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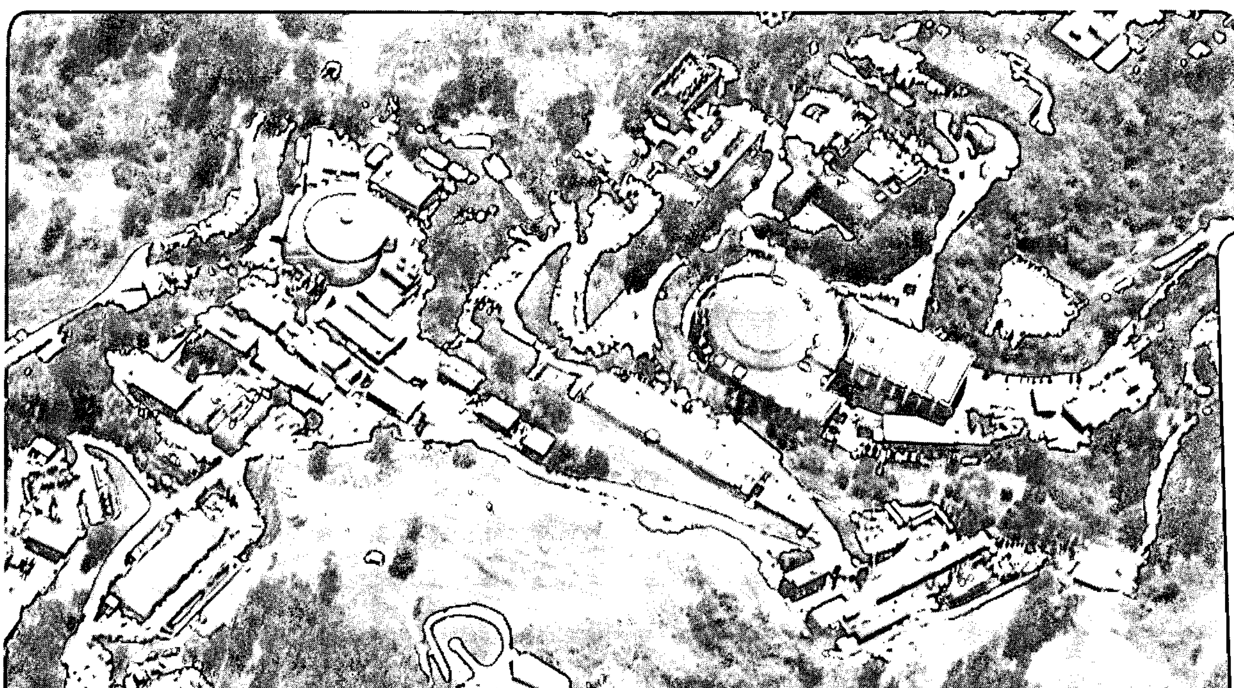

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How well does SDC need to measure cross-sections? *

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Abstract

This note addresses some of the issues concerned with the estimates for cross-sections at the SSC and LHC. I discuss the current expectations for how well rates can be calculated and attempt to extrapolate to the uncertainties at the time the SSC/LHC gives data. the question is relevant in deciding how well SDC should aim to be able to determine cross-sections

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When processes are observed at the SSC/LHC, we will be interested in how well the rates can be measured and calculated. By observing differences from expectations, hints for new physics can be found. I will only discuss rates for processes for which perturbative QCD can be used, since at present this is the only tool that we have with which we can hope to calculate rates with small uncertainties. The total cross section is not one of these processes.

A perturbative QCD estimate of a rate has the following generic form

$$\sigma \sim \sum_{ij} \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \hat{\sigma_{ij}}$$

Here $\hat{\sigma_{ij}}$ is the cross section for interaction of two partons of types i and j and momenta $x_1\sqrt{s}/2$ and $x_2\sqrt{s}/2$. It is expressed as a power series in the strong coupling constant, $\alpha_s(M)$. The parton distributions $f_i(x, Q^2)$ describe the probability of finding a parton inside the proton with momentum fraction x of the proton's momentum. Q and M are some energy scales characteristic of the momentum transfer in the parton process. I shall set $M = Q$ in the following, although strictly speaking this is not necessary. There are therefore two ingredients in a rate calculation; the partonic cross section and the distribution functions.

Distribution functions cannot be calculated in perturbative QCD, they must be extracted from data and then extrapolated using perturbative QCD to the range of Q needed [1]. The partonic cross-sections are to be computed in perturbative QCD. Note that the factorization scale Q appears both in the distribution functions and in the partonic cross section. Formally the result is independent of Q; as Q is altered the change due to f_i is cancelled by that due to $\hat{\sigma}$. The Q dependence is expected to be largest if $\hat{\sigma}$ is only known to lowest order in α_s.

At present, quark structure functions are well measured in deep-inelastic scattering for $x \gtrsim 7 \times 10^{-2}$. [2]. The dominant errors are systematic and are of order 2-3%. New data from HERA have started to emerge[4]. These have large errors at present, but will eventually be sensitive to $x \gtrsim 10^{-4}$. As Q increases, the evolution in perturbative QCD is such that the errors on distribution functions decrease. Since the distribution functions at some value of x and Q^2 are determined by those at $x \geq x_0$ and $Q_0^2 \leq Q^2$, at values of x larger than those currently accessible and energy scales appropriate to the SSC, it seems reasonable to assume that quark structure functions will be known to 2%. The errors on gluon (as well as the charm and strange sea) distribution are larger. Gluon distributions are not measured directly but must be inferred from the Q^2 behaviour of the measured structure functions. Constraints from other processes such as the production of prompt photons in hadron-hadron collisions are also used in most fits. [1].

A scheme has to be introduced to define structure functions beyond leading order. Two schemes have become common. In the DIS-scheme, structure functions are defined so that the deep-inelastic structure function $F_2(x,Q^2)$ is directly related to the distribution functions via $F_2(x,Q^2)$ scheme is somewhat easier to use in calculations and is therefore preferred by some theorists. It is important to remember that both $\hat{\sigma}$ and $f_i(x,Q^2)$ depend upon the choice of scheme and the same scheme must be used for both. They should both be used at the same order in perturbation theory. Both leading order (LO) and next to leading order (NLO) sets of structure functions are available. At present there are no next-to-next to leading order (NNLO) ones since the order α_s^3 Altarelli-Parisi splitting functions are not known. It is reasonable to expect that they will be known by the time the SSC gives data, since they are needed for NNLO QCD tests at HERA.

Since all processes are calculated in terms of the strong coupling constant $\alpha_s(Q)$, the uncertainty in α_s translates to an error in the rates. Currently $\alpha_s(M_Z) = 0.119 \pm 0.005$ [3]. The fractional error decreases (increases) slowly as Q increases (decreases). The dominant errors on these current measurements are systematic (mainly theoretical). Conservatively one can assume that these errors will fall somewhat in the next 10 years. I will therefore assume a 2.5% error on α_s which translates to an error on $\hat{\sigma}$ of 2.5n% for a process that starts in order α_s^n.

The parton cross section $\hat{\sigma}$ is computed as a function of α_s. This is usually expressed as a power series (perturbation expansion). However there are some processes for which this result leads to a result that is not reliable. This happens, for example, in the case of the transverse momentum (p_t) distribution ($d\sigma/dp_t$) of a W in the case where p_t/M_W is small. $\hat{\sigma}$ contains $\alpha_s \ln p_t/M_W$. The large logarithm invalidates the perturbative expansion. In this case, one can sum the terms of order $\alpha_s^n \ln^n p_t/M_W$ and end up with a reliable result [7]. The cross-section for $t\bar{t}$ production at CDF and D0 for top masses around 120 GeV is dominated by *gluon* − *gluon* collisions. $\hat{\sigma}$ for this process has a large correction at threshold. Attempts to sum these effects to all orders in perturbation theory are not very successful. [6] By contrast the $b\bar{b}$ total cross-section receives substantial contributions from the region where $\hat{s}/m_b^2$ is large (here $\sqrt{\hat{s}}$ is the center of mass energy of the *gluon* − *gluon* initial state) and logarithms of this quantity render the perturbation series in $\alpha_s(m_b)$ unreliable [5].

From the foregoing discussion, it will be clear that uncertainties are smallest in processes that start at a low order in α_s, are controlled by moderate values of x, and involve quark initial states. $\hat{\sigma}$ is also not likely to contain potentially large logarithms if the process is not near a kinematic boundary. An ideal example is the production of a new gauge boson (produced by $q\bar{q}$ annihilation) of mass of order 1 TeV. The rate is controlled by $x \sim 0.02$. One of the most worst examples is bottom quark production. The process is gluon initiated, at small Q ($\sim m_b$), small x ($\sim 10^{-4}$), and has a $\hat{\sigma}$ for which perturbative QCD is unreliable.

In order to illustrate the uncertainties in the rates, I will consider three processes, *viz.* the production of a single W or Z, the production of top quarks and the jet cross-section. All of these are large enough at the SSC; that the cross-section measurements will not limited by statistics.

A detailed discussion of the cross-sections for W and Z production has been presented by van Neerven and Zijlstra [8]. I shall use their results in the following discussion. The lowest order process contribution to $\hat{\sigma}$ is $q\bar{q} \to W$ and is of order α_s^0. We can therefore expect that the process has a good chance of being precisely calculated. The partonic cross-sections are known through order α_s^2 (NNLO). There is a problem in that at present no NNLO distribution functions are available. The authors of ref [8] ascribe an uncertainty to this. I assume that it will not be a problem by the time we have data. The results are summarized in the following table which shows the rate in nanobarns at the SSC for W^+ and W^- production (summed). The rates are shown in the $\overline{MS}$ and DIS schemes with $Q = M_W$. The values are given in leading order, next to leading order and next to next to leading order for the same set of distribution functions.

	$\overline{MS}$	DIS
LO	262	259
NLO	287	297
NNLO	279	302

The NLO and NNLO corrections are not dominated by soft (near-threshold) effects so summing these effects to all orders in α_s is not useful. [1]. One can see an approximately 3% difference from the choice of scheme at NLO and about twice this at NNLO. There is some hope that the latter difference could reduce when full NNLO structure functions appear. One has to estimate the uncertainty from the unknown higher order QCD corrections. One can extrapolate from the numbers given above. Again a value of order 3% seems reasonable.

The predicted rate changes as Q is varied from the value $Q = M_W$ used above. At NNLO the rate is vary stable with respect to these changes. Varying Q in the range $M_W/4$ to $4M_W$ causes the rate to change by $\pm 1\%$.

Estimating the errors from structure functions is the most difficult. The process probes values of x in the range $4 \times 10^{-6} \leq x \leq 1$. The range can be narrowed somewhat by only measuring the cross-section for W's produced centrally. In any event SDC cannot detect those bosons at very large rapidity (y). If we restrict to bosons with $|y| \leq 1.5$, then x lies in the range $4.4 \times 10^{-4} \leq x \leq 1.$. This is the range of x that will be covered by HERA. It is not reasonable to use the current structure functions to estimate uncertainties that will exist 10 years from now. By that time data from HERA will have reduced the uncertainties. W production at CDF and D0 is sensitive to $2 \times 10^{-3} \leq x \leq 1.$; a region which is probed by current experiments. The uncertainties due to structure functions in this case are approximately 2%. [8]. Hence, if HERA is capable of a precision comparable to existing deep-inelastic scattering experiments, it is reasonable to expect a comparable level of uncertainty in the SSC/LHC rates.

To summarize, I would expect uncertainties of order 3% (higher order QCD corrections), 3% (structure functions), 1% (choice of Q) and 2% (scheme). Adding these in quadrature gives a total error of 5%. Note that we expect to have of order 1 million reconstructed $Z \to e^+e^-$ events per 1 fb^{-1} of data.

[1]This is in contrast to Drell-Yan production of $\mu^+\mu^-$ at fixed target experiments at Fermilab.[9] Here the NLO effects are large and are dominated by soft gluon effect which can be resummed.

We can therefore expect that even in the early stages of SSC operation, the cross-section measurements will be dominated by systematic errors.

Processes that involve the pair production of electroweak bosons are likely to predicted with comparable accuracy. They are initiated by $q\overline{q}$ collisions, start at order α_s^0, and are in ranges of x where the structure functions are well known.

Other processes are unlikely to be so well predicted. In the case of top quark production which proceeds via $gg \rightarrow t\overline{t}$, the uncertainties will be larger. The process begins in order α_s^2 so an uncertainty of 5% can be expected from the error on α_s alone. As discussed above, the uncertainty on the gluon distribution function is larger than that of the quarks. An uncertainty of order 6% from this source is probably optimistic. The sensitivity to the choice of M is reduced in NLO [5], but an uncertainty of order 15% remains.[10]. The total expected uncertainty is therefore of order 20%.

All of the uncertainties that apply to top quark production also apply to jet cross sections. In addition, there are other problems caused by the definition of a jet. A jet in perturbative QCD is defined in terms of a group of partons that give a total transverse momentum of P_t with in cone in rapidity-azimuth space of size $\Delta R = \sqrt{(\delta y)^2 + (\delta\phi)^2}$. The jet rate depends on ΔR, but more serious is the matching of perturbative calculations to the physical jets of hadrons that are detected. It is issues of this type that dominate the measurements of α_s using event shapes in hadronic decays of the Z at LEP.[11]

In conclusion, I would expect that, by the time data is available, we will be able to calculate processes where only weak bosons are produced with an uncertainty of order 5%. This therefore sets the target of the precision with which SDC should be able to measure these cross-sections.

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