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ZERO-ENTROPY BOOTSTRAP AND THE FINE STRUCTURE CONSTANT*

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ABSTRACT

"Topological supersymmetry", a newly-recognized feature of the zero-entropy S-matrix bootstrap, leads to a zero-entropy coupling constant g_0 whose order of magnitude is controlled by the relation

$$4N_g(4N_g - 1) g_0^2/4\pi \sim 1,$$

where $N_g = 4$ is the number of quark generations. A unification principle, suggested by electromagnetic extension of topological theory and encouraged by this relation, is: $g_0 = e$. The posture of this unifying S-matrix condition with respect to field-theoretical grand unification is discussed.

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The observed value of the fine structure constant $e^2/4\pi \approx 1/137$ -- a small but not very small dimensionless number--has remained mysterious from the Lagrangian viewpoint through an epoch in which extensive field-theoretical successes have occurred. Bootstrap S-matrix theory is supposed to explain all particle properties through requirements of consistency, but until recently electromagnetism has been treated as separate from a strong interaction bootstrap. The use of combinatorial topology in bootstrap theory has now begun to give an S-matrix unification of strong, electromagnetic and weak interactions.^(1,2) A topological basis has been found for the observed particle multiplets, which in field theory are arbitrarily associated with $(SU_3) \times (SU_2) \times U_1$. (The symmetries of topological S-matrix theory are discrete, not continuous.) The status of leptons as particles, together with quarks--which cannot be particles--is topologically understood and 4 quark-lepton generations have been predicted. Electric charge quantization and the observed charge ratios of leptons and hadrons have been explained; the weak-interaction extension of topological theory currently under development⁽³⁾ will have for low q^2 at least as much predictive content as standard Weinberg-Salam theory.⁽⁴⁾ Now the so-far-developed electroweak topological theory does not explain the value of the fine-structure constant, but the topology of "minimal" electroweak interactions is so close to that of "zero-entropy" strong interactions that one may look forward to extension of electroweak-coupling universality--which is required by unitarity. Precisely, if g_0 is the zero-entropy coupling constant, it is an attractive conjecture that unitarity will demand $g_0 = e$; we then expect to calculate the value

of the fine-structure constant through the zero-entropy bootstrap. This note explains why the prospects for success in such an enterprise are promising.

A key consideration is the notion of topological supersymmetry, which vastly simplifies the bootstrap problem and simultaneously generates a dimensionless number of the order of magnitude 10^2 . The nonlinear bootstrap problem is concentrated at the topological expansion's base level, where all complexity indices are equal to zero--a level characterized as "zero entropy".⁽¹⁾ Higher levels are computed through linear relations from lower levels. Only at zero entropy are the relations nonlinear; the bootstrap problem resides at zero entropy. We now sketch the connection between zero entropy and topological supersymmetry.

Over the years it has become recognized that zero topological complexity implies many bootstrap simplifications.⁽⁵⁾ Recently Stapp⁽⁶⁾ found the topological representation of spin, which turned out to possess the feature that at zero entropy spin can be decoupled from momentum (like an internal quantum number) and treated simply as a 2-valued index on a quark line. At the same time it was found⁽¹⁾ that the zero-entropy Hilbert space is restricted to "particles" built from 2 quark lines ("mesons"), 3 quark lines ("baryons") and 4 quark lines ("baryoniums"). Consistency requirements preclude a larger number of quark lines for "elementary hadrons". The structure of the "quantum surface"--to which quark lines are attached-- can be represented by associating to each quark line, in addition to the 2-valued

spin index, a 2-valued charge index⁽²⁾ and a 4-valued "generation" index (called "edge flavor" in Ref. (1)). The distinction between quark and antiquark is represented by a direction on the quark line. In other words the original Harari-Rosner pattern⁽⁷⁾ survives at zero entropy with $2 \times 2 \times 4 = 16$ different quarks. It is this moderately-large multiplicity that promises to fix the small but not very small value of the fine structure constant.

What is meant by "topological supersymmetry"? The form of the discontinuity equations that connect (and hopefully determine) zero-entropy amplitudes, does not change when applied to differing numbers of quark lines--2, 3 or 4. Figure (1), for example, shows the zero-entropy topology of two-particle discontinuities of 'propagators'. These representations reflect the Stapp rule that dependence on quark spin at zero entropy factors away completely from momentum dependence--which may be associated with the solid-line Landau graph in Fig. 1. Stapp factorization⁽⁶⁾ turns out to mean that the same momentum functions consistently belong to all six topologies in Fig. 1, independently of the spin and flavor of the quark lines, with a corresponding symmetry for any N-point function, $N(\geq 3)$ being the number of external lines in the Landau graph. In other words it is consistent with Stapp's rules to assume that an N-line zero-entropy connected part is associated with a momentum function, $M_N(p_1 \dots p_N)$ that is independent of external quark lines. A zero-entropy symmetry between states of spin $0, \frac{1}{2}, 1, \frac{3}{2}, 2$ is thereby implied, together with the more familiar idea of flavor symmetry.

Hadron symmetry between spin 0 and spin 1 and between spin $\frac{1}{2}$ and spin $\frac{3}{2}$ has a well-explored phenomenological basis, and recently Gauron,

Nicolescu and Ouvry⁽⁸⁾ have uncovered evidence supporting the spin 0-spin $\frac{1}{2}$ aspect of topological supersymmetry. It was in fact this discovery that triggered the present note. We remark that experimental implications of topological supersymmetry with respect to spin 2--baryonium--urgently deserve attention. Implicit is the prediction of a large family of baryoniums, with properties closely and predictably related to those of mesons and baryons.

With topological supersymmetry the zero-entropy bootstrap problem reduces to a set of relations between the ("spinless") connected parts $M_N(p_1, \dots, p_N)$. If each of these amplitudes is enlarged by a factor

$$f^{N-2/2}, \text{ where } f = (-16)^2 - 16 = 16 \times 15, \quad (1)$$

then the relations become those of a spinless flavorless ordered S matrix as described in Ref. (5). The factor f reflects the quark multiplicity. The point is that at zero entropy any closed loop in a Landau graph carries either one or two closed quark lines, as in the example of Fig. 1. The multiplicity of a loop with one quark line is 16 and that of a loop with 2 quark lines is $(16)^2$. Each closed quark loop comes with a factor (-1) because of the fermion character⁽⁶⁾, so the total effective multiplicity of each closed Landau loop is

$$f = -16 + (-16)^2 \quad (2)$$

Equivalently, each closed Landau loop can be given a weight $+1$, as in the

absence of any internal degrees of freedom, if at the same time each amplitude M_N is enlarged by the factor $f^{N-2/2}$.

The present form of topological bootstrap theory assumes there is only one real pole (one "stable particle") in the equivalent spinless-flavorless ordered S matrix.⁽¹⁾ The associated mass m_0 sets the scale, and we define a dimensionless coupling constant g_0 by

$$g_0 \equiv M_3/m_0. \quad (3)$$

Because of the absence of any other scale, the corresponding dimensionless coupling constant g_R for the "renormalized" bootstrap problem must be of order unity. With attention to phase-space, one expects

$$\frac{g_R^2}{4\pi} \sim 1 \quad (4)$$

Since from Eq. (1),

$$g_R = f^{1/2} g_0 \quad (5)$$

it follows that

$$\frac{g_0^2}{4\pi} \sim (16 \times 15)^{-1} \quad (6)$$

The exact value of g_R requires a solution of the zero-entropy bootstrap equations. These equations are simple in carrying no flavor or spin complications and in representing only planar singularities* but are still nonlinear and their solution will require care and ingenuity. (If indeed the solution is destined to yield the value of the fine structure constant, then a large degree of attention is justified.)

The reader may feel bewildered by the suggestion that g_0 , a strong-interactions low- p_1 coupling, is equal to e . The idea behind

*

There are believed to be no Regge branch points, only Regge poles.⁽⁵⁾

the topological expansion, however, is that topological components of higher complexity are less important at low $p_{\perp}$ than those of lower complexity because they have less phase coherence. The greater the complexity the higher is the degree of cancellation between different components. Zero entropy components add together at small $p_{\perp}$ with a maximum degree of coherence. Roughly speaking, one arrives at low- $p_{\perp}$ "strong binding" by adding one hundred or so separate contributions with the same phase, each contribution being of order $e^2/4\pi$. A related effect is that each permutation of the three quarks in a baryon counts as a separate topology,* while each physical baryon is a sum over the six permutations.⁽¹⁾ A physical baryonium is similarly a sum over 4 permutations, and topological degrees of freedom in addition to quark permutations require also meson-meson couplings to be a sum over different topologies.⁽¹⁾ Physical strong-interaction coupling constants correspondingly tend to be larger than g_0 ; there is no corresponding effect for the electromagnetic coupling e , as shown in Ref. (2).

A hurdle still to be cleared by topological bootstrap theory is the explanation of flavor symmetry breaking. Already-understood topological complexity generated by unitarity breaks apart the supersymmetric spin multiplets, but the breaking of flavor symmetry is not yet understood. Until this phenomenon is under control, one must stand prepared for some important element in topological theory that might undermine the scenario presented here. We nevertheless close with some remarks relating the presently understanding of topological bootstrap theory to the field-theoretical notions of asymptotic freedom and grand unification.

As discussed in Refs. (1) and (2), increase of $p_{\perp}$ or q^2 decreases the

* The quark-permutation degree of freedom has in Ref. (1) been called "topological color".

importance of low topological complexity. In the high- $p_{\perp}$ domain where QCD perturbation theory is useful, the S-matrix topological expansion is dominated by high-complexity. The corresponding components exhibit less coherence, and weakening of physical strong-interaction couplings results. There occurs no corresponding weakening of electroweak couplings (there is here no zero-entropy coherence to diminish), so one qualitatively anticipates from topological theory an effect in the same direction as asymptotic freedom. We do not presently, however, attribute significance to "unification" at high $p_{\perp}$. The "unification" expressed by $g_0 = e$ yields its simplest physical consequences at low $p_{\perp}$ and low energy. At the present stage of understanding, S-matrix theory suggests an indefinite progression with increasing energy, at both low and high $p_{\perp}$, of the importance of evermore complex topologies. At low $p_{\perp}$ the dominant complexification is soft photon emission and absorption,⁽²⁾ leading to the classical domain of objective reality where equations of motion in continuous variables become a good approximation.* At high $p_{\perp}$ the dominant complexification is of a different character but may nevertheless permit approximate meaning for the continuum dynamics of QCD. At zero entropy there is a conserved "topological-color" degree of freedom⁽¹⁾ that, under complexification, promises to be a source of the gluon concept. In bootstrap theory gluons are absent at zero entropy, but certain types of complexity induced by unitarity correspond to the exchange of "color" and spin between quarks that maintain their flavor.

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A natural supposition is that continuous time is not a "primitive" notion, but an approximate representation of "entropy" growth in discrete topologies of high complexity.

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FIGURE CAPTION

Topological representation of two-particle discontinuities of zero-entropy "propagators". The solid lines belong to the momentum-carrying Landau graph. The dotted lines are Harari-Rosner quark lines.

(a)

(b)

(c)

(a')

(b')

(c')

Figure 1

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