Modeling Wireless Ad Hoc Networks with Directional Antennas

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***Abstract*— This paper presents the first analytical model of wireless ad hoc networks that considers the impact of realistic antenna-gain patterns on network performance. As such, our modeling approach allows the study of ad hoc networks in which nodes are equipped with directional antennas. This modeling capability stands out from all previous analytical models, which have only dealt with omnidirectional or over-simplified antenna gain patterns, and which have not addressed the specific mechanisms of the medium access control (MAC) protocols used (e.g., the backoff mechanism). A new analytical model for the IEEE 802.11 DCF MAC is introduced that allows the study of different carrier-sensing mechanisms, such as the *directional virtual carrier sensing* (DVCS) protocol that we use to validate our analytical model and show its applicability. Our numerical results show that our new analytical model predicts the results obtained by discrete-event simulations very accurately, and does it with a processing time that is orders of magnitude faster than the time required by simulations.**

I. Introduction

Many protocols for wireless ad hoc networks with directional antennas have been proposed, mostly on the medium access control (MAC) layer (e.g., [1], [2], [3], [4]). The vast majority of the performance-evaluation work on MAC protocols that exploit directional antennas has used discrete-event simulations to model protocol behavior. Only a few works have attempted to model ad hoc networks with directional antennas *analytically*. Section II summarizes this prior work, which has been very limited in that all the analytical models proposed to date have assumed (a) simplistic ways in which packets are offered to a shared channel, ignoring the specific mechanisms used in MAC protocols (e.g., backoff mechanisms), and (b) the use of over-simplified antenna gain patterns, like the "pie-slice" and "cone-plus-ball" antenna models [5], [6], [7], [8]. In the simplistic antenna models used to date, all directions within a certain angle sector have constant gain, while no power is radiated/absorbed along the other directions ("pie-slice"), or a lower constant gain is assumed for the directions outside the angle sector to represent the back and side lobes of the antenna pattern ("cone-plus-ball"). In reality, no physical antenna can provide such constant gain for a given angle sector, and real antenna patterns are far more complex than "pie-slices" or "cone-plus-ball" models. In fact, real antenna patterns have non-negligible gains in *all* directions, and often have significant side and back lobes that can contribute considerably to the amount of perceived noise, leading to performance degradations such as the ones observed by Ramanathan et al. [9] in a real-life ad hoc network testbed. Consequently, conclusions about capacity improvements based on such over-simplified antenna models (e.g., [7] and [10]), may not necessarily reflect the true potentials or limitations of the use of directional antennas in ad hoc networks.

In this paper, we present the first analytical modeling of wireless ad hoc networks that considers the impact of realistic antenna gain patterns on network performance. In particular, we focus on the modeling of wireless ad hoc networks with directional antennas.

Section III summarizes the operation of the *directional virtual carrier sensing* (DVCS) protocol [11], and outlines the main problems that arise in developing an analytical model for this and similar schemes based on directional antennas.

Section IV presents our new analytical model, which is derived from our previous work [12], and captures the interactions between the physical and the MAC layers while taking into account the radio connectivity among nodes. The extensions and modifications of our new analytical model include: (a) taking into account the impact of packet flow distribution among multiple receivers; (b) expressing the impact of frame size distribution; (c) modeling the impact of the carrier sensing mechanism in carrier-sensing MAC protocols; and (d) providing richer interference matrices that explicitly model the impact of a node's transmission on the SINR degradation of every other node, i.e., the effect of *capture* with respect to every potential interferer is treated individually. We attain a linear approximation for the probabilities of successful handshakes among transmitters and receivers by taking advantage of the fact that any MAC protocol must attempt to avoid having interfering transmissions around the recipient of a frame transmission.

Section V introduces a new model for the IEEE 802.11 DCF MAC that allows the study of different carrier-sensing mechanisms, including DVCS. We apply our analytical model to DVCS to compare its accuracy against results obtained with discrete-event simulations using realistic antenna patterns. The numerical results presented in Section VI indicate that our analytical model predicts simulation results very accurately, with processing times that are orders of magnitude faster than simulations. We show that, because simplified "pie-slice" antenna patterns do not have side lobes, average throughput is dramatically over-estimated. In the cases we modeled, "pie-slice" patterns renders 200% higher throughput than what is obtained using realistic antenna patterns. Section VIII summarizes our conclusions.

II. Related Analytical Work

Most of the work in ad hoc networks with directional antennas has used discrete-event simulations to model protocol operation and evaluate its performance (e.g., [13], [4] and references therein). As far as the analytical modeling is concerned, most of the previous work has concentrated on single-hop networks [14], [15], [16], [17]. To date, very few attempts have been done to the analytical modeling of multihop ad hoc networks with directional antennas. Chang and Chang [5] were arguably the first to analyze the use of directional antennas in slotted ALOHA and non-persistent CSMA. Their modeling approach closely follows the formalism used by Takagi and Kleinrock [18], in which nodes are spatially located according to a two-dimensional Poisson distribution, radio links are error-free (no physical layer aspects whatsoever), no acknowledgment traffic exists, and the transmitters have "instantaneous" knowledge of successful receptions. They use the "pie-slice" antenna gain model for the directional antennas. Later, Zander [6] proposed a slightly different approach to model slotted ALOHA, with the same antenna model and spatial distribution assumptions, but included a propagation model and treated packet receptions with a simple threshold model based on signal-to-noise ratio (SNR).

Wang and Garcia-Luna-Aceves [8] introduced the modeling of collision avoidance MAC protocols with directional antennas. Based on the IEEE 802.11 DCF MAC, they modeled and analyzed many of its variants with respect to the directional transmission of control and data packets coupled with omnidirectional or directional receptions (and hybrid operations). Their modeling approach also follows the work by Takagi and Kleinrock [18] with respect to the spatial distribution of nodes and absence of physical layer aspects, and the approach of Wu and Varshney [19] for the modeling of the node activity with a Markov chain with only three states. The limitations of this effort is that it assumes the "pie-slice" antenna model, and that transmission probabilities are taken from a "range of small values", thereby disregarding the fact that transmission probabilities are protocol-dependent (i.e., they do not consider the binary exponential backoff operation of the IEEE 802.11 and such parameters as contention window size or retry limits).

III. The Directional Virtual Carrier Sensing (DVCS) Protocol

A number of MAC protocols for ad hoc networks with directional antennas have been proposed in the past few years. In many cases, however, the proposed protocols have failed to provide a new MAC protocol in its strict sense, i.e., no new paradigms or mechanisms at the MAC *level* were presented. Instead, they have essentially investigated how directional transmissions/receptions of control/data frames impact network performance under a given MAC protocol originally designed to work with omnidirectional antennas (e.g., IEEE 802.11). Examples of such proposals include [2], [3], [10], and [8]. Few works have tried to propose new MAC-level mechanisms that attempt to take full advantage of the capabilities provided by directional antennas. One of the first such attempts was the *directional virtual carrier sensing protocol* (DVCS) proposed by Takai et al. [11]. We selected this protocol to instantiate our modeling approach to ad hoc networks with directional antennas, because of the richness of the DVCS operation.

The main idea behind DVCS is to allow contention-based MAC protocols to determine direction-specific channel availability. For that, DVCS uses the *directional network allocation vector* (DNAV), a directional version of the network allocation vector (NAV) of the IEEE 802.11, which contains information about the total duration of a transaction that is about to happen over the channel. During this time, the node cannot transmit any frame to the channel, reserving it for others do it. In DVCS, the DNAV reserves the channel for others only in a *range of directions*. To accomplish this, DVCS requires minimal information from the underlying physical device, such as the angle of arrival (AOA) and the antenna gain for each signal, features that can be readily available at the physical layer.

Multiple DNAVs can be set for a node, and each DNAV is associated with a direction and a width. Every node using DVCS maintains a unique timer for each DNAV, and updates the direction, width, and expiration time of each DNAV every time the physical layer gives newer information on the corresponding ongoing transmission. DVCS determines that the channel is available for a specific direction when no DNAV covers that direction.

In DVCS, each node caches estimated AOAs from neighboring nodes every time it hears any signal, regardless of whether the signal is addressed to it. If a node has data to send, it first checks if AOA information for the particular neighbor has been cached. If yes, it beamforms the directional antenna towards the cached AOA direction to send an RTS frame. Otherwise, the frame is transmitted in omnidirectional mode. Updates are done every time the node receives a newer signal from the neighbor, and it invalidates the cache if it fails to get the CTS back from the neighbor after four failed directional transmissions of RTS frames. Subsequent RTS frames are sent omnnidirectionally.

When the node receives an RTS frame from a neighbor, it adapts its beam pattern to maximize the received power and locks the pattern for the CTS transmission. If the node transmitted an RTS frame to a neighbor, it locks the beam pattern after it receives the CTS frame from the neighbor. The beam patterns at both sides are used for both transmission and reception, and are unlocked after the ACK frame is transmitted. We refer the reader to [11] for more details.

Many issues arise in the modeling the behavior of an ad hoc network with directional antennas accurately. In particular, the analytical model must incorporate the impact of realistic antenna-gain patterns under specific radio channel models, as well as include the intricate interactions between the MAC layer and the many aspects of the underlying PHY layer. Furthermore, the model should capture each node's dynamics resulting from its interaction with *every other node in the network*, a core phenomenon within a wireless ad hoc network.

In the case of an ad hoc network using DVCS, the analytical model must reflect how the DNAV works and which events at the PHY layer are critical at each step of the protocol operation. For instance, during a node's carrier sensing operation, only signal transmissions perceived within the *range of directions* of the target receiver can interfere with the node's backoff operation. In addition, because the node in backoff switches to omnidirectional mode, the relative antenna gains between this node and all the other nodes will be different from the case when this node is actually transmitting/receiving a frame. In the latter case, the node selects/forms a directional beam pattern to transmit/receive a frame, which will lead to different relative antenna gains with respect to every other node in the network.

In the following, we augment the modeling framework originally introduced in [12] to include, among other things, a general treatment of the impact of the carrier sensing activity in general ad hoc networks. The model we introduced in [12] captures the interactions between the PHY and MAC layers and takes into account the radio connectivity among the nodes. In addition, because the DVCS is built upon the IEEE 802.11 DCF MAC, we present in Section V an analytical model for the IEEE 802.11 that captures the impact of the carrier sensing mechanism, allowing the modeling of DVCS and its DNAV mechanism.

IV. The Analytical Model

This section revisits the analytical model for multihop ad hoc networks we originally introduced in [12], extending it with the new features summarized in Section I.

A. Model Preliminaries [12]

Our modeling approach focuses on the essential functionalities provided by the PHY and MAC layers, as well as their interaction. On the one hand, the MAC layer provides a scheduling discipline for nodes to access the shared channel(s), and this discipline renders probabilities with which nodes will attempt to transmit. On the other hand, the PHY layer encodes information attempting to ensure that the transmitted frames are received correctly; the likelihood with which a transmission attempt is successful depends on how well the signaling used defends against channel impairments (establishing thus, the interdependency between both layers). In the specific case where a reliable delivery service is required at the MAC layer, a node retransmits a frame according to some specific rule until the frame is successfully transmitted or discarded after a certain number of failed attempts. Each of these attempts is characterized by a *successful handshake probability*. Hence, MAC protocols can be seen as stochastic dynamic systems whose feedback information are the *successful handshake probabilities*, denoted by q_i, and the system outputs are the *scheduling rates* or *transmission probabilities*, denoted by τ_i, which are related by some function $h_i(\cdot)$. Assuming a steady-state operation, and traffic saturation at all nodes [12], the MAC functionality can be represented by some time-invariant (linear or non-linear) function $h_i(\cdot)$ as follows

$$\tau_i = h_i(q_i), \quad i \in V, \tag{1}$$

where V denotes the finite set of $|V| = n$ nodes spanning the network under consideration.

The PHY/MAC layer interactions also depend on the connectivity among nodes, which, in turn, depends on many aspects of the radio wave propagation phenomena such as the radio frequency in use, transmit power, antenna type, transmitter/receiver distance, and the like. In addition, because the radio channels in ad hoc networks are broadcast in nature, the quality of a radio link also depends on the transmission activity of *all other nodes* in the network. Hence, at the PHY layer, the successful reception of every bit of information transmitted by a node i to a node r is invariably related to the *signal-to-interference-plus-noise density ratio* SINR_i^r at node r (assuming a spread-spectrum system), which is given by (using a conventional matched filter receiver):

$$\mathrm{SINR}_i^r = \frac{P_i^r L_i}{\sum_{\substack{j \in V \\ j \neq i}} \chi_j P_j^r + \sigma_r^2}, \tag{2}$$

where P_k^r denotes the received signal power at node r for a signal transmitted by node $k \in V$, L_i is the spreading gain (or bandwidth expansion factor) of the spread-spectrum system, σ_r^2 is the background or thermal noise power at the front end of the receiver r, and χ_j is an on/off indicator, i.e.,

$$\chi_j = \begin{cases} 1, & \text{if } j \text{ transmits at the same time,} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Each time node i transmits a frame to node r, the (instantaneous) multiple access interference (MAI) level at node r depends on which nodes in V are transmitting concurrently with node i, as indicated by the variable χ_j, a Bernoulli-distributed random variable with probability τ_j, the probability that node j transmits a frame at any time, according to the MAC protocol in place. Because $|V| = n$, there are exactly 2^{n-2} combinations of potential active nodes (interferers) transmitting concurrently with i (excluding both transmitter i and receiver r). Let $\{c_{ik}^r\}_{k=1,\ldots,2^{n-2}}$ denote the set of such combinations, and $\mathcal{C}_i^r$ be a random variable that indicates the occurrence of a specific combination c_{ik}^r of interferers. We also assume, for simplicity, that when node i transmits a frame to node r, the set of interferers remains the same throughout the entire transmission of the frame. In reality, of course, some nodes may become active or inactive during the course of a frame transmission. However, this assumption is reasonable if frames are short and transmission rates are high. Consequently, we assume that during the reception of a frame—and for a fixed set of interferers—the bit-to-bit variations that may occur in SINR_i^r results from RF propagation effects only.

B. The Successful Handshake Probability

One of the limitations of our prior model [12] is the fact that it only reflects a "snapshot" of the network, in the sense that each sender has a unique (one-hop) receiver for its frames *all* the time. In reality, however, each packet that arrives at a node's input queue(s) may be potentially addressed to a distinct receiver within the node's one-hop neighborhood. Each time a node attempts to transmit a frame, the conditions for having a successful handshake depend on the selected target receiver, because its location determines the impact of potential interferers. In addition, each packet may be assigned a different priority, depending on such factors as its target destination, data content, or traffic type. Even if a single FIFO queue is used, the steady-state distribution of the aggregate flow at a node's input queue may depend on how traffic multiplexing occurs within a certain time period. Therefore, within a sufficiently long time interval, every time a node $i \in V$ decides to transmit a frame, it will do so by selecting a target receiver according to some probability that is a function of the factors we have mentioned. Hence, we extend our model to reflect such dynamics. We first take a closer look at the successful handshake probability with respect to a specific receiver, and make clear the dependencies regarding the sets of interferers and frame sizes, which were not shown in our prior model [12]. Following that, we generalize the formalism to the case of multiple receivers and the impact of flow distribution.

1) The Case of a Single Receiver: Two events must occur for a frame to be transmitted successfully from a node $i \in V$ to another node $r \in V$: the successful reception of i's frame at r, and the successful reception of r's acknowledgment at node i. Following the convention in [12], if the sender's frame is generically referred to as "DATA," and the corresponding acknowledgment as "ACK," we are interested in finding out the probability of the joint event {DATA successful, ACK successful}. Therefore, if we denote by q_i^r the probability that *node i transmits a frame successfully to its intended receiver* r, then

$$
\begin{aligned}
q_i^r &= P\{\text{DATA successful, ACK successful}\} \\
&= \textstyle\sum_k \sum_l P\{\text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r, \text{ACK suc.}, \mathcal{C}_r^i = c_{rl}^i\} \\
&= \textstyle\sum_k \sum_l P\{\text{ACK suc.}|\mathcal{C}_r^i = c_{rl}^i, \text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r\} \\
&\quad P\{\mathcal{C}_r^i = c_{rl}^i|\text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r\} P\{\text{DATA suc.}|\mathcal{C}_i^r = c_{ik}^r\} \\
&\quad P\{\mathcal{C}_i^r = c_{ik}^r\} \\
&= \textstyle\sum_l \sum_k f(c_{rl}^i) f(c_{ik}^r) P\{\mathcal{C}_r^i = c_{rl}^i|\text{DATA succ.}, \mathcal{C}_i^r = c_{ik}^r\} \\
&\quad P\{\mathcal{C}_i^r = c_{ik}^r\},
\end{aligned} \tag{4}
$$

where $f(c_{rl}^i) = P\{\text{ACK suc.}|\mathcal{C}_r^i = c_{rl}^i, \text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r\}$ and $f(c_{ik}^r) = P\{\text{DATA suc.}|\mathcal{C}_i^r = c_{ik}^r\}$ denote the probabilities of *successful frame reception* (ACK and DATA), which are functions of the MAI level introduced by the specific set of interferers c_{rl}^i and c_{ik}^r, and PHY-layer aspects such as modulation and demodulation schemes, channel coding, receiver design, and the like.

In Eq. (4), the probability that the set of active interferers $\mathcal{C}_r^i$ at node i is c_{rl}^i is conditioned on the fact that the set of active interferers $\mathcal{C}_i^r$ at node r is c_{ik}^r. This is because, in general, the set of active interferers during a DATA frame reception at node r may not necessarily be the same as the set of active interferers during the ACK reception at node i. Moreover, the likelihood that a node becomes active when r is sending its ACK back to i can change if it knows that some set of nodes has already become active during the transmission of i's DATA frame to r. Such dependencies obviously rely on the type of MAC protocol in place. For instance, we could have a contention-based MAC protocol (e.g., ALOHA) where nodes would attempt to transmit packets independently of each other without listening to the channel (carrier sensing). In this case, the fact that some nodes have already become active during the transmission of the DATA frame does not prevent other inactive nodes to become active during the ACK transmission. However, in a carrier-sensing MAC protocol, the nodes that have detected the transmission of a DATA frame in the channel defer their transmission to a future time that hopefully not coincide with the transmission of the subsequent ACK frame (in general, ACK frames are very short).

The previous development [12] considered the case where both DATA and ACK frames are of fixed size. However, if both (or any) of the DATA and ACK frames are of variable size, and their size distributions are known, Eq. (4) can be easily modified to include such cases. For instance, if the DATA frame size is B bytes long, with probability mass function $P\{B = b\}$, and the ACK frames are assumed to be of fixed size (which is generally the case), Eq. (4) becomes

$$
\begin{aligned}
q_i^r &= P\{\text{DATA successful, ACK successful}\} \\
&= \sum_{k,l,b} P\{\text{DATA suc.}, B = b, \mathcal{C}_i^r = c_{ik}^r, \text{ACK suc.}, \mathcal{C}_r^i = c_{rl}^i\} \\
&= \sum_{k,l,b} f(c_{rl}^i) f(c_{ik}^r) P\{\mathcal{C}_r^i = c_{rl}^i|\text{DATA suc.}, B = b, \mathcal{C}_i^r = c_{ik}^r\} \\
&\qquad P\{\mathcal{C}_i^r = c_{ik}^r|B = b\} P\{B = b\},
\end{aligned} \tag{5}
$$

where now $f(c_{rl}^i) = P\{\text{ACK suc.}\,|\,\mathcal{C}_r^i = c_{rl}^i, \text{DATA suc.}, B = b, \mathcal{C}_i^r = c_{ik}^r\}$ and $f(c_{ik}^r) = P\{\text{DATA succ.}|B = b, \mathcal{C}_i^r = c_{ik}^r\}$.

2) The Case of Multiple Receivers: The Impact of Flow Distribution: So far, we have only considered the computation of the successful handshake probability when a node has a packet to transmit to a specific receiver, i.e., the computation of the successful handshake probability *conditioned* on a specific target receiver r. Now, let $\mathcal{R}_i$ denote the set of nodes to which node i has packets to transmit during the course of an observation time interval T_{obs}, and let ρ_i^r be the probability that node $r \in \mathcal{R}_i$ is the intended receiver of i at any instant within this time interval. The steady-state probability q_i that node i *transmits a frame successfully* is then given by

$$
\begin{aligned}
q_i &= P\{\text{ successful handshake}\} \\
&= \textstyle\sum_{r \in \mathcal{R}_i} P\{\text{successful handshake} \cap r \text{ is the receiver}\} \\
&= \textstyle\sum_{r \in \mathcal{R}_i} P\{\text{successful handshake} \mid r\} P\{r \text{ is the receiver}\} \\
&= \textstyle\sum_{r \in \mathcal{R}_i} q_i^r \rho_i^r.
\end{aligned} \tag{6}
$$

C. Making the Case for Carrier-Sensing MAC Protocols

In the specific case of carrier-sensing MAC protocols, an obvious equally-important "feedback information" is the probability that the channel is perceived busy while the node is attempting to transmit its own frame. In fact, the probability τ_i that a node i transmits a frame *at any time* should be a function of not only the successful handshake probability (to which the MAC protocol generally responds with some specific action), but also to the probability that the channel is perceived busy, in which case the busier the channel, the lower the probability that a frame is transmitted at any time. Therefore, by introducing a second feedback variable, and letting g_i denote the probability that node i perceives the channel busy, the transmission probability τ_i rewrites as

$$\tau_i = h_i(q_i, g_i), \quad i \in V. \tag{7}$$

We next show how g_i and q_i can be computed from a linear system of equations.

D. The Impact of Network Topology: The Linear Model

Let us first consider the probability τ_i that node i transmits a frame at any given time, as expressed in Eq. (7). The transmission probability τ_i should attain its maximum at $q_i = 1$ and $g_i = 0$ because then all frame exchanges are successful and the channel is always perceived idle. Therefore, if $h_i(q_i, g_i)$ is a function with a continuous n^{th} derivative throughout the plane $[0,1] \times [0,1]$, the Taylor series expansion of $h_i(q_i, g_i)$ around $q_i = 1$ and $g_i = 0$ is given by

$$h_i(q_i, g_i) = h_i(1,0) + \left.\frac{\partial h_i}{\partial q_i}\right|_{\substack{q_i=1\\ g_i=0}} (q_i - 1) + \left.\frac{\partial h_i}{\partial g_i}\right|_{\substack{q_i=1\\ g_i=0}} g_i + \cdots \tag{8}$$

Thus, a first-order approximation of $h_i(q_i, g_i)$ is simply

$$\tau_i = h_i(q_i, g_i) \approx a_{i0} + a_{i1} q_i + a_{i2} g_i, \tag{9}$$

where

$$a_{i1} = \left.\frac{\partial h_i}{\partial q_i}\right|_{\substack{q_i=1\\ g_i=0}}, a_{i2} = \left.\frac{\partial h_i}{\partial g_i}\right|_{\substack{q_i=1\\ g_i=0}}, \text{ and } a_{i0} = h_i(1,0) - a_{i1}. \tag{10}$$

Because τ_i tends to *decrease* as g_i increases (the channel gets busier), and to *increase* as q_i increases (handshakes are more successful), we should expect

$$\tau_i \approx a_{i0} + a_{i1} q_i - a_{i2} g_i, \tag{11}$$

where $a_{i1} \geq 0$ and $a_{i2} \geq 0$. The MAC operation represented by $h_i(\cdot)$ refers to each node's own output, which is supposed to be a function of the MAC parameters. If all nodes operate according to the same MAC protocol, under the same parameters, we have that $a_{i0} = a_0$, $a_{i1} = a_1$ and $a_{i2} = a_2$, $\forall\, i \in V$. Hence, Eq. (11) reduces to

$$\tau_i \approx a_0 + a_1 q_i - a_2 g_i. \tag{12}$$

Taking into account all nodes in the topology, Eq. (12) can be written in matrix notation as

$$\boldsymbol{\tau} = a_0 \mathbf{1} + a_1 \mathbf{q} - a_2 \mathbf{g}, \tag{13}$$

where $\boldsymbol{\tau} = [\tau_1 \ \tau_2 \ \ldots \ \tau_n]^T$, $\mathbf{1} = [1 \ 1 \ \ldots \ 1]^T$, $\mathbf{q} = [q_1 \ q_2 \ \ldots \ q_n]^T$, and $\mathbf{g} = [g_1 \ g_2 \ \ldots \ g_n]^T$.

Let us now consider the computation of the probability q_i^r that node i has a successful handshake with node r. From Eqs. (4) and (5), one of the key problems in computing q_i^r is the knowledge of the joint probabilities containing combinations $\{c_{ik}^r\}_{k=1,\ldots,2^{n-2}}$ of active interferers, as opposed to the successful reception probability $f(\cdot)$, which is known for any given MAI level. The exact functional form of the joint probabilities depends on the specific MAC protocol in use. However, finding an exact functional forms is inherently a difficult task, given the complexity of the interactions among the nodes, specially under multihop scenarios. Moreover, because the number of possible combinations of active interferers increases exponentially with the number of nodes, computing the probabilities of all the joint events becomes prohibitive with just a few tens of nodes. Fortunately, we can attain a linear approximation to the computation of q_i^r for *any* multihop ad hoc network by taking advantage of the *necessary* behavior of *any* MAC protocol. More specifically, we exploit the following three considerations:

C1) **High contention**: When contention among nodes is high, the probability τ_k that node $k \in V$ attempts to transmit a frame, at any time, is expected to be arbitrarily small.

C2) **MAC efficiency**: If nodes j and k can interfere with each other's frame exchanges *significantly*—directly or indirectly—e.g., they are located up to two "hops" away from each other, it is expected that the MAC protocol operates in a way that the *probability that node j schedules a transmission at a particular time instant, given that node k has already scheduled a transmission at the same time instant, is arbitrarily small*. This is what the desired operation of *any* MAC protocol should be: to prevent interfering nodes from scheduling transmissions at about the same time, "clearing the floor," and guaranteeing that a successful handshake occurs. The more efficient the MAC protocol is, the smaller this conditional probability becomes.

C3) **Impact of "Cascade Effect"**: If two nodes j and k are very distant from each other, such that their signal powers cannot interfere significantly with each other's frame exchanges (or have their signal powers blocked somehow), their *decisions* to initiate a frame transmission at any time are considered to be independent of each other. In reality, though, two nodes very distant from each other can still have an *indirect* impact on each other's transmission due to a "cascade effect": a node's decision to transmit a frame depends on its neighbors' decisions to transmit, which, in turn, depends on their neighbor's neighbors decisions to transmit, and so on. However, for all practical purposes, transmission decisions can be considered independent if nodes are sufficiently distant from each other (e.g., more than two hops away). Otherwise, consideration C2 applies.

Given the above considerations, we can now focus on the computation of the probabilities of the joint events represented by the combinations $\{c_{ik}^r\}_{k=1,\ldots,2^{n-2}}$. Among all possible combinations, we are particularly interested in two cases: (1)

the combination corresponding to the event where "*no node transmits at the given time instant*," and (2) the combinations corresponding to the events where "*only node $k \in V$ transmits at the given time instant*." If the event "*no node transmits at the given time instant*" occurs, the MAI level at node r is null and, consequently, the SINR_i^r is the highest possible, maximizing the conditional probability $f(\cdot)$ of a successful frame reception at r. In the second case, where "*only node $k \in V$ transmits at the given time instant*," the impact of node k alone is taken into account on the degradation of SINR_i^r during a frame reception at node r, i.e., the "weight" of the interference caused by node k at node r is evaluated.

Let $A_i,\ i \in V$, represent the event that "node i transmits a frame at the given time instant". Hence, $P\{A_i\} = \tau_i$. Now, the event "no node transmits at the given time instant" is represented by the joint event $\bigcap_{i=1}^n \overline{A_i}$, where $\overline{A_i}$ is the complement set of A_i. From DeMorgan's rules,

$$P\Big\{\bigcap_{i=1}^n \overline{A_i}\Big\} = P\left\{\overline{\bigcup_{i=1}^n A_i}\right\} = 1 - P\Big\{\bigcup_{i=1}^n A_i\Big\}. \quad (14)$$

where

$$P\Big\{\bigcup_{i=1}^n A_i\Big\} = \sum_{i=1}^n P\{A_i\} - \sum_{j,k} P\{A_j \cap A_k\} + \quad (15)$$
$$+ \sum_{j,k,l} P\{A_j \cap A_k \cap A_l\} + \cdots + (-1)^{n+1} P\Big\{\bigcap_{i=1}^n A_i\Big\},$$

Eq. (15) requires the knowledge of all probabilities of simultaneous transmissions among all possible subsets of nodes. However, according to consideration C2, *if nodes j and k can interfere to each other's frame exchanges significantly*, it is expected that the MAC protocol will do its best to prohibit both nodes from transmitting at about the same time, i.e., we should expect that $P\{A_j|A_k\} \ll 1$. In addition, because $P\{A_k\} \ll 1$ under high contention (from consideration C1), for any two nodes j and k we should have

$$P\{A_j \cap A_k\} = P\{A_j \,|\, A_k\} P\{A_k\} \approx 0. \quad (16)$$

On the other hand, from consideration C3, *if nodes j and k are very distant (or physically blocked) from each other*, their scheduling can be considered practically independent. In such cases, and under high contention,

$$P\{A_j \cap A_k\} = P\{A_j\} P\{A_k\} = \tau_j \tau_k \approx 0. \quad (17)$$

Extending our reasoning for the probabilities of the joint events that three or more nodes transmit at the same time in Eq. (15), we can approximate Eq. (14) by

$$P\Big\{\bigcap_{i=1}^n \overline{A_i}\Big\} \approx 1 - \sum_{i=1}^n P\{A_i\}. \quad (18)$$

Note that, if nodes are allowed to transmit independently of each other, regardless of the state of their neighbors (e.g., ALOHA), all the joint probabilities in Eq. (15) reduce to product forms as in Eq. (17) and, under high contention, Eq. (18) approaches an equality. In the other extreme case, if the MAC protocol is scheduled-based and can guarantee that one and only one node transmits at any given time, i.e., $A_i \cap A_j = \emptyset$ for all $i \neq j$, we simply have

$$P\Big\{\bigcup_{i=1}^n A_i\Big\} = \sum_{i=1}^n P\{A_i\}, \quad (19)$$

and, in this case, Eq. (18) holds with equality. Such a situation should be expected from an efficient MAC protocol operating in a single-channel fully-connected network, for example. Indeed, such exclusiveness of events is what is generally aimed in the operation of an efficient MAC protocol with respect to nodes within some neighborhood sharing the same channel in a wireless multihop ad hoc network.

Now, let us consider the events where "just one node transmits at the given time instant". For the sake of argument, let us assume that node 1 is the transmitting node. Therefore, we are interested in computing the probability of the event $\left(\bigcap_{i=2}^n \overline{A_i}\right) \bigcap A_1$, which is

$$\begin{aligned} P\Big\{\big(\textstyle\bigcap_{i=2}^n \overline{A_i}\big) \bigcap A_1\Big\} &= P\left\{\textstyle\bigcap_{i=2}^n \overline{A_i} \,\middle|\, A_1\right\} P\{A_1\} \\ &= \left[1 - P\left\{\textstyle\bigcup_{i=2}^n A_i \,\middle|\, A_1\right\}\right] P\{A_1\} \\ &= P\{A_1\} - P\left\{\textstyle\bigcup_{i=2}^n A_i \,\middle|\, A_1\right\} P\{A_1\}. \quad (20) \end{aligned}$$

where

$$P\left\{\bigcup_{i=2}^n A_i | A_1\right\} = \sum_{i=1}^n P\{A_i|A_1\} - \sum_{j,k} P\{A_j \cap A_k|A_1\} +$$
$$+ \cdots + (-1)^{n+1} P\left\{\bigcap_{i=2}^n A_i \,\middle|\, A_1\right\}. \quad (21)$$

Using the same arguments based on considerations C1 to C3, and observing that all terms in Eq. (21) are multiplied by $P\{A_1\} \ll 1$ in Eq. (20), we obtain

$$P\Big\{\Big(\bigcap_{i=2}^n \overline{A_i}\Big) \bigcap A_1\Big\} \approx P\{A_1\}. \quad (22)$$

Having considered approximations to the probabilities of some of the events $\{c_{ik}^r\}_{k=1,\ldots,2^{n-2}}$, we now consider the computation of the conditional probabilities relating the two sets of active interferers $\mathcal{C}_r^i$ and $\mathcal{C}_i^r$ in Eqs. (4) or (5).

As mentioned in Section IV-B, the set of active interferers during the reception of an ACK may be dependent on which nodes were active during the transmission of the previous DATA frame. Again, such dependencies are intrinsically related to the MAC protocol in use and, for this reason, we will focus on two cases that represent the two extreme behaviors observed in the operation of a MAC protocol. In the first case, we assume that the set of active nodes $\mathcal{C}_r^i$ during the reception of an ACK is the same as the set $\mathcal{C}_i^r$ of active nodes during reception of the DATA frame. The rationale for this assumption is that ACK frames are usually of fixed size and much smaller than DATA frames. Therefore, during the reception of an ACK, it is very likely that the nodes that were active during the DATA frame transmission may still be active during the subsequent ACK transmission (specially if DATA frames are of variable sizes). In this case, we have

$$P\{\mathcal{C}_r^i = c_{ik}^r |\, \text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r\} = 1. \quad (23)$$

Even in the case when DATA frames are of fixed size, nodes may become active at slightly different times from than the instant that a node started transmitting a DATA frame. This could happen because of the delays generally incurred when switching among modes (receive/transmit, idle/transmit, etc.). In this case, interfering nodes could become active when a DATA frame was already being transmitted in the channel, overlapping with both DATA and subsequent ACK frames, specially if ACKs are sent back without much delay, almost immediately after receiving the DATA frame.

In the other extreme case, both sets $\mathcal{C}_r^i$ and $\mathcal{C}_i^r$ can be considered independent of each other, i.e.,

$$P\{\mathcal{C}_r^i = c_{rl}^i \,|\, \text{DATA suc.}, \mathcal{C}_i^r = c_{ik}^r\} = P\{\mathcal{C}_r^i = c_{rl}^i\}, \quad (24)$$

in which case we have a scenario where interfering nodes can become active independently of what happened during the previous DATA frame transmission. This is likely the most aggressive scenario as far as interference is concerned, because it allows for the occurrence of sets of interferers that would not normally happen in the operation of many MAC protocols.

For the computation of q_i^r to be complete, we need to consider the case when DATA frames are of variable size, as expressed in Eq. (5). Again, for tractability, we assume that the set of active interferers is independent of the frame size, i.e.,

$$P\{\mathcal{C}_i^r = c_{ik}^r | B = b\} = P\{\mathcal{C}_i^r = c_{ik}^r\}. \quad (25)$$

Given all previous considerations, we can now compute an approximation for q_i^r. Let us consider the case where the set $\mathcal{C}_r^i$ of active nodes during reception of an ACK is the same as the set $\mathcal{C}_i^r$ of active nodes during reception of the DATA frame. From Eq. (5),

$$\begin{aligned} q_i^r =& \sum_{k,l,b} f(c_{rl}^i) f(c_{ik}^r) P\{\mathcal{C}_r^i = c_{rl}^i | \text{DATA suc.}, B = b, \mathcal{C}_i^r = c_{ik}^r\} \\ & \times P\{\mathcal{C}_i^r = c_{ik}^r | B = b\} P\{B = b\} \\ =& \sum_k \sum_b f(c_{rk}^i) f(c_{ik}^r) P\{\mathcal{C}_i^r = c_{ik}^r\} P\{B = b\} \\ =& \sum_k \overline{f(c_{rk}^i)} \, \overline{f(c_{ik}^r)} P\{\mathcal{C}_i^r = c_{ik}^r\}, \end{aligned} \quad (26)$$

where $\overline{f(\cdot)}$ indicates the average of $f(\cdot)$ with respect to the frame size B. If we take just the two sets of events previously considered, q_i^r is approximated by

$$\begin{aligned} q_i^r \approx& \ \overline{f(c_{i0}^r)} \, \overline{f(c_{r0}^i)} P\{\text{no node transmits}\} + \\ & \sum_k \overline{f(c_{rk}^i)} \, \overline{f(c_{ik}^r)} P\{\text{only node } k \text{ transmits}\} \end{aligned} \quad (27)$$

The probability of the events above are given by Eqs. (18) and (22) respectively, where $P\{A_k\} = \tau_k, \ \forall \ k \in V$. Hence,

$$q_i^r \approx \pi_i^r \left(1 - \textstyle\sum_{k \in V} \tau_k\right) + \textstyle\sum_{k \in V} \pi_{ik}^r \tau_k, \quad (28)$$

where $\pi_i^r = \overline{f(c_{i0}^r)} \, \overline{f(c_{r0}^i)}$, and $\pi_{ik}^r = \overline{f(c_{rk}^i)} \, \overline{f(c_{ik}^r)}$. Collecting the terms in τ_k, we have

$$\begin{aligned} q_i^r \approx& \ \pi_i^r - \sum_{k \in V} \pi_i^r \tau_k + \sum_{k \in V} \pi_{ik}^r \tau_k = \pi_i^r - \sum_{k \in V} (\pi_i^r - \pi_{ik}^r) \tau_k \\ =& \ \pi_i^r - \sum_{k \in V} c_{ik}^r \tau_k, \end{aligned} \quad (29)$$

where $c_{ik}^r = (\pi_i^r - \pi_{ik}^r)$. It is interesting to observe that c_{ik}^r in Eq. (29) provides the "weight" of node $k \in V$ with respect to its impact on the successful handshake probability between nodes i and r. More specifically, in the event that "no node transmits," the product of probabilities of successful frame reception $f(\cdot)$ attains its maximum, π_i^r, because there is no MAI. If the potential interferer k is far from both nodes i and r, the contribution of its signal power to the MAI level will be practically null and, in this case, $\pi_{ik}^r \to \pi_i^r$, i.e., $c_{ik}^r \to 0$, canceling the impact of node k in the successful handshake probability q_i^r. On the other hand, if node $k \in V$ is very close to both (or either) nodes i and r, its signal power can be sufficiently high to deteriorate the SINR at both (or either) nodes, compromising their handshake. In that case, $\pi_{ik}^r \to 0$, i.e., $c_{ik}^r \to \pi_i^r$, and the "weight" of node k is equivalent to saying that "node k should not transmit when nodes i and r are attempting to establish a handshake."

Finally, let us consider the impact of the traffic distribution, as described in Section IV-B. For that, we take the probability of successful handshake to a specific receiver, as computed in Eq. (29), and substitute into Eq. (6), which gives the overall steady-state probability q_i that node i has a successful handshake with the nodes $r \in \mathcal{R}_i$. Hence,

$$\begin{aligned} q_i =& \sum_{r \in \mathcal{R}_i} \Big(\pi_i^r - \sum_{k \in V} c_{ik}^r \tau_k\Big) \rho_i^r \\ =& \sum_{r \in \mathcal{R}_i} \pi_i^r \rho_i^r - \sum_{r \in \mathcal{R}_i} \sum_{k \in V} c_{ik}^r \tau_k \rho_i^r = \pi_i - \sum_{k \in V} c_{ik} \tau_k, \end{aligned} \quad (30)$$

where $\pi_i = \sum_{r \in \mathcal{R}_i} \pi_i^r \rho_i^r$ and $c_{ik} = \sum_{r \in \mathcal{R}_i} c_{ik}^r \rho_i^r$.

1) Impact of Carrier Sensing: So far, we have only considered the computation of the successful handshake probability q_i, $i \in V$. This probability, as mentioned before, is related to which nodes $k \in V$ can interfere with the successful reception of a frame at both sender and receiver (DATA and ACK frames). On the other hand, when a node is performing carrier sensing, the nodes that have a direct impact on the perception of the availability of the channel will not necessarily resemble the set of nodes that can interfere with frame receptions. In fact, it may happen that a number of nodes belong to both sets and, in this case, an appropriate "weight" should be given to these nodes, because they have a heavier impact on an individual frame transmission probability of a node $i \in V$. In fact, it is by taking this approach that we are able to incorporate the impact of the carrier sensing range mechanism in the interference matrix .

If $\mathcal{S}_i^r \subseteq V$ denotes the set of nodes whose activity has a direct impact on the perception of node i about the state of the channel being busy when i wants to transmit to r, then the probability g_i^r that node $i \in V$ perceives the channel busy at any given time when i is attempting to transmit a frame to r is given by the probability that *at least one* of the nodes in $\mathcal{S}_i \subseteq V$ transmits during the time node i is performing carrier sensing, i.e.,

$$\begin{aligned} g_i^r =& \ 1 - P\Big\{\textstyle\bigcap_{k \in \mathcal{S}_i^r} \overline{A}_k\Big\} \approx 1 - \Big[1 - \textstyle\sum_{k \in \mathcal{S}_i^r} P\{A_k\}\Big] \\ =& \ \textstyle\sum_{k \in \mathcal{S}_i^r} \tau_k = \sum_{k \in V} d_{ik}^r \tau_k, \end{aligned} \quad (31)$$

where

$$d_{ik}^r = \begin{cases} 1, & \text{if } k \in \mathcal{S}_i^r \subseteq V \\ 0, & \text{otherwise.} \end{cases}$$

Following the same approach to consider the impact of traffic distribution, we have

$$\begin{aligned} g_i &= \sum_{r\in\mathcal{R}_i}\Big(\sum_{k\in V} d_{ik}^r \tau_k\Big)\rho_i^r = \sum_{k\in V}\Big(\sum_{r\in\mathcal{R}_i} d_{ik}^r\rho_i^r\Big)\tau_k \\ &= \sum_{k\in V} d_{ik}\tau_k, \end{aligned} \quad (32)$$

Observe that, if node $k \in V$ is such that $d_{ik}^r = 1,\ \forall r \in \mathcal{R}_i$, then $\sum_{r\in\mathcal{R}_i} d_{ik}^r\rho_i^r = 1$, indicating that node k has a direct impact on the carrier sensing mechanism when i attempts to transmit a frame to all of its potential receivers in $\mathcal{R}_i$.

2) Interference Matrix: Finally, from Eqs. (12), (30), and (32), the probability τ_i that node $i \in V$ transmits a frame at any given time is given by

$$\begin{aligned} \tau_i &= a_0 + a_1\pi_i - a_1\sum_{k\in V} c_{ik}\tau_k - a_2\sum_{k\in V} d_{ik}\tau_k \\ &= \pi_i - \sum_{k\in V}\phi_{ik}\tau_k, \quad \forall\, i \in V, \end{aligned} \quad (33)$$

where $\pi_i = a_0 + a_1\pi_i$ and $\phi_{ik} = a_1c_{ik} + a_2d_{ik}$. In matrix notation, we have

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \vdots \\ \pi_n \end{bmatrix} - \begin{bmatrix} 0 & \phi_{12} & \phi_{13} & \cdots & \phi_{1n} \\ \phi_{21} & 0 & \phi_{23} & \cdots & \phi_{2n} \\ \phi_{31} & \phi_{32} & 0 & \cdots & \phi_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \phi_{n3} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \vdots \\ \tau_n \end{bmatrix},$$

leading to the linear system

$$(\mathbf{I} + \mathbf{\Phi})\boldsymbol{\tau} = \boldsymbol{\pi}, \quad (34)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{\Phi}$ is the $n \times n$ matrix in Eq. (34), and $\boldsymbol{\pi} = [\pi_1\,\pi_2\ldots\pi_n]^T$. As defined in [12], the matrix $\mathbf{\Phi}$ is called the **interference matrix**, and it conveys all the information regarding how each node interferes with every other node for the given PHY and MAC layers.

The linear system in Eq. (34) has a solution if and only if the vector $\boldsymbol{\pi}$ is in the column space of the matrix $\mathbf{\Psi} = \mathbf{I} + \mathbf{\Phi}$, i.e., it is a linear combination of the columns of $\mathbf{\Psi}$. Following the same reasoning as in [12], it can be shown that the system of Eq. (34) has a solution regardless of network topology. This is guaranteed in the following theorem (we omit the proof for the sake of space; it follows the same steps as in [12]):

Theorem 1: Given $n > 1$, if $a_1 + a_2 < (n-1)^{-1}$ the matrix $\mathbf{\Psi} = \mathbf{I} + \mathbf{\Phi}$ is nonsingular regardless of network topology.

From Theorem 1, the *transmission probability vector* $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = (\mathbf{I} + \mathbf{\Phi})^{-1}\boldsymbol{\pi}, \quad (35)$$

from which a number of performance metrics of can be obtained, as shown in [12] and [20].

V. Modeling the IEEE 802.11 DCF MAC

In the recent past, there has been a number of attempts to model the IEEE 802.11 DCF MAC [21]. One of the most prominent works is the one by Bianchi [22], who presented a way to evaluate the saturation throughput of fully-connected networks based on the modeling of the binary exponential backoff algorithm, heart of the IEEE 802.11 DCF MAC. Bianchi modeled the backoff time counter operation as a bi-dimensional discrete-time Markov chain, assuming that each frame "collides" with a constant and independent probability p at each transmission attempt, regardless of the number of re-transmissions already undertaken. This probability was named the *conditional collision probability*, meaning the probability of a collision experienced by a frame being transmitted on the channel. The steady-state probability τ that a node transmits a frame at any time can be obtained from the Markov chain as a function of the conditional collision probability p, as well as many parameters of the IEEE 802.11 DCF backoff algorithm.

Building upon Bianchi's model, some works have tried to deal with other aspects of the IEEE 802.11 DCF not previously considered. Ziouva and Antonakopoulos [23] provided a general model for CSMA/CA protocols based on the IEEE 802.11. They assumed a backoff algorithm close to the one in the 802.11, and included the "freezing" activity of the 802.11 backoff algorithm, defining the probability of *detecting the channel busy*. Because their model targeted general CSMA/CA protocols, their Markov chain does not accurately reflects the IEEE 802.11 DCF operation. Ergen and Varaiya [24] followed Ziouva's approach with respect to the impact of the carrier sensing mechanism, and focused on the IEEE 802.11 itself. In their model, however, they make the simplifying assumption that the conditional collision probability is *equal* to the probability of detecting the channel busy. In practice, the model by Ziouva and Antonakopoulos [23] makes this same simplifying assumption, particularly when the number of nodes in the network is large.

One drawback of these models (including Bianchi's) is the fact that they do not consider the finite retry limits of the IEEE 802.11 DCF, which proposes that DATA and RTS frames must be retransmitted a finite number of times. Instead, they all assume that frames are retransmitted *infinitely* in time, until they are successfully transmitted. Wu et. al. [25] incorporated the finite retry limit into Bianchi's model. The same exact Markov model was also proposed by Gupta and Kumar [26]. However, both models failed to incorporate the impact of the carrier sensing mechanism. Most importantly, a major limitation of all the aforementioned models—and of many others that appeared in the literature so far—is the fact that they *implicitly* assume a fully-connected (single-hop) segment of the network, under perfect channel conditions (i.e., no hidden terminal problems, no PHY-layer aspects, etc.). It is not true, for instance, that the conditional collision probability is the same (or similar) to the probability of detecting the channel busy. Each of these probabilities reflect totally different phenomena at the PHY layer. Detecting that a channel is busy only requires that some energy level be perceived at a node (as a result of some transmission(s)). On the other hand, the conditional collision probability relates to a more complex process, and deals with the ability of the nodes to correctly receive a frame, which depends on many PHY-layer parameters such as the modulation/demodulation scheme, receiver design, etc. In fact, by its very definition, all the previous models mistakenly define p, which actually should be defined as the *probability of a failed handshake*, since a sender's frame could still be correctly received at the receiver's side, but not its acknowledgment. Such separation of roles can only be possible by explicitly considering PHY-layer aspects into the model.

Let $b_j(t)$ be the stochastic process representing the backoff time counter for a node $j \in V$ at a time t, and $s_j(t)$ be the stochastic process representing j's backoff stage $[0, m]$ at time t, for which the maximum window size is $W_i = 2^i W_{\min}$, $i \in [0, m]$. If we assume that each handshake *fails* with a constant and independent probability p_j, regardless of the number of retransmissions experienced, and that a node detects the channel busy with a constant and independent probability g_j, then the process $\{s_j(t), b_j(t)\}$ can be modeled with the discrete-time Markov chain depicted in Fig. 1. Notice that the independence assumptions are with respect to the *number of retransmissions*, but both p_j and g_j are *dependent on PHY-layer aspects*, and are computed according to the developments in Section IV.

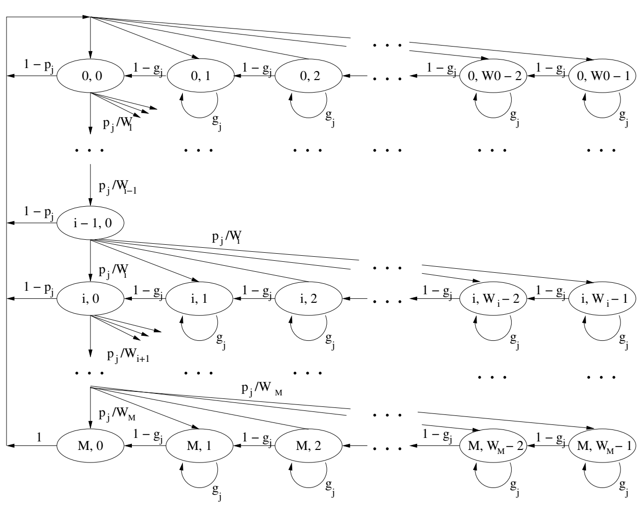

Fig. 1. Markov chain model representing the binary exponential backoff algorithm of the IEEE 802.11 DCF MAC.

In the Markov chain, the only non-null one-step transition probabilities are

$$\begin{array}{lll} P\{i,k|i,k+1\} = 1 - g_j, & k \in [0, W_i - 2], & i \in [0, M] \\ P\{i,k|i,k\} = g_j, & k \in [1, W_i - 1], & i \in [0, M] \\ P\{i,k|i-1,0\} = p_j/W_i, & k \in [0, W_i - 1], & i \in [1, M] \\ P\{0,k|i,0\} = (1-p_j)/W_0, & k \in [0, W_0 - 1], & i \in [0, M-1] \\ P\{0,k|M,0\} = 1/W_0, & k \in [0, W_0 - 1]. & \end{array} \tag{36}$$

The first and second equations indicate that the backoff counter is decremented if the channel is sensed idle (with probability $1 - g_j$), and frozen if the channel is sensed busy (with probability g_j). The third equation indicates that, after an unsuccessful handshake at stage $i-1$, a backoff interval is chosen within the interval $[0, W_i - 1]$ for stage i. The fourth equation indicates that a packet has experienced a successful handshake and a new packet starts at backoff stage 0 with a backoff window size chosen in $[0, W_0 - 1]$. The last equation describes that a new packet starts at backoff stage 0 after either a successful handshake or an unsuccessful handshake (the packet was dropped).

Let $b_{i,k} = \lim_{t\to\infty} P\{s(t) = i, b(t) = k\}, i \in [0, M], k \in [0, W_i - 1]$ be the stationary distribution of the Markov chain. We note that

$$b_{i,0} = p_j\, b_{i-1,0} \Longrightarrow b_{i,0} = p_j^i b_{0,0}, \quad 1 \le i \le M. \tag{37}$$

For $k \in [1, W_i - 1]$, we have

$$b_{i,k} = \frac{W_i - k}{(1-g_j)W_i} \cdot \begin{cases} \sum_{l=0}^{M-1} (1-p_j) b_{l,0} + b_{M,0}, & i = 0 \\ p_j\, b_{i-1,0}, & i \in [1, M]. \end{cases} \tag{38}$$

From Eq. (37), and noting that $\sum_{l=0}^{M-1} (1-p_j) b_{l,0} + b_{M,0} = b_{0,0}$, Eq. (38) can rewritten as

$$b_{i,k} = \frac{W_i - k}{(1-g_j)W_i} b_{i,0}, \quad i \in [0, M],\ k \in [1, W_i - 1]. \tag{39}$$

From Eqs. (37) and (39), all values of $b_{i,k}$ can be expressed as functions of $b_{0,0}$, which can be found from the normalization condition $\sum_{i=0}^{M} \sum_{k=0}^{W_i - 1} b_{i,k} = 1$, yielding

$$b_{0,0} = \frac{2(1-g_j)(1-p_j)(1-2p_j)}{(1-p_j^{M+1})(1-2p_j)(1-2g_j) + \kappa W}, \tag{40}$$

with $\kappa = (1-p_j)\left[1 - (2p_j)^{M+1}\right]$ if $m = M$, and $\kappa = 1 - p_j\{1 + (2p_j)^m [1 + p_j^{M-m}(1-2p_j)]\}$ if $m < M$.

Finally, by taking $\tau_j = \sum_{i=0}^{M} b_{i,0}$, we obtain

$$\tau_j = \frac{2(1-g_j)(1-p_j^{M+1})(1-2p_j)}{(1-p_j^{M+1})(1-2p_j)(1-2g_j) + \kappa W}, \tag{41}$$

with κ assuming the above values depending on whether $m \le M$. It is interesting to note that, if $M \to \infty$ and $g_j = 0$, the Markov chain in Fig. (1) reduces to the one used by Bianchi [22] for the case in which $m < M$. Accordingly, by making $M \to \infty$ and $g_j = 0$ in Eq. (41), we obtain the same expression derived by Bianchi [22].

Given the expression for the transmission probability τ_j, we can pursue its linearization, following the steps in Section IV-D. Because $q_j = 1 - p_j$, we have

$$\tau_j = \frac{2}{(W+1)^2} + \frac{2W}{(W+1)^2} q_j - \frac{2(W-1)}{(W+1)^2} g_j, \quad \forall j \in V, \tag{42}$$

which is in the desired form of Eq. (12), with $a_0 = 2/(W+1)^2, a_1 = 2W/(W+1)^2$, and $a_2 = 2(W-1)/(W+1)^2$.

VI. Model Validation

We evaluate the accuracy of our analytical model in predicting the performance of multihop ad hoc networks operating with directional antennas under the *directional virtual carrier sensing protocol* (DVCS) MAC protocol [11]. We refer the reader to [11] for more details on the operation of DVCS.

A. Simulation and Modeling Setup

We implement the analytical model in Matlab™ 7.0 [27], and conduct discrete-event simulations in Qualnet™ v3.5 [28]. For the directional antennas, we use a switched-beam antenna system with the default antenna gain pattern provided by Qualnet™, as shown in Figure 2. As in [11], we only focus on *interference reduction*, i.e., power control is enforced at the transmitter, decreasing the transmit power to compensate for the gain yielded towards the antenna boresight (15 dBi). Consequently, no communication range extension is in place[1].

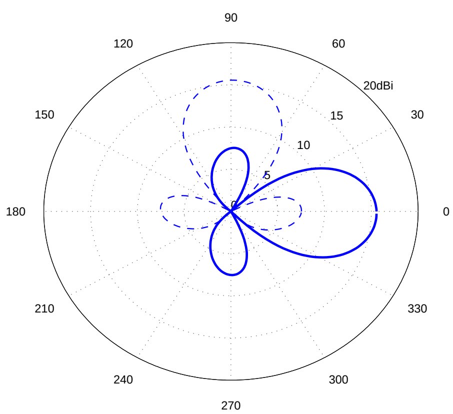

Fig. 2. Antenna gain pattern used for simulations and the analytical model. The figure shows two antenna patterns with boresight angles at 0 and 90 degrees, respectively.

The selected path loss propagation model is the two-ray ground reflection model. No shadowing or small-scale multipath fading is considered, and bit errors are treated independently, as it is done in Qualnet™. Nodes are randomly placed in an area of 1500×1500 m. For the physical layer, we use the direct sequence spread spectrum (DSSS) IEEE 802.11 PHY, with a raw bit rate of 1 Mbps under DBPSK modulation. Table VI-A summarizes the rest of the parameters used for the PHY and MAC layers.

Given that the focus of this paper is on the accurate modeling of MAC protocols operating with directional antennas and the modeling of DVCS more specifically, we will not consider the impact of traffic patterns involving different receivers per transmitter. Hence, each node has a single, one-hop receiver for its packets throughout the simulation time, to which it will send fixed-size packets of 1500 bytes (IP packet) generated from a CBR source. The source rate is high enough to saturate all nodes. Each simulation run corresponds to 5 minutes of data traffic, and the experiment is repeated for 20 seeds, with each trial corresponding to a different initial transmission time for each node. Initial transmission times are randomly chosen within the interval $[0, 0.01]$ s. This is done to allow the IEEE 802.11 exponential backoff algorithm to be triggered at different time instants at each node, so that different state evolutions occur within the same topology.

[1]There is an inherent trade-off between communication range extension and interference reduction [10], [8].

TABLE I
Simulation Parameters: MAC and PHY layers.

MAC		PHY	
$W_{\min}$	32	Temperature (Kelvin)	290
$W_{\max}$	1024	Noise factor	10
MAC Header (bytes)	34	Transmission power (dBm)	15
ACK (bytes)	38	Receive Sensitivity (dBm)	-91.0
CTS (bytes)	38	Receive Threshold (dBm)	-81.0
RTS (bytes)	44	Packet reception model	BER
Slot Time (μsec)	20	Directional Trans. Power (dBm)	0.0
SIFS (μsec)	10	RX Directional Sensitivity	-75.0
DIFS (μsec)	50	DNAV delta angle (degrees)	37.0

Because we analyze a static scenario, we let all RTS transmissions to be directional, as opposed to the default specification of DVCS, which proposes RTS to be transmitted in omnidirectional mode after 4 consecutive failed attempts, in which case it is assumed that the failure to get a CTS back is due to an inaccurate knowledge of the right direction to transmit (due to mobility).

B. Numerical Results

One of the advantages of our modeling approach is the ability to obtain *per node* performance metrics for a given network topology under a specific radio propagation model and detailed PHY/MAC layers [12]. The power of our modeling approach is best appreciated in the computation of the average throughput for a given network scenario. Figure 3 shows the average throughput results computed for 10 random topologies with 100 nodes each. As we can see, the model is able to predict the average throughput very accurately. Figure 4 contains a

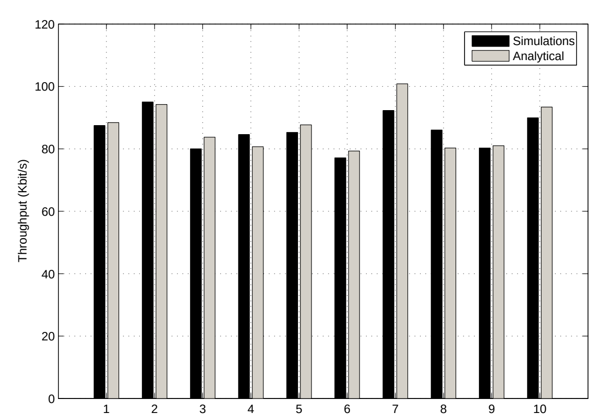

Fig. 3. Average throughput for 10 random topologies with 100 nodes.

histogram for the average number of nodes within a certain percentage prediction error. The percentage prediction error is computed with respect to the maximum range of throughput values observed in simulations for each node in each randomly generated topology. The histogram is the average of the 10 histograms over all topologies. According to our results, the percentage prediction error is within 20% for about 82% of the nodes. As highlighted in [12], the strength of our analytical model become also apparent when we compare the time required to obtain the above results through simulations and with the analytical model. Each simulation run in Qualnet for the 100-node scenario corresponding to 5 min of data traffic takes about 3060 s (51 min) in a Sun Blade 100 machine running Solaris 5.9. For the 20 seeds used, this corresponds to 17 hours of simulation. Using Matlab 7.0 in this same machine, our analytical model generates results in about 22 s. This corresponds to a time saving of 2800 times. The accuracy and speed attained by our analytical model compared to simulations enables the study of many physical-layer parameters associated with antenna-gain patterns on the performance of a MAC protocol.

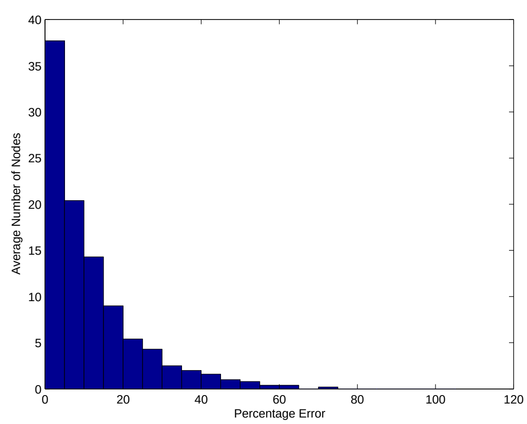

Fig. 4. Percentage error prediction histogram over 10 random topologies with 100 nodes.

VII. Realistic versus Simplified Antenna-Gain Patterns

In this Section, we show how misleading it can be the modeling of ad hoc networks with directional antennas based on over-simplified antenna-gain patterns like, for instance, the “pie-slice” or the “cone-plus-ball” antenna models. For this purpose, we use the same switched-beam antenna system as before, but with antenna-gain patterns that follow a “pie-slice” antenna model, shown in Figure 5.

Because the previous switched-beam antenna system had 8 main patterns (from which the system selects the best pattern to transmit/receive), the pie-slice antenna model also has 8 patterns, each corresponding to a sector of 45 degrees. Inside each sector, the gain is 15 dBi, whereas outside the sector the gain is -41.84 dBi, which is the minimum observed gain in the antenna-gain pattern of Figure 2.

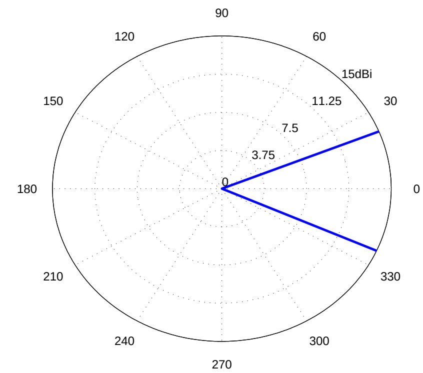

Fig. 5. Pie-slice antenna-gain pattern used for comparison with the realistic antenna-gain model shown in Fig. 2.

Keeping the same MAC- and PHY-layer parameters as used in Section VI, we compute the average throughput for 10 random topologies with a 100 nodes each. The numerical results corresponding to both antenna-gain patterns are shown in Figure 6. As we can see, throughput results for

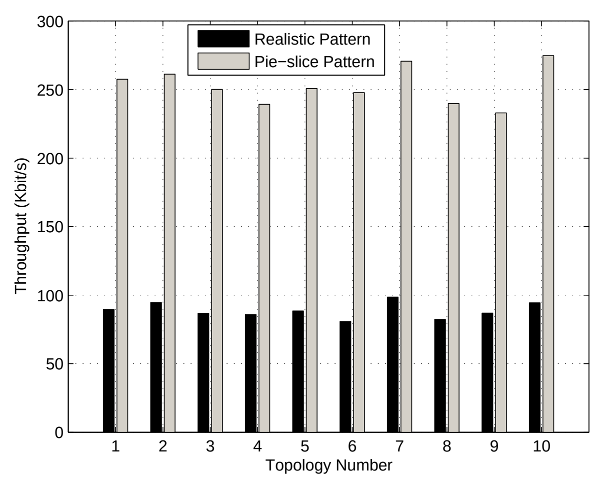

Fig. 6. Average throughput results for 10 topologies with 100 nodes each comparing the predicted performance using the “pie-slice” antenna-gain pattern and a realistic one.

ad hoc networks using over-simplified antenna gain patterns are too optimistic. The results suggest an increase in average throughput of more than 200% for the pie-slice antenna patterns. Obviously, such high average throughput is a direct consequence of the fact that no side lobes are present in the pie-slice model, and also non-negligible gains are present at *all directions* in the realistic antenna gain pattern. Therefore, the use of over-simplified antenna gain patterns in the study of ad hoc networks with directional antennas do not reflect the true potentials and limitations that this technology can achieve.

VIII. Conclusions

This paper presented the first analytical modeling of wireless ad hoc networks that considered the impact of realistic antenna gain patterns on network performance. As such, our modeling approach allows the study of ad hoc networks equipped with *directional antennas*, i.e., antenna systems that are able to transmit/receive energy over preferred directions. We showed how to make such a complex modeling problem tractable by considering the *needed behavior* of *any* medium access control (MAC) protocol. The result is an analytical model that allows a comprehensive treatment of ad hoc networks at the physical (PHY) and MAC layers, capturing the interactions between both layers, and taking into account the radio connectivity among the nodes—all conveniently conveyed through the use of *interference matrices*. We also introduced a new model for the IEEE 802.11 DCF MAC that takes into account advanced carrier-sensing mechanisms, such as the *directional virtual carrier sensing* (DVCS) protocol. We validated our analytical model by comparing its predictions with results obtained through a state-of-the-art simulation package. The results obtained show that our analytical model predicts simulation results very accurately, with a processing time that is orders of magnitude faster than simulations. Furthermore, we show that the "pie-slice" models for antenna patterns used in the past exaggerate the throughout attained in a network with directional antennas.

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