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Los Angeles

A Study of Investment Capacity and an Essay about Interest Rates

A dissertation submitted in partial satisfaction

of the requirements for the degree

Doctor of Philosophy in Management

by

Clinton Tepper

2021

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2021

ABSTRACT OF THE DISSERTATION

A Study of Investment Capacity and an Essay about Interest Rates

by

Clinton Tepper
Doctor of Philosophy in Management
University of California, Los Angeles, 2021
Professor Lars A. Lochstoer, Co-Chair
Professor Ivo I. Welch, Co-Chair

In Chapter 1, I find increases in investor exposure to prominent systematic trading strategies, such as momentum, correlate with lower returns to these strategies. A 1 percentage point increase in gross momentum exposure as a percentage of market capitalization corresponds with a permanent 1.1 percentage point decline in future annual momentum returns. The result is based on a new measure of total investment in a strategy that distills aggregate dollar exposure levels from stock trading volumes. The approach circumvents market clearing for zero cost strategies by defining aggregate exposure as the absolute sum of long plus short exposure. Estimating aggregate investment in momentum over time reveals a nearly 10-fold increase in momentum exposure as a percentage of market capitalization from 1980 to 2010. The association of a permanent decline in returns with increases in exposure extends to other strategies, including long-run reversal and idiosyncratic volatility.

In Chapter 2 (with Daniel Feenberg and Ivo Welch), I discuss how contrary to common perception, many fixed-income investors have not suffered unusually low real interest rates in and after the Great Recession of 2008. This is because taxable investors must first pay taxes on *nominal* interest returns, before inflation further reduces their earned *real* interest rates. To obtain the same real after-tax yield, investors need more than one-to-one compensation for inflation. As a result, long-term Treasury bonds have been no less attractive for taxable investors in 2016 (with a 0.5% post-tax real yield) than they were in 2006 (0.5%) and 1976 (–1.7%), and only moderately lower than yields in 1966 (0.9%), and 1956 (0.8%), although they are much less attractive than they were in 1996 (2.4%) and 1986 (2.9%). Short-term Treasury bond yields have been on the low side but have also not been particularly unusual.

The dissertation of Clinton Tepper is approved.

Tyler S. Muir

Mark J. Garmaise

Ivo I. Welch, Committee Co-Chair

Lars A. Lochstoer, Committee Co-Chair

University of California, Los Angeles

2021

To Kati

for love and being a partner

and Boone

our son

and Deborah, Stewart, and Sam

my mom, dad, and brother.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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CHAPTER 1

Capacity

1.1 Introduction

How does exposure to a trading strategy affect that strategy's future returns? The ability of investors to influence asset prices is well-documented empirically within the microstructure literature (De Long et al. 1990; Shleifer and Vishny 1997; Pontiff 2006). Mclean and Pontiff (2016) and Chordia, Subrahmanyam, and Tong (2014), among others, presented evidence that characteristics which correlate with higher future returns often lose their predictive power over time. Smith and Timmermann (2021) found that the future returns associated with characteristics depend on the economic regime. Mclean and Pontiff (2016) demonstrated a relationship between the disclosure and publication of a strategy and a drop in future returns. In this analysis, I investigate the relationship between aggregate strategy exposure and returns.

The contribution of this paper is twofold. The first contribution is methodological. I propose a measure for the dollar amount of assets invested in a strategy. As discussed in Section 1.1.2, the new measure avoids many of the severe limitations of existing measures. I estimate the aggregate amount of investor assets exposed to momentum as a monthly series and find that momentum exposure rises from about \$4 billion in 1963 (1.2% of market capitalization) to \$3.4 trillion (9% of market capitalization) at the end of 2020. Exposure increases rapidly around the publication of Jegadeesh and Titman (1993). Overall, momentum exposure represents an economically meaningful component of aggregate investment portfolios.

The second contribution answers the initial question. I analyze the link between the amount of exposure subscribed to a strategy and its future returns. Most of the analysis focuses on the momentum strategy. I find an economically and statistically significant negative long-run relationship between the level of momentum investment and future returns. A 1 percentage point increase in momentum exposure as a percent of market capitalization correlates with a 3.3 percentage point decline in 3-year momentum factor returns.

The purpose of the analysis is to study the relationship between the amount of assets subscribed to a strategy and its future returns. While the analysis primarily focuses on momentum, the approach is general enough to apply to most well-defined systematic investment strategies. To show this, I estimate the relationship between exposure levels and returns for long-run reversal and idiosyncratic volatility strategies.

1.1.1 Defining aggregate strategy exposure

Before proceeding further, the reader may benefit from a discussion on the meaning of investor exposure to a strategy. I define investment exposure to a strategy, such as momentum, as the dollar long exposure plus the dollar short exposure. Market clearing implies these two halves should be equal. A simple example may add clarity. See Section 1.2.1 for a more formal definition in the context of the estimation procedure.

Consider two stocks, ‘U’ and ‘D’. Stock ‘U’ is high momentum, while stock ‘D’ is low momentum. A hedge fund invests \$100 in ‘U’ and −\$100 in ‘D’. This implies the fund has \$200 in gross momentum exposure. Now suppose the fund increases its investment in momentum by 50%. The fund would then have \$150 invested in ‘U’ and −\$150 in ‘D’ for \$300 in exposure.

In contrast with studies of net exposure by Lewellen (2011) and Blitz (2017), this paper considers aggregate gross investor exposure. Suppose the hedge fund trades exclusively with a second fund. The second investor then holds a −\$300. For the purposes of this study, the total investor exposure to momentum is then \$600, the absolute sum of the exposure of the first and second investors. Interpreting exposure in this way renders questions about net exposure and market clearing ancillary to the relationship between investment and returns. Loosely, the gross exposure represents the polarization of exposures to momentum. I use the terms gross exposure and strategy assets interchangeably.

1.1.2 Why is the proposed measure needed?

Analyzing the relationship between investment and returns requires a measure of said investment. A simple approach might be to total the assets under management (AUM) of funds classified as investing in such strategies. While obviously limited to only funds which report such information, several additional shortcomings severely curtail the usefulness of a classification based approach as even a partial metric of aggregate investor exposure.

The AUM of a fund is at best a coarse measure for the dollars exposed to a strategy. In the case of mutual funds, closet indexing limits the level of exposure for a particular fund to a particular factor (see Petajisto (2013), Cremers et al. (2016), and Vogel (2017), among others). Instead of providing "pure" market neutral exposure to a characteristic-based strategy, mutual funds may "tilt" toward value or some other characteristic. The extent to which such a tilt translates to exposure varies from fund to fund.

This issue is a pernicious case of a more general problem with AUM as a measure, namely that both actual and guideline levels of risk and exposure vary both within a strategy and from fund to fund. Hedge funds and market-neutral "alternative" mutual funds may offer wildly disparate levels of exposure to otherwise identical variants. For instance, Two Sigma and AQR, two large and popular hedge fund managers, offer a menu of volatility levels for a given underlying investment strategy. Though mutual funds face regulatory burdens in taking on leverage, variable levels of cash holdings create a similar effect. While careful accounting and adjustment for risk differences and exposure levels between funds is possible in principle, such information is often unavailable, incomplete, or inaccurate. Hedge fund disclosures are voluntary, while mutual funds, by definition, do not disclose the extent to which they are closet indexing.

Aggregate fund AUM has several further shortcomings as a measure. Misclassification of investment styles reduces precision (Chen, Cohen, and Gurun 2020). AUM based measures of strategy assets miss the exposure of a large percentage of hedge funds and institutional allocators that make direct investments. Omissions may create selection bias in cases where responses are voluntary. Finally, the frequency of the disclosures may be inconsistent across different types of investors.

13-F filings present another approach for aggregating strategy exposures. A literature review did not uncover any papers which had completed the specific exercise of calculating strategy exposure, although numerous studies applied 13-F data to other purposes. Among those that used the data in a way related to investment strategies, Koijen and Yogo (2019) used 13-F data to compute subsets of investor flows related to characteristics along with demand for characteristics in investor portfolios. Lewellen (2011) found some institutional investors collect returns from momentum. Gompers and Metrick (2001), among others, reported institutional preference for particular characteristics, although Lewellen (2011) described an almost negligible net effect across institutions. Cao et al. (2018) used 13-F data to identify institutional ownership, and discovered stocks held by hedge funds tend to exhibit greater mispricing and subsequent future alpha as compared to stocks held more by other types of institutional investors.

Moreover, 13-F filings present their own set of difficulties. In rough order of their significance in the context of calculating strategy assets, these include position netting within large financial conglomerates, omission of short positions, different reporting dates across institutions, coverage of only about two-thirds of the market (Blume and Keim 2012), confidential disclosure (Aragon, Hertzel, and Shi 2013), and filing errors. Finally, quarterly reporting implies substantial noise when using 13-F filings to calculate strategy assets for shorter-term strategies.

Finally, while not a direct measurement of aggregated strategy assets, a substantial strand of literature examines the co-movement of assets in the context of crowded trading. Pairwise correlations of adjusted returns between assets with similar characteristics measure the consequences of crowded investing (Baltas 2019; Huang, Lou, and Polk 2018; Lou and Polk 2019). Such effects could be downstream products of substantial aggregate investment into particular strategies. However, outcomes of arbitrage activity depend on the liquidity of the underlying assets, and such liquidity varies over time. Hence these measures are at best ordinal measures of investor strategy exposure.

1.1.3 Variation in expected returns

Certain stock characteristics explain the cross-section of stock returns. Early seminal works include Banz (1981), De Bondt and Thaler (1985), Fama and French (1992), Jegadeesh and Titman (1993), and Daniel and Titman (1997), while Feng, Giglio, and Xiu (2020), Harvey, Liu, and Zhu (2016), Hou, Xue, and Zhang (2020), and Mclean and Pontiff (2016) provided recent reviews of the cross-sectional explanatory power of characteristic- and factor-based trading strategies. The existence of predictive power is regardless of whether a particular strategy delivers excess returns due to an underlying risk factor (Fama and French 1993) or represents an anomalous source of mispricing correlated with the characteristics themselves (Daniel and Titman 1997). At the same time, the magnitude and significance of the variation explained by these characteristics seems to change over time (Chordia, Subrahmanyam, and Tong 2014; Mclean and Pontiff 2016; Smith and Timmermann 2021).

Moreover, stock characteristics predict returns over time, particularly when aggregated to the level of the market. Shiller, Fischer, and Friedman (1984), Fama and French (1988), and Campbell and Thompson (2008) showed that dividend ratios predict future stock market returns. While the predictive power of this and other predictors vary depending on the tests employed and the use of out of sample testing (Campbell and Thompson 2008; Welch and Goyal 2008), the existence of long-run return predictability in some capacity is well-established (Campbell and Thompson

2008; Cochrane 2008; Lewellen 2015; Shiller 1981). Marrying cross-sectional and time series predictability, Haddad, Kozak, and Santosh (2020) reported that predictions from the dividend price ratio extend to characteristic-based strategy portfolios. Not only are market returns predictable, but the returns of strategies based on characteristics are also predictable.

The measure proposed in this paper predicts returns of characteristic based strategies, but in a manner up-stream from the valuation ratios studied in Haddad, Kozak, and Santosh (2020). Linking the proposed measure of strategy exposure with future returns provides a mechanism by which the publication dates considered in Mclean and Pontiff (2016) correlate with declines in returns.

1.2 Estimating strategy assets

1.2.1 Strategy exposure and growth

A trading strategy is a set of N_t stock weights over time, denoted as length N_t vector $w_t = \omega\left(S_{t-1}\right)$. The weights are a function of S_{t-1}, the set of observable characteristics at time t. For example, in the case of a typical value-weighted momentum strategy, this would be the past year of returns and the stock's market capitalization. Regardless of the strategy particulars, the weights represent a crucial input to the estimation procedure.

Without loss of generality with respect to the estimation procedure, assume each strategy k has zero net dollar exposure such that $\sum_{i \in 1:N_t} w_{it}^k = 0$. From this point forward, consider a single trading strategy $k = 1$ and hence drop the superscript. Define the weights such that the weights of stocks held long and short add to 1 and

-1 respectively so that $\sum_{i \in 1:N_t} |w_{it}| = 2$.

A large quantity of single strategy investors $j \in 1 : J$ commit A_{jt} to a zero-cost trading strategy. At the beginning of each period, investor j starts with strategy positions of $A_{jt-1} w_{it-1}$. At the end of the period, each investor picks a new commitment level implying positions of $A_{jt} w_{it}$.

Summing across investors gives the total strategy exposure

$$\text{gross exposure} = 2A_t = 2 \sum_j |A_{jt}| \tag{1.1}$$

where the factor of 2 accounts for the long and short legs of the portfolio.

Analogous to the positions of individual investors, $A_{t-1} w_{it-1}$ denotes the aggregate strategy positions at the start of the period. Positions at the end of the period change to $A_t w_{it}$, reflecting both changes to weights brought about due to changing characteristics as well as changes to aggregate investor allocations. The following decomposition makes this clear:

$$\begin{aligned} \underbrace{A_t w_{it} - A_{t-1} w_{it-1}}_{\text{Changes in positions}} &= \underbrace{A_{t-1} \left(w_{it} - w_{it-1} \right)}_{\text{Rebalancing}} + \underbrace{\left(A_t - A_{t-1} \right) w_{it-1}}_{\text{Allocations} \mid \text{old weights}} \\ &+ \underbrace{\left(A_t - A_{t-1} \right) \left(w_{it} - w_{it-1} \right)}_{\text{Net of allocations and rebalancing}} \end{aligned} \tag{1.2}$$

Note the last interaction term is not necessarily small, as a stock may shift from a long position to a short position, and aggregate allocations may move substantially. The changes in position identified by Equation 1.2 should not be confused with active trading volume implied by a strategy, which must also account for stock returns over the period as described in Section 1.2.2. Rather, Equation 1.2 reflects the total changes in positions as defined by $\omega\left(S_{t-1}\right)$ and the absolute sum of commitment levels across all investors.

Finally, denote the period-over-period growth in strategy assets as

$$G_t \equiv \frac{A_t}{A_{t-1}} \tag{1.3}$$

1.2.2 Definition of trading volume

Suppose stocks realize three types of volume: rebalancing V_{it}^{R}, allocation flows V_{it}^{F}, and orthogonal trading V_{it}^{ε}. Define V_{it}^{R+F} as the net volume from rebalancing and allocation flows. Total volume decomposes as follows:

$$V_{it} = V_{it}^{R+F} + V_{it}^{\varepsilon} \tag{1.4}$$

1.2.2.1 Rebalancing

Strategy investors rebalance after the realization of returns. Rebalancing trades result from changes in weights as well as deviations of portfolios from present target weights. Suppose net allocative flows, discussed in the next section, are zero. In contrast with Equation 1.2, rebalancing in the context of volume is net of individual stock returns. Thus the total rebalancing volume is given by

$$V_{it}^{R} = \left|A_{t-1}w_{it} - A_{t-1}w_{it-1}R_{it}\right| \tag{1.5}$$

where w_{it} is the portfolio weight of stock i at time t, A_{t-1} is the beginning of period aggregate long (or short) exposure, and R_{it} is gross stock return.

For intuition on Equation 1.5, consider a universe with two stocks, ‘U’ and ‘D’. ‘U’ is high momentum and has a weight of 1.0 while ‘D’ is low momentum with an initial weight of -1.0. A single investor gathers strategy exposure such that $A_0 = \$100$. The investor thus holds \$100 in ‘U’ and $-\$100$ in ‘D’. Now suppose over the period stock ‘U’ returns 50%, while stock ’D’ is flat. As weights are unchanged, rebalancing flows consist of a single \$50 sale of stock ‘U’, implying trading volume for stock ‘U’ of \$50. Absent any other activities, the end of period portfolio contains \$100 invested in ‘U’ and $-\$100$ in ‘D’, the same as the beginning of the period.

The example also illustrates how rebalancing accounts for profit-taking of gains and compensation for losses. The decision to categorize these actions as rebalancing flows is arbitrary but has no influence on the overall trading volume implied by the data, as discussed in the next section.

1.2.2.2 Flows

Simultaneous with rebalancing, investors choose a new allocation, A_t. The volume created by the strategy reflects allocations net of rebalancing flows. For example, if rebalancing would ordinarily indicate a sale for a share of a particular stock and allocation flows imply a purchase of a share, the net volume created is zero. This leads to the “netted” volume below:

$$V_{it}^{R+F} = \left| \underbrace{A_{t-1}\left(w_{it} - w_{it-1}R_{it}\right)}_{Rebalancing} + \underbrace{w_{it}\left(A_t - A_{t-1}\right)}_{Flows} \right| \tag{1.6}$$

$$= \left| A_t w_{it} - A_{t-1} w_{it-1} R_{it} \right| \tag{1.7}$$

$$= A_{t-1} \left| G_t w_{it} - w_{it-1} R_{it} \right| \;\; (\text{for } A_{t-1} > 0) \tag{1.8}$$

Consider the example from the previous section, where stock ‘U’ returns 50%. Suppose that instead of keeping allocations constant, the investor increases their exposure over the period from $A_0 = \$100$ to $A_1 = \$150$. Then the net trade for stock ‘U’ is zero, while the investor sells or shorts −\$50 of stock ‘D’. While allocative flows to stock ‘U’ are \$50, this positive flow is “netted” against the \$50 in proceeds from returns, leading to no net trade for this stock. Since stock ‘D’ remained flat, the position experiences no such netting, and the investor makes the −\$50 allocative trade. The investor at the end of the period holds \$150 invested in ‘U’ and −\$150 ‘D’.

1.2.2.3 Other Trading

All trading which does not fall into the previous two types of volume is considered other trading. This includes both noise trading and controls.

$$V_{it}^{\varepsilon} = \lambda_t + \varepsilon_i \tag{1.9}$$

The base case allows for time fixed effects, but those implementing the estimation may insert additional controls if desired. Time fixed effects, as opposed to a single intercept, hold particular appeal in accounting for volume trends.

1.2.2.4 Total Volume

The payoff of building up volume in the manner described is a clean decomposition of trading volume for each stock. Totaling up the three types of volume gives

$$\begin{aligned} V_{it} =& V_{it}^{R+F} + V_{it}^{\varepsilon} \\ =& \left| A_t w_{it} - A_{t-1} w_{it-1} R_{it} \right| + ... + \varepsilon_{it} \end{aligned} \tag{1.10}$$

where the ellipsis represents any added controls. If A_t is positive, the relationship simplifies to

$$V_{it} = A_{t-1} \left| G_t w_{it} - w_{it-1} R_{it} \right| + ... + \varepsilon_{it} \tag{1.11}$$

Equations 1.10 and 1.11 provide the framework for estimating the time series of strategy assets A_t.

1.2.3 Estimating strategy assets

The goal is to pick the values of A_t for all t and any control parameters that allow Equation 1.10 to best approximate the volume.

$$\min_{A_t,\lambda_t \forall t} \mathbb{E}\left(V_{it} - |A_t w_{it} - A_{t-1} w_{it-1} R_{it}| - \lambda_t\right)^2 \tag{1.12}$$

Equation 1.12 minimizes the sum of squared errors ε_{it} in Equation 1.10 using the time controls of Equation 1.9.

Defining the initial value for strategy assets as positive and plugging in sample averages for the expectation makes the problem described by Equation 1.12 equivalent to

$$\min_{A_0,G_t,\lambda_t \forall t} \sum_{t,i} \left(V_{it} - A_0 \left|\prod_{s=1}^{t-1} [G_s] \left(w_{it} G_t - w_{it-1} R_{it}\right)\right| - \lambda_t\right)^2 \tag{1.13}$$

Here A_0 is the initial estimation of strategy assets, while G_t is the growth of strategy assets at time t as previously discussed.

Estimation of the $2T + 1$ parameters in 1.13 is impractical as the problem is not convex. See Section 1.A in the appendix for a discussion. Instead of a brute force approach, make assumptions and economic inferences necessary for G_t to be observed. In particular, assume that growth in the gross strategy exposure of a sampling of investors , in this case mutual funds, corresponds to growth in aggregate strategy assets. Then estimating Equation 1.13 only entails determination of A_0 and the fixed effects λ_t.

The assumption that growth in strategy exposure for mutual funds proxies for overall growth in strategy investment is strong but reasonable. At the end of 2020, mutual funds held a collective $14 trillion in domestic equity assets, or somewhat under half of the US stock market capitalization. Regulatory hurdles to shorting do not preclude investment in market neutral strategies when such market neutral strategies are overlayed on the market portfolio.

Net exposure to market neutral strategies serves as a check. Market clearing implies that the sum of positions equals the market, meaning investors should hold low net exposure to strategies orthogonal to the market. A representative sample of investors should therefore hold minimal net exposure to such market neutral strategies. Such an implication is testable for the mutual fund industry if the strategy exposures are known. This idea is examined in the end of Section 1.3.4. On average, net exposures are substantially lower than gross exposures.

1.3 Detailed empirical procedure

This section describes the precise procedure used to estimate momentum assets. Where appropriate, these procedures also apply for estimating idiosyncratic volatility and long-run reversal. Readers interested in estimating the measure for a particular strategy can use the approach for momentum as a worked example whose steps can be applied to any strategy that can be defined in terms of systematic rules for calculating weights. These readers may choose to skip the data and sample formation sections and use data as appropriate for their circumstances. All code is posted online and/or available upon request.

1.3.1 Stock Data

Daily stock data from the Center for Research on Security Prices (CRSP) serve as the principal input to the estimation procedure. The focal period is limited by the mutual data discussed in Section 1.3.3 and begins in December 1962 and ends in December 2020. The CRSP stock data however begin a few years earlier than this to compute the backward-looking momentum, reversal, and idiosyncratic volatility characteristics. Stocks accepted into the sample must consist of common shares traded on major exchanges.[1]

Returns consist of either the CRSP holding period return (RET) or the delisting return (DLRET). If both fields are populated, use their geometric sum. Similarly for price, the field PRC is used or, if not available, the field DLPRC. If both fields are available, use their average. Use the returns to compute a geometrically compounded total return index for each stock.

Next, remove duplicate records based on CRSP PERMNO and DATE. If the duplicate is due to multiple distributions, keep the record with a DISTCD of 1232 if any, otherwise keep the record with the lowest non-missing DISTCD. If any duplicates remain, keep one of the records only if each of the CRSP fields used is identical, otherwise drop the records.

To compute shares outstanding, multiply the CRSP SHROUT field by 1000. Market capitalization is then this value multiplied by the price as previously calculated. Each stock's daily dollar volume is estimated as the average of the opening and closing price multiplied by the share volume. If only the closing price is available, that price is used in the calculation.

While the focus of this study is on momentum, this is the point where idiosyncratic volatility is computed. See Section 1.7.1 for details.

Stocks must have valid return data, shares outstanding, prices, and share volume.

[1]Keep stocks with the CRSP field SHRCD equal to 10 or 11 and EXCHCD equal to 1, 2, or 3.

If any of these fields are missing, drop them at this point. Estimation of the measure requires liquid trading volume. To drop the most illiquid stocks, impose the following selection criteria:

1. First, for the last market capitalization of each month, compute the 10th percentile market capitalization among NYSE stocks and drop stocks with a market capitalization below this point.

2. Second, drop stocks with a price less than $5.

3. Third, drop stocks with fewer data points than the median number of trading days minus one across all stocks. This last filter effectively drops stocks with more than a day of missing data in a particular month, leading to the removal of about 20 stocks in a given month on average.

The criteria are deployed at each month. To reduce look-ahead bias, delay the removal of stocks that fit the removal criteria by one month. Otherwise, a stock assigned a weight at time $t-1$ that fits the removal criteria at time t would not realize a return, potentially biasing the results.

Next, calculate the momentum characteristic as the trailing 12-month return less the trailing one-month return using the previously computed total return index. Each record must have a valid value for 1) the current date, 2) the date one month prior to the current date, and 3) one year prior to the current date. Calculate the reversal characteristic analogously as the five-year return less the one-year return.

Aggregate the daily CRSP data up to a monthly frequency by compounding the returns, summing the volume and taking the end of month values for each characteristic. Complete remaining data processing using the aggregated monthly data. Drop all records with a missing value for the characteristic of interest.

Table 1.1 lists summary statistics for each strategy considered. While the samples are similar, differences in the look-back period of the characteristics (one year for momentum, 5 years for reversal, 60 days for idiosyncratic volatility) lead to different start dates for the sample. The rolling exposure versions start with an additional 18-month lag, as described in Section 1.3.4.

Table 1.1
Estimation summary statistics

	$\Sigma_t(N_t)(000s)$	T	start	end	$\mu(MC)$	$\mu(V)$	$\sigma(V)$
MOM (full)	1353	697	Dec-1962	Dec-2020	4.03	0.58	3.54
MOM (roll)	1333	679	Jun-1964	Dec-2020	4.08	0.59	3.57
REV (full)	945	697	Dec-1962	Dec-2020	5.31	0.73	4.10
REV (roll)	930	679	Jun-1964	Dec-2020	5.39	0.74	4.13
$IVOL$ (full)	1568	697	Dec-1962	Dec-2020	3.55	0.52	3.30
$IVOL$ (roll)	1546	679	Jun-1964	Dec-2020	3.60	0.53	3.33

All data are from CRSP using the selection criteria described in Section 1.3.1. Sample size differs due to the differences in the formation process of the focal characteristics, e.g., momentum uses one year of data, while reversal uses five. Each sample ends in December 2020. $\Sigma_t(N_t)$ is stock-months, T is number of months, $\mu(MC)$ is the average market capitalization, and V is stock volume. All values except for T and $\Sigma_t(N_t)$ are in units of billions of dollars.

1.3.2 Portfolio formation

To assign the weights, follow a standard double-sort procedure, similar to that of the characteristic sorts of Fama and French (1993) but with a few differences. Specifically, within each month, sort stocks by the focal characteristic into terciles, with cutoffs at the 30% and 70% quantile. In contrast with the characteristic sorts of Fama and

French (1993), use the entire sample to determine the breakpoints. This accounts for the higher average market capitalization and liquidity of the sample.

For momentum, the sorts create high and low momentum buckets from ranking the 12-month preceding return less the most recent month. Independently, sort stocks by market capitalization into two groups with a cutoff at the median, thereby creating a group of large capitalization and small capitalization stocks. The intersection of these groups creates six portfolios. The stocks within each of these portfolios are capitalization-weighted. In another difference from Fama and French (1993), re-sort and form the portfolios at each month instead of each year.

The overall return of the characteristic portfolio is then the average of the present returns of the previous period's high characteristic groups less the average of the two low characteristic groups. Equivalently, assign each stock in the high characteristic group a weight equal to half of their within-portfolio weight. Likewise assign each stock in the low characteristic group a weight of minus half their within-portfolio weight. This implies the long weights add to 1.0 and the short weights add to -1.0. Explicitly setting the weights in this way defines the strategy in the notation of Equation 1.13. When calculating the returns, lag the weights by one prior to computing the returns to ensure that the portfolios are formed prior to the realization of returns as below.

$$r_t^{CHAR} = \sum_i w_{it-1} r_{it} \tag{1.14}$$

1.3.3 Mutual fund data

Mutual fund data are sourced from CRSP. The CRSP monthly mutual fund data contain AUM data of variable frequency starting in December 1962, thereby creating a natural start date for the overall sample.[2] To select the mutual funds used in the sample, load the CRSP fund summary table. Selected funds must have a most recent CRSPOBJCD starting with "ED", which implies a domestic equity fund.

Valid funds must have a non-missing return field and at least 12 months of data. Large AUM reversals could potentially distort growth calculations if not recognized. Following the procedure discussed in the data appendix of Pastor, Stambaugh, and Taylor (2015), define the variables

$$aumgrowth_{it} \equiv \left| \frac{aum_{it} - aum_{it-1}}{aum_{it-1}} \right| \tag{1.15}$$

$$reversal_{it} \equiv \frac{aum_{it+1} - aum_{it}}{aum_{it} - aum_{it-1}} \tag{1.16}$$

then set aum_t to missing if $aumgrowth_{it} \geq 0.5$, $-0.75 \leq reversal_{it} \geq 1.25$, and $aum_t \geq \$10mn$.

The mutual fund AUM data are often patchy, often reported on a quarterly or annual basis early in the sample. To account for this, use the monthly fund return to estimate the AUM when the gap between two AUM measurements is 13 months or less. A second interpolation scheme that linearly adds the return discounted net deposits between dates, as opposed to just multiplying the most recent AUM by the returns, led to substantively identical results. These are available upon request.

[2]The earliest CRSP monthly mutual fund data begin in December 1961, but less than 10 funds in the sample have complete data over the initial 12 months, so this first year was omitted.

1.3.4 Estimation of mutual fund exposure

The next step is to estimate mutual fund strategy exposures. These are necessary to compute the observable inputs G_t for Equation 1.13. While adding up the assets of funds that run a particular strategy such as momentum in their objectives might seem a simple solution, such a procedure has several fatal flaws. Chen, Cohen, and Gurun (2020) found that 30% of bond mutual funds misclassify their own investment style, a conclusion that seems likely to extend to other asset classes. But even if the strategy classifications were accurate, variable exposure levels imply that such a totaling would lead to substantial distortions. See Section 1.1.2 for a more in depth discussion on variable risk levels.

A solution is to borrow a page from the practitioner literature (e.g. Vogel (2017) and Blitz (2017)) and weight each fund by its factor exposure. Past totaling of exposure indicates relatively low net exposure across the focal investment universe (Blitz 2017). Because of market clearing, such a result is not surprising if the sample is large enough to be representative. Yet aggregate gross exposure, calculated as the sum of the absolute value of each individual fund's exposure, should prove substantial. This more appropriate than net exposure and reflects the obtained and provided strategy exposure.

To estimate mutual fund strategy exposures, regress the returns of the fund on the strategy return. For momentum, these regressions take the following forms

$$r_{nt} - r_t^f = \beta_n MOM_t + \alpha_n + control(s) \text{ (full sample)} \tag{1.17}$$

$$r_{ns} - r_s^f = \beta_{nt} MOM_s + \alpha_{nt} + control(s) \text{ (18-month rolling)} \tag{1.18}$$

$$s \in \{(t - 18mo) : (t - 1))\}$$

The controls consist of the returns of the three Fama-French return factors as tabulated on Ken French's website. Equation 1.17 regresses the entire return history of the fund on the momentum factor, thereby implicitly assuming that the aggregate fund exposure does not change over time. Equation 1.18 computes time-varying exposures via 18-month rolling regressions. The rolling regressions exclude the most recent return to reduce look-ahead bias.

To compute the ultimate series of mutual fund strategy exposures, multiply the absolute value of the regression coefficients $|\beta_{nt}|$ by each fund's AUM and total the results at each point in time. Then aggregate strategy AUM is calculated as follows:

$$fundAUM_t = \sum_n |\beta_{nt}| \, AUM_{nt} \tag{1.19}$$

$$G_t = \frac{fundAUM_t}{fundAUM_{t-1}} \tag{1.20}$$

Note that for the full sample specifications β is constant across time (but not across funds). Dropping the absolute value in Equation 1.19 allows for analysis of the net exposure, although this is not used to estimate the growth. As will be discussed, the net exposure is generally less than the growth exposure, confirming the results of (Blitz 2017).

An alternative specification below yields similar results for most purposes due to the low net exposure. An exception is an examination of contemporaneous return shocks on assets described in Section 1.4.

$$fundAUM_t = \sum_n \beta_{nt} AUM_{nt} \times \iota \left(\beta_n \geq 0.0\right) \tag{1.21}$$

Table 1.2a contains summary statistics for estimating mutual fund exposures by strategy. The table presents all three strategies, including momentum assets, reversal, and idiosyncratic volatility. The summary statistics show that the rolling regressions generally capture larger absolute gross exposures on average, both in absolute terms and as a ratio of total domestic equity fund assets. Asset growth averages between 1-2% growth per month, albeit with standard deviations of 5-15% per month.

Table 1.2b considers the net exposures of mutual fund assets. Net exposure is not used to estimate growth in strategy exposure but serves to assess the degree to which mutual funds summarize both the long and short sides of strategy trades. As expected, net exposure is much smaller than gross exposure over the sample. This is true both with respect to the average fund β and the aggregated strategy asset exposures. While the standard deviations indicate that the net exposure can represent a meaningfully large quantity at a particular point in time, the low mean net exposures suggest that such imbalances are mostly temporary.

1.4 Stock volume regressions

This section considers and assesses the measure in the context of explaining stock volume. Readers purely interested in the results of the procedures described in Sections 1.2 and 1.3 may skip to Section 1.5 without loss of continuity.

The estimate of G_t from the mutual fund data is the result of the procedure described in Section 1.3.4. Extraction of the aggregate strategy assets across all

Table 1.2
Mutual fund and exposure summary statistics

(a) *Mutual fund absolute (gross) exposures*

	funds (000s)	$\mu(\lvert\beta^{mf}\rvert)$	$\sigma(\lvert\beta^{mf}\rvert)$	$\mu(A^{mf})$	$\mu(A^{mf}_{abs}/A^{tot})$	$\sigma(A^{mf}_{abs}/A^{tot})$	$\mu(g)$	$\sigma(g)$
MOM (full)	3220	8.16	7.60	125.09	6.34	0.98	1.11	5.17
MOM (roll)	2741	15.61	16.74	272.62	15.36	6.05	1.75	12.86
REV (full)	3220	14.33	13.70	226.44	9.42	1.81	1.16	5.07
REV (roll)	2741	29.11	30.55	487.39	24.40	7.06	1.77	13.72
$IVOL$ (full)	3220	9.97	9.17	177.66	8.63	0.76	1.09	4.96
$IVOL$ (roll)	2741	17.51	18.69	300.02	17.82	5.79	1.53	12.42

This table contains summary statistics regarding the procedure for estimating mutual fund assets. The number of funds corresponds to the number of mutual fund-months from the CRSP mutual fund database, divided by 1000. $|\beta^{mf}|$ is the absolute value of the coefficient from the regressions described by Equations 1.17 and 1.18, aggregated over funds and periods. A^{mf}_{abs} is aggregated absolute mutual fund asset exposure, totaled over domestic equity funds as in Equation 1.19 (the exposure beta times the fund beta summed across funds). A^{tot} normalizes these assets by all domestic equity funds. g is the net growth of this exposure as described with Equation 1.20. All values except $\mu(A^{mf})$ multiplied by 100, with $\mu(A^{mf})$ in units of billions of dollars.

(b) *Mutual fund net exposures*

	funds (000s)	$\mu(\beta^{mf})$	$\sigma(\beta^{mf})$	$\mu(A^{mf})$	$\mu(A^{mf}_{net}/A^{tot})$	$\sigma(A^{mf}_{net}/A^{tot})$
MOM (net, full)	3220	−0.17	11.15	−7.38	1.54	1.70
MOM (net, roll)	2741	0.91	22.87	25.49	4.48	7.47
REV (net, full)	3220	3.25	19.56	21.86	1.32	1.34
REV (net, roll)	2741	2.92	42.10	−34.95	1.50	11.86
$IVOL$ (net, full)	3220	−2.12	13.38	5.54	0.12	1.69
$IVOL$ (net, roll)	2741	−4.94	25.13	−66.25	−5.60	8.11

This table is the same as the previous table but the exposure β^{mf} corresponds to net rather than gross exposure. The aggregate exposure A^{mf}_{net} is then $\Sigma_i AUM_{it} \times \beta^{mf}_i$, that is, A_{net} is the total from Equation 1.19 sans the absolute value.

stock market participants entails the following regression:

$$V_{it} = A_0 \left| \prod_{s=1}^{t-1} [G_s] \left(w_{it} G_t - w_{it-1} R_{it} \right) \right| + \lambda_t + \epsilon_{it} \tag{1.22}$$

The sample begins in 1962, so A_0 corresponds to half of the gross strategy assets as of 1962, or 18 months past 1962 for the rolling regressions. The quantities w_{it} and R_{it} correspond to the weight and return of of stock i at time t.

For intuition on Equation 1.22, consider that the coefficient A_0 corresponds to half of gross aggregate strategy exposure as of the beginning of the sample.[3] Cumulatively compounding this quantity with the previously estimated growth G_t gives assets over time, A_t. Multiplying A_t by the weights gives portfolio positions in dollars. The absolute changes in positions correspond to the volumes implied by the strategy. Each dollar in A_0 therefore implies a contribution to trading volume for each stock over the sample. In the absence of controls or intercepts, estimating A_0 is thus equivalent to projecting stock trading volume onto the volume created by one dollar invested in the strategy. In this way A_0 maximizes the fit between observed trading volume and volume implied by the right-hand side of Equation 1.22.

The results of the regression in Equation 1.22 serve as a first test of the measure. Table 1.3a shows the coefficient estimates as well as the results of several tests. Table 1.3b shows the same information as Table 1.3a but using the rolling beta regressions to estimate strategy growth.

Specifications (1) and (2) use the average volume and average cross-sectional volume respectively as counterfactual predictions. Specification (3) uses a single in-

[3]It is half of gross strategy assets because the portfolio is 100% levered.

Table 1.3
Explanatory volume regressions

(a) *Full sample*

	(1)	(2)	(3)	(4)	(5)	(6)
intercept	0.59		0.39		0.43	
	(13.72)		*(13.71)*		*(11.88)*	
A_0^{MOM} (estimated) L+S			4.26	3.85		
			(8.83)	*(7.60)*		
A_0^{MOM} $(G = 1\ \forall t)$ L+S					572.39	594.34
					(4.24)	*(4.39)*
R^2	0.00	3.99	8.86	10.85	2.31	6.46
$R^2 - R^2(mean)$	0.00	0.00	8.86	6.85	2.31	2.46
time FE		X		X		X
N	1342196	1342196	1342196	1342196	1342196	1342196

This table contains regressions of stock-specific volume on stock-specific trading as described in Equation 1.22. A_0 corresponds to momentum strategy asset exposure across all investors in 1962, the beginning of the sample. The last row contains the focal coefficients of a similar regression, but this time assuming $G = 1$. Standard errors clustered by firm and month.

(b) *18-month rolling*

	(1)	(2)	(3)	(4)	(5)	(6)
intercept	0.60		0.40		0.43	
	(13.74)		*(13.68)*		*(11.81)*	
A_0^{MOM} (estimated) L+S			6.87	6.25		
			(7.84)	*(6.81)*		
A_0^{MOM} $(G = 1\ \forall t)$ L+S					593.66	614.04
					(4.28)	*(4.42)*
R^2	0.00	3.95	8.53	10.59	2.41	6.50
$R^2 - R^2(mean)$	0.00	0.00	8.53	6.63	2.41	2.55
time FE		X		X		X
N	1322697	1322697	1322697	1322697	1322697	1322697

This table is the same as the previous table but uses 18-month rolling regressions. The estimate A_0 corresponds to momentum assets in 1964, the beginning of the rolling regression sample.

tercept while Specification (4) employs time fixed effects as in Equation 1.22. The fixed effects remove volume trends, but could potentially remove some relevant investor allocations. Both approaches are highly significant. The specifications used for all of the main results use the time fixed effects, although the effect on the results relative to just using an intercept is small.

Specifications (5) and (6) test the fundamental relationship between changes in strategy weights and volume without using any of the growth values extracted from the mutual fund data. That is, run the regression of 1.22 forcing G_t to 1.0. The significance of the results implies that absolute time deviations in weights correspond with trading volume. At the same time, the lower R^2 relative to Specifications (3) and (4) show that the growth values of mutual fund strategy exposure contribute to the predictive power of the regressions.

As another test of the measure, consider the individual regression cross-sections. The regression of Equation 1.22 should provide a valid estimate of the strategy exposure at the beginning of the sample (1962) even if the data used for each individual cross-section correspond to a single month. For example, consider the weights and returns of November 2011 along with the lagged October 2011 weights. Plugging into Equation 1.22 given the growth values from the mutual funds and excluding all other weights and returns yields a single cross-sectional estimate of A_0 corresponding to 1962 assets. Computing the mean of each cross-sectional estimate provides another procedure for estimating A_0. The results, presented in Section 1.D in the appendix, echo those of Specifications (4) and (6) of Table 1.3.

1.5 Aggregate momentum exposure

1.5.1 Evolution of momentum investment over time

The first payoff from the estimation process is the evolution of momentum strategy assets over time. A changing quantity of investor exposure to momentum could reflect a greater demand for the strategy, or it could also correspond to an increase in the mispricings as described in behavioral models such as those of Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999). The implications of different explanations are analyzed through the lens of return predictability in Section 1.6.

To create a time series of momentum assets, cumulatively compound the coefficient from the volume regressions of Section 1.4 with the growth estimates of Section 1.3.4 such that

$$A_t = A_0 \prod_{s=1}^{t} G_t \tag{1.23}$$

where A_t is the time series of momentum exposure.

1.5.2 Dollar investment in momentum

Figure 1.1 shows the results of this exercise. Momentum assets follow an approximately log-linear trend from the beginning of the sample to the end. The aggregate market capitalization of the sample serves as a reference. As will be explored in greater depth, the quantity of momentum assets seems to increase over time as a percentage of the market.

The high correlation between momentum assets and the market capitalization

Figure 1.1
Investment in momentum and market capitalization over time
The dark line plots the aggregate momentum exposure across all investors.

is not surprising, both from the standpoint of investor behavior and the mechanics of the estimation process. The momentum exposure of investors that maintain a persistent fraction of their risk in the momentum portfolio would track the market. However, a simple mechanical relationship is perhaps the primary driver.

To see this, consider a high and low momentum stock, “U” and “D” respectively. A momentum investor places \$100 in U and −\$100 in D. The investor therefore has \$200 in gross momentum exposure. Now suppose the market returns 20%. The portfolio’s momentum returns in this scenario are zero. However, the aggregate momentum exposure increases by 20% to \$240. Market return shocks, or other orthogonal shocks, would therefore directly affect momentum exposure without active intervention by investors.

Table 1.4 empirically documents the relationship between return factors and momentum asset growth. The extremely high t-stats on the market factor illustrate the mechanical relationship discussed. While market returns are the principal driver of momentum asset growth, the level of momentum exposure should correlate with momentum returns.

While the primary specification used in most parts of this paper estimates momentum growth from the absolute values of exposures in Equation 1.19, for this part of the study the alternative specification of Equation 1.21 provides more useful insights. Specifically confining the analysis to only funds with long momentum exposure should lead to a positive relationship between momentum assets and momentum growth.

The first three specifications of Table 1.4 show this to be the case. Even controlling for the market, the relationship between momentum asset growth and momen-

Table 1.4
Contemporaneous momentum asset growth on return factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$MktRF_t$	1.08	0.56	1.20	1.05	1.09	0.99	1.00
	(38.86)	*(6.69)*	*(19.28)*	*(38.54)*	*(39.46)*	*(35.88)*	*(35.51)*
MOM_t	0.20					0.15	
	(4.55)					*(5.33)*	
MOM_t^L		0.46					
		(6.08)					
MOM_t^S			0.12				
			(2.49)				
UMD_t					0.18		0.15
					(4.45)		*(4.86)*
SMB_t						0.21	0.21
						(4.50)	*(4.50)*
HML_t						−0.22	−0.22
						(−5.32)	*(−5.43)*
Months	696	696	696	696	696	696	696
R^2	74.28	75.30	72.83	72.44	74.30	76.93	76.98

Each column corresponds to a contemporaneous regression of momentum asset growth on return factors. The MOM_t refers to the momentum returns corresponding to the measured strategy (similar to UMD), while the other factors correspond to the Fama-French 3-factor model plus momentum. MOM_t^L and MOM_t^S correspond to returns from the long and short legs of the momentum portfolio. T-stats, shown in parenthesis, are calculated from a moving-block bootstrap with lags given by the $N^{\frac{1}{3}}$. In contrast with other tables, the exposure growth used on the left side of the regression corresponds to funds with long momentum exposure only.

tum returns is significant and positive. The lack of a 1:1 relationship might seem puzzling at first. This again is a mechanical effect with a real-world interpretation.

Consider a fund with $100 invested in a market basket of stocks, except that the fund manager "tilts" the portfolio weights to overweight high momentum stocks by $5 and underweight low momentum stocks by $5. This implies the fund has $10 of gross exposure to momentum. Now suppose momentum returns 20%, so the fund gains $2 in profits. If the fund manager rebalances the portfolio to the pre-realization proportions, momentum assets only increase by 20¢. The assumption of persistent exposure levels to momentum implies a rebalancing behavior on the part of the manager. The use of time-varying estimates of momentum exposure relaxes this assumption on an intermediate time horizon.

While the relationship between momentum returns and momentum assets is stronger when examining the long positions, the short portfolio is also significant. Specification (6) shows inclusion of additional return factor controls does not reduce the significance of the overall relationship. Finally, an alternative momentum proxy UMD also retains a highly significant correlation with momentum asset growth. This is expected given that the construction procedures for MOM and UMD are very similar (see Section 1.3.2), and the two series realize a > 0.95 correlation.

1.5.3 Relative investment in momentum

The nearly one-for-one relationship between market returns and momentum assets suggests using their ratio as a normalized measure of the level of market momentum exposure. While Figure 1.1 conveys an increase in momentum assets relative to the market, Figure 1.2 shows this growth directly.

Figure 1.2
Momentum assets as a percentage of market capitalization
The dark line shows the level of momentum assets as a percent of the market capitalization of the sample. The red line shows the publication date of Jegadeesh and Titman (1993), an early seminal paper on momentum.

For the first part of the sample, the fraction of momentum assets is variable but persistent.[4] Several years before the publication of Jegadeesh and Titman (1993), the fraction of market investment in momentum started to increase rapidly, with a notably high rate of increase at around the time of publication. The rate of increase then tapered off, with an overall decline since the financial crisis. The results are consistent with discourse around momentum in the late 1980s and early 1990s contributing to the rise in momentum assets.

A negative relationship between returns and momentum assets would be consistent with and support the findings of Mclean and Pontiff (2016) that returns decline around publication dates. The analysis in the subsequent sections analyze the relationship between assets and returns, with findings that broadly support a negative relationship between aggregate momentum exposure and expected future returns.

Finally, Table 1.5 provides summary statistics for the estimated aggregate exposures and trading volumes. The zero-cost momentum strategy considered delivers excess returns of about 7% per year. Aggregate strategy exposure averages to about 5% of sample capitalization over the period. The implied trading volume V^{RF} corresponds to allocative and rebalancing volume as estimated via the measure, as discussed in Sections 1.2.2 and 1.4, with the total volume provided as a reference. The active nature of the momentum strategy and the quantity of assets imply substantial trading volume. While not further studied in this paper, analysis of the trading

[4] A possible exception is a small run-up in the late 1960s potentially corresponding to the publication of Levy (1967), an early paper with a momentum-like metric, though the exposure levels seem to have reversed in subsequent years. The Levy (1967) strategy with 57 Web of Science citations as of September 2021 didn't realize the same level of popularity as the later Jegadeesh and Titman (1993) paper which received 3522 Web of Science citations. It seems reasonable to speculate the difference in impact between the two papers extends to practitioners.

volumes associated with momentum and other characteristics discussed later could lead to greater understanding of the motivations behind trading patterns observed in the market.

Table 1.5
Momentum strategy summary statistics

	T	$\mu(r)$	$\sigma(r)$	$\mu(A)$	$\mu(A/M)$	$\sigma(A/M)$	$\mu(V^{RF})$	$\mu(V^{M})$
MOM (full)	697	0.57	3.85	648.6	4.93	3.48	346.5	1134.5
MOM (roll)	679	0.56	3.89	605.1	4.67	3.23	338.6	1164.6

All data are from CRSP, using the selection criteria described in Section 1.3.1. Monthly return r and the ratio of aggregate strategy exposure to market capitalization A/M are multiplied by 100. Volume and aggregate strategy exposure are shown in units of billions of dollars. Averages and standard deviations are taken over the aggregated time-series. Aggregate momentum exposure A is estimated as described in Section 1.2. V^{RF} corresponds to the volume from the first term of Equation 1.22 aggregated for each month. V^{M} is the sum of total volume across the sample for each month.

1.6 Momentum exposure and returns

This section examines the fundamental research question as it relates to the momentum strategy, specifically, how does aggregate investment in the momentum strategy relate to long-run future returns? If such an effect exists, it would support models where investors exert a limited influence on prices and, in the presence of rising investment levels, provide a mechanism behind the observed characteristic return decay of Chordia, Subrahmanyam, and Tong (2014) and Mclean and Pontiff (2016).

As with any series of quantities in the absence of prices, the measure of gross strategy exposure alone cannot disentangle quantity changes driven by increased demand from those driven by increased supply. However, the different mechanisms have different implications with respect to future returns.

Demand based explanations would correspond to theories where arbitrageurs with limited resources profit from mispricings, as in Shleifer and Vishny (1997). A finding that returns decline with increases in exposure would support this mechanism, in that an increase in arbitrageurs willingness to absorb exposure would manifest as acceptance of lower returns from arbitrage activities.

A supply based explanation would differ in its implications with respect to future returns. Behavioral models of the types postulate mechanisms for irrational behavior to generate the momentum anomaly. Daniel, Hirshleifer, and Subrahmanyam (1998) shows how momentum can result from overconfidence, while Hong and Stein (1999) demonstrates how momentum can result from underreaction to news events. Regardless of the source, greater investor irrationality would imply more opportunity for investors to profit from the mispricings, leading to higher future returns that could in turn attract greater investment in the momentum strategy.

1.6.1 Empirical setup and null hypothesis

The baseline return specifications use two related but distinct proxies for investment. The first version is the previously discussed ratio of momentum assets to market assets. The return regressions are therefore

$$r_{t:(t+s)}^{MOM} = \beta \frac{\text{Mom. Assets}_{t-1}}{\text{Mkt. Assets}_{t-1}} + \alpha + \varepsilon_t \tag{1.24}$$

where β is the percentage point change in log returns for a given percentage point increase in momentum assets relative to the market.

An additional specification is similar to the baseline but uses the absolute sum of the market capitalization of stocks in the short and long leg of the strategy as the denominator.

$$r^{MOM}_{t:(t+s)} = \beta \frac{\text{Mom. Assets}_{t-1}}{Mkt_{t-1}\ (\text{High mom}) + Mkt_{t-1}\ (\text{Low mom})} + \alpha + \varepsilon_t \tag{1.25}$$

This version accounts for differences in the relative size of the active portion of the portfolios over time. It has the advantage of potentially better removing some of the variation in exposure created by orthogonal increases in market capitalization of constituents than the first specification. A drawback of this version is a less parsimonious interpretation of the regression coefficient.

As illustrated by Figure 1.2, the level of exposure to momentum is a persistent time series, and therefore improper accounting for serial correlation could lead to erroneously significant results. A more pernicious issue arises with the contemporaneous cross-correlations between shocks to returns and shocks to momentum assets. Table 1.4 shows the contemporaneous relationship is substantial and significant.

A two-pronged approach addresses these issues. First, in contrast with the measure used in Figure 1.4, both long and short exposure of mutual funds contribute to the estimation of aggregate momentum growth. Table 1.2 shows that the net exposure is substantially lower than gross exposure. This reduces the contemporaneous effect of momentum returns on aggregate exposure. Second, Stambaugh (1999) provides a convenient framework for handling residual autocorrelation and cross-correlations in conjunction with the bootstrapping procedure of Kothari and

Shanken (1997). The below framework provides the process for simulating returns under the null hypothesis.

$$R_t = \gamma^R + \varepsilon_t^R \tag{1.26}$$

$$M_t^{tot} = \phi^M M_{t-1}^{tot} + \gamma^M + \varepsilon_t^M \tag{1.27}$$

$$\begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^M \end{bmatrix} \sim MV\left(\mathbf{0}, \begin{bmatrix} \sigma_R^2 & \sigma_{RM} \\ \sigma_{RM} & \sigma_M^2 \end{bmatrix}\right)$$

Here M is the ratio of the measure described in Equation 1.24, R is the single month time t return, ε is the return shock to be simulated, and all other variables are parameters estimated from the data. Note that Equation 1.26 represents the null hypothesis of no return predictability. While this null hypothesis corresponds to Equation 1.24, analogous equations apply to other specifications. Estimate the standard errors with the following procedure:

1. Estimate the return predictability regressions of Equation 1.24 and 1.25
2. Estimate the parameters of the null hypothesis
3. Simulate the null hypothesis $10,000$ times, starting with the first point of the data
4. Compute the rolling return windows, and run the predictability regressions on the simulation results
5. Compute the standard errors of the results under the null and the implied t-statistics

Section 1.B in the appendix contains the Newey-West standard errors for the subsequent regression specifications. In almost all cases, using classical standard errors leads to much higher t-stats and corresponding significance relative to the bootstrapping approach.

1.6.2 Long-run momentum results

Consider a rolling time series of 72-month momentum return. Plotting this series against the negated measure provides a qualitative indication of the degree to which changes in the ratio of momentum assets to market assets varies with returns. Figure 1.3 shows the results. Movement in the negative measure of momentum exposure tracks the rolling 72-month returns, suggesting a negative relationship between aggregate market momentum exposure and returns.

Table 1.6a presents the results of the return predictability regressions as described with Equations 1.24 and 1.25. The results are economically meaningful. For every 1 percentage point of additional momentum assets, annual momentum returns decline by approximately 1.1%. While the t-stats are low for the measure when normalized by the market, significance rises to about 95% when normalizing by the capitalization of the long and short legs. The low t-stats of Specifications 1, 3, and 5 is a product of the strong persistence of $\frac{\text{Mom. Assets}_{t-1}}{\text{Mkt. Assets}_{t-1}}$ which effectively reduces the size of the data set.

The rolling beta specifications add valuable variation by allowing the mutual fund exposures to vary over time. The measure series may better reflect the ongoing exposure of the individual funds despite the reduced number of data points used for each point-in-time estimate of fund exposure.

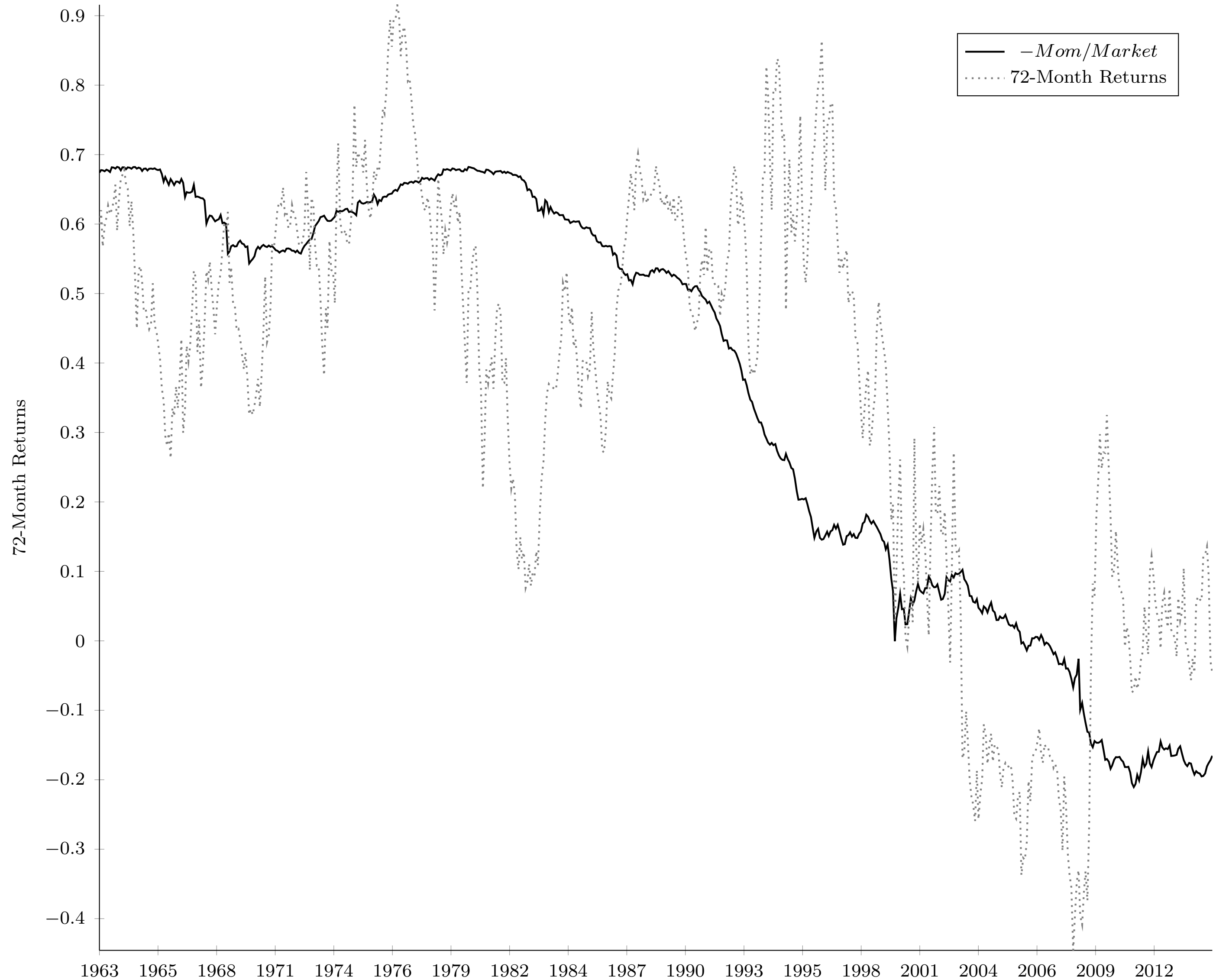

Figure 1.3
Returns and negative momentum exposure
The dark line shows the negative ratio of momentum assets to market, lagged, scaled, and centered to the same mean and standard deviation of the rolling cumulative 72-month returns. The movement of the dark line should be interpreted with respect to variation of the returns.

Table 1.6
Long-run return predictability regressions

(a) *Full sample*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M^{tot}_{t-1}	-3.33		-6.58		-9.22	
	(-1.21)		*(-1.15)*		*(-1.18)*	
M^{L+S}_{t-1}		-1.62		-3.12		-4.48
		(-1.87)		*(-1.97)*		*(-2.23)*
Months	661	661	625	625	601	601
R^2	29.67	28.71	47.78	43.41	55.54	52.37

Each column corresponds to a regression of future momentum returns on momentum assets. The interpretation is as follows: a 1 percentage point increase in momentum assets as a percentage of market capitalization corresponds to a -3.3 percentage point decline in three-year log momentum returns. The top row corresponds to the number of months in the return window. M^{tot} is momentum assets in dollars normalized by market capitalization, M^{L+S} is momentum assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	−3.25		−5.71		−9.16	
	(−2.08)		*(−2.11)*		*(−2.75)*	
M_{t-1}^{L+S}		−1.68		−2.81		−4.61
		(−2.19)		*(−2.20)*		*(−2.98)*
Months	643	643	607	607	583	583
R^2	22.13	22.96	27.95	26.03	34.44	33.25

This is the same as the prior table but with momentum exposure growth computed using 18-month rolling betas.

The aggregate effect of these differences improves the power of the estimate, with higher t-stats and comparable coefficient magnitudes. These are shown in Table 1.6b. The improved power extends to both versions of momentum exposure and all time horizons. The results are even more notable, given that the 18-month rolling results computes the exposure growth with reduced look-ahead bias as described in Section 1.3.4.

The gradual decay in returns is consistent with lines of research exploring slow-moving capital (Duffie 2010). It is likewise consistent with a limited ability for investors to exploit anomalies and correct prices (De Long et al. 1990; Shleifer and Vishny 1997; Pontiff 2006). At a more granular level, high momentum exposure relative to the market may increase transaction costs, reducing the ability of investors to profit from observed indications of the momentum characteristic (Korajczyk and Sadka 2004). On the other hand, the results are not consistent with a supply based explanation. An increase in investment due to more attractive opportunities generated by the irrational behavior described in Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999) would imply positive future returns, as opposed to the observed negative returns.

1.6.3 Short-run test results

Any short-run effect is interesting in its own right as a quantification of the one month effect of acquiring momentum exposure. At the same time, the evidence for a long-run effect suggests that a pure regression in differences is likely to be mis-specified. Running the short-run regressions in logs further provides a growth interpretation to the coefficients.

$$R_t^{MOM} = \beta_G GM_{t-1}^{TOT} + \beta_M M_{t-1}^{TOT} + \varepsilon_t \tag{1.28}$$

$$\log R_t^{MOM} = \beta_G \log G_{t-1}^{MOM} + \beta_M \log M_{t-1}^{TOT} + \varepsilon_t \tag{1.29}$$

Table 1.7
Short-run return predictability regressions

(a) *Full sample*

	1-month log R_t				1-month log R_t ($\times 100$)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	-0.47	-0.40					
	(−0.94)	*(−0.79)*					
M_{t-1}^{tot}		-0.10					
		(−1.34)					
GM_{t-1}^{L+S}			-0.25	-0.21			
			(−0.95)	*(−0.79)*			
M_{t-1}^{L+S}				-0.05			
				(−1.70)			
$\log G_{t-1}^{MOM}$					-4.79	-4.87	-4.83
					(−1.68)	*(−1.68)*	*(−1.66)*
$\log M_{t-1}^{tot}$						-0.41	
						(−1.54)	
$\log M_{t-1}^{L+S}$							-0.35
							(−1.54)
Months	695	695	695	695	695	695	695
R^2	0.12	0.87	0.13	0.75	0.40	1.13	0.98

For Specifications 1-4, the interpretation is as follows: a 1 percentage point increase in momentum assets as a percentage of market capitalization corresponds to a 10bp decline in one-month returns. For the log regressions, a 10% increase in momentum assets corresponds to a 4bp decline in returns. M^{tot} is momentum assets normalized by market capitalization, M^{L+S} is momentum assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	1-month log R_t				1-month log R_t (×100)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM^{TOT}_{t-1}	−0.31	−0.26					
	(−1.54)	*(−1.24)*					
M^{tot}_{t-1}		−0.09					
		(−1.72)					
GM^{L+S}_{t-1}			−0.15	−0.13			
			(−1.51)	*(−1.24)*			
M^{L+S}_{t-1}				−0.04			
				(−1.59)			
$\log G^{MOM}_{t-1}$					−2.08	−1.89	−1.92
					(−1.74)	*(−1.54)*	*(−1.56)*
$\log M^{tot}_{t-1}$						−0.41	
						(−1.63)	
$\log M^{L+S}_{t-1}$							−0.34
							(−1.45)
Months	677	677	677	677	677	677	677
R^2	0.34	0.86	0.33	0.77	0.44	0.96	0.84

This is the same as Table 1.7a but with momentum exposure growth computed using 18-month rolling betas.

Table 1.7a presents the results of these regressions. Specifications 1-4 show that, as with the longer-run results, the magnitude of the coefficients on the level data indicate a substantial effect. For the short-run growth component, a 1 percentage point increase in momentum assets as a percent of the market corresponds with an immediate 0.4 percentage point decline in log returns. For the longer run level component, a 1 percentage point increase in momentum assets correspond to an approximate 1 percentage point decline in returns over a year. The significance of Specifications 1-3 is low, though Specifications 5-7 and Table 1.7b realize higher t-statistics.

Specifications 5-7 provide a log-interpretation of the results. A 10% increase in momentum assets correlates with a 0.5% decline in log returns. A sustained 10% increase in momentum assets as a percentage of the market corresponds with about a 0.5 percentage point decline in returns over a year.

Using the variation provided with time-varying fund exposures improves the power of the regressions, as shown in Table 1.7b. These results generally mirror the results of Table 1.7a but with a moderate increase statistical power, particularly with the unlogged results.

The results of the short-run regressions generally complement the long-run results. The growth results suggest a negative short-run market impact following an increase in investor exposure. Ostensibly this is a counterintuitive result, given that buying a "high" momentum stock should correlate with a positive return. However, such a positive effect would happen almost contemporaneously with the purchase, while these regressions quantify the returns from the subsequent month. At this time, higher valuations for "high" momentum stocks, and vice versa for low momentum stocks, may play a larger role in the future returns.

Barber and Odean (2013) found a positive relationship between buying behavior over one week and subsequent one-week returns but a negative relationship between buying behavior over a year and the subsequent year's returns. The results in this paper are consistent with the longer-run results of Barber and Odean (2013). With higher frequency mutual fund data, the procedure in this paper could be used to likewise study the shorter-run effects.

1.6.4 Return chasing

The measure of gross momentum exposure serves as a vehicle for studying a variety of additional flow-performance relationships. For instance, the relationship between past returns and investment may provide researchers with insight into investors' behavior.

Specifically, high momentum returns may predict growth in momentum strategy assets, as individuals predict that high strategy returns will persist. Barber, Odean, and Zhu (2009) and Barber and Odean (2013), among others, presented evidence for this at the stock level. At the same time, individual retail investors tend to sell winning stocks in their portfolio while holding on to stocks with poor returns (Odean 1998), a manifestation of the disposition effect of Shefrin and Statman (1985). Barber, Odean, and Zhu (2009) found evidence for both of these effects, including a tendency for investors to chase returns in addition to selling recent winners. If these conclusions carry over to trading strategies, the direction of the relationship between flows and past returns will depend on whether return-chasing effects overcome the disposition effect. At the mutual fund level, Berk and Binsbergen (2015) presents evidence for inflows to mutual funds following periods of high returns in support of the equilibrium model of Berk and Green (2004).

The one-for-one movement of aggregate momentum exposure with the market described in Section 1.5 presents a challenge with respect to disentangling investor behavior. High momentum returns seem an unlikely signal for market returns, and as such the portion of exposure growth that corresponds to the market return. The market return portion of the exposure growth is a result of the mechanical relationship described in Section 1.5.2. The procedure described accounts for this relationship by netting out market returns from exposure growth.

As an initial qualitative test, Figure 1.4 graphs the rolling 1-year trailing returns along with 1-year strategy exposure growth net of the market. The graph seems to indicate a positive relationship between lagged returns and future exposure growth, a result indicative of return chasing.

To study the relationship more formally, consider the following regression:

$$\sum_{s\in 1:12} \log G^{MOM}_{t+s-1} - \sum_{s\in 1:12} \log R^{m}_{t+s-1} = \beta \sum_{s\in 1:12} \log R^{MOM}_{t-s} + \alpha + \varepsilon_t \tag{1.30}$$

where G^{MOM} is the month-over-month growth of aggregate momentum exposure, R^{MOM} is momentum returns, and R^m is the market return. Equation 1.30 regresses the growth of momentum exposure net of the market return on past returns. Other windows of returns and growth are analogous.

Table 1.8 presents the results. The full sample results of Table 1.8a show clear evidence of return chasing. The effect is small, in that a 10% momentum return over the past three years corresponds with a 1.47% increase in aggregate momentum

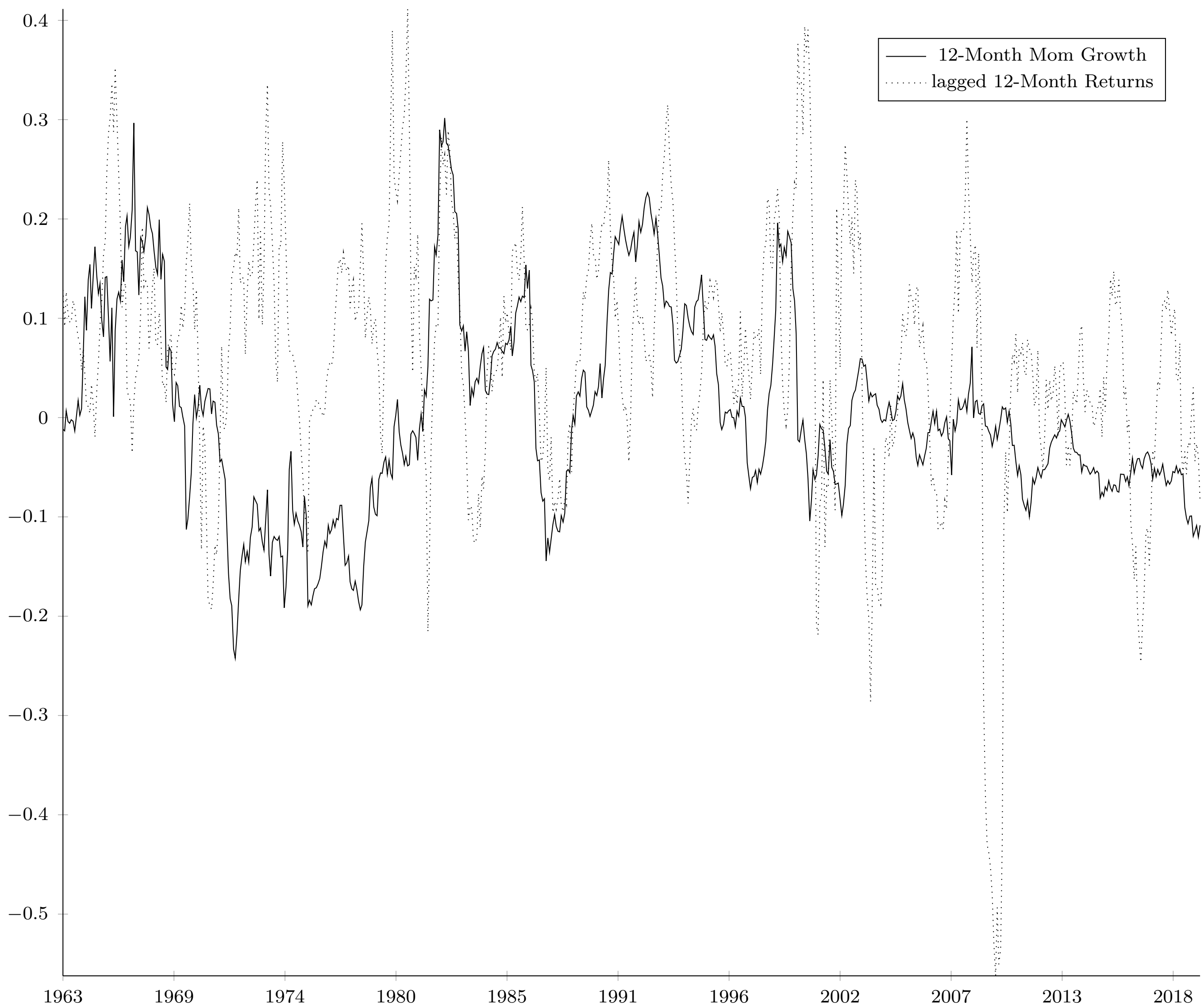

Figure 1.4

Trailing 1-year momentum returns and forward 1-year gross-exposure growth

This figure graphs lagged 12-month returns on the same scale as 12-month aggregate momentum exposure growth. Exposure growth is net of market returns.

Table 1.8
Fund exposure growth on lagged momentum returns

(a) *Full sample*

	1-month		12-month		36-month	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log R^{MOM}(12mth)$	0.33		8.39		33.73	
	(0.48)		*(1.26)*		*(2.52)*	
$\log R^{MOM}(36mth)$		1.02		14.73		39.33
		(2.42)		*(3.11)*		*(3.22)*
Months	685	661	674	650	650	626
R^2	0.03	0.72	1.25	9.10	3.34	10.64

This table presents regressions of log gross momentum exposure growth on past log momentum returns. The log momentum exposure growth is net of the value-weighted market returns of the stock universe. All coefficients are multiplied by 100. Interpret the coefficients to mean that a 10 percentage point three-year momentum return corresponds to a 1.47% increase in aggregate momentum exposure over the following year. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	1-month		12-month		36-month	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log R^{MOM}(12mth)$	5.20		−12.10		40.61	
	(1.61)		*(−0.38)*		*(0.64)*	
$\log R^{MOM}(36mth)$		2.52		13.91		−11.68
		(1.25)		*(0.61)*		*(−0.20)*
Months	667	643	656	632	632	608
R^2	0.36	0.20	0.17	0.52	1.24	0.24

This is the same as Table 1.8b but with momentum exposure growth computed using 18-month rolling betas.

exposure over the following 12 months. The link is stronger when allowing for a longer window of past returns. The magnitude of the relationship also increases with the window of time after the return realization. These results are generally consistent with the results of Barber, Odean, and Zhu (2009), and show that the return chasing effect seems to overwhelm the disposition effect.

In contrast, allowing for rolling mutual-fund exposures in Table 1.8b eliminates the return-chasing effect. One possible explanation is fund managers tilt exposure away from momentum after capturing momentum returns. This would suggest a fund manager-level disposition effect and would be consistent with the evidence presented in Jin and Scherbina (2011) and Cici (2012). Compositional effects present an alternative explanation. For instance, mutual fund investors could chase returns for funds with high momentum exposure but fail to rebalance their portfolios towards high momentum funds in the subsequent periods. Further analysis is left for future study.

1.7 Other strategies

The procedure for estimating strategy assets is general enough to apply to a wide variety of investment approaches. The subsequent analysis considers strategies measured on a monthly time horizon in CRSP, although the procedure could be rolled up to quarterly or annual strategies using accounting data. If a different method was used to capture strategy growth that did not rely on monthly mutual fund AUM, the procedure could estimate the aggregate exposure of shorter-term term strategies using daily or even intra-day data.

The relationship between the aggregate levels of investor exposure to a strategy and that strategy's future returns may depend on the underlying characteristic and/or the portfolio formation process. In addition, the sensitivity of future returns to investment may depend on the quantity of traders willing to take opposite positions (Kyle 1985). Suppose the demand effect described at the beginning of Section 1.6 is dominant over the supply effect. Then in the presence of positive excess strategy returns, the coefficient of the regression of future strategy returns on exposure should be negative. Otherwise, investors in the strategy would realize greater returns over long horizons at the cost of their trading partners as the strategy became more accessible. A dominant supply effect would lead to the opposite conclusion.

Consider two alternative strategies of different investment horizons: long-run reversal and idiosyncratic volatility. For each strategy, estimate the strategy assets over the data sample, then study the relationship between the strategy and returns. The results generally show that increasing assets correlate with reduced future returns.

1.7.1 Idiosyncratic volatility

Define idiosyncratic volatility (IVOL) similar to how it is defined in Fama and French (2016). For each stock and each month end, regress the prior 60 days of daily excess returns on MKTRF, SMB, and HML from the Fama-French 3-factor model as described by Equation 1.31.

$$R_{nt} - R_t^f = \beta^{MKT}\left(R_t^{MKT} - R_t^f\right) + \beta^{SMB} R_t^{SMB} + \beta^{HML} R_t^{HML} + \varepsilon_{nt} \tag{1.31}$$

The standard deviation of the residuals form the raw IVOL characteristic. Similar to Fama and French (2016), sort stocks by market capitalization into 5 quintiles. Within each quintile, designate the top 20% by idiosyncratic volatility as high IVOL, and the bottom 20% as low IVOL. This implies 5 high IVOL and 5 low IVOL sub-portfolios The size sorts add weight to small cap stocks despite the more limited sample, an important component of the characteristic portfolios (Ang et al. 2006; Fama and French 2016). Value weight stocks within each of the 10 focal sub-portfolios. The portfolio weights are then given by one fifth of the within-portfolio weights for the five low IVOL portfolios less one fifth of the within portfolio weights of the five high IVOL portfolios. Note the sorts differ from Fama and French (2016) in that the quintiles are defined using the entire portfolio as opposed to just NYSE stocks.

To construct mutual fund exposure, follow the steps of Sections 1.3.4 and 1.4. The controls for Equations 1.17 and 1.18 in this case consist of the Fama-French three-factor model plus momentum. The inclusion of MKTRF, SMB, UMD, and HML as controls is not redundant when accounting for mutual fund exposure to IVOL and may improve the quality of the exposure estimates. For instance, Fama and French (2016) found that RMW and CMA are far better at explaining the idiosyncratic volatility anomaly than any of the factors in the three-factor model. They further identified momentum as having additional explanatory power over RMW and CMA. Novy-Marx (2014) likewise finds gross profitability has high explanatory power with respect to the returns of defensive stocks, which often have low volatility. Novy-Marx (2013) finds that momentum is not redundant when used in a model gross profitability.

1.7.1.1 Investor exposure to idiosyncratic volatility

Figure 1.5 shows investment in idiosyncratic volatility over time. Exposure to IVOL remains relatively flat for the first 20 years of the sample, then increases until the financial crisis, after which exposure seems to decline. In contrast with momentum, the focal publication date for idiosyncratic volatility arrives after investors already accumulated significant exposure. A portion of the leading increase could be explained by the dissemination of the working paper version (Ang et al. 2004), though significant increases seem to precede even this earlier date. The results suggest that investors representing a meaningful quantity of assets already had knowledge of the strategy, or a strategy with a similar trading profile, prior to the publication of Ang et al. (2006).

Table 1.9 shows the strategy's summary statistics. Idiosyncratic volatility delivers slightly negative returns with high return volatility. Trading volume is higher than for the other strategies, likely a combination of the moderately higher assets under management and low signal persistence relative to the one-year and 5-year return lookbacks of the previously considered strategies.

Figure 1.5
Idiosyncratic volatility exposure as a percentage of market capitalization
The dark line shows the level of IVOL assets as a percent of the market capitalization of the sample. The red line shows the publication date of Ang et al. (2006), an early paper that focuses on the idiosyncratic volatility anomaly.

Table 1.9
Idiosyncratic volatility strategy summary statistics

	T	$\mu(r)$	$\sigma(r)$	$\mu(A)$	$\mu(A/M)$	$\sigma(A/M)$	$\mu(V^{RF})$	$\mu(V^{M})$
$IVOL$ (full)	697	-0.07	5.78	725.5	5.26	3.95	514.3	1173.2
$IVOL$ (roll)	679	-0.10	5.84	712.1	5.68	3.99	523.4	1204.2

This table presents summary statistics pertaining to the idiosyncratic volatility strategy. The quantities presented are calculated in the same manner as those described in Table 1.5.

1.7.1.2 Idiosyncratic volatility and returns

As with momentum, the results generally indicate a negative relationship between the aggregate exposure of investors to idiosyncratic volatility and the strategy's subsequent returns. The strength of the relationship is lower than momentum but greater than reversal, as will be discussed in the subsequent section.

Figure 1.6 graphically shows the relationship. Despite the higher variability of returns, the exposure of investors with momentum seems to correlate with a decline in momentum returns. The divergence at the time of the financial crisis represents an intriguing caveat to the broader trend and could reflect a re-pricing of volatility. Additional data could reveal whether this effect is transient or permanent.

The full-sample long-run regression results shown in table 1.10a quantify the effects observed in Figure 1.6. The coefficients indicate a negative relationship with a provocative 0.8% percentage point decline in annual returns corresponding with a 1 percentage point increase in aggregate gross exposure to IVOL as a percentage of market capitalization. The statistical significance of the effect is low in the full-

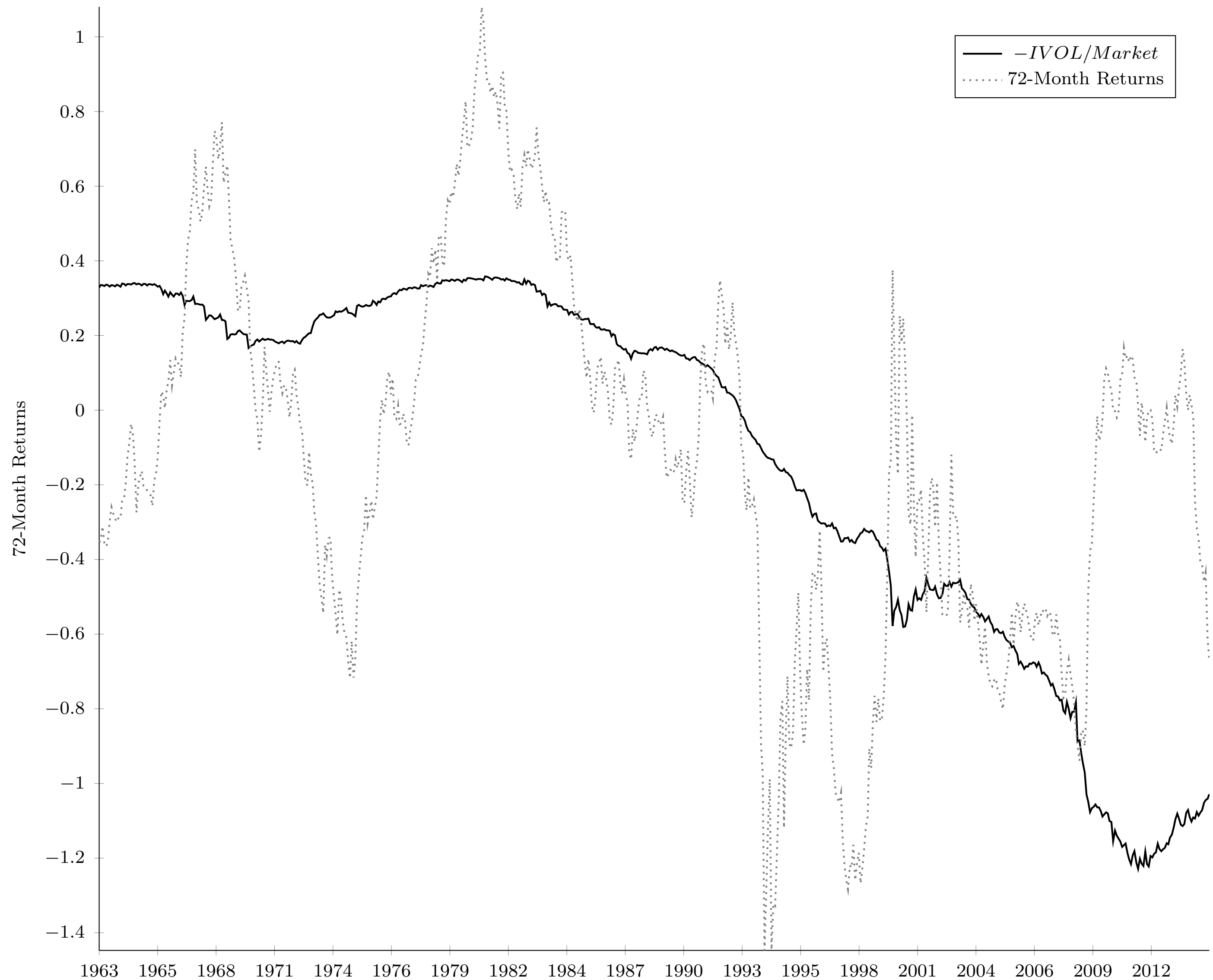

Figure 1.6
Returns and negative idiosyncratic volatility exposure
The dark line shows the negative ratio of idiosyncratic volatility to market, lagged, scaled, and centered to the same mean and standard deviation of the rolling cumulative 72-month log returns. The movement of the dark line should be interpreted with respect to its co-variation with the returns.

Table 1.10
Long-run return regressions on idiosyncratic volatility exposure

(a) *Full sample*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	−2.58		−5.92		−8.97	
	(−0.66)		*(−0.73)*		*(−0.80)*	
M_{t-1}^{L+S}		−1.22		−2.67		−3.99
		(−1.24)		*(−1.48)*		*(−1.75)*
Months	661	661	625	625	601	601
R^2	7.19	9.61	18.16	22.63	27.35	33.88

Each column corresponds to a regression on future IVOL returns. The interpretation is as follows: a 1 percentage point increase in IVOL assets as a percentage of market capitalization corresponds with a 2.6 percentage point decline in three-year IVOL returns. The top row corresponds to the number of months in the return window. M^{tot} is IVOL assets in dollars normalized by market capitalization, and M^{L+S} is IVOL assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	-2.86		-5.27		-7.87	
	(−1.34)		*(−1.41)*		*(−1.71)*	
M_{t-1}^{L+S}		-1.34		-2.43		-3.55
		(−1.71)		*(−1.83)*		*(−2.19)*
Months	643	643	607	607	583	583
R^2	9.16	12.41	17.11	22.59	25.33	32.72

This is the same as Table 1.10a but with IVOL exposure growth computed using 18-month rolling betas.

sample results despite the high economic magnitude. The alternate normalization specification M^{L+S} realizes significance at the 90% and 95% level when examining 72 month and 96 month forward returns.

The significance notably increases when allowing for time-varying exposures in Table 1.10b, particularly for the longer time horizons. The magnitudes of the coefficients remain approximately constant between the two estimation procedures.

The short-run results are largely consistent with the long-run results. From an economic standpoint, the coefficients of Tables 1.11a and 1.11b line up with those of Tables 1.10a and 1.10b. However, low statistical significance reduces the weight of this evidence.

Overall, the evidence for a negative relationship between investor IVOL exposure and IVOL returns is suggestive but not conclusive. The coefficient point estimates suggest a substantial impact from investment. Most of the specifications are sig-

Table 1.11
Short-run return regressions on idiosyncratic volatility exposure

(a) *Full sample*

	1-month log R_t				1-month log R_t $(\times 100)$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	-1.07	-0.99					
	(-1.48)	(-1.35)					
M_{t-1}^{tot}		-0.09					
		(-0.86)					
GM_{t-1}^{L+S}			-0.35	-0.32			
			(-1.19)	(-1.09)			
M_{t-1}^{L+S}				-0.04			
				(-1.18)			
$\log G_{t-1}^{IVOL}$					-5.68	-5.75	-5.73
					(-1.25)	(-1.24)	(-1.24)
$\log M_{t-1}^{tot}$						-0.47	
						(-1.07)	
$\log M_{t-1}^{L+S}$							-0.40
							(-1.14)
Months	695	695	695	695	695	695	695
R^2	0.31	0.63	0.20	0.51	0.22	0.60	0.56

For Specifications 1-4, the interpretation is as follows: a 1 percentage point increase in IVOL assets as a percentage of market capitalization corresponds with a 9bp decline in one-month IVOL returns. For the log regressions, a 10% increase in IVOL assets corresponds with a 5bp decline in returns. M^{tot} is IVOL assets normalized by market capitalization, and M^{L+S} is IVOL assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	1-month $\log R_t$				1-month $\log R_t$ $(\times 100)$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM^{TOT}_{t-1}	−0.03	0.00					
	(−0.11)	*(0.01)*					
M^{tot}_{t-1}		−0.05					
		(−0.76)					
GM^{L+S}_{t-1}			−0.02	−0.00			
			(−0.13)	*(−0.02)*			
M^{L+S}_{t-1}				−0.02			
				(−0.79)			
$\log G^{IVOL}_{t-1}$					−1.36	−1.21	−1.24
					(−0.70)	*(−0.62)*	*(−0.63)*
$\log M^{tot}_{t-1}$						−0.38	
						(−0.90)	
$\log M^{L+S}_{t-1}$							−0.32
							(−0.87)
Months	677	677	677	677	677	677	677
R^2	0.00	0.12	0.00	0.12	0.07	0.25	0.23

This is the same as Table 1.11a but with idiosyncratic volatility exposure growth computed using 18-month rolling betas.

nificant at time horizons greater than or equal to 72-months. The high economic magnitudes and indicative statistical significance together suggest that further study is warranted, perhaps with more data to achieve greater statistical power. Another potential avenue might be to account for the potential repricing of idiosyncratic volatility following the financial crisis.

1.7.2 Long-run reversal

Define the long-run reversal in a manner similar momentum, except sort stock returns based on their preceding 5-year return less the most recent year. Use this reversal characteristic to divide stocks into three portfolios. High reversal stocks have returns in the 30th percentile or lower, while low reversal stocks realize returns in the 70th percentile or higher. Separately and independently, sort stocks into large and small market capitalization based on the median size. The product of these two sorts forms six portfolios. Within each portfolio, value weight the constituents. The overall portfolio weight for stocks in the long leg is the average of the two low reversal portfolios, while the weight in the short leg is the average of the two high reversal portfolios.

With respect to estimating the aggregate investor holdings of the strategy, the procedure generally follows that described in Sections 1.3.4 and 1.4, with a couple of differences. Rather than controlling for the market and HML and SMB when running the regressions described by Equations 1.17 and 1.18 to compute mutual fund exposure, control only for the market. This accounts for the high redundancy between long-run reversal and the HML/SMB factors as described in Fama and French (1996).

1.7.2.1 Investor exposure to long-run reversal

Figure 1.7 shows the level of reversal assets as a ratio to the market capitalization of the sample. Investors' reversal exposure remains low with high persistence prior to about 1980. As with momentum, reversal assets begin to increase in the years before De Bondt and Thaler (1985), with substantial tapering after the financial crisis.

Table 1.12 provides summary statistics for the long-run reversal strategy. Compared with momentum, long-run reversal delivers lower returns and implies lower trading volumes over the sample period. Aggregate exposure as a percentage of the market is slightly lower than that of momentum.

Table 1.12
Reversal strategy summary statistics

	T	$\mu(r)$	$\sigma(r)$	$\mu(A)$	$\mu(A/M)$	$\sigma(A/M)$	$\mu(V^{RF})$	$\mu(V^{M})$
REV (full)	697	0.15	2.43	444.3	3.39	2.84	135.1	996.5
REV (roll)	679	0.15	2.45	504.8	4.19	3.35	179.4	1022.8

This table presents summary statistics pertaining to the reversal strategy. The quantities presented are the same as those of Table 1.5.

1.7.2.2 Reversal exposure and returns

The long-run reversal results generally mirror the momentum results, albeit with lower statistical power. Figure 1.8 shows that as with momentum, returns generally decline with investor reversal exposure. Exposure levels seem to capture substantially less of the variation compared with momentum, particularly in the first half of the sample. A relationship only seems to appear when reversal exposures rise

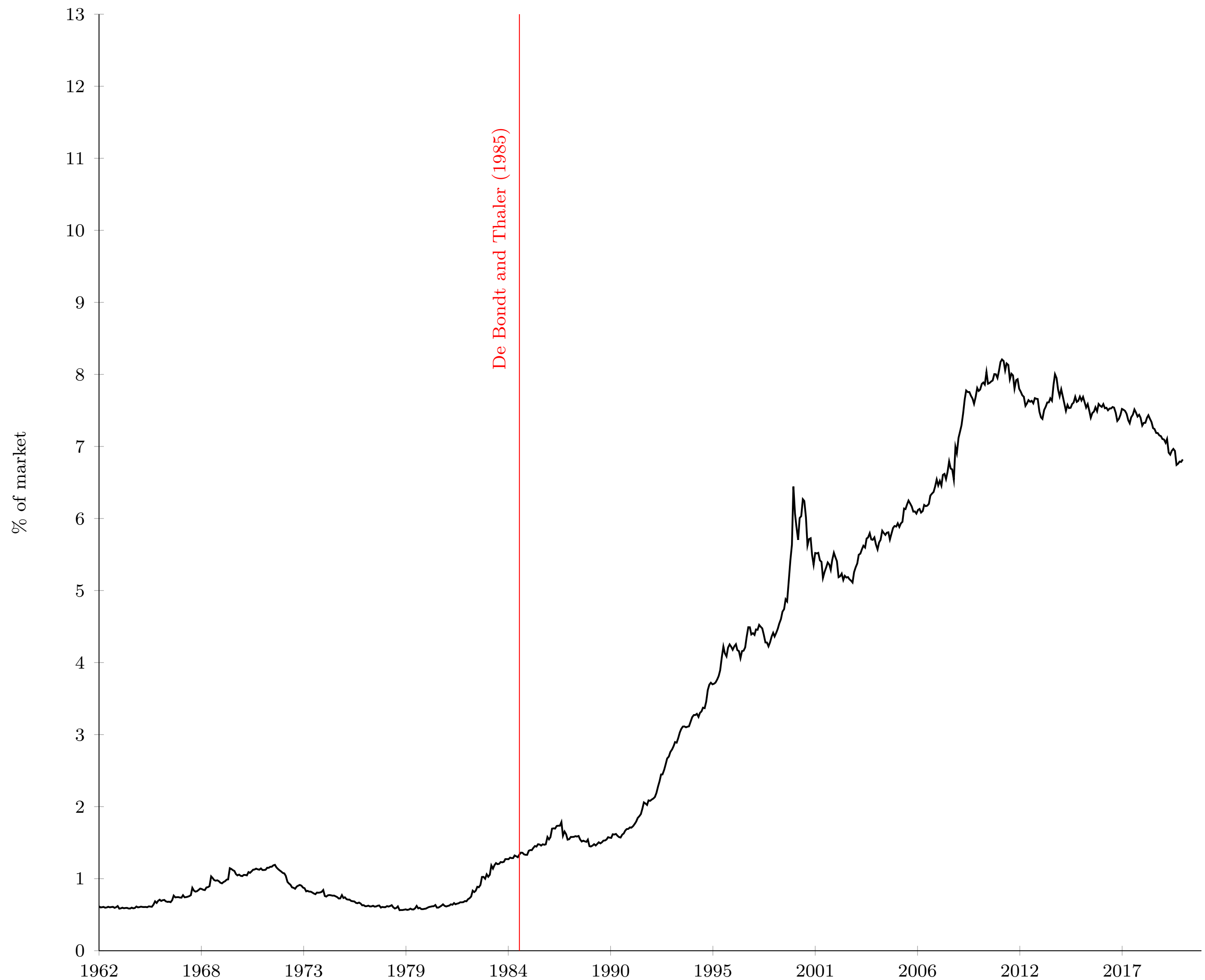

Figure 1.7
Reversal exposure as a percentage of market capitalization
The dark line shows the level of reversal assets as a percent of the market capitalization of the sample. The red line shows the publication date of De Bondt and Thaler (1985), an early seminal paper on reversal.

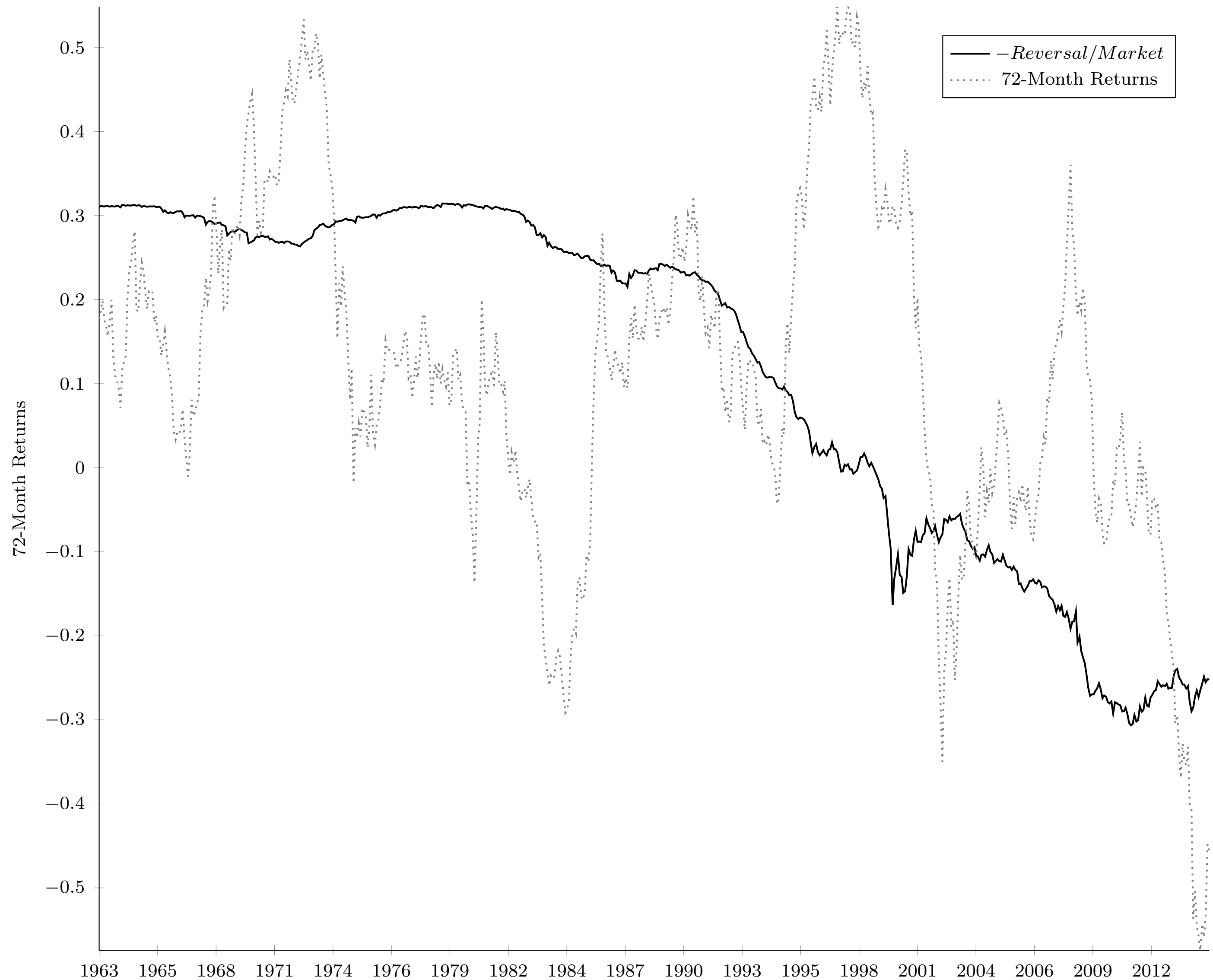

Figure 1.8
Returns and negative reversal exposure
The dark line shows the negative ratio of reversal assets to market, lagged, scaled, and centered to the same mean and standard deviation of the rolling cumulative 72-month returns. The movement of the dark line should be interpreted with respect to variation of the returns.

significantly.

Table 1.13
Long-run return regressions on reversal exposure

(a) *Full sample*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M^{tot}_{t-1}	−1.55		−2.59		−3.19	
	(−0.70)		*(−0.57)*		*(−0.51)*	
M^{L+S}_{t-1}		−1.16		−1.75		−1.96
		(−1.20)		*(−0.95)*		*(−0.82)*
Months	661	661	625	625	601	601
R^2	5.56	10.11	10.20	15.14	13.31	15.23

Each column corresponds to a regression on future reversal returns. The interpretation is as follows: a 1 percentage point increase in reversal assets as a percentage of market capitalization corresponds with a 1.5% percentage point decline in log three-year reversal returns. The top row corresponds to the number of months in the return window. M^{tot} is reversal assets in dollars normalized by market capitalization, while M^{L+S} is reversal assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

Table 1.13a quantifies the return predictability relationship. The coefficients are economically meaningful, with a 1% increase in reversal assets corresponding with a 0.5% decline in annual reversal returns. The lower frequency relationship translates to less statistical power in Table 1.13a, with consistent yet insignificant coefficients. Table 1.13b shows the 18-month rolling regressions on mutual fund exposures moderately improve statistical power. The coefficients remain negative

(b) *18-month rolling*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	−0.64		−1.94		−2.36	
	(−0.70)		*(−1.25)*		*(−1.25)*	
M_{t-1}^{L+S}		−0.61		−1.33		−1.51
		(−1.25)		*(−1.65)*		*(−1.54)*
Months	643	643	607	607	583	583
R^2	1.43	4.36	8.44	13.42	9.80	12.04

This is the same as Table 1.13a but with reversal strategy growth computed using 18-month rolling betas.

and economically material, if somewhat attenuated relative to the full-sample results.

The results of the short run regressions shown in Table 1.14a continue to show low statistical significance despite meaningfully large coefficient estimates. The results indicate about a 0.5% annual decline in returns per 1% addition of reversal assets as a percentage of market capitalization. The 18-month rolling regressions show moderately increased significance, particularly when reversal assets are normalized by the sum market capitalization of the long and short legs of the portfolio. Finally, note that the coefficient on short term changes in reversal assets is positive. Thus, investors purchasing high reversal stocks and shorting low reversal stocks could lead to immediate short term gains in the subsequent month, although the effect again has low statistical significance.

Overall, evidence for a relationship between investor exposure to the long run reversal strategy and future returns is suggestive of a substantial economic effect

Table 1.14
Short-run return regressions on reversal exposure

(a) *Full sample*

	1-month log R_t				1-month log R_t ($\times 100$)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	-0.14	-0.09					
	(-0.33)	*(-0.22)*					
M_{t-1}^{tot}		-0.05					
		(-0.80)					
GM_{t-1}^{L+S}			-6.00	-2.12			
			(-0.24)	*(-0.08)*			
M_{t-1}^{L+S}				-0.03			
				(-1.20)			
$\log G_{t-1}^{REV}$					1.56	1.55	1.56
					(0.87)	*(0.84)*	*(0.86)*
$\log M_{t-1}^{tot}$						-0.12	
						(-0.87)	
$\log M_{t-1}^{L+S}$							-0.16
							(-1.16)
Months	695	695	695	695	695	695	695
R^2	0.01	0.31	0.01	0.49	0.11	0.35	0.54

For Specifications 1-4, the interpretation is as follows: a 1 percentage point increase in reversal assets as a percentage of market capitalization corresponds with a 5bp decline in one-month reversal returns. For the log regressions, a 10% increase in reversal assets corresponds with a 1bp decline in returns. M^{tot} is reversal assets normalized by market capitalization, and M^{L+S} is reversal assets normalized by the market value of the long and short legs. T-stats shown in parenthesis were calculated by simulating a multivariate AR(1) under the null hypothesis.

(b) *18-month rolling*

	1-month $\log R_t$				1-month $\log R_t$ $(\times 100)$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	0.09	0.10					
	(0.76)	*(0.87)*					
M_{t-1}^{tot}		−0.03					
		(−0.95)					
GM_{t-1}^{L+S}			4.23	5.65			
			(0.66)	*(0.84)*			
M_{t-1}^{L+S}				−0.02			
				(−1.36)			
$\log G_{t-1}^{REV}$					1.12	1.17	1.20
					(1.55)	*(1.57)*	*(1.61)*
$\log M_{t-1}^{tot}$						−0.13	
						(−0.89)	
$\log M_{t-1}^{L+S}$							−0.19
							(−1.37)
Months	677	677	677	677	677	677	677
R^2	0.08	0.25	0.06	0.40	0.35	0.52	0.74

This is the same as Table 1.14a but with reversal exposure growth computed using 18-month rolling betas.

but is not statistically significant. The specific characteristic of the strategy and the long holding period could insulate the strategy from transaction costs and the price impact of acquiring positions. Alternatively, the low variation relative to the momentum strategy could simply reduce the quality of the return predictability estimate, with future data improving certainty with respect to the effect.

1.8 Conclusions

I propose a new measure of aggregate investor exposure to systematic investment strategies. The measure is general enough to apply to most strategies defined as a set of formal investment rules. It works by estimating the amount of volume a strategy should imply, and exploiting both cross-sectional and time-series variation in the volume to identify the level of investor exposure to a characteristic. The measure successfully explains variation in volume across stocks, providing a degree of validation. Both the level and the growth of strategy assets provide incremental and statistically significant explanatory power.

Following the estimation procedure for the momentum characteristic leads to a time series of investor momentum exposure. Momentum assets greatly increase as a percent of the market starting in the mid-1980s, with the greatest prolonged rate of increase at about the time of the publication of Jegadeesh and Titman (1993). Overall, investor exposure rises from about 1% of market capitalization at the beginning of the sample, to over 10% of market capitalization in the mid-2000s, before tapering off somewhat in the past decade. Similar series of investor exposure built for idiosyncratic volatility and long-run reversal show similar trends in aggregate gross exposure.

I use the estimated time series of exposures to study the relationship between exposure and future returns. The results have implications with respect to the ability of investors to access systematic strategies and their return expectations. Regressions of momentum exposure on future returns indicate about a 1.1 percentage point decline in annual returns for each percentage point of additional momentum exposure as a percentage of the sample's market capitalization. The effect is statistically significant and present across a variety of specifications. It indicates that decay in returns coincides with increases in characteristic investment, providing strong circumstantial evidence of a "capacity effect." A real world implication is increased retail access to systematic strategies does not imply access to those strategies' past return distributions.

Extending the analysis to long-run reversal and idiosyncratic volatility shows that the negative relationship between exposure and future returns is not confined to momentum. Limited statistical significance means that the relationships outside of momentum should be treated as suggestive, particularly for long-run reversal. At the same time, the large economic magnitudes imply that further study could lead to important links between the prevalence of these strategies and future returns.

APPENDICES

1.A Analysis of convexity in the objective function

The problem described by Equation 1.13 presents challenges due to the absolute value function. Yet conditional on the growth of exposure G_t for all t, the problem collapses to ordinarily least squares. This property suggests a two step approach:

1. Estimate G_t for all t in a manner to be determined

2. Then use OLS to estimate an implied A_0 and any relevant affine controls

Using this procedure, the challenge of Equation 1.13 is confined to estimating the growth of strategy assets. Unfortunately, conditioning on A_0 to estimate G_t gives a non-linear and non-smooth problem that is not generally convex.

The lack of convexity arises due to properties of the absolute value function. To see this, consider the below simplified problem ($a \neq 0$):

$$x = argmin_x \left(|ax - b| - c\right)^2 \tag{1.32}$$

$$= argmin_x \left(ax - b\right)^2 + c^2 - 2c\left|ax - b\right| \tag{1.33}$$

If $c \leq 0$, any non-zero value of $ax - b$ will lead to an evaluation greater than c^2. Hence $x = \frac{b}{a}$ at the minimum, a unique solution. Unfortunately, the condition that $c \leq 0$ is unlikely to hold for all terms, as V_{it} may be more or less than λ.

If $c > 0$, the function is no longer convex, a property starkly illustrated by plugging in $a = 1$, $b = -3$, $c = 1$ and plotting as in Figure 1.9.

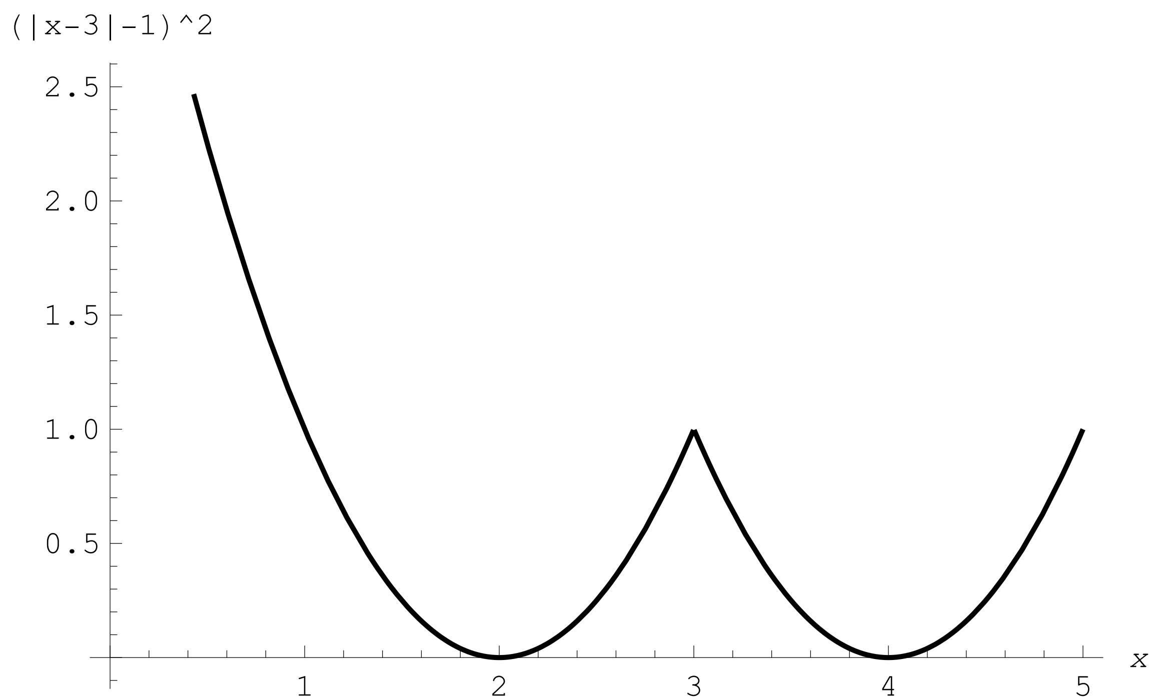

Figure 1.9
Non-convexity of the quadratic objective function

For a more formal analysis, note that if each term of the problem is shown to be convex, then the problem is convex. But if any terms are non-convex, the problem convexity is not guaranteed. Start by noting that each term of Equation 1.13 can be written in the form $f(x) = (|ax - b| - c)^2$. Then use the following proposition to frame the analysis:

Proposition 1. *A function of the form $f(x) = (|ax - b| - c)^2$ where $a \neq 0$ is convex if and only if $c \leq 0$.*

PROOF: Expanding the function gives

$$f(x) = (ax - b)^2 - 2c\,|ax - b| + c^2 \tag{1.34}$$

hence the "if" portion follows immediately from the sum of individually convex functions. For only if, consider that a convex function $f(\cdot)$ satisfies the following properties by definition:

$$\forall \lambda \in [0,1],\ x_1, x_2 \in \mathbb{R} \tag{1.35}$$

$$x_\lambda \equiv \lambda x_1 + (1 - \lambda)\, x_2 \tag{1.36}$$

$$f(x_\lambda) \leq \lambda f(x_1) + (1 - \lambda)\, f(x_2) \tag{1.37}$$

Let $x_\lambda^* = \frac{b}{a}$, $x_1^* = \frac{b-c}{a}$, and $x_2^* = \frac{b+c}{a}$. Then plugging in

$$f(x_\lambda^*) = c^2 \tag{1.38}$$

$$f(x_1^*) = f(x_2^*) = \begin{cases} 0 & \forall c > 0 \\ 4c^2 & \forall c \leq 0 \end{cases} \tag{1.39}$$

Thus, convexity fails to hold for all c>0.

Proposition 1 shows that each term of the problem in Equation 1.13 may or may not be convex depending on the precise weights, returns, and volume for a particular stock over a particular period. This means the optimization problem is non-linear, non-smooth, and not generally convex.

1.B Classical standard errors

High persistence in the explanatory variables along with potential contemporaneous cross-correlations motivate the parametric bootstrapping procedure described in Section 1.6.1. The procedure estimates standard errors and associated significance that are generally more conservative than those measured via classical techniques. To show this, the following tables contain t-stats from both the bootstrap standard errors and classical Newey-West standard errors. Following Andrews (1991) Equations 6.4 and 6.2 with a Bartlett kernel and assuming a true AR(1) persistence of 0.99 implies an optimal lag length of about $25T^{1/3}$. This is about 216 months or 18 years, roughly one-third of the sample. While quite long, this conservative lag length seems appropriate given the persistence shown in Figure 1.2.

Table 1.15 and Table 1.16 contain the Newey-West classical t-stats along with the bootstrap t-stats for comparison. For almost all of the estimates, the Newey-West t-stats substantially exceed the bootstrap t-stats. The only exceptions are with respect to the unlogged differenced variables in the short-run regressions. For these variables, the lag length is likely high given the expected lower persistence relative to the more interesting level variables, although the results are still shown for consistency.

Table 1.15
Classical standard errors for long-run return predictability regressions

(a) *Full sample*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	−3.33		−6.58		−9.22	
hac (t)	(*−5.84*)		(*−6.03*)		(*−6.54*)	
ar1 (t)	(*−1.21*)		(*−1.15*)		(*−1.18*)	
M_{t-1}^{L+S}		−1.62		−3.12		−4.48
hac (t)		(*−6.34*)		(*−5.90*)		(*−6.28*)
ar1 (t)		(*−1.87*)		(*−1.97*)		(*−2.23*)
Months	661	661	625	625	601	601
R^2	29.67	28.71	47.78	43.41	55.54	52.37

Each column corresponds to a regression of future momentum returns on momentum assets. It is the same as Table 1.6a but includes t-stats calculated using classical Newey-West standard errors with a lag length of $25T^{1/3}$. For reference, the table also contains the parametric AR(1) bootstrap t-stats from Table 1.6a.

(b) *18-month rolling*

	36-month		72-month		96-month	
	(1)	(2)	(3)	(4)	(5)	(6)
M_{t-1}^{tot}	−3.25		−5.71		−9.16	
hac (t)	*(−4.77)*		*(−4.12)*		*(−3.72)*	
ar1 (t)	*(−2.08)*		*(−2.11)*		*(−2.75)*	
M_{t-1}^{L+S}		−1.68		−2.81		−4.61
hac (t)		*(−5.33)*		*(−3.79)*		*(−3.45)*
ar1 (t)		*(−2.19)*		*(−2.20)*		*(−2.98)*
Months	643	643	607	607	583	583
R^2	22.13	22.96	27.95	26.03	34.44	33.25

This is the same as the prior table but with momentum asset growth computed using 18-month rolling betas.

Despite the high t-stats, the Newey-West significance should be interpreted with extreme caution for two reasons. For the full-sample results, the Andrews (1991) heuristic for determining the lags breaks down when plugging in the OLS estimate of persistence for the ratio of momentum exposure to market capitalization. The regression coefficient of 0.9998 is approximately a unit root. At the same time, the alternate normalizer of long plus short capitalization estimated with time-varying fund exposures leads to an AR(1) persistence of about 0.967. Other persistence estimates of measure levels range in between. None of these values seem particularly differentiated, yet each lead to wildly different lag estimates. Second, the HAC-corrected errors fail to account for contemporaneous cross-sectional correlations between returns. This may lead to the distortions in the estimates discussed in Stambaugh (1999).

Table 1.16
Classical standard errors for short-run return predictability regressions

(a) *Full sample*

	1-month $\log R_t$				1-month $\log R_t$ $(\times 100)$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	−0.47	−0.40					
hac (t)	*(−0.74)*	*(−0.62)*					
ar1 (t)	*(−0.94)*	*(−0.79)*					
M_{t-1}^{tot}		−0.10					
hac (t)		*(−5.12)*					
ar1 (t)		*(−1.34)*					
GM_{t-1}^{L+S}			−0.25	−0.21			
hac (t)			*(−0.76)*	*(−0.64)*			
ar1 (t)			*(−0.95)*	*(−0.79)*			
M_{t-1}^{L+S}				−0.05			
hac (t)				*(−4.10)*			
ar1 (t)				*(−1.70)*			
$\log G_{t-1}^{MOM}$					−4.79	−4.87	−4.83
hac (t)					*(−1.98)*	*(−2.08)*	*(−2.06)*
ar1 (t)					*(−1.68)*	*(−1.68)*	*(−1.66)*
$\log M_{t-1}^{tot}$						−0.41	
hac (t)						*(−4.36)*	
ar1 (t)						*(−1.54)*	
$\log M_{t-1}^{L+S}$							−0.35
hac (t)							*(−3.27)*
ar1 (t)							*(−1.54)*
Months	695	695	695	695	695	695	695
R^2	0.12	0.87	0.13	0.75	0.40	1.13	0.98

This table is the same as Table 1.7a but includes classical Newey-West standard errors with a lag length of $25T^{1/3}$. For reference, the table also contains the parametric AR(1) bootstrap errors from elsewhere in the paper.

(b) *18-month rolling*

	1-month log R_t				1-month log R_t ($\times 100$)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
GM_{t-1}^{TOT}	−0.31	−0.26					
hac (t)	(−*1.38*)	(−*1.14*)					
ar1 (t)	(−*1.54*)	(−*1.24*)					
M_{t-1}^{tot}		−0.09					
hac (t)		(−*6.33*)					
ar1 (t)		(−*1.72*)					
GM_{t-1}^{L+S}			−0.15	−0.13			
hac (t)			(−*1.43*)	(−*1.18*)			
ar1 (t)			(−*1.51*)	(−*1.24*)			
M_{t-1}^{L+S}				−0.04			
hac (t)				(−*5.38*)			
ar1 (t)				(−*1.59*)			
$\log G_{t-1}^{MOM}$					−2.08	−1.89	−1.92
hac (t)					(−*2.58*)	(−*2.29*)	(−*2.29*)
ar1 (t)					(−*1.74*)	(−*1.54*)	(−*1.56*)
$\log M_{t-1}^{tot}$						−0.41	
hac (t)						(−*3.88*)	
ar1 (t)						(−*1.63*)	
$\log M_{t-1}^{L+S}$							−0.34
hac (t)							(−*2.82*)
ar1 (t)							(−*1.45*)
Months	677	677	677	677	677	677	677
R^2	0.34	0.86	0.33	0.77	0.44	0.96	0.84

This is the same as Table 1.16a but with momentum asset growth computed using 18-month rolling betas.

1.C Relationship between aggregate mutual fund assets, strategy assets, and returns

Much of the variation captured by the change in momentum mutual fund exposure stems from broad investor allocations to the domestic equity mutual funds. Diversified allocations still increase momentum exposure, so such allocations are not in-of-themselves a problem. But the effect poses the question of whether the momentum exposure itself has incremental explanatory power relative to broad mutual fund inflows. This section analyzes this problem.

If momentum exposure as estimated using the measure described in Section 1.2 is additive relative to aggregate mutual fund inflows, then using momentum exposure instead of domestic equity assets should lead to more precise estimates. Confounding a parsimonious approach, the two statistics have very high multi-colinearity, reducing the power of direct analysis.

As a first pass, consider the univariate regressions of Table 1.17. The first two rows in both panels show the 36-month and 96-month results identical to Table 1.6. Increases in either momentum exposure and domestic equity mutual funds lead to declines in returns. For the full sample results, the significance of mutual fund inflows in predicting future momentum returns lies between the two versions of momentum exposures. In contrast, momentum exposures demonstrate substantially better efficacy when allowing for time-varying fund exposures to compute momentum exposure growth.

Simple multivariate regressions might seem like an obvious test. However, the high multicolinearity leads to nonsensical and insignificant estimates for the individ-

Table 1.17
Univariate return predictability regressions with mutual fund assets

(a) *Full sample*

	36-month			96-month		
	(1)	(2)	(3)	(4)	(5)	(6)
M^{tot}_{t-1}	−3.33			−9.22		
	(−1.21)			*(−1.18)*		
M^{L+S}_{t-1}		−1.62			−4.48	
		(−1.87)			*(−2.23)*	
MF^{tot}_{t-1}			−0.97			−2.96
			(−1.58)			*(−1.58)*
Months	661	661	661	601	601	601
R^2	29.67	28.71	31.83	55.54	52.37	59.96

Each column corresponds to a regression of future momentum returns on momentum assets. The top row corresponds to the number of months in the return window. M^{tot} is momentum assets in dollars normalized by market capitalization, and M^{L+S} is momentum assets normalized by the market value of the long and short legs. MF^{tot} is total assets invested in domestic equity mutual funds as a percentage of the market. T-stats shown were calculated by simulating a multivariate AR(1) under the null.

(b) *18-month rolling*

	36-month			96-month		
	(1)	(2)	(3)	(4)	(5)	(6)
M^{tot}_{t-1}	−3.25			−9.16		
	(−2.08)			*(−2.75)*		
M^{L+S}_{t-1}		−1.68			−4.61	
		(−2.19)			*(−2.98)*	
MF^{tot}_{t-1}			−0.96			−3.00
			(−1.51)			*(−1.57)*
Months	643	643	643	583	583	583
R^2	22.13	22.96	30.77	34.44	33.25	60.33

This is the same as the prior table but with momentum asset growth computed using 18-month rolling betas. While the use of rolling betas does not directly affect MF^{tot}, the coefficients differ slightly relative to Table 1.17 due to the loss of 18 months of data.

ual coefficients. The first three specifications in both panels for Table 1.17 show the results of these regressions. Coefficients are mostly insignificant, particularly in the full-sample results. The momentum exposure measure has the wrong sign in most specifications.

To account for the multicolinearity, Specifications 4-6 run the same regressions using only the first principal component. This is a particularly apt approach if both mutual fund assets and momentum exposures proxy for a single effect. Run in this manner, the momentum measure is significant in the full sample results with t-stats of about 1.8, despite attenuation in the magnitude of the coefficients. The measure realizes t-stats between 1.2 and 1.5 allowing for rolling betas. The coefficients for aggregate mutual fund assets correspond to t-stats of about 1.5, an improvement over the ordinary least squares regressions.

In summary, both coefficients seem to hold meaningful predictive power with respect to returns, albeit potentially proxying for the same underlying source of variation. The lack of differentiation in the relative efficacy of the two measures merits further investigation, as it implies neither is a perfect or near-perfect proxy for the underlying effect.

1.D Cross-sectional estimates of strategy assets

This section supplements Section 1.4 with an alternative approach to computing A_0, the strategy assets in 1962. To see how this works, consider the below modification of Equation 1.22:

Table 1.18
Multi-variate return predictability regressions with mutual fund assets

(a) *Full sample*

	ols			pca-1		
	36-mth	72-mth	96-mth	36-mth	72-mth	96-mth
M^{tot}_{t-1}	0.87	4.07	4.62	−0.25	−0.55	−0.82
	(0.06)	*(0.15)*	*(0.14)*	*(−1.77)*	*(−1.75)*	*(−1.82)*
MF^{tot}_{t-1}	−1.22	−3.24	−4.36	−0.90	−1.88	−2.70
	(−0.36)	*(−0.49)*	*(−0.50)*	*(−1.50)*	*(−1.45)*	*(−1.49)*
Months	661	625	601	661	625	601
R^2	31.93	52.83	60.49	31.77	51.91	59.76

Each column corresponds to a regression on future momentum factor returns. The last three specifications use only the first principal component on the right-hand side of the regression, while the first three are standard OLS multi-variate regressions. M^{tot} is momentum assets in dollars normalized by market capitalization, and MF^{tot} is domestic mutual fund assets over market assets. T-stats shown were calculated by simulating a multivariate AR(1) under the null.

(b) *18-month rolling*

	ols			pca-1		
	36-mth	72-mth	96-mth	36-mth	72-mth	96-mth
M^{tot}_{t-1}	−0.28	1.95	1.13	−0.19	−0.42	−0.57
	(−0.17)	*(0.71)*	*(0.34)*	*(−1.52)*	*(−1.41)*	*(−1.25)*
MF^{tot}_{t-1}	−0.90	−2.45	−3.22	−0.92	−1.93	−2.88
	(−1.33)	*(−1.70)*	*(−1.60)*	*(−1.46)*	*(−1.45)*	*(−1.57)*
Months	643	607	583	643	607	583
R^2	30.82	52.27	60.53	30.81	50.70	60.08

This is the same as Table 1.18a, but the betas used to compute momentum assets for M^{tot} are now calculated out of sample with rolling 18-month betas used to compute fund loadings on the momentum factor. While the use of rolling betas does not directly affect MF^{tot}, the coefficients differ slightly relative to Table 1.17 due to the loss of 18 months of data.

$$V_{it} = A_0^t \left| \prod_{s=1}^{t-1} [G_s] \left(w_{it} - w_{it-1} R_{it}\right) \right| + \lambda_t + \epsilon_{it} \tag{1.40}$$

$$\bar{A}_0 = \frac{1}{T} \sum_t A_0^t \tag{1.41}$$

Here, each A_0^t corresponds to an estimate of 1962 assets, implying T cross-sectional estimates of the coefficients. Averaging these coefficients should yield a similar estimate of A_0 to that of specification (4) of table 1.3.

Table 1.19 shows the results. Specification (1) shows the estimate of A_0 as highly significant, regardless of whether the growth G_t is calculated using full-sample or rolling betas. Running the regressions with $G_t = 1$ in specification (2) also produces

Table 1.19
Explanatory cross-sectional volume regressions

(a) *Full sample*

	(1)	(2)
A_0^{MOM} (estimated) L+S	3.45	
	(10.86)	
intercept	0.72	0.71
	(4.19)	*(4.19)*
A_0^{MOM} $(G = 1\ \forall t)$ L+S		656.62
		(4.18)
N	696	696

This table computes the mean and standard error for the individual cross-sectional estimates of A_0. Each cross-sectional estimate corresponds to an estimate of momentum strategy assets in 1962. Standard errors computed using the Newey-West procedure with 24 lags.

(b) *18-month rolling*

	(1)	(2)
A_0^{MOM} (estimated) L+S	6.77	
	(9.00)	
intercept	0.70	0.73
	(4.23)	*(4.22)*
A_0^{MOM} $(G = 1\ \forall t)$ L+S		674.02
		(4.22)
N	678	678

This table is the same as the previous table but uses 18-month rolling regressions. The estimate A_0 corresponds to momentum assets in 1964, the beginning of the rolling regression sample.

a significant coefficient, albeit with less significance. These results, as with those of specification (6) of Table 1.3, show that much of the variation comes from the definition of the strategy via the weights.

Figure 1.10 presents graphically the individual cross-sectional estimates used to compute Table 1.19a. As implied by Equation 1.22, the estimates are discounted at the growth rates back to 1962. That is, the results are *not* a time series of momentum assets, but instead represent repeated estimates of 1962 assets. While the individual estimates vary, they mostly stay within a persistent range, and the vast majority of the estimates are statistically greater than zero. This again shows how the cross-sectional estimates contribute to estimating the level of momentum assets.

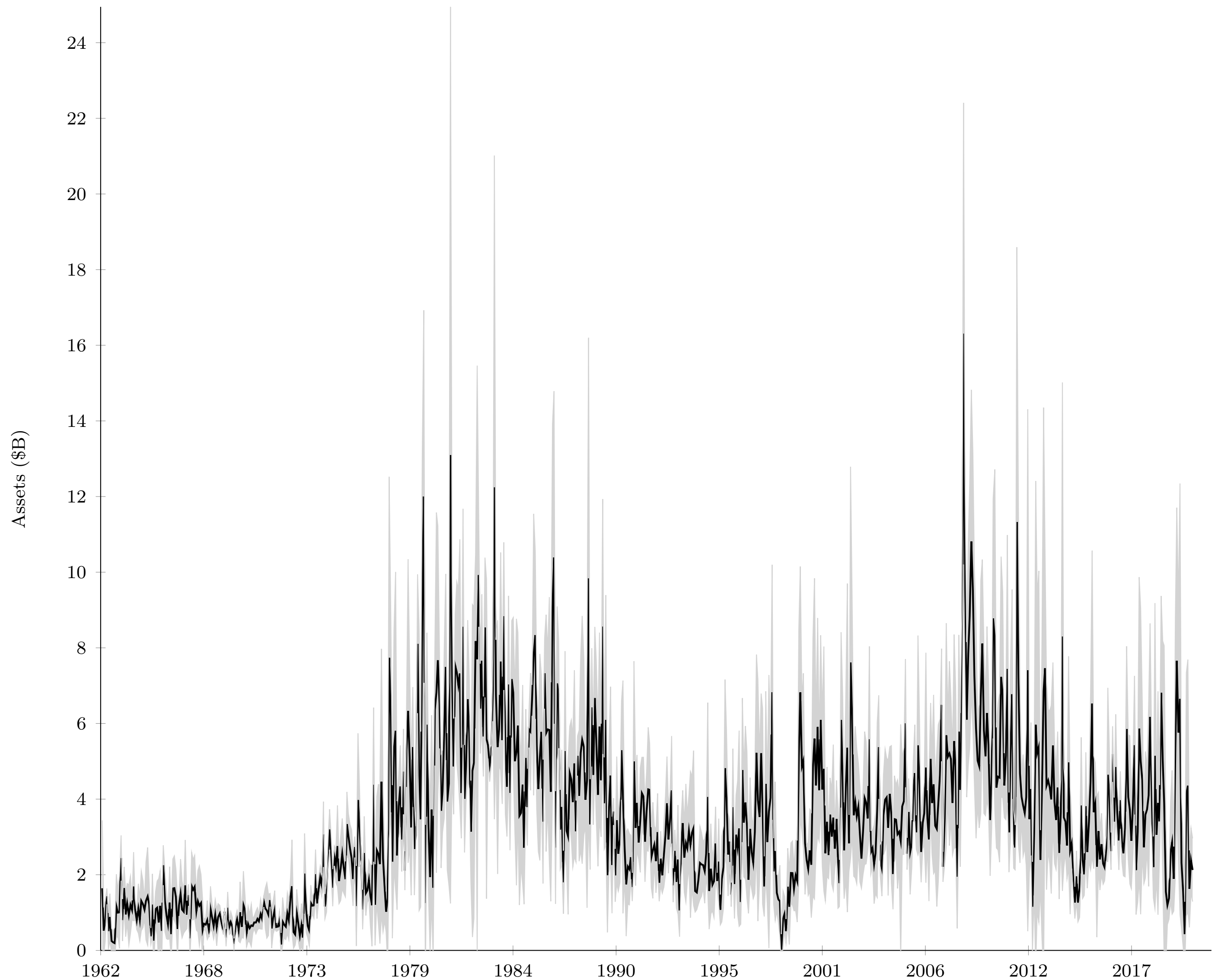

Figure 1.10
Estimates of 1962 Assets (cross-sectional) with standard errors
The center line corresponds to individual cross-sectional estimates of 1962 momentum assets. The outer shadings correspond to the 2σ error bars of each cross-sectional estimate. This graph is *not* a time series of momentum assets. It is a series of estimations using data from each month to compute the assets in 1962.

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CHAPTER 2

Are Interest Rates Really Low?

With Daniel Feenberg *(NBER)* **and Ivo Welch** *(UCLA Anderson School)*

[Figure 2.1: **Nominal Interest Rates on Short-Term and Long-Term Treasury Notes**]

Figure 2.1 plots the post-war history of nominal interest rates on short-term Treasury bills and long-run Treasury bonds. The low interest rates towards the end of the sample—seemingly the lowest since the Korean War—have raised widespread concern. High-ranking fed officials, like Eric Rosengren (Boston), John Williams (San Francisco), Chris Neely (St. Louis), Narayana Kocherlakota (Minneapolis), or Jerome Powell (Federal Reserve Chair), are on record discussing how low interest rates have caused investors to "reach for yield," and thus how low interest rates have at least been partly responsible for the high stock market. They are not alone. The perception that interest rates have been unusually low, perhaps because short-term yields have hit their nominal bound of zero, is also pervasive among journalists, foreign financial and non-financial policy makers, retail and professional investors—and academics. For example, the secular stagnation theory in Summers (2014) uses a decline in the real interest rate to make the case for unconventional monetary policy. Taylor (2014) posits that the Fed held interest rates too low for too long before the crisis. And so on.

Yet, there is an important aspect often overlooked. Most financial economists use canonical benchmark models, which are themselves based on "perfect market" assumptions—the equivalent of a friction-free environment in physics. Such a market ignores not only market power and information differences, but also all trading costs, liquidity, and taxation. Thus, many academic papers begin with the qualification that they assume away these "complications," and then they proceed to their analysis. In other words, many academic papers about interest rates and the economy reflexively ignore taxation. For many purposes, this does not greatly distort the insights of the model. However, in the case of assessing whether interest rates are low or high, it does. Taxation of nominal yields is a first-order concern.

For example, consider that the 20-year Treasury bond promised 5.0% in 2006 but "only" 2.2% in 2016. The prevailing inflation rates of 3.2% and 1.2%, respectively, eroded much of the difference, leaving real interest rates of 1.8% and 1.0%. Yet, even this difference is irrelevant for the average taxable retail investors in the U.S. The prevailing average marginal tax rate on interest in the economy was about 25% in both years. Assessed on 5.0% and 2.2%, investors had to pay taxes of 1.25% and 0.55% in 2006 and 2016, respectively, leaving them with 3.75% and 1.65% in nominal after-tax terms. In real terms, investors in 2016 thus earned about 0.5%, approximately the same as the 0.5% that they earned in 2006. We will point out that similar calculations for the post World-War-2 sample show that short-term post-tax post-inflation interest have been on the low side since the financial crisis, but not unusually so by historical standards.

Of course, not all bondholders pay taxes on interest receipts. About 40% of government bonds are held by other U.S. government institutions themselves. Of the remaining 60%, about half is held by foreigners. Most foreigners are exempt from U.S. taxes on interest. Therefore, such foreigners may effectively earn higher rates of return on U.S. Treasuries than their U.S. retail investors counterparts. The remaining 30% of government debt could be held either in tax-exempt vehicles (such as in charitable endowments or in 401-K plans) or in taxable investment accounts.[1] Moreover, not all taxable investors are equally taxed. High-income investors generally pay higher taxes, as do investors in blue states. A single unique tax rate does not exist, much less a data source that makes it possible to extract the complete distribution of relevant tax rates. However, there are two reasonable methods to assess the financial-market relevant taxation on interest payments.

The first method relies on a large sample of anonymized and disclosure-proofed tax returns over-weighted with high income returns. While the samples are heavily redacted, the masking does not affect our ability to calculate average marginal tax rates by income type.[2] Simplifying, our model assesses an investor-averaged rate at which their last "marginal" dollar in interest receipts was taxed. The model is not only imperfect, but, as already noted, also applies only to the U.S.-taxed subset of investors, assuming that they did not have unusual but representative holdings in the Treasury bond market. Moreover, our main analysis considers only Federal taxation—the web appendix shows that state taxes would add a further four to five

[1] In all cases, the U.S. effectively pays more in effective interest to foreign and untaxed investors than it pays to domestic taxable investors. From the perspective of the U.S. government, the real interest rate it pays is a weighted average of its non-tax-paying and tax-paying bond holders.

[2] A description of the methods can be found in Feenberg and Coutts (1993) and http://users.nber.org/~taxsim/allyup.

percent on average. Including them would only strengthen our claim.

[Figure 2.2: **Long-Term AAA Municipal and Corporate Yields**]

The second method relies on the interest rate differential between taxable Treasury bonds and tax-exempt highly-rated municipal bonds. If a Treasury bond offers 5% and an otherwise equivalent perfectly safe "Muni" bond offers 4%, it follows that an investor with a 20% tax rate on interest income is indifferent between holding either of the two. Unfortunately, municipal bonds are never exactly the same as Treasury bonds. The issuing municipal entity may default on repayment. Moreover, Munis have much lower liquidity (resellability before expiration), a problem that suddenly (and perhaps unexpectedly) became more acute in the financial crisis of 2009. Fortunately, it is possible to remove an estimate of the credit and liquidity spread components by comparing highly-rated muni bonds to highly-rated corporate bonds. Putting together Treasury, municipal AAA-rated, and corporate AAA-rated bonds allows extracting an "effective" implied tax rate on interest payments:

$$\text{Effective Tax Rate} = 1 - \frac{\text{Muni AAA Yield}}{\text{Treasury Yield } + \text{Corp AAA Yield Spread}} .$$

Of course, this implied effective tax rate itself depends on the supply and demand of tax-exempt and taxable investors and investment alternatives, both domestically and beyond. For example, if only the highest-taxed 1% of investors were holding Treasuries, the financial market series would indicate implied tax rates close to the maximum personal interest tax rate.[3] If only tax-exempts were holding Treasuries,

[3]The average economy-wide tax rate was significantly below the marginal economy-wide tax

the effective tax rate series would indicate values close to zero.

[Figure 2.3: **Tax Rates**]

Figure 2.3 shows that the two tax-rate calculations are not perfectly congruent but generally economically quite similar when both are available.[4] The tax-rate of 20-25% on interest was also remarkably typical and stable over the sample. However, this tax rate was also very different from the top statutory rate on interest and/or ordinary income, which could reach as high as 90% in the 1950s! Even the 1986 Reagan Tax Reform Act (with lower tax rates, but fewer loopholes, exceptions, and exemptions) did not drastically reduce the effective tax rate.

The final series necessary to compute investor-relevant real interest rates is inflation. The relevant inflation rate should be the prevailing contemporary expected inflation rate over the life of the bond. Our proxy is the CPI-based geometric inflation rate in the year before, the same year, and the year after the interest rates are measured (reflecting at least some expectation of the future). Again, our key inference remains largely the same if we use just the contemporaneous interest rate or a multi-year ex-ante or ex-post inflation rate.

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rate for interest income, suggesting that it was higher-earning and higher-taxed individuals who earned and paid taxes on interest receipts.

[4]Year-to-year changes in the tax rates correlate far less. This would be a concern for many other economic studies, but it is not of great concern in our own study.

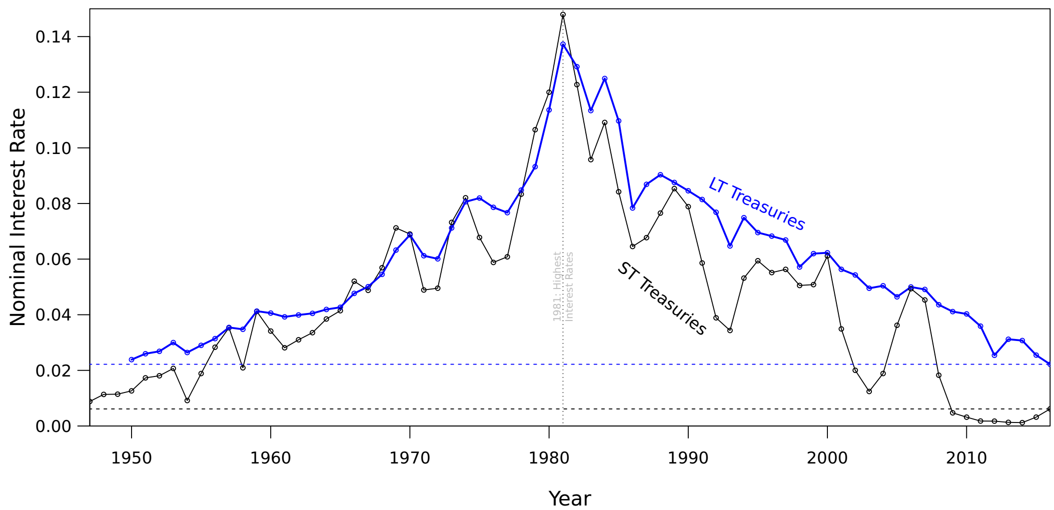
[Figure 2.4: **Post-Tax Real Yields on Short-Term 1-Year Treasuries**]

[Figure 2.5: **Post-Tax Real Yields on Long-Term 20-year Treasuries**]

Figures 2.4 and 2.5 make our paper's key point. They plot the time-series of after-tax inflation-adjusted real interest rates on Treasury yields. The graphs show that real interest rates have not been unusually low after the crisis when put into the perspective of post-war history. The post-2008 interest rates have been well within the "ordinary" range. The 2016 yield on short-term Treasuries is only –0.40 standard deviations below its historical mean. The 2016 yield on long-term Treasuries is likewise –0.2 standard deviations below its historical mean.

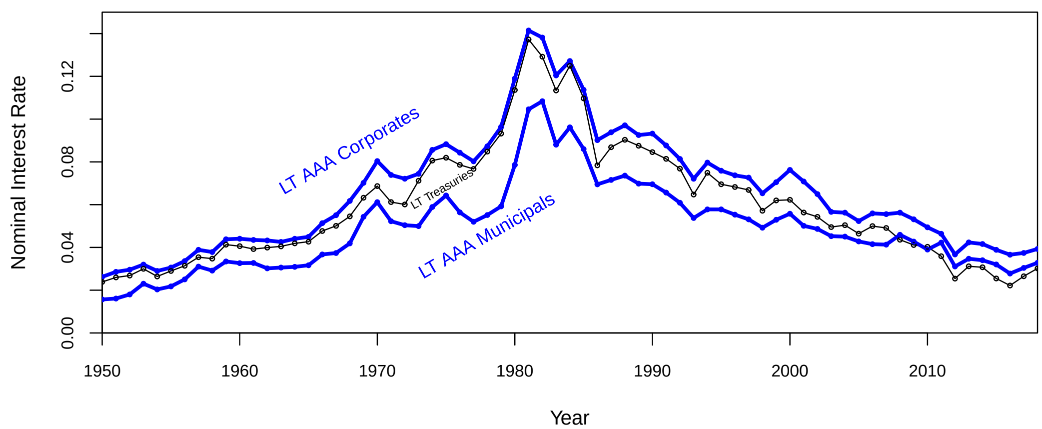
We can conclude that the prevailing popular notion—that low interest rates have (and should have been) driving investors towards stocks and other risky investments—seems exaggerated. Of course, this is not to say that naïve investors after the financial crisis of 2008 may not have suffered from money illusion and fled the bond market to "reach for yield" in the stock market, after all. It is to say that sophisticated taxable investors should not have reached for yield any more than usual. For them, the "real" short- and long-term real interest rates in the wake of the Great Recession should have looked by-and-large mundane.

Figure 2.1
Nominal Interest Rates on Short-Term and Long-Term Treasury Notes



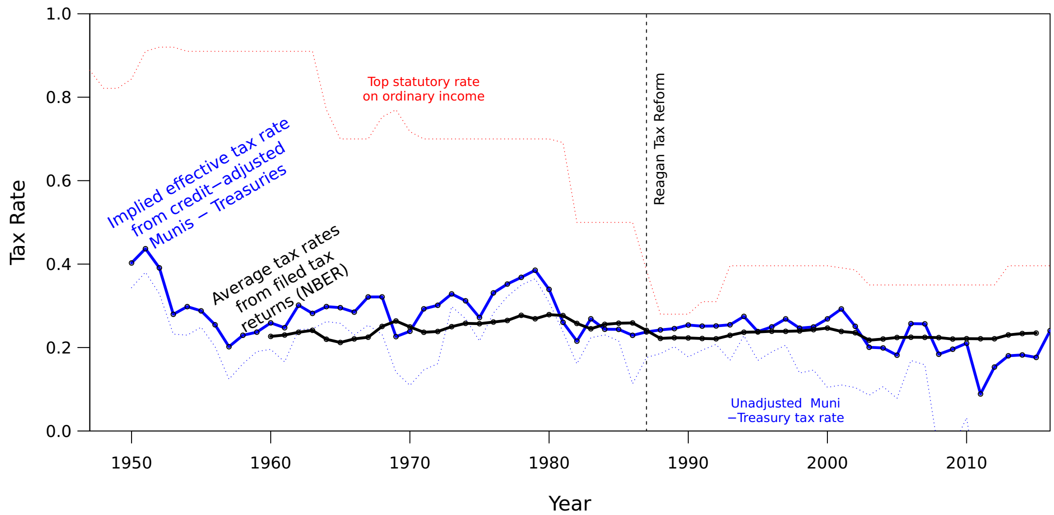
The graph shows that short-term Treasuries, represented by the 1-Year Treasury Note, have offered considerably lower yields than long-term Treasuries (20-Year) only in the second half of the sample. Interest rates peaked in 1981. All series originated from Global Financial Data (GFD), specifically series IGUSA1D and IGUSA20D, respectively, corresponding to 1-Year and 20-Year Treasuries. The data series are also listed in our web appendix.

Figure 2.2
Long-Term AAA Municipal and Corporate Yields



The graph shows that (1) a mixture of long-term (20+ year) AAA-rated corporate bonds offered increasingly higher promised (not expected!) yield spreads from 1965 to 1978 and after 1985; and (2) long-term 20-year AAA-rated municipal bonds have offered lower promised (not expected!) yield spreads until about 2008. All series originated from Global Financial Data (GFD). The corporate yield series is MO-CAAAD, the "Moody's Corporate AAA Yield" index. The municipal bond yield series is MOWAAAW, the "Moody's 20Y AAA Muncipal Bond Yield." The series are also listed in our web appendix.

Figure 2.3
Tax Rates



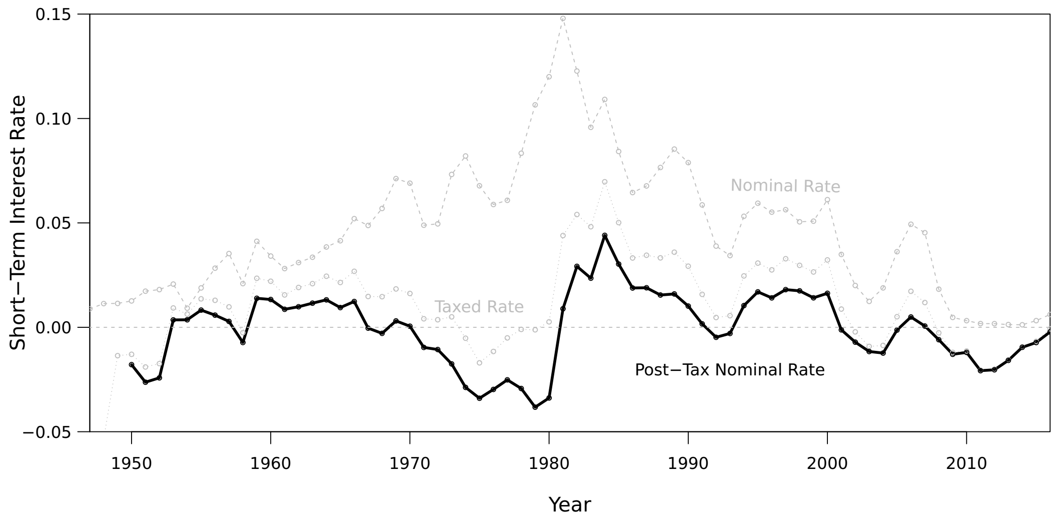
The black line shows the interest income weighted average marginal Federal (without any state) income tax rates on interest income, as calculated from the Taxsim model. This interest tax series is now available at http://users.nber.org/~taxsim/marginal-tax-rates/af84.html.

The blue lines are tax rates computed from the yield differential of non-taxable 20-year maturity-matched municipal bonds (series MOWAAAW) from GFD, and 20-year Treasury bonds (series code IGUSA20D), adjusted for the credit and liquidity spread (the difference between taxable Treasuries and 20+ year AAA corporate bonds (series MOCAAAD).

Despite originating from completely different methods, the NBER and financial tax series suggest similar marginal average level tax rates on interest—relatively stable and about 20% to 25%.

PS: The red dotted line suggests that the top statutory tax-rate (IRS Historical Table 23 Series 5, available at https://www.irs.gov/statistics/soi-tax-stats-historical-table-23) should not be used as a proxy for the effective tax rate in the economy. It was neither greatly reflective of observed paid tax rates nor of the pricing of financial instruments. The blue dotted line suggests that an unadjusted Muni minus Treasury spread that is not credit- and liquidity- adjusted would yield highly misleading estimates, e.g., non-sensible negative estimates after the financial crisis. Our web appendix investigates this further using tracking regressions.

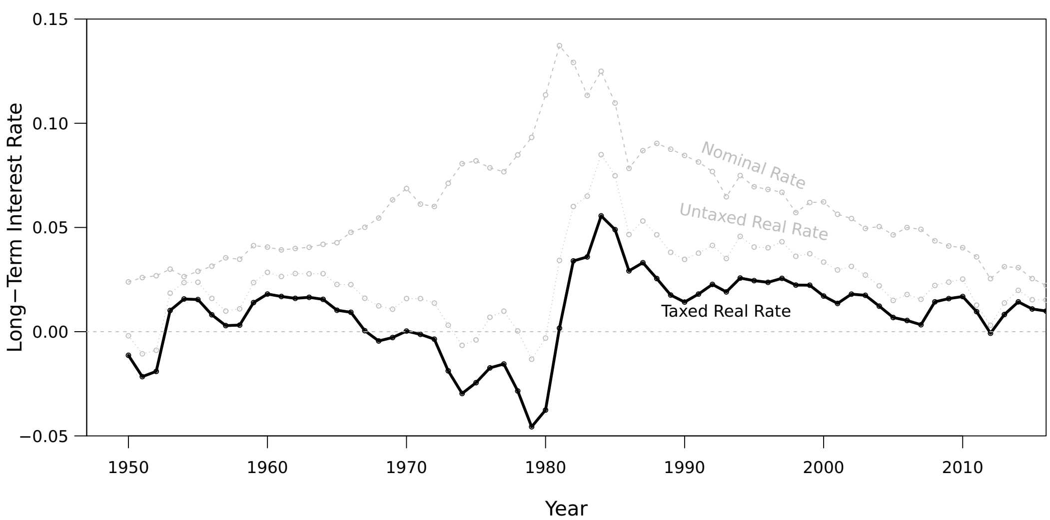
Figure 2.4
Post-Tax Real Yields on Short-Term 1-Year Treasuries



This graph plots short-term Treasury yields after smoothed inflation and Federal taxes have been removed. Calculating the smoothed inflation rate involves averaging the previous, current, and subsequent year's CPI rate. The tax rate used in this graph is from the tax rate implied by the spread between the 20-year Treasury and municipal bonds, after adjusting for credit and liquidity, as measured by the spread between 20+ year AAA corporate bonds and 20-year Treasuries.

After-tax short-term (1-year) real Treasury yields are about –0.7% as of 2016—but this is still higher than the rates from 1971-1980 and from 2002-2004. At –0.7%, the rate is more ordinary than extraordinary. The short-term real-after-tax rate in 2016 is **–0.4** standard deviations relative to its historical series since 1950, corresponding to the 38th percentile.

Figure 2.5
Post-Tax Real Yields on Long-Term 20-year Treasuries



The figure is analogous to Figure 2.4, except that the focus is on longer-term Treasuries. After-tax long-term (20-year) real Treasury yields are well in line with common yield patterns—except for the 1982 to 1987 period which showed remarkably high yields. The long-term real-after-tax rate in 2016 is **−0.2** standard deviations relative to its historical series since 1950, corresponding to the 36th percentile.

APPENDICES

2.A The Tax Code

2.A.1 The NBER Taxsim Model

Each year since 1960 (except 1961, 1963, and 1965), the *Statistics of Income Division* of the IRS has released a public use file derived from the individual income tax filings for that year. Although the files are redacted to maintain confidentiality, they fairly represent the distribution of income and tax by component. We then use the NBER tax calculator to determine the tax liabilities from these files. This calculator takes into account the numerous details of the tax law, including the maximum tax on earned income, the minimum tax, special treatment of capital gains, the net investment income tax, income averaging, phase-ins and phase-outs of itemized deductions and income-based clawbacks of various credits deductions, and many other complexities, all of which can potentially influence the required tax.

The average marginal rate on *interest* income differs from the average marginal rate on *ordinary* income, not only because interest income is distributed differently, but also because some some features of the tax law treat interest income differently. (Even the maximum tax on earned income can effect the tax rate on interest income through the stacking rule.) Nevertheless, interest income was mostly treated the same as ordinary income. From 1971 to 1981, the maximum tax rate on earned income resulted in a statutory difference between 8% to 20% for top income tax brackets. Specifically, the rule capped the tax on earned income at 60% in 1971 and 50% from 1972-1981 IRS (2016). Despite the seemingly significant statutory difference, the difference between the average marginal rates of interest and earned income reached a maximum of 3.1%.

Several other differences could lead to variations between taxes on earned and unearned income. The earned income tax credit can also lead to some differences in interest income treatment, primarily effecting lower tax brackets. Payroll (FICA) taxes exclusively effect earned income. A 3.8% surcharge on investment income was instituted in 2013. Finally, owners of Treasuries issued prior to 1941 typically received a credit of 3% of interest paid or, prior to 1955, would not pay the 3% "normal" tax on such income.

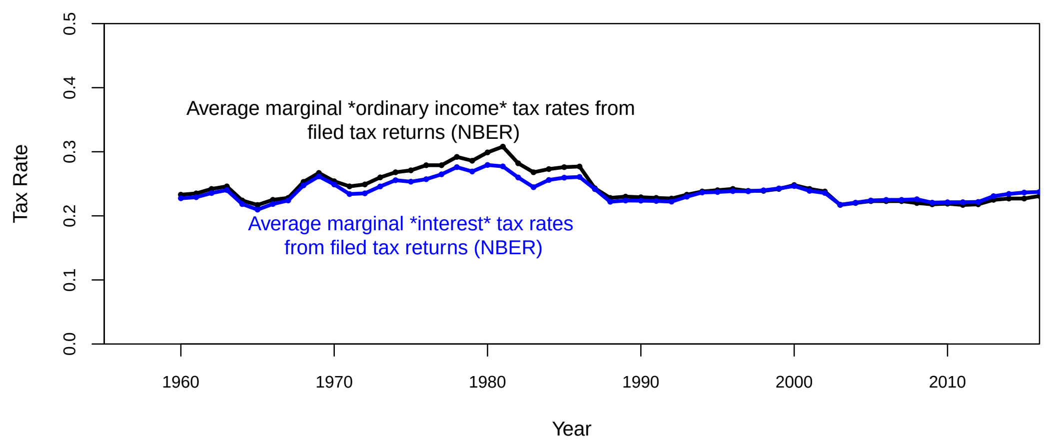
The average marginal rate[5] is computed over a finite difference in interest income of 1% of base interest income.

2.A.2 Average Average Taxes versus Average Marginal Taxes

Most of the preceding discussions about statutory tax rates focused on the average marginal tax rate (on interest rates) of investors. The average total effective tax rate is an alternative taxation metric. Of course, each individual taxable investor should factor only the marginal tax rate into the decision making process. The usefulness of the average average tax rate would be only in assessing the tax burden in the economy. The graph below shows both the total average *marginal* statutory tax rate (not just for interest) and the total average *average* tax rate. Because of progressivity, the average marginal rate has always been about 10 percentage points higher than the average average rate. The difference was consistent over time.

[5]**Naming Convention:** The first *average* pertains across individuals. The second *average* or *marginal* pertains to one single investor's average or marginal tax over her own total income.

Figure 2.6
Average Marginal Total *Ordinary* vs *Interest* Tax Rates



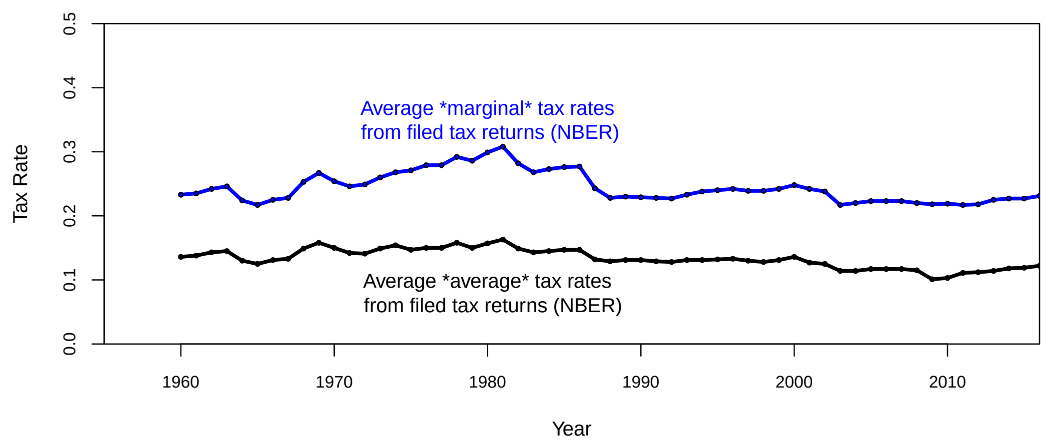
This graph plots the average marginal tax rate for all ordinary income as well as the average marginal interest-only tax rate over time. The differences between the graphs are generally small and transitory.

2.A.3 Inclusion of State Taxes

The tax data series inclusive of state taxes begins in 1978 and terminates in 2008. Because the point of this paper is about assessing the magnitude of current interest rates against interest rates from a historical perspective, the combined Federal plus state income tax data was simply too incomplete to be suitable for the results in the main text. In the appendix here, we can give some idea of the effect in the subsample in which both Federal and state tax data were available.

Our graph shows that state taxes increase the average marginal tax rate by about four to five percentage points. Although meaningful to investors, the impact on our main results is small: An after-tax marginal yield of about 1.5% would only be

Figure 2.7
Marginal and Average Tax Rates



This graph plots the average marginal tax rate and the total average effective rate over time. At all times, the average effective rate is significantly smaller than the average marginal rate.

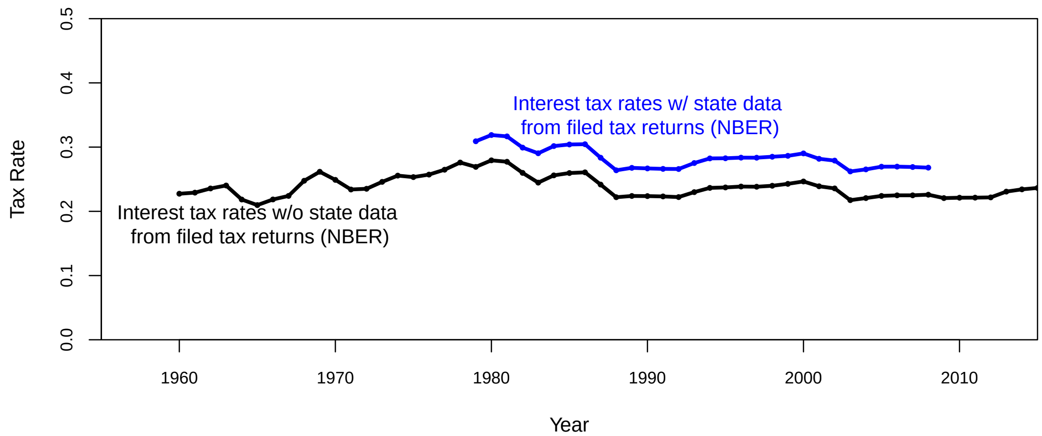
reduced by about 0.1%. Moreover, because the effect stays relatively even over time, the effect of state taxes on the relative ranking of current interest rates relative to historical interest rates is likely to be modest.

2.A.4 Tax Rates and Aggregate Substitution Between Taxable and Tax-Exempt Bonds

Remarkably, the evidence does not suggest large substitution effects by investors from taxable Treasuries towards non-taxable municipals in high-effective-tax environments.

In more detail, if municipal bond prices fail to adjust for changes in investor

Figure 2.8
Total Taxes With and Without State Taxes



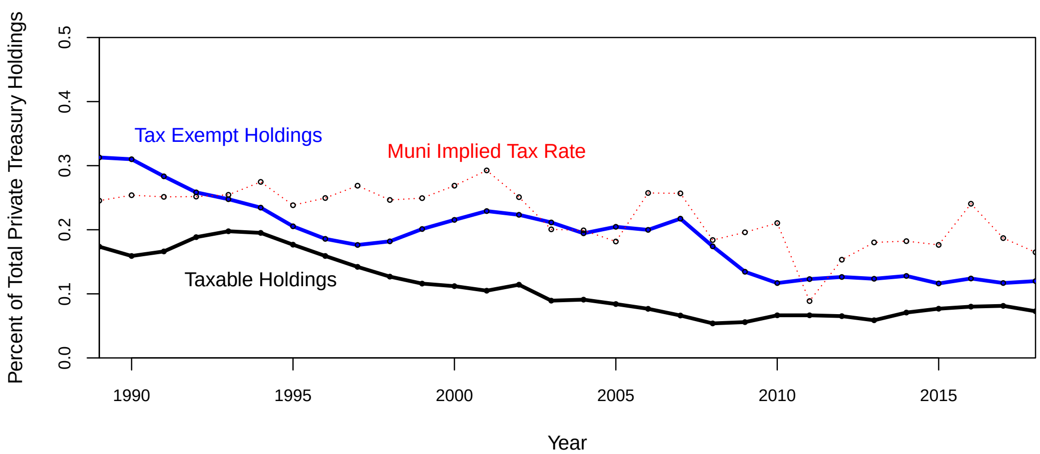
This graph plots the average marginal tax rate inclusive of and excluding state taxes. Excluding state taxes predictably leads to a lower effective tax rate. The effect is relatively uniform over time.

marginal tax rates, investors would likely substitute between municipal bonds and Treasuries until prices equilibrated. In other words, absent price adjustments, higher marginal tax rates should incentivize (some) taxable investors to substitute from taxable towards non-taxable municipal bonds. This should further increase the spread between the two. It should also induce the ratio of holdings of Treasury bonds by taxable investors over non-taxable investors to decrease.

However, we see little evidence of such substitution. In detail, as a crude measure, the plot below divides holders of privately held Treasuries into taxable, tax-exempt, and unclassified (not shown) investors as a total percentage of all Treasuries outstanding. The plot also includes the municipal implied tax rate. Unfortunately, this does not isolate the causal relationship between the municipal bond implied tax rate and Treasury flows, other than accounting for AAA corporate credit/liquidity spreads and inflation. Hence the absence of substitution flows when municipal bond implied tax rates changed provides little evidence that investors cared to substitute.

Furthermore, Treasury ownership data is both coarse and of limited availability prior to 1990. Observation of the actual municipal and Treasury holdings of the counter-parties, and their respective marginal tax rates, would be a more direct means of observing the influence of marginal tax rates on security selection. We neither have the data nor is this our primary focus, so we leave this to others for future study.

Figure 2.9
Percent Holdings of Treasury Securities by Taxable and Tax-Exempt Investors



This graph plots the relative holdings of Treasury securities by tax-exempt investors and taxable investors. The series are derived from the "Estimated Ownership of Treasury Securities" tables available at https://www.fiscal.treasury.gov/fsreports/rpt/treasBulletin/treasBulletin_home.htm. Taxable securities are defined as the sum of holdings of depository institutions and insurance companies. The sum of pension fund holdings and state and local government holdings together proxy tax-exempt holdings. Divide these values by total private Treasury holdings to calculate the values shown. The municipal bond implied tax rate is based on the spread of 20-year maturity municipal bonds and 20-year Treasuries after a credit and liquidity adjustment based on 20 to 30 year AAA corporate spreads over 20-year Treasuries.

2.B The Credit- and Liquidity-Adjustment For Munis

2.B.1 Tracking Regressions

Our control approach to credit and liquidity spreads has been to presume that, for equal maturity bonds,

$$\text{T-Bond Yield} - \text{Muni Yield} \approx \text{Credit and Liquidity Spread} + \text{Tax Spread} + \text{etc}$$

We used the *20+ year AAA Moody's corporate spread over 20-year Treasuries* as our proxy for the credit and liquidity spread of *AAA Moody's 20-year municipal bonds over 20-year Treasuries.*

We can check whether the credit and liquidity characteristics between municipal bonds and corporate bonds were not one-to-one and/or greatly influenced by other time-varying effects, i.e., different from the implied tax, liquidity, and credit effects. We can use tracking regressions to test the efficacy of our corporate spread in representing the credit and liquidity components of municipal bonds, after adjusting for taxes. Similar behavior of AAA corporate spreads and AAA municipal bond spreads over time, with a coefficient of around 1, would support the hypothesis of an effective proxy of credit and liquidity effects by the corporate credit and liquidity spread (See our calculation of municipal implied tax rates in the main text). In brief, our results suggest good evidence in favor of an approximate 1-to-1 covariation of corporate spreads with municipal spreads. This mitigates concern about omitted (time-varying) distorting effects that would have been picked up by corporate bond spreads.

Our specification is a tracking regression of

$$[\text{20y T-Bond} - \text{20y Muni Yield}] \approx \gamma_0 + \gamma_1 \times [\text{20y T-Bond} - \text{20y Corporate AAA Yield}]$$

$$+\gamma_2 \times [\text{20y T-Bond} \ \cdot \tau] \ + \ \overline{\gamma} \cdot \ \text{Covariates}$$

Covariates includes term structure effects and inflation, but could include other series (e.g. the S&P 500). Only the specification which is not differenced shows the CPI effect as significant.

Each regression decomposes the municipal treasury spread into a risk-less tax component and a corporate credit/liquidity component. Regressing the municipal bond spread against these components suggests how the corporate spread explains the residual difference between Treasuries after-tax and municipal bond rates. Some specifications include additional covariates to verify robustness. Finally, the relevance of the corporate spread is tested for resilience against several alternative statutory rates.

Table 2.1
Time Series Regressions in Levels: T-Bond Minus Muni Bond Spreads Explained By Statutory Taxes and Credit/Liquidity Spreads

Dependent Variable: 20-Year T-Bond Minus 20-Year AAA Muni Yield Spread, ×100					
20Y T-Bond Minus AAA Corp	114.9	108.9	126.1	112.0	should be 100 (%)
	(15.8)	(14.2)	(21.3)	(15.8)	
20Y T-Bond*NBER Interest Tax	103.2	84.6			controls for real tax effects
	(10.0)	(10.8)			
20Y T-Bond*NBER Gains Tax			115.0	80.6	
			(18.4)	(18.1)	
10 Minus 20-Year T-Bond		15.9		25.8	
		(12.5)		(16.3)	
CPI Inflation		6.4		11.0	
		(2.3)		(4.1)	
Intercept	0.4	0.4	0.6	0.6	
	(0.2)	(0.2)	(0.2)	(0.2)	
Num Years	59	59	59	59	
R^2	0.91	0.93	0.81	0.88	

(Newey-West standard errors in parentheses)

Regressing the 20-year T-Bond – municipal bond spread against the corporate spread suggests a one-to-one relationship between corporate and municipal spreads. The independent variables, listed on the left hand side, include the focal 20-year T-Bond – 20+ year AAA corporate spread, as well as some additional covariates. The tax rate multiplied by the 20-year Treasury bond accounts for the tax-exempt status of muncipal bonds. The regression includes two different tax rates for robustness. Several other covariates were also included. If the relationship holds perfectly, the coefficient on the corporate AAA spread would be 100 percentage points. The regression results show that the spread is close to 100 in standard error terms.

Table 2.2
Time Series Regressions in Differences: T-Bond Minus Muni Bond Spreads
Explained By Statutory Taxes and Credit/Liquidity Spreads

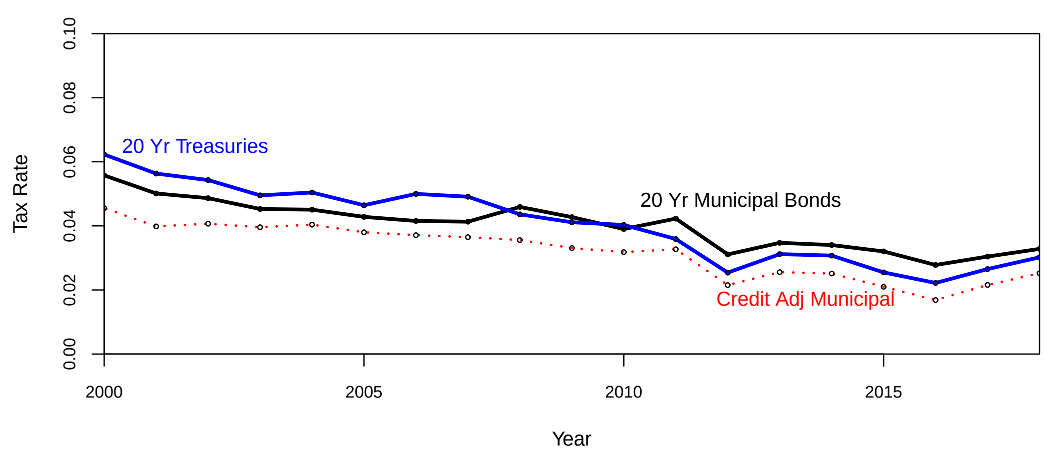
Dependent Variable: 20 Y T-Bond Minus 20Y Muni Yield Spread					
20Y T-Bond Minus AAA Corp	103.1	104.1	102.6	104.8	should be 100 (%)
	(16.9)	(18.0)	(15.6)	(16.4)	
20Y T-Bond*NBER Interest Tax	46.2	37.9			controls for real tax effects
	(20.2)	(27.6)			
20Y T-Bond*NBER Gains Tax			50.8	41.0	
			(14.6)	(17.8)	
10 Minus 20-Year T-Bond		1.5		7.9	
		(16.1)		(17.1)	
CPI Inflation		2.4		2.4	
		(3.9)		(3.1)	
Intercept	−0.0	−0.0	−0.0	−0.0	
	(0.0)	(0.0)	(0.0)	(0.0)	
Num Years	58	58	58	58	
R^2	0.63	0.64	0.63	0.64	

This is the same as the previous table except that the dependent and independent variables are differenced. Ideally, covariates other than the corporate spread and the tax effect should be near zero. In the level version presented in the previous table, the CPI is significant. Here, even the CPI term is always insignificant. The coefficient on the *20Y Treasury Note – 20+ corporate AAA yield spread* remains substantially unchanged.

2.B.2 The Crossing of the Muni and Treasury Yields after the Crisis

During the financial crisis, the price of Treasuries rose above the price of AAA municipal bonds. A simple figure can show that this "anomaly" likely originated from changes in the market-wide prices for credit and liquidity, and not from a negative implied tax rate:

Figure 2.10
Treasury and Municipal Bond Yields



This graph shows that the raw municipal bond yield rose above the equivalent 20-year Treasury yield around the time of the financial crisis. The dotted line shows that this effect disappears after accounting for the credit and liquidity components of the municipal bond yield. The adjusted municipal bond yield is calculated by 20 Yr AAA Muni Yield $-(1-\tau)\times($>20 Yr AAA Corp Rate$-$ 20 Yr Treasury Yield), where τ is the 20-year municipal bond implied tax rate from the standard formula $1 - \frac{\text{20 Yr Muni AAA Yield}}{\text{20 Yr Treasury Yield + 20+ Yr Corp AAA Spread}}$. Using the statutory tax rate instead of the municipal implied yield leads to a very similar result.

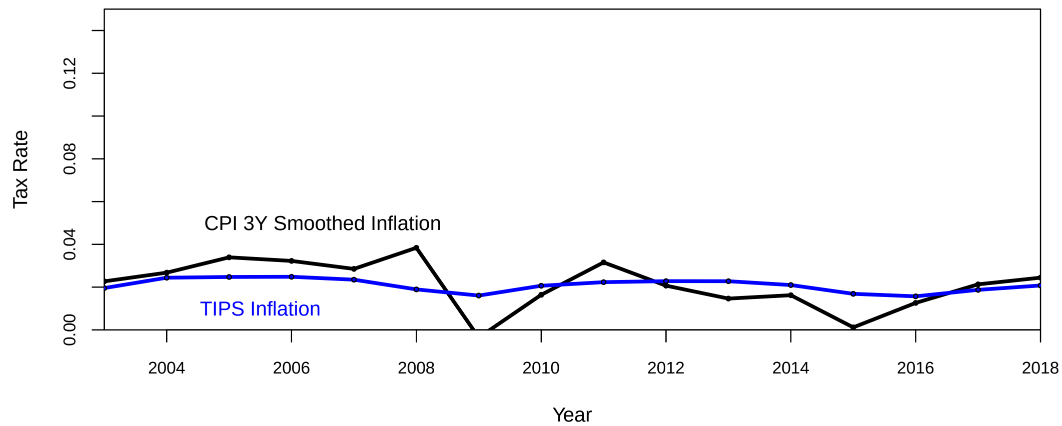
2.C Rates of Return and Inflation

2.C.1 Breakeven Inflation

Treasury Inflation Protected Securities (TIPS) seem superficially attractive as a measurement of inflation. Their yield can be inverted to imply a break-even level of inflation which would make an investor indifferent between TIPS and the corresponding nominal treasury security. Unfortunately, there are two problems with using TIPS in the main results. First, the inflation rate implied by TIPS can sometimes differ greatly from other empirical market metrics, as discussed in great detail in Fleckenstein, Longstaff, and Lustig (2014). Second, a continuous series for TIPS is only available beginning in 2003.

The following graph compares the 3-year CPI inflation used in our primary analysis with the TIPS-implied inflation rate from 2003 onward. Recall that our 3-year CPI inflation rate in the main text averaged CPI inflation across prior, current, and subsequent years. The graph suggests that the CPI smoothed rate is reasonably close to the TIPS implied rate in most years *for our purposes.*

Figure 2.11
Smoothed CPI Inflation vs Breakeven Inflation



This graph plots both the 10-year TIPS implied break-even inflation rate and the 3-year smoothed CPI inflation rate (averaged over t–1, t, and t+1). The breakeven rate is substantially similar to the CPI measure except in 2009. This suggests that the main results would be minimally affected by using breakeven as opposed to smoothed CPI inflation.

Using breakeven rates in our calculations would likely not change our conclusion that real after-tax rates are not extraordinarily low. In fact, breakeven inflation rates in 2016 were slightly lower than they have been at any other point in the 14-year sample. Naively plugging in the 2016 breakeven inflation rate instead of smoothed CPI rate to calculate the 2016 real after-tax interest yield gives an indication as to the impact on the results. Using the higher breakeven rate leads to a value for the real after-tax 20-year bond of approximately 0, corresponding to the 27th percentile of after-tax rates dating back to 1950. Although lower than the 36th percentile calculated in the main results, the value is hardly without precedent.

2.D Data

2.D.1 Summary Statistics

This section provides additional detail on sources and derivations behind the key data series. The bulk of the statutory tax rates originated from the NBER TAXSIM database at [http://users.nber.org/~taxsim/]. The average marginal tax rate is available at [http://users.nber.org/~taxsim/allyup/], where the procedure for calculating the average marginal rate from the raw tax filings is extensively documented. Feenberg and Coutts (1993) contains additional information on the TAXSIM model. The *topord* series originates from the IRS's website [https://www.irs.gov/statistics/soi-tax-stats-historical-table-23]. Treasury holding data came from the Department of Treasury (DOT) Treasury Bulletin Publication, specifically from the 2017, 2012, 2007, 2002, 2000, and 1996 publications.

Global Financial Data (GFD) provided most of our financial series. The nominal one-year Treasury yields came from series IGUSA1D. The 20-year Treasury yields came from series IGUSA20D. The principle AAA municipal bond yields are found under GFD ticker MOMAAAW. This series is also a Moody's index. Finally, CPI data and information on breakeven inflation rates originated from FRED.

Many of the remaining series are derived. To calculate the real Treasury yields, the CPI index for each year was averaged over the past and subsequent years, thus forming a three-year moving average. The equation $\frac{1+\text{ Treasury Yield}}{1+\text{ CPI rate}} - 1$ provides the real Treasury yield from the nominal rate, while the after-tax real Treasury rate similarly stems from $\frac{1+\text{ Treasury Yield}\cdot(1-\tau)}{1+\text{ CPI rate}} - 1$.

The implied municipal bond tax rate may be calculated from the below equation.

The corporate spread consists of the GFD series MOCAAAD net of the 20-year T-Bond nominal rate. The *0credit* series re-calculates the municipal bond implied tax rate with the corporate credit and liquidity spread set to zero:

$$\text{Effective Tax Rate} = 1 - \frac{\text{Muni AAA Yield}}{\text{Treasury Yield} \ + \ \text{Corp AAA Spread}}$$

The next table links each figure presented in the main section and appendix with its respective data series. The subsequent table summarizes the source or derivation for data series used in the main section and appendix. The last table in this section contains basic summary information for each data series. In order of the column headings, the summary table lists the number of data points from 1950 onward, the mean, standard deviation, percentile measures, percentile of the 2016 data point (if available), and number of data points greater than the 2016 value $+2\sigma$.

Table 2.3
Data Series Descriptions (1/2)

cpi	Consumer price index for all urban consumers (avg)
inflation	Consumer price index for all urban consumers (avg YoY)
tipsinflation	10-year breakeven inflation rate (avg)
inflation3y	Average inflation over three years
tnote1	United States 1-year Treasury (avg)
tnote20	United States 20-year Treasury (avg)
muni20	Moody's 20-year AAA municipal bond yield (avg)
corplt	Moody's 20+ year corporate AAA yield (avg)
topord	Top marginal tax rate
avgord	Federal average marginal tax rate (deflated)
avgordtotal	Federal average average tax rate (deflated)
interesttax	Federal average marginal tax rate on interest (deflated)
interesttaxstate	Federal + state average marginal tax rate on interest (deflated)
munitax0credit	Implied muncipal bonds tax, no credit or liquidity adj
munitax	Implied muncipal bonds tax with credit and liquidity adj
tnote1r	1-year Treasury yield after inflation
tnote1at	1-year Treasury yield after-taxes and inflation
tnote20r	20-year Treasury yield after inflation
tnote20at	1-year Treasury yield after-taxes and inflation
tbankamt	Depository Treasury holdings (avg quarterly)
tinsuranceamt	Insurance Treasury holdings (avg quarterly)
tpensionlocalamt	Public pension Treasury holdings (avg quarterly)
tpensionprivateamt	Private pension Treasury holdings (avg quarterly)
tlocalgovamt	Local government Treasury holdings (avg quarterly)
ttotalprivateamt	Total non-Federal Treasury holdings (avg quarterly)
hightaxholdings	Taxable Treasury holdings
taxexemptholdings	Tax-exempt Treasury holdings
residualholdings	Unclassified Treasury holdings
muni20minusspread	The credit and liquidity spread after-taxes

Table 2.4
Data Series Descriptions (2/2)

Series	Vendor	Series Code	Formula
cpi	FRED	CPIAUCNS	
inflation	FRED	CPIAUCNS	
tipsinflation	FRED	T10YIE	
inflation3y			$\left(\frac{\text{cpi}_{t+1}}{\text{cpi}_{t-2}}\right)^{1/3}$
tnote1	GFD	IGUSA1D	
tnote20	GFD	IGUSA20D	
muni20	GFD	MOMAAAW	
corplt	GFD	MOCAAAD	
topord	IRS	Historical Table 23 (6)	
avgord	NBER TAXSIM		
avgordtotal	NBER TAXSIM		
interesttax	NBER TAXSIM		
interesttaxstate	NBER TAXSIM		
munitax0credit			$1-\frac{\text{muni20}}{\text{tnote20}}$
munitax			$1-\frac{\text{muni20}}{\text{tnote20}+(\text{corplt}-\text{tnote20})}$
tnote1r			$\frac{1+\text{tnote1}}{1+\text{inflation3y}}-1$
tnote1at			$\frac{1+\text{tnote1}(1-\text{munitax})}{1+\text{inflation3y}}-1$
tnote20r			$\frac{1+\text{tnote20}}{1+\text{inflation3y}}-1$
tnote20at			$\frac{1+\text{tnote20}(1-\text{munitax})}{1+\text{inflation3y}}-1$
tbankamt	DOT	Table OFS-2 (4)	
tinsuranceamt	DOT	Table OFS-2 (8)	
tpensionlocalamt	DOT	Table OFS-2 (7)	
tpensionprivateamt	DOT	Table OFS-2 (6)	
tlocalgovamt	DOT	Table OFS-2 (10)	
ttotalprivateamt	DOT	Table OFS-2 (3)	
hightaxholdings			$\frac{\text{tbankamt}+\text{tinsuranceamt}}{\text{ttotalprivateamt}}$
taxexemptholdings			$\frac{\text{tpensionlocalamt}+\text{tpensionprivateamt}+\text{tlocalgovamt}}{\text{ttotalprivateamt}}$
residualholdings			$1-\text{hightaxholdings}-\text{taxexemptholdings}$
muni20minusspread			$\text{muni20}-(\text{corplt}-\text{tnote20})(1-\text{munitax})$

Table 2.5
Data Series Usage Index

Figure or Table	Data Series Used
Nominal Interest Rates	tnote20, tnote1
LT AAA municipal and Corp. Rates	corplt, tnote20, muni20
Tax Rates	munitax, munitax0credit, interesttax, topord
Post-tax Real Rates (ST)	tnote1, tnote1r, tnote1at
Post-tax Real Rates (LT)	tnote20, tnote20r, tnote20at
Average Marg. Ordinary and Interest Taxes	avgord, interesttax
Average Average and Average Marginal Taxes	avgord, avgordtotal
Taxes Net and Gross of State Taxes	interesttax, interesttaxstate
Treasury Holdings by Investor Tax Status	hightaxholdings, taxexemptholdings, munitax
Tracking Regressions	tnote20, tnote10, muni20, corplt, interesttax, gainstax, inflation
Treasury and municipal Rates	muni20, tnote20, muni20minusspread
Smoothed CPI vs Breakeven Inflation	inflation3y, tipsinflation

Table 2.6
Data Series Summary Statistics

	Summary Statistics			Percentiles						
	N	mean	σ	0 %	25 %	50 %	75 %	100 %	$\%_{2016}$	$> x_{2016} + 2\sigma$
cpi	69	112	76	24	33	104	177	251	0.971	none
inflation	69	0.035	0.028	−0.004	0.016	0.029	0.043	0.135	0.159	7
inflation3y	69	0.035	0.025	0.003	0.020	0.028	0.042	0.117	0.087	10
tnote20	69	0.059	0.028	0.022	0.039	0.054	0.077	0.137	0.014	17
tnote1	69	0.047	0.033	0.001	0.020	0.045	0.065	0.148	0.116	14
tnote10	67	0.057	0.029	0.018	0.037	0.051	0.075	0.139	0.030	17
muni20	69	0.048	0.020	0.016	0.033	0.046	0.058	0.108	0.116	11
corplt	69	0.066	0.028	0.026	0.043	0.062	0.081	0.141	0.116	11
topord	69	0.574	0.223	0.280	0.386	0.500	0.718	0.920	0.464	14
avgord	57	0.244	0.024	0.217	0.227	0.239	0.260	0.308	0.404	7
avgordtotal	57	0.133	0.015	0.101	0.125	0.131	0.147	0.163	0.246	5
munitax0credit	69	0.153	0.145	−0.257	0.110	0.186	0.232	0.381	0.029	58
munitax	69	0.261	0.061	0.089	0.229	0.254	0.293	0.437	0.333	5
gainstax	59	0.193	0.037	0.141	0.168	0.183	0.222	0.257	0.661	none
tnote1r	69	0.011	0.020	−0.021	−0.005	0.010	0.027	0.070	0.232	7
tnote1at	69	−0.001	0.017	−0.038	−0.012	0.001	0.012	0.044	0.377	3
tnote20r	69	0.023	0.019	−0.013	0.011	0.023	0.035	0.085	0.232	5
tnote20at	69	0.008	0.019	−0.046	0.000	0.012	0.018	0.056	0.362	2
interesttax	59	0.238	0.017	0.210	0.224	0.236	0.247	0.279	0.559	3
interesttaxstate	30	0.283	0.016	0.262	0.268	0.282	0.290	0.319		
tbankamt	28	269	118	115	182	257	326	582	1.000	none
tinsuranceamt	28	200	64	107	151	196	240	334	1.000	none
tpensionlocalamt	28	171	26	128	151	166	187	217	0.714	none
tpensionprivateamt	28	208	128	117	136	147	185	543	1.000	none
tlocalgovamt	28	453	138	241	335	430	588	697	1.000	none
ttotalprivateamt	28	5013	3014	1947	2991	3401	6999	11548	1.000	none
tipsinflation	16	0.021	0.003	0.016	0.019	0.021	0.023	0.025	0.062	7
hightaxholdings	28	0.112	0.048	0.054	0.069	0.098	0.159	0.198	0.393	4
taxexemptholdings	28	0.195	0.058	0.115	0.132	0.200	0.225	0.313	0.179	5
residualholdings	28	0.694	0.098	0.514	0.646	0.695	0.800	0.820	0.750	none
muni20minusspread	69	0.043	0.020	0.014	0.029	0.040	0.054	0.101	0.058	15

2.D.2 Data

Table 2.7
Key Series Comprising Graphs in the Paper, Quoted in Percent

	(Figure 2.3)				(Figures 2.1, 2.4, and 2.5)					
	Tax Rates				1-Year T-Note			20-Year T-Bond		
year	munitax	0credit	interesttax	topord	Nom.	Real	Txd-Real	Nom.	Real	Txd-Real
1950	40.3	34.3		84.4	1.3	−1.3	−1.8	2.4	−0.2	−1.1
1951	43.7	38.1		91.0	1.7	−1.9	−2.6	2.6	−1.1	−2.2
1952	39.1	32.9		92.0	1.8	−1.7	−2.4	2.7	−0.9	−1.9
1953	28.0	23.2		92.0	2.1	0.9	0.4	3.0	1.8	1.0
1954	29.8	23.0		91.0	0.9	0.6	0.4	2.6	2.4	1.6
1955	28.8	24.9		91.0	1.9	1.4	0.8	2.9	2.4	1.5
1956	25.5	20.2		91.0	2.8	1.3	0.6	3.1	1.6	0.8
1957	20.2	12.4		91.0	3.5	1.0	0.3	3.5	1.0	0.3
1958	22.9	16.1		91.0	2.1	−0.3	−0.7	3.5	1.1	0.3
1959	23.7	19.0		91.0	4.4	2.7	1.6	4.1	2.4	1.4
1960	25.9	19.6	22.7	91.0	3.4	2.2	1.3	4.1	2.8	1.8
1961	24.8	16.6	22.9	91.0	2.8	1.6	0.9	3.9	2.6	1.7
1962	30.2	24.3	23.6	91.0	3.1	1.9	1.0	4.0	2.8	1.6
1963	28.2	24.5	24.0	91.0	3.4	2.1	1.2	4.0	2.8	1.6
1964	29.8	26.2	21.8	77.0	3.8	2.4	1.3	4.2	2.8	1.6
1965	29.6	25.8	21.0	70.0	4.1	2.1	0.9	4.3	2.3	1.0
1966	28.5	23.0	21.8	70.0	5.2	2.7	1.2	4.8	2.3	0.9
1967	32.1	25.4	22.4	70.0	4.9	1.5	−0.0	5.0	1.6	0.0
1968	32.1	23.2	24.8	75.2	5.7	1.5	−0.3	5.5	1.2	−0.4
1969	22.6	14.1	26.2	77.0	7.1	1.8	0.3	6.3	1.1	−0.3
1970	23.9	10.9	24.9	71.8	6.9	1.6	0.1	6.9	1.6	0.0
1971	29.3	14.6	23.4	70.0	4.9	0.4	−1.0	6.1	1.6	−0.1
1972	30.1	16.1	23.5	70.0	5.0	0.4	−1.1	6.0	1.4	−0.4
1973	32.9	29.8	24.6	70.0	7.3	0.5	−1.8	7.1	0.3	−1.9
1974	31.2	26.9	25.6	70.0	8.2	−0.5	−2.9	8.1	−0.7	−3.0
1975	27.2	21.6	25.3	70.0	6.8	−1.7	−3.4	8.2	−0.4	−2.4

(Table continued on following page.)

	(Figure 2.3)				(Figures 2.1, 2.4, and 2.5)					
	Tax Rates				1-Year T-Note			20-Year T-Bond		
year	munitax	0credit	interesttax	topord	Nom.	Real	Txd-Real	Nom.	Real	Txd-Real
1976	33.1	28.3	25.7	70.0	5.9	−1.2	−3.0	7.9	0.7	−1.7
1977	35.2	32.2	26.5	70.0	6.1	−0.5	−2.5	7.7	1.0	−1.6
1978	36.8	35.0	27.6	70.0	8.3	−0.1	−2.9	8.5	0.0	−2.8
1979	38.5	36.5	26.9	70.0	10.7	−0.1	−3.8	9.3	−1.3	−4.6
1980	33.9	30.8	27.9	70.0	12.0	0.3	−3.4	11.4	−0.3	−3.8
1981	26.1	23.8	27.7	69.1	14.8	4.4	0.9	13.7	3.4	0.2
1982	21.5	16.1	26.0	50.0	12.3	5.4	2.9	12.9	6.0	3.4
1983	26.9	22.4	24.5	50.0	9.6	4.8	2.4	11.3	6.5	3.6
1984	24.4	23.0	25.6	50.0	10.9	7.0	4.4	12.5	8.5	5.6
1985	24.4	21.6	26.0	50.0	8.4	5.0	3.0	11.0	7.5	4.9
1986	22.9	11.3	26.1	50.0	6.5	3.3	1.9	7.8	4.7	2.9
1987	23.7	17.6	24.2	38.5	6.8	3.5	1.9	8.7	5.3	3.3
1988	24.2	18.6	22.2	28.0	7.7	3.3	1.5	9.0	4.7	2.5
1989	24.5	20.2	22.4	28.0	8.5	3.6	1.6	8.8	3.8	1.8
1990	25.4	17.7	22.4	28.0	7.9	2.9	1.0	8.5	3.5	1.4
1991	25.1	19.4	22.3	31.0	5.9	1.6	0.2	8.1	3.8	1.8
1992	25.2	20.8	22.2	31.0	3.9	0.5	−0.5	7.7	4.1	2.3
1993	25.5	16.9	23.0	39.6	3.4	0.6	−0.3	6.5	3.5	1.9
1994	27.5	22.8	23.6	39.6	5.3	2.5	1.0	7.5	4.6	2.6
1995	23.8	16.9	23.7	39.6	5.9	3.1	1.7	7.0	4.1	2.4
1996	25.0	19.0	23.9	39.6	5.5	2.8	1.4	6.8	4.0	2.4
1997	26.9	20.6	23.8	39.6	5.6	3.3	1.8	6.7	4.3	2.6
1998	24.7	13.9	24.0	39.6	5.1	3.0	1.7	5.7	3.6	2.2
1999	24.9	14.6	24.3	39.6	5.1	2.7	1.4	6.2	3.7	2.2
2000	26.9	10.5	24.6	39.6	6.1	3.2	1.6	6.2	3.3	1.7

(Table continued on following page.)

	(Figure 2.3)				(Figures 2.1, 2.4, and 2.5)					
	Tax Rates				1-Year T-Note			20-Year T-Bond		
year	munitax	0credit	interesttax	topord	Nom.	Real	Txd-Real	Nom.	Real	Txd-Real
2000	26.9	10.5	24.6	39.6	6.1	3.2	1.6	6.2	3.3	1.7
2001	29.3	11.0	23.9	39.1	3.5	0.9	−0.1	5.6	3.0	1.4
2002	25.1	10.4	23.6	38.6	2.0	−0.2	−0.7	5.4	3.1	1.8
2003	20.1	8.6	21.7	35.0	1.2	−0.9	−1.2	5.0	2.7	1.7
2004	19.9	10.6	22.1	35.0	1.9	−0.9	−1.2	5.0	2.2	1.2
2005	18.2	7.8	22.4	35.0	3.6	0.5	−0.1	4.6	1.5	0.7
2006	25.7	16.9	22.5	35.0	4.9	1.7	0.5	5.0	1.8	0.5
2007	25.7	15.9	22.5	35.0	4.5	1.2	0.1	4.9	1.6	0.3
2008	18.4	−5.3	22.6	35.0	1.8	−0.3	−0.6	4.4	2.2	1.4
2009	19.6	−3.9	22.1	35.0	0.5	−1.2	−1.3	4.1	2.4	1.6
2010	21.0	3.2	22.1	35.0	0.3	−1.1	−1.2	4.0	2.5	1.7
2011	8.9	−17.7	22.1	35.0	0.2	−2.1	−2.1	3.6	1.3	1.0
2012	15.3	−22.3	22.2	35.0	0.2	−2.0	−2.0	2.5	0.3	−0.1
2013	18.0	−11.3	23.1	39.6	0.1	−1.6	−1.6	3.1	1.4	0.8
2014	18.2	−10.7	23.4	39.6	0.1	−0.9	−1.0	3.1	2.0	1.4
2015	17.6	−25.7	23.6	39.6	0.3	−0.7	−0.7	2.5	1.5	1.1
2016	24.1	−25.3	23.7	39.6	0.6	−0.5	−0.7	2.2	1.0	0.5
2017	18.7	−14.7	23.8	39.6	1.2	−0.7	−0.9	2.7	0.7	0.2
2018	16.5	−8.7	21.4	37.0	2.3	0.0	−0.3	3.0	0.7	0.2

These tables present the key data series used in our main analysis. The series "munitax" corresponds to the tax rate implied by the spread of 20-year AAA municipal bonds over 20-year Treasuries after adjusting for credit and liquidity using AAA 20+ year corporate bonds. "0Credit" is the same series absent the credit and liquidity adjustment. "Interesttax" contains the average tax on interest income. "Topord" is the top statutory tax rate. Real Treasury yields refer to the nominal yield less the average of the previous, current, and subsequent years' CPI inflation rates. "Txd-real" refers to the real yield after the application of the tax listed in the first column.

2.E Literature

2.E.1 Other Academic Papers Relating to Taxes and Municipal Bonds

- Elton et al. (2001) discusses the components of the corporate bond less Treasury yield spread. They use a transition matrix approach to estimate a default premium, Fama-French factors to estimate a risk premium, and a range of state tax rates to determine the tax effects. The authors quantify a significant risk premium, in addition to notable tax and default effects.

- Severn and Stewart (1992) analyze the tax effects on the Treasury corporate spread. They find that in most states circa 1992, investors benefit from holding state-tax deductible treasuries. However, the implied tax rate determined from the spread varies widely over their sample range.

- Elton and Green (1998) study tax and liquidity effects in the pricing of treasuries using inter-dealer data. They identify a small on-the-run premium due to their value in the repo market. Their analysis also uncovers limited evidence of a small tax effect on prices.

- Green and Odegaard (1997) test a null hypothesis of no tax effect in the relative pricing of treasuries by estimating a structural model. They find evidence of a tax effect prior to 1986 but no tax effect in subsequent years. Their tests exploited the difference in the tax treatments of discount and premium bonds, with the difference mostly eliminated from 1986 tax legislation.

- Elton, Gruber, and Blake (2005) compare the price effects of distributions made by tax-exempt closed end funds versus those of taxable closed end funds. They find ceteris paribus that the price drops by a greater amount for tax-exempt funds relative to taxable funds, with the price of tax-exempt funds falling by more than the dividend. The authors also determine that tax estimates from the implied dividend rate vary with the capital gains rate.

- Chalmers (1998) takes an innovative approach in isolating the default risk from municipal bond yields. Comparing the yields of pre-refunded default free municipal bonds with ordinary municipal bonds, he finds that default risk does not explain the difference in after-tax yields between Treasuries and municipal bonds. Municipal bond yields are generally higher than would be predicted by default risk and tax effects.

- Starks, Yong, and Zheng (2006) examine the behavior of investors in municipal bond closed-end funds near year end. Their regressions associate January effect abnormal returns with tax-loss harvesting. Municipal bond CEFs are chosen to isolate tax-sensitive investors. The results support the tax sensitivity of municipal bond fund investors.

- Elton and Gruber (1970) create a parsimonious model for the effect of taxes on price variation at the time of a dividend distribution. They find that the drop increases with the capital gains rate and decreases with the ordinary tax rate. The authors interpret the result as a clientele effect, where investors pick firms with dividend policies that correspond with the respective investor's tax situation.

- Harris and Piwowar (2006) quantifies liquidity effects in municipal bond markets. They uncover evidence that municipal bond transactions are expensive, particularly for taxable investors. The identified costs decline with credit quality.

- Ang, Bhansali, and Xing (2010) back out implicit tax rates on municipal bonds by studying transactions of discount securities. They calculate that discount municipal bonds trade at a higher yield after accounting for default risk and liquidity effects. The results imply tax rates higher than 70% for inter-dealer transactions.

2.E.2 Some Academic Papers Emphasizing Unusually Low Interest Rates

- Maggio and Kacperczyk (2017) discuss the effect of very low interest rates on the product offerings of financial institutions. Their analysis principally concerns money market funds and the degree to which such funds "reach for yield." Generally funds affiliated with large institutions are more likely to exit the market, while funds manged by independent investment firms demonstrate increased tendency to invest in riskier assets.

- Fischer (2016) examines the persistence of the zero lower bond and the implications of the persistence on policy. He discusses the effects of negative interest rates and other central bank monetary tools. The paper also includes his views on stability regulation.

- Gust et al. (2017) quantify the impact of the zero lower bound by estimating a DSGE model. They incorporate five types of shocks into the model, including

TFP , fiscal, monetary and two types of financial shocks. Their model shows that the lower bound led to an extra 2% in output contraction given a total estimated contraction of 6%.

- Negro et al. (2017) likewise calibrate a DSGE model, focusing their analysis on financial frictions and the effects of government policy. They demonstrate that government intervention mitigated reduced a potential –5.8% drop in output to –4.4%. The authors further discuss the amplifying effects of the zero lower bound on several components of the crisis, including deflation expectations and a decline in demand.

- Gourinchas and Rey (2016) take a global perspective, analyzing the implications of low real and natural rates across advanced economies. They identify two periods of low consumption wealth ratios, including the 1920s and the 2000s. Using predictive regressions, they uncover evidence of the ratio as a leading indicator of low real rates, and further estimate that real interest rates will remain low until 2021.

- Filipović, Larsson, and Trolle (2017) present a term structure model engineered to account for the current environment of low interest rates and the issues created by the zero lower bound. The results are achieved via a Linear-Rational Square Root Model. Their approach contributes effective simulation of persistent low interest rates.

- Korinek and Simsek (2016) focuses on the effectiveness of macroprudential policies. They propose a model where tighter borrowing constraints drive the economy into a liquidity trap and further force households with accumulated borrowing to de-lever. The inefficiencies brought by the liquidity trap imply more aggressive policy measures to insure borrows and a higher inflation target.

- Dell'Ariccia, Laeven, and Suarez (2017) analyze the effect of low short term rates by measuring the riskiness of new loans from 1997-2011. They find evidence that reduced short term interest rates leads to more aggressive risk taking. Moreover, the empirical results indicate that the negative effect increases with bank capital.

- Summers (2014) famous secular stagnation theory uses the decline in the real interest rate to make the case for unconventional monetary policy. He begins by arguing that economic growth over the past several decades failed to meet expectations. He then connects the underwhelming post-crisis recovery with a reduction in the real interest rates. The analysis drives a recommendation to boost demand via private and public investment.

- Eggertsson, Mehrotra, and Robbins (2017) quantifies Summers's hypothesis of a low negative real interest rate with an overlapping generations model. They find a natural rate between –1.5% and –2.2% and further simulate a permanently negative neutral rate using standard macro parameters. Major contributors to the decline in the natural rate include reductions in productivity growth and the fertility rate.

- Holston, Laubach, and Williams (2017) apply a Kalman filter to economic data in order to estimate, among other measures, the natural rate of interest. They identify a decline in the neutral interest rate in the US and three other developed economies. Their approach estimates that the neutral rate fell in the US between 1.5% and 2% between 2007 and 2016. Their work implies a greater frequency of periods where monetary policy is constrained by the lower bound.

- Mehrotra (2017) studies how low growth and real interest rates influence the cost of debt servicing. He finds that the real rate of interest is less than GDP growth, suggesting a negative debt service cost. The results are tempered by his calculation that debt servicing costs could turn positive with approximately a 30% probability.

- Taylor (2014) uses the Taylor rule as a benchmark to posit that the Fed held rates too low for too long before the crisis. Moreover, the low short-term rates fueled origination of adjustable rate mortgages with low teaser rates. Taylor further claims that increased regulation, quantitative easing, and zero-rate forward guidance impaired the post-crisis recovery.

- Cochrane (2013) examines the results of New Keynesian DSGE models, and finds that the policy recommendations following a period of negative interest rates can change depending on the selected equilibrium. His critique centers on the premise that the choice of equilibria and hence the predictions of the models are suspect.

- Eggertsson and Krugman (2012) discusses the consequences of rapid deleveraging event in an economy where agents have substantial debt. Their results suggest that in a period of low interest rates, government spending should inordinately increase output. The authors advocate a higher inflation target as a policy measure for overcoming the liquidity trap.

2.E.3 Various Officials and Others Emphasizing Unusually Low Interest Rates

- Smialek and Mayeda (2017) "Boston Fed President Eric Rosengren told Bloomberg Television on Friday in Boston that he frets low rates spur a reach for yield, leaving investors more exposed to a shock."

- McGeever (2017) notes that Hyun Song Shin, head of research at the Bank for International Settlements, are pushing investors farther out on the term structure as they chase for yield.

- In Miller (2017) John Williams of the San Francisco Fed discusses how persistently low rates could incentivize investors to take greater risk in their search for yield.

- Weissmann (2016) reports on how then candidate Donald Trump stated that Yellen is using low interest rates to keep the stock market high.

- In Federal-News-Service (2014), Representative Dennis Ross (R-FL) discusses how the Fed is forcing people to buy stocks by keeping rates low. Yellen also discusses how low the interest rates acts as an incentive for individuals to invest in higher yielding securities.

- Appelbaum (2017) summarizes Yellen's discussions on how the Fed's yield lowering mechanisms aided the economy and increased growth.

- Neely (2014) from the St. Louis Fed writes on how QE reduced yields and increased the price of equities.

- In Crutsinger (2015), Yellen cites low rates of return on bonds as a cause of high equity prices.
- Hilsenrath (2016) writes in the Wall Street Journal that Fed officials believe low rates may cause investors to under price risk and thus create a financial bubble.
- Belz (2014) reports how Narayana Kocherlakota of the Minneapolis Fed views low interest rates as one of many contributors to high asset prices.
- Powell (2017) Powell's speech to the 77th Annual Meeting of the American Finance Association includes the view that low rates have supported asset prices, albeit not to the point of creating a bubble. He also discusses how long-term nominal and real rates have declined for the past 30 years.

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