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Semicircle: An exact relation in the Integer and Fractional Quantum Hall Effect

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We present experimental results on the quantized Hall insulator in two dimensions. This insulator, with vanishing conductivities, is characterized by the quantization (within experimental accuracy) of the Hall resistance in units of the quantum unit of resistance, h/e^2. The measurements were performed in a two dimensional hole system, confined in a Ge/SiGe quantum well, when the magnetic field is increased above the $\nu = 1$ quantum Hall state. This quantization leads to a nearly perfect semi-circle relation for the diagonal and Hall conductivities. Similar results are obtained with a higher mobility n-type modulation doped GaAs/AlGaAs sample, when the magnetic field is increased above the $\nu = 1/3$ fractional quantum Hall state.

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In the extreme quantum limit, when the magnetic field (B) exceeds a critical field B_c, the quantum Hall series is terminated by an insulator. This insulator is characterized by a diverging diagonal resistivity, ρ_{xx}, as the temperature (T) vanishes. In general this insulating behavior occurs when the lowest resolved energy level exceeds the Fermi energy. For samples exhibiting only integer plateaus the transition occurs typically beyond $\nu = 1$. [1] For higher mobility samples this transition can occur beyond the $\nu = 1/3$ state [2] or the $\nu = 1/5$ state. [3] In addition to these primary fractions transitions to insulating behavior have been observed originating from many other fractions. [4] In this article we will only concentrate on the transitions from the states $\nu = 1$ to insulator and $\nu = 1/3$ to insulator. For both these transitions it has been shown previously that the transition point can be obtained from the B-field, $B = B_c$, where a T-independent ρ_{xx} is observed. [5] Our main focus here is the determination of ρ_{xy} and the inter-relation between the diagonal, σ_{xx}, and Hall, σ_{xy} conductivities around the transitions, with emphasis on extending previous works to deep into the insulating phase.

In order to measure ρ_{xy} we average over both B-field directions, which minimizes the contribution of the diverging ρ_{xx}. In fig. 1 we have plotted ρ_{xx} and ρ_{xy} as a function of B. The transition point $B_c = 6.06$ T is easily identified with the crossing of the ρ_{xx} traces obtained at different T's. The remarkable feature in this figure is the accuracy of the quantization beyond B_c, i.e., deep inside the insulating phase. The deviation from its quantized value between 6 and 8 T (inside the insulator) is less than 0.5% for T=1.8 K. Over the entire T range (below 2 K) ρ_{xy} deviates by less than 2% from h/e^2, while ρ_{xx} is highly insulating.

Following Dykhne and Ruzin's [6] calculation for a two phase model, where the transition region is described by a semi-circle relation between σ_{xx} and σ_{xy}, we plotted in the inset of fig. 1 σ_{xx} as a function of σ_{xy}. In their calculation $\sigma_{xx}^2 + (\sigma_{xy} - \sigma_0/2)^2 = (\sigma_0/2)^2$, where $\sigma_0 = e^2/h$ for the $\nu = 1$ to insulator transition and $\sigma_0 = e^2/3h$ for the $\nu = 1/3$ to insulator transition. Our experimental plot is close to a perfect semi-circle centered around $\sigma_{xy} = e^2/2h$ with radius $e^2/2h$, independent of T. These results were obtained in a two-dimensional hole gas confined in a Ge/SiGe quantum well, described in more details in ref. [7]. More details on the T and low B-field dependences can be found in ref. [8].

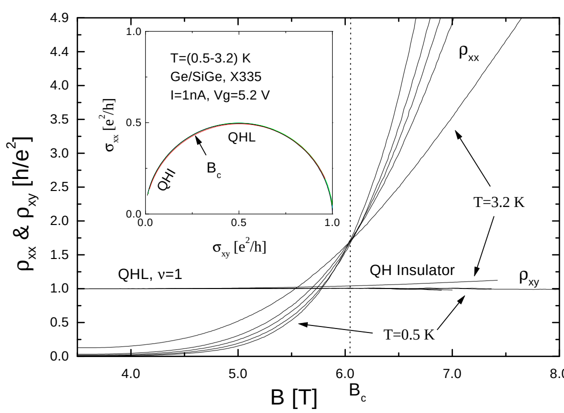

FIG. 1. The Hall and diagonal resistivities as a function of B for different T's, which are 0.5, 0.8, 1.2, 1.8 and 3.2 K. In the inset we have plotted σ_{xx} as a function of σ_{xy} for the same T's. B_c= 6.06 T.

We repeated these measurements with an n-type modulation doped GaAs/AlGaAs sample, which exhibits a transition from the $\nu = 1/3$ quantum Hall state to insulator. We obtain similar results for this system, as demonstrated in fig. 2, but with a semi-circle centered at 0.345 $e^2/2h$, instead of 0.333 $e^2/2h$ as predicted in ref. [6]. By increasing T (corresponding to the dotted lines) deviations from the semi-circle are becoming significant. In the inset we have

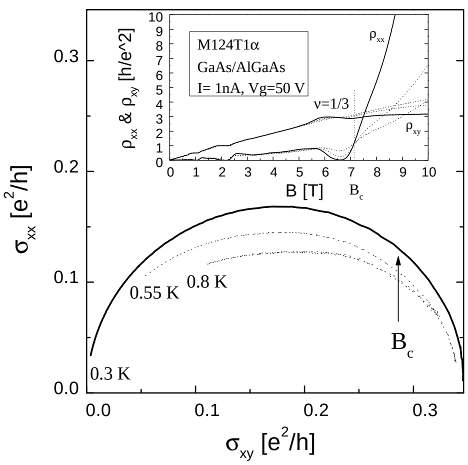

FIG. 2. The solid line presents σ_{xx} as a function of σ_{xy} at 0.3 K and the dotted lines are at 0.55 and 0.8 K. In the inset we have plotted the corresponding ρ_{xx} and ρ_{xy} versus B. $B_c = 7.2$ T.

plotted the corresponding ρ_{xx} and ρ_{xy} traces. Here again ρ_{xy} remains within 6 % of its quantized value $3h/e^2$, for the lowest measured T (0.3 K). For higher T 's (dotted lines), the deviations are larger.

The quantization of ρ_{xy} is intimately related to the existence of the semi-circle. Indeed, when obtaining conductivities from resistivities by matrix inversion, where $\rho_{xy} = h/\nu e^2$, we immediately obtain $\sigma_{xx}^2 + (\sigma_{xy} - \nu e^2/2h)^2 = (\nu e^2/2h)^2$, independently of ρ_{xx}.

There has been several theoretical attempts to estimate ρ_{xy} in the insulating phase. First Viehweger and Efetov [9] calculated the reactive part of ρ_{xy} and obtained $\rho_{xy} \sim B$. When calculating the diffusive part of ρ_{xy} a finite ρ_{xy} was also obtained [10]. More recently Shimshoni and Auerbach [11], inspired by a recent experimental work on duality [12], used a semi-classical network model and obtained a quantized ρ_{xy} in the insulating phase. The origin of the quantum decoherence in their model is, however, not clear. In a purely quantum mechanical numerical calculation in the lowest Landau level, i.e., for the $\nu = 1$ case, a semi-circle and a quantized ρ_{xy} was obtained very recently. [13]

An important point is the robustness of the quantization on different parameters. We therefore made measurements with different T 's, currents (I), contacts, density and samples. The results are summarized in the table. The deviation from the quantized value is obtained from the data for B-fields as deep as we could reliably measure inside the insulating phase.

Properties for $B > B_c$ (X335)	ρ_{xy} $[h/e^2]$
T (40 mK - 2 K)	$1 \pm 2\%$
I (0.1 nA - 1 μA)	$1 \pm 5\%$
Density (0.75-1.5) $\times 10^{11}$ cm^{-2}	$1 \pm 2\%$
Exchanging I-V contacts	$1 \pm 5\%$
Different sample (X334)	$1 \pm 4\%$
FQHE $\nu = 1/3$ (M124)	$3 \pm 6\%$

Summarizing, we have experimentally analyzed the nature of a quantized Hall insulator formed when the B-field is increased above the $\nu = 1$ (or $\nu = 1/3$) to insulator transition, resulting in $\rho_{xy} = e^2/h \pm 2$ % (or $\rho_{xy} = 3e^2/h \pm 6$ %). This verifies the semi-circle relation. The experiments presented covered a certain range of parameters, in particular in temperature, current, density and disorder. Different behaviors outside of the considered range are however possible and would be interesting to investigate in future work.

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