.99
$n=2$	**0.000925**	**0.011028**	**0.051716**	**0.141773**	**0.331384**	**0.704922**	**0.966359**
$n=3$	0.000214	0.005598	**0.050227**	**0.165386**	**0.385776**	**0.750317**	**0.972762**
$n=4$	0.000127	0.007565	**0.070361**	**0.209925**	**0.442245**	**0.783465**	**0.976880**
$n=6$	0.000121	**0.010175**	**0.090318**	**0.250271**	**0.490137**	**0.810020**	**0.980100**
$n=12$	**0.010000**	**0.100000**	**0.300000**	**0.500000**	**0.700000**	**0.900000**	**0.990000**
$\lvert q_0^n\rvert$ for given n and δ							
δ	0.01	0.1	0.3	0.5	0.7	0.9	0.99
$n=2$	**0.000839**	**0.009584**	**0.039064**	**0.088720**	**0.168391**	**0.290770**	**0.363919**
$n=3$	0.001377	0.014565	**0.049087**	**0.091050**	**0.140768**	**0.198557**	**0.227281**
$n=4$	0.002014	0.020207	**0.061095**	**0.102617**	**0.144773**	**0.187568**	**0.207034**
$n=6$	0.000128	**0.001282**	**0.003846**	**0.006409**	**0.008973**	**0.011537**	**0.012690**
$n=12$	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Table 3: Numerical Results on p_0^n and $|q_0^n|$ for Different n and δ ($M = 12$)

Our numerical exercises on p_0^n (top half of Table 3) illustrate some interesting results. Recall that p_0^n is the discounted summation of the probability of meeting zero c-type players in the group in all future periods once the player initiates a defection in the current period. Given any group size n, Table 3 reveals that p_0^n increases with δ. ($p_0^n = 0$ for $\delta = 0$ and $p_0^n = 1$ for $\delta = 1$.) Intuitively, this pattern is due to the feature of the contagious equilibrium – defection will spread to the entire population once defection is initiated, and it takes some time for a defection to spread. Therefore, when the player cares more about the future, the discounted summation of the probability of meeting zero c-type player becomes larger.

Next we ask: given a fixed δ, how does p_0^n change with increases in the group size n? The results reported in Table 3 suggest that the answer depends on δ. When δ is small, p_0^n follows a non-monotonic pattern; it decreases with n first and then increases with n, reaching $p_0^n = \delta$ when $n = M$. However, when δ is large enough (in our numerical example when $\delta \geq 0.5$), p_0^n is monotonically increasing as the group size n increases.

We next consider the numerical results for $q_0^n = \delta e_n H_n (I - \delta A_n)^{-1} \rho_0^n$, which is the decrease in the discounted summation of the probability of meeting zero c-type players in the group when the d-type player reverts back to playing cooperation instead of defection given that the group size is n and there are $k = n$ d-type players. Again we see (in the bottom half of Table 3) that q_0^n is increasing in δ given a certain group size. When δ is large enough (δ greater than 0.9 in Table 3), $|q_0^n|$ is monotonically decreasing as the group size n increases. Therefore, the equilibrium condition off the equilibrium path becomes less restrictive as n increases.[9]

Given the numerical results in Table 3, we conjecture that there exists a threshold value for the discount factor, $\bar{\delta}$ such that, for any $\delta > \bar{\delta}$, p_0^n is monotonically increasing in n and $|q_0^n|$ is monotonically decreasing in n. To verify this hypothesis, we plot p_0^n and q_0^n as continuous functions of δ in Figure 1. Indeed, we can see that when $\delta > .35$ (approximately) p_0^n is increasing in n. When $\delta > 0.8$ (approximately), $|q_0^n|$ is decreasing in n. (For $n = 12$, $p_0^n = \delta$ and $|q_0^n| = 0$ for any δ.)

In order to check whether this (partial) monotonicity result is only true for p_0^n and $|q_0^n|$ or it also applies to the original equilibrium conditions, we performed a similar exercise using the original equilibrium conditions 1 and 2. Recall that equilibrium condition 1 is:

$$(n-1)\beta \quad \geq \quad p_0^n \alpha + p_1^n(\alpha + \beta) + \ldots + p_{n-1}^n(\alpha + (n-1)\beta),$$

and equilibrium condition 2 is:

$$\alpha \quad \geq \quad q_0^n \alpha + q_1^n(\alpha + \beta) + \ldots + q_{n-1}^n(\alpha + (n-1)\beta).$$

Define

$$p^n \equiv 1 - \frac{\sum_{j=1}^{n-1} j p_j^n}{n-1}$$

and

$$q^n \equiv \frac{\sum_{j=1}^{n-1} j q_j^n}{n-1}.$$

Then with the normalization condition $\beta = (n-1)^{-1}$, equilibrium condition 1 becomes

$$\alpha \leq p^n$$

and equilibrium condition 2 becomes

$$\alpha \geq q^n.$$

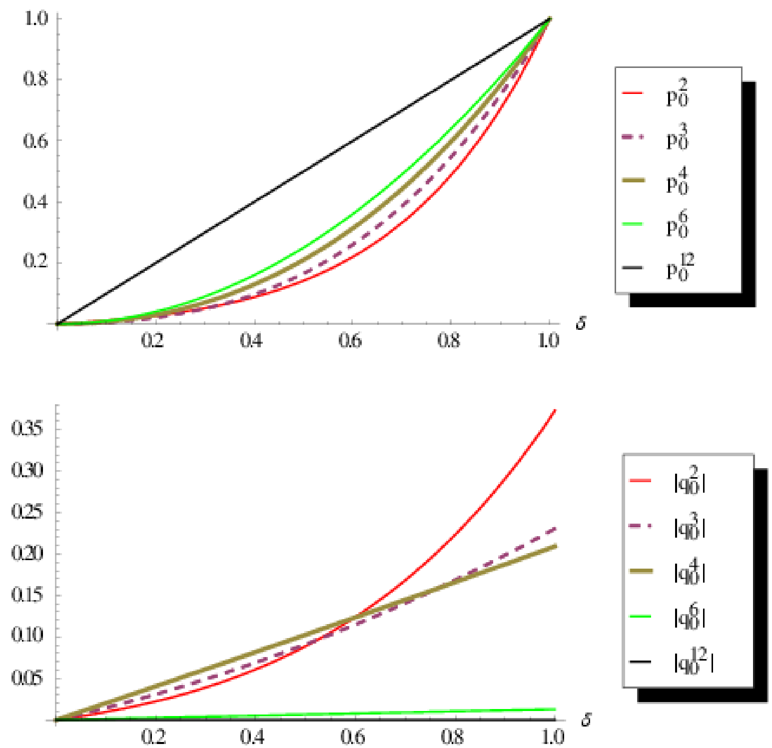

Figure 1: p_0^n and $|q_0^n|$ as functions of δ ($M = 12$)

Table 4 provides numerical results on p^n and q^n for different values of δ and n. In all cases, the contagious equilibrium always exists, i.e., the numerical value in each cell for p^n is always larger than the value in the corresponding cell for q^n. Thus by choosing α between q^n and p^n both equilibrium conditions 1 and 2 always hold. We observe that the result that cooperation is monotonically easier to sustain as n increases appears to hold more strongly under these original equilibrium conditions. That is, we find that p^n is monotonically increasing in n given any δ, and q^n is monotonically decreasing in n if δ is sufficiently large enough (greater than 0.5). Thus, the cutoff value for the discount factor $\bar{\delta}$ is smaller when we use the original equilibrium conditions. This observation is verified in Figure 2.

Summarizing, our main finding is that, for a fixed population M and for δ sufficiently high, the conditions under which the contagious strategy sustains play of the cooperative strategy in an n-player Prisoner's Dilemma game by all anonymously and randomly matched players in each period is monotonically more easily satisfied as the group size, $n \to M$.

4 Group Size and Information Sharing

In this section we establish an equivalence result between the n-player PD game and the more traditional 2-player PD game. In the first setting, a finite population of size M is randomly assigned

[9]In all of the cases reported in Table 3, it is always the case that $q_0^n < 0$ and $q_j^n > 0$ for $j = 1, \ldots, n-1$.

p^n for given n and δ							
δ	0.01	0.1	0.3	0.5	0.7	0.9	0.99
$n = 2$	0.000925	0.011028	0.051716	0.141773	0.331384	0.704922	0.966359
$n = 3$	0.001859	0.022562	0.100818	0.239648	0.460745	0.788192	0.977226
$n = 4$	0.002785	0.033155	0.137519	0.298782	0.524404	0.821914	0.981270
$n = 6$	0.004600	0.050899	0.185380	0.363489	0.585281	0.850813	0.984588
$n = 12$	0.010000	0.100000	0.300000	0.500000	0.700000	0.900000	0.990000
q^n for given n and δ							
δ	0.01	0.1	0.3	0.5	0.7	0.9	0.99
$n = 2$	0.000839	0.009584	0.039064	0.088720	0.168391	0.290770	0.363919
$n = 3$	0.001195	0.012416	0.040398	0.072725	0.109571	0.151112	0.171385
$n = 4$	0.000926	0.009293	0.028096	0.047192	0.066581	0.086265	0.095219
$n = 6$	0.000059	0.000590	0.001771	0.002952	0.004132	0.005313	0.005844
$n = 12$	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 4: Numerical Results on p^n and q^n for Different n and δ ($M = 12$)

into groups of size n in each period and then each group plays an n-person PD game, just as described in the previous sections. In the second setting, the finite population of size M is randomly assigned into groups of size n in a first-round matching, and then the n players in each group are further randomly paired with each other in a second-round matching prior to playing the 2-person PD game as shown in Table 5, where rows represent the representative player's choice and columns represent the opponent player's choice.[10] We will show that there exists an equivalence between these two settings when information on the outcome of play is shared between the n players who are matched in the first round of the second setting. Therefore, under this information sharing assumption, the existence and monotonicity results shown in the previous sections should extend directly to the second setting involving the 2-player PD game.

	Opponent's Choice	
	D	C
C	0	$(n-1)\beta$
D	α	$\alpha + (n-1)\beta$

Table 5: The Payoff Matrix for the 2-Player Prisoner's Dilemma Game

4.1 Payoff Equivalence

Suppose that in the second setting there are i cooperators in the group of size n other than the (representative) player himself. Then the player's expected payoff from choosing C is

[10] For simplicity, we assume that n is an even number.

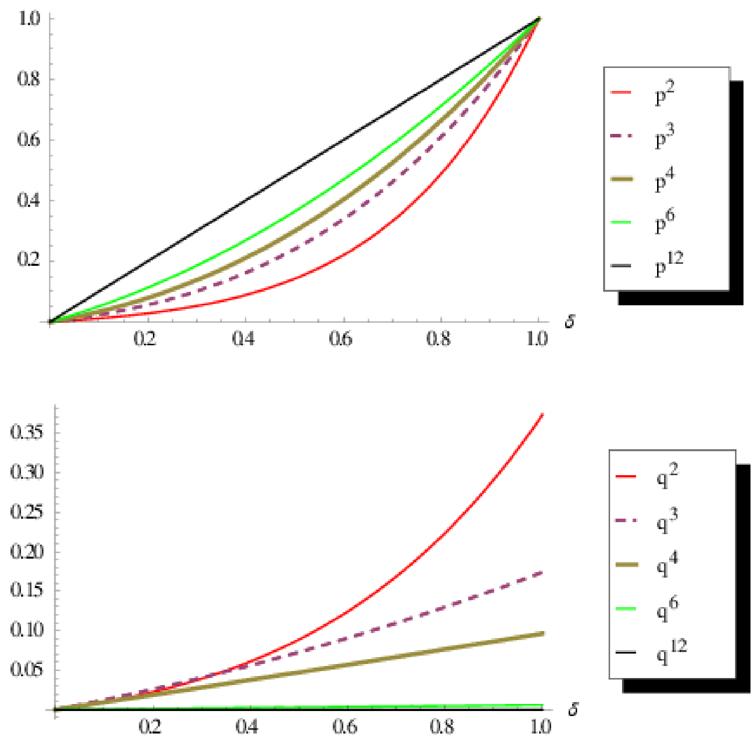

Figure 2: p^n and q^n as functions of δ ($M = 12$)

$$EU(C|i) = \frac{i}{n-1} \cdot (n-1)\beta + \frac{n-1-i}{n-1} \cdot 0 = i\beta.$$

The expected payoff for the player from choosing D is

$$EU(D|i) = \frac{i}{n-1}[\alpha + (n-1)\beta] + \frac{n-1-i}{n-1} \cdot \alpha = \alpha + i\beta.$$

Compared with the payoffs for the n-person Prisoner's Dilemma as shown in Table 2, we find that the player's (expected) payoffs in this 2-person PD game are exactly the same as for the n-player PD game provided that the latter game has the same number of cooperators $0 \leq i \leq n-1$ in the group of size n.

4.2 Strategy Equivalence

In the case of the 2-person Prisoner's Dilemma game, if players' strategies depend only on their own personal histories, then the strategy space will be different relative to the n-person PD game, where it is possible to condition behavior on the outcome of interactions with $n-1$ other players. However, if we allow players in the 2-person game to observe other pairs' payoff outcomes in the matching group of size n, and we allow all players' strategies to depend on the entire history of such payoff information, then we can easily establish a strategy equivalence between the two different settings as well.

Assume that in the second setting, in each period the players in each group of size n can not only observe the outcome of their own pair, but can also observe the outcome of other pairs in their n-player group. However, they are not able to observe the outcome of any pair outside of their n-player group. Furthermore, let us revise the contagious strategy in the second setting as follows: Define a player as a "c-type" if in all previous repetitions of the game this player and all of other $n-1$ group members in all previous periods have never chosen D, i.e., the outcome where all pairs this player can observe in every previous period has been (C, C). Otherwise, the player is a "d-type" player. The "contagious strategy" is defined in the same manner as before: A player chooses C if he is c-type and chooses D if he is d-type.

By allowing the player to observe the outcome of other pairs in the same n-player group and to allow the contagious strategy to depend on the outcome of all pairs the player can observe, it is straightforward that the contagious process in the second setting is same as in the first setting. Therefore, the results in the first setting will carry over to the second setting.

The equivalence results between the first and second settings have an interesting implication. In the second setting we first randomly assign players into groups of size n, and then further pair players in each group randomly. It is easy to see that this procedure still produces uniform random matching for each pair. Thus the monotonicity result obtained previously implies that in the uniform random matching game involving play of the 2-person PD game, it is easier to sustain a contagious equilibrium if players are allowed to observe more information on other players' outcomes. Kandori (1992) has a similar result for the case of full information, where he shows that cooperation is much easier to sustain when all M players can observe outcomes experienced by all other players, compared to the case where players can only observe their own private history. This is analogous to the comparison of groups of size 2 with groups of size M in our setting. Our monotonicity result thus establishes a connection between private information and full (population-wide) information, by showing that cooperation can be monotonically easier to sustain when partial information on the outcome of play of other community members is steadily increased (i.e., when the group size n increases gradually from 2 to M).

5 An Application to a Public Goods Game

In this section we show that with a slightly different normalization of the payoff matrix, the n-person Prisoner's Dilemma game can be re-interpreted as a public goods game so that our previous monotonicity result continues to hold in this public goods game version of the stage game.

As before, we assume a population of M players, who are anonymously and randomly assigned to groups of size n in each period to play an n-player public goods game. Here we study a binary choice version of the classic public good game (Issac and Walker (1988)) where each player is endowed with a single token and must decide whether or not to invest that token in his own privately held account or in a public account. Each token invested in the public account yields a payoff of μ for each group member. A token invested in the private account yields an additional payoff of γ, but only to the player associated with that private account. Table 6 represents the payoff matrix for the player from choosing to invest in the public account (C) or in the private account (D) given the number of other contributors to the public account in the group of size n. The standard public good game setup has $\mu > 0$ and $\gamma > 0$, so that non-contribution to the public good is always a dominant strategy in the one-shot, n-player game, and finally that $\gamma + \mu < n\mu$, which implies that the social optimum is achieved when all n players contribute to the public good. Notice that these restrictions also satisfy assumptions A1-A3, as defined in Section 2 for an n-player Prisoner's Dilemma game.

number of contributors in the group	0	1	2	...	$n-1$
C (invest in the public account)	μ	2μ	3μ	...	$n\mu$
D (invest in the private account)	$\gamma+\mu$	$\gamma+2\mu$	$\gamma+3\mu$	...	$\gamma+n\mu$

Table 6: The Payoff Matrix for the n-Player Public Goods Game

When this public goods game serves as the stage game played by a population of M players, who are randomly divided up into groups of size n in every period, the sufficient conditions to sustain the contagious equilibrium are very similar to those shown before. On the equilibrium path we must have:

$$n\mu \quad \geq \quad p_0^n(\gamma+\mu) + p_1^n(\gamma+2\mu) + \ldots + p_{n-1}^n(\gamma+n\mu),$$

while off the equilibrium path we require that:

$$\gamma \quad \geq \quad q_0^n(\gamma+\mu) + q_1^n(\gamma+2\mu) + \ldots + q_{n-1}^n(\gamma+n\mu).$$

Define

$$\tilde{p^n} \equiv \frac{n-1}{n}p^n = \frac{n-1}{n} - \frac{\sum_{j=1}^{n-1} jp_j^n}{n}$$

and

$$\tilde{q^n} \equiv \frac{n-1}{n}q^n = \frac{\sum_{j=1}^{n-1} jq_j^n}{n}.$$

Then with the normalization that $\mu = 1/n$, equilibrium condition 1 becomes

$$\gamma \leq \tilde{p^n}$$

and equilibrium condition 2 becomes

$$\gamma \geq \tilde{q^n}.$$

Based on the previous numerical results (Table 4), it is easy to show that the monotonicity pattern still holds for the public goods game when δ is sufficiently large, with the threshold value for $\bar{\delta}$ slightly increased.

6 The Experiment

In this section we report on a simple, individual decision-making experiment that tests the monotonicity results of Propositions 1-2. In our experimental design, we always consider a community of size $M = 12$ and we compare the cooperation rates between groups of size $n = 2$ and $n = 6$. With the normalized payoff in Table 2 and $\beta = \frac{1}{n-1}$, the stage game payoff for the 2-person PD and the 6-person PD are shown in Tables 7 and 8, respectively.

Other Player's Choice	D	C
C	0	1
D	α	$\alpha + 1$

Table 7: The Payoff Matrix for the 2-Player Prisoner's Dilemma Game

Number of Others Playing C	0	1	2	3	4	5
C	0	0.2	0.4	0.6	0.8	1
D	α	$\alpha + 0.2$	$\alpha + 0.4$	$\alpha + 0.6$	$\alpha + 0.8$	$\alpha + 1$

Table 8: The Payoff Matrix for the 6-Player Prisoner's Dilemma Game

To implement an infinite-horizon n-player PD game in the laboratory, we use the standard random termination methodology (Roth and Murnighan 1978) in which subjects participate in supergames that consist of an indefinite number of rounds, where the probability of continuation from one round to the next is a known constant equal to the discount factor, $\delta \in (0, 1)$. For our experiment, we chose to set $\delta = 0.75$. With this choice, the expected duration of a supergame is 4 rounds. To provide subjects with experience (and to propely induce the discount factor of .75) we have subjects participate in multiple supergames in our experiment.

The numerical results in the previous sections show that, as the group size n increases, the sufficient conditions both on the equilibrium path and off the equilibrium path are less restrictive. Table 9 summarizes the numerical results for the sufficient conditions to sustain the contagious equilibrium for groups of size $n = 2$ and 6 when $\delta = 0.75$ as in our experiment.

Group Size	On-equm Path	Off-equm Path
$n = 2$	$\alpha \leq 0.403$	$\alpha \geq 0.194$
$n = 6$	$\alpha \leq 0.648$	$\alpha \geq 0.004$

Table 9: Sufficient Conditions for Contagious Equilibrium (Numerical Results for $\delta = 0.75$)

Given the numerical results of Table 9 we chose to set $\alpha = 0.5$, which satisfies the sufficient condition on the equilibrium path for a group size of $n = 6$ but not for a group size of $n = 2$. Notice that our choice of $\alpha = .5$ satisfies the sufficient condition off the equilibrium path for both group sizes, $n = 6$ and $n = 2$.

We chose $\delta = .75$ and $\alpha = .5$ for several reasons. First, we wanted a parameterization that would allow for several rounds of repeated play and that could sustain the contagious strategy as an equilibrium in a community of a fixed population size under a larger group size but not under a smaller group size so as to test our main monotonicity result. Second, we chose to focus on the on-equilibrium-path condition rather than the off-the-equilibrium-path condition; if α was instead chosen in such a way that the on-the-equilibrium-path condition (but not the off-equilibrium-path condition) was always satisfied for both group size treatments, e.g., a choice of $\alpha = 0.1$, then we might observe that subjects seldom chose to defect (with the consequence that they were seldom actually off the equilibrium path) under either group size, making it difficult to detect any treatment effect.

Given our choice of α and δ, we are able to test the following hypotheses using our experimental data:

Hypothesis 1: The overall cooperation level is higher with a larger group size of $n = 6$ than with a smaller group size of $n = 2$.

Hypothesis 2: The cooperation level when subjects are c-type (on-equilibrium-path) is higher with a group size of $n = 6$ than with a group size of $n = 2$.

Hypothesis 3: The cooperation level when subjects are d-type (off-equilibrium-path) is no different between a group size of $n = 6$ and a group size of $n = 2$.

An important feature of our experimental design is that each 12-player community consists of just one human subject who interacts with 11 other "robot" players as opposed to allowing 12 human subjects to interact with one another. We employ this design in order to avoid the coordination problem of strategy selection among 12 human players and thereby remove strategic uncertainty. As with other folk-theorem type results, the contagious equilibrium is not the unique equilibrium of the infinitely repeated n-player PD game we implement in our experiment. There exist many other non-cooperative equilibria including the one where all players choose to defect in every round of the supergame. Empirically, when players face both the selection of their own strategy and the uncertainty of strategy selection by other players, the outcome of play can be far from that predicted by the contagious equilibrium.[11] Furthermore, we would expect that this problem of strategic uncertainty is naturally more severe as the group size n gets larger. For these reasons, we chose to eliminate the strategic uncertainty dimension from our experimental design by having our players interact with robot players programmed to play according to the contagious strategy so as to provide a cleaner test of our monotonicity results.[12]

In our experiment, we explicitly told our subjects that in each round of a supergame (or "sequence" as it was referred to in the experiment) they would be randomly matched with $n - 1$ other robot players (out of a total population of 11 robot players) and not with any other human subjects. Further, since $n < M$, there would be robot-robot group interactions that the human subject is not a part of. Subjects were further instructed that the robots in each community played according to the rules of the contagious strategy. Specifically subjects were told:

> "The robots are programmed to make their choices according to the following rules:

- - choose X in the first round of each new sequence;

[11] See for example, Duffy et al. (2013) and Duffy and Ochs (2009).

[12] We note that this type of experimental design involving robot players has not previously been implemented to test the contagious equilibrium prediction.

- - if, during the current sequence, any of a robot's group members, including you or any other robot players have chosen Y in any prior round of that sequence, then the robot will switch to choosing Y in all remaining rounds of the sequence;
- - otherwise, the robot will continue to choose X."

Here X refers to the cooperative action C, while Y refers to the defect action D.[13] Thus subjects had complete knowledge of the strategies to be played by their opponents. We did not provide subjects with any further information, such as the number of periods it might take for them to meet a defecting player once they (the human subject) had initiated a defection, as this calculation was one that we wanted subjects to make on their own.

One may be concerned that explicitly telling subjects the contagious strategy used by robots will induce a kind of experimenter demand effect, by which the human subjects will also follow this same contagious strategy. However, if that were the case, then our hypotheses that the group size n matters will not find any support since subjects in both treatments $n = 6$ and $n = 2$ were told that the robots would follow the contagious strategy. On the other hand, if we observe a higher cooperation rate under a group size of $n = 6$ than under a group size of $n = 2$, then it implies that subjects rationally choose to follow the contagious strategy more frequently when the equilibrium conditions are satisfied.

The experiment was conducted at the Experimental Social Science Laboratory (ESSL) of the University of California, Irvine using undergraduate students with no prior experience with our experimental design. Instructions were read aloud and then subjects completed a brief comprehension quiz. The instructions used in the $n = 6$ treatment are provided in Appendix C; instructions for the $n = 2$ treatment are similar. We conducted session 1 using a group size of $n = 2$ with 12 subjects and session 2 using group size of $n = 6$ with 10 subjects.[14] Since each subject interacted with an independent group of 11 other robots all playing according to the contagious strategy, each subject's behavior amounts to a single, independent observation. Thus the number of subjects we have for each session corresponds to the number of independent observations.

To allow subjects to gain some experience with play of the repeated game against the robot players, we had the subjects participate in multiple supergames. As noted above, subjects were informed that at the start of each and every new supergame (sequence) all of the 11 robot players in their community of size 12 would start out as c-types playing the cooperative strategy (X) and that robot players would only change to playing the defect strategy (Y) if they became d-types during that supergame. We did not fix the number of supergames played in advance. Instead, during the experiment, when the total number of rounds (over all supergames) exceeded 75, the supergame in progress was determined to be the last supergame (we did not inform subjects of our stopping rule); when that final supergame was completed, the session was declared to be over. Following the

[13] We used the neutral labels X and Y rather than (Cooperate and Defect) in our experimental implementation of the n-player game.

[14] Using the mean and standard deviation of the cooperation rate by human subjects in each treatment given by our data, the estimated statistical power for a t-test is 0.9. The required sample sizes is 11 for each treatment for statistical power of 0.9 and 9 for each treatment for statistical power of 0.8.

completion of the experiment, three sequences were randomly selected from all played and subjects were paid their total earnings from those three sequences in addition to a \$7 show-up fee.[15] Both sessions were completed within the two hour time-horizon for which we recruited subjects.

Table 10: Session Description

	No. of Subjects	No. of Supergames	No. of Rounds	Avg. Earnings
n=2	12	20	76	USD 15.33
n=6	10	18	75	USD 22.59

Figure 3 shows the average cooperation rate per round over time. The blue line (with diamonds) is the average cooperation rate of the human subjects only, while the red line (with squares) is the average cooperation rate by the 12-member community as a whole, consisting of 1 human subject and 11 robots. The start of each new supergame is indicated by a vertical line.

The top panel of Figure 3 shows average cooperation rates for the $n = 2$ group size. We observe a decline in cooperation over time by the human subjects in almost all supergames lasting more than 2 rounds, which indicates that more human subjects began to switch from cooperation to defection from round 2 if they did not choose to defect from the beginning of the supergame. Consistently, the cooperation rate at the community level shows a similar pattern but remains above the cooperation rate by the human subjects alone, as it takes some time for the contagious strategy, as played by the robot players to spread throughout the population of size 12. Across all supergames of the session with $n = 2$, there is no obvious learning effect or convergence.

The bottom panel of Figure 3 shows average cooperation rates for the $n = 6$ group size and exhibits a very different pattern of cooperation rates over time. Indeed, the overall cooperation rate is higher in Session 2 ($n = 6$) than in Session 1 ($n = 2$). Although there is also a decline of cooperation in the sugergames at the beginning of the session, the cooperation rate becomes high at around 90% following the fourth supergame of the session (approximately after the first one-third of the session) and remains high for the remaining supergames of that session. This finding indicates that, given the payoff parameters we have chosen and the strategy followed by the robots, most subjects learn over time that it is in their best interest to follow the contagious strategy when n is large ($n = 6$), relative to the case where n is small ($n = 2$). The comparison of the cooperation rates between group of size $n = 2$ and $n = 6$ indicates that the human subjects responded to the incentives we provide. They choose to start defecting more frequently when the contagious effect of a single defection was much slower under the $n = 2$ treatment and this tendency to defect was not diminished by experience. By contrast, when the contagious effect of a defection was more immediate, as in the $n = 6$ treatment, they learned to avoid triggering a wave of defection.

Table 11 reports the cooperation rates calculated based on the human subjects' choices and on community-wide action choices (humans plus robot players) over all rounds of all supergames. We further calculated the cooperation rates over the first and second halves of each session. For the

[15] We chose to pay for three randomly sequences, as opposed to just one, so as to minimize the possibility that a "short" (e.g., 1-round) supergame was chosen for payment.

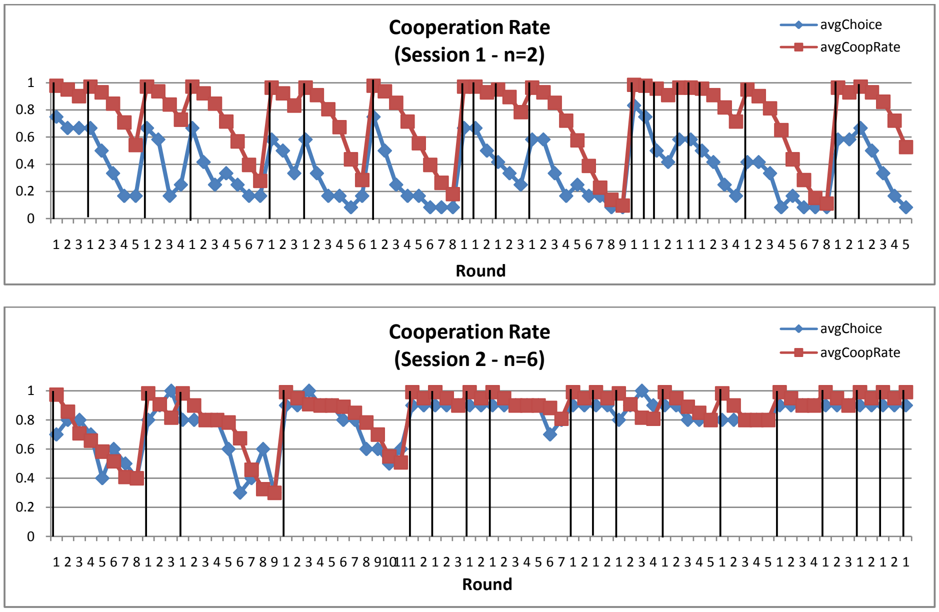

Figure 3: Cooperation Rate by Human Subjects and Communities Over Time

human subjects' choices only, the cooperation rate in the $n = 6$ treatment is always significantly higher than under the $n = 2$ treatment ($p < 0.01$) according to a two-sided Mann-Whitney test. At the community-level, the cooperation rates over the first half of the experiment are not significantly different between the $n = 2$ and $n = 6$ treatments, but over the second half, cooperation rates are significantly higher under the $n = 6$ treatment than under the $n = 2$ treatment ($p < 0.01$). Comparing the cooperation rate in the first and second half of the session, there is no significant difference in these cooperation rates when $n = 2$. However, cooperation rates in the second half of the $n = 6$ treatment are significantly greater than in the first half of that treatment, using a two-sided Wilcoxon test for matched pairs ($p = 0.08$ for the cooperation rate by human subjects and $p = 0.02$ for cooperation rate by communities).

Table 11: Mann-Whitney Tests on Cooperation Rate

	Human Subjects			Communities		
	Whole Session	1st half	2nd half	Whole Session	1st half	2nd half
n=2	0.364	0.373	0.355	0.743	0.754	0.733
n=6	0.800	0.732	0.870	0.846	0.776	0.918
p-value	0.006	0.008	0.003	0.015	0.391	0.005
No. of Obs.	12 (n=2) vs. 10 (n=6)					

Table 12 examines the extent to which the human subjects in each treatment have followed the contagious strategy as defined in the model. Recall that a player is defined to be on the equilibrium path in the first round of a supergame or when the player has never experienced a defection by his group members or himself in the past rounds of a supergame. Otherwise, a player is off the equilibrium path. In our data, the average frequency with which a player is on the equilibrium path when the group size $n = 2$ is significantly lower than that when the group size $n = 6$, using a two-sided Mann-Whitney test ($p < 0.01$). We further calculate the cooperation rates when each human subject is on the equilibrium path and off the equilibrium path. When the human subjects are on the equilibrium path, the cooperation rate when $n = 2$ is significantly lower than when $n = 6$ (two-sided Mann-Whitney test, $p < 0.01$). When the human subjects are off the equilibrium path, the cooperation (defection) rates under the two different group sizes are not significantly different. Finally, for each human subject, we also calculated the frequency with which that subject played the contagious strategy, which is given by: (freq. on equilibrium path)×(cooperation rate on equilibrium path) + (freq. off equilibrium path)×(defection rate off equilibrium path) We find that the frequency of contagious strategy play is also significantly higher when $n = 6$ than when $n = 2$ (two-sided Mann-Whitney test, $p < 0.05$). We conclude that this evidence supports our hypotheses 2 and 3.

Table 12: Strategy Analysis of the Human Subjects

	Frequency On Equm Path	Frequency Off Equm Path	Cooperate Rate On Equm Path	Defect Rate Off Equm Path	Frequency using Contagious Strategy
n=2	50.99%	49.01%	56.50%	94.46%	79.49%
n=6	82.27%	17.73%	86.49%	74.48%	91.60%
p-value	0.008	0.008	0.008	0.118	0.014
No. of Obs.	12 vs. 10	12 vs. 10	12 vs.10	11 vs. 8	12 vs. 10

Summarizing our experimental results, we find that the behavior of the human subjects is consistent with our theoretical predictions on the impact of group size for cooperative play. Given the same payoff parameter α, cooperation rates increase when the group size increases. Under a small group size, the cooperation rate declines over time and this pattern repeats itself across supergames even as subjects gain repeated experience with the environment. By contrast, under the large group size, subjects learn to stick with cooperation after experiencing the much quicker consequences of triggering a contagious wave of defection in their community.

7 Conclusions

We have examined the effect of group size, n, on the equilibrium conditions needed to sustain cooperation via the contagious strategy as a sequential equilibrium in repeated play of an n-player Prisoner's Dilemma game, given a finite population of players of size $M \geq n$ and random and anonymous matching of players in each repetition of the game. We find that, if agents are sufficiently patient, the equilibrium conditions, both on the equilibrium path and off the equilibrium path, become less restrictive, and thus more easily satisfied as the group size $n \to M$. This result arises from the faster speed with which a contagious wave of defections can occur as the group size becomes larger. We show that our results continue to hold in settings where players have information about the outcomes of play in their n-player matching group but are randomly paired to play the classic 2-person prisoner's dilemma game or where the payoff matrix of the n-player game is altered slightly to capture the incentive structure of the standard public goods game. Finally, we provide experimental evidence in support of our theoretical prediction that cooperation rates are higher under a large group size compared to a small group size.

Our findings serve to highlight the implications of Kandori's (1992) idea that a social norm of cooperative behavior among anonymous strangers can be policed by community-wide enforcement. Specifically, community–wide enforcement becomes easier to sustain as the speed with which information travels becomes faster, which is here proxied by increases in the group size, n. Centralized communication or monitoring mechanisms might also perform the same role played by larger group sizes in easing the conditions under which a social norm of cooperation is sustained in a large population of players.

In our experiment, we have removed strategic uncertainty by having human participants play with robot players who always play according to the contagious strategy. By contrast, Camera et al. (2012) find large heterogeneity in strategies adopted by participants in a 2-player Prisoner's Dilemma game with random and anonymous matching. When there is heterogeneity in strategies or a belief that strategies are heterogeneous, the incentives for agents to play according to the contagious strategy may be greatly altered or even non-existent. Consequently, we may observe experimental results that are far from the cooperative equilibrium even if the model parameterization satisfies the equilibrium conditions for such a cooperative social equilibrium to exist. A good next step would be to study reducing the fraction of robot players playing according to the contagious strategy in the population and (perhaps gradually) replace them with human subjects who can choose to play according to any strategy. The research question would be what (minimal) number of robot contagious strategy players, r, is needed in a population of size M in order that the remaining $M-r$ human subject players to learn to coordinate on a cooperative social norm equilibrium. We leave these questions to future research.

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Appendix A

We first show that the equilibrium conditions in Lemma 1 are equivalent to the equilibrium conditions provided by Kandori (1992) when the group size $n = 2$. Translating our notation to that used by Kandori (1992), $C_{n-1} = 1$, $u_n = (-l, 1)^T$, $v_n = (0, 1+g)^T$, $Z_n = (i_M - \rho\ \rho)$ ($\rho = \frac{1}{M-1}(M-1, M-2, \ldots, 1, 0)^T$, in which the ith element of ρ is the conditional probability that a d-type player meets a c-type when there are i d-types, and i_M is a $1 \times M$ vector with all elements equal to 1), $A_n = A$, $B_n = B$, $H_n = H$. Thus condition 1, equation 1 can be written as:

$$\begin{aligned} 1 &\geq (1-\delta)e_1(I-\delta A)^{-1}(i_M - \rho\ \rho)\begin{pmatrix} 0 \\ 1+g \end{pmatrix} \\ &= (1-\delta)e_1(I-\delta A)^{-1}\rho(1+g), \end{aligned}$$

which is the same as equilibrium condition 1 in Kandori (1992). Condition 2, equation 2 can be written as

$$e_k(i_M - \rho\ \rho)\begin{pmatrix} l \\ g \end{pmatrix} \geq \delta e_k H(I-\delta A)^{-1}(i_M - \rho\ \rho)\begin{pmatrix} 0 \\ 1+g \end{pmatrix},$$

i.e.,

$$(\frac{M-k}{M-1})g + (\frac{k-1}{M-1})l \geq \delta e_k H(I-\delta A)^{-1}\rho(1+g),$$

which is the same as equilibrium condition 2 in Kandori (1992).

Lemma 2 *Define* $p_j^n \equiv (1-\delta)e_1(I-\delta A_n)^{-1}\rho_j^n$ *and* $q_j^n \equiv \delta e_n H_n(I-\delta A_n)^{-1}\rho_j^n$ $(j = 0, \ldots, n-1)$*, then* $\sum_{j=0}^{n-1} p_j^n = 1$ *and* $\sum_{j=0}^{n-1} q_j^n = 0$.

Proof. By definition, p_j^n denotes the discounted summation of the probability of meeting j c-type players in the group once a defection has started when the group size is n, and q_j^n denotes the change in the discounted summation of the probability of meeting j c-type players in the group when the d-type player reverts back to playing cooperation instead of defection given that the group size is n and there are $k = n$ d-type players. Notice that by definition the summation of the elements in each row of matrix Z_n, A_n, and B_n is always equal to 1. Denote i_k as a $1 \times k$ vector with all elements equal to 1. Thus $Z_n i_n = i_M$, $A_n^t i_M = i_M$ and $B_n^t i_M = i_M$ for any group size n and $t = 0, 1, \ldots, \infty$. Therefore we have

$\sum_{j=0}^{n-1} p_j^n = (1-\delta)e_1(I-\delta A_n)^{-1}Z_n i_n = (1-\delta)e_1(I-\delta A_n)^{-1}i_M = (1-\delta)\sum_{t=0}^{\infty}\delta^t e_1 A_n^t i_M = 1,$

$\sum_{j=0}^{n-1} q_j^n = e_n H_n(I-\delta A_n)^{-1}Z_n i_n = e_n H_n(I-\delta A_n)^{-1}i_M = \sum_{t=0}^{\infty}\delta^t e_n(B_n - A_n)A_n^t i_M = 0.$ ■

Proof. [Theorem 1]

We first show that $\lim_{\delta\to 1}(I-\delta A_n)^{-1}\rho_j^n < \infty$ for $j = 1, \ldots, n-1$. (Therefore, $\lim_{\delta\to 1} p_j^n = 0$ for $j = 1, \ldots, n-1$ and $\lim_{\delta\to 1} p_0^n = 1$.) The proof is similar as in Kandori's (1992) proof for Theorem 1. Since $X_t = M$ is the absorbing state and the Mth element of ρ_j^n is zero for $j = 1, \ldots, n-1$,

$$(I-\delta A_n)^{-1}\rho_j^n = \sum_{t=0}^{\infty}\delta^t A_n^t \rho_j^n = \sum_{t=0}^{\infty}\delta^t \tilde{A}_n^t \rho_j^n = (I-\delta\tilde{A}_n)^{-1}\rho_j^n, \text{ for } j = 1, \ldots, n-1$$

where $\tilde{A_n}$ is a matrix obtained by replacing the last column of A_n by zeros. Given this, we have only to show the existence of $(I - \tilde{A_n})^{-1}$. Since the number of d-types never declines, $\tilde{A_n}$ is upper-triangular and so is $(I - \tilde{A_n})$. The determinant of an upper-triangular matrix is the products of its diagonal elements, which are all strictly positive for $(I - \tilde{A_n})$. Therefore, $\lim_{\delta \to 1} p_j^n = \lim_{\delta \to 1} (1 - \delta) e_1 (I - \delta A_n)^{-1} \rho_j^n \to 0$ and $q_j^n = \delta e_n H_n (I - \delta A_n)^{-1} \rho_j^n$ is finite for $j = 1, \ldots, n-1$.

Now the r.h.s. of equilibrium condition 1, $(1 - \delta) e_1 (I - \delta A_n)^{-1} Z_n v_n = p_o^n D_o + \sum_{j=1}^{n-1} p_j^n D_j \leq D_o + \sum_{j=1}^{n-1} p_j^n D_j$, where the inequality comes from $\sum_{j=0}^{n-1} p_j^n = 1$ and $p_o^n \leq 1$. Therefore, equilibrium condition 1 is satisfied if $C_{n-1} - D_o \geq \sum_{j=1}^{n-1} p_j^n D_j$, which is satisfied when $C_{n-1} - D_o$ and δ are sufficiently large.

Similarly, the r.h.s. of equilibrium condition 2, $\delta e_n H_n (I - \delta A_n)^{-1} Z_n v_n = q_o^n D_o + \sum_{j=1}^{n-1} q_j^n D_j$, is finite because $\sum_{j=0}^{n-1} q_j^n = 0$ and so $q_o^n = - \sum_{j=1}^{n-1} q_j^n$. Therefore, equilibrium condition 2 is satisfied when $v_n - u_n$ is sufficiently large. ■

Appendix B: Fixing the Number of Groups, $m = M/n$

In this appendix we examine the case where M and n are varied in such a way that the number of groups $m = M/n$ is held constant. In particular, we compare the equilibrium conditions in three cases where $m = 3$: 1) $M = 12$ and $n = 4$; 2) $M = 9$ and $n = 3$; and 3) $M = 6$ and $n = 2$. Our aim here is to understand whether variations in the group size n continue to matter for satisfaction of the equilibrium conditions needed for cooperation to be sustained as a social norm, when the number of groups is held constant. The following numerical results are obtained holding fixed $\delta = 0.9$.

Equilibrium Condition 1

First we consider equilibrium condition 1 for each of the three cases where $M/n = 3$:

$M = 6$ and $n = 2$: $\beta \geq 0.773647\alpha + 0.226353(\alpha + \beta)$;

$M = 9$ and $n = 3$: $2\beta \geq 0.774597\alpha + 0.0696314(\alpha + \beta) + 0.155771(\alpha + 2\beta)$;

$M = 12$ and $n = 4$: $3\beta \geq 0.783465\alpha + 0.030344(\alpha + \beta) + 0.0546601(\alpha + 2\beta) + 0.131531(\alpha + 3\beta)$.

Imposing the normalization condition $\beta = (n - 1)^{-1}$, these conditions can be simplified as follows:

$M = 6$ and $n = 2$: $\alpha \leq 0.773647$ for $\beta = 1$;

$M = 9$ and $n = 3$: $\alpha \leq 0.809415$ for $\beta = 1/2$;

$M = 12$ and $n = 4$: $\alpha \leq 0.821913$ for $\beta = 1/3$.

We observe that when the number of groups $m = M/n$ is fixed (at 3) the results are very similar to those in the case where the population size M is fixed: equilibrium condition 1 is observed to become less restrictive as the group size n becomes larger. Intuitively, again, this is driven by the faster contagious process with a larger group size. The extent of the tendency for cooperation to become more easily sustainable as n increases is smaller than in the case where M is fixed, since in the latter case the contagious process becomes faster not only due to a larger group size but also due to there being a smaller number of groups as n increases.

Equilibrium Condition 2

Finally, we consider equilibrium condition 2 for each of the three cases where $M/n = 3$:

$M = 6$ and $k = n = 2$: $\alpha \geq -0.270439\alpha + 0.270439(\alpha + \beta)$;

$M = 9$ and $k = n = 3$: $\alpha \geq -0.219582\alpha + 0.15486(\alpha + \beta) + 0.064722(\alpha + 2\beta)$;

$M = 12$ and $k = n = 4$: $\alpha \geq -0.187568\alpha + 0.126324(\alpha + \beta) + 0.0512613(\alpha + 2\beta) + 0.00998238(\alpha + 3\beta)$.

If we further impose our payoff normalization, then we have:

$M = 6$ and $k = n = 2$: $\alpha \geq 0.270439$ for $\beta = 1$;

$M = 9$ and $k = n = 3$: $\alpha \geq 0.142152$ for $\beta = 1/2$;

$M = 12$ and $k = n = 4$: $\alpha \geq 0.086265$ for $\beta = 1/3$.

As with equilibrium condition 1, the results for equilibrium condition 2 under a fixed ratio for M/n are similar to those found under a fixed M. Equilibrium condition 2 becomes less restrictive with increases in the group size, n.

Appendix C: Instructions (Treatment $n = 6$)

[Instructions for the $n = 2$ treatment are similar]

Overview

This is an experiment in economic decision-making. The Department of Economics has provided funds for this research. You are guaranteed $7 for showing up and completing this experiment. During the course of this experiment, you will be called upon to make a series of decisions. The decisions you make determine your additional earnings for the experiment, beyond the $7 show-up payment. Your total earnings, including the show-up payment, will be paid to you in cash and in private at the end of the session. We ask that you not talk with one another and that you silence any mobile devices for the duration of this experiment.

Specifics

In today's experiment, you will play with 11 computerized robots denoted by R1, R2, ..., R11. You and the 11 robots consist of a single community. In this room, we have ____ human participants. So in total we have ___ communities. The play in each community will not influence other communities in any way.

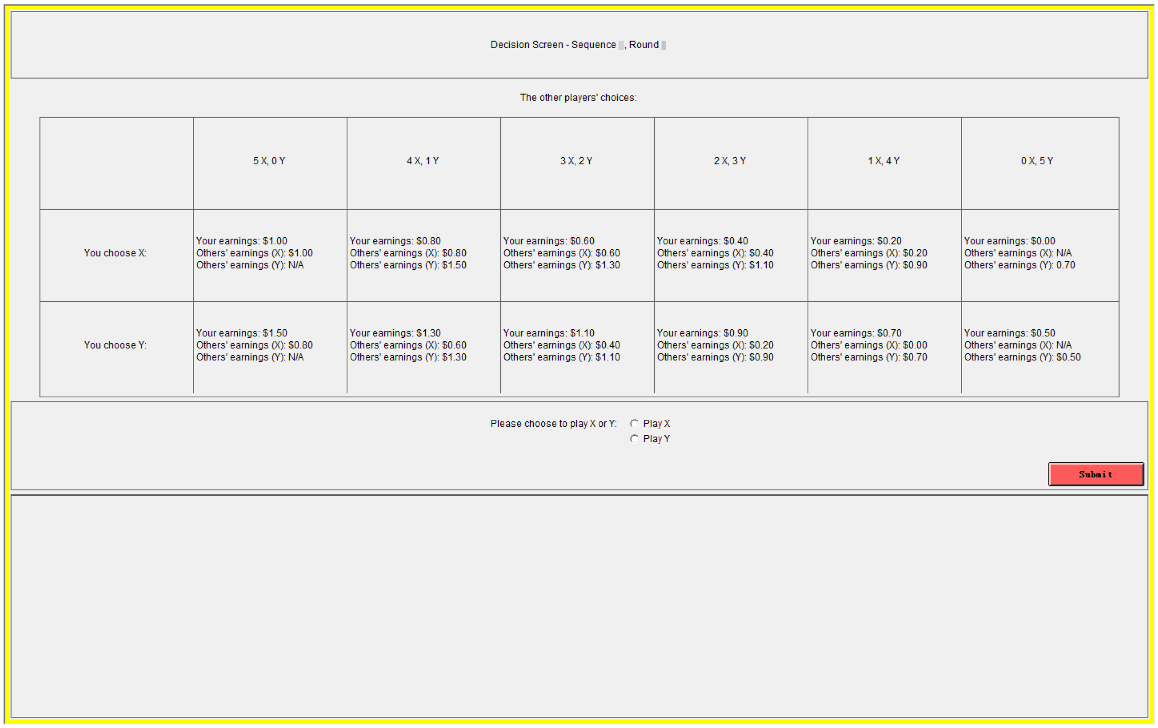
The experiment consists of a number of "sequences". Each sequence consists of an indefinite number of rounds. At the start of each round of a sequence, you and every robot in your community will be randomly and anonymously assigned to one of two groups of size 6. All possible divisions of the 12 community members into two groups of size 6 are equally likely at the start of each new round. Thus, the composition of your 6 group members is very likely to change from one round to the next. For example, you may have robots R1, R3, R4, R6, R10 in your group in one round, and have robots R2, R3, R6, R8, R9 in your group in another round. However, you will not know which robot is in your group in a given round since the matching is anonymous. Note also that in addition to the group you are in, there will be a second, 6-member group in your community consisting of all robot players.

In each round, all members of both 6-member groups for that round must simultaneously choose between two options: X or Y. Your earnings for the round will be decided by your own choice and by how many of the other 5 robot members in your group choose X and Y in the round. The same is true for the robot players in your group and the robot players in the other group. Specifically, the determination of a player's earning (you or the robots) is explained in the payoff table that appears on your decision screen which is shown in Table 1 on the next page.

In Table 1, the second and third rows show your earnings (in dollars) from choosing option X or option Y respectively, given the number of the 5 robots in your group who choose option X or option Y, as indicated by the first row.

Example:

Suppose that 3 of the 5 robot group members choose X while 2 choose Y: "3X,2Y". If you choose X, your earnings for the round is $0.60 (row two, column four). If you choose Y, your earnings for the round is $1.10 (row three, column four).



Decision Screen - Sequence , Round

The other players' choices:

	5 X, 0 Y	4 X, 1 Y	3 X, 2 Y	2 X, 3 Y	1 X, 4 Y	0 X, 5 Y
You choose X:	Your earnings: $1.00 Others' earnings (X): $1.00 Others' earnings (Y): N/A	Your earnings: $0.80 Others' earnings (X): $0.80 Others' earnings (Y): $1.50	Your earnings: $0.60 Others' earnings (X): $0.60 Others' earnings (Y): $1.30	Your earnings: $0.40 Others' earnings (X): $0.40 Others' earnings (Y): $1.10	Your earnings: $0.20 Others' earnings (X): $0.20 Others' earnings (Y): $0.90	Your earnings: $0.00 Others' earnings (X): N/A Others' earnings (Y): 0.70
You choose Y:	Your earnings: $1.50 Others' earnings (X): $0.80 Others' earnings (Y): N/A	Your earnings: $1.30 Others' earnings (X): $0.60 Others' earnings (Y): $1.30	Your earnings: $1.10 Others' earnings (X): $0.40 Others' earnings (Y): $1.10	Your earnings: $0.90 Others' earnings (X): $0.20 Others' earnings (Y): $0.90	Your earnings: $0.70 Others' earnings (X): $0.00 Others' earnings (Y): $0.70	Your earnings: $0.50 Others' earnings (X): N/A Others' earnings (Y): $0.50

Figure 4: Payoff Table

Note that you and your robot group members all face the same payoff table as shown in Table 1. This payoff table not only shows "Your earnings" but it also shows how "Others' earnings" are affected by your choice and the choices of others. To see this, suppose again that 3 of the others choose X and 2 of the others choose Y: "3X,2Y". In this case, if you choose X, then the earnings for the round for the 3 other group members who (like you) choose X is $0.60, and the earnings for the round for the 2 other group members who choose Y is $1.30. If, in this same scenario (3 others choose X and 2 others choose Y: "3X,2Y"), you instead choose Y, then the earnings for the round for the 3 other group members who choose X is $0.40, and the earnings for the round for the 2 other group members who (like you) choose Y is $1.10.

Robot Rules

The robots are programmed to make their choices according to the following rules:

- choose X in the first round of each new sequence;

- if, during the current sequence, any of a robot's group members, including you or any other robot players have chosen Y in any prior round of that sequence, then the robot will switch to choosing Y in all remaining rounds of the sequence.

- otherwise, the robot will continue to choose X.

Notice that the choice of each robot may be different in each round if their experience in previous rounds of the sequence is different. As a human participant, you are always free to choose X or Y. Notice further that no robot will start choosing Y unless the human subject in their community has previously chosen Y in a round of the current sequence.

When you make your choice in each round, you will be reminded of the results in all previous rounds of the current sequence. You will be shown your own choice, the number of your robot group members who chose X, the number of your robot group members who chose Y, your earnings for the round and your cumulative earnings for the current sequence of rounds. To complete your choice in each round, simply click on the radio button next to option X or Y (as shown at the bottom of Table 1) and then click the red Submit button. You can change your mind anytime prior to clicking the Submit button. After you have clicked the red Submit button, the computer program will record your choice and the choices made by your robot group members and determine your earnings for the round. Then the results of the round will appear on your computer screens. You will be reminded of your own choice and will be informed about how many of your robot group members have chosen X or Y, as well as the payoffs that you have earned for the round. Please record the results of the round on your RECORD SHEET under the appropriate headings. Immediately after you have received this information on choices and payoffs for the round, a random number from 1 to 100 will be drawn by the computer to determine whether the sequence continues or not. If a number from 1 to 75 is chosen, the sequence will continue with another round. If a number from 76 to 100 is chosen, the sequence ends. Therefore, after each round there is 75% chance that the sequence will continue with another round and a 25% chance that the sequence will end. Suppose that a number less than or equal to 75 has appeared. Then you will play the same game as in the previous round, but you and your robot community members will be randomly and anonymously assigned to two new groups of size 6. The robots always play according to the robot rules stated above, even in the all-robot group. You only see the outcome of your own group's decisions each round. After you have made your decision for the new round and learned the outcome, record the results and your earnings for the round on your record sheet under the appropriate headings. Then another random number will be drawn to decide whether the sequence continues for another round. If a number greater than 75 appears, then the sequence ends. Depending on the time available, a new sequence may be played. The new sequence will again consist of an indefinite number of rounds of play of the same game as described above. Recall that, according to the robot rules, all robots play X in the first round of each new sequence, and will continue doing so until they experience play of Y by one or more players in their group, at which point they will switch over to playing Y for the duration of that sequence.

Earnings

Following completion of the last sequence, the experimenter will randomly draw three sequences to determine your earnings from today's experiment. Your earnings from each of the three chose sequences will be the accumulated earnings from all the rounds played in those three sequences. Since you don't know which three sequences will be chosen for payment you will want to do your best in every round of every sequence. At the end of the session you will be shown your total earnings on your computer screen. This total amount will include your $7 show-up payment. Please write down the total amount you are owed on your receipt.Then you will be paid your earnings in cash and in private.

Final Comments

First, do not discuss your decisions or your results with anyone at any time during the experiment.

Second, each of you will play with 11 robots in today's session. Your play will not affect the earnings of any other human subject participant.

Third, your identity and the identity of your robot group members are never revealed.

Fourth, at the end of every round of a sequence, there is 75% chance that the sequence will continue with another round and a 25% chance that the sequence will end.

Fifth, remember that at the start of each round of a sequence you and your 11 robot community members are randomly and anonymously assigned to one of two groups of size 6, so the composition of your robot group members is very likely to change from one round to the next.

Sixth, the robots always play according to the rules stated in the instructions. Specifically, they always start out a sequence playing X and only switch to playing Y if a member of their group has chosen to play Y in any prior round of the sequence; otherwise they continue to play X.

Finally, we will randomly draw three sequences at the end of today's session to determine your earnings. Your total earnings will be the sum of your accumulated earnings from the three chosen sequences.

Questions?

Now is the time for questions. Does anyone have any questions?