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UNIVERSITY OF CALIFORNIA,
IRVINE

Automated Identification of Near-Stationary Traffic States and Calibration of Unifiable
Multi-Lane Multi-Class Fundamental Diagrams

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Civil and Environmental Engineering

by

Qinglong Yan

Dissertation Committee:
Professor Wenlong Jin, Chair
Professor R. Jayakrishnan
Professor Stephen Ritchie

2018

DEDICATION

To my family

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CURRICULUM VITAE

Qinglong Yan

EDUCATION

Doctor of Philosophy in Transportation Systems Engineering	2018
University of California, Irvine	<i>Irvine, California</i>
Master of Science in Statistics	2018
University of California, Irvine	<i>Irvine, California</i>
Master of Science in Transportation Systems Engineering	2014
University of California, Irvine	<i>Irvine, California</i>
Bachelor of Engineering in Engineering Mechanics	2012
Wuhan University	<i>Wuhan, Hubei</i>

RESEARCH EXPERIENCE

Graduate Research Assistant	2014–2018
University of California, Irvine	<i>Irvine, California</i>

TEACHING EXPERIENCE

Teaching Assistant	2016–2017
University of California, Irvine	<i>Irvine, California</i>

PROFESSIONAL EXPERIENCE

Algorithm Developer Intern	2017–2018
CLR Analytics Inc.	<i>Irvine, California</i>
Traffic Engineer Intern	2013–2014
Irvine Traffic Research and Control Center	<i>Irvine, California</i>

REFEREED JOURNAL PUBLICATIONS

Yan, Q.-L., Sun, Z., Gan, Q.-J., and Jin, W.-L. (2018). Automatic identification of near-stationary traffic states based on the PELT changepoint detection. *Transportation Research Part B: Methodological*, 108, 39-54

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Yan, Q.-L. and Jin, W.-L. (2016). Automatic near-stationary traffic state identification based on PELT changepoint detection. Presented at Transportation Research Board 95th Annual Meeting. (No. 16-2847)

PROFESSIONAL AFFILIATIONS

Student Member, Transportation Research Board

ABSTRACT OF THE DISSERTATION

Automated Identification of Near-Stationary Traffic States and Calibration of Unifiable
Multi-Lane Multi-Class Fundamental Diagrams

By

Qinglong Yan

Doctor of Philosophy in Civil and Environmental Engineering

University of California, Irvine, 2018

Professor Wenlong Jin, Chair

Experience of daily commuters shows that stationary traffic patterns can be observed during peak periods in urban freeway networks. Such stationary states play an important role in various traffic flow studies. Conceptually, studies on the impact of capacity drop and design of traffic control strategies have been built on the assumption of stationarity. Mathematically, the existence and stability of stationary states in general road networks have been proved. Empirically, near-stationary states have been utilized for calibration of fundamental diagrams and investigation of traffic features at freeway bottlenecks. Therefore, an imperative need for real-world near-stationary data has been realized to better understand, investigate, and explore such above studies. However, there lacks an efficient method to identify near-stationary states.

To fill the gap, in this research, an automated method has been developed to efficiently identify near-stationary states from large amounts of inductive loop-detector data. The method consists of four steps: first, a data pre-processing technique is performed to select healthy datasets, fill in missing values, and normalize vehicle counts and occupancies; second, a PELT changepoint detection method is adopted to detect changes in means and partition time series into candidate intervals; third, informative characteristics of each candidate,

including duration and gap, are defined and calculated; finally, near-stationary states are selected from candidates through duration and gap criteria.

A game theory approach is further designed to directly calibrate parameters of the above method. First, a multi-objective optimization problem is formulated to consider the quantity and quality of near-stationary states as the objective functions. Then the problem is converted into a non-cooperative game with at least one Nash equilibrium. To solve the game and obtain a unique solution, an alternated hill-climbing search algorithm is developed.

Furthermore, two calibration schemes for multi-lane and multi-class fundamental diagrams are respectively designed by utilizing near-stationary states. Such multi-commodity fundamental diagrams possess unifiable and non-FIFO properties and can capture interaction among different commodities. Calibration and validation results show that both the calibrated unifiable multi-lane and multi-class fundamental diagrams are well-fitted, physically meaningful, and have robust performance on the estimation of commodity flow-rates.

Chapter 1

Introduction

1.1 Research background

Experience of daily commuters shows that stationary traffic states exist during peak periods in urban freeway networks. Such stationary patterns can be observed from snapshots of speed profiles in the Los Angeles freeway network during the morning peak hours on July 17, 2018, as shown in Figure 1.1. As can be seen, in the network, congested segments, sizes of queues, and bottleneck locations remain relatively time-independent during the peak period from 7:30 to 9:00 a.m.

Stationary states play an important role in various transportation studies. Conceptually, traffic patterns have been assumed to be stationary in many studies on analysis, control, management, planning, and design of road networks during peak periods, such as modeling of capacity drop at lane-drop, sag, and tunnel bottlenecks (Jin, 2017b,a, 2018) and traffic flow dynamics at merging junctions (Jin and Zhang, 2003; Jin, 2010), development of the unifiable multi-commodity kinematic wave theory (Jin, 2017d), design of traffic control strategies (Payne and Thompson, 1974; Yang and Yagar, 1995; Yang and Lam, 1996), and postulation

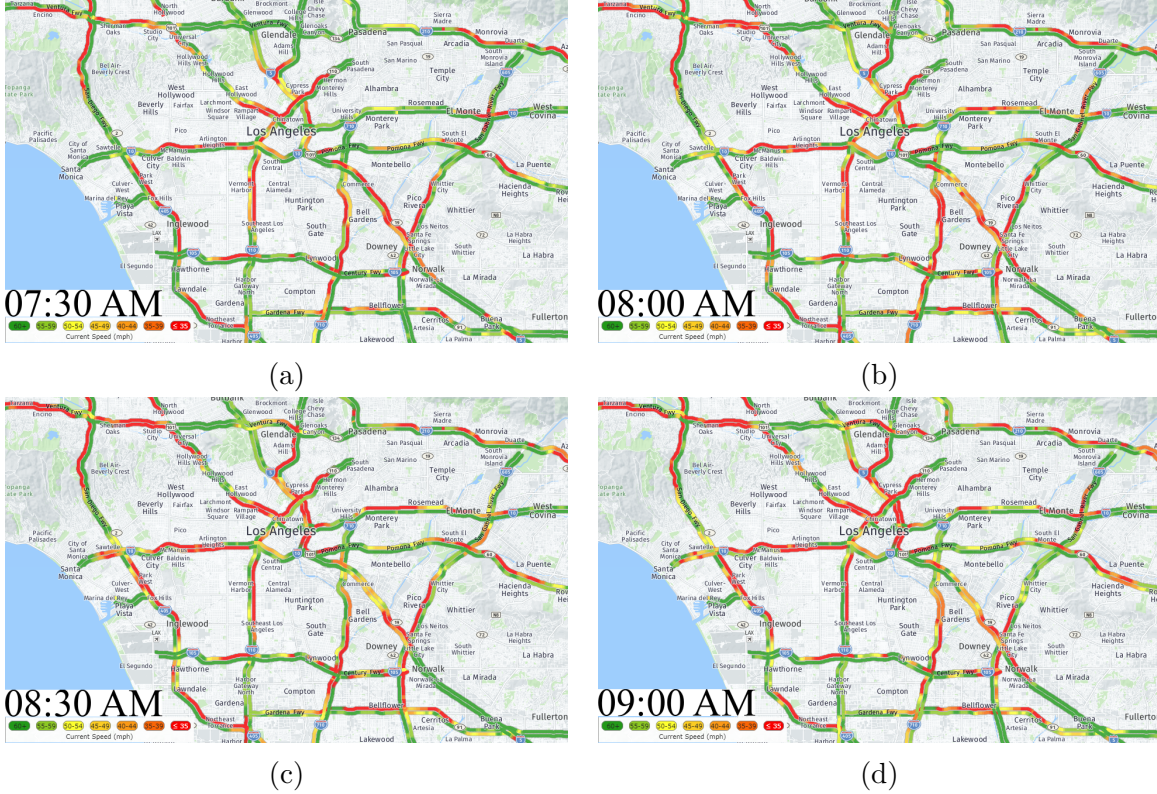


Figure 1.1: Stationary traffic patterns in the Los Angeles freeway network during the morning peak period (7:30-9:00 a.m.) on July 17, 2018.

of network-wide macroscopic fundamental diagrams (Godfrey, 1969). Mathematically, the existence and stability of stationary states in general road networks have been proved by a series of recent studies (Jin, 2012, 2015, 2017c). Empirically, near-stationary states have been widely utilized for calibration of fundamental diagrams (Del Castillo and Benitez, 1995; Cassidy, 1998), investigation of traffic features at freeway bottlenecks (Cassidy and Bertini, 1999), quantification of capacity drop magnitudes (Jin et al., 2015), detection of freeway incidents (Lin and Daganzo, 1997), evaluation of freeway merge ratios (Cassidy and Ahn, 2005; Bar-Gera and Ahn, 2010), and estimation of vehicle left-lane changes in laterally unbalanced traffic (Gan and Jin, 2015).

Therefore, an imperative need for real-world near-stationary data has been realized to better understand, investigate, and explore the above literature. However, existing studies on the identification of near-stationary states (Del Castillo and Benitez, 1995; Cassidy, 1998) are

primarily designed by visual and manual inspection and thus are time-consuming and labor-intensive when dealing with large amounts of data across numerous days, so there lacks an automated method that can be applied to efficiently identify near-stationary states from inductive loop-detector data.

In particular, traffic on freeways can be categorized into multiple commodities according to their associated lanes and vehicle type characteristics. Different commodities have dissimilar impacts on the mobility, safety, environmental and human health, and costs (Ramaswamy and Ben-Akiva, 1990; Dong et al., 2015). In the near future, as connected and autonomous vehicles become increasingly prominent in traffic streams, their improvement on lane performance and influence on other types of vehicles will also be worth investigating. Theoretically, many multi-commodity traffic behavioral and dynamics models (Newell, 1998; Daganzo, 2002a,b; Wong and Wong, 2002; Laval and Daganzo, 2006; Jin, 2017d) have been developed to study traffic flow phenomena among different commodities, such as the moving bottleneck and lane-changing maneuver. Therefore, a well-defined multi-commodity fundamental diagram model serves as an important starting point to better understand the above studies. However, existing studies on the calibration of multi-lane and multi-class fundamental diagrams (Li and Zhang, 2011; Coifman, 2014b) neglect two important factors: (i) the unifiable property such that a fundamental diagram of total traffic should also exist as well as multi-commodity fundamental diagrams and (ii) interaction among multiple commodities, so there lacks a well-designed calibration scheme for unifiable multi-lane and multi-class fundamental diagrams under near-stationary states.

1.2 Research objective

This research aims to develop a complete methodology to efficiently identify near-stationary traffic states from real-world data and design a calibration scheme for unifiable multi-lane

and multi-class fundamental diagrams under near-stationary states. More specifically, the objectives are summarized as follows:

- **Develop an automated method that can be applied to efficiently identify near-stationary states from inductive raw loop-detector data.**

The method should be performed automatically and also computationally efficient. In addition, near-stationary states should possess the features such that (i) both vehicle counts and occupancies are nearly constant over time and (ii) durations last sufficiently long in both free-flow and congested periods, which can become the identification criteria. However, the challenge is to first automatically partition time series into multiple candidate intervals for the further identification.

- **Develop a well-designed approach to calibrate parameters appearing in the above method.**

To form a complete methodology, parameter calibration is a critical step. In the process of calibrating parameters, two important features of near-stationary states should be considered: one is the quantity as the above method is expected to identify as many existing near-stationary states from daily time series as possible; the other is the quality because identified near-stationary data should have excellent performance on the calibration of fundamental diagrams. With both the features, a multi-objective optimization problem can actually be constructed for parameter calibration. However, the challenge is to efficiently solve the problem and obtain a unique solution without imposing any subjective preference on the quantity and quality.

- **Develop a calibration scheme for unifiable multi-lane fundamental diagrams in near-stationary states.**

The objective is to design a scheme for calibration of multi-lane fundamental diagrams in near-stationary states. Such multi-lane fundamental diagrams should possess

unifiable and non-FIFO properties. In addition, the interaction among different lanes should be reflected such that the speed on one lane should not only be dependent of its own density but also other lane densities. Before calibration, it is important to empirically verify both the properties and a validation procedure is also needed to evaluate the performance of calibrated unifiable multi-lane fundamental diagrams.

- **Develop a calibration scheme for unifiable multi-class fundamental diagrams in near-stationary states.**

The calibration scheme designed above should also be applicable to unifiable multi-class fundamental diagrams. However, conventional inductive loop detectors cannot measure traffic flow characteristics of different types of vehicles. To address the issue, an emerging inductive signature data source and related technologies can be adopted to estimate individual vehicle speeds and classifications and then generate traffic flow characteristics of multiple vehicle classes.

1.3 Research outline

This dissertation research is organized into seven chapters. Chapter 1 introduces the background and objectives of the research. Chapter 2 provides literature reviews with respect to existing studies on the identification of near-stationary states and calibration of multi-lane and multi-class fundamental diagrams.

Chapter 3 proposes an automated method for efficiently identifying near-stationary states from inductive loop-detector data. The chapter starts with definitions of steady, stationary, and equilibrium states and discussions of their logical relationships. Then a four-step method is developed to automatically identify near-stationary states: first, a data pre-processing technique is performed to select healthy datasets, fill in missing values, and normalize vehi-

cle counts and occupancies; second, the PELT changepoint detection method is applied to detect changes in means and partition time series into multiple candidate intervals; third, informative characteristics of each candidate interval are defined and calculated; finally, near-stationary states are selected from candidates through duration and gap criteria. In addition, two (direct and indirect) verification methods are provided to verify the validity of identified near-stationary states. Finally, a comparison between near-stationary and consecutive time series data with various aggregation intervals is made on the calibration of fundamental diagrams.

Chapter 4 proposes a game theory approach to directly calibrate parameters of the above four-step method, and this work will further automate the method and make it more widely applicable. First, two critical parameters, the penalty ratio and gap threshold, are calibrated by solving a multi-objective optimization problem with respect to two conflicting, unimodal objectives: the number of near-stationary states (quantity) and linearity of the near-stationary pattern (quality) in the congested regime. Then we convert the problem into two non-cooperative game with two players and define Nash equilibrium solutions in the context of the game. To solve the game and obtain optimal parameters, we develop an alternated hill-climbing search algorithm to imitate the game process and efficiently search for a unique Nash equilibrium solution. In an extended paradigm, we further build a five-player game to additionally calibrate minimum acceptable durations of near-stationary states.

Chapter 5 proposes a calibration scheme for multi-lane fundamental diagrams in near-stationary states. First, the theories of unifiable multi-commodity fundamental diagrams with absolute and relative speed ratios are reviewed, respectively. Then some guidelines for verification of unifiable and non-FIFO properties are provided. In a case study, we empirically verify the validity of unifiable and non-FIFO properties, calibrate a fundamental diagram of total traffic, and fit absolute and relative speed ratios. Finally, the calibrated unifiable multi-lane fundamental diagrams by two different formulations are compared with

the observed lane flow-rates in near-stationary states.

Chapter 6 adopts the same calibration scheme for unifiable multi-class fundamental diagrams. To create traffic flow characteristics of different types of vehicles, we adopt inductive signature data as the data source and apply related signature-based technologies to individual vehicle speed estimation and vehicle type classification. Then Edie's formulas are adopted to convert individual vehicle data into informative time series data of multiple vehicle classes. Finally, similar calibration and validation procedures as above are performed to calibrate unifiable multi-class fundamental diagrams and validate their performance on the estimation of near-stationary flow-rates of multiple vehicle classes.

Chapter 7 finally summarizes the dissertation and discusses some future research directions.

Chapter 2

Literature Review

In this chapter, a review of existing methods for identifying near-stationary states is presented. In addition, existing methods for calibration of multi-lane and multi-class fundamental diagrams are reviewed.

2.1 Near-stationary state identification

Stationary and steady states are literally similar but have distinct definitions in different fields. In mathematics and statistics, a stationary time series is one whose statistical properties such as the mean, variance, and autocorrelation do not change over time (Hamilton, 1994). In chemistry and other related disciplines, a steady state is a situation in which all state variables of a system or a process are constant over time (Cao and Rhinehart, 1995). Here the concept of steady states is less strict than of stationary states above as it only requires the mean to be constant. Unlike the above fields, both stationary and steady states exist in traffic flow theories and are defined in a space-time domain, where a stationary state is a state such that all traffic flow characteristics, including the flow-rate, density, and speed,

in the space-time domain are time-independent, and a steady state is achieved if all the traffic flow characteristics are constant (both time- and location-independent) in the space-time domain.

In reality, however, measurements cannot be strictly constant over space or time because of inevitable noise and disturbances in real-world and experimental data. Therefore, it is almost impossible to find strictly steady or stationary states, and states can only be near-steady or near-stationary.

Among existing methods for detecting near-steady states, a majority of them are designed on the basis of a common structure, called the moving window strategy, and the only difference is that different methods have different detection criteria loaded on the moving window. For the strategy, it is carried out by the following steps: (1) initially, a window with a minimum acceptable length starts at the first node of a time series process; (2) if the near-steady-state criterion is not met, the window will be moved forward to append the newest node and discard the oldest one; (3) if the criterion is met, the window will be expanded by appending new incoming nodes as long as the criterion continues to be met. Once the criterion is not met, the near-steady state detected in the previous time step and relevant parameters will be archived, and the window will be reset and moved to cover the most recent new nodes. The above steps can be visualized in Figure 2.1. Note that a challenge of applying the strategy is to choose an appropriate window length. Users have to define a length longer than autocorrelation persistence. If the window is short, noise in the data would have an adverse effect on the criteria. However, if the window is long, recognition of changes and detected near-steady states would be delayed, and the computational effort would also be increased.

For the methods that adopt the Student's t-test as the detection criterion, in (Holly et al., 1989), a linear regression was applied to a data window and then a t-test was performed on the regression slope. If the slope was significantly different from zero, the process was not

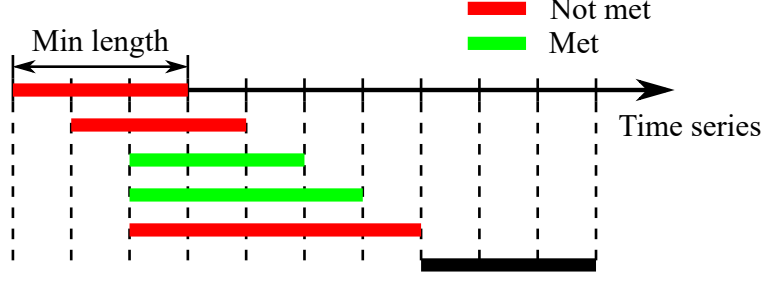


Figure 2.1: Process of the moving window strategy.

in a near-steady state. Alternatively, in (Narasimhan et al., 1987), data from two recent moving windows were considered as two independent samples and the corresponding means and pooled standard deviation were calculated. Then a t-test was performed to testify if the means were unchanged. In addition, Kelly and Hedengren (2013) adopted a residual Student's t-test to identify near-steady states if over 95% of the values in the window failed to be significantly different from the mean.

Another method (Crow et al., 1960) is to perform the F-test type statistic, a ratio of two different estimates of variance on the same set of data. Inside the most recent window, the average is first calculated; the numerator estimate of variance is calculated as the mean-square deviation from the average, and the denominator estimate of variance is calculated as the mean of squared differences of successive data. If the process is near-steady, the statistic would be equal to one. Later, (Cao and Rhinehart, 1995, 1997) modified the above method by incorporating an exponentially weighted moving-average filter to calculate the mean and two different estimates of variance, where the numerator variance is calculated based on the filtered squared deviation from the previously filtered value and the other based on the filtered squared difference of successive data. By doing so, data are treated sequentially for the detection of near-steady states without the need to select a data window.

In (Kim et al., 2008), the standard deviation estimated in a data window was used as a criterion such that the process could be identified as a near-steady state if all the values fell within plus or minus three standard deviation limits of the mean. Such a method is also a

type of statistical process control (Oakland, 1986). In (Simon and Litt, 2011), a data window can be detected as a near-steady state if the standard deviation did not exceed a predefined threshold.

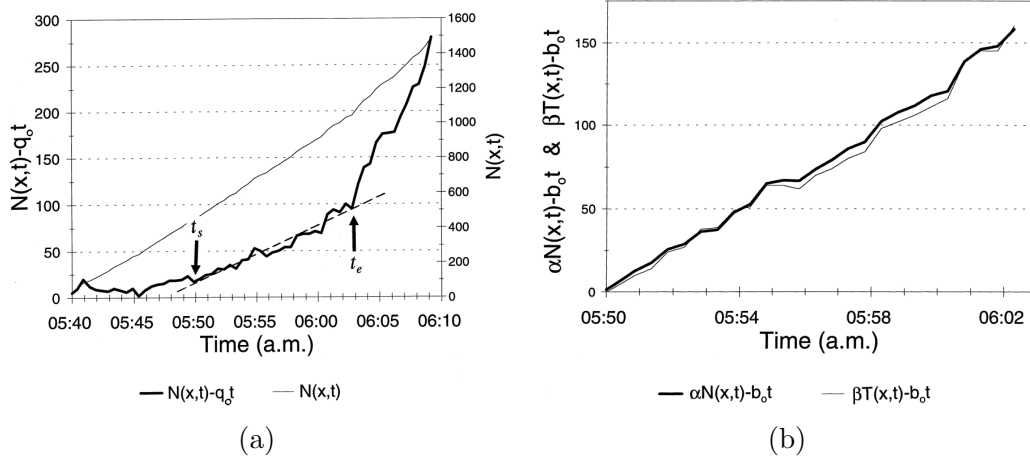


Figure 2.2: Visualization of criterion (a) one and (b) two in Cassidy's work.

In the field of traffic, existing studies on the identification of near-stationary states are usually motivated by the need for calibrating fundamental diagrams. In (Del Castillo and Benitez, 1995), potentially near-stationary intervals lasting at least 4 or 5 minutes were first visually selected from speed series and verified by the criterion such that the standard deviation was less than 15% of the mean. Then, a nonparametric test based on Kendall's tau-test was carried out to further extract near-stationary states whose speeds and vehicle counts were trend-free. In addition, Cassidy (1998) proposed a method to manually identify near-stationary states from each consecutive 30-min period through the following three criteria: (i) cumulative total vehicle counts subtracting a linear function of time did not create any deviation more than 10 vehicles from their best-fit line, as shown in Figure 2.2a; (ii) the transformed curves of cumulative total vehicle counts and occupancies were approximately superimposed, as shown in Figure 2.2b; and (iii) the interval should last at least 4 minutes in high-occupancy periods and 10 minutes otherwise. However, since the aforementioned methods use visual inspection and are time-consuming and labor-intensive when dealing with a large amount of data across numerous days, there lacks an automated method that

can be applied to efficiently identify near-stationary states.

2.2 Calibration of multi-lane and multi-class fundamental diagrams

Traffic on roads can be categorized into multiple commodities according to their lanes and vehicle classes. For an M -commodity traffic, the flow-rate, density, and speed of commodity m are denoted by q_m , k_m , and v_m , respectively, where $m = 1, \dots, M$. Correspondingly, the flow-rate, density, and speed of total traffic are denoted by q , k , and v , respectively. In addition, the commodity density proportion is denoted by $p_m = k_m/k$, absolute speed ratio by $\eta_m = v_m/v$, and absolute speed ratio by $\pi_m = v_m/v_r$, where $r \in \{1, \dots, M\}$ is a reference commodity.

Two important properties exist in the multi-commodity traffic: one is the FIFO property such that all commodities have the same speed at the same time and location, i.e., $v_1 = \dots = v_M$; the other is the unifiable property such that there exists a bivariate fundamental diagram between flow-rate and density of total traffic, i.e., $q = Q(k)$. In addition, commodities should be mutually interacted such that one commodity speed is dependent of not only its own commodity density but also the other commodity densities, i.e., $v_m = V_m(k_1, \dots, k_M)$.

Among existing studies on modeling multi-lane and multi-class traffic, most of them are neither FIFO nor unifiable and neglect mutual interaction among commodities for simplicity (Munjal and Pipes, 1971; Michalopoulos et al., 1984; Daganzo, 2002a; Wong and Wong, 2002; Ngoduy and Liu, 2007). The general form of multi-commodity fundamental diagrams in such above studies can be written by

$$q_m = Q_m(k_m), \quad \forall m = 1, \dots, M. \quad (2.1)$$

Jin (2017d) proposed a novel multi-commodity kinematic wave model in which the multi-commodity traffic is unifiable and may violate the FIFO property. In addition, the proposed model can capture interaction among different commodities. The general form of the unifiable multi-commodity fundamental diagrams can be written by

$$q_m = p_m \cdot Q(k) \cdot H_m(k, p_1, \dots, p_{M-1}), \quad \forall m = 1, \dots, M, \quad (2.2)$$

which is comprised of three multiplicative components: commodity density proportion, fundamental diagram of total traffic, and commodity absolute speed ratio function, where the last one is formulated with respect to the total density and $M - 1$ commodity density proportions.

In (Jin and Yan, 2018), the above unifiable multi-commodity fundamental diagrams were reformulated in terms of commodity relative speed ratios, which are physically, economically, and behaviorally meaningful since they can characterize relative aggressiveness and values of times among drivers:

$$q_m = \frac{p_m \Pi_m(k, p_1, \dots, p_{M-1})}{\sum_{m'=1}^M p_{m'} \Pi_{m'}(k, p_1, \dots, p_{M-1})} \cdot Q(k), \quad \forall m = 1, \dots, M. \quad (2.3)$$

Empirically, only a handful of existing studies focus on the calibration of multi-lane and multi-class fundamental diagrams. In (Li and Zhang, 2011), equilibrium states were first identified from time series loop-detector data and then multi-lane fundamental diagrams were fitted by following (2.1). Specifically, a piecewise linear fit was adopted to fit the identified equilibrium states. The calibrated multi-lane fundamental diagrams do not possess the unifiable property and cannot capture mutual interaction among different lanes.

In (Coifman, 2014b), vehicles were measured individually and assigned into bins with similar lengths and speeds. Then the multi-class fundamental diagrams were fitted by connecting

median points across all speed bins for each given vehicle length. The calibrated multi-class fundamental diagrams still follow (2.1) without considering the unifiable property and mutual interaction among different vehicle classes.

Chapter 3

A Four-Step Method for Identifying Near-Stationary Traffic States

3.1 Introduction

Both observations and theories confirm that traffic states are relatively stationary during peak periods on an urban road network (Jin, 2015). The existence of such stationary states has been an underlying assumption of many studies on analysis, operations, control, and management of transportation networks, such as detection of freeway incidents and active bottlenecks in (Lin and Daganzo, 1997; Cassidy and Bertini, 1999), quantification of capacity drop and its impacts at lane-drop bottlenecks in (Jin et al., 2015; Jin, 2017b), estimation of the number of vehicle left lane changes in (Gan and Jin, 2015), and calibration of lane-specific fundamental diagrams and lane flow distributions in (Li and Zhang, 2011; Duret et al., 2012). In addition, the static traffic assignment problem was formulated to determine link flow patterns and travel times based on the assumption that the network was in stationary states (Beckmann et al., 1956; Sheffi, 1985). Some traffic control strategies, such as ramp metering

(Payne and Thompson, 1974), signal control (Yang and Yagar, 1995), and congestion pricing (Yang and Lam, 1996), were also developed in such stationary networks. Godfrey (1969) postulated a macroscopic fundamental diagram when traffic was stationary on a signalized urban network, and it has been verified later by Geroliminis and Daganzo (2008). In (Jin and Zhang, 2003; Jin, 2010), traffic flow at merging junctions was studied in stationary states, and such stationary patterns have been visually observed by Cassidy and Ahn (2005) and Bar-Gera and Ahn (2010). In a series of recent studies by Jin (2012, 2015, 2017c), the existence and stability of stationary states in general road networks have been proved within the framework of kinematic wave theories.

Therefore, to identify stationary states from field data has become a major task prior to empirically studying such literature. An appropriate data source for identifying stationary states is raw data measured by inductive loop detectors. Such data comprise of time series of traffic flow measurements for each lane in short consecutive intervals, e.g., 30 seconds (Hall, 1996; Chen et al., 2001). However, strictly stationary states do not exist in the raw loop-detector data due to the inevitable noise and fluctuations. Therefore, in practice, stationary states can only be nearly stationary.

Existing studies on the identification of such near-stationary states are usually motivated by the need for calibrating fundamental diagrams. In (Del Castillo and Benitez, 1995), potentially near-stationary intervals lasting at least 4 or 5 minutes were first visually selected from speed series and verified by the criterion such that the standard deviation was less than 15% of the mean. Then, a nonparametric test based on Kendall’s tau-test was carried out to further extract near-stationary states whose speeds and vehicle counts were trend-free. In addition, Cassidy (1998) proposed a method to manually identify near-stationary states from each consecutive 30-min period through the following three criteria: (i) cumulative total vehicle counts subtracting a linear function of time did not create any deviation more than 10 vehicles from their best-fit line; (ii) the transformed curves of cumulative total vehicle

counts and occupancies were approximately superimposed; and (iii) the interval should last at least 4 minutes in high-occupancy periods and 10 minutes otherwise. However, since the aforementioned methods use visual inspection and are time-consuming and labor-intensive when dealing with a large amount of data across numerous days, there lacks an automated method that can be applied to efficiently identify near-stationary states.

In this chapter, we attempt to fill this gap. To automate the identification process, one important step is to automatically partition the time series into multiple candidate intervals that can potentially be near-stationary states. We will apply a changepoint detection method based on the pruned exact linear time (PELT) search algorithm (Killick et al., 2012) to detect changes in mean values and divide the time series into candidate intervals. The mean values inside these candidate intervals are approximately constant over time. Among many existing changepoint detection methods (Scott and Knott, 1974; Auger and Lawrence, 1989; Jackson et al., 2005; Chen and Gupta, 2011), the PELT-based method is preferred because (i) it is formulated with only one parameter, the penalty threshold; (ii) the PELT search algorithm is based on dynamic programming, which can guarantee to find the global optimum; and (iii) a built-in pruning technique in the algorithm can reduce the time complexity to be linear in the number of data points without affecting the exactness of partitioning, which is computationally efficient for a large amount of data. For each candidate interval, we calculate its characteristics, including vehicle count and occupancy gaps and the duration. Then we adopt Cassidy’s criteria to pick out near-stationary states. Here we modify Cassidy’s criteria to be compatible with the PELT changepoint detection method.

The rest of the chapter is organized as follows. In Section 3.2, we present definitions of stationary, steady, and equilibrium states and their logical relationships. In Section 3.3, we present an automated four-step method for identifying near-stationary states from raw loop-detector data. In Section 3.4, we empirically verify the validity of identified near-stationary states. In Section 3.5, we compare between near-stationary and time series data

with different aggregation intervals on the calibration of fundamental diagrams. In Section 3.6, we conclude with some discussions.

3.2 Theoretical discussions on stationary, steady, and equilibrium states

In this section, we define stationary, steady, and equilibrium states and discuss their logical relationships by specific traffic scenarios. Then we provide some guidelines for identifying near-stationary states from raw loop-detector data.

3.2.1 Definitions and logical relationships

In an arbitrary space-time domain, D , as shown in Figure 3.1, the flow-rate, density, and speed at location x and time t are denoted by $q(x, t)$, $k(x, t)$, and $v(x, t)$, respectively, where $(x, t) \in D$. Given the traffic flow variables, we define stationary, steady, and equilibrium states as follows:

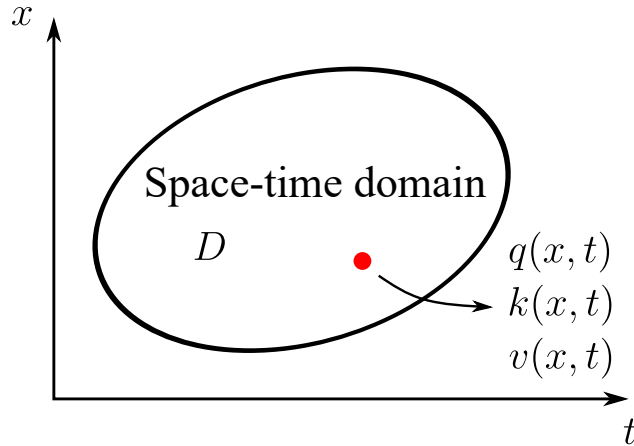


Figure 3.1: An arbitrary space-time domain.

Definition 3.1. A traffic state in the space-time domain is a *stationary state* if both flow-

rates and densities in the state are time-independent, or equivalently,

$$\frac{\partial q(x, t)}{\partial t} = \frac{\partial k(x, t)}{\partial t} = 0, \quad \forall (x, t) \in D. \quad (3.1)$$

Definition 3.2. A traffic state in the space-time domain is a *steady state* if both flow-rates and densities in the state are constant (both time- and location-independent), or equivalently,

$$\frac{\partial q(x, t)}{\partial t} = \frac{\partial k(x, t)}{\partial t} = \frac{\partial q(x, t)}{\partial x} = \frac{\partial k(x, t)}{\partial x} = 0, \quad \forall (x, t) \in D. \quad (3.2)$$

Definition 3.3. A traffic state in the space-time domain is an *equilibrium state* if the state can fall on one or generally multiple location-dependent fundamental diagrams¹, or equivalently,

$$q(x, t) = \phi_x(k(x, t)), \quad \forall (x, t) \in D. \quad (3.3)$$

Hereafter we omit (x, t) unless necessary. According to the above definitions, some inferences with respect to these three types of static states can be drawn as follows:

1. From (3.1) and the traffic conservation law, $\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0$, we have $\frac{\partial q}{\partial x} = -\frac{\partial k}{\partial t} = 0$, $(x, t) \in D$. Thus the flow-rate is constant in a stationary state.
2. Based on the traffic constitutive law, $q = kv$, the speeds are time-independent in a stationary state, and constant in a steady state.
3. A steady state is always stationary, but a stationary state may not be steady.
4. Steady states always belong to equilibrium states, i.e., a steady state always falls on a fundamental diagram, as an equilibrium flow-density relationship can be derived from

¹A fundamental diagram is assumed to be time-independent.

most first- and higher-order continuum and car-following traffic flow models in steady states (Jin, 2016). However, the statement may not be true if reversed.

Therefore, the logical relationships among stationary, steady, and equilibrium states can be described by the Venn diagram in Figure 3.2, where four marked non-overlapping regions are generated in the diagram. Examples of the four types of states are given in the following:

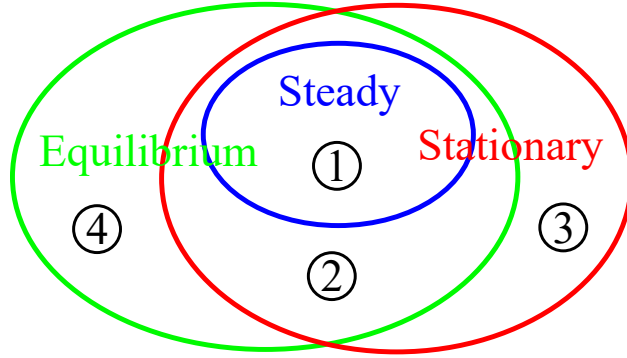


Figure 3.2: Logical relationships among stationary, steady, and equilibrium states.

- **Type 1: steady**

From a microscopic point of view, a steady state can be reached when all vehicles have the same spacing and speed in a certain space-time domain, as shown in Figure 3.3a. Such a steady state is also stationary and can always fall on an equilibrium fundamental diagram, as shown in Figure 3.3b².

- **Type 2: stationary and equilibrium but not steady**

In the LWR model (Lighthill and Whitham, 1955; Richards, 1956), a traffic state with a zero-speed shock wave always belongs to the type. As shown in Figure 3.4a, such a state is comprised of two steady states with the same flow-rate but different densities and speeds upstream and downstream of the zero-speed shock wave location, which is apparently not steady but satisfies the features of a stationary state. In addition, the state is also an equilibrium state as both of the steady states can fall on

²A triangular fundamental diagram is used as an example.

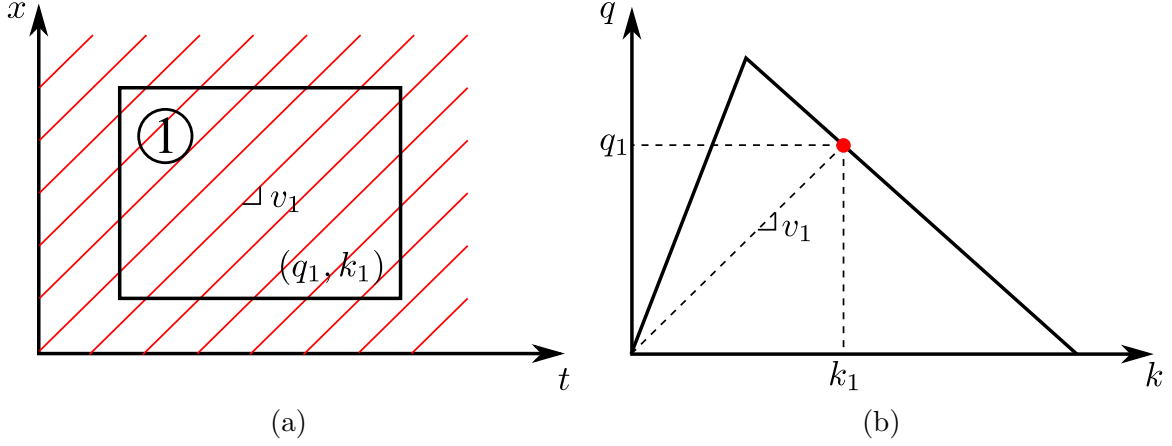


Figure 3.3: A steady state (Type 1) in (a) $x-t$ and (b) fundamental diagram

a common fundamental diagram, as shown in Figure 3.4b. As another example, for an inhomogeneous road with a continuous lane-drop bottleneck, as shown in Figure 3.5a by Jin (2017a), the traffic state within the lane-drop zone is stationary and also can fall on location-dependent fundamental diagrams. However, the state with continuously decreasing densities and increasing speeds over space is not steady. This example is shown in Figure 3.5b (red arrow).

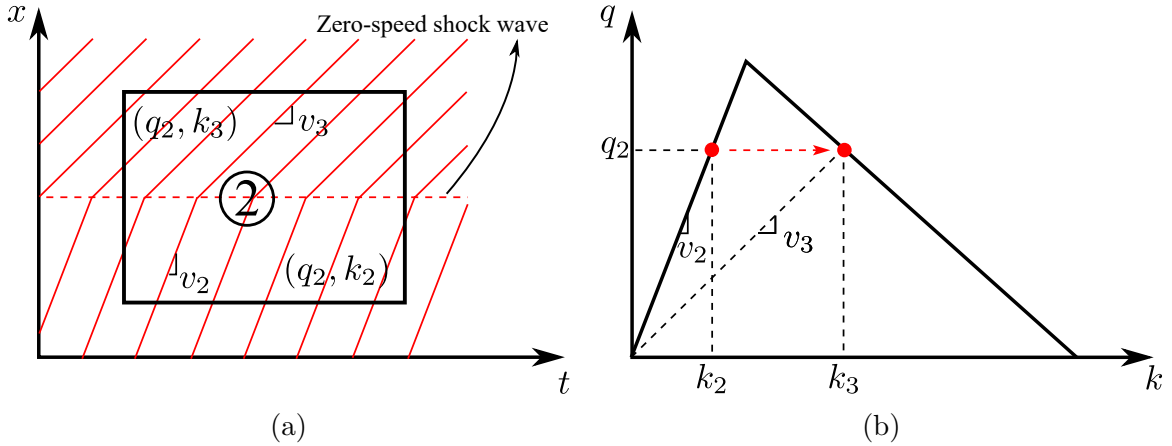


Figure 3.4: A stationary and equilibrium but not steady state (Type 2): A traffic state with a zero-speed shock wave in (a) $x-t$ and (b) fundamental diagram

- **Type 3: stationary but not equilibrium**

Again, as discussed in (Jin, 2017a), when capacity drop occurs, the traffic state within the downstream bounded acceleration zone is also stationary but not an equilibrium

state as it appears in the form a horizontal line within the fundamental diagram of the downstream link, as shown in Figure 3.5b (blue arrow).

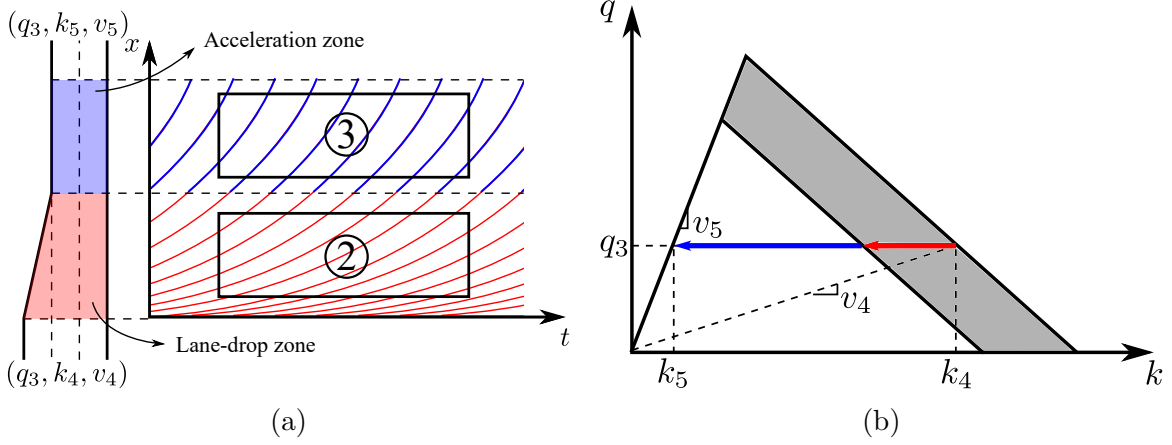


Figure 3.5: A stationary and equilibrium but not steady state (Type 2) and a stationary but not equilibrium state (Type 3) at a lane-drop bottleneck (a) $x-t$ and (b) fundamental diagrams

- **Type 4: equilibrium but not stationary**

In the LWR model, any traffic state is equilibrium and satisfies the fundamental diagram, but may not be stationary unless there exists a zero-speed shock wave or a steady state. An example is shown in Figure 3.6.

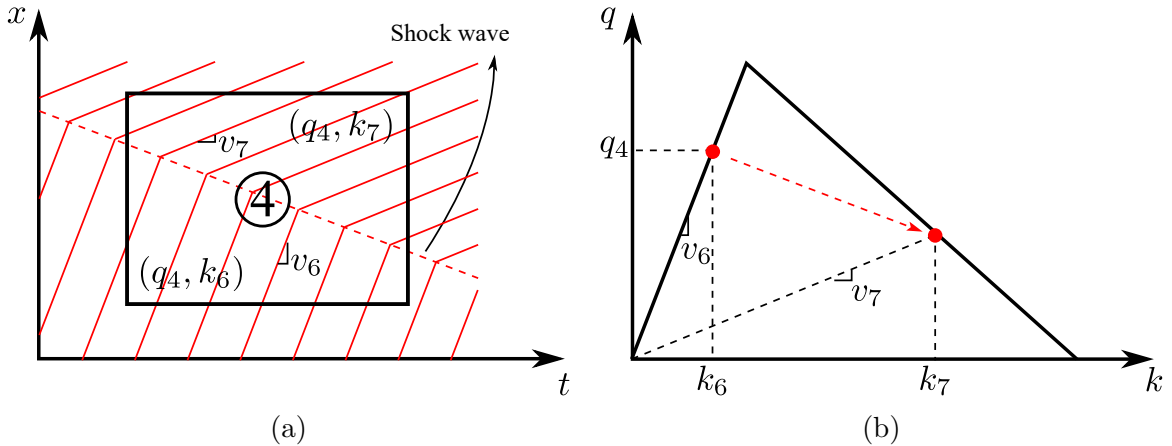


Figure 3.6: An equilibrium but not stationary state (Type 4): A general shock wave in (a) $x-t$ and (b) fundamental diagram

Note that (i) the states of types 3 and 4 can also occur inside the capacity reduction and

downstream acceleration zones of an inhomogeneous road with a sag or tunnel bottleneck, respectively, as discussed in (Jin, 2018); (ii) many other traffic states that can be described by higher-order continuum or car-following models (Jin, 2016) are neither equilibrium nor stationary.

3.2.2 Near-stationary states and some guidelines for their identification

In this chapter we are interested in identifying stationary states from raw loop-detector data. Here the raw data is already aggregated for each 30-second interval, which is considered to be a good balance between maximizing the number of vehicles and minimizing the likelihood of non-stationary states within it (Coifman, 2014b).

From Definition 3.1, a stationary state depends on the choice of a space-time domain. First, as a loop detector covers a short road segment of about 6 feet, it is natural to choose a rectangle covering the space-time domain, as shown in Figures 3.3-3.6. Second, the time interval should be sufficiently long such that the number of data points is sufficient for statistically testing the stationarity. Following (Cassidy and Windover, 1995), we set the interval to be at least 4 minutes during high-occupancy (more congested) periods and 10 minutes during low-occupancy (less congested) periods.

Compared with vehicle counts and occupancies, space-mean speeds are less oscillatory and have been utilized to identify near-stationary states Del Castillo and Benitez (1995); Li and Zhang (2011). However, the speeds are always nearly constant under non-stationary free-flow traffic conditions (Persaud and Hurdle, 1988). Thus here we follow (Cassidy and Windover, 1995) to use both vehicle counts and occupancies to characterize traffic states.

However, due to inevitable noises and fluctuations, strictly stationary states hardly exist

in reality. Thus we resort to identify near-stationary states. Also following (Cassidy and Windover, 1995), we consider a state nearly stationary if measured vehicle counts and occupancies are nearly constant, or equivalently, both cumulative vehicle count and occupancy curves are approximately linear.

As discussed in the preceding subsection, stationary states measured by a loop detector may not be on a fundamental diagram (see the stationary state of Type 3 in Figure 3.5b) or may be on multiple location-dependent fundamental diagrams (see the stationary state of Type 2 in Figure 3.5b). In these cases, the flow-density relation may not be a simple function, as illustrated in (Persaud and Hurdle, 1988). In addition, even though a loop detector covers a short segment, it cannot report vehicle counts and occupancies over space but only aggregated measurements. Thus in this chapter we choose a loop detector whose stationary states are of Type 1 (steady states) with location-independent traffic flow measurements and on a fundamental diagram. For example, near a lane-drop bottleneck shown in Figure 3.5a, a loop detector should be upstream to the bottleneck, not inside or downstream to it.

3.3 A four-step method for identifying near-stationary states

In this section, a four-step method for automatically identifying near-stationary states from raw loop-detector data is presented. The steps include: (i) data pre-processing, (ii) PELT changepoint detection, (iii) candidate interval characterization, and (iv) near-stationary state selection. A flow chart of the proposed method is shown in Figure 6.3.

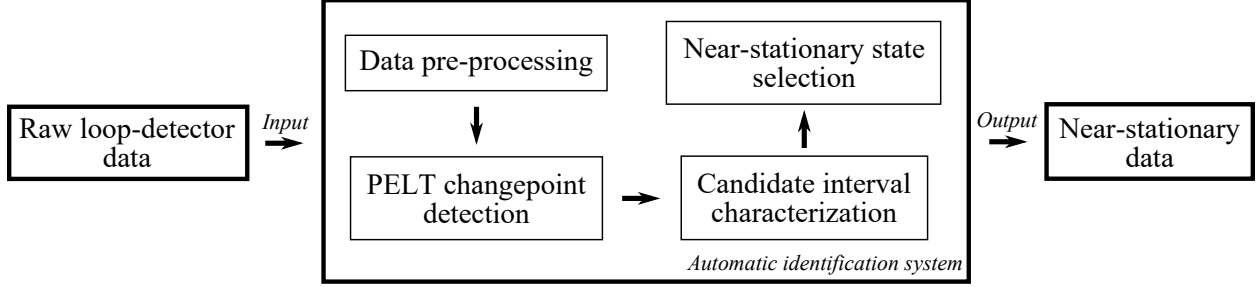


Figure 3.7: Flow chart of the proposed method.

3.3.1 Data pre-processing

Given a daily set of raw loop-detector data that contains multiple time series of lane vehicle counts and occupancies, it can be selected as a healthy dataset with sufficient congestion if satisfying all the following criteria:

- Observation rates for all lanes are higher than 95%;
- The longest hole of missing values is shorter than 3 minutes. If the criterion is met, all the missing values are filled in by the locally weighted regression scatterplot smoothing method (Cleveland, 1979) with a smoother span of 0.25;
- There exists at least one time interval with a length of at least 5 minutes such that all averaged occupancies across lanes in the interval are higher than 0.3.

Next, in the chosen dataset, lane vehicle counts and occupancies are averaged across lanes, and the averaged vehicle count and occupancy series are denoted by $c_{1:n} = (c_1, \dots, c_n)$ and $o_{1:n} = (o_1, \dots, o_n)$, respectively, where n is the sample size of each series. Then, both of the series are normalized to the same scale with a mean of zero and a standard deviation of one:

$$\tilde{c}_i = \frac{c_i - \bar{c}}{s_c}, \quad \forall i = 1, \dots, n, \quad (3.4a)$$

$$\tilde{o}_i = \frac{o_i - \bar{o}}{s_o}, \quad \forall i = 1, \dots, n, \quad (3.4b)$$

where $\bar{c}$, $\bar{o}$, s_c , and s_o are means and standard deviations of the averaged vehicle count and occupancy series, respectively; $\tilde{c}_i$ and $\tilde{o}_i$ are the normalized vehicle count and occupancy at time point i , respectively. Note that, since both normalized series have a common scale, they do not necessarily need to be differentiated and can share the same parameters in the following steps.

3.3.2 PELT changepoint detection

Suppose that a number of changepoints, $m_{\tilde{c}}$, exist in the normalized vehicle count series, $\tilde{c}_{1:n} = (\tilde{c}_1, \dots, \tilde{c}_n)$, and their ordered positions are $\tau_{1:m_{\tilde{c}}}^{\tilde{c}} = (\tau_1^{\tilde{c}}, \dots, \tau_{m_{\tilde{c}}}^{\tilde{c}})$, which are integers between 1 and $n - 1$ inclusive. In addition, two boundary changepoints are artificially added and their positions are denoted by $\tau_0^{\tilde{c}} = 0$ and $\tau_{m_{\tilde{c}}+1}^{\tilde{c}} = n$, respectively. Thus, the series is partitioned into $m_{\tilde{c}} + 1$ intervals. In interval j , where $j = 1, \dots, m_{\tilde{c}} + 1$, the normalized vehicle counts are denoted by $\tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}}$.

To detect the changepoint positions, an optimization problem is formulated with the following objective function:

$$\min \sum_{j=1}^{m_{\tilde{c}}+1} [\mathcal{C}(\tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}})] + \beta m_{\tilde{c}}, \quad (3.5)$$

where $\mathcal{C}$ is the cost function for an interval; β is the penalty parameter to avoid overfitting. In particular, a double negative log-likelihood function is used to specify the cost function. By assuming that (i) normalized vehicle counts in each interval are independent and identically

distributed as a Normal distribution, and (ii) the variance does not change across intervals³, i.e., $\tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}} \stackrel{iid}{\sim} N(\mu_{\tilde{c},j}, \sigma_{\tilde{c}}^2)$, $\forall j = 1, \dots, m_{\tilde{c}} + 1$, the cost function can be derived as follows:

$$\begin{aligned} \mathcal{C}(\tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}}) &= -2 \log L(\mu_{\tilde{c},j}, \sigma_{\tilde{c}}^2 \mid \tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}}) \\ &= -2 \log \prod_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} \frac{1}{\sqrt{2\pi\sigma_{\tilde{c}}^2}} \exp\left[-\frac{1}{2\sigma_{\tilde{c}}^2}(\tilde{c}_i - \mu_{\tilde{c},j})^2\right], \quad \forall j = 1, \dots, m_{\tilde{c}} + 1, \quad (3.6) \\ &= (\tau_j^{\tilde{c}} - \tau_{j-1}^{\tilde{c}}) \log(2\pi\sigma_{\tilde{c}}^2) + \frac{1}{\sigma_{\tilde{c}}^2} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} (\tilde{c}_i - \mu_{\tilde{c},j})^2 \end{aligned}$$

where the maximum likelihood estimate of $\mu_{\tilde{c},j}$ is

$$\hat{\mu}_{\tilde{c},j} = \frac{1}{\tau_j^{\tilde{c}} - \tau_{j-1}^{\tilde{c}}} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} \tilde{c}_i; \quad (3.7)$$

$\sigma_{\tilde{c}}^2$ is unknown but fixed.

Note that, for the finally derived equation in (3.6), the first term can be removed as it is linear in the difference of two consecutive changepoint positions and turns out to be a constant after being substituted into (3.5). In addition, the coefficient of the second term can be removed as it would not affect the optimal partitioning for any choice of $\sigma_{\tilde{c}}^2 > 0$ by appropriately tuning β . Thus, (3.6) can be simplified as:

$$\mathcal{C}(\tilde{c}_{(\tau_{j-1}^{\tilde{c}}+1):\tau_j^{\tilde{c}}}) = \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} (\tilde{c}_i - \mu_{\tilde{c},j})^2, \quad \forall j = 1, \dots, m_{\tilde{c}} + 1. \quad (3.8)$$

To solve the optimization problem and search for optimal changepoint positions, the PELT algorithm based on dynamic programming and a pruning technique⁴ is applied. Thus, the

³To detect changes in mean, the variance needs to be held constant.

⁴For the algorithm pseudocode, the reader is referred to (Killick et al., 2012)

optimal changepoint positions for the normalized vehicle count series can be obtained:

$$\hat{\tau}_{1:m_{\tilde{c}}}^{\tilde{c}} = (\hat{\tau}_1^{\tilde{c}}, \dots, \hat{\tau}_{m_{\tilde{c}}}^{\tilde{c}}). \quad (3.9)$$

The following theorem proves that, actually, (3.9) is also the optimal changepoint positions for the averaged vehicle count series through an appropriate choice of the penalty.

Theorem 3.1. *Data normalization does not affect the number and positions of detected changepoints of an original series by rescaling the penalty with a sample standard deviation.*

Proof. Here the averaged and normalized vehicle count series are taken as an example for the proof and the former can be considered as an original series. Substituting (3.7) and (3.8) into (3.5), the objective function for the normalized series can be specified as

$$\min \sum_{j=1}^{m_{\tilde{c}}+1} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} \left(\tilde{c}_i - \frac{1}{\tau_j^{\tilde{c}} - \tau_{j-1}^{\tilde{c}}} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} \tilde{c}_i \right)^2 + \beta m_{\tilde{c}}. \quad (3.10)$$

Then, substituting (3.4a) into (3.10), one equivalently has

$$\min \frac{1}{s_c} \left[\sum_{j=1}^{m_{\tilde{c}}+1} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} \left(c_i - \frac{1}{\tau_j^{\tilde{c}} - \tau_{j-1}^{\tilde{c}}} \sum_{i=\tau_{j-1}^{\tilde{c}}+1}^{\tau_j^{\tilde{c}}} c_i \right)^2 + \beta' m_{\tilde{c}} \right], \quad (3.11)$$

where $\beta' = s_c \beta$ is the rescaled penalty. Since the multiplier, $\frac{1}{s_c}$, is a positive constant, (3.11) can be considered as an objective function for the original series with a penalty of β' , which would result in exactly the same number and positions of detected changepoints as those in the normalized series. $\square$

Similarly, the above method with the same β is applied on the normalized occupancy series, $\tilde{o}_{1:n} = (\tilde{o}_1, \dots, \tilde{o}_n)$, and its optimal changepoint positions can be denoted by

$$\hat{\tau}_{1:m_{\tilde{o}}}^{\tilde{o}} = (\hat{\tau}_1^{\tilde{o}}, \dots, \hat{\tau}_{m_{\tilde{o}}}^{\tilde{o}}). \quad (3.12)$$

After that, (3.9) and (3.12) are combined and the changepoint positions are sorted in ascending order:

$$\hat{\tau}_{1:m} = \cup(\hat{\tau}_{1:m_{\tilde{c}}}^{\tilde{c}}, \hat{\tau}_{1:m_{\tilde{o}}}^{\tilde{o}}) = (\hat{\tau}_1, \dots, \hat{\tau}_m), \quad (3.13)$$

where m is the number of combined changepoints and $m \leq m_{\tilde{c}} + m_{\tilde{o}}$ as the changepoint positions from two series may overlap. Consequently, the daily time series is partitioned into $m + 1$ intervals whose means of both normalized/averaged vehicle counts and occupancies over time are approximately constant, relative to the choice of β . Such intervals are referred to as candidate intervals and have potential opportunities to be selected as near-stationary states.

3.3.3 Candidate interval characterization

For candidate interval p , where $p = 1, \dots, m + 1$, some characteristics are defined and calculated as follows:

- The *starting* and *ending positions* are calculated by $r_p = \hat{\tau}_{p-1} + 1$ and $e_p = \hat{\tau}_p$, respectively, which can be used to locate the candidate interval in a daily time series. In addition, they can also be easily converted into any time format, e.g., hh:mm:ss.
- The *length* is the number of time points in the candidate interval, which is calculated by $l_p = e_p - r_p + 1$. Then, the corresponding *duration* can be written by $d_p = \frac{1}{60}\theta l_p$ in minutes, where θ is the data aggregation interval, e.g., 30 seconds.
- The *candidate flow-rate* is calculated by $q_p^c = \frac{60}{d_p} \sum_{i=r_p}^{e_p} (s_c \tilde{c}_i + \bar{c})$ in vehicles per hour per lane (vphpl), and the *candidate occupancy* is $o_p^c = \frac{1}{l_p} \sum_{i=r_p}^{e_p} (s_o \tilde{o}_i + \bar{o})$ per lane (pl).
- The *vehicle count gap* is the maximum absolute deviation of cumulative normalized

vehicle counts from their best-fit line by least-squares estimation:

$$g_p^{\tilde{c}} = \max_{r_p \leq i \leq e_p} |N_{i,p}^{\tilde{c}} - \hat{N}_{i,p}^{\tilde{c}}|, \quad (3.14)$$

where $N_{i,p}^{\tilde{c}}$ is the cumulative normalized vehicle count at time point i and calculated by

$$N_{i,p}^{\tilde{c}} = \sum_{t=r_p}^i \tilde{c}_t, \quad \forall i = r_p, \dots, e_p; \quad (3.15)$$

$\hat{N}_{i,p}^{\tilde{c}}$ is the corresponding fitted value and calculated by

$$\hat{N}_{i,p}^{\tilde{c}} = \hat{a}_{1,p} + \hat{a}_{2,p}i, \quad \forall i = r_p, \dots, e_p, \quad (3.16)$$

where $\hat{a}_{1,p}$ and $\hat{a}_{2,p}$ are the least-squares estimates. Similarly, the *occupancy gap* can be written by $g_p^{\tilde{o}} = \max_{r_p \leq i \leq e_p} |N_{i,p}^{\tilde{o}} - \hat{N}_{i,p}^{\tilde{o}}|$.

Note that, the vehicle count gap defined above is different from the one in Cassidy's first criterion (Cassidy, 1998), where a linear function of time was subtracted from cumulative total vehicle counts to promote visual inspection, and the straight best-fit line was fitted manually. In the following, the theorem proves that there exists a relation between the newly defined and Cassidy's vehicle count gaps by assuming that the best-fit line in the latter is also fitted by least-squares estimation.

Theorem 3.2. *Under the assumption that the straight line in Cassidy's first criterion is fitted by least-squared estimation, the vehicle count gap defined in (3.14) is equivalent to Cassidy's vehicle count gap multiplied by an inverse product of the number of lanes and standard deviation of the averaged vehicle count series.*

Proof. For brevity, $\tilde{c}$ and p are omitted in this proof unless necessary. According to (3.15)

and (3.16), the least-squares criterion can be written by:

$$\min \sum_{i=r}^e (N_i - a_1 - a_2 i)^2. \quad (3.17)$$

By solving (3.17), one has

$$\hat{a}_1 = \frac{\sum_{i=r}^e i \sum_{i=r}^e i N_i - \sum_{i=r}^e i^2 \sum_{i=r}^e N_i}{(\sum_{i=r}^e i)^2 - l \sum_{i=r}^e i^2}, \quad (3.18a)$$

$$\hat{a}_2 = \frac{\sum_{i=r}^e i \sum_{i=r}^e N_i - l \sum_{i=r}^e i N_i}{(\sum_{i=r}^e i)^2 - l \sum_{i=r}^e i^2}. \quad (3.18b)$$

From (3.4a) and (3.15), the cumulative total vehicle counts subtracting a linear function of time can be written by

$$M_i = \sum_{t=r}^i L c_t - q_0 i = L s_c N_i + (L \bar{c} - q_0) i - L \bar{c} (r - 1), \quad \forall i = r, \dots, e, \quad (3.19)$$

where L is the number of lanes; q_0 is a constant. The corresponding fitted value of (3.19) can be written by

$$\hat{M}_i = \hat{b}_1 + \hat{b}_2 i, \quad \forall i = r, \dots, e, \quad (3.20)$$

where $\hat{b}_1$ and $\hat{b}_2$ are the least-squares estimates. According to (3.19) and (3.20), Cassidy's vehicle count gap can be written by

$$g' = \max_{r \leq i \leq e} |M_i - \hat{M}_i|, \quad (3.21)$$

and the corresponding least-squares criterion is

$$\min \sum_{i=r}^e (M_i - b_1 - b_2 i)^2. \quad (3.22)$$

Again, by solving (3.22), one has

$$\hat{b}_1 = \frac{\sum_{i=r}^e i \sum_{i=r}^e i M_i - \sum_{i=r}^e i^2 \sum_{i=r}^e M_i}{(\sum_{i=r}^e i)^2 - l \sum_{i=r}^e i^2}, \quad (3.23a)$$

$$\hat{b}_2 = \frac{\sum_{i=r}^e i \sum_{i=r}^e M_i - l \sum_{i=r}^e i M_i}{(\sum_{i=r}^e i)^2 - l \sum_{i=r}^e i^2}. \quad (3.23b)$$

From (3.18a), (3.18b), (3.19), (3.23a), and (3.23b), the following relations among the parameter estimates can be obtained:

$$\hat{b}_1 = Ls_c \hat{a}_1 - L\bar{c}(r-1), \quad (3.24a)$$

$$\hat{b}_2 = Ls_c \hat{a}_2 + (L\bar{c} - q_0). \quad (3.24b)$$

Substituting (3.19), (3.20), (3.24a), and (3.24b) into (3.21), one finally has

$$\begin{aligned} g' &= \max_{r \leq i \leq e} |Ls_c N_i + (L\bar{c} - q_0)i - L\bar{c}(r-1) - [Ls_c \hat{a}_1 - L\bar{c}(r-1)] - [Ls_c \hat{a}_2 + (L\bar{c} - q_0)]i| \\ &= Ls_c \max_{r \leq i \leq e} |N_i - \hat{N}_i|, \\ &= Ls_c g \end{aligned} \quad (3.25a)$$

or equivalently,

$$g = \frac{1}{Ls_c} g'. \quad (3.25b)$$

Therefore, the vehicle count gap in (3.14) is equivalent to Cassidy's vehicle count gap multiplied by an inverse product of the number of lanes and standard deviation of the averaged vehicle count series. $\square$

Remark 3.1. A general form of Theorem 3.2 can be stated as: given two sets of data, $x_1, \dots, x_n$ and $y_1, \dots, y_n$, where the latter is a linear transformation of the former such that

$y_i = \rho_1 x_i + \rho_2, \forall i = 1, \dots, n$, then the gap of the latter is equivalent to that of the former multiplied by ρ_1 , i.e., $g_y = \rho_1 g_x$.

Note that, the characteristics of candidate intervals defined in this step build a bridge to connect the PELT changepoint detection method in step 2 and modified Cassidy's near-stationary state selection criteria in the next step, which makes them mutually compatible in the framework. In addition, the characteristics can be used to archive identified near-stationary states in the well-organized form of data frames, which will be shown later.

3.3.4 Near-stationary state selection

Near-stationary states are selected from the candidate intervals by the following two criteria:

- **Gap criterion**

$$\max\{g_p^{\bar{c}}, g_p^{\bar{o}}\} \leq \frac{\xi}{s_c}, \quad \forall p = 1, \dots, m+1, \quad (3.26)$$

where ξ is the gap threshold in vehicles per lane.

- **Duration criterion**

$$d_p \geq \lambda_n I(o_p^c < \gamma_h) + \lambda_h I(o_p^c \geq \gamma_h), \quad \forall p = 1, \dots, m+1, \quad (3.27)$$

where λ_n and λ_h are the minimum acceptable durations in normal- and high-occupancy periods, respectively; I is the indicator function; γ_h is the high-occupancy threshold.

Note that, the gap criterion (3.26) is performed to select candidate intervals such that the maximum of vehicle count and occupancy gaps is smaller than a given threshold, ξ/s_c , which may vary across days with different s_c 's. It can make sure that both normalized/averaged

vehicle counts and occupancies in the selected intervals are nearly, which ensures the validity of characteristics in the selected intervals. Based on Theorem 3.2, it can be seen that ξ is mathematically equivalent to Cassidy's vehicle count gap threshold averaged by the number of lanes. This criterion can be considered as a combination of Cassidy's first and second criteria since the transformed cumulative total vehicle count and occupancy curves can be approximately superimposed if both of them are approximately linear. This modification also resolves the difficulty to quantify the superimposition in Cassidy's second criterion.

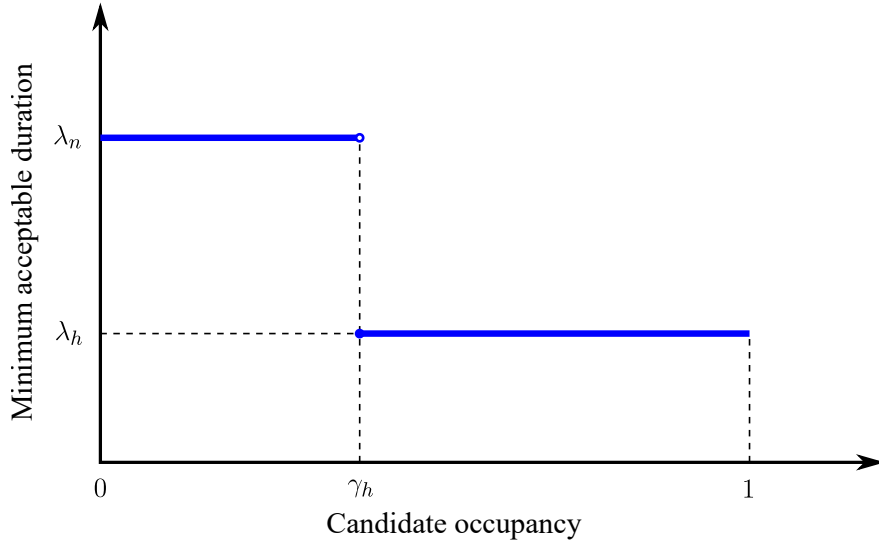


Figure 3.8: Relation between minimum acceptable duration and candidate occupancy.

In addition, the duration criterion (3.27) is consistent with Cassidy's third criterion, but γ_h is added to clearly distinguish high- from normal-occupancy periods. It is performed to further select candidate intervals that can last at least λ_h minutes in high-occupancy periods ($o_p^c \geq \gamma_h$) and λ_n minutes otherwise ($o_p^c < \gamma_h$), which ensures the stability of characteristics in the selected intervals. Here the relation between minimum acceptable duration and candidate occupancy is actually a step function, as shown in Figure 3.8. In practice, the oscillation among the traffic flow measurements increases with the occupancy, so candidate intervals in high-occupancy periods would be less stable and last shorter than those in normal-occupancy periods. Thus it is reasonable that $\lambda_h < \lambda_n$ so as to identify sufficient near-stationary states in high-occupancy periods.

Finally, the filtered candidate intervals through above two criteria can be considered as near-stationary states.

3.4 Verification of identified near-stationary states

In this section, we apply the proposed four-step method on a real-world study site to identifying near-stationary states from a large amount of raw loop-detector datasets. In addition, we present two verification ways, direct and indirect methods, to verify the validity of identified near-stationary states.

3.4.1 Study site and data description

Figure 3.9 shows a multi-lane freeway segment on the eastbound State Route 91 in Buena Park, California. The segment includes one isolated high occupancy vehicle (HOV) lane⁵, four general purpose lanes, an on-ramp from Valley View Street, and an off-ramp to Knott Avenue. The study site of interest is the upstream mainline loop-detector station with ID 1203506. The station is isolated from that on the HOV lane and deployed 70 feet upstream of the on-ramp merge gore. Such a location is chosen to minimize the chance of non-steady near-stationary states to be identified, as analyzed in Section 3.2. Daily observation of real-time traffic conditions on Google Maps shows that heavy congestion can appear at this location during peak periods on weekdays.

Raw loop-detector data measured at the location are provided by the PeMS data warehouse (<http://pems.dot.ca.gov/>), where lane vehicle counts and occupancies across four lanes ($L = 4$) are reported every 30 seconds ($\theta = 30$). Thus, in each daily dataset, the total

⁵A project (http://www.dot.ca.gov/dist12/SR_91_Pavement_Rehab.php) had been implemented on this section during July, 2012 to October, 2013 to make the HOV lane continuously accessible.

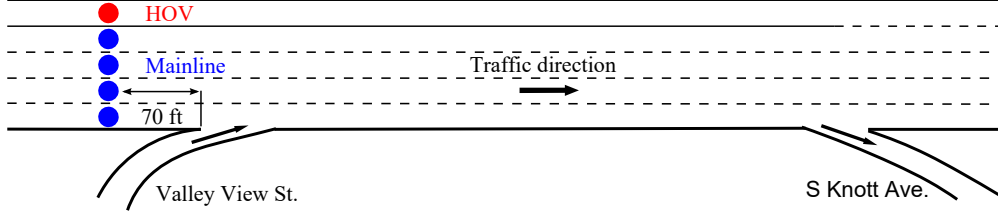


Figure 3.9: Study site on SR-91E.

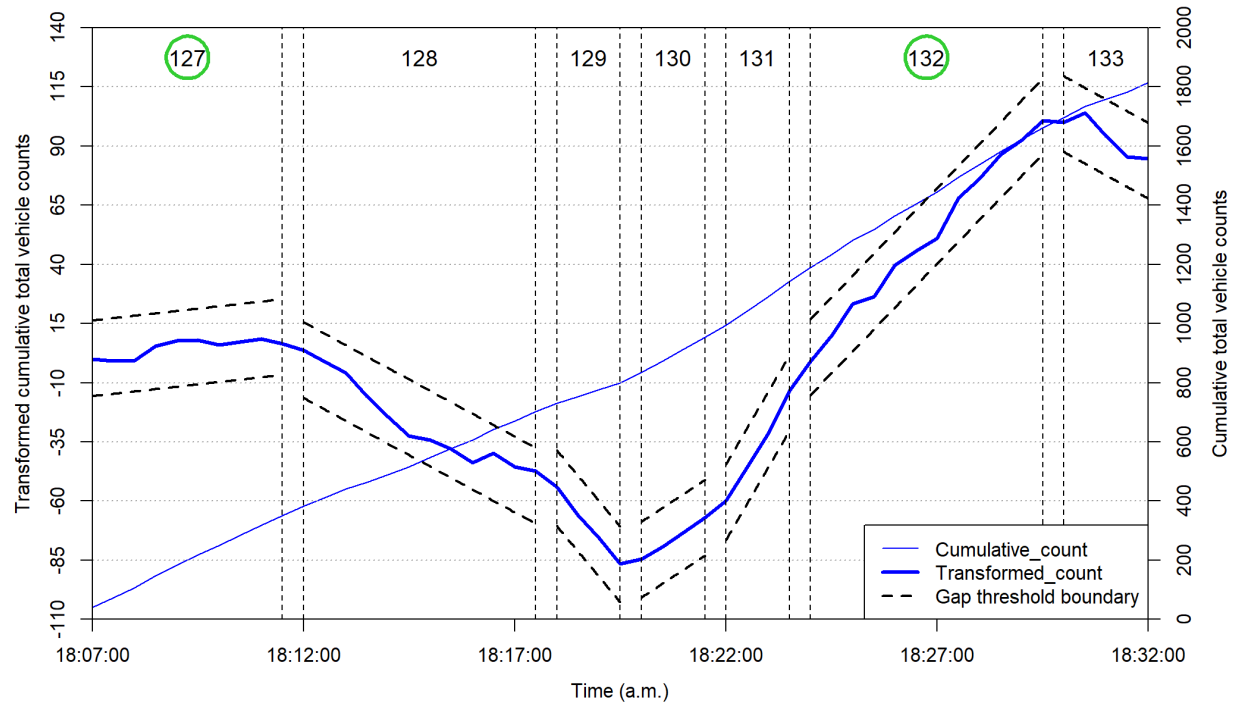
number of measurement points is 2880 ($n = 2880$). The data sample used to identify near-stationary states is collected from nine months (Jan., Feb., Mar., Apr., May, July, Sept., Oct., and Nov.) in 2011 with the sample size of 273 days.

3.4.2 Determination of the parameters

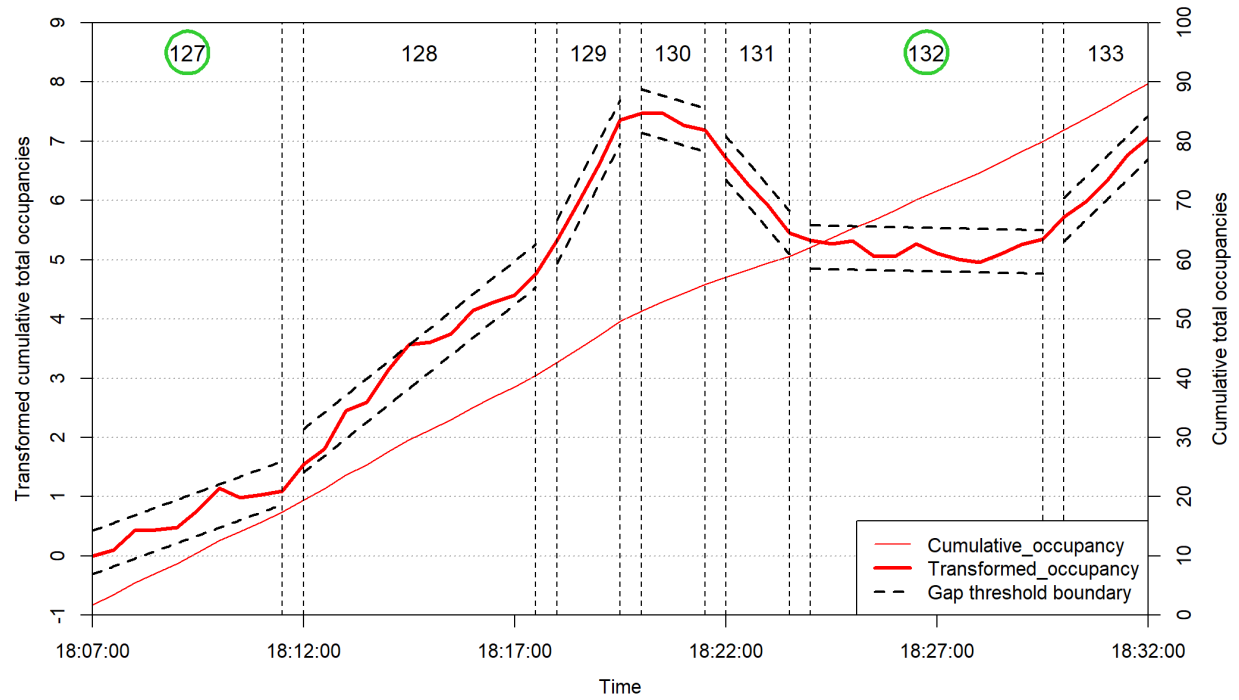
In total, five parameters exist in the four-step method and are determined as follows: (i) here we represent the penalty by $\beta = \alpha \log(n)$, where α is the penalty ratio with a feasible region usually between 0 and 1, and set $\alpha = 0.1$ based on the experimental analysis of the datasets; (ii) in (Cassidy, 1998), Cassidy claimed that an interval whose transformed cumulative total vehicle counts across three travel lanes created a deviation of more than 10 vehicles from the best-fit line would be considered to be non-stationary. Thus we set the gap threshold by $\xi = \lceil 10/3 \rceil = 4$ vehicles per unit lane; (iii) again, following Cassidy's suggestion, we set $\lambda_n = 10$ and $\lambda_h = 4$ minutes; (iv) we set $\gamma_h = 0.4$ by engineering judgment.

3.4.3 Direct and indirect verification methods

Given all the determined parameters, finally, 861 near-stationary states have been identified from 107 qualified daily datasets at the study location. Before they can be utilized for practical problems, it is important to verify the validity of them. Here two verification methods are provided.



(a)



(b)

Figure 3.10: Curves of original (thinner) and transformed (thicker) cumulative total (a) vehicle counts and (b) occupancies.

The first is to adopt Cassidy’s visualized method to examine the performance inside each identified near-stationary state, which is referred to as direct verification. To give an example, a 25-min peak period is selected between 18:07:00 and 18:32:00 a.m. on Mar. 10, 2011. Figure 3.10a and 3.10b are the original and transformed curves of cumulative total vehicle counts and occupancies in the period, respectively, where the difference between the original and transformed curves is on the order of 95%. The period has been partitioned into 7 candidate intervals with ID from 127 to 133, whose characteristics and status are recorded in Table 3.1. Among them, candidate intervals 127 and 132 have been selected as near-stationary states. It can be seen that all their transformed curves are approximately linear and within the gap threshold boundaries. In addition, their durations both satisfy the duration criterion by checking Table 3.1. On the contrary, candidate interval 128 was rejected by the gap criterion, as relatively large variations can be observed in the transformed occupancy plot. The others were rejected by the duration criterion due to their short durations.

Table 3.1: Characteristics and status of selected candidate intervals.

ID	Starting time	Ending time	Duration (min)	Flow-rate (vphpl)	Occupancy (pl)	Count gap	Occupancy gap	Status
127	18:07:00	18:12:00	5	1053	0.435	0.183	0.711	Accepted
128	18:12:00	18:18:00	6	880	0.481	0.416	0.879	Rejected by gap
129	18:18:00	18:20:00	2	720	0.567	0.038	0.085	Rejected by duration
130	18:20:00	18:22:00	2	1163	0.394	0.021	0.164	Rejected by duration
131	18:22:00	18:24:00	2	1418	0.296	0.086	0.068	Rejected by duration
132	18:24:00	18:30:00	6	1300	0.403	0.284	0.506	Accepted
133	18:30:00	18:32:30	2.5	918	0.491	0.295	0.129	Rejected by duration

The second method is to examine the overall performance of all identified near-stationary states in terms of the fundamental diagram calibration, which is referred to as indirect verification. Figure 3.11a is a flow-occupancy relation formed by the identified near-stationary states. Compared to those formed by 30-sec raw data and candidate intervals in Figure 3.11b, the near-stationary relation is almost scatter-free and composed of two approximately linear patterns. However, we find some mixed points appearing between two patterns as the chosen location is so close to the on-ramp merge gore that non-steady near-stationary states

(Type 3) can hardly be avoided.

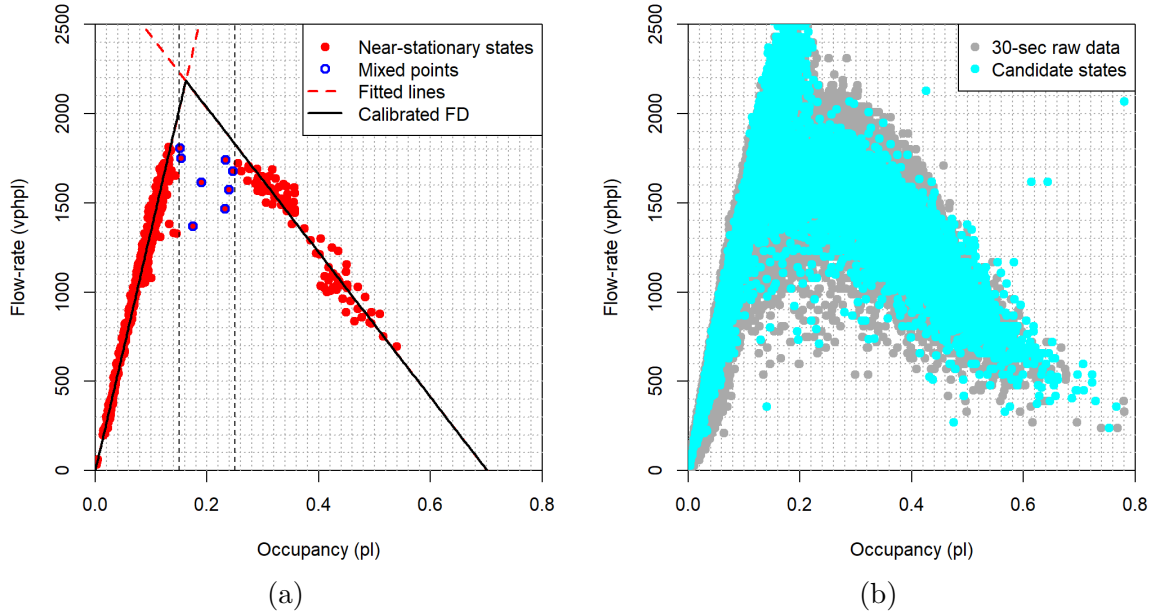


Figure 3.11: (a) Flow-occupancy relation and calibrated fundamental diagram in near-stationary states; (b) Flow-occupancy relations in 30-sec raw data and candidate intervals.

To calibrate the fundamental diagram, we set two occupancy thresholds, 0.15 and 0.25, to discard those mixed points that fail to fall on the fundamental diagram. Then we apply a constraint linear regression through the origin and simple linear regression to fitting the patterns in the free-flow and congested regimes, respectively, as shown in Figure 3.11a. Their corresponding R^2 's are 0.98 and 0.93, respectively, which indicates that the estimated linear models fit the near-stationary states very well. In the figure, the black piece-wise linear curve is the calibrated triangular fundamental diagram with the capacity, 2186 vphpl, critical occupancy, 0.16 pl, and jam occupancy, 0.70 pl, which are reasonable values. In particular, the calibrated jam occupancy matches Coifman's analytical finding (Coifman, 2014a) such that the jam occupancy for stationary traffic should be around 0.8 on average.

In summary, through the above direct and indirect verification methods, it can be concluded that the identified near-stationary states are valid with high quality and the calibrated fundamental diagram is well-fitted and physically meaningful.

3.5 Comparison of different types of data on the fundamental diagram calibration

Theoretically, a well-defined fundamental diagram should be constructed in steady states. However, most of the existing studies on the calibration of fundamental diagrams use aggregated time series data, which are comprised of near-stationary, non-stationary, and transitional states. In this section, a comparison of near-stationary and aggregated time series data is made on the calibration of fundamental diagrams.

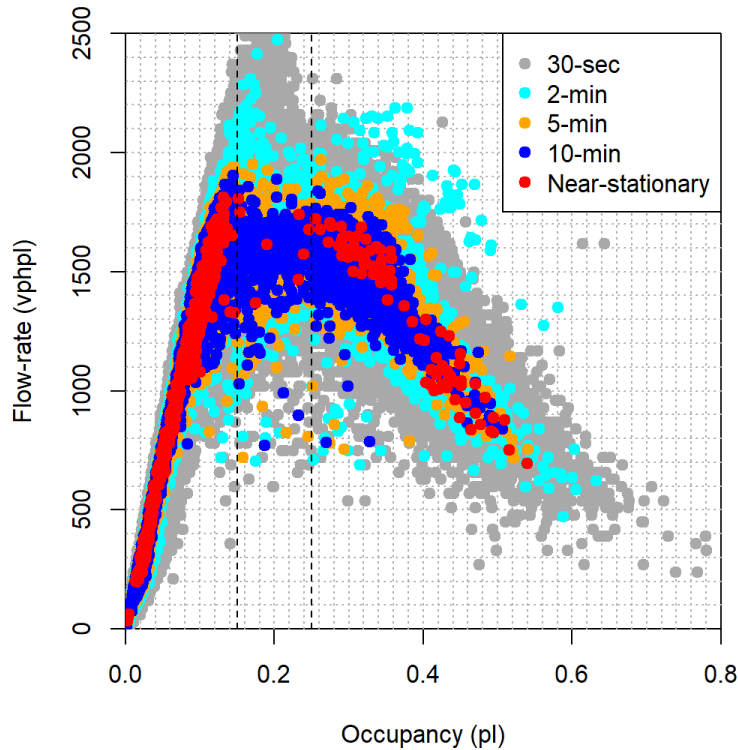


Figure 3.12: Flow-occupancy relations by different types of data.

As shown in Figure 3.12, flow-occupancy relations are formed by 30-sec, 2-min, 5-min, 10-min, and near-stationary data, respectively. Through the visual comparison, it can be found that the patterns formed by the aggregated time series data are more scattered in the congested regime. In addition, we find that the near-stationary pattern actually does not lie on the center of the other patterns formed by aggregated data, so simply increasing the length

of aggregation interval can reduce scatteredness but cannot produce patterns similar to the near-stationary pattern.

To evaluate the performance of different types of data on the calibration of fundamental diagrams, we particular compare slopes of linear regressions for the congested pattern. The reason is that such a slope is proportional to the shock wave speed. If it cannot be calibrated accurately, the associated parameters of the fundamental diagram including critical and jam occupancies and the capacity would also be negatively affected. The pooled t-test proposed by Andrade and Estévez-Pérez (2014) is adopted for the comparison and the results are shown in Table 3.2. It can be found that (i) compared to R^2 of the near-stationary pattern in the congested regime, R^2 's of the other types of data are extremely low, which indicates that linear associations by the aggregated time series data are not clear; (ii) from the p-values and 95% confidence intervals, the regression slopes by the aggregated time series data are significantly different from the slope by near-stationary states. Therefore, utilizing such aggregated time series data to calibrate fundamental diagrams may result in problematic calibrated parameters.

Table 3.2: Comparison results.

Interval	Congested regime			
	R^2	Pooled t-test		
		p-value	Decision	95% CI
Near-stationary	0.93	-	-	-
30-sec	0.37	0	Reject	[-2.15E+03, -9.29E+02]
2-min	0.56	0	Reject	[-1.46E+03, -7.01E+02]
5-min	0.59	0	Reject	[-1.38E+08, -7.65E+02]
10-min	0.56	0	Reject	[-1.49E+03, -9.00E+02]

3.6 Conclusion

In this chapter, after defining stationary, steady, and equilibrium states and discussing their relations, some guidelines were presented for identifying near-stationary states. Then a novel four-step method was proposed for automatically identifying near-stationary states from raw loop-detector data. In the proposed method, a data pre-processing technique was first performed to select healthy datasets with sufficient congested periods, fill in missing values, and normalize averaged vehicle counts and occupancies to the same scale. Then, the PELT changepoint detection method was applied to partition daily time series into multiple candidate intervals whose means of normalized vehicle counts and occupancies were approximately constant over time. After defining and calculating characteristics of the candidate intervals, including starting and ending positions/time, length, duration, candidate flow-rate and occupancy, and vehicle count and occupancy gaps, the near-stationary states were finally selected through the modified Cassidy’s gap and duration criteria. An R package of the proposed method, ANSSI (Automatic Near-Stationary State Identifier), is available online (<https://github.com/QinglongYan/ANSSI>).

In a case study, the proposed method was applied to identify near-stationary states from a large amount of 30-sec loop-detector datasets at a freeway location. Two (direct and indirect) verification methods are provided to verify the validity of the identified near-stationary states. Results are summarized as follows: (i) the transformed curves of cumulative total vehicle counts and occupancies for each individual were approximately linear by visual inspection; (ii) the flow-occupancy relation was almost scatter-free and composed of two approximately linear patterns without any non-steady near-stationary points; (iii) the calibrated triangular fundamental diagram was well-fitted and physically meaningful; (iv) the calibrated model was not the best fit for the 10-min flow-occupancy relation. The above results showed that the identified near-stationary states were valid with high quality. In addition, we compared between near-stationary and various aggregated time series data on the calibration of funda-

mental diagrams and the results showed that the calibrated parameters by these aggregated time series data were significantly different from those by near-stationary states.

Some contributions of this proposed method can be summarized as follows:

- The method is automated. In the proposed method, all the four steps can be automatically performed without any visual inspection or manual operation.
- The method is computationally efficient. In the second step, the applied PELT search algorithm based on dynamic programming and a pruning technique has a linear running time on average, which significantly reduces the computational complexity.
- The overall framework for identifying near-stationary states is novel. Among the four steps, the defined characteristics of candidate intervals play a critical role in connecting the PELT changepoint detection and modified Cassidy's criteria and make them mutually compatible. In addition, the proposed identification process, preprocessing - partitioning - characterizing - selecting, can be considered as a general framework for future studies on the identification of near-stationary states.

However, the study is only a starting point for identifying near-stationary states. Some potential improvements can be made on the proposed methods:

1. In the PELT changepoint detection (step 2), one assumption is made such that the variances of normalized vehicle counts and occupancies are constant over time in order to detect changes in mean. However, in practice, the oscillation in the traffic flow measurements is usually stronger during peak periods. Thus, such an assumption would generate more candidate intervals with shorter durations, which are more likely to be rejected by the duration criterion, and result in less identified near-stationary states during peak periods, for example, as shown in Figure 3.10. To improve it, one attempt is to separate peak and off-peak periods in advance and then identify near-stationary

states from separated periods with different parameters, respectively. Alternatively, after the separation, one can normalize averaged vehicle counts and occupancies in each period, reconnect them into an adjusted normalized series with approximately constant variances over time, and then continue with the same identification procedures.

2. In the duration criterion (3.27), the minimum acceptable duration has a sudden drop at $\gamma = 0.4$ from normal- to high-occupancy periods, as shown in Figure 3.8. This would result in less identified near-stationary states near the drop occupancy, as shown in Figure 3.11a. To improve it, the duration criterion can be modified so that the minimum acceptable duration can decrease more smoothly as the candidate occupancy increases.
3. In the case study, it can be noticed that only a few near-stationary states but with high quality ($R^2 = 0.93$) were identified in the congested regime, as shown in Figure 3.11a. Actually, the quantity and quality of identified near-stationary states are highly dependent of the choices of the penalty and gap threshold but not the proposed four-step method. With different parameter choices, it is totally possible to identify more near-stationary states in the congested regime by sacrificing a little quality.

Chapter 4

Parameter Calibration in the Four-Step Method: A Game Theory Approach

4.1 Introduction

Among the parameters of the four-step method, λ_n and λ_h can be determined by following Cassidy's suggestion (Cassidy, 1998); γ_h can be set by engineering judgment; whereas α and ξ need to be further calibrated. As particular attention is paid to two important performances of near-stationary states during peak periods: quantity and quality, which can be respectively defined as the number and linearity of near-stationary states in the congested regime, therefore, the parameter calibration can be formulated as a multi-objective optimization problem with the performance as two objective functions and parameters as two decision variables.

To solve the multi-objective optimization problem, some classical approaches such as the

weighted sum (Zadeh, 1963), ϵ -constraint (Haimes, 1971), and goal programming (Lee et al., 1972) methods can scalarize the problem into a single-objective optimization problem by innovating artificial fix-ups with respect to the objective functions and result in a unique solution. Alternatively, other posteriori methods through the genetic (Deb et al., 2002), differential evolution (Abbass et al., 2001), and other meta-heuristic algorithms (Deb, 2001; Reyes-Sierra et al., 2006) can directly solve the problem and eventually produce a set of pareto-optimal solutions. However, such above methods cannot lead to a unique solution without imposing a subjective preference between the quantity and quality in advance. Even though one unique solution can be manually selected from a pareto-optimal set by the posteriori methods, searching for multiple solutions instead of one would be more computationally expensive.

A further investigation of the problem shows that (i) α has more effect on the quantity, (ii) ξ has more effect on the quality, and (iii) their relationships are both approximately unimodal. Inspired by the above properties and the conflict between quantity and quality, in this chapter, we will propose a preference-free approach based on the game theory to reinterpret the problem as a non-cooperative game with two players, each controlling a decision variable and related objective function, and attempt to calibrate the parameters by simulating the bargaining process to find a unique Nash equilibrium solution.

In the early 1950s, Nash (1951) first presented the non-cooperative game theory “based on the absence of coalitions in that it is assumed that each participant acts independently without collaboration or communication with any of the others”, and proved that at least one equilibrium point, which is later known as the Nash equilibrium, exists and can be reached when no player can gain by a unilateral change of strategy holding the strategies of the others unchanged. Later, Sefrioui and Perlaux (2000) introduced the non-cooperative game theory associated with genetic algorithms into solving multi-objective optimization problems and demonstrated better computational efficiency and convergence properties compared to

the posteriori methods. In recent decades, the approach has been applied to some practical problems in different fields (Periaux et al., 2001; Li et al., 2012).

Compared to the pioneer studies, however, our problem is more complicated: the quantity and quality are measured through the four-step method and their own definitions using real-world data, so they cannot be formulated as any specific functional forms with respect to α and ξ and the relations have to be discrete with chaos and uncertainty. Thus some resulting challenges have to be faced such that (i) multiple Nash equilibria may exist in the game and some of them are inferior, (ii) the game may fail to converge because of an inappropriate initial condition, and (iii) without utilizing the unimodal properties, the genetic algorithms adopted in previous studies are not computationally efficient for our case. Therefore, in this chapter, we will develop a robust search algorithm to efficiently find a reasonable and unique Nash equilibrium solution to our problem.

Furthermore, considering the limitation of the four-step method that few near-stationary states can be identified in transitional and extremely high occupancy periods, in the last part of the chapter, we will adjust the gap criterion in step 4 of the method and extend the problem to a five-player game to obtain a more complete near-stationary pattern in the congested regime.

The rest of the chapter is organized as follows. In Section 4.2, we formulate a multi-objective optimization problem on the parameter calibration of α and ξ . In Section 4.3, we present a game theory approach to convert the problem to a non-cooperative game with two players and define Nash equilibrium solutions. In Section 4.4, we develop an alternated hill-climbing algorithm to search for a unique Nash equilibrium solution. In Section 4.5, we apply the approach and search algorithm to calibrating and validating the parameters using raw loop-detector data. In Section 4.6, we adjust the gap criterion in the four-step method and extend the problem to a five-player game. In Section 4.7, we conclude the chapter with some discussions.

4.2 A multi-objective optimization problem for calibration of penalty ratios and gap thresholds

In this section, we formulate the parameter calibration as a multi-objective optimization problem with two objective functions and two decision variables. In addition, some properties of the objective functions are discussed.

4.2.1 A multi-objective optimization problem

Among the five parameters of the four-step method, we consider λ_n , λ_h , and γ_h as three fixed parameters, whereas α and ξ are considered as two random variables. Then, for a set of identified near-stationary states with a cardinality of $n(\alpha, \xi)$, which is a function of the variables, the flow-rate and occupancy of the i^{th} near-stationary state are also dependent of the variables and can be denoted by $q_i(\alpha, \xi)$ and $o_i(\alpha, \xi)$, respectively, where $i = 1, \dots, n(\alpha, \xi)$. Hereafter (α, ξ) is omitted unless necessary.

In the process of identifying near-stationary states, two important performances of near-stationary states during peak periods, including the quantity and quality, are highly concerned. For the quantity, it is defined as the number of near-stationary states in the congested regime and dependent of both α and ξ :

$$N = \sum_{i=1}^n I(o_i \geq \tilde{o}_c) \equiv N(\alpha, \xi), \quad (4.1)$$

where $\tilde{o}_c$ is the prior critical occupancy and $I(o_i \geq \tilde{o}_c)$ is an indicator function such that it equals one if $o_i \geq \tilde{o}_c$, i.e., the i^{th} near-stationary point falls within the congested regime, and zero otherwise.

In addition, the quality is defined as the degree of linear association between flow-rate and

occupancy of near-stationary points in the congested regime by assuming that a well-defined flow-occupancy relation is triangular. Statistically, the quality can be measured by the coefficient of determination and also dependent of both α and ξ :

$$R^2 = 1 - \frac{\sum_{j=1}^N (q_j^c - \hat{q}_j^c)^2}{\sum_{j=1}^N (q_j^c - \bar{q}^c)^2} \equiv R^2(\alpha, \xi), \quad (4.2)$$

where q_j^c is the flow-rate of the j^{th} near-stationary state in the congested regime ($j = 1, \dots, N$), $\hat{q}_i^c$ is the corresponding fitted flow-rate estimated by the simple linear regression through the ordinary least-squares criterion, and $\bar{q}^c$ is the average of all near-stationary flow-rates in the congested regime, i.e., $\bar{q}^c = \frac{1}{N} \sum_{j=1}^N q_j^c$. In particular, R^2 has a property such that $0 \leq R^2 \leq 1$, where $R^2 = 0$ and $R^2 = 1$ represent the lowest and highest qualities, respectively.

In practice, if N is excessively small, the measured R^2 would not be statistically valid and the further calibrated fundamental diagram using such unrepresentative data may become problematic. For example, R^2 always equals one if only two near-stationary states are identified in the congested regime regardless of their positions, but the result does not imply the highest quality. Therefore, to make the measurement of the quality more reasonable and meaningful, (4.2) is further modified by

$$R_{mod}^2 = R^2 \cdot I(N \geq \rho) \equiv R_{mod}^2(\alpha, \xi), \quad (4.3)$$

where ρ is the minimum representative size. Note that (i) R_{mod}^2 is named the modified R-squared to distinguish it from the adjusted R-squared in statistics, where the latter has a totally different formulation and is usually used for model selection in multiple linear regression; (ii) R_{mod}^2 can guarantee that the measured result has the lowest quality if $N < \rho$;

Our ultimate goal is to find an appropriate penalty ratio and gap threshold that maximize the performances of identified near-stationary states. Mathematically, it can be formulated

as a multi-objective optimization problem:

$$\begin{aligned}
& \underset{\alpha, \xi}{\text{Maximize}} && N(\alpha, \xi), R_{\text{mod}}^2(\alpha, \xi), \\
& \text{subject to} && \alpha \in A, \\
& && \xi \in \Xi,
\end{aligned} \tag{4.4}$$

where $N(\alpha, \xi)$ and $R_{\text{mod}}^2(\alpha, \xi)$ are the objective functions, α and ξ are the decision variables, and A and Ξ are the corresponding decision variable spaces.

Note that, one benefit of choosing (4.3) as one objective function is that, in the process of maximizing R_{mod}^2 , the portion of non-steady near-stationary states (see the stationary state of Type 3 in Figure 3.5b) in the congested regime can be implicitly reduced, if any, because such states, similar to non-stationary states, fail to fall on the fundamental diagram and have a negative effect on R_{mod}^2 .

4.2.2 Properties of the objective functions

Both the objective functions are dependent of the same two decision variables, but each of them has a unique property with one of the variables. For $N(\alpha, \xi)$, it is more affected by and approximately unimodal in α . In particular, as α is employed to limit the number of detected changepoints in step 2 of the four-step method, when α is small, a large number of changepoints would be detected and partition daily time series into numerous candidate intervals with short durations. Such candidate intervals are highly likely to be rejected by the duration criterion in step 4 and result in only a few near-stationary states and thus a small N . On the other hand, when α is large, a small number of changepoints would be detected to partition daily time series into only a few candidate intervals with potentially large vehicle count and occupancy gaps. Such candidate intervals are highly likely to be rejected by the gap criterion in step 4 and eventually result in a small N as well. From the

analysis above, it can be inferred that $N(\alpha, \xi)$ would first increase and then decrease with α for any fixed ξ and the relationship should be approximately unimodal.

For $R_{\text{mod}}^2(\alpha, \xi)$, it is more affected by and approximately unimodal in ξ . To clarify, when ξ is small, only a few candidate intervals with small gaps can finally be selected as near-stationary states through the gap criterion. Such sparse near-stationary points can hardly form a complete linear pattern in the congested regime and would result in a small R_{mod}^2 . On the other hand, when ξ is large, a great number of candidate intervals with large gaps would be mistakenly identified as near-stationary states, form a scattered pattern in the congested regime, and eventually cause a small R_{mod}^2 as well. Therefore, the above analysis indicates that the relation between $R_{\text{mod}}^2(\alpha, \xi)$ and ξ is approximately unimodal.

Note that even though ξ also has a possible effect on $N(\alpha, \xi)$ as larger ξ would always produce greater N , we do not pair them up because the best choice of ξ to maximize $N(\alpha, \xi)$ for any fixed α is always the upper bound of Ξ , which would correspond to the smallest R_{mod}^2 and thus is inappropriate.

4.3 A game theory formulation of the multi-objective optimization problem

In this section, we first build a two-player game to give a new interpretation to the multi-objective optimization problem. Then we define and discuss Nash equilibrium solutions in the context of the game. Finally, we give a mathematical example to demonstrate the game theory approach.

4.3.1 Two-player game

Based on the aforementioned properties of the objective functions and the fact of conflict between quantity and quality, the multi-objective optimization problem can be reinterpreted as a two-player game with the following rules: (i) the game is a finite non-cooperative game such that the players act independently without any collaboration with each other and both of them have finite strategies to choose; (ii) the game is a non-zero-sum game such that the common objective of both players is to maximize their own payoffs; (iii) the game is in an extensive form such that the players take turns to choose their best strategies in a periodic sequence of rounds and the payoff function for each of them is not only dependent of his/her current strategy choice but also the preceding strategy of the other; and (iv) the game has perfect information such that both players clearly know the preceding strategy made by the other in any round of the game.

More specifically, in our game, player 1 is assigned to choose strategy α from strategy space A and his/her payoff function is $N(\alpha, \xi^*)$, where ξ^* is the preceding best strategy determined by player 2. Similarly, for player 2, his/her strategy ξ can be chosen from strategy space Ξ and the corresponding payoff function is $R_{\text{mod}}^2(\alpha^*, \xi)$, where α^* is the preceding best strategy of player 1. In each round of the game, aiming at ξ^* , player 1 attempts to choose the best $\alpha = \alpha^* \in A$ to maximize $N(\alpha, \xi^*)$; and then facing the chosen α^* , player 2 would select the best $\xi = \xi^* \in \Xi$ to maximize $R_{\text{mod}}^2(\alpha^*, \xi)$. In Figure 4.1, a flow chart is shown to describe the game process between the two players.

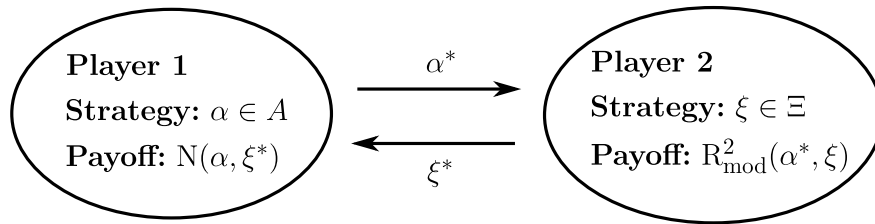


Figure 4.1: Process of the two-player game.

Alternatively, the game can also be represented in a mathematical form as follows:

- Player 1:

$$\begin{aligned} & \underset{\alpha}{\text{Maximize}} \quad N(\alpha, \xi^*), \\ & \text{subject to} \quad \alpha \in A. \end{aligned} \tag{4.5}$$

- Player 2:

$$\begin{aligned} & \underset{\xi}{\text{Maximize}} \quad R_{\text{mod}}^2(\alpha^*, \xi), \\ & \text{subject to} \quad \xi \in \Xi. \end{aligned} \tag{4.6}$$

Note that (4.5) and (4.6) are not two separate single-objective optimization problems, but, by explicitly including the preceding strategy of the other into one's objective function, they can be considered as a social optimization problem, or a convoluted problem of conflicting optimizations (Fujiwara-Greve, 2015).

4.3.2 Nash equilibrium solutions

Proven by Nash (1951), a finite non-cooperative game involving two or more players always has at least one Nash equilibrium, at which no player can further improve his/her payoff by a unilateral change of strategy when the strategies of the others remain unchanged. Such a Nash equilibrium can be considered as a solution concept for non-cooperative games. In our two-player game, the Nash equilibrium can be defined as follows:

Definition 4.1. A pair of strategies chosen by two players, $(\alpha^*, \xi^*) \in A \times \Xi$, is a *Nash*

equilibrium if and only if

$$N(\alpha^*, \xi^*) \geq N(\alpha, \xi^*), \quad \forall \alpha \in A, \quad (4.7a)$$

$$R_{\text{mod}}^2(\alpha^*, \xi^*) \geq R_{\text{mod}}^2(\alpha^*, \xi), \quad \forall \xi \in \Xi, \quad (4.7b)$$

and the pair of payoffs, $(N(\alpha^*, \xi^*), R_{\text{mod}}^2(\alpha^*, \xi^*))$, is called the *Nash outcome* of the game.

Note that our game can have multiple Nash equilibria, among which some might be dominated by the others such that both quantities and qualities in their Nash outcomes perform worse than those of the others. For example, given three Nash equilibria with ID's, a , b , and c , and their corresponding Nash outcomes, $(30, 0.7)$, $(50, 0.9)$, and $(70, 0.8)$, by comparing their performances, we can find that a is dominated by both b and c , whereas b and c are mutually non-dominated. In our case, we only prefer admissible (or non-dominated) Nash equilibria, which should satisfy the following criteria:

Definition 4.2. Among multiple Nash equilibria of the game, a Nash equilibrium, $(\alpha^*, \xi^*) \in A \times \Xi$, is said to be *admissible* if there is no other Nash equilibrium, $(\bar{\alpha}^*, \bar{\xi}^*) \in A \times \Xi$, such that

$$N(\bar{\alpha}^*, \bar{\xi}^*) \geq N(\alpha^*, \xi^*), \quad \forall \alpha \in A, \quad (4.8a)$$

$$R_{\text{mod}}^2(\bar{\alpha}^*, \bar{\xi}^*) \geq R_{\text{mod}}^2(\alpha^*, \xi^*), \quad \forall \xi \in \Xi, \quad (4.8b)$$

with at least one of the inequalities strict.

In order to determine a unique solution to our game, when only one Nash equilibrium exists in the game, it is undoubtedly admissible and becomes the unique solution. However, when the game has two or more admissible Nash equilibria, we select a particular one by showing the preference between the quantity and quality. That is, if the quantity is more concerned, the admissible Nash equilibrium leading to the largest N will be chosen as a unique solution; in

contrast, if more attention is paid to the quality, the admissible Nash equilibrium resulting in the highest R_{mod}^2 will be finally selected; alternatively, one can also manually select a moderate solution that makes a trade-off between quantity and quality. In Figure 4.2, a Venn diagram is shown to describe the logical relationship among the Nash equilibrium, admissible Nash equilibrium, and our unique Nash equilibrium solution.

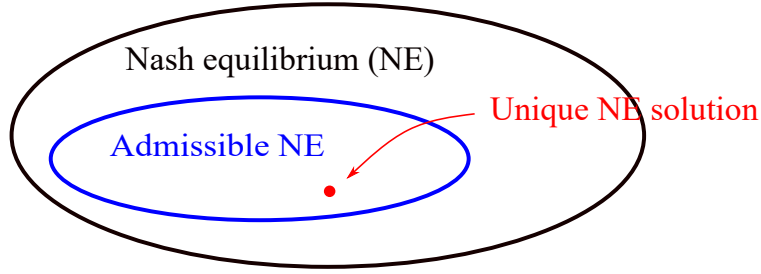


Figure 4.2: Logical relationship among three equilibria.

4.3.3 Example

Here we give an example to demonstrate our game theory approach by simply using mathematical functions. Let us suppose that we have a multi-objective optimization problem formulated as follows:

$$\begin{aligned}
 &\underset{x,y}{\text{Maximize}} && f_1(x,y) = x \cdot (1 - x + 0.8 \cdot y), \\
 &&& f_2(x,y) = y \cdot (1 - y + 0.5 \cdot x), \\
 &\text{subject to} && x \in X, \\
 &&& y \in Y,
 \end{aligned} \tag{4.9}$$

where $X = Y = [0, 1]$. In Figures 4.3a and 4.3b, we display the contour plots of two objective functions, respectively. As can be seen, in the feasible region $X \times Y$, $f_1(x, y)$ is strictly unimodal in x and increases with y , and $f_2(x, y)$ is strictly unimodal in y and increases with x , which almost match the properties previously presented in Section 4.2.

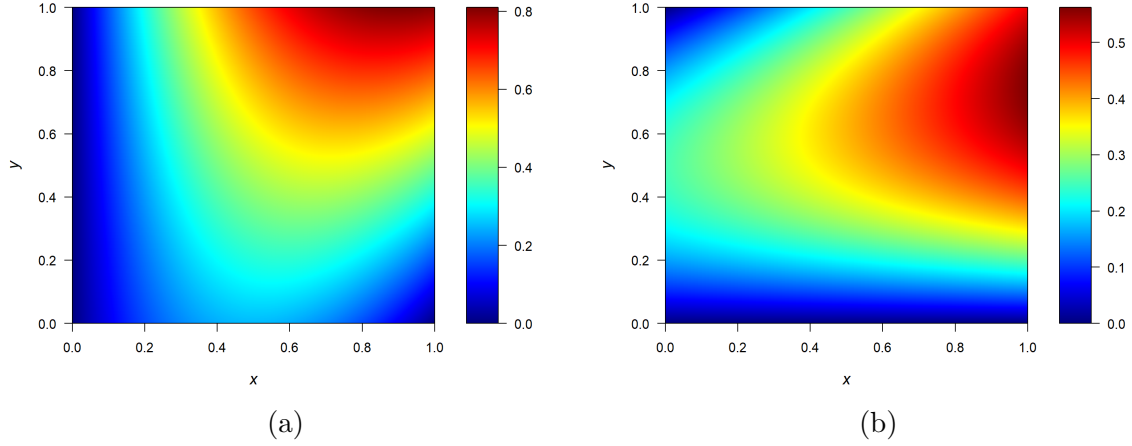


Figure 4.3: Contour plots of (a) $f_1(x, y)$ and (b) $f_2(x, y)$.

To convert the problem into a two-player game, we consider $x \in X$ and $y \in Y$ as the strategies and strategy spaces of players 1 and 2, respectively. Correspondingly, the payoff functions of two players can be written by

$$f_1(x, y^*) = x \cdot (1 - x + 0.8 \cdot y^*), \quad (4.10a)$$

$$f_2(x^*, y) = y \cdot (1 - y + 0.5 \cdot x^*), \quad (4.10b)$$

where x^* and y^* are the preceding best strategies made by two players, respectively.

To find all existing Nash equilibria, first, we define the *rational reaction set* as a set containing the best strategies of one player in the current round of a game for all possible strategies made by his/her opponent in the previous round. Then, in this example, the rational reaction sets of two players can be obtained when both derivatives of (4.10a) and (4.10b) with respect to their corresponding strategies equal zero:

$$S_1 = \{x \mid \frac{df_1(x, y^*)}{dx} = 0\}, \quad (4.11a)$$

$$S_2 = \{y \mid \frac{df_2(x^*, y)}{dy} = 0\}, \quad (4.11b)$$

which form two relations between the strategies of two players:

$$x = \frac{1 + 0.8 \cdot y^*}{2}, \quad (4.12a)$$

$$y = \frac{1 + 0.5 \cdot x^*}{2}. \quad (4.12b)$$

According to Definition 4.1, all Nash equilibria should be located at intersections of two relations. Finally, by solving (4.12a) and (4.12b), we achieve the unique Nash equilibrium solution:

$$x^* = \frac{7}{9}, \quad (4.13a)$$

$$y^* = \frac{25}{36}, \quad (4.13b)$$

which can also be visualized graphically in Figure 4.4.

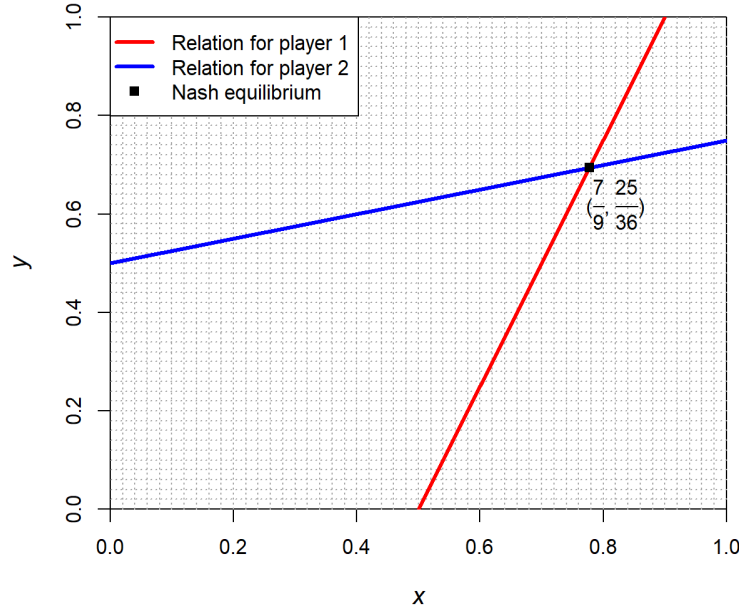


Figure 4.4: Relations of two rational reaction sets and the unique Nash equilibrium.

Note that (i) two linear relations can only lead to one intersection and thus one Nash equilibrium; (ii) when the payoff functions become more complex, the derived relations can be

non-linear and even discontinuous, which might result in more Nash equilibria.

Back to our problem, however, as $N(\alpha, \xi)$ and $R_{\text{mod}}^2(\alpha, \xi)$ are derived through a combination of the entire four-step method, a certain amount of raw loop-detector data input, and their own definitions in (4.1) and (4.3), they cannot be simply written as any specific functional forms with respect to α and ξ , and thus are non-differentiable. Therefore, it is almost impossible to find Nash equilibria by applying the method in the above example. In the next section, we will present an alternative search algorithm to find a unique Nash equilibrium solution.

4.4 An alternated hill-climbing search algorithm to solve the two-player game

A direct way to find a Nash equilibrium of our game is to make two simulated players play the game by following the established rules. During the process, once an equilibrium stage is reached such that no player can further improve his/her payoff by unilaterally changing the strategy holding the strategy of the other unchanged, it will be identified as a Nash equilibrium. Inspired by the above idea, in this section, we first develop a modified hill-climbing algorithm to efficiently search for the best strategy to maximize a payoff function that is approximately unimodal. Then we present an alternated search algorithm to identify a unique Nash equilibrium solution as well as the corresponding Nash outcome to our problem.

4.4.1 Modified hill-climbing search for one player

In each round of the game, an essential action for one player is to search for a strategy from the strategy space to maximize his/her own payoff function. Here let us suppose that we have

a payoff function, $G(z)$, which is approximately unimodal in z with one global maximum and multiple potential local maxima and flats, and a discrete strategy space with m strategies arranged in ascending order, $Z = \{z_1, \dots, z_m\}$.

Algorithm 4.4.1: Modified hill-climbing search

input : an approximately unimodal payoff function, $G(z)$;
a discrete strategy space, $Z = \{z_1, \dots, z_m\}$;
the length of examination, η .
output: the best strategy leading to the maximum payoff.
function MODIFIED-HILL-CLIMBING($Z, G(z), \eta$):
 $r \leftarrow 1$;
 $i \leftarrow 2$;
 while $i \leq m$ **do**
 if $G(z_i) \geq G(z_r)$ **then**
 $r \leftarrow i$;
 $i \leftarrow i + 1$;
 else if $i - r == \eta$ **then**
 $z^* \leftarrow z_r$;
 break;
 else
 $i \leftarrow i + 1$;
 end
 end
 return z^*
end

We start at the smallest strategy, z_1 , consider it as the currently best strategy, and search forward. If a successor leads to a better payoff that is greater than or equal to the current one, the current strategy will be replaced by it. In contrast, if the successor of the current strategy fails to result in a better payoff, we will not terminate the search but continue examining a fixed sequence of successors, whose length is denoted by η . Eventually, if no better strategy is found during the examination process, the current one will be identified as the best strategy that maximizes $G(z)$. We name our algorithm the *modified hill-climbing search* and the corresponding pseudocode is given in Algorithm 4.4.1.

Note that (i) unlike the conventional hill-climbing search and its variants (Russell and Norvig,

2016), our algorithm has three modifications: (1) we start searching at the left hill foot to guarantee that the search is always unidirectional and towards the global maximum, (2) we update the best strategy as long as the successor performs no worse than the current one to avoid falling into flats, and (3) the examination procedure can prevent the search from getting stuck in local maxima; (ii) compared to genetic algorithms adopted in Sefrioui and Perlaux (2000), our algorithm takes full advantage of the unimodal properties and can be more robust; (iii) when η is extremely small, our algorithm would perform similarly to the conventional hill-climbing search and may sink into a local optimum if any. On the other hand, when η is extremely large, the algorithm would be similar to a brute-force search. Therefore, an appropriate choice of η can make our algorithm both accurate and computationally efficient.

4.4.2 Alternation between two players

In our game, the strategy spaces of both the players have to be discrete: $A = \{\alpha_1, \dots, \alpha_{m_1}\}$ and $\Xi = \{\xi_1, \dots, \xi_{m_2}\}$, where the strategies in each space are created with a common difference and sorted in ascending order and the lower and upper bounds should be chosen appropriately so that peaks of the payoff functions can always be reachable.

Here we assign player 1 to start the game. In the first round, to choose α so that maximizing $N(\alpha, \xi^*)$, player 1 needs to make an initial guess for $\xi^* \in \Xi$ of player 2. As such an initial guess can determine the finally achieved Nash equilibrium in an extensive-form game if multiple Nash equilibria exist (Ritzberger et al., 2002), we decide to select multiple ξ^* 's widely located among Ξ , whose total number is denoted by θ_2 , to increase the likelihood of identifying all existing Nash equilibria. However, considering that some inappropriate initial choices might make the game fail to converge to an equilibrium stage, we force the players to terminate the game after finite rounds, whose total number is denoted by ϵ , even if a Nash equilibrium

Algorithm 4.4.2: Alternated hill-climbing search

input : the payoff functions, $N(\alpha, \xi^*)$ and $R_{\text{mod}}^2(\alpha^*, \xi)$;
the discrete strategy spaces, $A = \{\alpha_1, \dots, \alpha_{m_1}\}$ and $\Xi = \{\xi_1, \dots, \xi_{m_2}\}$;
the length of examination, η ;
the maximum number of rounds, ϵ ;
the number of initial guesses for ξ^* of player 2, θ_2 ;
the user preference, $P = 1$ (quantity) and $P = 0$ (quality).
output: a unique Nash equilibrium solution and the corresponding Nash outcome.

begin

$S_{ue} \leftarrow \{\}$;

for $i \leftarrow 1$ **to** θ_2 **do in parallel**

$d \leftarrow \left\lfloor \frac{n_\xi - \theta_2}{\theta_2 - 1} \right\rfloor$;

$r \leftarrow \left\lfloor \frac{n_\xi - \theta_2 - (\theta_2 - 1) \cdot d}{2} \right\rfloor + 1 + (d + 1) \cdot (i - 1)$;

$\xi^* \leftarrow \xi_r$;

$j \leftarrow 1$;

while $j \leq \epsilon$ **do**

$\alpha^* \leftarrow \text{MODIFIED-HILL-CLIMBING}(A, N(\alpha, \xi^*), \eta)$;

$\xi^* \leftarrow \text{MODIFIED-HILL-CLIMBING}(\Xi, R_{\text{mod}}^2(\alpha^*, \xi), \eta)$;

if $\xi^* == \xi_r$ **then**

$N^* \leftarrow N(\alpha^*, \xi^*)$;

$R_{\text{mod}}^{2*} \leftarrow R_{\text{mod}}^2(\alpha^*, \xi^*)$;

$S_{ue} \leftarrow S_{ue} \cup (\alpha^*, \xi^*, N^*, R_{\text{mod}}^{2*})$;

break;

else

$\xi_r \leftarrow \xi^*$;

$j \leftarrow j + 1$;

end

end

end

extract S_{aue} from S_{ue} by the non-dominated sorting algorithm (Deb et al., 2002);

if $P == 1$ **then**

| select $(\alpha^*, \xi^*, N^*, R_{\text{mod}}^{2*})$ whose N^* is the largest among S_{aue} ;

else

| select $(\alpha^*, \xi^*, N^*, R_{\text{mod}}^{2*})$ whose R_{mod}^{2*} is the highest among S_{aue} ;

end

return $(\alpha^*, \xi^*, N^*, R_{\text{mod}}^{2*})$

end

has not been found.

The same games with different initializations can be played in parallel without mutual interference. In each round, player 1 and player 2 take turns to select their best strategies, α^* and ξ^* , by applying the modified hill-climbing algorithm. After a periodic sequence of rounds, if ξ^* achieved from the current round equals that from the preceding round, the currently best strategies will be identified as a Nash equilibrium and reported associated with the corresponding Nash outcome. Then, with all obtained Nash equilibria, we apply the non-dominated sorting algorithm proposed by Deb et al. (2002) to further extracting admissible Nash equilibria. Finally, a unique Nash equilibrium solution is selected among the admissible Nash equilibria depending on the preference between quantity ($P = 1$) and quality ($P = 0$). The pseudocode of the alternated hill-climbing search algorithm is given in Algorithm 4.4.2.

Note that, in the pseudocode, (i) d is the maximum attainable difference in position between two consecutive initial ξ^* 's, and r is calculated in this way to guarantee that all initial ξ^* 's are uniformly picked from Ξ ; (ii) as θ_2 becomes larger, it is possible to find more Nash equilibria and eventually lead to a better solution, but the computational complexity would also be increased. Therefore, we adopt the parallel computing technique to our search algorithm so that the search processes with different initializations can be carried out concurrently.

4.5 Case study

In this section, we apply the proposed approach associated with the search algorithm to empirically calibrating the parameters using raw loop-detector data. In addition, we validate the calibration results by assessing the performances of identified near-stationary states and the calibrated fundamental diagram.

4.5.1 Study site and data

The selected study site and data are exactly the same as those in Chapter 3. To be specific, the study site is a freeway mainline loop-detector station covering four general-purpose lanes and deployed upstream to the on-ramp from Valley View Street on the eastbound State Route 91 in Buena Park, California. In addition, the data are 30-sec raw loop-detector data provided by the PeMS¹ data clearinghouse and collected from nine months in 2011 with a sample size of 273 days. During the selected period, the loop-detector station has been identified as an active bottleneck for over 60% of the days by the PeMS algorithm (Chen et al., 2004), so sufficient congestion periods can be expected in the data sample.

4.5.2 Calibration

To measure the objective/payoff functions, some relevant parameters are determined as follows. First, the three fixed parameters in the four-step method are set by $\lambda_n = 4$ and $\lambda_h = 10$ min, which follow Cassidy's suggestion (Cassidy, 1998), and $\gamma_h = 0.4$, which is determined by engineering judgment, respectively. Then we consider a flow-occupancy plane whose occupancy is greater than or equal to 0.18 as the congested regime, i.e., $\tilde{o}_c = 0.18$, and set $\rho = 6$, which indicates that identified near-stationary states in the congested regime with a size lower than 6 would be considered unrepresentative and result in $R_{mod}^2 = 0$. Finally, the decision variable/strategy spaces are set by $A = \{0.02, 0.03, \dots, 0.30\}$ with a common difference of 0.01 and cardinality of 29, and $\Xi = \{2, 2.1, \dots, 10\}$ with a common difference of 0.1 and cardinality of 81.

Next, to verify the unimodal properties, the outcomes of both the objective functions corresponding to all feasible solutions are manually calculated and shown in the form of contour plots within the feasible region, $A \times \Xi$, in Figure 4.5a and 4.5b, respectively. As can be seen,

¹Caltrans Performance Measurement System (<http://pems.dot.ca.gov/>)

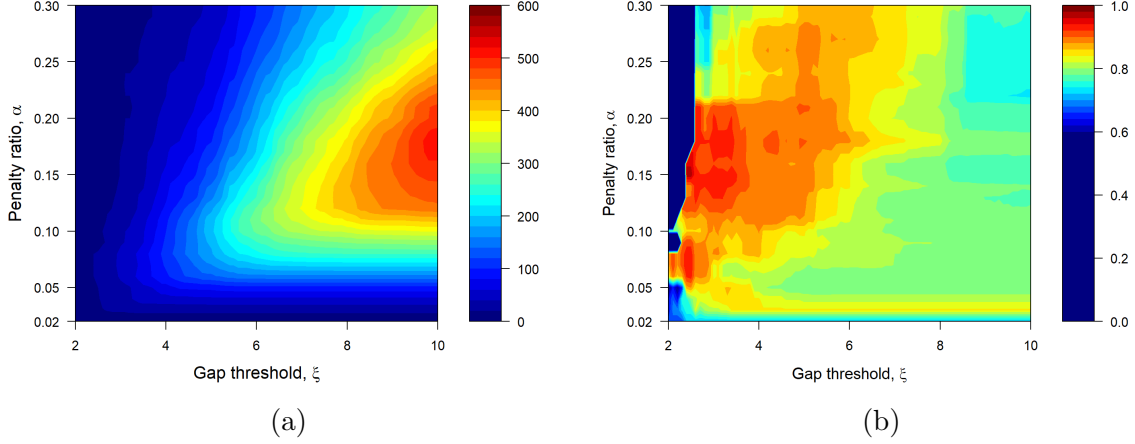


Figure 4.5: Contour plots of (a) $N(\alpha, \xi)$ and (b) $R^2_{\text{mod}}(\alpha, \xi)$.

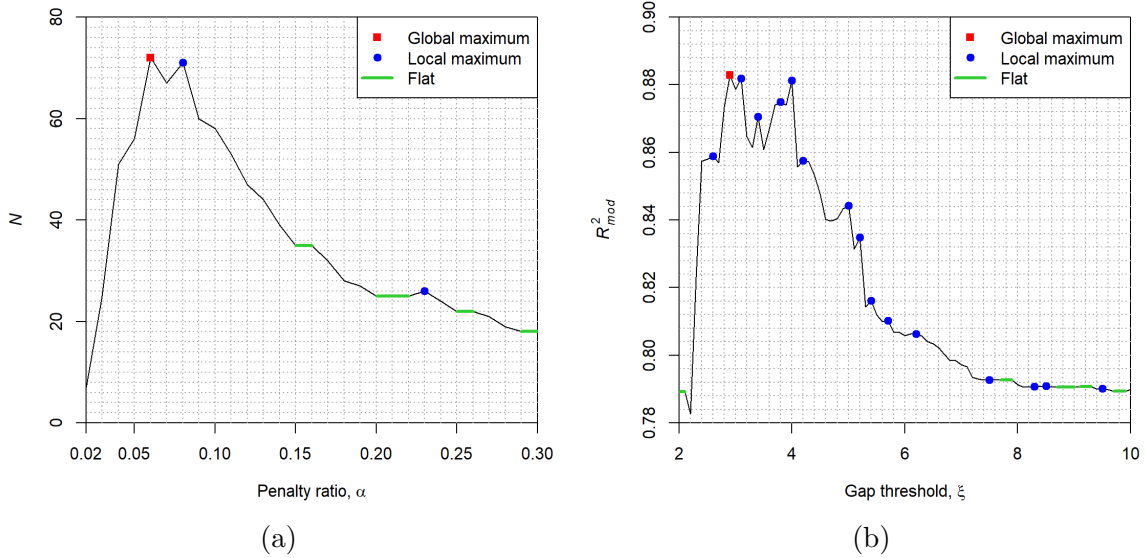


Figure 4.6: Bivariate relations of (a) $N(\alpha, \xi = 3.5)$ and (b) $R^2_{\text{mod}}(\alpha = 0.1, \xi)$.

$N(\alpha, \xi)$ is approximately unimodal in α for any certain ξ , and $R^2_{\text{mod}}(\alpha, \xi)$ is approximately unimodal in ξ for any certain α , which are consistent with the theoretical analysis in Section 4.2. In addition, we observe that all peaks of the objective functions are included in the feasible region, which confirms that our settings for A and Ξ are appropriate. Then, to show that both the objective functions have local maxima and flats, in Figure 4.6a and 4.6b, we depict the bivariate relations of $N(\alpha, \xi = 3.5)$ and $R^2_{\text{mod}}(\alpha = 0.1, \xi)$, respectively, as two examples and mark out the global, local maxima, and flats. From the figures, we can see that even though the unimodal properties hold, multiple local maxima and flats exist in the

relations, especially for the latter. Therefore, our modified hill-climbing algorithm will be superior to the conventional one when searching for the global maximum of a payoff function in such situations.

Prior to seeking a unique Nash equilibrium solution to our two-player game, first, the parameters needed for performing the alternated search algorithm are set by $\eta = 10$, $\epsilon = 20$, $\theta_2 = 10$, and we concern the quantity of identified near-stationary states more than quality if multiple admissible Nash equilibria are found, i.e., $P = 1$. Then the same games with 10 different initial ξ^* 's are played concurrently through the parallel computing to search for a unique solution. The search processes can be visualized by the sequential strategy choices made by player 2, as shown in Figure 4.7. We can see that all the games converge very fast within only 2 or 3 rounds and finally reach a common Nash equilibrium, $(\alpha^* = 0.07, \xi^* = 2.5)$, which naturally becomes the unique solution to our game as well as the multi-objective optimization problem.

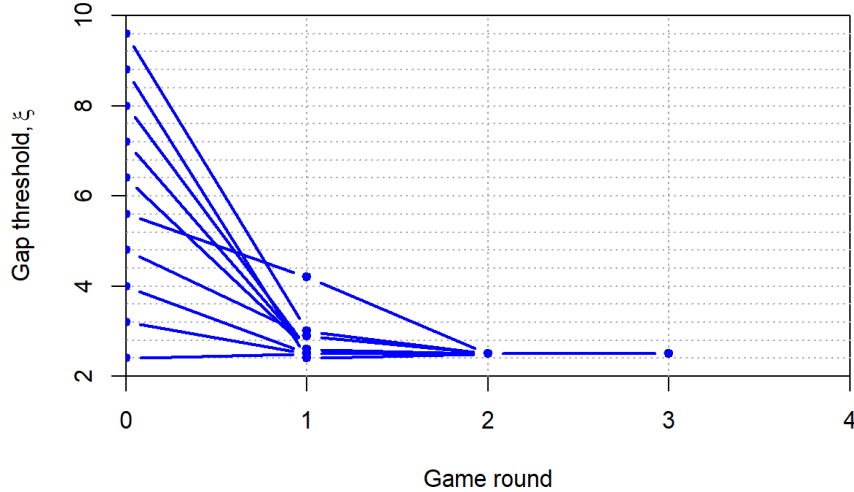


Figure 4.7: Convergence of the same games with different initializations.

In Figure 4.8, all the objective outcomes, pareto frontier, and the Nash outcome corresponding to our unique solution, $(N^* = 24, R_{mod}^{2*} = 0.94)$, are shown in the objective region, $N \times R_{mod}^2$. Here the pareto frontier is comprised of 53 pareto-optimal solutions that are not dominated by any other member belonging to $A \times \Xi$ and extracted by the non-dominated

sorting algorithm (Deb et al., 2002). As can be seen, the pareto frontier lies on the outermost layer of the objective outcome pattern and the Nash outcome falls within it, which indicates that our alternated hill-climbing search algorithm is able to find one of the global optimum solutions to the multi-objective optimization problem. In addition, we find that our Nash equilibrium solution spontaneously gives more weight to the quality as the Nash outcome is located around the bottom edge of the pareto frontier.

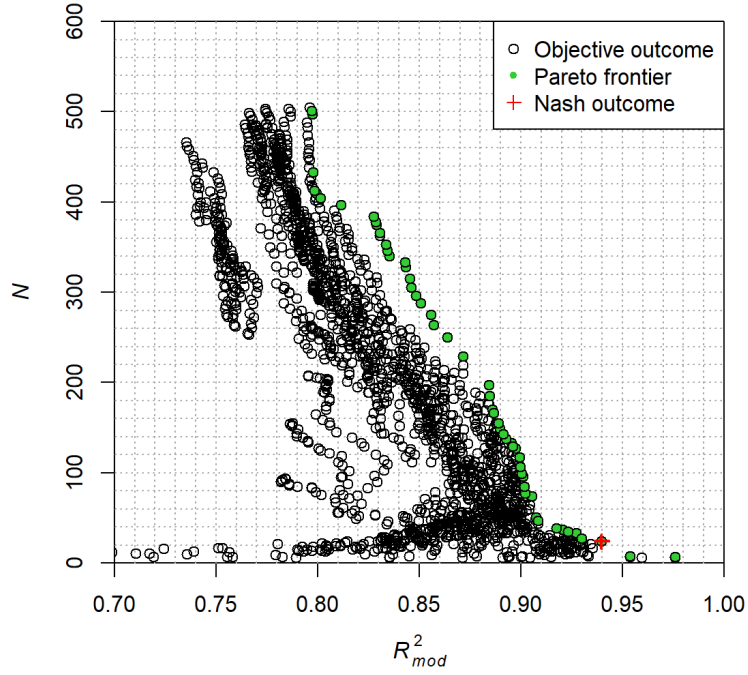


Figure 4.8: Objective outcomes, pareto frontier, and Nash outcome in the objective region.

4.5.3 Validation

With the achieved solution, 184 near-stationary states are identified from the 30-sec raw loop detector data of 107 days through the four-step method. In Figure 4.9, we show two flow-occupancy relations formed by the raw loop-detector and identified near-stationary data, respectively. By comparison, we can see that the near-stationary relation is almost scatter-free and consists of two patterns that are approximately linear and clearly separated without any mixed points (non-steady near-stationary states) falling between them.

Then, to calibrate a triangular fundamental diagram, we apply a constraint linear regression through the origin and simple linear regression to fitting the patterns in the free-flow and congested regimes, respectively. In Table 4.1, the calibrated parameters of the fundamental diagram as well as the Nash equilibrium and outcome are compared with the manual results in Chapter 3. We find that the quality has been apparently inflated but the quantity decreases as a result of the trade-off. In addition, the calibrated fundamental diagram parameters by both approaches, including the capacity, critical, and jam occupancies, are almost the same and physically meaningful.

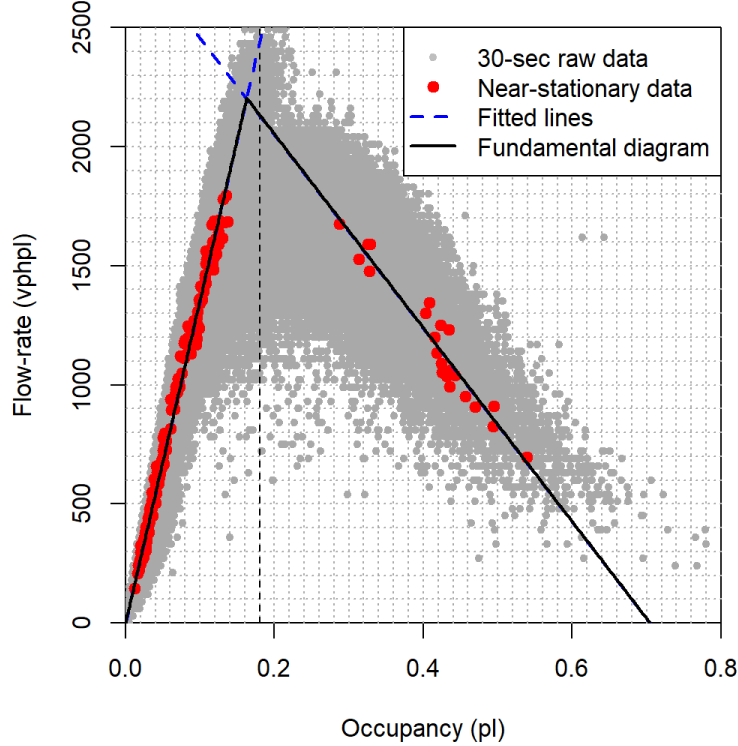


Figure 4.9: Near-stationary relations and calibrated fundamental diagram

Table 4.1: Nash equilibria, outcomes, and calibrated parameters of fundamental diagrams by different algorithms.

Algorithm	Nash equilibrium		Nash outcome		Calibrated parameters of FD		
	α^*	ξ^*	N^*	R_{mod}^{2*}	capacity	critical occ	jam occ
Manual	0.1	4	100	0.88	2186	0.16	0.70
Game theory	0.07	2.5	24	0.94	2201	0.16	0.70

Through the case study, in summary, our game theory approach associated with the alternated hill-climbing search algorithm can provide a valid calibration of the penalty ratio and gap threshold in the four-step method. In addition, both the resulting near-stationary states and calibrated fundamental diagram perform well.

4.6 An extended five-player game to calibrate penalty ratios, gap thresholds, and minimum acceptable durations

One limitation of the four-step method that has been pointed out in Section 3.6 and can also be observed in Figures 3.11a and 4.9 is that identified near-stationary states rarely appear around the occupancy of 0.4 and beyond 0.6. This is because that, for the duration criterion in step 4, the minimum acceptable duration has a sudden drop from the normal- to high-occupancy period at $\gamma_h = 0.4$ and a fixed $\lambda_h = 4$ min is too strict to identify near-stationary states from periods with extremely high occupancies.

To solve the limitation and make the near-stationary flow-occupancy pattern more complete in the congested regime, first, we retrofit the relation between minimum acceptable duration and candidate occupancy from a step-wise to quadratic form. As shown in Figure 4.10, the modified relation has a more smoothly quadratic decrease in the transitional occupancy period between γ_n and γ_x and smaller minimum acceptable duration in the extremely high-occupancy period. Then we consider γ_n , γ_h , and γ_x as the fixed parameters but λ_n , λ_h , and λ_x as three additional decision variables to provide the minimum acceptable durations with more flexibility. Thus the multi-objective optimization problem can be reformulated as

follows:

$$\begin{aligned}
& \underset{\alpha, \xi, \lambda_n, \lambda_h, \lambda_x}{\text{Maximize}} && N(\alpha, \xi, \lambda_n, \lambda_h, \lambda_x), \quad R_{\text{mod}}^2(\alpha, \xi, \lambda_n, \lambda_h, \lambda_x), \\
& \text{subject to} && \alpha \in A, \\
& && \xi \in \Xi, \\
& && \lambda_n \in \Lambda_n, \\
& && \lambda_h \in \Lambda_h, \\
& && \lambda_x \in \Lambda_x,
\end{aligned} \tag{4.14}$$

where Λ_n , Λ_h , and Λ_x are the corresponding decision variable spaces.

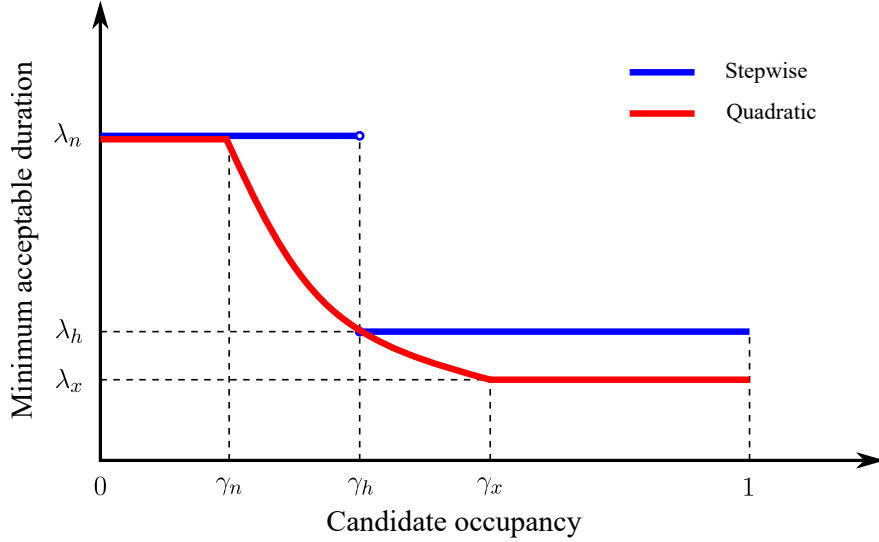


Figure 4.10: Relations between minimum acceptable duration and candidate occupancy.

Similar to ξ , we find that all λ_n , λ_h , and λ_x have more effects on $R_{\text{mod}}^2(\alpha, \xi, \lambda_n, \lambda_h, \lambda_x)$ and their relationships are all approximately unimodal. More specifically, when the three variables are small, a great number of unstable near-stationary states with short durations would be identified, form a dispersed flow-occupancy pattern in the congested regime, and finally cause a small R_{mod}^2 ; on the other hand, when the variables are large, only a few near-stationary states with long durations can be identified and the formed flow-occupancy relation in the congested regime would be sparse and incomplete with a small R_{mod}^2 . There-

fore, only moderate choices of the variables can lead to high R_{mod}^2 and make the relationships approximately unimodal.

Based on the new unimodal properties, we can arrange three additional players to join the game with their strategies and strategy spaces, $\lambda_n \in \Lambda_n$, $\lambda_h \in \Lambda_h$, and $\lambda_x \in \Lambda_x$, respectively, and payoff functions, $R_{mod}^2(\alpha^*, \xi^*, \lambda_n, \lambda_h^*, \lambda_x^*)$, $R_{mod}^2(\alpha^*, \xi^*, \lambda_n^*, \lambda_h, \lambda_x^*)$, and $R_{mod}^2(\alpha^*, \xi^*, \lambda_n^*, \lambda_h^*, \lambda_x)$. As before, the five-player game still follows all the established rules, and in each round, the players take turns to choose the best strategies to maximize their payoff functions given the strategy choices of the other four players in the preceding round. A flow chart of the game process can be visualized in Figure 4.11. Note that even though players 2-5 share the same objective function, their payoff functions are totally different because of different strategies and strategy spaces.

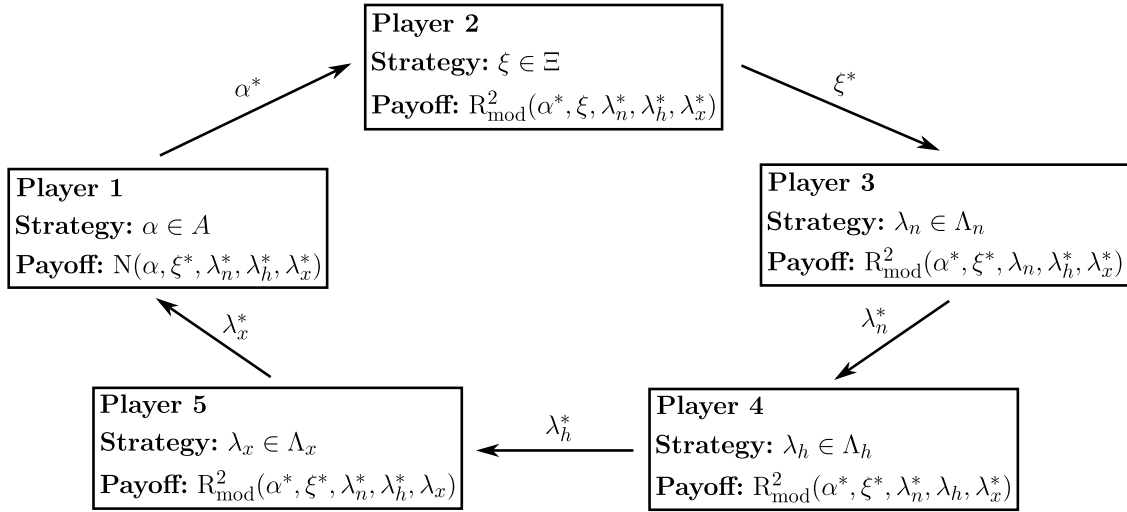


Figure 4.11: Process of the five-player game.

To achieve a unique Nash equilibrium solution through the five-player game, the alternated hill-climbing algorithm in Algorithm 4.4.2 is still applicable, but two adjustments should be made such that (i) to start the game, player 1 needs to make multiple initial guesses for strategy choices of the other four players; and (ii) a Nash equilibrium is reached if the best strategies of players 2-5 remain the same in two consecutive rounds.

Back to the case study, we hold the parameters that have already been set unchanged and determine the new parameters as follows: we set $\gamma_n = 0.2$ and $\gamma_x = 0.6$ by engineering judgment; the discrete strategy spaces of the three new players are set by $\Lambda_n = \{8.5, 9, \dots, 11.5\}$, $\Lambda_h = \{2.5, 3, \dots, 5.5\}$, and $\Lambda_x = \{1, 1.5, \dots, 4\}$ with a common difference of 0.5; the numbers of initial guesses for strategies of players 2-5 are chosen as $\theta_2 = 6$ and $\theta_3 = \theta_4 = \theta_5 = 2$ with a total of 48 different combined initializations.

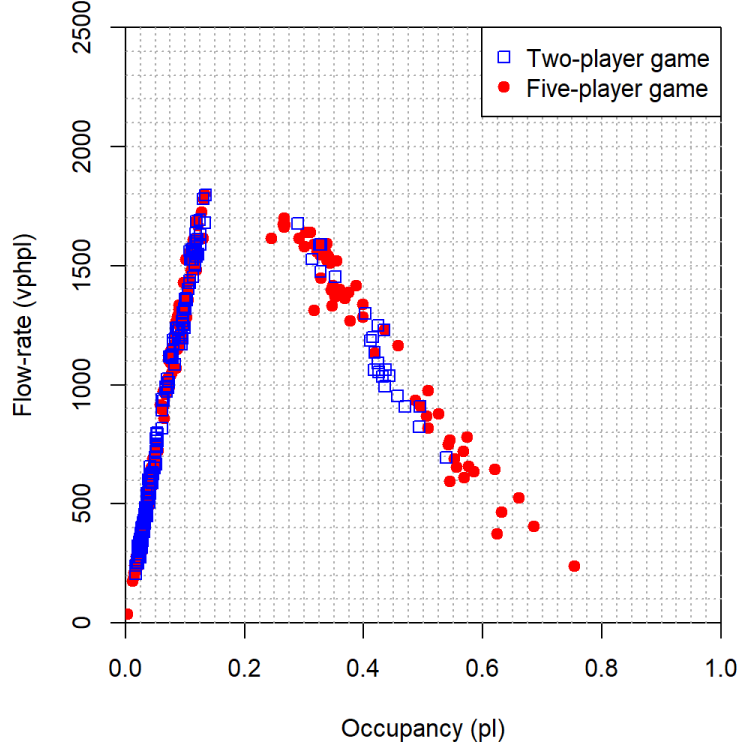


Figure 4.12: Flow-occupancy relations generated by two-player and five-player games.

Finally, the alternated search algorithm finds two Nash equilibria, which are both admissible, and a unique Nash equilibrium solution is $(\alpha^* = 0.05, \xi^* = 2.2, \lambda_n^* = 9, \lambda_h^* = 5, \lambda_x^* = 1)$ with the Nash outcome of $(N^* = 58, R_{mod}^{2*} = 0.96)$. Figure 4.12 shows the near-stationary flow-occupancy relations generated by the two-player and five-player games, respectively. It can be seen that the new relation can still form two scatter-free and approximately linear patterns with few mixed points between them. More importantly, we find that the relation by the five-player game has a more consistent and complete linear pattern in the congested regime, which fills the gaps around the occupancy of 0.4 and beyond 0.6 that appear in the old

one, larger quantity, and even higher measured quality. A comparison between two- and five-player games on the calibration of the parameters is shown in Table 4.2.

Table 4.2: Comparison of parameter calibration between two- and five-player games.

Algorithm	Nash equilibrium					Nash outcome	
	α^*	ξ^*	λ_n^*	λ_h^*	λ_x^*	N^*	R_{mod}^{2*}
Two-player game	0.07	2.5	10 (manual)	4 (manual)	-	24	0.94
Five-player game	0.05	2.2	9	5	1	58	0.96

In summary, by adding flexibility to the minimum acceptable durations through the extended five-player game, the overall performance of identified near-stationary states in the congested regime becomes greater. Therefore, such an extension is reasonable.

4.7 Conclusion

This chapter is a follow-up study of the automated four-step method to calibrate parameters of the method so as to further identify near-stationary traffic states from inductive loop-detector data. In the chapter, we first formulated the calibration of the penalty ratio and gap threshold as a multi-objective optimization problem with the unimodal properties. Then we presented a game theory approach to reinterpret the problem as a non-cooperative two-player game and defined Nash equilibrium solutions. We further developed an alternated hill-climbing algorithm to seek a unique Nash equilibrium solution to our problem. In a case study, we applied our proposed approach and search algorithm using raw loop-detector data and showed that (i) the calibrated parameters are valid, (ii) the identified near-stationary states have a high quality and reasonable quantity, and (iii) the calibrated fundamental diagram is well-fitted and physically meaningful.

To the best of the author’s knowledge, we are the first to introduce the non-cooperative game

theory associated with the Nash equilibrium into the identification of near-stationary traffic states and further fundamental diagram calibration, and our game theory approach as well as the exclusive search algorithms has some significant advantages as follows: (i) while being concerned about both the quantity and quality of near-stationary states during peak periods, the approach does not impose any subjective preference on the conflicting performances and can guarantee to reach at least one Nash equilibrium based on the mechanism of finite non-cooperative games; (ii) by utilizing the unimodal properties, the modified hill-climbing algorithm is robust and efficient to find the best strategies in each round of the game; (iii) the alternated search algorithm takes into account the potential possibilities of the existence of multiple Nash equilibria and convergence failure and can search for a unique Nash equilibrium solution by playing the same games with different initializations concurrently.

In this chapter, we give a paradigm in Section 4.6 to show that by adjusting the relation between minimum acceptable duration and candidate occupancy in the gap criterion of the four-step method and extending the game to a non-cooperative game with five players, we can empirically obtain a more complete near-stationary flow-occupancy relation. In the future, on the basis of the solid framework of the game theory approach, some other extensions and improvements are also worth trying. For example, note that in the extended five-player game, players 2-5 actually share the same objective function with respect to the quality. Thus we can still maintain a two-player structure, but the difference is that player 2 has a strategy vector, $[\xi, \lambda_n, \lambda_h, \lambda_x]$, instead of just ξ , with a corresponding payoff function, $R_{\text{mod}}^2(\alpha^*, [\xi, \lambda_n, \lambda_h, \lambda_x])$, and attempts to choose the best strategy combination that maximizes the payoff in each round of the game. In this way, we actually build a non-cooperative game between player 1 and 2 superficially but a cooperative subgame within player 2. Such a mixed game can also be solvable through our alternated search framework but the modified hill-climbing technique needs to be further adjusted because of the multidimensional payoff function of player 2. In addition to the example, we will be interested in developing new payoff functions related to other performances of near-stationary states and smoother

relationships between minimum acceptable duration and candidate occupancy.

Chapter 5

Calibration of Unifiable Multi-Lane Fundamental Diagrams

5.1 Introduction

Traffic on freeways can be categorized into multiple commodities according to their associated lanes and vehicle type characteristics. Different commodities usually perform differently and have dissimilar impacts on the mobility, safety, environmental and human health, and costs. For example, daily commuters typically experience traffic situations where priority vehicles on high-occupancy vehicle (HOV) and toll (HOT) lanes travel faster than regular vehicles on general purpose lanes, aggressive drivers on median lanes faster than conservative drivers on shoulder lanes, and passenger cars faster than trucks (Daganzo, 2002a). In addition, trucks have always been linked with adverse effects on freeway incidents (Dong et al., 2015), emissions (Sprung et al., 2018), travel time variability (FHWA, 2017), and pavement deterioration (Ramaswamy and Ben-Akiva, 1990). Furthermore, utilizations of HOV and HOT lanes have become a big concern in recent years (Murray et al., 2001; Kwon

and Varaiya, 2008). In the near future, as connected and autonomous vehicles become increasingly prominent in traffic streams, their influences on lane performance and other types of vehicles will also be worth investigating. Therefore, to better understand such above public impacts, concerns, and issues, a well-defined, physically meaningful, and well-calibrated multi-lane multi-class fundamental diagram model plays a critical role in serving the various needs.

From a theoretical point of view, the multi-lane multi-class fundamental diagrams are important phenomenological models to study moving bottlenecks (Gazis and Herman, 1992; Newell, 1998), lane-changing maneuvers (Laval and Daganzo, 2006), lane-drop effects (Munjal and Pipes, 1971; Munjal et al., 1971), and impacts of heterogeneous drivers and vehicles (Wong and Wong, 2002; Logghe and Immers, 2008) in traffic dynamics. Empirically, only a handful of studies attempt to calibrate multi-lane (Li and Zhang, 2011) or multi-class (Coifman, 2014b) fundamental diagrams. Such above studies have a common simplification such that commodity speeds are assumed to be dependent of either its own commodity density or the total density and neglect the unifiable property such that a flow-density relation of total traffic also exists as well as commodity fundamental diagrams. However, in reality, due to car-following and lane-changing mechanisms among vehicles, one commodity's speed and flow-rate should generally be affected by all prevailing commodities' densities instead of its own. In addition, a fundamental diagram of total traffic was observed in many empirical studies (Hall et al., 1986; Cassidy, 1998) and should be taken into account when calibrating multi-commodity fundamental diagrams.

To fill the gap, we adopt a novel multi-commodity kinematic wave theory proposed in (Jin, 2017d; Jin and Yan, 2018) to calibrate multi-lane and multi-class fundamental diagrams. In the theory, the multi-commodity fundamental diagrams possess the unifiable property and FIFO (first-in-first-out) can be violated such that different commodities can have different speeds at the same time and location. In addition, the absolute and relative speed ratios

defined in the theory can capture interaction among different commodities.

This chapter will focus on the calibration of unifiable multi-lane fundamental diagrams. The rest of the chapter is organized as follows. In Section 5.2, we review the novel theory of unifiable multi-commodity fundamental diagrams. In Section 5.3, some guidelines for verification of unifiable and non-FIFO properties are presented. In Section 5.4, we empirically verify the unifiable and non-FIFO properties and calibrate a fundamental diagram of total traffic as well as relative lane speed ratios. In Section 5.5, we calibrate lane absolute speed ratios by multivariate linear regression. In Section 5.6, the calibrated unifiable multi-lane fundamental diagrams are validated. Finally in Section 5.7, we conclude the chapter with some discussions.

5.2 Review of unifiable multi-commodity fundamental diagrams

For an M -commodity traffic flow, the flow-rate, density, and (space mean) speed of commodity m at location x and time t are denoted by $q_m(x, t)$, $k_m(x, t)$, and $v_m(x, t)$, respectively, where $m = 1, \dots, M$. Correspondingly, the total traffic flow-rate, density, and speed are denoted by $q(x, t)$, $k(x, t)$, and $v(x, t)$, respectively. Hereafter (x, t) is omitted unless necessary. Further the flow-rate and density proportions of m are denoted by ϕ_m and p_m , respectively, and absolute and relative speed ratios of m by η_m and π_m , respectively, and all of them are

non-negative. That is,

$$\phi_m = \frac{q_m}{q}, \quad (5.1a)$$

$$p_m = \frac{k_m}{k}, \quad (5.1b)$$

$$\eta_m = \frac{v_m}{v}, \quad (5.1c)$$

$$\pi_m = \frac{v_m}{v_r}, \quad (5.1d)$$

where $r \in \{1, \dots, M\}$ is a reference commodity.

Given the above variables, a multi-commodity traffic flow system can be built based on the following rules:

(R1) Additive relations between commodity and total flow-rates and densities:

$$q = \sum_{m=1}^M q_m, \quad (5.2a)$$

$$k = \sum_{m=1}^M k_m, \quad (5.2b)$$

which lead to

$$\sum_{m=1}^M \phi_m = 1, \quad (5.2c)$$

$$\sum_{m=1}^M p_m = 1. \quad (5.2d)$$

(R2) Commodity and total traffic constitutive laws:

$$q_m = k_m v_m, \quad \forall m = 1, \dots, M, \quad (5.3a)$$

$$q = k v, \quad (5.3b)$$

which lead to

$$\phi_m = p_m \eta_m, \quad \forall m = 1, \dots, M. \quad (5.3c)$$

(R3) Commodity and total traffic speed functions:

$$v_m = V_m(k_1, \dots, k_M) = V_m(k, p_1, \dots, p_{M-1}), \quad \forall m = 1, \dots, M, \quad (5.4a)$$

$$v = V(k, p_1, \dots, p_{M-1}) \equiv \sum_{m=1}^M p_m V_m(k, p_1, \dots, p_{M-1}). \quad (5.4b)$$

Note that (i) (5.1b) leads to $k_m = p_m k$, so the state variables, $(k_1, \dots, k_M)$, can be replaced by $(k, p_1, \dots, p_{M-1})$ without loss of generality, as shown in (5.4a); (ii) (5.4b) can be derived by (5.1c), (5.2c), and (5.3c); (iii) ϕ_m , η_m , and π_m can also be represented as functions with respect to the state variables:

$$\phi_m = \Phi_m(k, p_1, \dots, p_{M-1}), \quad (5.5a)$$

$$\eta_m = H_m(k, p_1, \dots, p_{M-1}), \quad (5.5b)$$

$$\pi_m = \Pi_m(k, p_1, \dots, p_{M-1}). \quad (5.5c)$$

In the multi-commodity traffic system, two important properties are defined as follows:

Definition 5.1. A multi-commodity traffic flow is *first-in-first-out (FIFO)* if and only if all commodities have the same speed, which also equals the total traffic speed, at the same location and time:

$$v_1 = \dots = v_M = v, \quad (5.6a)$$

or equivalently, all commodity absolute and relative speed ratios equal one:

$$\eta_m = \pi_m = 1, \quad \forall m = 1, \dots, M. \quad (5.6b)$$

Definition 5.2. A multi-commodity traffic flow is *unifiable* if and only if the total traffic speed can be simplified as a function of the total density:

$$v = V(k, p_1, \dots, p_{M-1}) = V(k), \quad (5.7a)$$

or equivalently, there exists a fundamental diagram of total traffic between total flow-rate and density:

$$q = Q(k) \equiv kV(k). \quad (5.7b)$$

Note that (i) (5.6a) and (5.7a) lead to

$$v_m = V(k), \quad \forall m = 1, \dots, M, \quad (5.8)$$

which indicates that all commodity speeds are the same and only dependent of the total density when a multi-commodity traffic flow is both unifiable and FIFO; (ii) (5.7b) can be derived from most first- and higher-order continuum and car-following traffic flow models in steady states (Jin, 2016), so a necessary condition to trigger the unifiable property is that the total traffic needs to remain steady.

Through derivations, finally, the general form of unifiable multi-commodity fundamental diagrams associated with absolute speed ratios can be written by

$$q_m = p_m \cdot Q(k) \cdot H_m(k, p_1, \dots, p_{M-1}), \quad \forall m = 1, \dots, M, \quad (5.9)$$

and that associated with relative speed ratios by

$$q_m = \frac{p_m \Pi_m(k, p_1, \dots, p_{M-1})}{\sum_{m'=1}^M p_{m'} \Pi_{m'}(k, p_1, \dots, p_{M-1})} \cdot Q(k), \quad \forall m = 1, \dots, M. \quad (5.10)$$

Note that (5.9) and (5.10) are formulated differently but totally equivalent. In addition, the relative speed ratios in (5.10) are physically, behaviorally, and economically meaningful since they can characterize drivers' relative aggressiveness, values of times, and other features.

5.3 Guidelines for verification of unifiable and non-FIFO properties

Prior to calibration of the multi-commodity fundamental diagrams, it is important to verify the validity of unifiable and non-FIFO properties because (i) if the unifiable property does not hold, the fundamental diagram of total traffic would not exist and thus neither (5.9) or (5.10) is valid; (ii) if the FIFO property is not violated, all commodity absolute and relative speed ratios should be equal to one and thus both (5.9) and (5.10) would be simplified as $q_m = p_m \cdot Q(k)$, $\forall m = 1, \dots, M$.

First, to verify that a multi-commodity traffic is unifiable, the following four criteria must be satisfied:

- **Stationarity:** both commodity and total traffic states should be near-stationary states, which can be identified by the automatic four-step method proposed in Chapter 3.
- **Interactivity:** all included commodities should be mutually interacted. More specifically, the change of one commodity density should affect speeds of the other commodities, or equivalently, one commodity speed should be dependent of not only its own

commodity density but also densities of the other commodities.

- **Fundamentality:** based on Definition 5.2, total traffic states should fall on a well-defined fundamental diagram between flow-rate and density. Empirically, the quantity and quality defined in Chapter 4 can be adopted to measure performances of the flow-density (or flow-occupancy) relation formed by near-stationary states of total traffic.
- **Variability:** at least one commodity density proportion should vary significantly over the total density rather than all remaining entirely constant. To clarify, as shown in (5.7a), if all commodity density proportions are constant, the total traffic speed is naturally only dependent of the total density and further simplification is redundant. In this case, the multi-commodity traffic can only be conditionally unifiable. Therefore, near-unifiable states can only be reached when the variability criterion is satisfied in addition to the above three.

With the verified unifiable property, the non-FIFO property can be met if (5.8) is violated. That is, we shall empirically observe that (i) the relation between commodity speed and total density is more scattered compared to the speed-density relation of total traffic, and (ii) there exists at least two commodities whose relations between commodity speed and total density do not overlap each other. Alternatively, the non-FIFO property can also be verified individually if

$$\eta_m \neq 1, \quad \exists m = 1, \dots, M, \quad (5.11a)$$

or equivalently,

$$\pi_m \neq 1, \quad \exists m = 1, \dots, M. \quad (5.11b)$$

That is, at least one absolute or relative speed ratio should not be constant at one over the

total density.

5.4 Empirical verification of unifiable and non-FIFO properties in multi-lane traffic

In a multi-lane traffic scenario, we present an empirical verification of the unifiable property and further calibrate a fundamental diagram of total traffic. Then we provide three ways to visually verify the validity of the non-FIFO property and calibrate relative speed ratios across lanes.

5.4.1 Study site and data processing

The study site is the same as before, as shown in Figure 3.9, and here we take into account the loop detectors on both general purpose (GP) and high occupancy vehicle (HOV) lanes. In addition, the raw loop-detector data of GP and HOV lanes are collected from the same period as before.

In the selected sample, the GP loop-detector station is identified as active bottlenecks for over 60% of the days by the PeMS algorithm (Chen et al., 2004). Thus sufficient congestion periods can be expected on GP lanes. However, the loop detector on the HOV lane is rarely marked as bottlenecks among the days, which infers that significantly different utilizations and traffic conditions may occur on HOV and GP lanes at the same times, especially during peak periods.

Moreover, SR-91 serves as a major freight route that allows large trucks to operate on the mainline lanes. According to Caltrans truck volumes reported for 2008, trucks comprise about 8% of total daily traffic around our study site. Based on the regulations and lane

restrictions in California, these trucks are restricted to a speed limit of 55 miles per hour and must use two right-hand GP lanes. Therefore, vehicle types and mobilities would be different on different lanes.

We consider the traffic across four GP lanes as a reference and apply the automatic four-step method to identify near-stationary states from the raw loop-detector data aggregated over four GP lanes. Among the parameters, we set $\lambda_n = 10$ and $\lambda_h = 4$ by following Cassidy’s suggestion (Cassidy, 1998) and $\gamma_h = 0.4$ by engineering judgment; in addition, we manually set $\alpha = 0.12$ and $\xi = 5.5$ to guarantee that enough near-stationary states can be identified in the congested regime. Finally, 1530 time intervals among filtered 107 days are identified as near-stationary states. Among them, near-stationary occupancies are converted into densities by assuming that the effective vehicle length is constant at 25 ft.

5.4.2 Verifying the unifiable property

Figure 5.1 shows a flow-density relation in near-stationary states averaged over four GP lanes. We can clearly see that the relation is comprised of two patterns that are approximately linear and almost scatter-free. We set two occupancy thresholds at 0.15 and 0.25 to discard the mixed points between two patterns. The quantity and R^2 of two separate patterns are stored in Table 5.1. As can be seen, both of the patterns have enough near-stationary data with high qualities.

To verify the interactivity, we consider two additional scenarios: one is total traffic with truck GP lanes (right-most two lanes); the other is total traffic combining all GP and HOV lanes. Their flow-density relations in near-stationary states are shown in Figures 5.2a and 5.2b, respectively. We can see that the free-flow pattern of the former scenario is more dispersed than that of the reference and the congested pattern of the latter scenario is more scattered than that of the reference. Therefore, the covered lanes in these two scenarios are

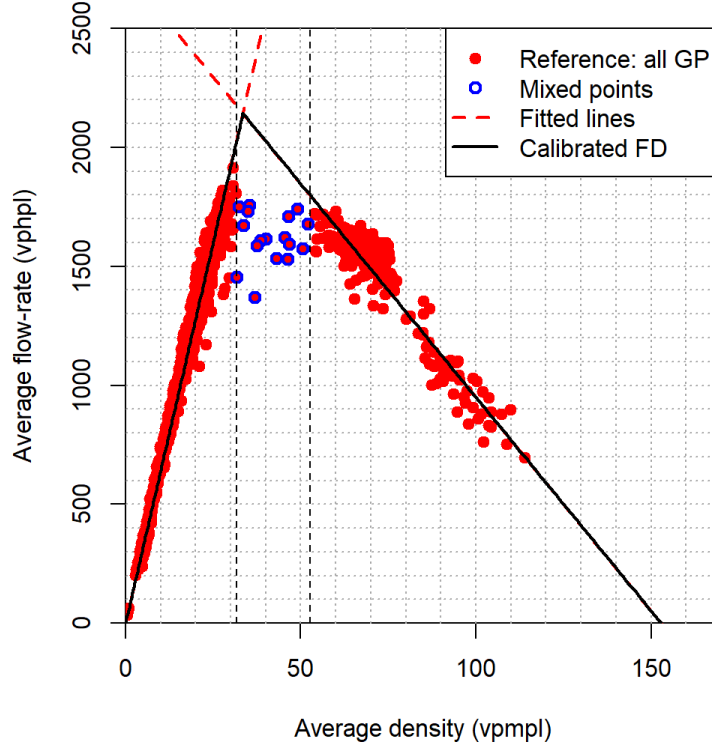


Figure 5.1: Near-stationary flow-occupancy relation across four GP lanes and the calibrated fundamental diagram.

less interacted compared to those in the reference.

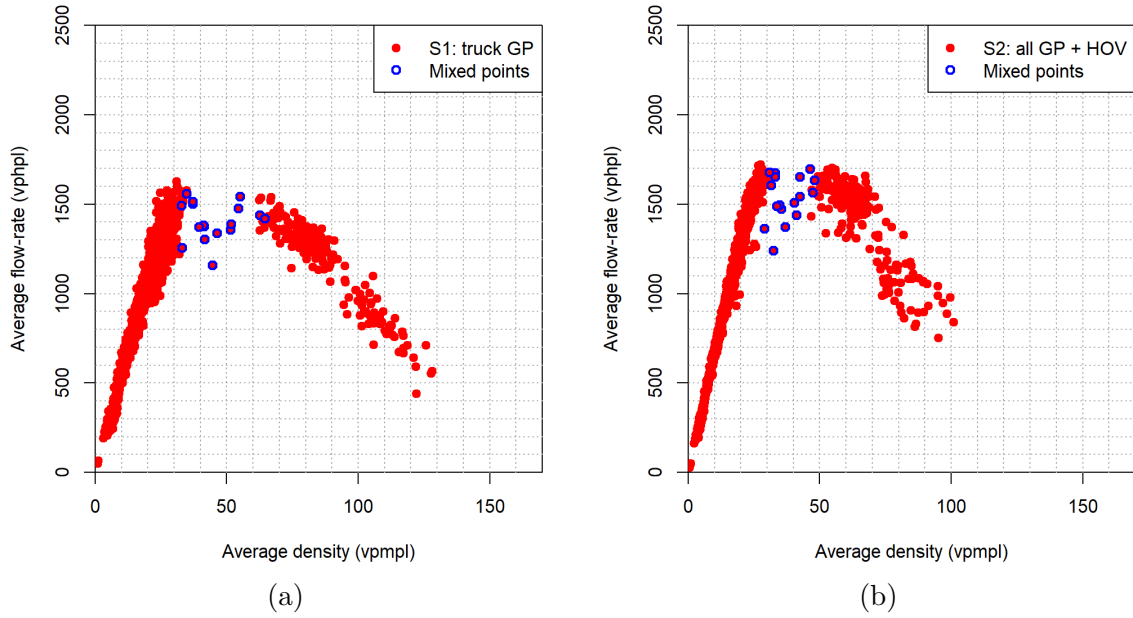


Figure 5.2: Near-stationary flow-density relations across (a) truck GP lanes and (b) all GP + HOV lanes.

Furthermore, we attempt to examine the effect of gap and duration criteria on the stationarity. As shown in Figures 5.3 and 5.4, near-stationary flow-density relations are formed with stricter gap, looser gap, stricter duration, and looser duration, respectively. Based on the results in Table 5.1, we can find that stricter gap and duration can lead to better quality but worse quantity; in contrast, looser gap and duration can result in more near-stationary data but with lower R^2 .

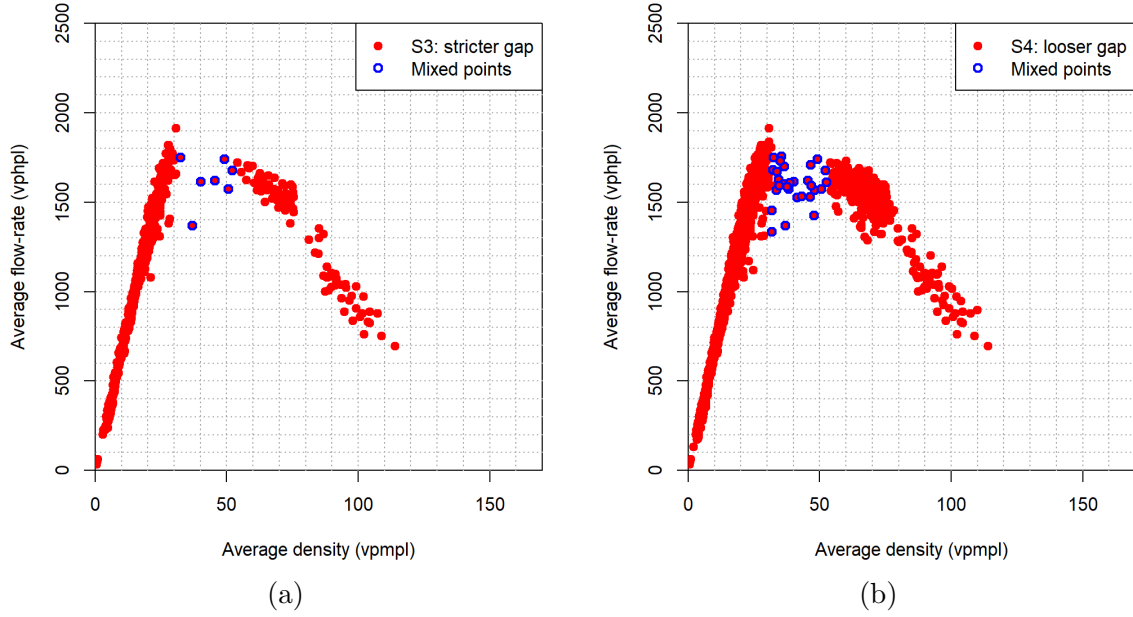


Figure 5.3: Near-stationary flow-density relations with (a) stricter gap and (b) looser gap criterion.

To verify the variability, we cut the total near-stationary densities into consecutive bins and calculate the median lane density proportions in each bin. As shown in Figure 5.5, all four lane density proportions change over the total density bins instead of being constant, so the variability criterion has been verified.

With the above verifications, the reference with four GP lanes can be considered as a good choice of total traffic. To calibrate a total traffic fundamental diagram in near-stationary states, we apply the constraint linear regression through the origin to the free-flow pattern and simple linear regression to the congested pattern. Then the piece-wise linear curve under the intersection of the fitted lines is the calibrated fundamental diagram of total traffic, which

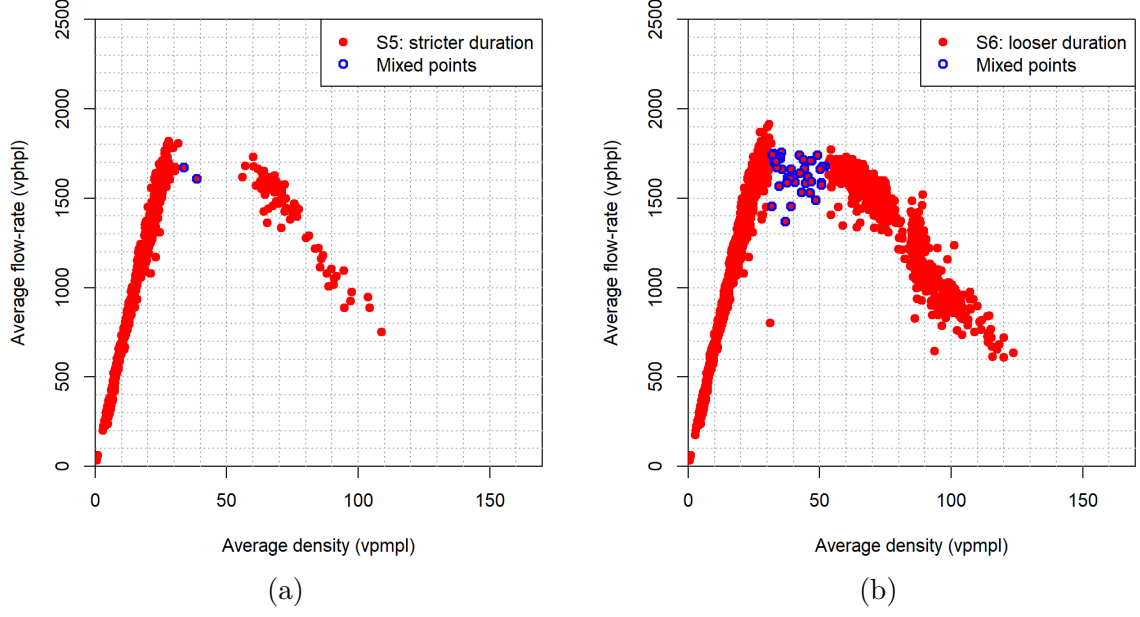


Figure 5.4: Near-stationary flow-density relations with (a) stricter duration and (b) looser duration criterion.

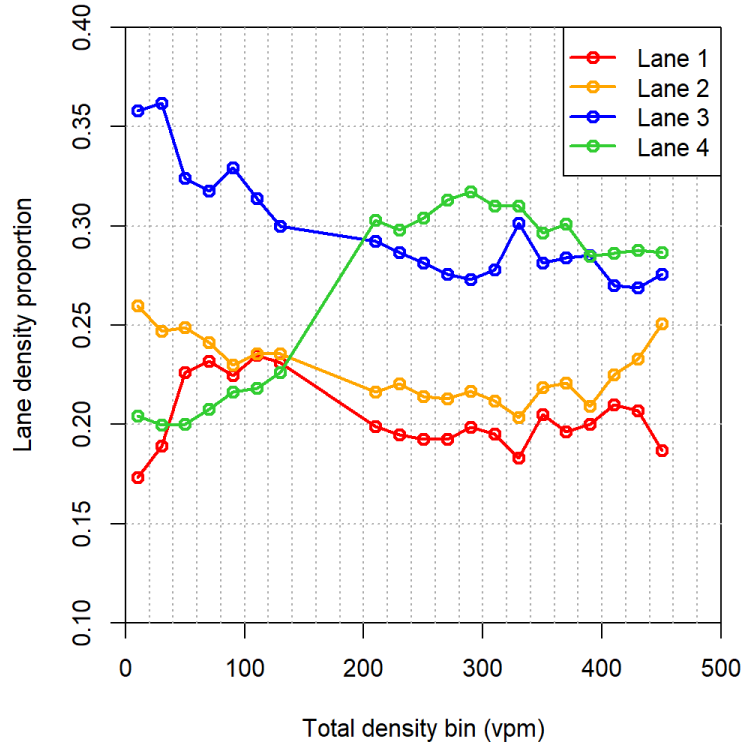


Figure 5.5: Verification of the variability.

can be written by

$$\hat{Q}(k^s) = \min\{\hat{\mu}k^s, \hat{\omega}(\hat{\kappa}_j - k^s)\} \quad (5.12)$$

Table 5.1: Quantity and quality of congested near-stationary states in different scenarios.

Scenario	Lane	Parameter				Quantity		R-squared	
		β	ξ	λ_n	λ_h	Free-flow	Congested	Free-flow	Congested
Reference: all GP	1-4	0.12	5.5	10	4	1290	223	0.980	0.907
1: truck GP	3-4	0.12	5.5	10	4	1290	223	0.946	0.907
2: all GP + HOV	0-4	0.12	5.5	10	4	1290	223	0.984	0.750
3: stricter gap	1-4	0.12	4.2	10	4	616	97	0.982	0.938
4: looser gap	1-4	0.12	8.0	10	4	2360	396	0.977	0.862
5: stricter duration	1-4	0.12	5.5	13	7	880	77	0.984	0.912
6: looser duration	1-4	0.12	5.5	8	2	1663	565	0.972	0.885

with the calibrated free-flow speed, $\hat{\mu} = 64$ mph, shock wave speed, $-\hat{\omega} = -18$ mph, jam density, $\hat{\kappa}_j = 612$ vpm, critical density, $\hat{\kappa}_c = \frac{\hat{\omega}}{\hat{\mu} + \hat{\omega}} \hat{\kappa}_j = 134$ vpm, and capacity, $\hat{C} = \hat{\mu} \hat{\kappa}_c = 8574$ vph, which are all physically reasonable.

5.4.3 Verifying the non-FIFO property

With the unifiable property has been verified, we can verify the non-FIFO property if (5.8) is violated. In Figure 5.6, we depict relations between lane and total traffic speeds and total density in near-stationary states, where four GP lanes from median to shoulder are ordered by lane 1-4, respectively. In the following, we present some findings from the figure to support the FIFO violation:

1. All the relations among different lanes do not fully overlap each other even though they have similar shapes. More specifically, in the congested regime, lane speeds drop from lane 1 to 4 on average and differences among them also decrease along the total density; in the free-flow regime, lane speed patterns also decline from lane 1 and differences among them are more distinct, but speeds on lane 4 is surprisingly higher than those on lane 3 on average.

2. The relations of lanes are also different from the speed-density relation of total traffic, whose free-flow pattern lies between lane 2 and 4 with a nearly constant magnitude, and congested pattern lying between lane 2 and 3 is almost scatter-free.
3. Compared to the relation of total traffic, all the lane relations are much more scattered, especially in the congested regime, which implies that lane speed functions should not just depend on the total density.

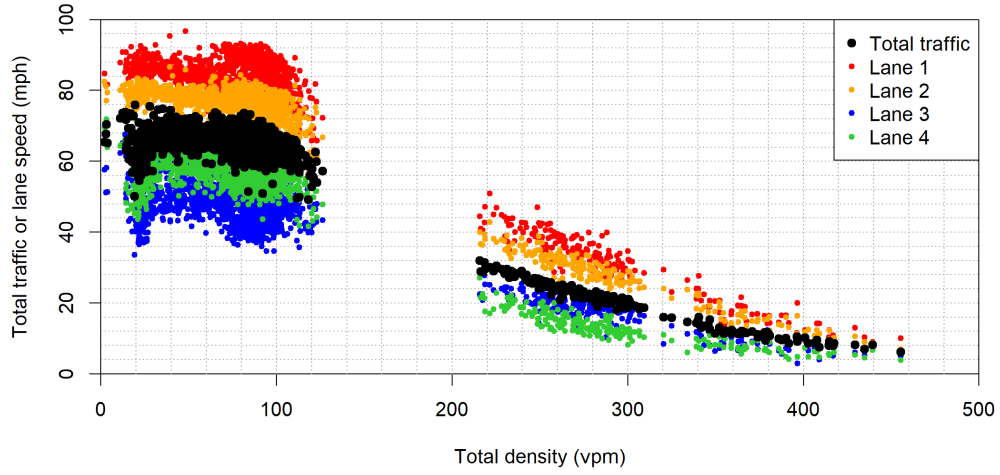


Figure 5.6: Relations between total traffic/lane speed and total density in near-stationary states

In addition, the non-FIFO property can be directly confirmed by showing that at least one lane absolute speed ratio is not constantly equal to one, i.e., (5.11a). As shown in Figure 5.7, distributions of observed lane absolute speed ratios in near-steady states across lanes and regimes are displayed in the form of box plots. Obviously, almost all the box plots do not even cross the horizontal line at one, and they span a wide range of lane absolute speed ratios, especially for those in the congested regime. To give more details, we also find that (i) all absolute speed ratios of lane 1 and 2 are distributed above one, while those of lane 3 and 4 are mostly smaller than one; (ii) in general, medians of lane absolute speed ratios decrease from lane 1 to 4 in both free-flow and congested regimes, while lane 4 is an exception in the free-flow regime, whose median is greater than that of lane 3; (iii) for lane 1-3, medians of

lane absolute speed ratios increase from the free-flow to congested regime, while the median for lane 4 has a significant drop.

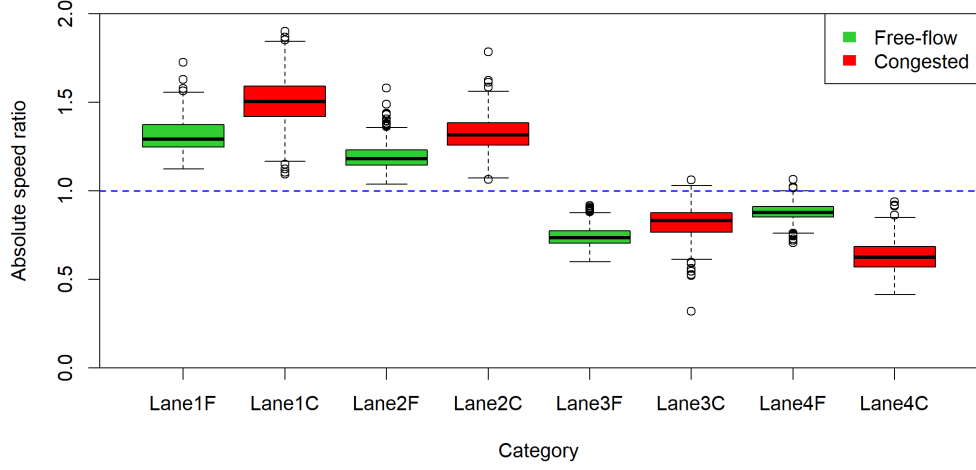


Figure 5.7: Box plots of lane absolute speed ratios in free-flow and congested regimes.

For the third way to verify the non-FIFO property, we consider lane 1 as the reference lane, so the relative speed ratio of lane 1 is always equal to one. Then the relative speed ratios of lanes 2-4 can be calculated and are displayed along the total density in Figure 5.8. As can be seen, the relative speed ratios of lane 2 are greater than those of lanes 3 and 4, and most of them are below one, which are realistic as vehicles on median lanes usually drive faster than those on shoulder lanes. More importantly, we find that all the relative speed ratios along the total density are nearly constant in the free-flow and congested regimes, respectively, even though the data are a little scattered in the heavily congested period.

To verify that all the relative speed ratio patterns have no trend, we apply the simple linear regression to them and the 95% confidence intervals of the regression slopes are calculated, as shown in Table 5.2. We can see that all the confidence intervals are extremely narrow and located around zero even though some of them do not cover zero. Therefore, we can conclude that all the patterns are almost trend-free in their respective regimes and thus nearly constant with acceptable variations. Then the mean relative speed ratios are calculated and stored in Table 5.2.

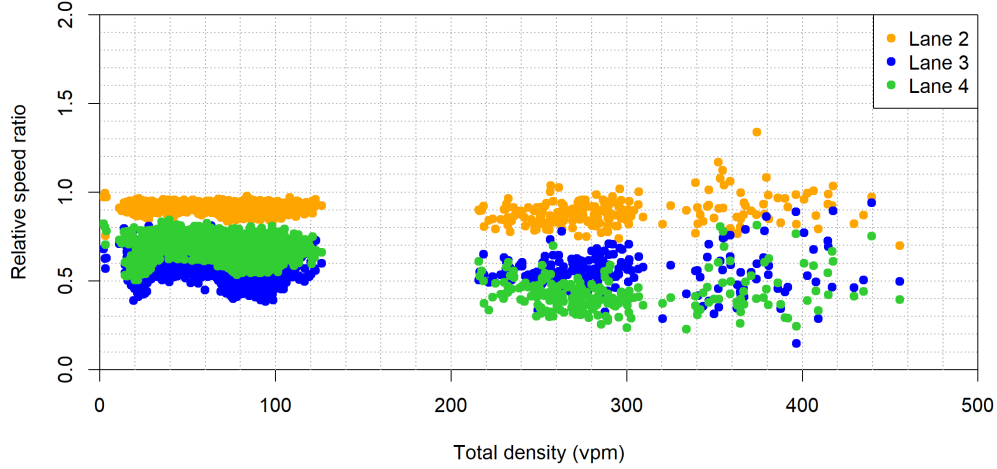


Figure 5.8: Relative speed ratios of lanes 2-4 along the total density.

Table 5.2: 95% confidence intervals of the regression slopes and mean relative speed ratios.

Lane	95% CI of the slope		Mean relative speed ratio	
	Free-flow	Congested	Free-flow	Congested
2	[-0.00034, -0.00025]	[0.00020, 0.00057]	0.908	0.886
3	[-0.00044, -0.00014]	[-0.00012, 0.00041]	0.569	0.550
4	[-0.00030, -0.00004]	[0.00003, 0.00053]	0.674	0.429

5.5 Calibration of unifiable multi-lane fundamental diagrams

Since the relative speed ratios of lanes 2-4 are approximately constant, the unifiable multi-lane fundamental diagram with relative speed ratios in the form of (5.10) can be directly calibrated. To calibrate the unifiable multi-lane fundamental diagrams with absolute speed ratios, we need to first calibrate the absolute speed ratio functions.

We apply the multivariate linear regression to calibrate lane absolute speed ratio functions. Here we take the absolute speed ratio of lane 1 as an example. The base model is designed to regress the absolute speed ratio of lane 1 on the total density and three density proportions across lane 1-3, where the density proportion of lane 4 is excluded to avoid the

multicollinearity problem. Considering some possibilities such that (i) relations between response and predictors are non-linear, (ii) parameters of predictors change from free-flow to congested regime, and (iii) mutual interaction effects exist among predictors, we can further extend the base model by adding quadratic, piece-wise, and interaction terms to build more advanced models that have potential to fit the near-steady data better.

As we realize that a huge magnitude difference exists between lane speed ratio and total density, which would result in a parameter estimate of the total density extremely close to zero, we scale the total density into a range of 0 and 1 by a scaled factor of the inverse of the estimated jam density. In addition, to reduce the correlation between a multiplicative term, e.g., a quadratic or interaction term, and its two component variables so that minimizing the occurrence of the multicollinearity problem, we further center the scaled total density and all three lane density proportions by subtracting their means, respectively. Finally, the transformed predictors are the centered scaled total density denoted by k^{scs} with the corresponding critical density by $\hat{\kappa}_c^{scs}$, and three centered lane density proportions by p_1^{sc} , p_2^{sc} , and p_3^{sc} .

For a certain model, parameters of predictors are estimated by the ordinary least squares criterion; the goodness of fit is measured by R^2 ; and we apply the t-test to testify the significance of any predictor given that all the other predictors are retained in the model. The statistical results of the fitted base model, Mod 1, and advanced models, Mod 2-10, are shown in Table 5.3, where the t-test results are represented by significance codes of p-values with the largest significance level of 0.05. In Mod 1, all the predictors are statistically significant and the parameter estimates are negative for p_1^{sc} and p_2^{sc} and positive for k^{scs} and p_3^{sc} , which are consistent with the above findings by visual inspection. On the basis of Mod 1, Mod 2-6 additionally include quadratic, piece-wise, and interaction terms regarding k^{scs} and p_m^{sc} ($m = 1, 2, 3$), respectively, and all of them have better goodness of fit. In particular, the predictors in Mod 2 and 5 are all statistically significant, while Mod 3, 4, and 6 have

few insignificant predictors. In addition, based on Mod 3, Mod 7-10 take into account the change of parameters of quadratic and interaction terms from free-flow to congested regime, respectively, where again, all of the models fit the data better than Mod 3, Mod 7-9 have few insignificant predictors, but almost half of the predictors in Mod 10 are insignificant.

Table 5.3: Parameter estimates and corresponding p-value significance codes in Mod-1 through Mod-10 for lane 1.

Parameter	Lane 1									
	Mod 1	Mod 2	Mod 3	Mod 4	Mod 5	Mod 6	Mod 7	Mod 8	Mod 9	Mod 10
Intercept	1.34***	1.35***	1.34***	1.33***	1.34***	1.33***	1.33***	1.34***	1.34***	1.34***
k^{scs}	0.42***	0.84***	0.66***	0.22***	0.37***	0.24***	0.22*	0.63***	0.66***	0.65***
p_1^{sc}	-1.45***	-1.69***	-0.48***	-1.85***	-1.77***	-1.90***	-0.62***	-0.48**	-0.56***	-0.56***
p_2^{sc}	-1.71***	-1.25***	-0.66***	-1.31***	-0.93***	-1.22***	-0.95***	-1.13***	-0.39**	-1.20***
p_3^{sc}	0.88***	1.45***	1.94***	1.52***	1.61***	1.54***	2.31***	2.92***	2.26***	2.94***
$(k^{scs})^2$		-1.07***					-4.29***			
$(p_1^{sc})^2$		13.41***					5.58***			
$(p_2^{sc})^2$		33.86***					28.24***			
$(p_3^{sc})^2$		-12.19***					-9.96***			
$k^{scs} \cdot I(k^{scs} > \hat{k}_c^{scs})$			-0.57***				-0.33*	-0.64***	-0.57***	-0.72***
$p_1^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$			-7.95***				-9.05***	-3.49***	-7.51***	-2.07
$p_2^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$			-0.35				0.03	0.05	0.86	1.90
$p_3^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$			-0.85***				0.40	-2.67***	-1.24***	-4.18***
$k^{scs} \cdot p_1^{sc}$				-16.85***		-15.96***		-0.84		-2.09
$k^{scs} \cdot p_2^{sc}$				-0.98		-0.16		-8.50***		-10.82***
$k^{scs} \cdot p_3^{sc}$				-0.07		0.38		10.23***		11.54***
$p_1^{sc} \cdot p_2^{sc}$					67.24***	9.75*			2.05	-1.24
$p_1^{sc} \cdot p_3^{sc}$					24.27***	0.67			11.70***	-3.48
$p_2^{sc} \cdot p_3^{sc}$					17.29***	6.49*			-10.34**	-3.89
$(k^{scs} \cdot I(k^{scs} > \hat{k}_c^{scs}))^2$							4.70***			
$(p_1^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs}))^2$							-21.44*			
$(p_2^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs}))^2$							-12.68			
$(p_3^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs}))^2$							42.07***			
$k^{scs} \cdot p_1^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$								-13.47***		-14.11***
$k^{scs} \cdot p_2^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$								7.75*		6.87
$k^{scs} \cdot p_3^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$								-9.57***		-9.09***
$p_1^{sc} \cdot p_2^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$									50.87**	76.33***
$p_1^{sc} \cdot p_3^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$									-22.61*	-10.48
$p_2^{sc} \cdot p_3^{sc} \cdot I(k^{scs} > \hat{k}_c^{scs})$									23.73	-13.71
R^2	0.679	0.744	0.827	0.818	0.757	0.819	0.858	0.859	0.839	0.865

Significance codes: * when p-value < 0.05, ** when p-value < 0.01, and *** when p-value < 0.001.

Among the fitted models, we can see that, as they become more complex with more predictors included, they undeniably fit the data better but are more likely to contain more insignificant predictors and eventually cause the overfitting problem. Therefore, to select the best model,

we adopt two model selection criteria, adjusted R^2 and BIC (Bayesian information criterion), to balance between the goodness of fit and overfitting problem. In addition, we adopt the PRESS (predicted residual error sum of squares) to measure how well a model can predict with new data. As can be inspected in Figure 5.9, the three criteria show that Mod 8 relatively performs the best across four GP lanes and is finally selected as the calibrated absolute speed ratio function.

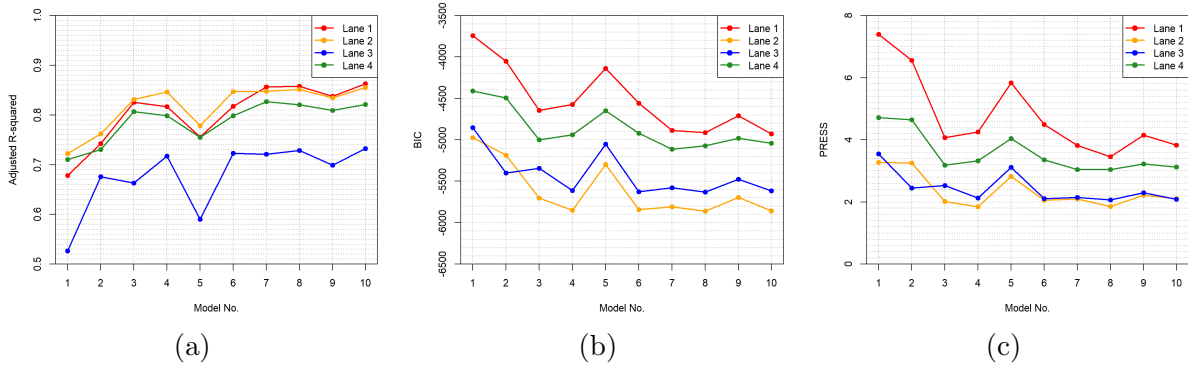


Figure 5.9: Model selection criteria: (a) adjusted R-squared, (b) BIC, and (c) PRESS.

5.6 Validation of unifiable multi-lane fundamental diagrams

In Figure 5.10, we display the observed near-stationary lane flow-rates of four lanes in the congested regime and corresponding estimated lane flow-rates by the formulation of unifiable multi-lane fundamental diagrams with constant relative speed ratios (denoted by CRSR) and the one with absolute speed ratio functions (denoted by FASR). Through the visual inspection, we find that the calibrated lane flow-rates by both the formulations can estimate the observed values well even though CRSR produces relatively larger errors on lanes 3 and 4.

Further we adopt the mean absolute percentage error (MAPE) as a merit to compare the

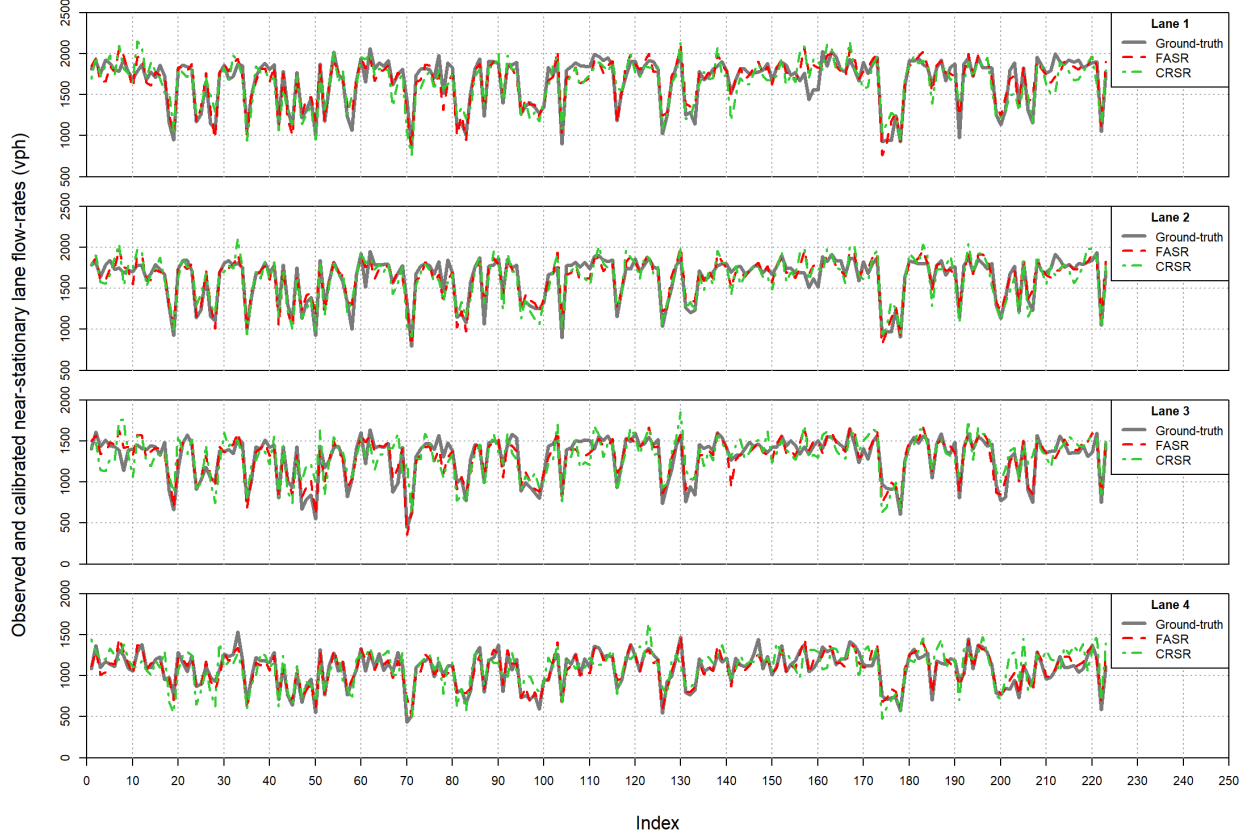


Figure 5.10: Comparison of near-stationary and calibrated lane flow-rates in the congested regime

estimation accuracy of two different formulations, and the measured results are shown in Table 5.4. From the results, we find that FASR performs slightly better than CRSR except for lanes 1-2 in the free-flow regime. However, the overall difference is not significant. In summary, the unifiable multi-lane fundamental diagrams by both the formulations perform well on the estimation of lane flow-rates, but CRSR needs much less computational effort on the calibration.

5.7 Conclusion

This chapter presented a calibration scheme for unifiable multi-lane fundamental diagrams in near-stationary states. First, the theories of unifiable multi-commodity fundamental di-

Table 5.4: MAPE of calibrated unifiable multi-lane fundamental diagrams with two different formulations.

Model		MAPE (%)				
		Lane 1	Lane 2	Lane 3	Lane 4	Average
FASR	(Free-flow)	4.52	4.22	6.18	6.14	5.27
CRSR		3.88	3.46	9.89	7.09	6.08
FASR	(Congested)	5.57	4.89	6.47	5.22	5.54
CRSR		7.86	6.14	12.77	12.84	9.90

agrams with absolute and relative speed ratios were reviewed, respectively. Then some guidelines for verification of unifiable and non-FIFO properties were presented. In a case study, we empirically verified the validity of the unifiable and non-FIFO properties, calibrated the fundamental diagram of total traffic as well as absolute and relative speed ratios, and finally generated two unifiable multi-lane fundamental diagrams with different formulations, both of which are unifiable, non-FIFO, and can reflect mutual interaction among different lanes. Through a comparison between the observed near-stationary lane flow-rates and two calibrated unifiable multi-lane fundamental diagrams, we found that our calibrated models were well-fitted, physically meaningful, and performed well on the estimation of lane flow-rates.

Chapter 6

Calibration of Unifiable Multi-Class Fundamental Diagrams Using Inductive Signature Technologies

6.1 Introduction

In this chapter, we apply the calibration scheme designed in the previous chapter to the multi-class traffic scenario. One challenge of the multi-class fundamental diagram calibration is vehicle classification. Among various data sources, global positioning system (GPS) and probe vehicle data are able to identify vehicle types (Sun and Ban, 2013; Simoncini et al., 2018). However, such data can only capture a small proportion of traffic and would cause potential sample bias used to infer population traffic flow characteristics. Another data source provided by dual-loop detectors can measure all vehicle lengths and divide total traffic into multiple commodities with different ranges of vehicle lengths (Coifman, 2014b), but the deployment of such dual-loop detectors is spatially limited and cannot be widely

applied. In this chapter, we adopt data from inductive single-loop sensors equipped with advanced signature detector cards, which can be widely deployed in the United States. These detector cards can record detailed inductive waveforms, called signatures, generated by individual vehicles traversing the single-loop sensor. Such signature data have distinctive features that can be utilized for vehicle body type classification (Hernandez et al., 2016) and speed estimation (Tok et al., 2009). Compared to previous studies that classify vehicles by length (Coifman and Kim, 2009; Lao et al., 2012), the vehicle classification by body type can reflect the effect of heterogeneous vehicles not only on mobility but also emission and safety concerns (Sprung et al., 2018; Dong et al., 2015). With the estimated speeds and body types of individual vehicles, we adopt the Edie’s formulas (Edie, 1963) to convert the individual data into informative 30-sec time series data and identify near-stationary states from the time series by the four-step method to prepare for the calibration.

The rest of the paper is organized as follows. In Section 6.2, we perform vehicle body type classification and speed estimation using inductive signature techniques and identify near-stationary states from converted time series data. In Section 6.3, we empirically verify the validity of unifiable and non-FIFO properties and calibrate commodity relative speed ratio functions. In Section 6.4, we validate the calibrated unifiable multi-class fundamental diagrams. Finally in Section 6.5, we conclude the chapter with some discussions.

6.2 Data preprocessing by inductive signature technologies

In this section, we first introduce our study site and signature data. Then we adopt two signature-based techniques to classify vehicles by body type and estimate vehicle speeds. Finally, we convert individual vehicle data to informative time series and identify near-

stationary states for further calibration.

6.2.1 Study site and signature data description

Our study site is a multi-lane freeway segment of Interstate 210 Eastbound (I-210E), which is a primary truck route, in Azusa, California, as shown in Figure 6.1. The segment includes a high occupancy vehicle (HOV) lane (denoted by lane 1), four general purpose lanes (denoted by lanes 2-5 from median to shoulder), an on-ramp from Azusa Ave., and a short-segment auxiliary lane. In addition, two inductive single-loop sensors equipped with advanced signature detector cards are deployed on lanes 4 and 5 at the end of the auxiliary lane, 530 ft downstream of the on-ramp merge gore, and 2450 ft upstream of the next off-ramp. The detector cards have a sampling rate of 1000 samples per second, which yields a temporal gap of 0.001 sec.

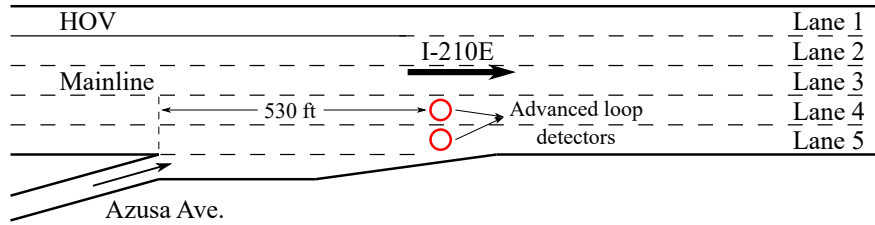


Figure 6.1: Study site on I-210E.

Each individual vehicle record collected from the advanced loop detector includes three major attributes: (i) the timestamp when the vehicle reaches the leading edge of the loop sensor, (ii) lane number, and (iii) inductive vehicle signature. Our data sample is collected from September to December in 2017 with a sample size of 42 days.

6.2.2 Body type classification and speed estimation of individual vehicles

We adopt the tiered classification approach by Hernandez (2014) to classify vehicles into different body types with their distinctive signature patterns. More specifically, in tier one, all the vehicles are classified into either single or multi-units through a decision tree classifier, where the latter class includes trucks with semi-, single-, and multi-trailers. The initial category is subsequently classified into passenger cars and single-unit trucks through a feed forward neural network. Thus three main vehicle types can be obtained: passenger cars, single-, and multi-unit trucks with corresponding example signature patterns shown in Figure 6.2. The overall correct classification rate can reach up to over 93%. Based on the classification, in our data sample, the observation rates of the three body types are 78%, 13%, and 9% during peak periods (7:00 - 9:00 AM and 5:00 - 7:00 PM) and 68%, 15%, and 17% during off-peak periods. These observation rates corroborate anecdotal knowledge that freight movements tend to avoid peak periods.

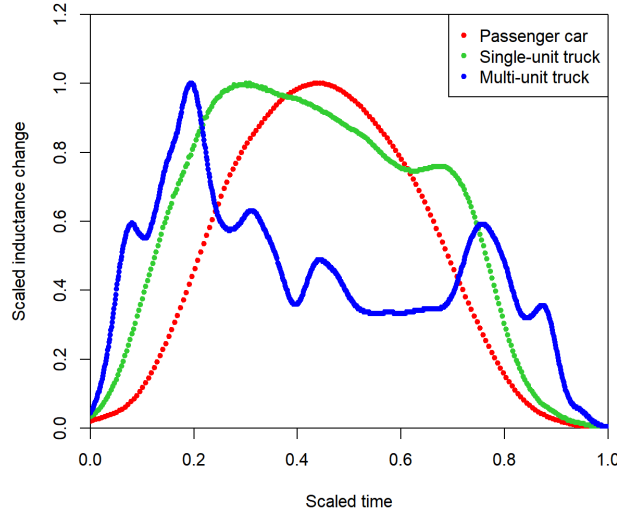


Figure 6.2: Inductive signatures of three different body types.

To further associate the three classes with two lanes, we finally categorize vehicles into six commodities: passenger car on lane 4 ($m = 1$; PC4), single-unit truck on lane 4 ($m = 2$;

ST4), multi-unit truck on lane 4 ($m = 3$; MT4), passenger car on lane 5 ($m = 4$; PC5), single-unit truck on lane 5 ($m = 5$; ST5), and multi-unit truck on lane 5 ($m = 6$; MT5).

In order to achieve a higher speed estimation accuracy with high truck flows, we redevelop the signature-based speed estimation model proposed by Tok et al. (2009) on a dual-loop detector site comprised of 16% of trucks. To be specific, first, we obtain the true individual vehicle speeds and lengths from the dual-loop sensors. Subsequently, we group vehicles based on length-related signature features by using the K-means clustering algorithm and develop the speed linear model within each vehicle group. Finally, we use 30 equally spaced and 30 first order magnitude values extracted from our normalized single-loop signatures to train a multi-layer perceptron (MLP) model constructed with 7 hidden layers, 50 neurons on each layer, and a rectified linear unit (ReLU) activation function. The MLP model is used to assign single-loop signatures to the pre-defined vehicle groups. Thereafter, the corresponding speed linear models will be applied and the individual vehicle speeds will be estimated. A flow chart of our speed estimation method is shown in Figure 6.3. The overall estimation error of the method is ± 2 mph.

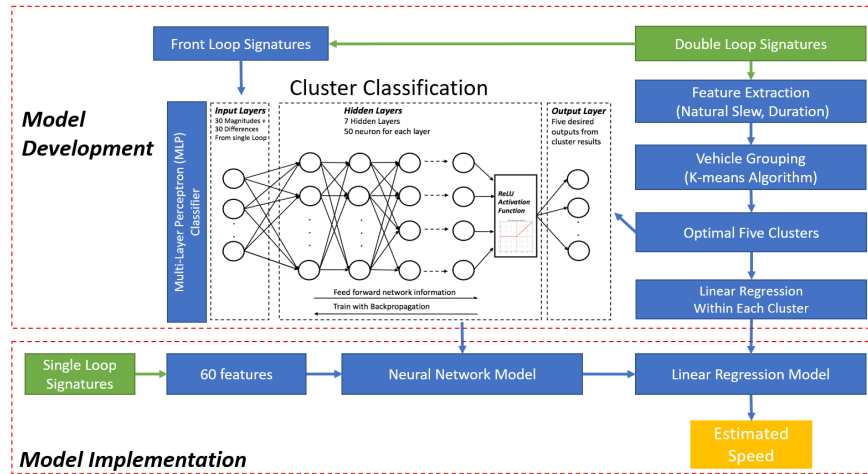


Figure 6.3: Flow chart of the individual vehicle speed estimation method.

6.2.3 Near-stationary state identification

We apply Edie's formulas (Edie, 1963) to convert the individual vehicle data into time series data. In particular, we define a space-time domain, $A = X \times T$, where $X = 6$ ft is the diameter of a single loop and $T = 30$ sec. Then the space mean speed and flow-rate of commodity m in the domain can be calculated by¹

$$v_m^A = \left(\frac{1}{I_m} \sum_{i=1}^{I_m} 1/s_{m,i} \right)^{-1}, \quad (6.1a)$$

$$q_m^A = \frac{I_m}{T}, \quad (6.1b)$$

where $s_{m,i}$ is the estimated speed of vehicle i commodity m in the domain ($i = 1, \dots, I_m$). Correspondingly, the commodity density can be calculated by (5.3a) and the flow-rate, density, and speed of total traffic by (5.2a), (5.2b), and (5.3b), respectively.

We apply the automated four-step method to identify near-stationary states from the 30-sec total flow-rate and density time series. Finally, we denote the identified near-stationary density by k^s and near-stationary commodity density proportions by p_m^s , $m = 1, \dots, 6$.

6.3 Calibration of unifiable multi-class fundamental diagrams

In this section, we first empirically verify the unifiable property and calibrate a fundamental diagram of total traffic. Then we verify the non-FIFO property. Finally, we apply multivariate linear regression and model selection criteria to fit multi-commodity relative speed ratio models.

¹Note that X is extremely short, so it is reasonable to assume that all counted vehicle trajectories in the domain can travel through the entire X within T (Jamshidnejad and De Schutter, 2015).

6.3.1 Verification of the unifiable property

The multi-commodity traffic must meet the four criteria stated in Section 5.3 to achieve the unifiable property. Among the criteria, the first two are satisfied because (i) the four-step method adopted to identify near-stationary states is designed by following the stationarity criterion, and (ii) the six commodities that form the total traffic are mutually correlated and interacted under the lane-changing and car-following maneuvers.

To verify the fundamentality, in Figure 6.4, total flow-density relations are depicted by 30-sec raw data (gray) and near-stationary states (red), respectively. Compared to the former, it can be seen that the near-stationary relation is much less dispersed and composed of two approximately linear patterns in the free-flow and congested regimes and some non-steady near-stationary points (black) falling between them.

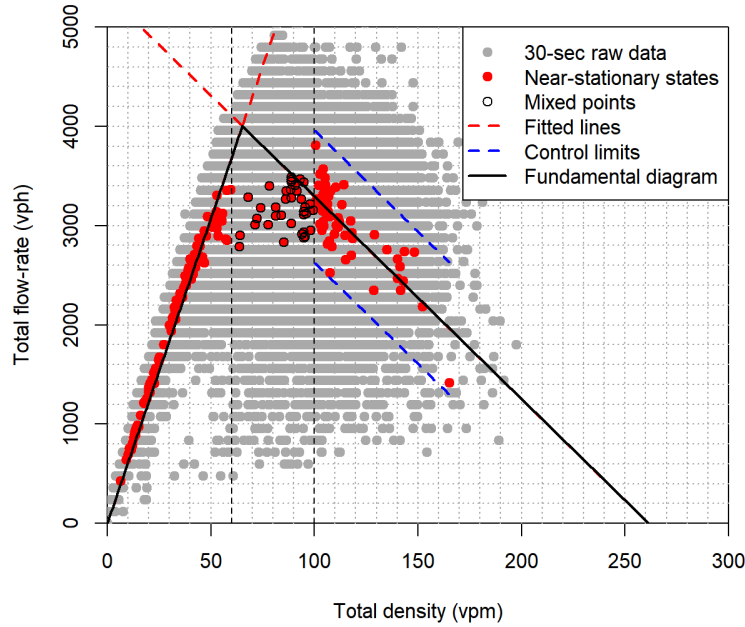


Figure 6.4: Near-stationary flow-density relation and calibrated fundamental diagram of total traffic.

We set two density thresholds at 60 and 100 vpm to discard the mixed points. The resulting total numbers of near-stationary points in the free-flow and congested regimes are 111 and 60, respectively. Next we adopt the coefficient of determination (R^2) to measure the degree

of linear association of two patterns and obtain the measured R^2 's of 0.98 (free-flow) and 0.66 (congested). From the results, the congested pattern performs a little worse than free-flow because (i) heavy congestion rarely occurs at the location, so the pattern is not complete, and (ii) the loop detectors are installed in the acceleration zone of a lane-drop bottleneck, so some identified near-stationary states are not near-steady. However, the value of 0.66 still indicates a reasonable linear association. To further evaluate the quality of the congested pattern, we apply linear regression to the pattern and achieve the upper- and lower-bound control limits, which are the estimated intercept plus/minus three times of its standard error. As can be seen, all near-stationary points are bounded by the control limits. Therefore, both the free-flow and congested patterns have robust quantities and qualities of near-stationary states.

Further to verify the variability, we cut the total densities into multiple short consecutive bins each covering 10 vpm, and show both the median and boxplot of lane-based and class-based density proportions along the total density bins, as shown in Figures 6.5a and 6.5b, respectively. We find that the commodity density proportions are not nearly constant but vary along the total density. In addition, for some bins in the free-flow regime, the total flow-rate and density are nearly constant, but the commodity density proportions are significantly oscillated. These findings indicate that the well-performed total flow-density relation in near-stationary states is not due to the constancy of commodity density proportions. Therefore, the variability criterion is met and these identified near-stationary states excluding those mixed points are all nearly unifiable states.

Two approximately linear patterns in Figure 6.4 imply that a triangular fundamental diagram would fit the data best. To calibrate the total fundamental diagram, we apply the constrained linear regression through the origin to the free-flow pattern and simple linear regression to the congested pattern. Thus the piece-wise linear curve under the intersection of the fitted

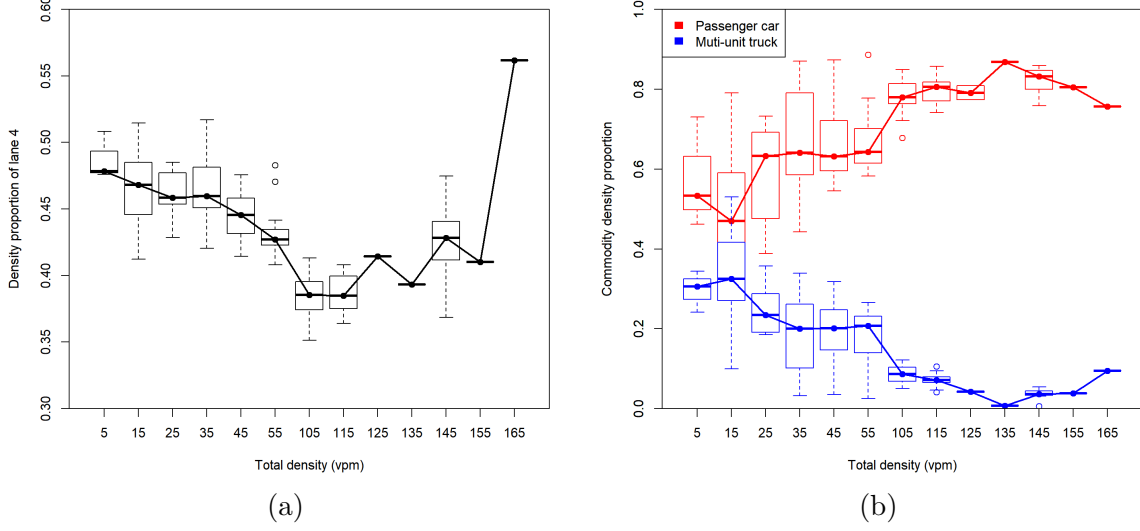


Figure 6.5: Density proportions of (a) lane 4 and (b) passenger car and multi-unit truck over total density.

lines is the calibrated fundamental diagram of total traffic, which can be written by

$$\hat{Q}(k^s) = \min\{\hat{\mu}k^s, \hat{\omega}(\hat{\kappa}_j - k^s)\} \quad (6.2)$$

with the calibrated free-flow speed, $\hat{\mu} = 61$ mph, shock wave speed, $-\hat{\omega} = -20$ mph, jam density, $\hat{\kappa}_j = 262$ vpm, critical density, $\hat{\kappa}_c = \frac{\hat{\omega}}{\hat{\mu} + \hat{\omega}} \hat{\kappa}_j = 65$ vpm, and capacity, $\hat{C} = \hat{\mu}\hat{\kappa}_c = 4005$ vph, which are all physically reasonable.

6.3.2 Verification of the non-FIFO property

Following Section 5.3, two methods are provided to verify the non-FIFO property. First, in Figure 6.6, we show speed-density relations of commodity and total traffic in near-stationary states. As the unifiable property has been verified, it can be seen that the total speed-density relation is almost scatter-free. However, the commodity relations are more dispersed especially in the free-flow regime. In addition, we can find that different commodity relations do not mutually overlap. Therefore, (5.8) is violated and the multi-commodity traffic is non-

FIFO.

Furthermore, we consider PC4 as a reference commodity. Hence, the relative speed ratios of the other five commodities can be calculated and are shown along the total density in Figure 6.7. We can see that the relative speed ratios of MT4, PC5, and MT5 are below one in the free-flow regime and almost all five commodity relative speed ratios are away from one in the congested regime, which match (5.11b) and indicate that the non-FIFO property is satisfied. Moreover, some additional findings can be observed such that (i) vehicles of the same class travel faster on lane 4 than 5 in both free-flow and congested regimes and (ii) passenger cars travel faster than trailer trucks in the free-flow regime but slower in the congested regime, which show the physical, behaviorally, and economical meaningfulness of commodity relative speed ratios. The reasons of the second finding might be that (i) passenger cars on shoulder lanes are mostly driven by conservative drivers who tend to be cautious of large trucks and drive slowly and (ii) most passenger cars entering the freeway from the on-ramp have a lower speed than mainline traffic and thus would interfere the overall speed of passenger cars.

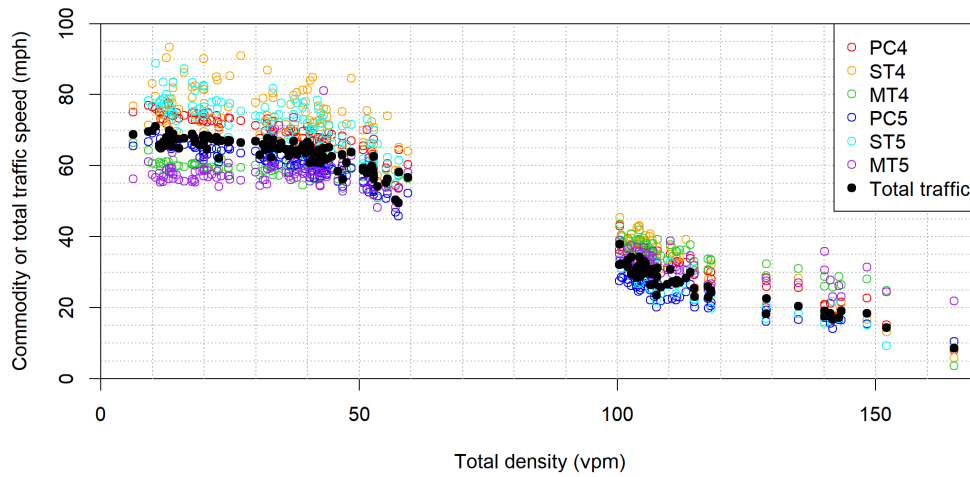


Figure 6.6: Relations between commodity or total traffic speed and total density.

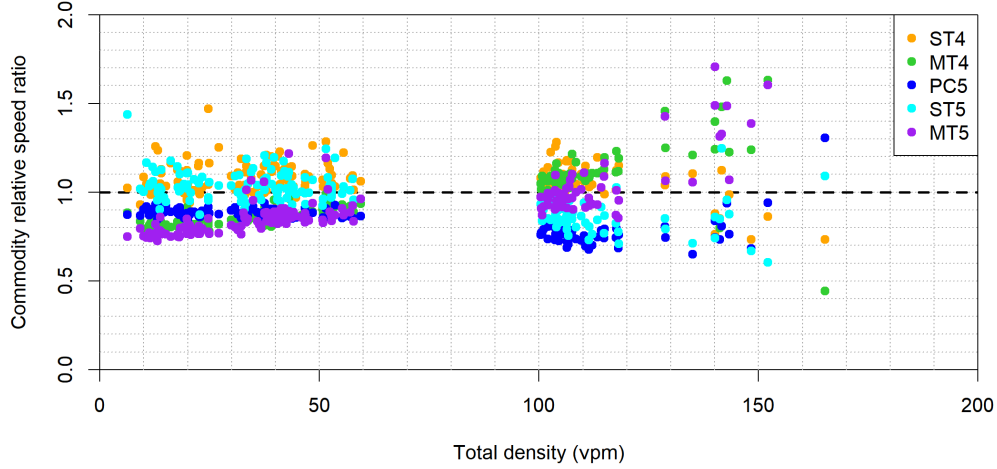


Figure 6.7: Commodity relative speed ratios over total density (Reference: passenger car on lane 4).

6.3.3 Calibration of commodity relative speed ratio functions

It can be clearly seen from Figure 6.6 that the relative speed ratios of five commodities are not constant, so they need to be further calibrated using the state variables. Here we apply multivariate linear regression to the calibration and regard the commodity relative speed ratios as responses and total density and five commodity density proportions ($m = 1, \dots, 5$; PC4, ST4, MT4, PC5, ST5) as basic predictors. In addition, considering the possibilities such that (i) relations between responses and predictors are non-linear, (ii) predictors have different effects on responses from free-flow to congested regime, and (iii) the effects of commodity density proportions on responses change over the total density, we will further add quadratic, piece-wise, and interaction terms to build some other candidate regression models.

As we realize that a huge magnitude difference exists between commodity density proportions and total density, which would result in a parameter estimate of the total density extremely close to zero, we scale the total density as well as the estimated critical density into a range

Table 6.1: Parameter estimates and significances of five candidate models for PC5.

Parameter	PC5				
	Model 1	Model 2	Model 3	Model 4	Model 5
Intercept	0.84***	0.84***	0.83***	0.87***	0.84***
k^{ssc}	-0.26***	-0.08*	-0.15*	-0.09	-0.27***
p_1^{sc}		1.62***	1.38***	0.35*	1.96***
p_2^{sc}		2.30***	1.85***	0.13	1.79***
p_3^{sc}		1.83***	1.80***	0.33	2.47***
p_4^{sc}		0.33*	0.44*	0.05	1.13***
p_5^{sc}		0.19	0.22	0.04	1.01***
$(k^{ssc})^2$			0.55*		
$(p_1^{sc})^2$			2.10*		
$(p_2^{sc})^2$			11.26		
$(p_3^{sc})^2$			-2.44		
$(p_4^{sc})^2$			-0.29		
$(p_5^{sc})^2$			2.55		
$k^{ssc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				-0.10	
$p_1^{sc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				4.71***	
$p_2^{sc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				5.12***	
$p_3^{sc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				6.70***	
$p_4^{sc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				2.85***	
$p_5^{sc} I(k^{ssc} > \hat{\kappa}_c^{sc})$				2.32***	
$k^{ssc} p_1^{sc}$					11.03***
$k^{ssc} p_2^{sc}$					11.02***
$k^{ssc} p_3^{sc}$					14.37***
$k^{ssc} p_4^{sc}$					6.24***
$k^{ssc} p_5^{sc}$					5.35***

Significance codes: * if p-value $\in [0, 0.001]$, ** if p-value $\in (0.001, 0.01]$,
and *** if p-value $\in (0.01, 0.05]$.

of 0 and 1 by a scaled factor of the inversed estimated jam density:

$$k^{ss} = \frac{1}{\hat{\kappa}_j} k^s, \quad (6.3a)$$

$$\hat{\kappa}_c^s = \frac{1}{\hat{\kappa}_j} \hat{\kappa}_c. \quad (6.3b)$$

In addition, to reduce the correlation between a multiplicative term, e.g., a quadratic or interaction term, and its two component variables so that minimizing the likelihood of multicollinearity problem, we further center the scaled total density and all five commodity density proportions by subtracting their means, respectively:

$$k^{ssc} = k^{ss} - \text{mean}(k^{ss}), \quad (6.4a)$$

$$\hat{\kappa}_c^{sc} = \hat{\kappa}_c^s - \text{mean}(k^{ss}), \quad (6.4b)$$

$$p_m^{sc} = p_m^s - \text{mean}(p_m^s), \quad \forall m = 1, \dots, 5. \quad (6.4c)$$

For a certain model, parameters of the predictors are estimated by the ordinary least squares criterion and we apply the t-test to testifying the significance of any predictor given that all the others are retained in the model. We take commodity PC5 as an example, and parameter estimates and significances of five candidate models are shown in Table 6.1. In particular, Model 1 only contains the transformed total density, which has a negative effect on the commodity relative speed ratios and is statistically significant. The same conclusion can also be drawn by visually inspecting Figure 6.6. In addition, the purpose of Model 2 is to testify if commodity density proportions still have effect on the response given that the total density is retained and the results show that most of them are statistically significant. For Models 3-5, we attempt to testify if the effects of the state variables on the response are non-linear, change from free-flow to congested regime, and are mutually interacted, respectively. From the results, most of the predictors in Models 3 and 4 are not significant but all of them in Model 5 are statistically significant, which implies that the effect of commodity density proportions on the response may change over the total density.

To select a relatively best model from the candidates, we adopt two model selection criteria, Bayesian information criterion (BIC) and predicted residual error sum of squares (PRESS), where the former aims to balance between goodness of fit and model complexity and latter

measures how well a model predicts with new data. For both of the criteria, lower values mean better performances. The measured results for five candidate models of all the five commodities can be visualized in Figure 6.8. As can be seen, Model 5 generally performs the best on both of the criteria among all the commodities (except for PRESS of MT4) and is selected as the best fitted model of commodity relative speed ratios.

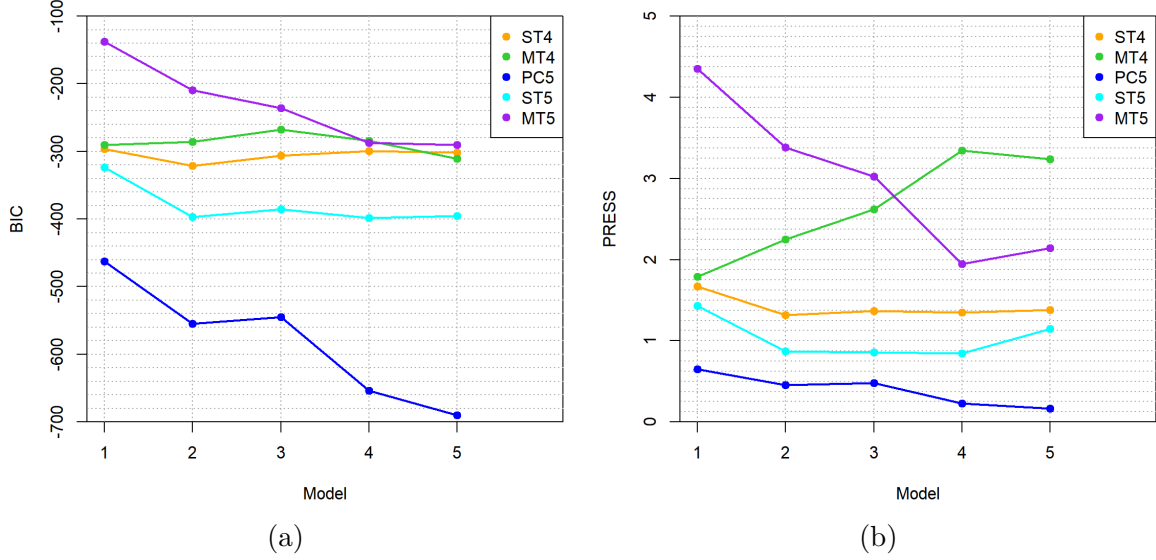


Figure 6.8: Model selection by (a) BIC and (b) PRESS.

Therefore, with the calibrated fundamental diagram of total traffic and fitted commodity relative speed ratios, the multi-class fundamental diagrams can be calibrated by (5.10).

6.4 Validation of unifiable multi-class fundamental diagrams

As shown in Table 6.2, the ranges, mean plus/minus standard deviation of fitted commodity relative speed ratio, and R^2 of fitted models are provided. We can see that all the fitted values are greater than zero and most of them are distributed away from one, which indicate that our fitted commodity relative speed ratios are physically reasonable and the calibrated

multi-commodity traffic still holds the non-FIFO property. In addition, most of the measured R^2 values are around 0.8, which shows that Model 5 fits most of the commodity data very well. However, the R^2 for ST4 is only 0.29. The reason is that the relative speed ratios of commodity ST4 are almost constant and independent of the state variables, as shown in Figure 6.7.

In Figure 6.9, we display the observed and calibrated near-stationary commodity flow-rates over the total density index. We can see that the trajectories almost overlap in the free-flow regime and only commodities PC4 and PC5 have some slight calibration errors in the congested regime. To measure the estimation accuracy, we adopt the mean absolute percentage error (MAPE) as the metric. The measured results are shown in Table 6.2. Similarly, the calibration accuracy performs slightly better in the free-flow regime. Overall, all the MAPE values are lower than 10%, which indicates that our calibrated unifiable multi-class fundamental diagrams have a robust performance on the estimation of commodity flow-rates.

Table 6.2: Performances of Model 5 and calibrated multi-commodity fundamental diagrams.

Commodity	Model 5			MAPE (%)		
	Range	Mean $\pm$ SD	R-squared	Free-flow	Congested	Total
PC4	-	-	-	6.77	6.70	6.75
ST4	[0.74, 1.17]	1.07 ± 0.05	0.29	8.55	8.48	8.53
MT4	[0.73, 1.39]	0.95 ± 0.15	0.76	6.23	8.88	7.16
PC5	[0.70, 1.28]	0.84 ± 0.07	0.87	6.71	6.64	6.68
ST5	[0.76, 1.25]	0.98 ± 0.10	0.73	6.74	9.07	7.55
MT5	[0.71, 2.56]	0.92 ± 0.19	0.84	7.52	8.86	7.99

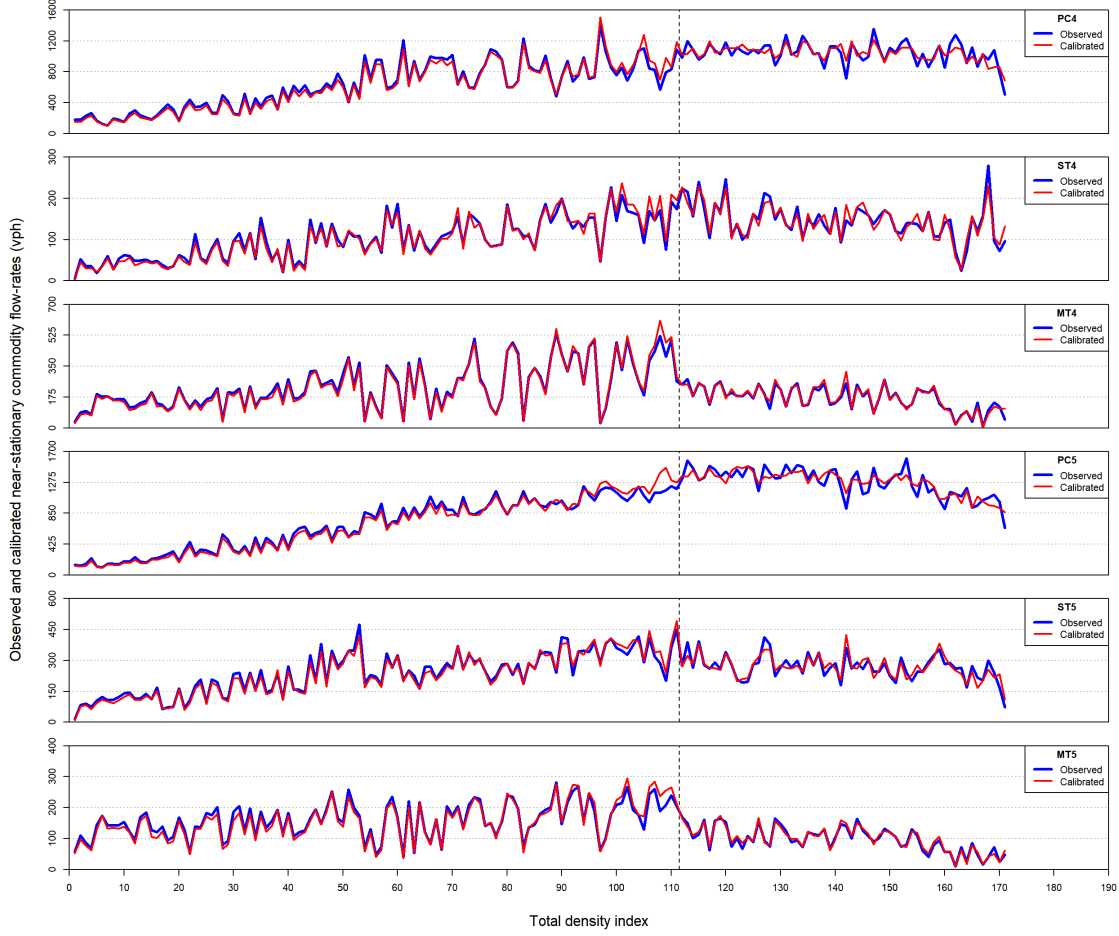


Figure 6.9: Observed and calibrated near-stationary commodity flow-rates.

6.5 Conclusion

This chapter aimed to calibrate multi-class fundamental diagrams using a novel theory and emerging data source. First, we reviewed the theory of multi-commodity fundamental diagrams with relative speed ratios that possess unifiable and non-FIFO properties and presented some guidelines for verification of the properties. Next, we adopted signature-based techniques to estimate individual vehicle speeds and classify vehicle body types from inductive signature loop-detector data and identified near-stationary states from converted informative 30-sec time series. Then we empirically verified the unifiable and non-FIFO properties, calibrated a total fundamental diagram, and modeled the commodity relative

speed ratio functions by multivariate linear regression and model selection criteria. Through the validation, we showed that the calibrated multi-class fundamental diagrams were well-fitted and had a robust performance on the estimation of commodity flow-rates.

Some highlights of this chapter are presented as follows:

1. The underlying theory of multi-commodity fundamental diagrams is well-defined, physically meaningful, and close to reality. First, the multi-commodity fundamental diagrams possess two important properties: (i) the unifiability shows that a fundamental diagram of total traffic should exist as well as commodity fundamental diagrams; (ii) the non-FIFO property shows that commodities have different speeds at the same time and location. In addition, the contained commodity relative speed ratios can characterize drivers' relative aggressiveness and values of time and thus are physically, behaviorally, and economically meaningful. More importantly, the commodity fundamental diagrams can capture mutual interactions among prevailing commodities such that one commodity speed is dependent of density proportions of all the commodities and the total density.
2. An emerging inductive signature data source and related technologies are adopted. Vehicle signatures have distinctive features that can be utilized for individual vehicle speed estimation and body type classification with high estimation accuracy. Thus when the data are converted into time series, both total and commodity flow-rates, densities, and speeds can be measured and serve the need for our multi-class fundamental diagram calibration.
3. To the best of our knowledge, we are the first to calibrate unifiable multi-class fundamental diagrams by inductive signature data and also the first to introduce the emerging data source into traffic flow theory studies. In addition, the calibration and validation methods are well-designed, solid, and robust, and results show that our

calibrated unifiable multi-commodity fundamental diagrams are well-performed and physically meaningful.

In the future, we will be interested in incorporating the calibrated unifiable multi-lane and multi-class fundamental diagrams into multi-commodity kinematic wave theory (Jin, 2017d; Jin and Yan, 2018) and solving problems in multi-commodity traffic flow dynamics, such as moving bottlenecks, lane-drop, and lane-changing effects. In addition, we will be interested in utilizing the calibrated unifiable multi-lane and multi-class fundamental diagrams to investigate and resolve some public impacts and concerns on the mobility, safety, emissions, and costs in multi-commodity traffic.

Chapter 7

Conclusion

In this chapter, a summary of the research is first presented. Then some future research directions are discussed.

7.1 Summary

Through the research, a complete methodology for automatically identifying near-stationary traffic states from inductive loop-detector data is developed. In addition, two calibration schemes for unifiable multi-lane and multi-class fundamental diagrams are designed in near-stationary states.

In Chapter 3, an automated four-step method was developed to efficiently identify near-stationary states from large amounts of inductive loop-detector data. The chapter started with some theoretical discussions of stationary, steady, and equilibrium states and their logical relationships. Then the four-step method was elaborated: in step one, a data preprocessing technique was developed to select healthy datasets with sufficient congestion periods, fill in missing values, and normalize vehicle counts and occupancies to the same scale; in step

two, the PELT changepoint detection method was adopted to detect changes in means and partition daily time series into multiple candidate intervals whose mean values of both vehicle counts and occupancies are relative constant over time; in step three, informative characteristics of each candidate interval, included starting and ending time, candidate flow-rate and occupancy, duration, and vehicle count and occupancy gaps were defined and calculated; finally in step four, the gap and duration criteria were formulated to select near-stationary states from candidates. In addition, two verification methods, direct and indirect approaches, were proposed to evaluate the performance of identified near-stationary states. The results showed that the identified near-stationary states were valid and the calibrated triangular fundamental diagram was well-fitted and physically meaningful. Finally, near-stationary states were compared with aggregated time-series data on the calibration of total traffic fundamental diagrams.

In Chapter 4, a game theory approach was proposed to directly calibrate parameters of the above four-step method from data. First, two important parameters, the penalty ratio and gap threshold, were calibrated by solving a multi-objective optimization problem with respect to two conflicting, unimodal objectives: the number of near-stationary states (quantity) and linearity of the near-stationary pattern (quality) in congested traffic. Then the problem was converted into a non-cooperative game with two players and Nash equilibrium solutions were defined in the context of the game. To solve the game and obtain optimal parameters, an alternated hill-climbing algorithm was developed to imitate the game process and search for a unique Nash equilibrium solution. Through a case study, we demonstrated that the game theory approach associated with the search algorithm was effective in finding optimal parameters, which led to better quantity and quality compared to those with the predetermined ones in Chapter 3. Finally, in an extended paradigm, we further built a five-player game to calibrate minimum acceptable durations of near-stationary states and obtained a wider, more continuous range of congested near-stationary states with better quantity and quality.

In Chapter 5, a calibration scheme for unifiable multi-lane fundamental diagrams was designed in near-stationary states. First, a detailed review of unifiable multi-commodity fundamental diagrams with commodity absolute (Jin, 2017d) and relative (Jin and Yan, 2018) speed ratios was given. Next some guidelines for verification of unifiable and non-FIFO properties were presented, including stationarity, interactivity, fundamentality, and variability. In an case study, we empirically verified the validity of the unifiable and non-FIFO properties in multi-lane traffic and showed that the calibrated unifiable multi-lane fundamental diagrams were well-fitted to near-stationary states, physically meaningful, and performed well on the estimation of lane flow-rates.

In Chapter 6, the same calibration scheme was applied to unifiable multi-class fundamental diagrams. Here we adopted the emerging inductive signature data as our data source and applied related signature-based technologies to individual vehicle speed estimation and classification. Then the Edie’s formulas were adopted to convert individual vehicle data into informative time series with traffic flow characteristics among different vehicle classes. Through the calibration and validation, we demonstrated that the calibrated unifiable multi-class fundamental diagrams fitted near-stationary data well and had a good performance on the estimation of flow-rates among different vehicle classes.

7.2 Future research directions

The automated four-step method and game theory approach constitute a complete methodology for identification of near-stationary states from inductive loop-detector data. This methodology lays a solid foundation for a series of automated near-stationary state identification on road segments, at junctions, and in urban freeway networks. In the future, utilizing near-stationary data to deeply understand, investigate, and explore various transportation problems will also be of interest:

1. In (Jin et al., 2015), lane-drop/merge bottlenecks were categorized into three types: (i) uncongested, when both upstream and downstream locations are uncongested, (ii) congested, when both upstream and downstream roads are congested, and (iii) active, when the upstream location is congested but downstream not. In addition, a phenomenological model of capacity drop was proposed to reconcile continuous fundamental diagrams with capacity drop. This study will motivate us to automatically identify and categorize bottlenecks, and quantify capacity drop magnitudes at active bottlenecks in urban freeway networks with near-stationary states.
2. Merging junctions are important network bottlenecks and a better understanding of traffic features at merging junctions has both theoretical and practical implications. In (Jin and Zhang, 2003; Jin, 2010), various distribution schemes for determining flows through a merge were studied. In the future research, one direction of interest is to verify the theories and empirically evaluate freeway merge ratios in near-stationary states.
3. In (Geroliminis and Daganzo, 2008), the existence of a macroscopic fundamental diagram (MFD) was verified in an urban area by using 5-min loop-detector and Global Positioning System data. The future interest will be to identify near-stationary states in a signalized road network and exploring the existence of an MFD particularly.

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