Submitted to Astrophysical Journal
as a Letter

UCRL-19871
Preprint

c. 2

OPTICAL TIMING OF THE CRAB NEBULA PULSAR NP 0532

Jerry Nelson, Richard Hills, David Cudaback, and Joseph Wampler

July 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

DISCLAIMER

UCRL-19871

OPTICAL TIMING OF THE CRAB NEBULA PULSAR NP 0532*

JERRY NELSON

Lawrence Radiation Laboratory
University of California, Berkeley

RICHARD HILLS
DAVID CUDABACK

Astronomy Department and Radio Astronomy Laboratory
University of California, Berkeley

JOSEPH WAMPLER

Lick Observatory, Board of Studies in Astronomy and Astrophysics
University of California, Santa Cruz

*Contributions from the Lick Observatory, No. 321.

ABSTRACT

Accurate pulse arrival times have been measured for NP 0532 during the period December 15, 1969 to May 3, 1970, and have been fitted to simple models of the pulsar braking mechanism. A good fit could not be obtained to all the data at once, because of deviations on a time scale of several days. However it was possible to divide the observing period into four shorter intervals in such a way that the data within each deviated only slightly from smoothly varying functions. The difference in the parameters of these four functions may indicate sudden events in the pulsar producing changes of order of 1 part in 10^9 in the pulsar frequency and 4 parts in 10^5 in the rate of change of frequency. In each case the difference in frequency from one interval to the next implies a slowdown of the pulsar.

We found that the average value of the "braking parameter" n in the equation $\frac{dE}{dt} = -A\omega^n$ was 3.63, but dividing the data into shorter intervals gave values between 0 and 5. We found no changes in the mean shape of the pulses, or the phase of the interpulse relative to the main pulse.

I. INTRODUCTION

Steadily increasing accuracy in measurements of the periods of pulsars has contributed much to current understanding of these objects. The discovery that the periods are increasing led to a wide acceptance of the rotating neutron star model (Gold 1969). Measurements of the second derivative of the period should enable one to distinguish between various possible mechanisms for the energy loss implied by the slowdown (Goldwire and Michel 1969; Boynton, Groth, Partridge and Wilkinson 1969a). However, the abrupt change in period observed in NP 0532 (Boynton et al. 1969b) and NP 0833 (Reichley and Downs 1969) and the reported observation of a quasi-sinusoidal oscillation in the arrival times of the radio pulses from NP 0532 (Richards, Pettingill, Counselman and Rankin 1970) indicate the need for detailed observations.

A program to obtain accurate arrival times of the optical pulses from NP 0532 is under way at Lick Observatory. In this Letter we report the results of the first observing period (December 15, 1969 to May 3, 1970).

We find that the shape of the light curve is highly stable and consequently the times of arrival of the pulses can be measured to within a small fraction of the pulse width. The accuracy of such a measurement is limited by the total number of photons received. With our present technique, one night's observation enables us to approach the basic limits of accuracy set by our atomic clock, which is about 1 μsec.

II. INSTRUMENTATION AND DATA ACQUISITION

Our data were obtained using the prime focus photometer on the 36-inch Crossley reflector. In order to reduce the sky background a 4" entrance aperture was used, except during poor seeing conditions, when a 10" diaphragm was used. An ITT FW-130 photomultiplier was used as a detector. During good observing conditions we typically detected between two and three photoelectrons from each primary pulsar pulse, which of course means that many pulses must be added together in order to measure an accurate arrival time.

The counts were stored in a 1024-channel multiscaler. The multiscaler was normally advanced at the rate of 25 μsec per channel, although occasionally runs were obtained with 5, 10, and 50 μsec advance periods. For each run the expected pulsar frequency was determined from an extrapolation of previous data and an accurate ephemeris of the earth's motion. A frequency synthesizer, which was set to this expected pulsar frequency and which was re-adjusted every three minutes, was used to generate a start sweep signal for the multiscaler. So long as the extrapolated ephemeris was correct to within 1 part in 10^9 this frequent resetting of the synthesizer prevented the relative phase of the pulsar and the start sweep signal from drifting by more than 0.5 μsec during a 15-minute run. The time of the first start sweep signal in each run was measured by a portable rubidium atomic clock. The succeeding pulsar signals were then superimposed in the multiscaler so that the time of arrival of the first pulsar pulse could be measured relative to the beginning of the sweep.

Channel advance signals were synchronized with the start sweep signal, and these signals, as well as the frequency synthesizer and all other timing circuitry, were driven by the atomic clock. The clock rate appeared to be determinable to within about one part in 10^{12}; the clock time was usually calibrated every week against a set of cesium clocks at Hewlett Packard's time center at Santa Clara, California. Their clocks are regularly calibrated against both NBS and USNO, so our time base (Universal Time Coordinated) is directly traceable to NBS and USNO to within about 1 μsec.

The systematic errors in the data were mainly due to the drift of the pulse during the run, caused by inaccuracies in setting the frequency synthesizer. This produced an error in determining the arrival time which was typically <.5 μsec. Errors in the electronics were usually <.2 μsec although from February 12 to February 27 an electronics problem in the start sweep-channel advance circuit prevented synchronization of the channel advance signal with the start sweep signal and caused timing errors which may have smeared the pulse by as much as ±1/2 channel.

This electronics system was developed, maintained, and repeatedly calibrated by John Saarloos of the Lawrence Radiation Laboratory, who worked heroic hours to achieve the excellent time calibration. Most of the observing at the telescope was done by Remington Stone of the Lick Observatory, and therefore consistency in the observational technique was assured. The work of these two men enabled the authors to concentrate on planning and analysis.

III. DATA REDUCTION

A. The Light Curve

To determine the arrival time of a pulsar pulse relative to the start of the multiscaler sweep, the observed light curve, contained in the 1024 channels of the multiscaler, was fit by an empirical function. Three parameters (the strength of the pulse, the height of the background, and the delay of the pulse from the start of the sweep) were determined for each run using the method of least squares. Five additional parameters, which describe the shape of the curve, were held constant after being chosen by an empirical fit to a light curve produced by averaging many runs. Figure 1 shows a typical run, 23 runs averaged together, and the empirical function. The functional form used to describe the light curve is given by the equation

$$L(t) = \frac{1 - 1{,}620t + 1{,}402{,}000t^2}{1 - 1{,}905t + 3{,}180{,}000t^2}\, e^{-350{,}000t^2}$$

where t is the time in seconds. The three parameters that characterized each run were determined by fitting a function of the form $aL(t-t_o)+b$; the arrival time is then given by t_o. (a and b measure the strength of the signal and background, respectively.) While not essential to obtain accurate arrival times, the form of L used did give a good χ^2 fit to each run. This shows that to within the measurement accuracy the shape of the average pulse did not change from day to day. A similar conclusion was reached for shorter time periods by Warner, Nather, and MacFarlane (1969) and by Wampler, Scargle, and Miller (1969).

The uncertainty in the determination of the arrival time due to the photon counting statistics, was found to be typically 5 μsec for each 15-minute run. This result was confirmed by Monte Carlo programs which also showed that the arrival times obtained were not affected by the variations in the size of the background. The background varied because of changes in the size of the diaphragm, the position and brightness of the moon and the hour angle of the pulsar.

The interpulse was also fitted to a similar function whenever it was included in the data. Typical uncertainties in the interpulse arrival time were ±20 μsec. We tested the hypothesis that the phase angle between the main pulse and the interpulse was constant. A χ^2 of 171 for 158 runs was obtained which indicates that the angle is fixed to within about .03 deg (2.5 μsec).

B. Reduction to an Inertial Frame of Reference

The times of arrival of the pulses at the solar system barycenter were calculated from the observed arrival times by means of the JPL ephemeris DE 69. We are grateful to J. D. Mulholland for providing this tape and the astronomical constants which have been determined for use with it. The ephemeris is the result of a numerical integration of the planetary motions with the parameters and initial conditions fitted to the best available optical, radar and spacecraft tracking data. Since the numerical procedures of integration, tabulation, and interpolation have all been carried out to greater accuracy than is needed for our purposes the only errors introduced by the ephemeris should be those arising from uncertainties in the starting conditions and planetary masses used for the integration. These error terms have periods equal to the orbital periods of the planets involved. The most important term is that due to the uncertainties in the orientation of the solar system with respect to the pulsar. Neither the solar longitude with respect to the celestial coordinate system nor the position of the pulsar in that system is known to an accuracy much better than 0.1". An error of this magnitude in the earth-sun-pulsar angle will produce a term of one year period and about 240 μsec amplitude in the barycenter arrival times. Because of the short period of the observations, terms of one year period will be masked by the fitting procedure.

An improved position for the pulsar may be obtained from measurements of Lick 36-inch refractor plates, but for the moment we have adopted the same position (due to Minkowski) as previous investigators (Table I). Uncertainties in the masses of the outer planets also lead to considerable errors in determining the position of the barycenter. But, because the periods are very long, only the absolute barycenter arrival times are significantly affected, not the pulsar frequency and its derivatives.

The reduction to the barycenter removes the largest part of the effect of the observer's motion from the data. However, in order to make the arrival times correspond to those which would be obtained by an observer in an inertial frame, it is necessary to apply two further corrections. The first of these accounts for the variation in the rate of a clock on earth relative to a clock outside the solar system. This variation results from the accelerations and changing gravitational potential experienced by the clock. The correction was determined by a numerical integration of the rate of the clock on earth relative to the coordinate time. This is given to sufficient accuracy by

$$\frac{dt_{coord}}{dt_{clock}} = 1 - \sum_i \frac{GM_i}{c^2 r_i} - \frac{V^2}{2c^2} + K,$$

where V is the velocity relative to the barycenter and M_i and r_i are the masses and distances from the earth of the sun and planets. The secular term was removed by adding to the clock rate the constant K, which corresponds to the rate of a clock with the same average velocity and potential as that on earth. The correction was adjusted to be zero at perihelion passage 1970. The most important part of this correction is a one-year sine wave of 1658 μsec amplitude due to the eccentricity of the earth's orbit, but the numerical integration also contains higher Fourier components and terms with periods of one day, one month, and the synodic periods of the outer planets, which have amplitudes between 1 and 20 μsec.

The second correction removes the delay caused by the effect of the sun's gravitational field on the light from the pulsar, assuming Einstein's general relativity. This was calculated using the expression

$$\Delta t = \frac{2m}{c} \ln \left(\frac{r_o + x_o}{r + x} \right)$$

where m is the Schwarzschild radius of the sun, r is the distance earth-sun and x the component of that distance in the direction of the pulsar. r_o and x_o are the corresponding distances for a reference point which was chosen to be 1 a.u. from the sun in the direction of the pulsar. This correction varies from near zero to 25 μsec over the period of observation.

We believe that these corrections remove all relativistic effects to the level of our experimental errors. We thank Chris Wilson for his help in the derivation of these corrections.

DATA ANALYSIS

Using an approximate knowledge of the pulsar frequency we were able to assign without ambiguity an integer pulse number to each pulse for which the barycentric arrival time had been calculated. We chose the convention that the first pulse to arrive at the barycenter after time t_o = MJD 40587.0 (the first instant of 1970) is pulse number one. t_o is used as the zero point in time for our models. We found that we were able to combine the data from each night's runs into a single data point (Table II). This was done by fitting a phase $\emptyset_o$ to each night's data using the model A (described below) with the other three parameters fixed. The goodness of these fits indicate that we observed no deviations from the model during a single night and that our estimates of the errors for each run were reasonable.

The models tested are based on the assumption that the energy of the pulsar is proportional to the square of the pulsar frequency ($E = 1/2\ I\omega^2$), and the rate of energy loss is some function G of the frequency $f = \omega/2\pi = d\emptyset/dt$. Initially we assume the moment of inertia is constant. The parameters obtained are given in Table III and the residuals, defined as the expected arrival time minus the observed arrival time, are displayed in Fig. 2. The models fitted to the data were:

A. the so-called "power-law slow down," $G(f) = -\alpha f^n$. This yields for the phase

$$\phi(t) = \phi_o + \frac{f_o^{3-n}}{(n-3)\beta}\left\{\left[1 + (n-2)\beta\, f_o^{n-2}(t-t_o)\right]^{\frac{3-n}{2-n}} - 1\right\},$$

where $\beta = \alpha/4\pi^2 I_o$. If $\gamma = \beta f_o^{n-2}$, the parameters ϕ_o, f_o, γ, n were determined by fitting this function to the data in a χ^2 sense. The residuals show systematic deviations in excess of 100 μsec but their structure does not appear to be periodic. It should be pointed out that while these deviations are much larger than the experimental errors, they could be accounted for by variations in the moment of inertia as small as one part in 10^9, or by changes in the rate of energy loss of one part in 10^4.

B. A Taylor series expansion of the phase. i.e.,

$$\phi(t) = \phi_o + f_o(t-t_o) + \frac{f_o'}{2}(t-t_o)^2 + \frac{f_o''}{6}(t-t_o)^3 + \ldots$$

where the coefficients are free parameters. By truncating this series to four terms one can approximate the solution to Model A. In this case

$$n-1 = \frac{f_o f_o''}{(f_o')^2}$$

(Goldwire and Michel 1969). One difficulty in this model is the correct assessment of the effects of truncating the series. Also, with this approximation, extrapolation can be misleading since polynomials do not have the desired asymptotic properties. This four-parameter polynomial actually yielded essentially the same results as the first model.

C. Model A plus a sine wave of one year period with arbitrary phase and amplitude, to account for uncertainties in the barycentric correction. Because of the short observing period the parameters of the sine wave cannot be meaningfully assessed, although there was some improvement in the fit over Model A.

D. Model A plus a sine wave of 77-day period and arbitrary amplitude and phase. A sine wave of this period and an amplitude of 380 μsec was found by Richards et al. (1970) in the radio pulse arrival times at Arecibo. The fit was improved somewhat, but the amplitude of the best fitting sine wave was only 30 μsec.

We searched for sine waves of other periods but we found none which dramatically improved the fit.

E. A somewhat more physical model, which assumes $dE/dt = -a_1f^2 - a_2f^4 - a_3f^6$. The f^2 term could be due to stellar wind torque, the f^4 term due to magnetic dipole radiation, and the f^6 term due to gravitational quadrupole radiation, and higher-order terms are neglected. An alternate way to obtain this expression is to consider the Taylor series expansion of dE/dt in f, and note that terms containing f^0 and all odd powers of f vanish because of the boundary conditions of the problem. This five-parameter model was fitted to the data and again no improvement over the first model was found in that the residuals were essentially unchanged. Actually, the five parameters were underconstrained and a unique solution could not be found: the data imposed only two effective constraints on the three parameters, a_1, a_2, and a_3, thus many sets of a's yield essentially the same fit. In fact it is possible to force a_2 to be zero (no magnetic dipole radiation) and still obtain a comparable fit. Thus while there appear to be theoretical reasons for expecting a high percentage of dipole radiation (Ostriker and Gunn 1969), our data give us no lower limit whatsoever on this.

F. Model A with n = 4. This corresponds to a braking mechanism which is exclusively magnetic dipole radiation. The fit to this model was much worse than to Model A, and the residuals show a cubic time dependence which results from the fact that the second derivative of the pulsar frequency is forced, by the choice of n, to take an erroneous value. However, examination of these residuals showed that there were apparently two discontinuous changes in the slope of the otherwise fairly smooth curve, occurring in the region of MJD 40656 and 40680. This suggested model G , in which we divide the observing period into shorter intervals.

G. By dividing the data at the points mentioned above and fitting a function of the same form as Model A to each of the three sections, we obtained a great reduction in χ^2. However, the fit to the first section was still much worse than the other two sections, so a further division at MJD 40625 was tried. This produced a further dramatic improvement in the fit. The total χ^2 for the fit of the four sections (51 data points) to the four different functions (16 parameters) is 131 compared to the value of 6000 obtained when using a single function to fit all the data. As a test of the significance of this result we broke the data into four different blocks, choosing the breaks so that the apparent events lay within the intervals. The uniqueness of the regions around MJD 40657 and 40680 was apparent, but the break in the first section of data at MJD 40625, while improving χ^2, could not be singled out as much more likely than breaks introduced at nearby points in the data. Although it is possible to obtain a fairly good fit to each of the four sections, the assumption of a single power law to describe the braking mechanism is of course not expected to cover phenomena related to sudden changes in the pulsar; this is reflected by the fact that the values of n obtained for the four short sections varied from 0 to 5.

Inspection of the parameters for Model G (see Table III) does not immediately reveal the nature of the changes in the parameters at the assumed breaks since the model uses t_o as the zero point in time rather than the time of the breaks. By calculating the frequency and its derivative at each breaking point, we found that both the frequency and its derivative decreased in magnitude at each point, the frequency by as much as 1 part in 10^9, its derivative by as much as 4 parts in 10^5.

Because we have found no simple model that gave a good χ^2 fit to the data, care should be exercised in the physical interpretation of the fitting parameters, particularly n.

V. CONCLUSIONS

The slowdown does not seem to follow simple radiation braking nor do we see the 77-day sine wave reported by Richards et al. (1970). However, the residuals from the fits to the models chosen do not vary randomly from one data point to the next. Instead they show a definite structure changing over a period of several days. This, together with the consistency of the data within each night's observing, suggests that these effects are not due to errors in the observations. Furthermore, we do not believe that the planetary ephemeris used could cause errors with this structure. Rather, we consider it most likely that the deviations are present in the light pulses emitted from the pulsar. There are many possible explanations of these deviations which, after all, represent phase shifts of less than 1 deg. These include wandering of the light emitting region with respect to the main body of the pulsar (but the constancy of phase between the main pulse and interpulse argues against this), changes in the moment of inertia, or changes in the rate of energy loss.

While our data do not show unambiguously that the deviations must be due to discrete changes in the pulsar as opposed to continuous variations, the results from model G encourage this view, particularly in light of the sudden change in frequency observed by Boynton et al. (1969b). The evidence is fairly strong for the second and third breaks, but the location of the first break is uncertain, and it cannot be considered a well-defined event. One is left to speculate as to whether more accurate and more regular observations might show a series of events of different magnitudes. The great complexity in the data indicates a strong need for continued, accurate observations. It seems likely that we will not be able to distinguish properly between the various braking mechanisms until we understand the abrupt changes in the pulsar frequency.

The encouragement and support of Professor Burton J. Moyer was invaluable in making this experiment possible. We thank Professor L. E. Cunningham for his advice and encouragement, and we are greatly indebted to Orin Dahl for many fruitful discussions on the data analysis, and Mike Raugh and Roger Chaffee for much programming assistance. We thank the Princeton group for preliminary comparisons on the ephemeris calculations.

This research was supported, in part, by the U.S. Atomic Energy Commission, the National Aeronautics and Space Administration (NSR 05-061-006), and the National Science Foundation (GP-6310).

REFERENCES

Boynton, P. E., Groth, E. J., Partridge, R. B., and Wilkinson, D. T. 1969a, *Ap. J.* (*Letters*), **157**, L197.

_____. 1969b, *I.A.U. Circ.* No. 2179.

Gold, T. 1969, *Nature*, **221**, 25.

Goldwire, H. C., and Michel, F. C. 1969, *Ap. J.* (*Letters*), **156**, L111.

Ostriker, J. P., and Gunn, J. E. 1969, *Ap. J.*, **157**, 1395.

Reichley, P. E., and Downs, G. S. 1969, *Nature*, **222**, 229.

Richards, D. W., Pettengill, G. H., Counselman, C. C., and Rankin, J. M. 1970, *Ap. J.* (*Letters*), **160**, L1.

Wampler, E. J., Scargle, J. D., and Miller, J. S. 1969, *Ap. J.* (*Letters*), **157**, L1.

Warner, B., Nather, R. E., and McFarlane, M. 1969, *Nature*, **222**, 233.

TABLE I

CONSTANTS FOR DATA REDUCTION

Coordinates of Pulsar

α = 5 hr 31 m 31.46 sec (1950)

δ = 21° 58' 54.8" (1950)

Coordinates of Telescope

$\rho \sin \phi'$ = .60335

$\rho \cos \phi'$ = .79619

Longitude = 8 hr 6 min 34.93 sec

Earth radius = 6378.1492 km

1 A.U. = 499.004773 sec (UTC)

Epoch

t_o = 40587.0 Modified Julian Day

= 2440587.5 Julian Day

Time interval used: Universal Time Coordinated second

TABLE II

PULSE TIMES, PHASES AND ERRORS

Time of Pulse* at Lick (MJD-40000)	Time of Pulse at Barycenter (MJD-40000)	Timing† Uncertainty (Microseconds)	Pulse Number	No. of Runs on Night
570.332986234200	570.338681758760	19.0	-43482887	7
572.299768953591	572.305461675645	2.4	-38349922	8
584.395833503211	584.401360119158	2.7	-6781905	7
585.366782745023	585.372285064535	2.1	-4247992	10
586.333565222125	586.339041746654	1.5	-1724960	11
587.315625207782	587.321073892697	1.7	837935	10
588.198611508178	588.204033737853	4.3	3142270	5
589.314351951027	589.319738925802	1.2	6054017	15
614.298958567764	614.303044239830	3.6	71254052	8
615.283680688575	615.287697005235	1.7	73823707	7
616.203125105741	616.207075586630	5.0	76223013	4
617.271875127488	617.275747747321	1.5	79011928	14
619.282986558981	619.286708983309	1.3	84259927	18
620.189236445120	620.192889674658	2.0	86624780	7
621.278125416837	621.281694225629	2.5	89466222	6
622.217708584260	622.221203532219	28.7	91918047	1
629.355208495275	629.358112539649	8.0	110543061	4
632.293403227639	632.296050477631	4.7	118210082	5
636.223958448118	636.226251473243	6.8	128466534	4
637.240625180206	637.242824756064	5.1	131119428	5
638.278125391538	638.280228877833	9.9	133826680	2
639.207291831597	639.209308748992	3.5	136251242	5
640.196875100868	640.198799198741	4.3	138833451	6
642.292014065875	642.293739720109	4.7	144300472	4
643.288541990041	643.290172464342	3.4	146900789	8
644.271180786207	644.272716935058	6.8	149464860	4
652.259375344289	652.260130155340	1.8	170308897	6
654.236458712036	654.237017298747	5.9	175467774	2
657.303125147257	657.303378112181	5.6	183469715	2
663.269097947377	663.268755574383	8.0	199036820	2
664.164930648541	664.164499163162	4.8	201374322	4
665.215625168443	665.215089169084	2.9	204115903	7
666.234375349119	666.233738209616	3.1	206774129	3
667.196875265927	667.196142808425	3.0	209285579	2
668.201042069348	668.200210350918	4.3	211905748	2
669.238541777384	669.237607738200	2.7	214612890	5
670.236458784992	670.235426648345	2.3	217216747	6
673.188542080923	673.187221681888	2.6	224919583	7
676.219097380817	676.217484509762	8.7	232827158	8
677.225347334792	677.223638279345	2.7	235452745	8
684.269097364107	684.266730981555	10.2	253831815	2
685.186458495882	685.184008978002	3.4	256225457	4
686.207292002084	686.204750578385	6.6	258889085	5
687.188541902222	687.185912923753	6.2	261449428	5
688.201041926368	688.198323365934	3.7	264091310	7
689.184375384542	689.181570648827	5.9	266657088	5
696.234375163800	696.230977408821	15.5	285052392	2
701.213194582884	701.209407367380	7.0	298043432	4
707.208680748041	707.204460807254	7.8	313687221	3
708.200347385788	708.196059993923	5.4	316274739	4
709.200347665803	709.195993485740	13.7	318884002	2

*Synthesized from the runs of that night.

†Weighted mean of statistical errors of individual runs of that night.

TABLE III

FITS OF MODELS TO PULSAR DATA

	Model Tested	χ^{2*}	Values of Fitted Parameters	Formal† Statistical Errors
A	$G(f) = -\alpha f^n$	5929	ϕ_o = .745952 cycles	1.9×10^{-5}
	$\gamma = \frac{\alpha}{4\pi^2 I_o} f_o^{n-2}$		f_o = 30.20586559715 Hz	2×10^{-11}
			$\gamma = 1.276572230.10^{-11} sec^{-1}$	4×10^{-19}
			n = 3.6272	6×10^{-4}
B	$\phi(t) = \phi_o + f_o(t-t_o) + \frac{f'_o}{2}(t-t_o)^2 + \frac{f''_o}{6}(t-t_o)^3$	5917	ϕ_o = .745950 cycles	1.9×10^{-5}
	$n = 1 + f_o f''_o/(f'_o)^2$		f_o = 30.20586559714 Hz	2×10^{-11}
			$f'_o = -3.85599686 \times 10^{-10}$ Hz/sec	1.2×10^{-17}
			$f''_o = 1.29291 \times 10^{-20}$ Hz/sec^2	2.9×10^{-24}
			n = 3.6266	6×10^{-4}
C	$G(f) = -\alpha f^n$	4270	ϕ_o = .424 cycles	1.3×10^{-2}
	+ a 1 year sine wave in the phase		f_o = 30.2058656299 Hz	2.4×10^{-9}
	$a \sin [\frac{2\pi}{\tau}(t-t_o) + b]$		$\gamma = 1.2765249 \times 10^{-11} sec^{-1}$	1.8×10^{-17}
	τ = 365.25 days		n = 3.097	2.6×10^{-2}
			a = .362 cycles	1.5×10^{-2}
			b = 2.052 rad.	1.4×10^{-2}

TABLE III (continued)

	Model Tested	χ^{2*}	Values of Fitted Parameters	Formal† Statistical Errors
D	$G(f) = -\alpha f^n$	4267	ϕ_o = .746506 cycles	2.4×10^{-5}
	+ a 77 day sine wave in the phase		f_o = 30.20586559642 Hz	3.3×10^{-11}
	$a \sin [\frac{2\pi}{\tau}(t-t_o) + b]$		$\gamma = 1.276571024 \times 10^{-11}$ sec^{-1}	6.2×10^{-19}
	τ = 77 days		$n = 3.6132$	8.3×10^{-4}
			$a = -9.87 \times 10^{-4}$ cycles	2.4×10^{-5}
			$b = 1.078$ rad.	2.6×10^{-2}
E	$G(f) = -I(a_1f^2+a_2f^4+a_3f^6)$	5982	ϕ_o = .745950 cycles	2.0×10^{-5}
	with		f_o = 30.20586559726 Hz	1.2×10^{-11}
	$\alpha_1 = -a_1f_o-a_2f_o^3-a_3f_o^5$		$\alpha_1 = -3.85599778 \times 10^{-10}$	4.5×10^{-18}
	$\alpha_2 = -a_1-3a_2f_o^2-5a_3f_o^4$		$\alpha_2 = -3.35917 \times 10^{-11}$	2.0×10^{-15}
	$\alpha_3 = -6a_2f_o-20a_3f_o^3$		$\alpha_3 = 1.0 \times 10^{-13}$	2.8×10^{-12}
F	$G(f) = -\alpha f^n$	4.0×10^5	ϕ_o = .745831 cycles	2.0×10^{-5}
	$n = 4$		f_o = 30.20586560704 Hz	1.2×10^{-11}
			$\gamma = 1.2765968455 \times 10^{-11}$ sec^{-1}	9.9×10^{-20}

TABLE III (continued)

	Model tested	χ^2*	Values of Fitted Parameters	Formal† Statistical Errors
	$G(f) = -\alpha f^n$		ϕ_o = .745560 cycles	2.2×10^{-5}
			f_o = 30.20586559662 Hz	4.7×10^{-11}
	Block 1 [570-622] 16 data points	39.2	$\gamma = 1.27656644 \times 10^{-11}$ sec^{-1}	1.9×10^{-18}
			n = 3.361	1.5×10^{-2}
			ϕ_o = .715 cycles	2.6×10^{-2}
	Block 2 [629-654] 12 data points	61.1	f_o = 30.205865621 Hz	1.7×10^{-8}
			$\gamma = 1.276613 \times 10^{-11}$ sec^{-1}	2.4×10^{-16}
			n = 4.27	3.1×10^{-1}
G			ϕ_o = .584	9.2×10^{-2}
	Block 3 [657-677] 12 data points	18.3	f_o = 30.205865704 Hz	3.9×10^{-8}
			$\gamma = 1.276708 \times 10^{-11}$ sec^{-1}	3.7×10^{-16}
			n = 5.11	3.3×10^{-1}
			ϕ_o = 3.26	2.5×10^{-1}
	Block 4 [684-709] 11 data points	12.4	f_o = 30.205864812 Hz	8×10^{-8}
			$\gamma = 1.276032 \times 10^{-11}$ sec^{-1}	5.6×10^{-16}
			n = .189	3.6×10^{-1}

*For models A through F these fits are for 51 data points, thus a good fit should have $\chi^2 \sim 50$.

†Since the χ^2 indicates a poor fit, these errors have little real significance.

FIGURE CAPTIONS

Fig. 1--Typical light curve for a 15-minute integration (a), 23 light curves averaged together (b), and a plot of the analytic function used to fit the main peak (c).

Fig. 2--Plots of the residuals obtained after fitting the data with the various models discussed in the text. The upper plot represents residuals for three models (A, B, and E) since for these models the plots are nearly identical. It should be noted that the residuals for models C and D are also similar to A. Residuals for model F clearly show the discontinuities in the data near MJD 40656 and 40680. The lower plot gives the residuals after breaking the data into four segments (model G). The dashed vertical lines denote the dates at which the data were divided.

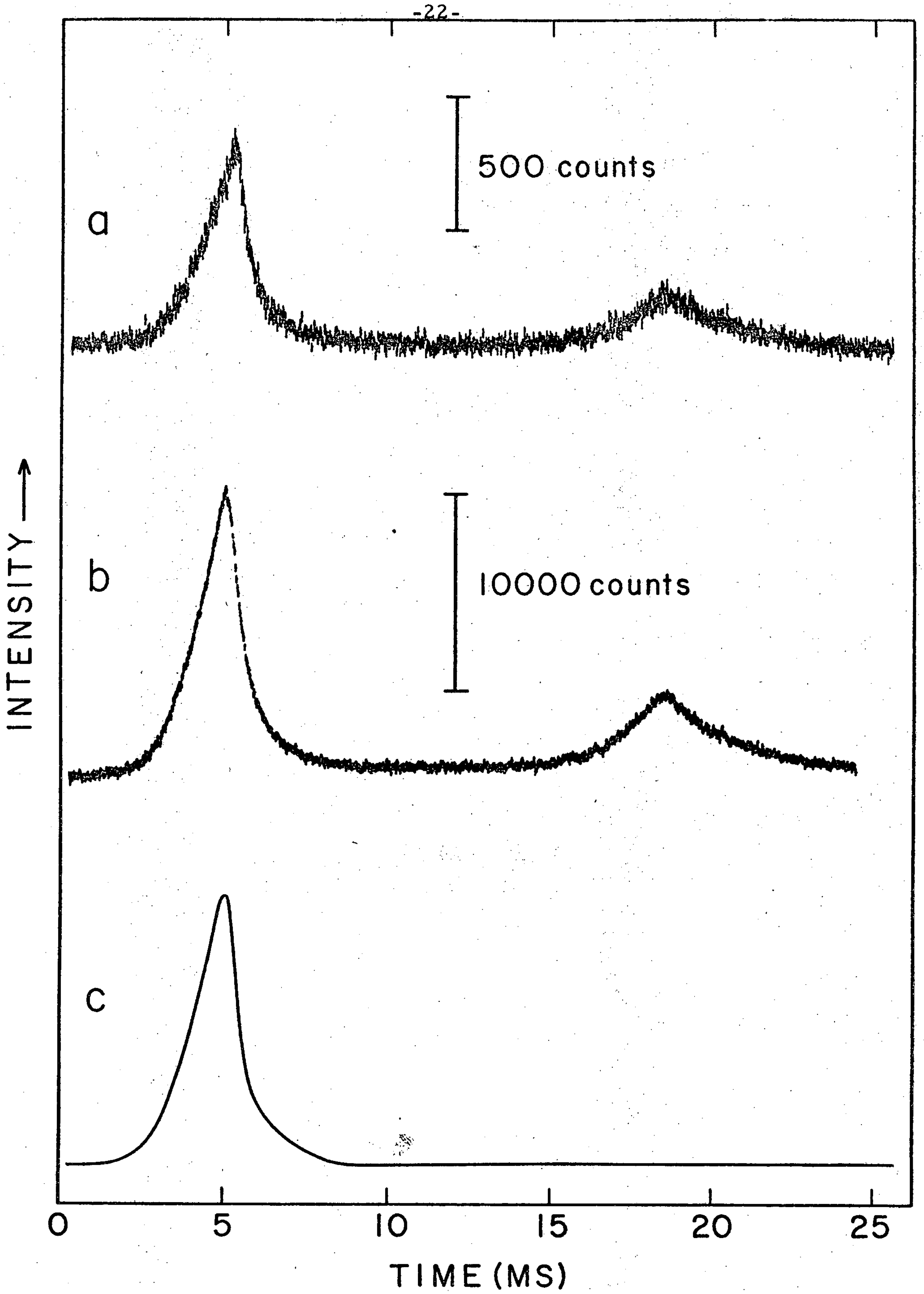

Fig. 1

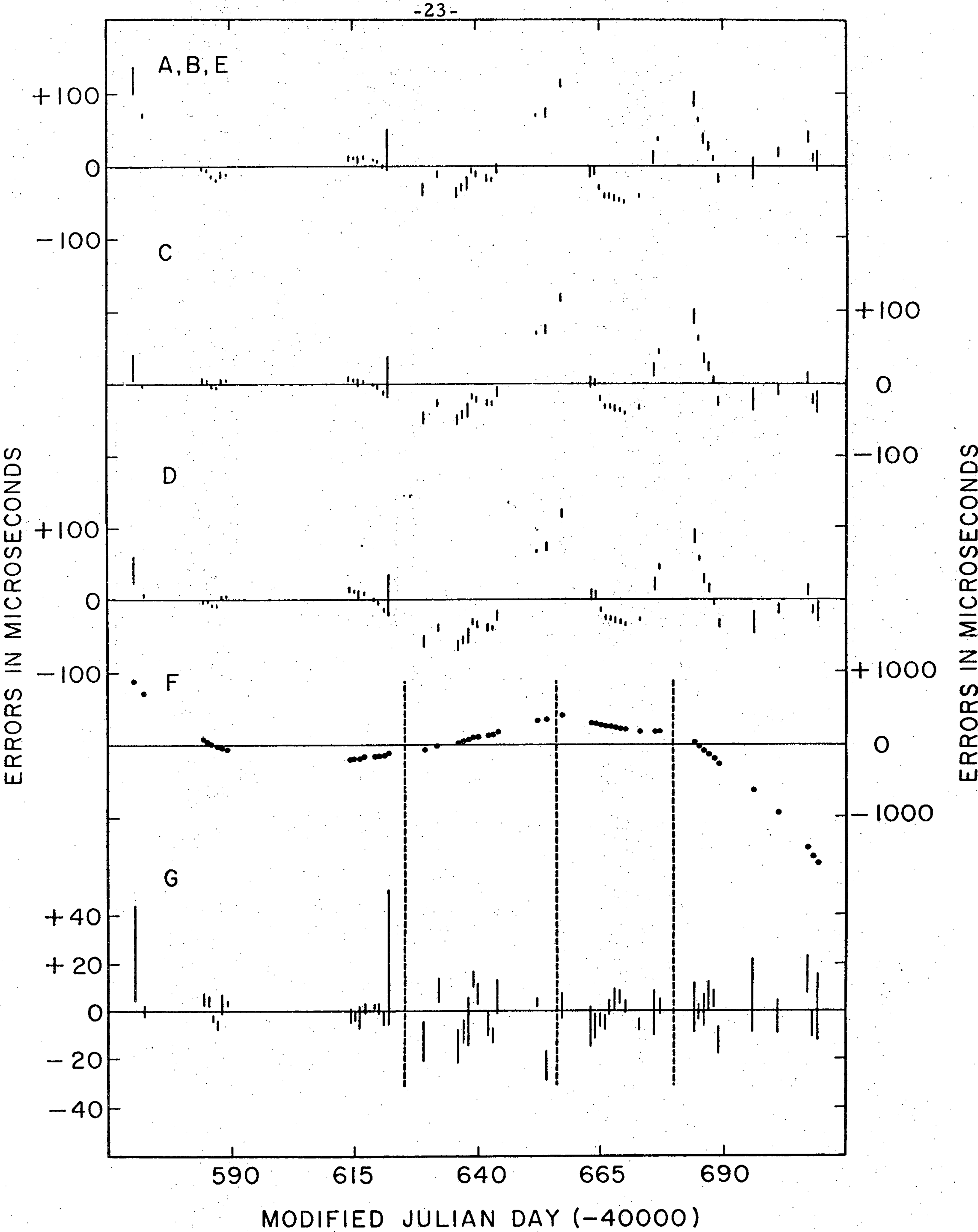
A, B, E
C
D
F
G
+100
0
-100
+40
+20
-20
-40
+1000
-1000
ERRORS IN MICROSECONDS
590
615
640
665
690
MODIFIED JULIAN DAY (-40000)

Fig. 2

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720