

UC Davis

UC Davis Electronic Theses and Dissertations

Title

Constraining Dark Photon Dark Matter With The Dark E-Field Radio Experiment

Permalink

<https://escholarship.org/uc/item/76t0n7kv>

Author

Levine, Joseph Michael

Publication Date

2025

Peer reviewed|Thesis/dissertation

Constraining Dark Photon Dark Matter With The Dark E-Field Radio Experiment

By

JOSEPH MICHAEL LEVINE
DISSERTATION

Submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Physics

in the

OFFICE OF GRADUATE STUDIES

of the

UNIVERSITY OF CALIFORNIA

DAVIS

Approved:

J. Anthony Tyson, Chair

S. Mani Tripathi

Brian Kolner

Committee in Charge

2025

Abstract

An abundance of astrophysical and cosmological data convincingly suggests that a large fraction of the Universe is comprised of cold matter, which has no known interaction with any of the forces of the standard model apart from gravity. This dark matter (DM) plays a central role in a wide range of phenomena that span cosmic time, from the anisotropies of the cosmic microwave background temperature to the formation of galaxies. Yet, despite its fundamental significance in our Universe and approximately 50 years of theoretical and experimental efforts, the source of this DM remains unknown.

Recently, many non-baryonic DM candidates have been proposed. One popular candidate, the weakly interacting massive particle (WIMP), was thought to be the perfect DM candidate for the past four decades. To date, an enormous WIMP-detection program has found nothing. Working under the guiding principle of “leave no stone unturned”, other, more exotic candidates are now being explored. To that end, new detector technology must be developed in order to probe the vast parameter spaces of many such DM candidates. This dissertation describes the progress of the Dark E-field Radio (DER) experiment in searching for one such candidate, the dark photon (DP). The DP is an extension of the Standard Model (SM) of particle physics. It is a gauge boson of a new $U(1)$, which could today have a relic density from the early Universe.

To explore the DP DM scenario, the DER experiment was designed to efficiently probe

a wide range of DP parameter space, leveraging a wide-bandwidth E-field antenna moved to multiple positions in a shielded room and a low noise amplifier. Following this, a real-time, wide-band spectrum analyzer serves as the data acquisition system. Borrowing from the field of Electromagnetic Compatibility (EMC), the concept of statistical uniformity in a reverberation chamber is introduced. Moving the antenna and averaging the spectra together averages over the resonances of the system, allowing for a much more stable calibration. The thermal noise emerging from the antenna results in a power spectrum that features prominent variations with frequency. The source of these variations is explored and tested.

Detailed system characterization was performed, including characterization of the antenna/room response, noise contributions (primarily due to the blackbody radiation of the room-temperature walls), shielding effectiveness of external radio frequency interference and various tests of the spectrum analyzer. To verify system performance, a small signal injection test was performed. The signal was detected after the predicted amount of averaging, confirming the effectiveness of the entire experiment — from the antenna/room system through data analysis.

So far, Run 1A of the DER experiment has been completed. It has been peer-reviewed and published (Levine et al. 2024). Many aspects of this data run and analysis are described herein, including data acquisition, reduction, storage and analysis, followed by the necessary calibration to process the null power spectrum into an exclusion limit on SM photon/DP coupling ε in the 0.2-1.2 μeV mass range (equivalent to the 50-300 MHz frequency range). A

95% exclusion limit on ε between 6×10^{-15} and 6×10^{-13} is reported, tracking the complex resonant mode structure in the shielded room. Run 1A extends the results of our 2021 pilot experiment (Godfrey et al. 2021), which was designed to demonstrate the feasibility of the DER technique. The pilot experiment was conducted over the same mass range as the experiment reported here, but did not use the calibration techniques to approximate statistical uniformity, nor did it fully account for the resonant enhancement of the cavity.

This detection technique can be applied at higher DP masses, with planned runs up to $60 \mu\text{eV}$. Statistical uniformity can be improved and automated using a mode stirrer, a common practice in the field of EMC. Additionally, a dual-conversion superheterodyne receiver has been designed and tested, greatly extending the maximum frequency of the spectrum analyzer. The estimated reach for these runs is described. New antennas and microwave electronics could enable sensitivity at an additional order of magnitude in mass beyond this. Additionally, smaller cryogenically cooled cavities and low noise amplifiers could improve sensitivity to ε by an order of magnitude. Considerations for designing these types of experiments are described.

This work was supervised by Professors J. Anthony Tyson, S. Mani Tripathi, and Brian H. Kolner.

Contents

Contents	v
List of Figures	viii
List of Tables	xiii
1 The Dark Matter Story	1
1.1 The History of Dark Matter	2
1.1.1 Early ideas in dark matter	2
1.1.2 Into the modern era	3
1.2 From “History” to “Current Events”	20
1.2.1 The modern picture of the Universe	20
1.2.2 The history of the Universe	27
1.3 Modern Evidence and Observations of Dark Matter	39
1.3.1 The cosmic microwave background	39
1.3.2 Gravitational lensing and mapping extragalactic dark matter	44
1.3.3 The bullet cluster and MOND	48
1.3.4 Dark matter galactic dynamics	49
1.4 MACHOs, WIMPs and WISPs, oh my!	51
1.4.1 WIMPs	54
1.4.2 WISPs	57
1.4.3 Dark photons	60
1.4.4 Dark photon physics	63
1.4.5 Searching for dark photon dark matter	67
2 Experiment Overview and Design	69
2.1 Sources of Power in Measured Spectrum	71
2.1.1 Thermal Noise	72
2.1.2 Dark Photon Signal	83
2.1.3 Radio frequency interference	85
2.1.4 Amplifier Chain noise	86
2.1.5 ADC Effects	91
2.2 Toy Analysis	92

2.2.1	Signal significance	95
2.2.2	Predicted time to detection	100
2.3	Thermal Noise in a Cavity: Thermal Variations	102
2.3.1	Theory of thermal radiation in a cavity	102
2.3.2	Inspection of thermal noise spectra	106
2.3.3	A simple resonator: shorted coax cable	107
2.3.4	A simpler resonator: shorted coax cable and circulator	114
2.3.5	A more complex resonator: antenna in room	116
2.3.6	Effective temperature of amplifier	118
2.3.7	Intentional breaking of thermal equilibrium	121
2.3.8	Relation of antenna S_{11} to thermal noise without a circulator	128
2.3.9	Discussion of thermal variations	131
2.4	Reverberation Chambers and Statistical Uniformity	132
2.4.1	Deterministic solutions to electromagnetic waves in cavities	134
2.4.2	Statistical approach to electromagnetic waves in cavities	139
2.5	System Design	140
2.5.1	Shielded room	143
2.5.2	Antenna	144
2.5.3	Terminator and fiber-optic switch control	148
2.5.4	Signal conditioning	148
2.5.5	Veto antenna	154
2.5.6	12 V power system	155
2.5.7	GPU-Based Real-Time Spectrum Analyzer	158
3	System Characterization and Data Acquisition System	162
3.1	Measurement of Amplifier Chain Performance	163
3.1.1	Y-factor method	163
3.1.2	Confirming gain with tracking generator measurement	171
3.1.3	System stability over a run period	172
3.2	Measurement of Shielding Effectiveness and Radio Frequency Interference	176
3.3	Spectrum Analyzer Characterization	182
3.3.1	Spectrum analyzer calibration	182
3.3.2	Spurious signal performance of the ADC	183
3.3.3	ADC clock performance	185
3.3.4	Real-time data collection efficiency	186
4	Data Acquisition, Data Analysis and Calibration	188
4.1	Data Acquisition	189
4.1.1	Raw data, S_o	191
4.2	Data Analysis	194
4.2.1	Fit background, $\hat{B}(\nu)$	196
4.2.2	Normalized spectrum, S_o^{norm}	198
4.2.3	Signal-matched filter	201

4.2.4	Monte Carlo: pseudo experiments	203
4.2.5	Rejection of a single candidate	207
4.3	Calibration	208
4.3.1	Average effective aperture, $\langle A_e(\nu) \rangle$	210
4.3.2	Simulation of $\langle \tilde{Q} A_e \rangle$	211
4.3.3	Correction and uncertainty of $\langle \tilde{Q} A_e \rangle$	214
4.4	Hardware Injection Test	217
4.4.1	Injection test prerequisites	218
4.4.2	Performing the injection test	223
4.4.3	Inspection of Data	228
5	Results	232
5.1	Discussion of Uncertainties	233
5.2	Exclusion Limit	233
6	Beyond 300 MHz	237
6.1	A First Attempt at Run 1B	240
6.1.1	Run 1B upgrades	240
6.1.2	Run 1B: data run and analysis	245
6.2	Mixer System: Run 2	250
6.2.1	Frequency mixing	250
6.2.2	Design requirements	256
6.2.3	The dual-conversion superheterodyne receiver	257
6.3	Possible Future Upgrades	272
6.3.1	Detector design principles to maximize the signal-to-noise ratio	272
6.4	Leveraging Axion Experiments for Dark Photon Searches	275
6.4.1	Reanalysis of axion haloscope data	275
6.4.2	Design and operation of axion experiments for dark photons	276
A	Overview of RTSA code base	277
A	Tips for Using the RTSA System	279
B	Data Structure and Processing	280
B.1	Writing data	280
B.2	Reading and averaging data	284
	Bibliography	286

List of Figures

1.1	Public commentary on the new theory of the expanding universe in the form of a Dutch newspaper comic, circa 1930	5
1.2	Hubble’s plot of distance vs. line-of-sight velocity for 24 objects	6
1.3	Image of M33 overlayed with a plot showing the expected and observed rotation velocities	14
1.4	Photograph of the Andromeda Nebula	15
1.5	Rotation velocity measurements of 67 individual regions of the Andromeda galaxy	16
1.6	Rotation curves of 25 galaxies, measured using 21 cm line radio observations . .	18
1.7	Modern compilation of galactic rotation curves	19
1.8	Evolution of the components of the universe as a function of the scale factor a .	27
1.9	Fermilab’s famous “history of everything” poster	29
1.10	Schramm plot showing the relative primordial abundances of the light elements compared to the baryon/photon ratio η predicted by standard BBN	35
1.11	Temperature map of the cosmic microwave background across the sky	38
1.12	Observed temperature power spectrum of the cosmic microwave background . .	41
1.13	Cosmic microwave background temperature spectra as cosmic parameters are varied	43
1.14	Cartoon depicting simple gravitational lensing around a massive compact object	45
1.15	Galaxy cluster CL0024+1654 acting as a gravitational lens and distorting a more distant galaxy	46
1.16	Three sequential images of supernova Refsdal showing the time delay of the received light that has been lensed through galaxy cluster MACS J1149+2223 . .	47
1.17	The bullet cluster. Two galaxy clusters colliding, shown in optical and X-ray, with contours showing mass density	48
1.18	Schematic diagram of the Milky Way	50
1.19	Estimates of local dark matter density $\rho_{\text{DM},\odot}$ from 1922 to 2021	51
1.20	Mass ranges for various dark matter candidates, experimental anomalies and search techniques	52
1.21	2023 exclusion limits on spin-independent (SI), WIMP-nucleon cross section vs. mass	55
1.22	2024 exclusion limits on spin-independent (SI), WIMP-nucleon cross section vs. mass	56
1.23	Feynman diagrams showing an axion decaying into two photons	60
1.24	Exclusion limits on axion-photon coupling $g_{a\gamma\gamma}$	61

1.25	Exclusion limits on dark photon - Standard Model photon coupling ε	68
2.1	Overview of the Dark E-field Radio experiment	70
2.2	Blackbody electric field spectral density from radio to UV frequencies	74
2.3	An antenna and matched resistor in cavities which are in thermal equilibrium.	77
2.4	Simulated antenna noise voltage at room temperature in the time domain	79
2.5	Simulated antenna noise power spectral density at room temperature in the frequency domain	80
2.6	Effect of averaging on simulated antenna noise power spectral density at room temperature in the frequency domain	82
2.7	Schematic of LNA adding input referred noise	86
2.8	Schematic of cascade of amplifiers and their added noise	89
2.9	Cascaded noise temperature for system with two amplifiers	90
2.10	Input-referred power spectral density S_i from simplified simulation	94
2.11	Noise spectrum containing sub-detection-threshold signal	98
2.12	Noise spectrum containing sub-detection-threshold signal, various number of averages	99
2.13	Predicted time to detection for the toy analysis spectra	101
2.14	Two cavities in thermal equilibrium of arbitrary shape, material and physical contents contain equivalent radiation fields	103
2.15	A reflective cavity with a small hole behaves like a blackbody	105
2.16	Output-referred Vivaldi antenna and terminator spectra as measured through Run 1A amplifier chain	107
2.17	Schematic diagram of a coax cable with a short on the end and a matched measurement device on the other	110
2.18	Power spectra of short and open terminations at the end of 1.85 m of RG400 coax cable	110
2.19	Schematic symbol of a circulator	111
2.20	Photo of Teledyne C-0S03A-3M RF circulator	112
2.21	3-port, frequency-dependent S-parameter data for Teledyne C-0S03A-3M circulator	113
2.22	Schematic diagram of a coax cable with a short on one end and a circulator on the other	115
2.23	Normalized power spectrum of 185 cm coax resonator as measured through circulator.	115
2.24	Schematic diagram of an experimental set-up using a circulator to isolate antenna from amplifier effects	117
2.25	Normalized power spectrum for Vivaldi antenna in the room with and without circulator	117
2.26	Schematic diagram of set-up to measure the noise emerging <i>out</i> of an amplifiers input	120
2.27	Normalized power spectrum of noise emerging <i>out</i> of the input of an amplifier's input	120
2.28	Setup to create hot and cold terminator	122

2.29	Spectra resulting from the antenna and circulator set up at a variety of temperatures above room temperature	124
2.30	Spectra resulting from the antenna and circulator set up with the terminator 210 K above and below room temperature	126
2.31	Sum of normalized spectra shown in Fig. 2.30	127
2.32	$1 - S_{11} ^2$ and the normalized antenna spectrum over plotted	129
2.33	Correlation of $1 - S_{11} ^2$ and the normalized antenna spectrum	130
2.34	Modal density for an electromagnetic cavity with dimensions of the shielded room	137
2.35	Pictures of conductor configurations in shielded room	138
2.36	Measured S_{11} with different conductors in shielded room	138
2.37	Schematic diagram of the RF receiver system	141
2.38	Photo of dark radio lab	142
2.39	AB-900A biconical antenna free space effective aperture, simulated, measured and theoretical	146
2.40	simulated AB-900A biconical antenna free space complex input impedance, vs measured biconical data	147
2.41	Measured gain (S_{21}) of the Run 1A band-pass filter	153
2.42	Schematic diagram of Run 1A Amplifier chain	154
2.43	Schematic diagram of Interlock board	156
2.44	Photo of lead-acid battery and slow turn on circuit	157
2.45	Real time DAQ data stream	160
3.1	Cartoon showing Y-factor data	164
3.2	Raw output power spectra used to compute the gain and noise temperature for the Run 1A amplifier chain using the Y-factor method	166
3.3	Y-factor data from the Run 1A amplifier chain at a single frequency (625 MHz)	167
3.4	Frequency dependent noise temperature of the Run 1A amplifier chain, measured using the Y-factor method	168
3.5	Frequency-dependent gain of the Run 1A amplifier chain, measured using the Y-factor method	169
3.6	Run 1A terminator, input-referred power spectral density	170
3.7	Gain vs. frequency of the Run 1A amplifier chain	171
3.8	Gain vs. LNA voltage of the Run 1A amplifier chain	173
3.9	Voltage at LNA vs. time with Run 1A setup	174
3.10	Frequency-averaged power spectral density vs. time of the terminator from Run 1A	175
3.11	Photo of setup to measure Shielding effectiveness of shielded room	178
3.12	Shielding effectiveness of shielded room, 50-300 MHz	179
3.13	Shielding effectiveness of shielded room, 125-3000 MHz	180
3.14	Spectrum from veto antenna during 300 MHz data run	181
3.15	Scan of terminated input of Teledyne spectrum analyzer system to measure spur performance	184
3.16	Acquisition efficiency for GPU-based Real-time spectrum analyzer	187

4.1	Run 1A averaged, output-referred antenna power spectrum S_o	192
4.2	Logarithmic scaled Run 1A averaged, output-referred, antenna and terminator power spectra.	193
4.3	Flow chart outlining the logic of signal processing in the detection algorithm of sections 4.2.1 through 4.2.3.	195
4.4	Fitting background $\hat{B}$ in the presence of a synthetic signal.	198
4.5	Normalized, output-referred power spectrum S_o^{norm}	200
4.6	S_o^{norm} after it has been passed through a matched filter.	202
4.7	Result of Monte Carlo pseudo experiments on signal detection.	205
4.8	Limit on output-referred total integrated signal power, P_o^{lim}	207
4.9	Screenshot of COMSOL simulation GUI for two-antenna validation	213
4.10	Simulated vs measured $\langle S_{21} \rangle$	214
4.11	Corrected average effective aperture.	216
4.12	Schematic of hardware injection test	219
4.13	Average S-parameters of hardware injection test	220
4.14	Histogram of frequencies used for hardware injection test	223
4.15	Output-referred power spectral density from the hardware injection test	225
4.16	Standard deviation of normalized, output-referred power spectral density from the hardware injection test	226
4.17	Standard deviation of normalized, output-referred power spectral density from the hardware injection test, computed with median absolute deviation	227
4.18	Output-referred power spectrum from hardware injection test, various levels of zoom	229
4.19	Normalized, output-referred power spectrum from hardware injection test, various levels of zoom	230
4.20	Matched filtered, output-referred power spectrum from hardware injection test, various levels of zoom	231
5.1	Zoom of 95 % exclusion limit on ε from Dark E-field Radio Run 1A	235
5.2	Dark photon limits of various experiments with this work shown in red	236
6.1	Modified Com-Power antenna stand holding Vivaldi antenna for Run 1B	242
6.2	Closeup picture of modified Com-Power antenna stand holding the Vivaldi antenna for Run 1B	243
6.3	Current state of mode stirrer construction	245
6.4	Run 1B averaged, output-referred antenna power spectrum S_o	247
6.5	Run 1B S_o^{norm} after it has been passed through a matched filter.	248
6.6	9-day averaged veto spectrum and Run 1B S_o^{norm} after it has been passed through a matched filter	249
6.7	Overview of basic mixer operation in both upconversion and downconversion mode	252
6.8	Illustration of the problem of images in frequency mixing	253
6.9	Schematic of a basic superheterodyne receiver system	254
6.10	Power spectra along various points of a superheterodyne receiver	255

6.11	Block diagram of the mixer system to be used in Run 2	257
6.12	Schematic of simplified dual conversion superheterodyne receiver	259
6.13	Cartoon illustrating the first mixer stage of dual conversion superheterodyne receiver system	260
6.14	Simulated frequency response of dual conversion superheterodyne receiver with signals around 1 GHz	261
6.15	Simulated frequency response of dual conversion superheterodyne receiver with signals around 8 GHz	262
6.16	Schematic of Run 2A dual conversion superheterodyne receiver	264
6.17	Picture of dual-conversion mixer system	266
6.18	Spur web, calculated using Marki Microwave's online calculator	270
A.1	Real-time heads-up window for data acquisition.	283

List of Tables

1.1	Thermal history of the universe in the standard cosmological model	28
2.1	Statistics of simple Monte Carlo simulation for probability of signal detection in the toy analysis	100
2.2	Direction, coordinate, and length measurements of the shielded room in lab 314	144
2.3	Specifications for the Pasternack PE15A1012-E low noise amplifier	149
2.4	Specifications for the Mini-Circuits ZKL-1R5+ as measured with a supply voltage of 9.05 V	150
2.5	Specifications for the custom, real-time spectrum analyzer used for Run 1A. . .	159
4.1	Run 1A details	190
4.2	Threshold parameters that are part of the detection algorithm and Monte Carlo	204
6.1	Overview of planned future runs	239
6.2	Run 1B details	246
6.3	Dual Conversion Mixer System Components	265
A.1	Example database file	281

Acknowledgments

There is a seemingly unending list of people who I need to thank for helping me reach this point. I have received so much support personally and professionally from so many people that it is inevitable I will leave people off. This acknowledgment should be read in the style of Sublime’s Thanx, with an instrumental dub playing in the background.

Ben, I would not have stuck with this experiment if it weren’t for you and your guidance. I miss our late nights, manic-depressive cycles of accidental discoveries and naming new techniques after ourselves. Thanks for handing me a working prototype of the experiment and all your work getting me up to speed running it. Thanks for showing me how to bang your head on a problem for a year even when everyone is telling you to stop wasting your time. My thermal-wiggles was your antenna-resistor, and I wouldn’t have seen that through without your inspiration. Thanks for being the first to proof read this and catch the first round of errors. I truly wouldn’t be here without you.

Tony, your belief in me and my abilities, even when they didn’t yet exist, has led to my growth in more ways than I can count. I don’t know of any other PhD student to receive “total academic freedom”. I would not be the researcher I am if you didn’t give me such a long leash to figure it out for myself, while offering your intuition and wisdom when requested. It has been a pleasure working with you.

Mani, you are a black-belt of political jujitsu. Your timeline for my PhD milestones was my saving grace. I could always count on our conversations at G street to resolve whatever issue I was facing at the time. Thanks for going to bat for me.

The rest of the dark radio group, thanks for your support. Norris, when you were interviewing schools, I gave you the run-down that you would have to learn the entire experiment in six months. You exceeded expectations, and you will do great! Paul, thanks for access to your museum of test equipment. Brian, your proofreading skills significantly improved my writing (though I'm sorry I left the section titled "toy analysis").

To my wonderful Amanda, thank you for always listening and supporting me through the good and bad times. Your perspective and positive outlook have made this whole process so much fun. I love you, and I'm looking forward to many more years together.

I was looking forward to grad school because so many people told me I would meet such cool people. To my cohort, you guys did not disappoint. Lena and Umut, it was solidified pretty quickly over Discord that we would be best of friends. Thanks for all the fun times. Looking forward to living it up in the nursing home. I also have to thank Umut for all the help with QFT over the years and all the help with Ch. 1. Gabe, I don't know how one person can be so good at so many things and still be patient enough to explain it three times. Thank you for all the brainstorming and thoughts you put into this experiment. Kelsey, Sunny, Kevin, Rashad, Rahul, Adam, Davis, Chris, Griffin, Bobby and Megan, it was a fun ride!

Thanks to my lifelong friends, Esias, Chris, Stine and Cameron. Thanks to my bands for giving me an outlet, and especially Stephen, Stan, Isaac and Kent. A big thank you to my COVID-pod of 2021 and house band: Kevin, Paige and Elsie. I wouldn't have made it through that first year without you!

There have been so many staff and faculty at UCD that either taught me or helped my

PhD experience run smoothly. Emilija, thanks for cheering me on from the sidelines and telling it like it is. My postdoc apps and talks would not have been as strong if it hadn't been for you. Thanks for all your support over the last year. Dave Hemmer, your oversaw my very first project as a grad student in the machine shop. It has been a lot of fun learning my way around the machine shop over the years. Nathan Domek, thanks for all the help with IT. Monda, I'm sorry I placed so many orders and then bugged you for tracking info, but thank you for processing them! Bill Tuck, you are indeed a wizard. I don't know how you consistently anticipate the three follow-up questions and answer them before I ask. Thanks to Rena Zeve for all of her work on the REU program, which got me here in the first place.

While I owe so much to everyone at CSU Chico, I have to single out Louis Buchholtz. There are once-in-a-lifetime teachers and once-in-a-lifetime people, and you are both. I can not begin to thank you for the hundreds of hours you invested in me. From tube men to twangers, matrices to politics, you were there for it all. You masterfully tutored me in every upper-division physics class, and I still use those notes regularly. Thank you for showing me the magic of physics.

Eric Ayars and Nick Nelson, you two taught me everything I knew about electronics, experimental methods and scientific computing. I wouldn't have made it without you. Tube Man Forever. To Rick Danner for teaching me basically of my undergrad math classes. Dave Kagan, WOW your quantum class was hard! The only thing harder was grading that much. I don't know how you did it. Thanks for trusting us to lead our own class. Rest in peace. Thanks to Eric Dietz for making E&M intuitive over three semesters. A big thank you to Joe and Hywan Pechkis for giving me my first shot at research. Edward Roualdes, you taught

me how to do a statistical bootstrap, which I have spent the past 5 years bastardizing and ultimately became the foundation of the analysis pipeline of this experiment. Anna Petrova-Mayor, thanks for letting me play with lasers. To my classmates, Kirk, Steven, April, Kyle, Forest, Faisal, and Daniel. And of course, our scout mother, Clare!

I had many great primary education teachers, but four really stood out in laying the foundation for my academic achievements. Matthew Keye, you made physics fun for 17-year-olds. That is nothing short of a superpower. Donna Williams, thank you for hanging out after school for homework hour. It made all the difference. Thanks to Mark Agüero and Phil Theodorou for giving us credit for starting a band in the practice room.

I owe my early passion for science to Bill Nye the science guy, everyone who worked on that show, the NSF for funding it, and PBS for showing it.

And, of course, I wouldn't have watched Bill Nye (or exist. . .) if it weren't for my parents. I wouldn't be where I am today if you two didn't inspire my sense of curiosity as a child. Thanks for helping 3-year-old me troubleshoot my "ice cream-making machines" (upside down trike touching various sticks throughout the house and yard). A special thanks to my mom for checking all the references in this dissertation.

Thanks to Ross, Baba, Papa Stan, Papa Shelly, Nana and Grandma Helen for their support and love through the years. Thanks to Janet for tutoring me in middle school math, as described in her poem "From Ds to PhDs". Thank you to Joel for teaching me to enjoy the process and not fixate on the goal. And to my extended family: My big sis Maishia and granny Belva. Your enthusiasm for reading this work is unmatched. It did a lot to keep me going!

A note on organization of associated code and software

In preparing this work, I spent a lot of time in front of computers. That effort produced lots of Python notebooks, analysis routines, Arduino sketches, COMSOL simulations, data acquisition routines, hardware control scripts, etc. These are all located in a directory called `darkRadio`, which I put under `git` control during my first year of graduate school. It is currently hosted at <https://github.com/josephmlev/darkRadio>. Where appropriate throughout this work, I have included links to the relevant code used to generate various figures. They appear in footnotes, usually in the section title.

While easy to navigate, `github` is a private company and not suitable for archival purposes. For this reason, I have uploaded the `darkRadio` directory in `.zip` format to `zenodo`. It is available at <https://zenodo.org/records/15066400>. The `github` links show the path, so after downloading the `.zip`, it should be simple to find the relevant location.

Chapter 1

The Dark Matter Story

The real dark matter was the friends we made
along the way.

Dylan Smith

There are hundreds of dissertations written on dark matter (DM) every year, not to mention the thorough reviews that are written every few years. The goal of this chapter is not to address every possible aspect of such a wide topic but rather to set the stage for the experiment outlined in the following chapters by answering the following questions:

- Why do we believe there to be DM in the first place? (Sec. 1.1 –Sec. 1.3)
- What could it be? What have we looked for so far? (Sec. 1.4)

With that in mind, in this first chapter I have opted to prioritize narrative over completeness. There will be a few digressions along the way. However, the goal of this chapter

is simply to motivate what brought society to the point of paying young scientists to put antennas in metal boxes and measure noise.

Section 1.1 begins chronologically, giving an overview of the history leading up to the general acceptance that there is some dark matter governing galactic dynamics. Section 1.2 describes the advances of the field in the era of “current events”, i.e. after the 1970s. A digression into cosmology in Sec. 1.2.1 helps set the stage for Sec. 1.3 which covers evidence that DM is a non-baryonic particle, beyond what is described by the Standard Model of particle physics. Section 1.4 covers a few of these candidates.

The structure and content of this section were greatly inspired by several dissertations [1–3] (also Tyler Erjavec’s dissertation which is in preparation), review articles [4–8] plus Sec. 27 of [9], and textbooks [10, 11]. I am grateful to these authors for their clear and concise summaries of the field.

1.1 The History of Dark Matter

1.1.1 Early ideas in dark matter

The fundamental idea that things exist that can not be easily observed by human senses is not a new one. Following a long history of discovery of previously unobserved phenomena, the study of dark matter (DM) began to take shape around the end of the 19th century with the discovery of dark regions among areas with a high density of stars [12]. It was Lord Kelvin who began a dynamical study of the motions of stars in order to tease out the weight

of the luminiferous aether (who’s “existence is a fact that can not be questioned” as stated in his 1901 lecture which is transcribed on page 260 of [13]). Eventually, the theory of the aether would give way to special relativity, which ruled out a potential candidate for the dark regions. This cycle of proposing DM candidates and their rejection continues to this day, and is the topic of this dissertation.

1.1.2 Into the modern era

1905, known as Einstein’s miracle year, ushered in a new era of “modern physics”. The aether that Lord Kelvin was trying to weigh was accepted as non-existent, but that didn’t answer questions regarding galactic dynamics, which were posed by those who believed in it. Fritz Zwicky is credited with the first “discovery” of DM in 1933 [14], though it was not widely accepted at that time. This lack of acceptance was in part due to his technique of *morphological analysis*, which is similar to arranging refrigerator magnets to arrive at creative solutions¹. The many “creative” solutions are simultaneously to the benefit and detriment of this technique. To quote Stephen Maurer [15],

When researchers talk about neutron stars, dark matter, and gravitational lenses, they all start the same way: “Zwicky noticed this problem in the 1930s. Back then, nobody listened...”.

It even seems as if Zwicky himself didn’t believe his own result, writing [14]²,

¹This analogy is from a class I took from Andrew Wetzel at U.C. Davis. In researching it for this dissertation, it is surprisingly accurate

²This quote has a few different translations from the German it was actually written in.

If this [experimental result] would be confirmed we would get the surprising result that dark matter is present in much greater amount than luminous matter.

The next subsection will detail Zwicky’s work, which led to this conclusion.

1.1.2.1 Dispersion velocity of galaxies within a cluster

Zwicky’s insight came from analyzing data collected by Hubble and Humason, published as *The Velocity-Distance Relation Among Extra-Galactic Nebulae* in 1931 [16]. Since this work represents such an important milestone (namely, that the universe is expanding) in science, a brief discussion is needed.

Hubble et al. and the expanding universe As is often the case in science, when many people are working on the same problem, credit is generally given only to one person for simplicity. In the case of the expanding universe, Hubble usually gets the credit. For a more detailed history of the discovery of the expanding universe, see Ch. 5 of [17]. Figure 1.1 is a comic from a Dutch newspaper providing some commentary from the public on this new idea circa 1930.

Hubble’s 1931 data were collected following his seminal paper *A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae*, published in 1929 [18]. In this first work, Hubble published his famous result that the universe is expanding.

In this paper, he lists 24 objects, which he calls Nebulae³, but they are now understood

³Hubble was working with a definition similar to the following (From Encyclopedia Britannica): “The term [Nebula] was formerly applied to any object outside the solar system that had a diffuse appearance rather than a pointlike image, as in the case of a star”.

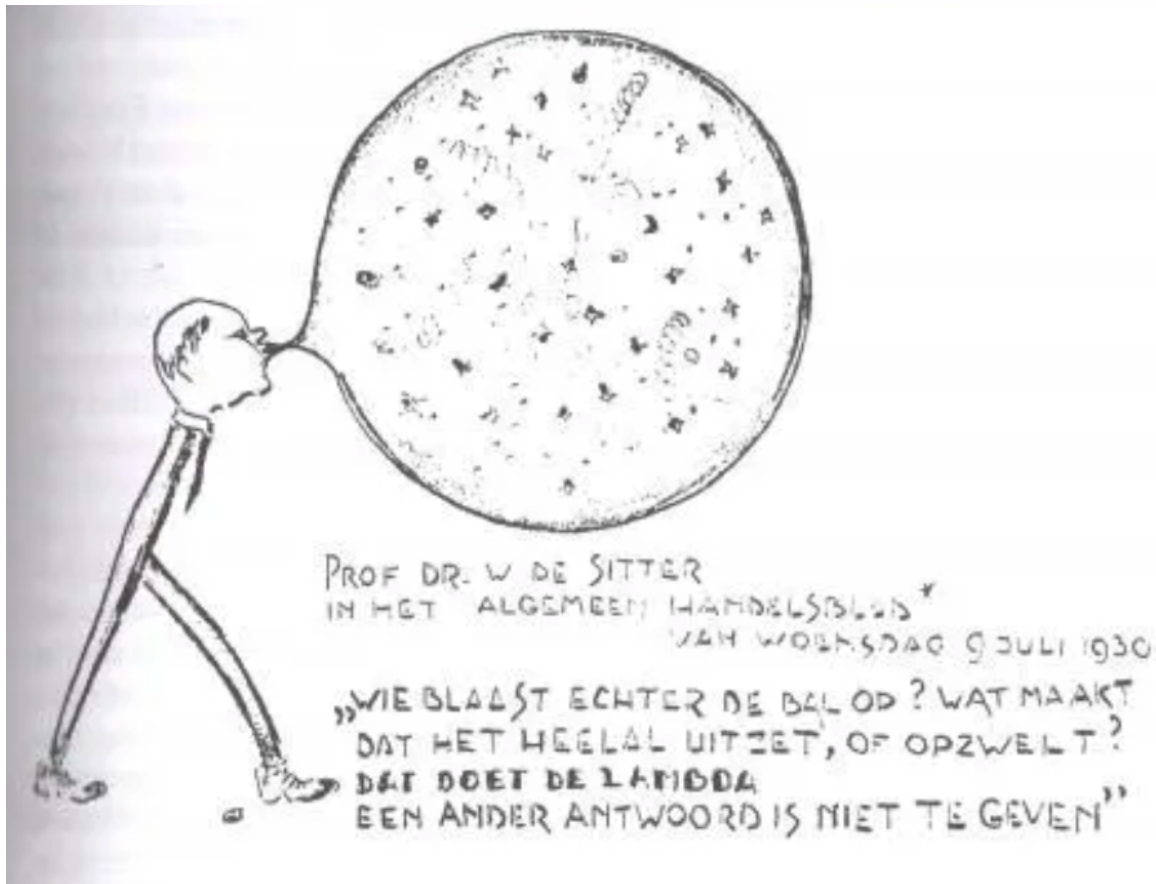


Figure 1.1: Public commentary on the new theory of the expanding universe in the form of a Dutch newspaper comic, circa 1930. The quote is translated as “What, however, blows up the ball? What makes the universe expand or swell up? That is done by the Λ . Another answer can not be given.” The comic was found in the archives of the Leiden Observatory and provided by H. van der Laan to P. J. E. Peebles, where he published it in his *Principles of Physical Cosmology* [17].

to be galaxies. For each of these galaxies, Hubble lists the radial⁴ velocity and distance for each of these objects. From these data, he produced the plot that is reproduced here in Fig. 1.2.

Figure 1.2 suggests a linear fit with a constant of proportionality, which would come to be

⁴Today we would call this the “line-of-sight” velocity, i.e. velocity calculated by the redshift.

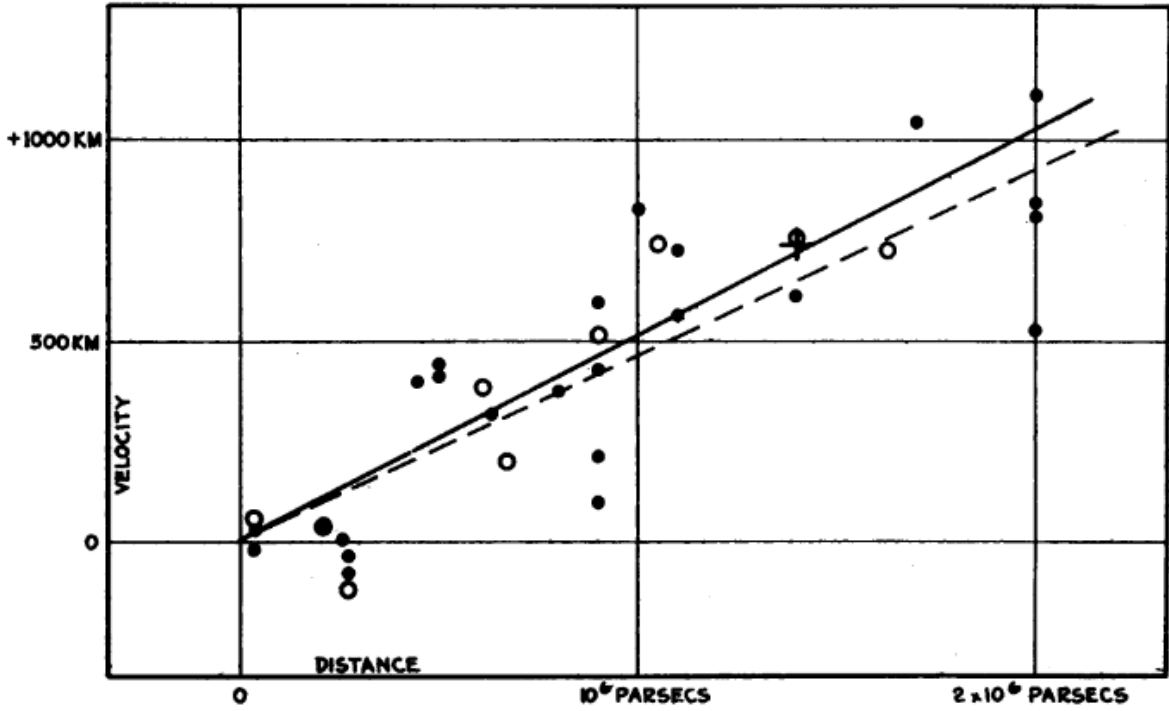


Figure 1.2: Hubble's plot of distance vs. line-of-sight velocity for 24 objects (black dots), reproduced from [18]. Note that the Y-axis should have dimensions of km/s. The line of best fit to these dots is shown as the black line. Also shown are 9 white dots, which show the 24 objects compressed into smaller groups according to similar velocity and distance. The best fit for the groups is shown as a dashed line. The slope of this plot gives the Hubble Constant H_0 , where the 0 subscript indicates the present value, i.e. at a redshift of 0. The fit here gives $H_0 \approx 500 \text{ km/s/Mpc}$, close to a factor of 10 higher than the modern accepted value. A plot of this type is now known as a Hubble plot.

known as the Hubble Constant H_0 . The simple equation which seems to govern line-of-sight velocities to galaxies is

$$v = H_0 d. \quad (1.1)$$

Equation 1.1 is known as the Hubble-Lemaître law⁵. A positive value of H_0 implies that the further an object is, the faster it is receding. In other words, the Universe is expanding! Hubble found H_0 to be 500 km/s/Mpc, significantly higher than the modern accepted value (either 67.27 ± 0.6 or 73.04 ± 1.04 km/s/Mpc. See Sec. 8.4.1 of [4] for a discussion on the so-called Hubble tension).

An expanding universe implies a beginning at some point in the past where the Universe was all at a singular point. In the modern view, this singularity is thought to have been the start of the universe and is known as the Big Bang. There are a few interesting quantities that can be pulled out of $H_0 \sim [1/\text{time}]$ through dimensional analysis.

- **Time: $1/H_0$** The most obvious manipulation of an inverse time is to take $1/H_0$, the Hubble time. Using a modern accepted value of $H_0 = 67.8$ km/s/Mpc, the Hubble time is about 14.5 Gyr. Assuming a constant rate of expansion, one Hubble time in the past, all objects would have 0 distance between each other. This would be the time since the Big Bang (i.e. the age of the Universe), but due to a non-constant expansion, the accepted value for the age of the universe is currently 13.8 Gyr.
- **Length: c/H_0** By incorporating the speed of light c we find the Hubble length to be c/H_0 . This is the distance at which an object is currently receding at the speed of light. Its value is about 4.5 Gpc or 14.5 billion light years. Note that the observable universe is significantly larger than this (~ 30 Gpc) because the furthest light we would be able to see started traveling when the universe was much smaller.

⁵It was called the Hubble law, until the International Astronomical Union (IAU) voted to recommend the change.

Zwicky and the virial theorem Following Hubble and Humason’s 1933 paper, Zwicky noticed an interesting anomaly in the data. Seven of the galaxies are in the Coma cluster and have a significant variation in velocity about the that predicted by Eq. 1.1. This is called the dispersion velocity. While the whole cluster is receding, the individual galaxies are flying around relative to their center of mass. Qualitatively, one would expect a cluster’s dispersion velocity and total mass to correlate; if a cluster didn’t contain much mass, a fast-moving galaxy would possess enough kinetic energy to escape the gravitation of the cluster⁶.

This intuition is quantified by the virial theorem (for a derivation, see Chapter 3 of Goldstein [19]). It was formalized 1870 by Rudolf Clausius (English translation can be found here [20]). The name *virial* was coined by Clausius. Kinetic energy was, at the time, referred to as *vis viva*, latin for “living force”. The plural of vis is virias, and since the theorem is concerned with many particles, each with their own *vis vita* the name *virial* theorem was chosen.

The theorem is derived briefly here (following chapter 3 of Goldstein[19]) since it is informative. However, the reader can skip to Eq. 1.7 for the result as it applies to galactic dynamics.

The “virial” of the system is defined as

$$G \equiv \sum_{i=1}^N \mathbf{r}_i \cdot \mathbf{p}_i \quad (1.2)$$

Where $\mathbf{r}_i$ is the position of the i th particle and $\mathbf{p}_i$ is it’s momentum, $m_i \mathbf{v}_i$.

⁶The hidden assumptions here are that the galaxy is bound to the cluster and that the cluster has had time to reach equilibrium. Both of these assumptions came under scrutiny. See [5] for a discussion.

By the product rule, the time derivative of G is

$$\dot{G} = \sum_{i=1}^N (\dot{\mathbf{r}}_i \cdot \mathbf{p}_i + \mathbf{r}_i \cdot \dot{\mathbf{p}}_i). \quad (1.3)$$

Since the first term is really $m_i \mathbf{v}_i \cdot \mathbf{v}_i$ it can be seen as twice the kinetic energy, $2T$. The second term can be simplified by Newton's second law, $\mathbf{F} = \dot{\mathbf{p}}$. So,

$$\dot{G} = 2T + \sum_{i=1}^N (\mathbf{r}_i \cdot \mathbf{F}_i). \quad (1.4)$$

Solving for the time-averaged, time-derivative of G ,

$$\begin{aligned} \bar{\dot{G}} &= \frac{1}{\tau} \int_0^\tau dt \dot{G} \\ &= \frac{G(\tau) - G(0)}{\tau} \end{aligned}$$

Where τ is the period the average is taken over.

In the case that the system is bound together (i.e. $\mathbf{r}_i, \mathbf{p}_i < \infty$), G is finite and as τ approaches ∞ , $\bar{\dot{G}}$ approaches 0

Returning to Eq. 1.3, we can now say

$$\bar{T} = -\frac{1}{2} \overline{\sum_{i=1}^N (\mathbf{r}_i \cdot \mathbf{F}_i)}, \quad (1.5)$$

A familiar statement of the virial theorem, where the term on the right hand side is known as *the virial of Clausius*.

While incredibly general (one can pull the ideal gas law out of this in just a few steps, see again Ch. 3 of Goldstein [19]), we are concerned not with a jar filled with gas in some lab, but with a “gas” of galaxies (among which all labs are contained!).

To specify the equation to that of galactic dynamics, recognize for a conservative central force, $\mathbf{F} = -\nabla U$ and $\mathbf{r} \cdot \mathbf{F} = rF$. If U is in a power law, i.e. of the form $k r^{n+1}$, it can quickly be seen that the virial of Clausius of Eq. 1.5 can be written $\frac{n+1}{2}\overline{U}$.

For a problem involving an inverse square force as we are concerned with, $n = -2$, and we arrive at the well-known result, which is usually just called “the virial theorem”, though as we have seen, it reaches much deeper than a simple statement of kinetic vs. potential energies for galaxies:

$$\overline{T} = -\frac{1}{2}\overline{U}. \quad (1.6)$$

Finally, following Edington’s 1916 paper [21] we can form a useful formula which ultimately gave a clue to the existence of DM. By setting $T = 1/2 M \overline{v^2}$ and $U = GM^2/2R$ where M is the total mass of a cluster or galaxy, v is it’s velocity, and R is it’s radius, we arrive at

$$M \approx \frac{2 R \overline{v^2}}{G}. \quad (1.7)$$

While at first glance, this appears nearly identical to the solution of circular orbits, it is important to recognize this is a *statistical* statement. There is a rough analogy here; Eq. 1.7 is to simple circular orbits as any statistical, quantum mechanical observable is to its classical analog, à la Ehrenfest’s theorem.

Using this result⁷, Zwicky attempted to weigh the Coma cluster. Assuming, as Zwicky did [14], the Coma cluster has a radius of 10^6 light years, contains 800 nebulae, each with

⁷Zwicky actually assumed a sphere of uniform density instead of a clustering of particles, which results in the prefactor of 5/6 in place of the 2 on the right-hand side of Eq. 1.7.

$10^9 M_\odot$ and a mass to light ratio $M_\odot/L_\odot = 1$, he computed an expected velocity dispersion of 80 km/s, a factor of nearly 400 less than the observed 1000 km/s.

Zwicky would return to the Coma cluster in 1937 [22] where he would update some of his assumptions. Debates about this missing mass problem continued, as detailed in [5]. An interesting argument in favor of the existence of DM is known as the timing argument. It was derived by Kahn and Woltjer in 1959 [23]. The basic idea is that given the negative red-shift (i.e. blue-shift) of the Andromeda galaxy, it and our Milky Way are approaching (at 125 km/s), an indication that they are bound system. Given that the period of this system must be less than the age of the universe (assumed to be 10^{10} yr), a lower bound can be set on the reduced mass of the system (since orbital period T is inversely proportional to the reduced mass). This value was a factor of 6 lower than the accepted reduced mass of the system.

In the years following the arguments described above, WWII brought unprecedented destruction, the reconstruction of which slowed the progress of all science that was not essential to the war effort, especially in Europe. The focus of astronomy and astrophysics largely shifted to stellar structure and evolution, largely due to the work and understanding of nuclear reactions around this time. The war also brought with it advances in radar technology. At the end of the war, the German occupation forces left large amounts of radar equipment strewn about Europe. This included a large number of 7.5 m Würzburg antennas designed to 54 cm aircraft radar, but were also sensitive to the famous Hydrogen 21 cm line [24]. This would play a major role in what was to come.

1.1.2.2 Galactic rotation curves

The next chapter in the story utilizes data on much smaller length scales than the clusters of the previous section. Clusters are on the order of several Mpc, while individual galaxies are closer to several tens of kpc. Similar to the cluster dynamics outlined in the previous section, galactic rotation curves rely on measurements of the dynamics of galaxies, in this case, their rotation. By measuring the redshift of the atomic lines of different elements across the radius of an edge on galaxy, the rotation velocity can be calculated. A brief toy calculation of the expected velocity profile is in order. For a more rigorous treatment, see Ch. 2 of Galactic Dynamics by Binney and Tremaine [25]. This reference includes several realistic mass distributions (disks!) and, importantly, computes gravitational potentials, which are far more useful for solving real problems in galactic dynamics. Nevertheless, this toy calculation will lay a foundation for the remainder of this section.

From introductory physics, equating the centripetal and gravitational forces yields a particle's velocity in a simple circular orbit around a mass M ,

$$v = \sqrt{\frac{GM}{r}}. \quad (1.8)$$

Newton has two useful theorems which simplify the problem in the case of spherically symmetric mass distributions. First, a body *inside* a spherical shell of mass experiences no net gravitational force due to this shell. Second, a body *outside* a spherical shell of mass experiences a net gravitational force that is equivalent to that of a point mass at the center of the shell with the same mass as the shell. These are informally referred to as the shell

theorems.

Using the first shell theorem means we are only interested in mass contained *within* a sphere of radius equal to the orbit of the test mass (again, assuming a spherically symmetric mass distribution). Any mass outside of this sphere can be decomposed into shells, and shells exert no net force on points inside of them. By the second shell theorem, all mass contained within this sphere acts as a point mass at the center. The mass M contained within a sphere of radius r for a spherically symmetric mass distribution of density $\rho(r)$ is given by

$$M(r) = 4\pi \int_0^r \rho(r') r'^2 dr'. \quad (1.9)$$

Now, if luminous stars contributed all of the mass to a given galaxy, the measurement of the surface brightness I (flux per area) would be a good indication of the mass distribution. Surface brightness profiles can be modeled by the Sérsic profile [10, 26],

$$I(r) = I_e \exp \left\{ -b(n) \left[\left(\frac{r}{R_e} \right)^{1/n} - 1 \right] \right\}, \quad (1.10)$$

where R_e is the half-light radius (within which 50% of the total flux is contained, usually on the order of 1-5 kpc for spiral galaxies), I_e is the surface brightness at this radius, n is the Sérsic index and $b(n) = 2n - 1/3 + 4/(405n)$. For spiral galaxies, using $n = 1$ generally provides a good fit, leading to the exponential profile,

$$I(r) \propto I_e e^{-r/R_e}, \quad (1.11)$$

Working under the assumption that $I(r)$ and $\rho(r)$ are proportional, one would expect at a large distance from the center ($r \gg R_e$), nearly all of the mass of the galaxy would be interior to a sphere of radius r , and $M(r)$ would reach an asymptotic value. In other words, the orbital velocity predicted by Eq. 1.8 is expected to fall off like $r^{-1/2}$.

Observations, however, did not agree with this prediction. A cartoon overlaying a photo of the spiral galaxy M33 with the predicted and observed rotation curves is shown in Fig. 1.3. The original data that Fig. 1.3 is based on are reproduced from Vera Rubin and Kent Ford's seminal 1970 paper [27] in Figs. 1.4 and 1.5.

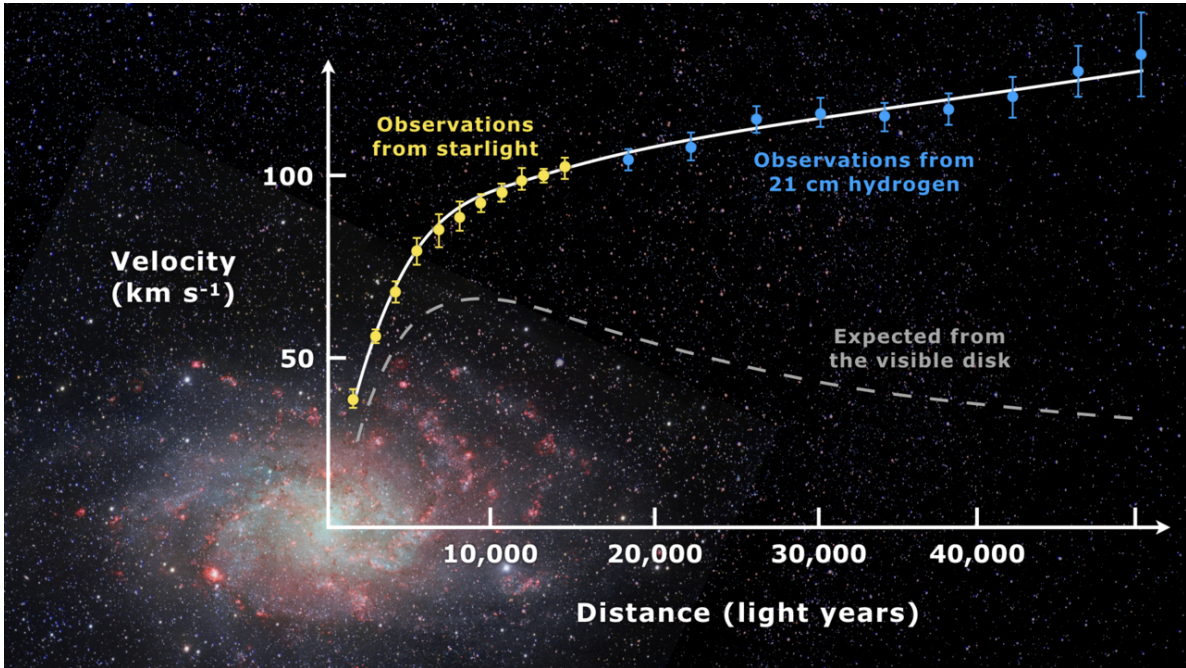


Figure 1.3: Image of M33 overlayed with a plot showing the expected and observed rotation velocities. At large radii, the rotation curve is predicted to fall off like r^{-1} . Radio telescopes allowed for the measurement of the 21 cm hydrogen line (blue points) to much greater distances than optical measurements of luminous stars (yellow points). Observations of both indicate a discrepancy between theory and observation, which can be explained by non-luminous DM. Note that 10,000 light years $\sim$ 3 kpc, so the horizontal scale extends to roughly 15 kpc. Figure reproduced from Wikipedia [28]

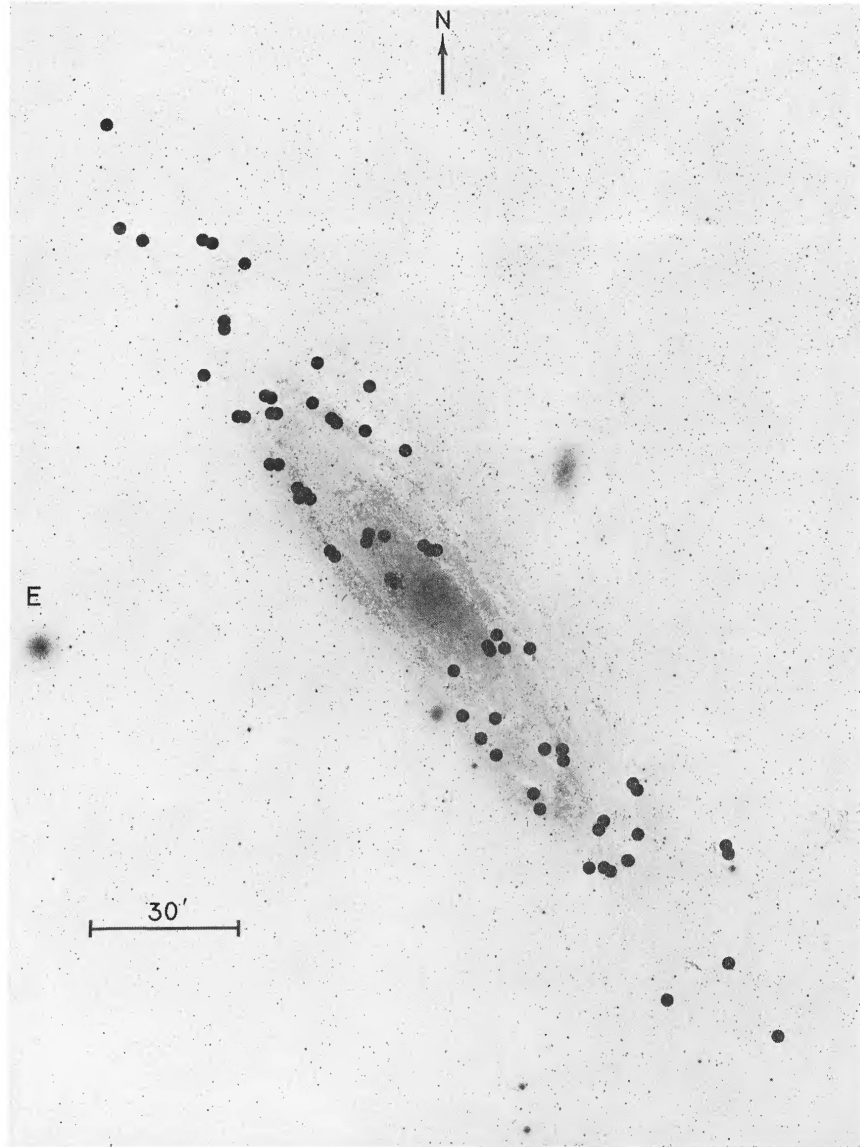


Figure 1.4: Photograph of the Andromeda galaxy. Black dots indicate locations where spectra were measured and used to compute the rotation velocities that are shown in Fig. 1.5. Note that 30' (arcminutes) is the approximate angular size of the full moon as viewed from earth. Reproduced from Vera Rubin and Kent Ford's seminal paper [27]. Credit for the photograph is given to Dr. S. van den Bergh.

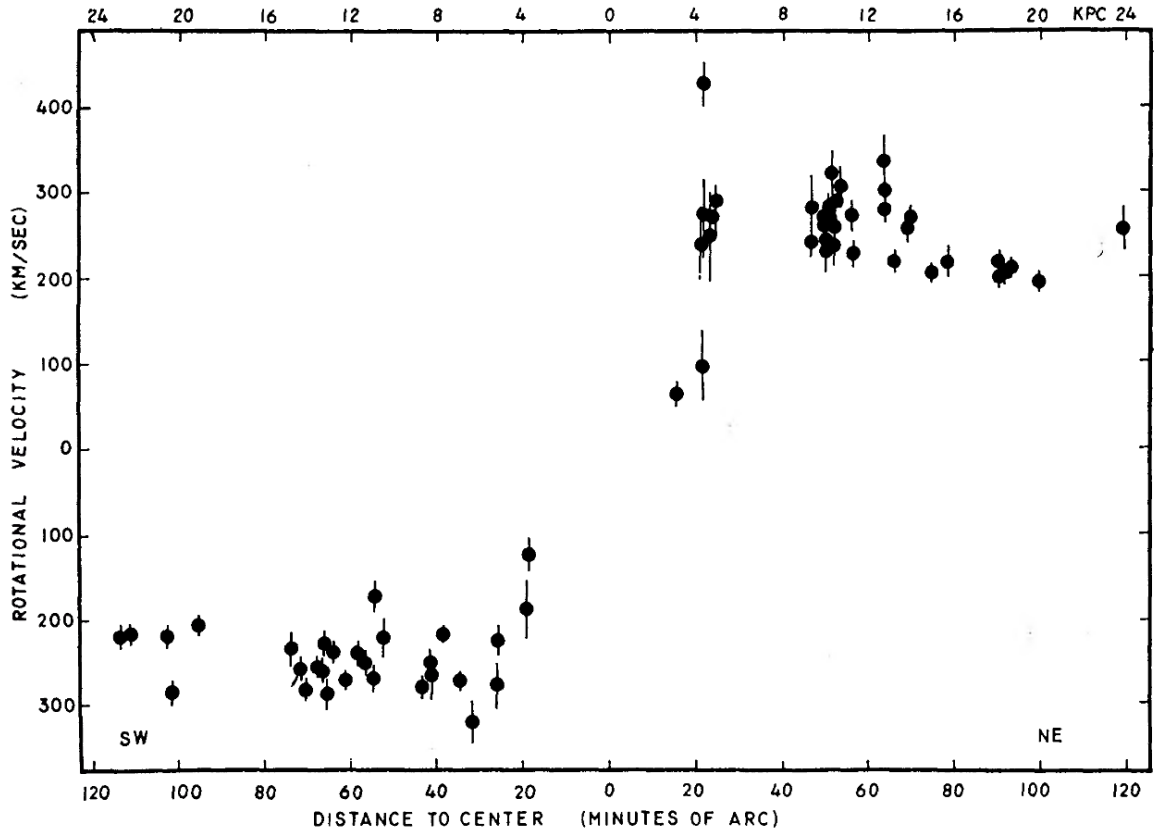


Figure 1.5: Rotation velocity measurements of 67 individual regions of the Andromeda galaxy. The locations of the point are shown in Fig. 1.4. Reproduced from [27].

A compilation of 25 rotation curves published in 1978 [29] from various galaxies (measured using 21 cm line radio observations) is reproduced in Fig. 1.6. This figure shows most of these galaxies have very flat rotation curves. While not immediately apparent from this figure, the peaks in velocity (when present) are further out than R_e , the half-light radius.

A modern compilation of rotation curves is shown in Fig. 1.7. This figure is reproduced from [4] and includes Rubin and Ford’s original 1970 curve of M31 [27], through a 2023 rotation curve of our Milky Way [30].

While Rubin and Ford are generally credited with this discovery, the actual narrative takes many twists and turns between mid-1960s through the early-1980s. The reader is encouraged to reference [5] for a thorough explanation of this history, leading to the results reproduced here.

We now return to our toy model with empirical evidence of flat rotation curves. While the exponential profile of Eq. 1.11 predicted a r^{-1} velocity profile for $r \gg R_e$, we know this is not observed. From Eq. 1.8 and the shell theorems, we can see that a flat velocity profile suggests the enclosed mass scales with the radius. By Eq. 1.9, the mass density $\rho(r)$ must scale with r^{-2} . **The conclusion is that galaxies seem to contain a significant amount of spherically symmetric, non-luminous mass, with density profiles that fall off like r^{-2} and extend to radii past the stars.** The 1979 review by Faber and Gallagher [33] presents a detailed outline of the DM problems and presents a fascinating look through the lens of what was known at the time.

The next section will move from the historical setting to the modern understanding of the Universe and our place in it.

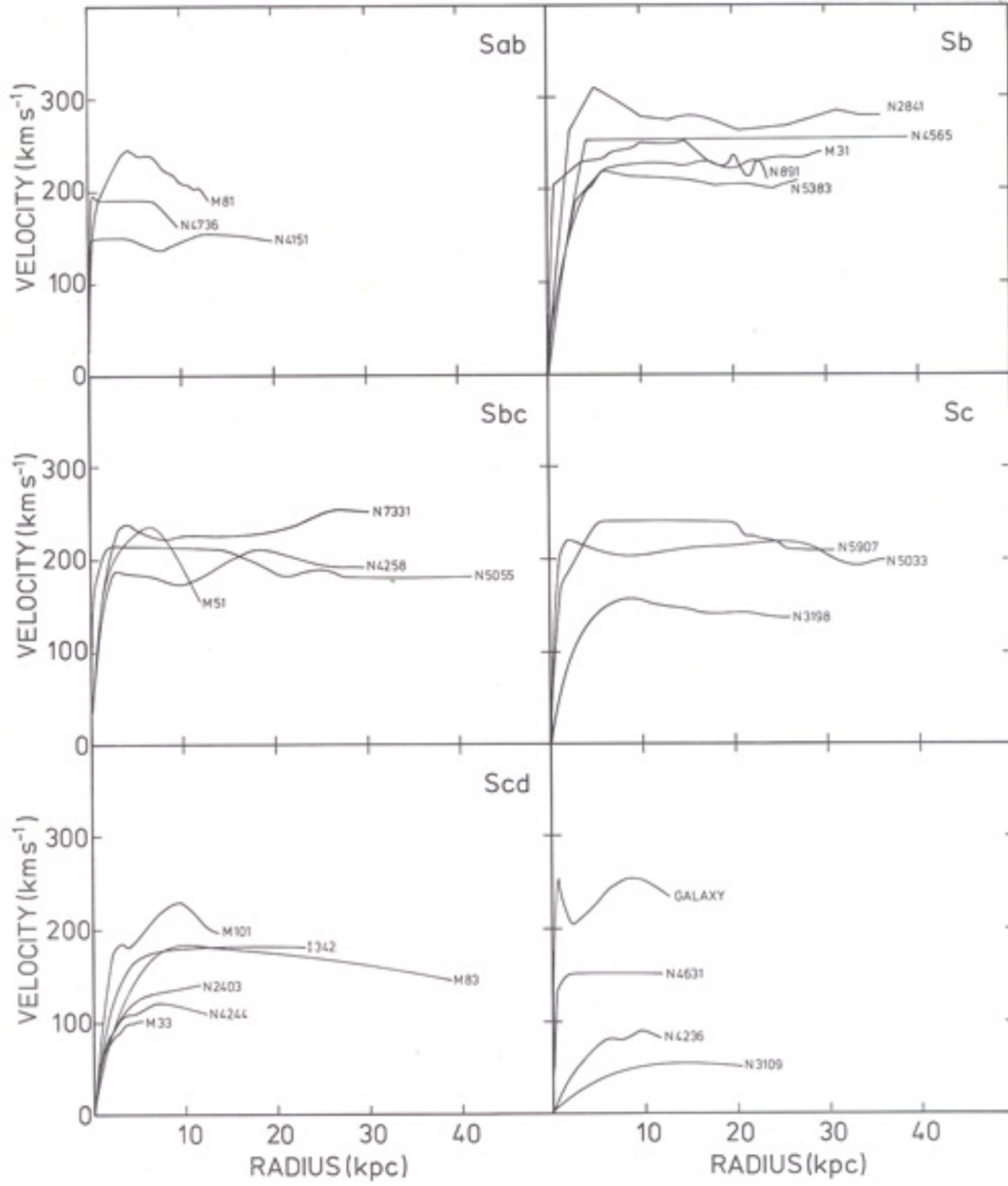


Figure 1.6: Rotation curves of 25 galaxies, measured using 21 cm line radio observations. Sorted by Hubble types. The velocity curves are flat or fall off much slower than would be expected from the half-light radii computed from visible star light. This figure is reproduced from Albert Bosma's 1978 thesis[29].

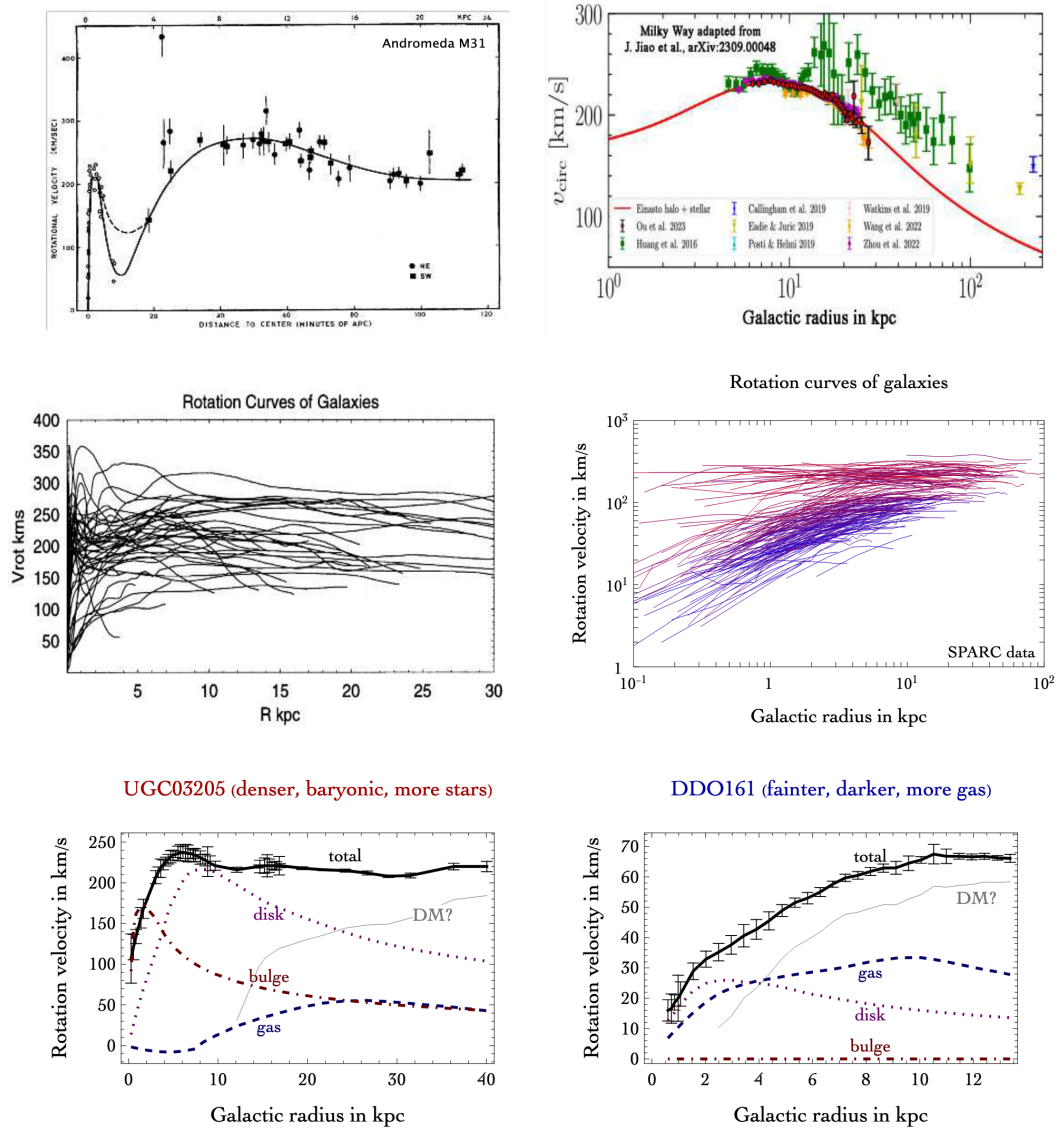


Figure 1.7: 2024 compilation of galactic rotation curves, reproduced from [4]. **Top left:** rotation curve of M31, published by Rubin and Ford in 1970 [27]. **Top right:** Rotation curve of our Milky Way, adapted from [30], including several older measurements. **Center left:** 50 rotation curves from 1999 [31]. **Center right:** 175 rotation curves from the SPARC catalog color-coded by surface brightness (red is brighter, blue is dimmer) [32]. **Bottom left and right:** two plots comparing contributions to the rotation curve from the observable starlight (decomposed between the disk and central bulge) and gas to the rotation velocity. These plots also show the additional contribution that would be required to match the observed rotation. See Sec. 4.3.4 of [10] for a discussion of this mass decomposition. These plots were prepared by the authors [4].

1.2 From “History” to “Current Events”

As pointed out by Trimble, “Practicing scientists will normally put the cut between history and current events at the time when they started reading the literature for themselves, probably early in graduate school.” [6]. While that puts my personal cut-off around 2020, the cut-off of the Dark E-field Radio group is closer to the mid-1960’s, a fruitful time for research into dark matter (DM) as we have seen.

A characteristic shift in the DM problem occurs in this era of current events, namely the 1980s. Instead of asking “is there DM (and if so how much)”, we ask “what is this stuff?!”. To be sure, the first question is still relevant today, and the ever more precise answer will be outlined in Sec. 1.3. This section is a bridge between the early observations hinting at the existence of DM and the modern understanding of our Universe.

1.2.1 The modern picture of the Universe

Cluster dynamics and rotation curves can be understood without much more than a semester of introductory physics. In order to set the stage for the evidence presented later in this section, this subsection will outline the modern picture of the Universe – how it began, where it is going and what it is made of, as well as some tools and conventions used to describe this complex dance. The main references used in this subsection are [10, 17, 34].

In short, the framework that describes our Universe is called Standard Cosmology. In this picture, our Universe emerged from a singularity of space-time with nearly infinite temperature and density approximately 13.8 billion years ago. Following a brief period of

rapid inflation, its evolution has been governed by the distribution of energy among a few components. Today, the composition of the universe is 4.9% baryonic matter, 26.5% cold DM and 68.5% dark energy (plus a tiny fraction of radiation) [7].

There are four main assumptions of Standard Cosmology [17]:

1. The Universe is expanding as was described by Eq. 1.1, the Hubble-Lemaître law.
2. On “large” scales, the universe is homogeneous and isotropic. This is known simply as the cosmological principle. Obviously, this is not valid locally, for you and I and our earth and sun are quite different than the six protons per cubic meter at 2.7 K that, on average, fill the Universe! “Large” in this context is on the order hundreds of Mpc or $\sim 1\%$ of the Hubble length; that is, when averaging over volume elements around this size, the universe appears homogeneous and isotropic.
3. The laws of physics are the same everywhere at all times. This assumption breaks down at the Planck scale, which would have occurred during the singularity that preceded the Big Bang [35]. However, for most epochs following this brief 10^{-43} s period of rapid exponential inflation (see Sec. 1.2.2.2), this is a reasonable assumption.
4. Immediately following this singularity, the mass-energy of the universe was contained principally in blackbody radiation. This radiation would continue to outweigh the matter for the first 50,000 years of the Universe.

The physics of a smooth expanding universe can be described by the kinematics of the dimensionless scale factor $a(t)$. This is the scale of the universe relative to today. It is

conventional to denote parameters at their present value with a 0 subscript, so the scale factor of the universe is $a_0 = a(t_0) = 1$. An overview of the basics of this theory is given in the remainder of this subsection, roughly following [10].

The so-called Hubble flow relates the relative velocity of two observers in an expanding universe separated by a distance r , so $v(t) = [\dot{a}/a] r(t)$. This is the Hubble-Lemaître law 1.1, where $H(t) \equiv \dot{a}/a$. a is related to the redshift z by

$$a = \frac{1}{1+z}. \quad (1.12)$$

The kinematics of a is given by the two Friedmann equations [36], derived from the Einstein equation [37]⁸,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}, \quad (1.13)$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar, G is Newton’s gravitational constant, $T_{\mu\nu}$ is the energy-momentum tensor and Λ is the cosmological constant. An insightful description of this equation is provided in [39]. Neglecting the final term ($\Lambda g_{\mu\nu}$), Eq. 1.13 relates the geometry of space-time on the left with the energy content contained within space-time on the right. The addition of the Λ term introduces a “vacuum-energy”, inherent to empty

⁸The equation shown here is not exactly as was derived in Einstein’s 1916 paper [37]. The notation is slightly different and the final term was not included at this point. Historical context through review of Einstein’s letters with colleagues is provided in [38].

space-time regardless of matter content⁹.

To derive the Friedmann equations, we assume the Universe obeys the cosmological principle defined above; it is homogeneous and isotropic. In other words, the Universe is filled with a uniformly distributed cosmological fluid composed of a mixture of a few parts (matter, radiation, dark energy, described below).

The first of the Friedmann equations is known simply as the Friedmann equation,

$$\dot{a}^2 = \frac{8\pi G\rho a^2}{3} - kc^2, \quad (1.14)$$

where c is the speed of light (usually set to 1, but I have retained it here), ρ is the total mass density of the cosmological fluid and k is the curvature. The curvature describes the geometric structure of the Universe, with $k > 0$ indicating a closed spherical geometry, $k = 0$ a flat Euclidean geometry, and $k < 0$ an open hyperbolic geometry¹⁰.

The second Friedmann equation is known as the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3P}{c^2} \right). \quad (1.15)$$

By combining the time derivative of Eq. 1.14 with Eq. 1.15, we get a continuity equation,

$$\dot{\rho} + 3\frac{\dot{a}}{a} \left(\rho + \frac{P}{c^2} \right) = 0. \quad (1.16)$$

⁹The comparison of observations of the vacuum energy with theoretical predictions represents a problem in modern cosmology. The problem is known as the cosmological constant problem (now the *old* cosmological constant problem), or the fine tuning problem. The theoretical value of Λ is around 120 orders of magnitude higher than its observational value. There are some contributions to this density that can partially cancel. Instead, the cancellation is not quite complete, but accurate to 120 decimal places. This is the topic of Weinberg's 1989 paper [40].

¹⁰This can be roughly understood in terms of projectiles on parallel trajectories. The projectiles would either converge, remain parallel or diverge for $k > 0$, $k = 0$ and $k < 0$, respectively. The two-dimensional analog is that of a sphere where $k > 0$; parallel projectiles will eventually converge

The equation of state relates pressure of one component of the fluid P_i to the density of that component through a dimensionless parameter w_i ,

$$P_i = w_i \rho_i c^2. \quad (1.17)$$

It can be seen from this equation how the sign of w_i characterizes the behavior of different components. For example, in the equation of state for dark energy, $w_\Lambda = -1$ corresponding to a negative pressure.

For a flat Universe with $k = 0$, a critical density can be defined from Eq. 1.14,

$$\rho^c(t) \equiv \frac{3H^2(t)}{8\pi G}. \quad (1.18)$$

This density at the present time is $\rho_0^c = 1.9 \times 10^{-29} h^2 \text{g/cm}^2$, where h (“little h”¹¹) is the dimensionless Hubble constant, so that $H_0 = 100 h \text{ km/s/Mpc}$, and the currently accepted value is ~ 0.7 . For reference, ρ_0^c is about 6 protons per cubic meter, as alluded to earlier in this subsection with the introduction of the cosmological principle.

Given this natural density scale, it is customary to represent dimensionless densities of the various constituents,

$$\Omega_i(t) \equiv \frac{\rho_i(t)}{\rho^c(t)}. \quad (1.19)$$

In units of ρ^c , Eq.1.14 can be written

¹¹For the history and current usage of little h see *Damn you little h!* by Darren J. Croton [41]

$$\Sigma_i \Omega_i = \frac{k}{\dot{a}} + 1, \quad (1.20)$$

from which it can be seen the total density Ω is only 1 for a flat universe (i.e. $k = 0$)

We can now define the three components of the cosmic fluid, classified by their equations of state, Eq. 1.17:

1. **Matter:** Most matter is characteristically non-relativistic, i.e. its rest mass is much larger than its momentum (with the proper factors of c inserted). In the context of the cosmic fluid, the equivalent statement is that $P \ll \rho c^2$. Inserting this relationship into the equation of state, we see $w_m = 0$. Matter can be broken down into baryons and non-baryonic cold DM.
2. **Radiation:** As opposed to non-relativistic matter, radiation is ultra-relativistic. From statistical mechanics considerations (counting states in a relativistic gas), we can show $w_r = 1/3$. Radiation obviously includes photons, but at earlier cosmic times when the universe was hotter, it included neutrinos. Radiation could also include hot DM, though it is generally accepted there is no such thing.
3. **Dark Energy:** Dark energy is a rather generic term and includes many models. The important feature common to these models is that $w_\Lambda = -1$, i.e. *negative* pressure. From the second Friedmann equation, Eq. 1.15, we can see that $\ddot{a} > 0$ would require $w < -1/3$. In order to account for the observed acceleration, dark energy is introduced. While much is unknown about dark energy, it is modeled as the energy associated with the cosmological constant Λ of Eq. 1.13.

Furthermore, we know the relative contributions of the relevant components today [42] (matter, baryons, dark energy, respectively), normalized to the critical density, Eq. 1.19;

$$\Omega_{m,0} = 0.3153 \pm 0.0073 \quad (1.21)$$

$$\Omega_{b,0} = 0.04924 \pm 0.00033 \quad (1.22)$$

$$\Omega_{\Lambda} = 0.6847 \pm 0.0073 \quad (1.23)$$

In other words, the universe is currently 68% dark energy, 31% matter and negligible radiation. Only 4.9% of the matter is baryons, and the rest is DM.

Having defined w for the three components, we can now describe how their density scales with a through the continuity equation, Eq. 1.16;

$$\rho_m = \rho_{m,0} a^{-3} \quad (1.24)$$

$$\rho_r = \rho_{r,0} a^{-4} \quad (1.25)$$

$$\rho_{\Lambda} = \rho_{\Lambda,0} \quad (1.26)$$

In other words, matter and radiation dilute in an expanding universe, while dark energy density appears to remain constant. This is shown in Fig. 1.8.

With this picture of a diluting and cooling Universe in mind, the next subsection will focus on the thermal history of the universe.

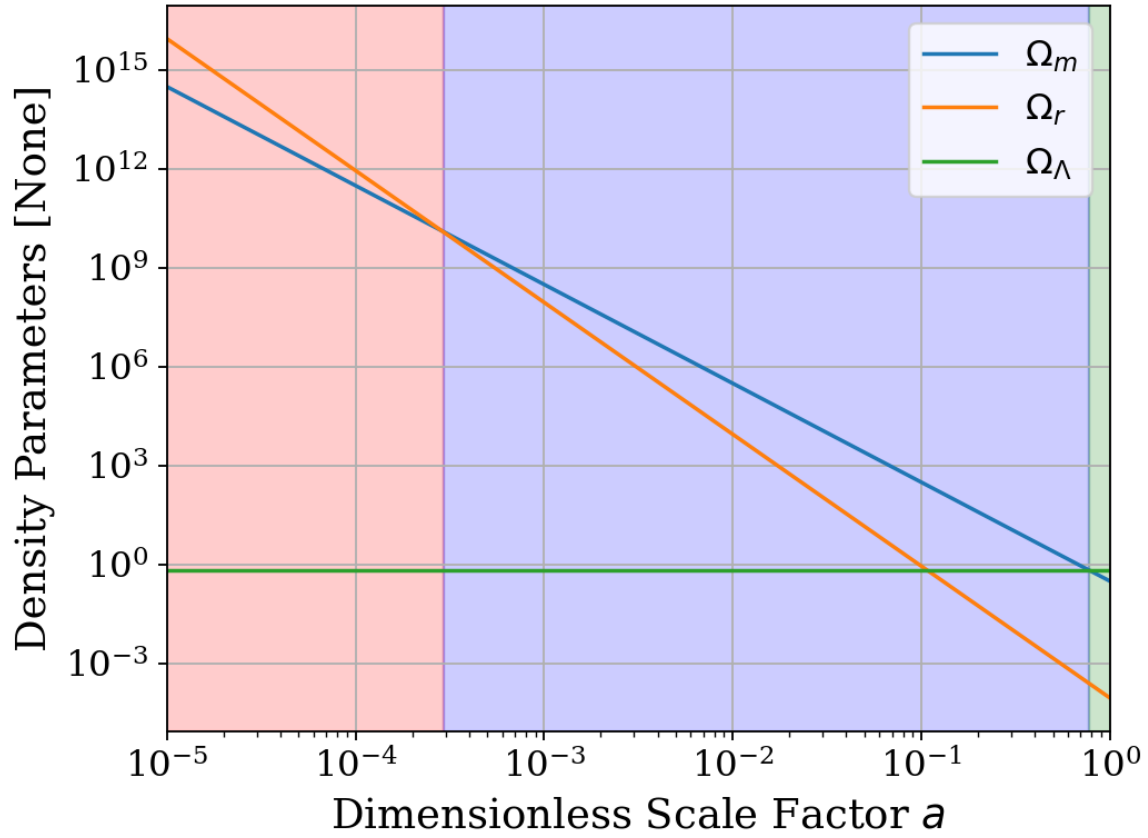


Figure 1.8: Evolution of the components of the universe as a function of the scale factor a . The three components are matter, radiation and dark energy, shown as blue, orange and green curves, respectively. They are shown here as density parameters (Eq. 1.19), meaning they have been normalized to the critical density (Eq. 1.18). The density of these components scale with a as shown in Eqs. 1.24 - 1.26. The three components each dominate at different times, shown as the shaded regions. Radiation was dominant until $a = 2.95 \times 10^{-4}$ [42]. Today, dark energy has a slight edge over matter. As the universe expands, energy density dilutes, and the universe cools.

1.2.2 The history of the Universe

As far as we know, the Universe began as a singularity where the laws of nature as we know them were not valid. From this point onward, it expanded and cooled, and its energy diluted.

This is summarized in Table 1.1, as well as Fig. 1.9. This section will provide background

on a few epochs important for understanding the evidence for DM. The remainder of this chapter will assume a basic understanding of this history, outlined in this sub-section.

Cosmic Time	Temp. [K]	Redshift $[z]$	Era
10^{-43} s	10^{32}	10^{32}	Planck time
10^{-36} – 10^{-34} s	10^{25} – 10^{28}	10^{25} – 10^{28}	Inflation
1 s	10^{10}	10^{10}	Neutrino decoupling
10–1000 s	10^8 – 10^9	10^8 – 10^9	Big Bang nucleosynthesis
5×10^4 yr	9×10^3	3400	Matter–radiation equivalence
0.4 Myr	3×10^3	1000	Recombination/photon decoupling
0.5–1 Gyr	19–30	6–10	Reionisation
9.5 Gyr	3.5	0.30	Λ –matter equivalence
13.8 Gyr	2.7	0	Present time

Table 1.1: Thermal history of the universe in the standard cosmological model. Reproduced from [10]

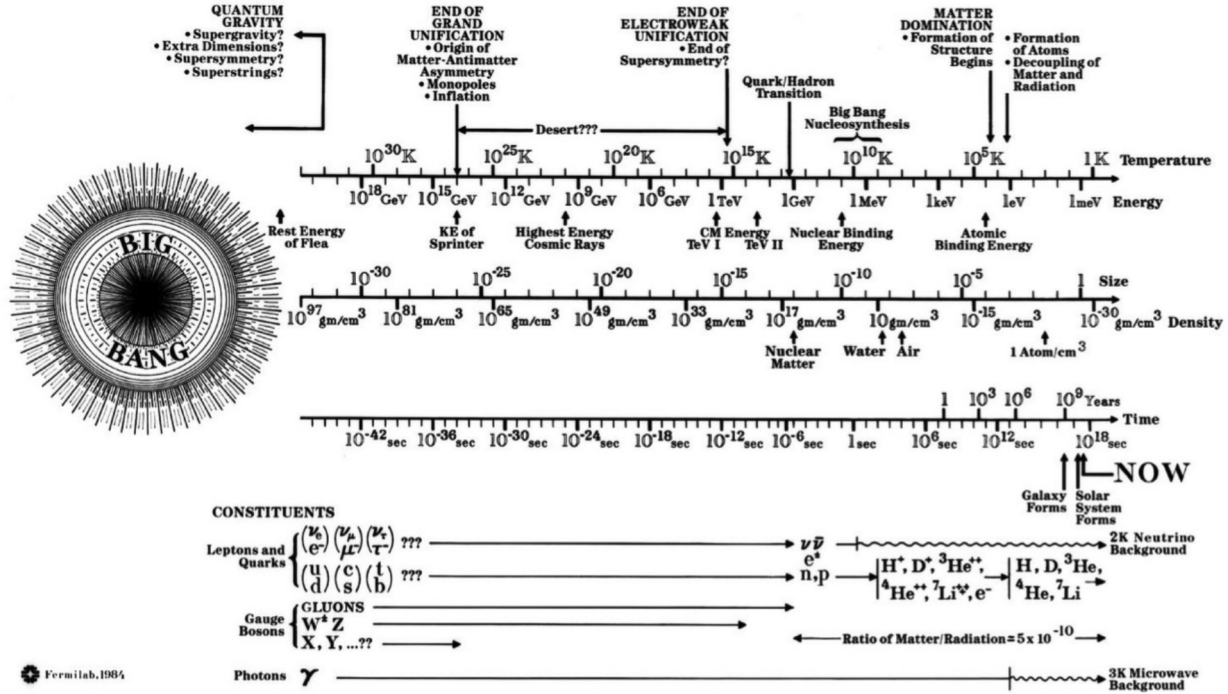


Figure 1.9: Fermilab’s famous “history of everything” poster. This is an excellent visual companion to Table 1.1. Figure reproduced from Michael Turner’s 2022 review on precision cosmology [43], where he attributes it to a collaboration between himself and Angela Gonzalez of Fermilab, circa 1984.

1.2.2.1 Planck Scale

Shortly after the Big Bang, the universe was so hot and dense that our current understanding of physics breaks down. This can be roughly understood by finding the wavelength corresponding to the Schwarzschild radius ($r_s = 2GE/c^4$) of an energetic particle. At such a high energy, the particle’s self-gravity would form a black hole with itself [35]. Planck units can be constructed through combinations of fundamental constants. The Planck scale in SI units are:

- **Length:** $l_p = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35} \text{ m}$

- **Time:** $t_p = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{ s}$
- **Mass:** $m_p = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-8} \text{ kg}$
- **Energy:** $E_p = m_p c^2 = \sqrt{\frac{\hbar c^5}{G}} = 1.96 \times 10^9 \text{ J}$
- **Temperature:** $T_p = \frac{E_p}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.42 \times 10^{32} \text{ K}$
- **Density:** $\rho_p = \frac{m_p}{l_p^3} = \frac{c^5}{\hbar G^2} = 5.16 \times 10^{96} \text{ kg/m}^3$

1.2.2.2 Inflation

Inflation is introduced to the cosmological picture to solve a few puzzles - the horizon, flatness and monopole problems, outlined in [44, 45]. The favored solution to these problems is known as inflation. In this model, there is a brief period (between approximately 10^{-36} and 10^{-34} seconds after the Big Bang) where the scale factor increased exponentially,

$$a(t) = a_e e^{H(t-t_e)}, \quad t < t_e \quad (1.27)$$

where t_e and a_e are the time and scale factor at the end of inflation, and H is the Hubble constant (assumed to be constant throughout inflation in this model). Inflation is generally thought to be driven by a scalar field ϕ trapped in a false vacuum state [46, 47]. This field is frequently referred to as the *inflaton*.

It is important to keep in mind that, at this point, the temperatures are far too hot to allow neutral atoms to form, so the Universe is an ionized plasma. The Universe is opaque to

radiation since the photons couple to the free charges and scatter. Thus, we can not directly observe any radiation until recombination ($z_* \sim 10^3$).

1.2.2.3 Big-bang nucleosynthesis

Big-bang nucleosynthesis (BBN) is the period in which the hot plasma cooled enough to allow the light elements to form. Light elements in the context of BBN generally include (in order of abundance) ^4He , ^2H (usually notated as D for Deuterium), ^3He , and ^7Li . Understanding BBN requires understanding the conditions preceding it. Furthermore, the actual useful predictions of BBN require numerically solving complex nuclear reaction networks to predict the elemental abundance. See for example [48]. The takeaway of what is presented here is that:

- The universe is cooling and expanding, reducing the temperature and density of nucleons. Furthermore, free neutrons are unstable. The net effect is a reduction of the reaction rate with time. BBN is a race against the clock.
- Once the clock runs out, we can compute the relative abundances of light elements. This can be compared to observations of abundances, confirming our understanding of the conditions of this period of the universe.
- Determining when the clock ran out is a good indicator of the baryon/photon ratio η . This can be independently confirmed from measurements of the cosmic microwave background (see Sec. 1.3.1).

Excellent overviews of the topic are given in [11, 46, 49], and up-to-date (2020) values are computed in [50].

Before BBN, the temperatures were high enough to keep neutrons and protons in equilibrium via the reactions

$$n + \nu_e \leftrightarrow p + e^- \quad (1.28)$$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e. \quad (1.29)$$

These reactions can continue as long as their rate $\Gamma \propto a^{-5}$ is large compared to the Hubble expansion rate $H \propto a^{-2}$. The reaction rate's dependence on a is determined because the cross section falls with T^2 , which itself is inversely proportional to a , and the number density falls like a^{-3} . The Hubble constant's dependence on a comes from Eq. 1.14 in a flat, radiation-dominated Universe.

The number densities of neutrons and protons are given by the Boltzmann equation, and their ratio is given simply by

$$\frac{n_n}{n_p} = e^{-(m_p - m_n)/kT} \quad (1.30)$$

where $m_{p(n)}$ is the mass of a proton (neutron). Note that $(m_p - m_n) \equiv Q_n = 1.29 \text{ MeV}$. Thus, when $T \gg Q_n$, $n_n/n_p \approx 1$. As the temperature falls below Q_n , however, the ratio quickly falls off. Before n_n can fall to zero, the reactions 1.28 and 1.29 freeze out as the expansion rate dominates the reaction rate. This happens at $T_f = 0.8 \text{ MeV}$, resulting in $n_n/n_p \approx 1/5$.

The free neutrons, however, are unstable and decay with a half-life $\tau_n = 879.4 \pm 0.6 \text{ s}$ ¹². Any nuclei that are going to form need to do so at times less than a few half-lives (and while there is sufficient temperature and density). If not for this time restriction, nuclear reactions would proceed until everything was ${}^{62}\text{Ni}$ as it has the highest binding energy per nucleon.

The first step in the true BBN period is the production of deuterium. With a binding energy of 2.22 MeV ($2.25 \times 10^{10} \text{ K}$), one would naively expect this is the temperature at which BBN begins. However, the universe is still dominated by radiation, and the photons have a blackbody spectrum with a long tail, and the deuterium must wait until $\sim 10^9 \text{ K}$.

After a significant amount of deuterium has formed, a large network of reactions is possible, and as these elements and isotopes form, the network grows even more. Reactions proceed as the neutrons are used up and the Universe is expanding, reducing temperature and number density. Most of the reactions that can occur in this time lead to ${}^4\text{He}$. The mass ratio of ${}^4\text{He}$ to the total baryon mass is defined as Y_p . For useful predictions, this reaction network must be solved numerically, resulting in relative abundances as a function of the baryon/photon ratio η , shown in Fig. 1.10. η is so important here because fewer photons allow for an earlier start to deuterium synthesis, resulting in a more efficient synthesis of heavier nuclei. The current accepted value¹³ for η and the light elemental abundances are [50]

¹²Similar to the Hubble tension, there is some tension in τ_n depending on how it is measured. I quote this value (the world average) following[50], which contains a discussion and many references related to the topic.

¹³Assuming 3 neutrino flavors.

$$\eta = (6.129 \pm 0.039) \times 10^{-10}, \quad (1.31)$$

$$Y_p = 0.24465 \pm 0.00410, \quad (1.32)$$

$$\text{D/H} = (2.53 \pm 0.15) \times 10^{-5}, \quad (1.33)$$

$${}^3\text{He/H} = (9.97 \pm 0.91) \times 10^{-6}, \quad (1.34)$$

$${}^7\text{Li/H} = (4.76 \pm 0.74) \times 10^{-10}, \quad (1.35)$$

in good agreement with observed values with the exception of lithium, which differs by a factor of ~ 5 . This disagreement is well known and may be due to the burning of lithium in stars. Even so, the agreement is considered excellent and a confirmation of the theory underlying BBN.

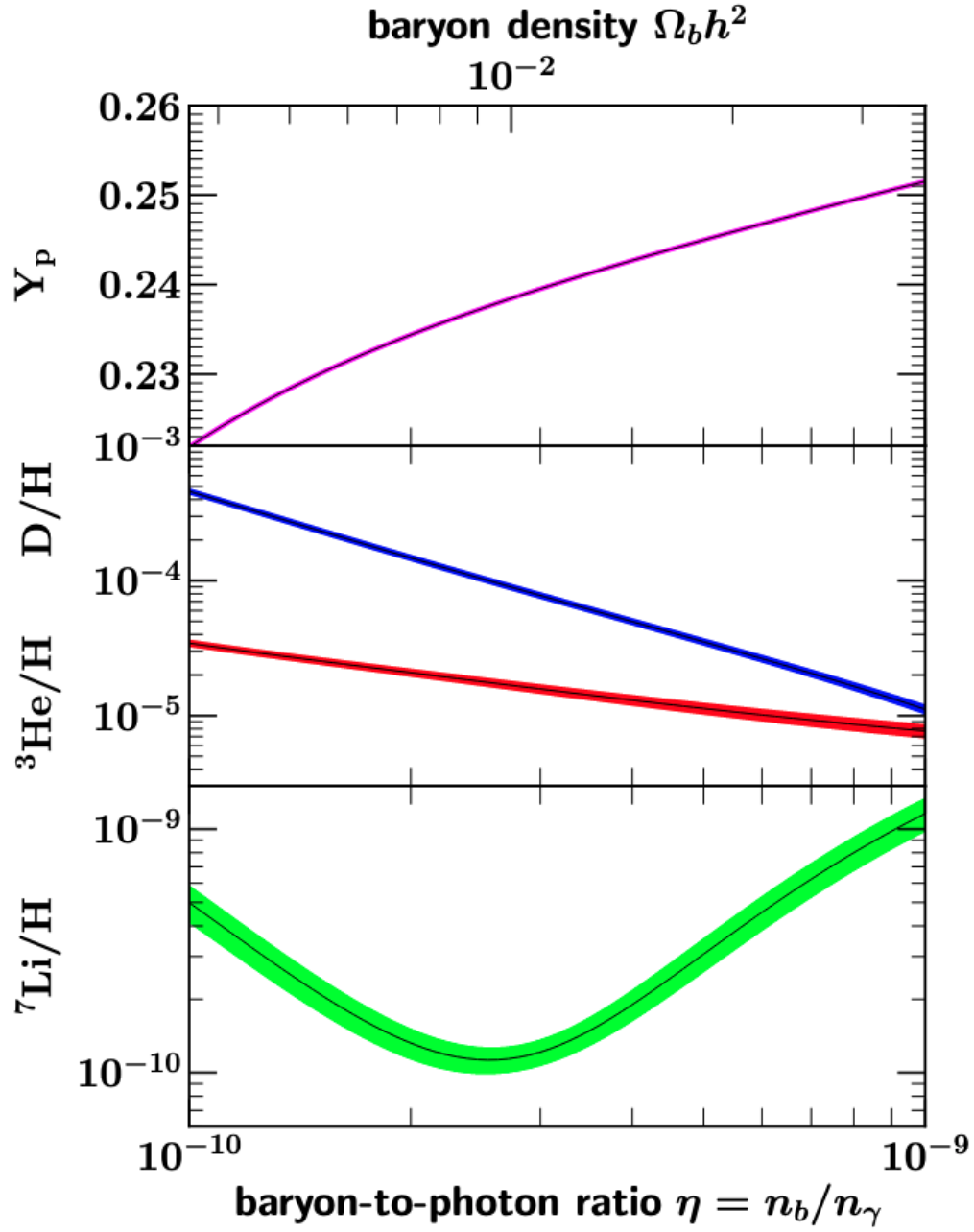


Figure 1.10: Schramm plot showing the relative primordial abundances of the light elements compared to the baryon/photon ratio η predicted by standard BBN. The ${}^4\text{He}$ density is given in ratio to the total baryon density Y_p , while the other elements are given as a number density relative to H. The curves assume 3 neutrino flavors and a neutron half-life $\tau_n = 879.4 \pm 0.6$ s. The shaded area around the curves are 1σ errors. Figure reproduced from [50]

1.2.2.4 Matter–Radiation Equivalence

Once matter began to take over after $z_{eq} = 3402 \pm 26$ [42]¹⁴, the negligible pressure of matter allowed small perturbations to begin to collapse under gravity. These over-dense structures contain the baryons that would eventually become you and I, as well as our Sun which fuels us.

1.2.2.5 Recombination/Photon Decoupling

As time goes on, the Universe continues to expand and cool. While the temperatures have dropped far below the nuclear energy scales (10s of MeV), they are still above ionization energy scales (10s of eV). Photons still possess sufficient energy and number density to prevent electrons from binding to nuclei, resulting in a universe that is full of an electron/light-nucleus plasma. Similar to the freeze-out of the reactions described by Eqs. 1.28 and 1.29, the high-energy tail of the blackbody photon spectrum means that the temperature must drop below 10s of eV that we naively expect. The photons decouple from the plasma, allowing electrons to combine with the nuclei, forming natural atoms. This recombination occurs at $z_* = 1098.92 \pm 0.25$ [42], corresponding to an energy of about 0.3 eV or a temperature of about 3000 K.

An important consequence of this temperature-dependent decoupling of recombination is that the opaque Universe becomes transparent at the same moment throughout. No early-

¹⁴The Planck collaboration reports 12 different values for z_{eq} in [42] for fits to different likelihoods (Camp-Spec or Plik) and data sets (TT, TE, EE, etc.). The values given here are from the Plik likelihood of the TT,TE,EE+lowE+lensing data, which “represent the legacy results on the cosmological Λ CDM parameters from the Planck satellite, and are currently the most precise measurements coming from a single CMB experiment.”

universe photons exist today since they were absorbed and remitted by the plasma, and so we can't directly observe anything before recombination. However, the blackbody photons of the 3000 K Universe decouple and free stream through expanding space. As the space expands, the photons redshift, but they reach us all the same. We observe them today as a bath of blackbody photons corresponding to 2.72548 ± 0.00057 K [51]. These photons are known as the cosmic microwave background (CMB), shown in Fig. 1.11. Because the Universe was opaque before recombination, the CMB contains the oldest photons, and so it is the farthest back that we are able to observe directly using electromagnetic radiation.

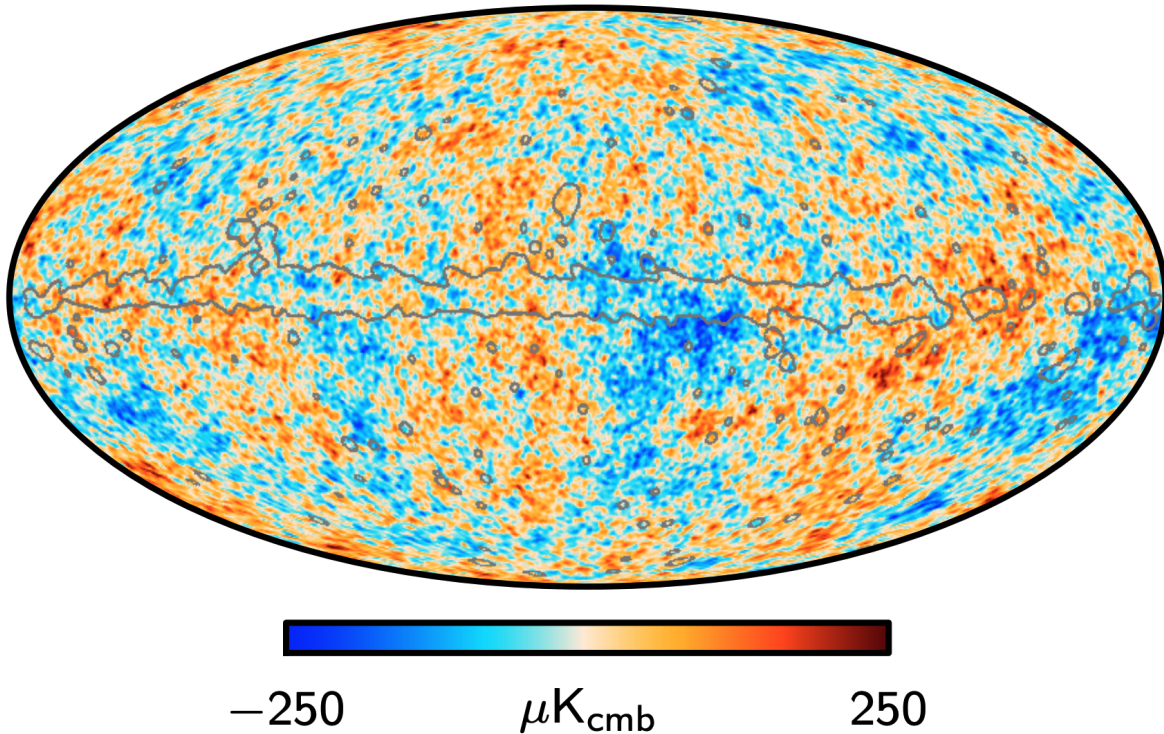


Figure 1.11: Temperature map of the cosmic microwave background across the sky. The color corresponds to the temperature variation about the mean, 2.7248 K . The temperature has been corrected for the dipole contribution due to the motion of the instrument relative to the CMB. This includes the motion of the satellite around the Earth, the Earth around the Sun, the Sun around the Galaxy and the Galaxy around the Local Group. The gray lines indicate the temperature mask where the galactic synchrotron radiation has been subtracted. Figure reproduced from [52].

1.2.2.6 Reionization through modern times:

Following recombination, structures that were seeded from small perturbations continue to collapse under gravity. Some of these over-dense regions form stars and galaxies that would produce high-energy photons, causing regions to reionize. The dynamics of collapse are difficult to explain if the matter can self-interact, as baryons would. This will be discussed in the following section

1.3 Modern Evidence and Observations of Dark Matter

This section is concerned with modern evidence for dark matter (DM), which tends to point to non-baryonic models, which the Dark E-Field Radio Experiment is searching for.

1.3.1 The cosmic microwave background

The cosmic microwave background (CMB, see Sec. 1.2.2.5) is a blackbody spectrum corresponding to a nearly uniform temperature everywhere we look in the sky. It is shown in the previous section, though with significant processing to bring out very tiny fluctuations in temperatures ($\delta T/T \sim 10^{-5}$). A consequence of this uniformity is that it took about 28 years and a space mission between the initial 1964/1965 discovery of the CMB [53] and the 1992 detection of any anisotropies [54] (the dipole contribution caused by our own motion was measured from an airplane in 1977 [55]). These fluctuations offer a probe into the fractional composition of the Universe during recombination. They provide extremely powerful evidence for the existence of DM, not to mention confirmation of the spatially-flat Λ CDM model and determination of its parameters. For a review of the subject see [56].

The information is contained in the statistics of the angular size of the temperature variations. To quantify this, the temperature map of Fig. 1.11 is expanded in spherical harmonics,

$$\frac{\delta T(\theta, \phi)}{T} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell,m} Y_m^{\ell}(\theta, \phi), \quad (1.36)$$

where ℓ and m are integers associated with θ and ϕ respectively, and Y_m^ℓ are the usual spherical harmonic functions. Note that the angular scale is related to ℓ approximately by π/ℓ ¹⁵. Using this, the CMB power spectrum is defined,

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell,m}|^2. \quad (1.37)$$

Finally, this is normalized so that

$$D_\ell^{TT} \equiv \frac{\ell(\ell + 1)C_\ell}{2\pi}. \quad (1.38)$$

Here, the TT superscript indicates that this is a temperature power spectrum, however, there several spectra for different polarizations. We will focus on the TT spectrum here. For more details, see [52].

Plotting D_ℓ^{TT} as a function of ℓ reveals several peaks, the positions and amplitudes of which reveal a great deal about the conditions of the Universe at the time of recombination. This power spectrum is shown in Fig. 1.12.

These angular fluctuations are the result of the red(blue)shift of photons traveling from a point of low (high) gravitational potential. Since the CMB photons decoupled from the cosmic plasma at roughly the same time, the temperature distribution tells us the gravitational potential $\delta\Phi$ as it was during recombination. The temperature is related to the potential as [57]

$$\frac{\delta T}{T} = \frac{\delta\Phi}{3c^2}. \quad (1.39)$$

¹⁵For reference, $\ell = 200$ is about 1° .

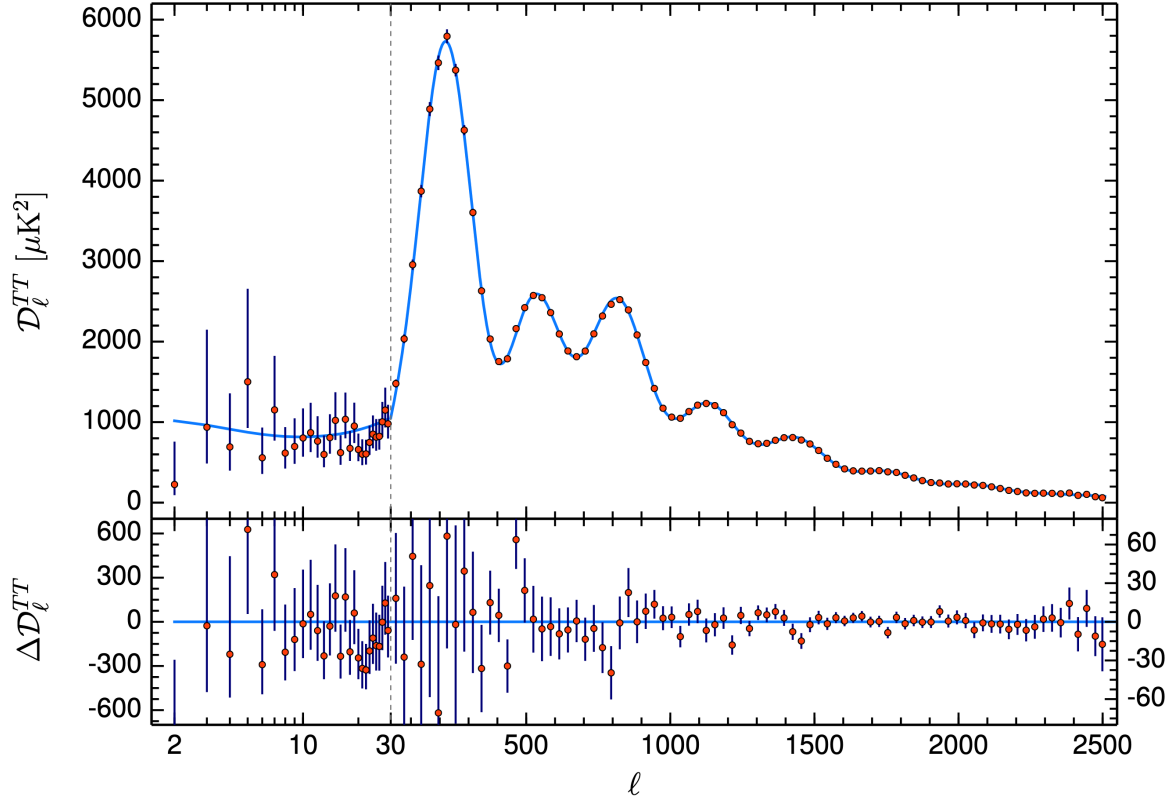


Figure 1.12: Top: Observed temperature power spectrum of the cosmic microwave background. The fit is for base Λ CDM, assuming a flat universe. Bottom: residuals between the fit and data. Note that the horizontal axis has a logarithmic scale for $\ell < 30$ and linear scale for $\ell > 30$. Also note that $\ell = 1$ (the dipole term) is suppressed due to the motion of the instrument relative to the CMB. Figure reproduced from Planck collaboration 2018 [42].

On angular scales larger than the Hubble distance at the time of recombination ($\theta_H \gtrsim 1^\circ$ or $\ell \lesssim 200$ as viewed today from Earth), different regions of the Universe have not been in causal contact since inflation and density fluctuations are imprints from this extremely early phase of the Universe.

At smaller angular scales ($\ell > 200$), the peaks of the power spectrum can be understood by considering the dynamics of the photon-baryon fluid prior to recombination. Recall the

equation of state (Eq. 1.17) – For radiation $w_r = 1/3$, while for matter $w_m = 0$. Therefore, a coupled photon-baryon fluid has a $0 < w_{\gamma,b} < 1/3$. The non-zero pressure-density relationship means that pressure will oppose the increase of density. Non-baryonic DM remains uncoupled to the photons and retains $w_{\text{dm}} = 0$. It does, however, interact with the fluid through gravity. The net result is a fluid that can behave as a harmonic oscillator; the gravitational potential and pressure combine to play the role of a sort of spring. Sound waves propagated through this system until the brief moment of recombination when the photons were released. The density waves have left a lasting imprint on the temperature fluctuations of the CMB via Eq. 1.39 as we observe today. The peaks in the CMB power spectrum are sound waves and their harmonics. Modeling this system reveals that the CMB we observe is extremely sensitive to cosmic parameters and, therefore, tells us a great deal about our Universe. The details of the dynamics of this system are beyond the scope of this dissertation, but an excellent review is provided by Hu and Dodelson [56]. Figure 1.13 is taken from this review and gives a sense of how different ratios of components would modify the power spectrum.

One further point is that the CMB power spectrum is sensitive to Ω_B , η , and Y_p , which can be independently predicted from the physics of Big Bang nucleosynthesis (Sec. 1.2.2.3). The excellent agreement between two independent probes of the physics of very different epochs greatly improves confidence in our understanding of both of these eras.

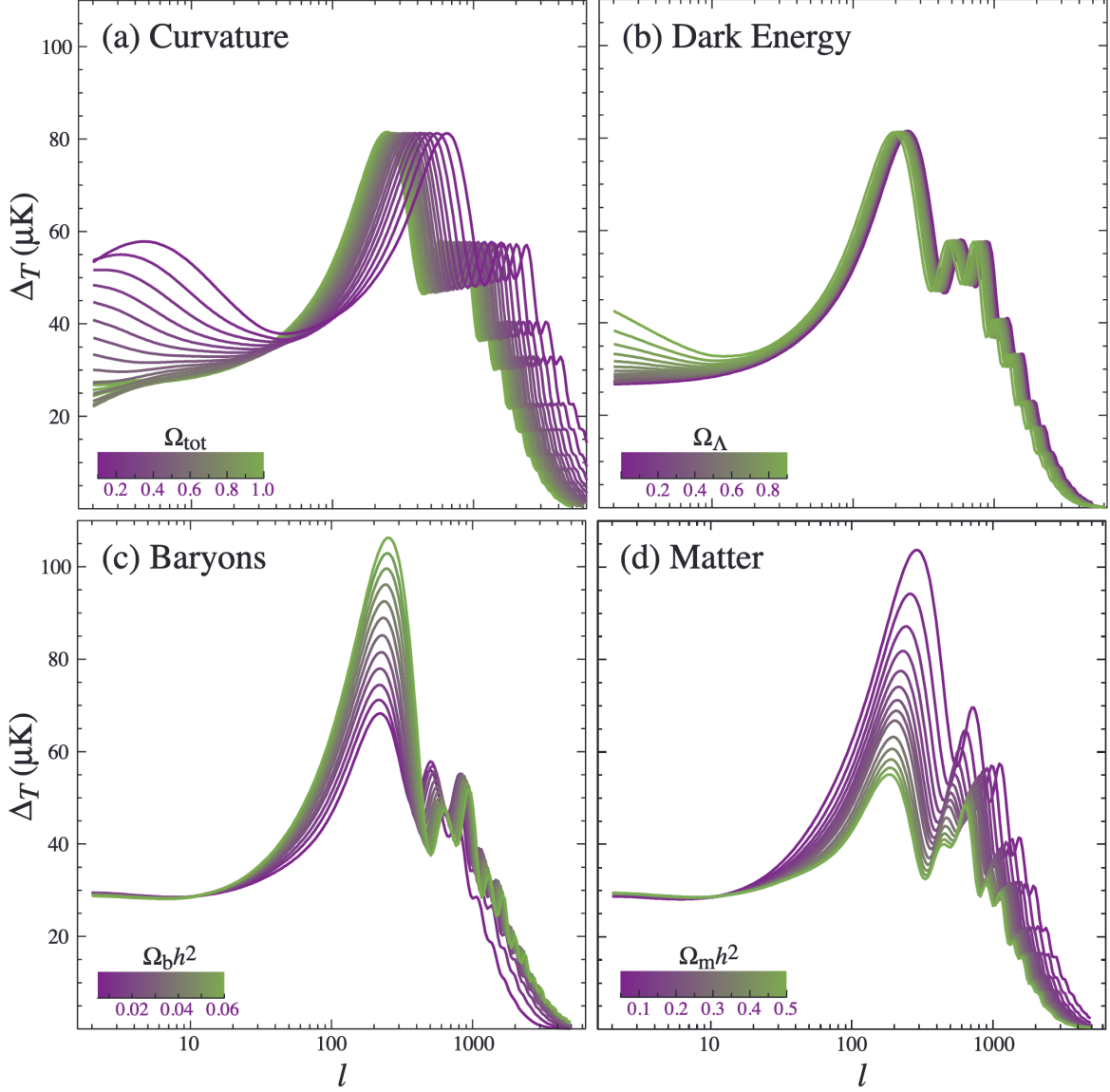


Figure 1.13: Cosmic microwave background temperature spectra as cosmic parameters are varied. The parameters are varied around a fiducial model for a flat universe ($\Sigma\Omega_i \equiv \Omega_{\text{tot}} = 1$) with $\Omega_\Lambda = 0.65$, $\Omega_b h^2 = 0.02$ and $\Omega_m h^2 = 0.147$. The redshift of reionization $z_{\text{ri}} = 0$ and the inflation energy scale $E_i = 0$. Figure reproduced from [56], where the choices of those parameters are explained.

1.3.2 Gravitational lensing and mapping extragalactic dark matter

Einstein's field equation (Eq. 1.13) relates mass-energy to the curvature of space-time. This provides an important probe into our universe. Observing apparent distortions of distant galaxies allows us to construct maps of mass in the foreground. A cartoon of this process is shown in Fig. 1.14. This effect was famously used as a confirmation of Einstein's general theory of relativity during the total solar eclipse of 1919 [58]. When the sun was eclipsed by the moon, faint stars were observed to be in positions a few arcseconds away from their known positions due to the curvature of space-time by the sun's mass.

Figure 1.15 shows a galaxy cluster that is being strongly lensed such that it appears there are many copies of the same galaxy. Images such as this allow for the reconstruction of detailed DM mass maps [59, 60]. Observations of DM warping space provide very strong evidence for its existence.

An interesting consequence of this detailed knowledge of DM mass distributions is the ability to predict the delay in arrival time of light from a distant supernova that takes multiple paths through the warped space-time. Supernova Refsdal was first observed in November 2014 [62] through the gravitational lens provided by galaxy cluster MACS J1149+2223 in the foreground. Another image of the supernova was predicted to reappear the following year, and it did in the correct position and time [63]. Three images of the same region of the sky at different times are shown in Fig. 1.16.

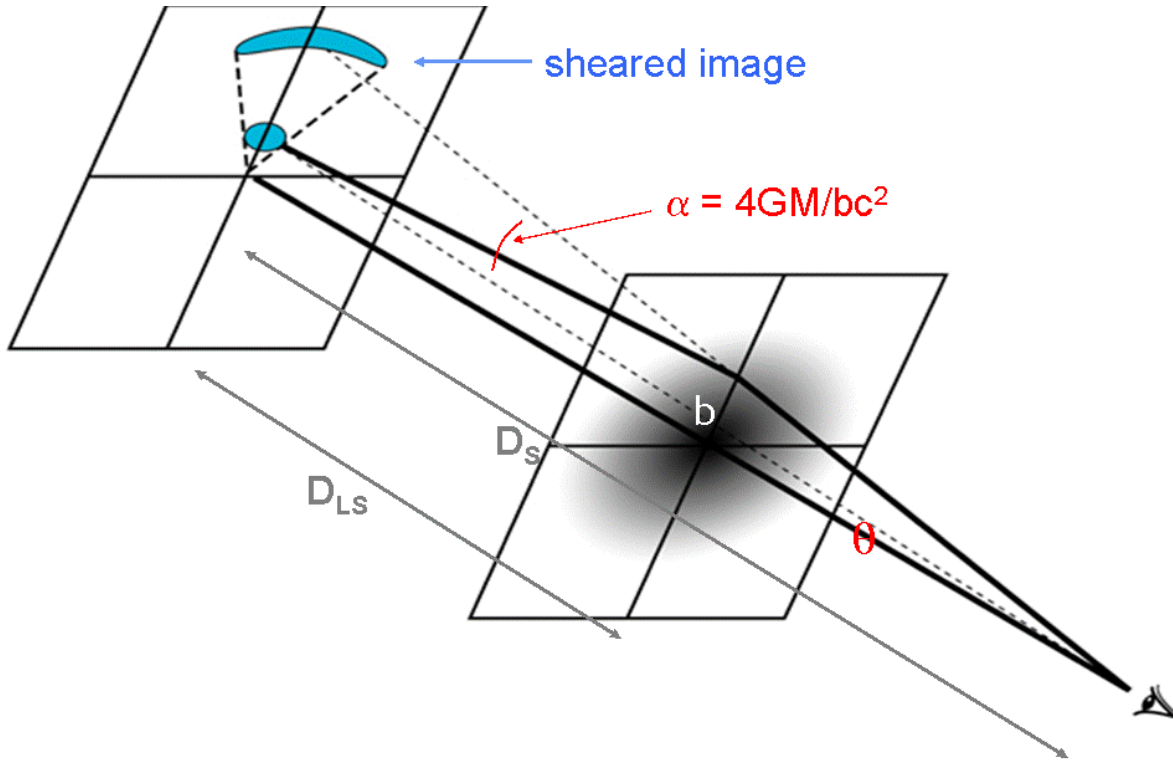


Figure 1.14: Cartoon depicting simple gravitational lensing of a background galaxy (blue, upper left) around a massive compact object (black, center). Light leaves the galaxy on the left, which is shown as a blue ellipse. As the light travels past the mass in the center of the image, it bends. When it reaches an observer (lower right), it appears distorted into a “banana” (labeled “sheared image”) and displaced by an angle α . Image reproduced from [59].



Figure 1.15: Galaxy cluster CL0024+1654 (yellow) acting as a gravitational lens and distorting a more distant galaxy (blue). Image reproduced from [61].

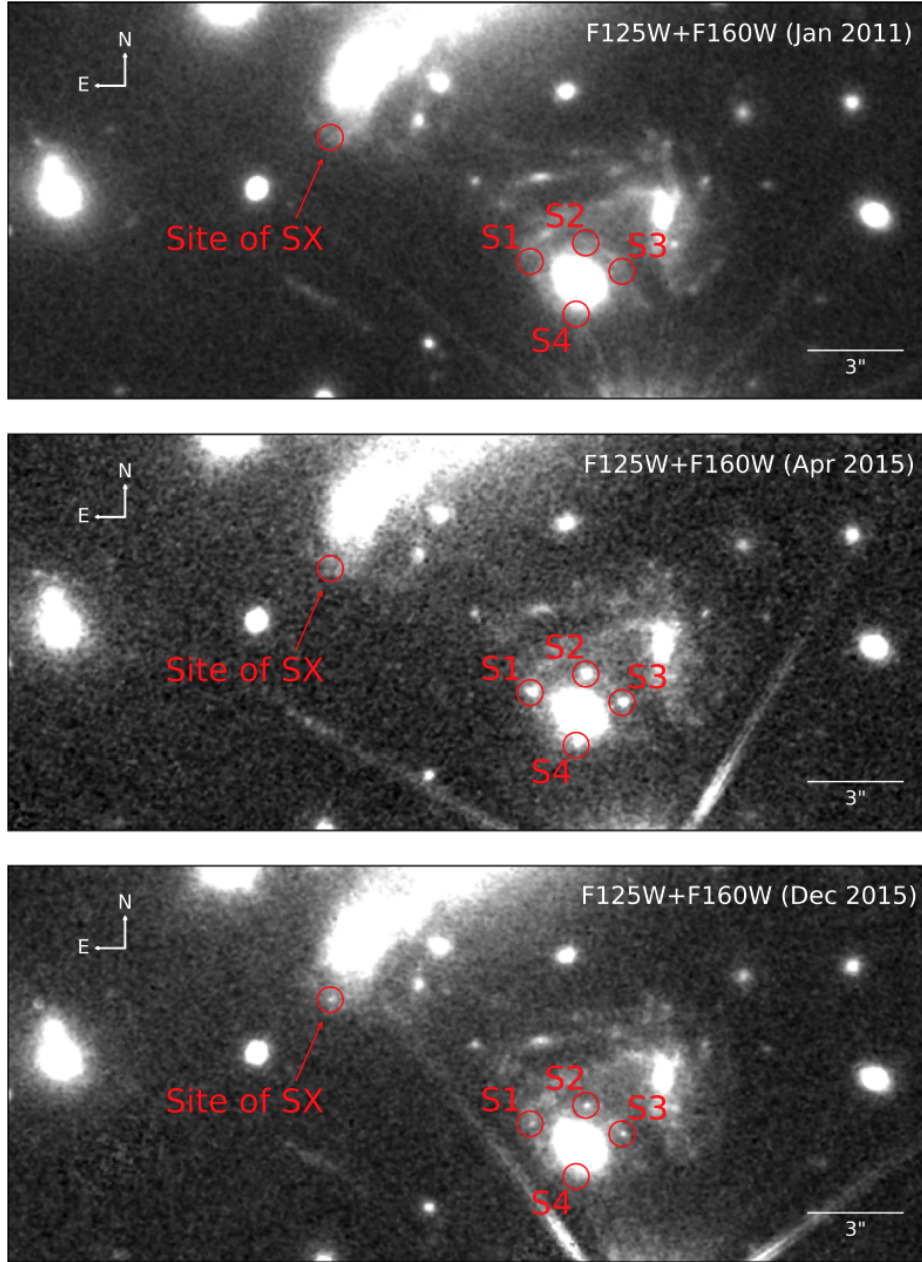


Figure 1.16: Three sequential images of supernova Refsdal showing the time delay of the received light that has been lensed through galaxy cluster MACS J1149+2223. The first panel is before the supernova. In the second, it appears four separate times (S1-S4). In the final panel, it is seen several months later (SX), and the previous four images have begun to fade. Figure reproduced from [63].

1.3.3 The bullet cluster and MOND

The bullet cluster is generally regarded as the smoking gun for the existence of DM. It was originally published in a paper confidently titled “A Direct Empirical Proof of the Existence of Dark Matter” [64]. The single figure from this paper is reproduced in Fig. 1.17. This figure is important because it shows a clear spatial separation between the mass density and the X-ray emissions from hot gas in two colliding clusters. This indicates that clumps of DM are able to pass through each other, while gas is not. This places strict limits on the self-interaction of DM.

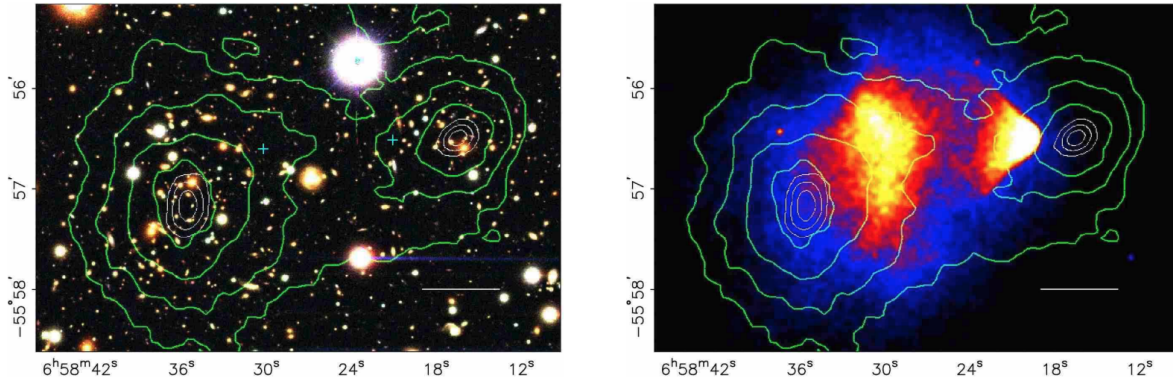


Figure 1.17: The bullet cluster. Two galaxy clusters colliding, shown in optical and X-ray, with contours showing mass density. The majority of the baryonic mass of the system is contained in hot gas, which emits X-rays and is imaged with the Chandra space telescope (heat map, right panel). The characteristic “V” shape of the right cluster’s gas indicates a shock wave caused by the interaction of the gas. However, as can be seen from the mass contours, the dark matter is spatially separated in the two clusters and remains mostly round, indicating a lack of DM/DM and DM/baryon interactions. This is regarded as the smoking gun for the existence of dark matter. Figure reproduced from [64].

At the time of publication, it was expected this observation would eliminate the theories of modified Newtonian gravity (MOND), which attempt to explain gravitational effects without

involving DM. However, since then updated MOND theories have popped up. The history of these theories and their trajectories through 2018 is reviewed in [5] and references therein. A 2023 analysis of wide binary stars observed by the GAIA satellite is consistent with Newtonian gravity and excluded MOND at 16σ [65].

1.3.4 Dark matter galactic dynamics

In order to understand how clumps of DM (known as halos) in the context of galaxies, it is important to consider how gas behaves when collapsing under gravity. The ability of gas to self-interact means the gas is able to radiate some of its energy away and cool. While this cooling means energy is lost, angular momentum is conserved. This process is known as dissipative collapse, and results in the disk-like structures of galaxies that we observe. A schematic of the Milky Way is shown in Fig. 1.18. The study of galaxy formation is complex and many details are omitted here. See, for example [10, 25] for an overview of the subject.

In contrast to the process of dissipative collapse, DM is effectively collisionless, at least on scales relevant to galactic dynamics. Clumps of dark matter (known as halos) conserve both angular momentum *and* energy, leading to DM halos which are roughly spherically symmetric. A review of this process and an excellent summary is provided in Ch. 2.1 of [4];

In summary, the classical theory of collapse of isolated, spherical, collision-less and dissipation-less systems predicts equilibrium DM halos that are singular isothermal spheres with exactly Maxwell-Boltzmann speed distributions. This is often called the *Standard Halo Model* (SHM).

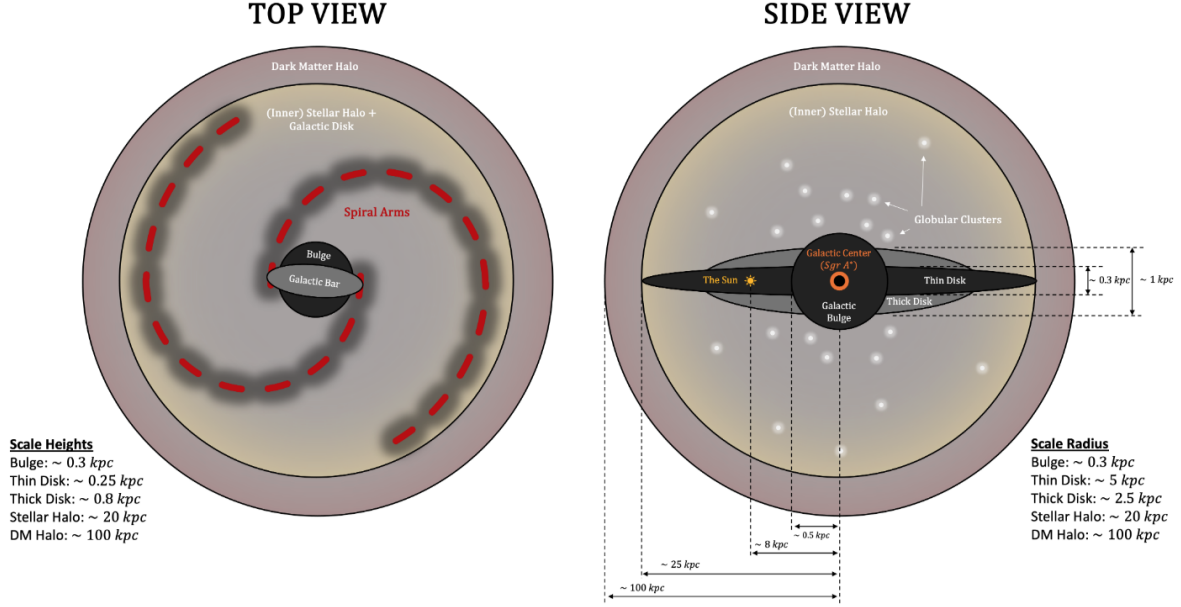


Figure 1.18: Schematic diagram of the Milky Way. The critical feature is the main disk formed by the gas and stars in contrast to the spherically symmetric halo formed by the DM. Figure reproduced from [66]

This SHM is helpful in determining the properties of our own dark matter halo, which are critical in direct detection experiments. The local energy density provides insight as to how much DM we can expect an experiment to have the opportunity to interact with. DM exclusion limits from direct detect experiments rely heavily on the density. Figure 1.19 shows estimates of the local dark matter density at our position in the Galaxy between 1922 and 2021. At the time of writing, common values of the local DM energy density ρ_{DM} range between 0.3 and $0.45 \text{ GeV}/\text{cm}^3$. It is understood that exclusion limits all depend on their assumed value of ρ_{DM} , and the scaling of the limit with ρ_{DM} is provided to update limits as improved measurements of ρ_{DM} become available. In this work, I will follow [67] and use $\rho_{\text{DM}} = 0.45 \text{ GeV}/\text{cm}^3$

The SHM also predicts a Maxwell-Boltzmann speed distribution. This provides a known, relative motion between our detectors and the halo resulting in a distribution of kinetic energies about the rest energy. The result is a small spread in observed energy of one part in 10^6 with a Rayleigh distribution.

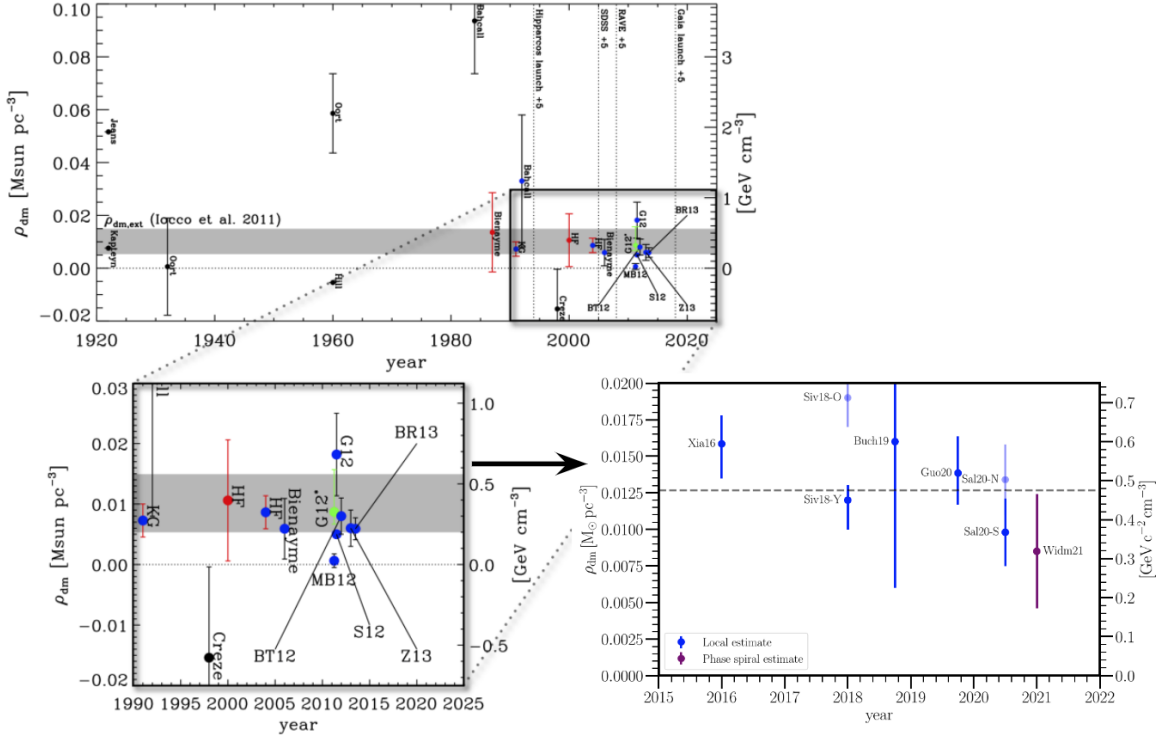


Figure 1.19: Estimates of local dark matter density at our position in the Galaxy ρ_{DM} from 1922 to 2021. Figures are reproduced from [68] and [69], but are combined here.

1.4 MACHOs, WIMPs and WISPs, oh my!

With the evidence in strong support of the dark matter theory laid out in the previous sections, this section will provide a roadmap through a few dark matter (DM) candidates of

interest. Figure 1.20 provides an overview of the possible mass scales relevant to many of the candidates under investigation. This section focuses on weakly interacting massive particles (WIMPs) and weakly interacting slim particles (WISPs). Within the WISP paradigm, only Axions and Dark Photons are discussed. For a detailed review of the vast DM landscape, see [4, 8]. For a review specific to particle-like DM see [7, 39]. Similarly, for wave-like DM see [70].

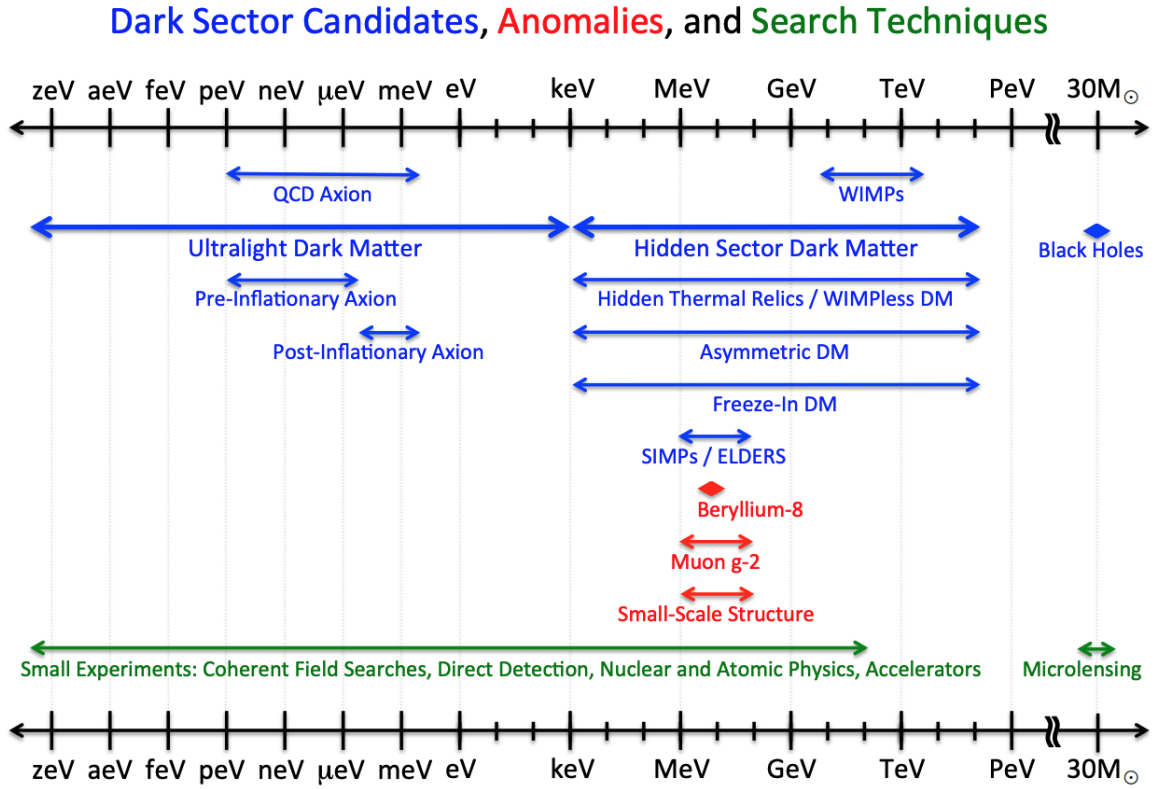


Figure 1.20: Mass ranges for various dark matter candidates (blue), experimental anomalies (red) and search techniques (green). The mass range covers 36 orders of magnitude before the scale break. Figure reproduced from [8].

The bounds on dark matter mass can be understood through the smallest observed structures known to contain DM; dwarf galaxies. These objects are roughly 1 kpc in diameter,

10 km/s velocity and $5 \times 10^5 M_\odot$ [4]. They are an excellent laboratory for DM since they contain few stars, and their mass is mostly made of DM. The lower bound on DM mass is set such that the de Broglie wavelength of a DM particle must fit within a dwarf galaxy. This lower bound has traditionally been estimated to be $m \gtrsim 10^{-22}$ eV, but has been recently calculated more rigorously as $m > 2 \times 10^{-21}$ eV (95% CL)[71]. The upper bound is set slightly lower than the mass of a dwarf galaxy so that there would be a few DM particles in the halo; $m \lesssim 10^4 M_\odot$ or $m \lesssim 10^{70}$ eV.

This enormous mass range of ~ 92 orders of magnitude (!) can be conveniently split into three regimes: waves, particles and composite objects. The local wave-particle boundary is at approximately 1 eV assuming $\rho_{\text{DM}} = 0.45 \text{ GeV/cm}^3$. At lower masses, the number density would be higher than ~ 1 per de Broglie wavelength at our local DM density, and the particles would condense into a field. At the particle-composite object boundary, DM particles are so massive and sparse that they no longer behave as a “cloud” of individual particles but instead as single massive particles like black holes or dense clusters. These latter objects are generically referred to as massive compact halo objects (MACHOs), though they are generally disfavored candidates since the rates of discrete microlensing events should be higher than we observe. The upcoming Vera Rubin Observatory survey will probe this parameter space further [72].

In the following subsections, various candidates and constraints will be explored from all three mass regimes outlined above.

1.4.1 WIMPs

For the past four decades, weakly interacting massive particles (WIMPs) were thought to be the most promising class of DM candidates[73]. WIMPs initially appeared to be an ideal solution, as particles with masses at the weak scale (roughly 10 GeV to 1 TeV) would naturally be produced in the early Universe with the correct relic abundance [39]. The big idea is *thermal production*, and is similar to Big Bang nucleosynthesis (BBN, see Sec. 1.2.2.3) in that a freeze-out occurs when the reaction rate drops below the Hubble rate. While the BBN freeze-out occurs around 10^5 eV, the WIMPs must freeze out around 100 GeV. Additionally, WIMPs offered a potential resolution to the long-standing supersymmetry motivated hierarchy problem [74], a central focus of particle physics over the same period. However, despite extensive experimental efforts, no new particles beyond the Standard Model (SM) Higgs boson have been conclusively detected at the weak scale.

WIMP direct detection is a class of experiments that attempt to observe the scattering of WIMPs off of target nuclei. Many such experiments have been completed, probing extremely small cross-sections over the predicted mass range. Figure 1.21 is reproduced from the Particle Data Group’s (PDG) 2024 review [9] and presents the exclusion limits of spin-independent (SI) WIMP cross section vs. mass as reported. An excellent review of the history and current progress of these experiments is given in [7]. An important consideration of these direct-detection experiments is the so-called neutrino fog, labeled *Neutrino coherent scattering* in Fig. 1.21. Solar neutrinos have a very small cross section, and are therefore not shielded; the mean-free-path of solar neutrino in lead is famously on the order of 1 light-year. In a

normal experiment, this tiny cross section means a neutrino will pass through the detector. Unfortunately, these experiments are not "normal"; their exposures are measured in tonne-years, and a given run will have a few thousand radioactive background events that can not be vetoed [75].

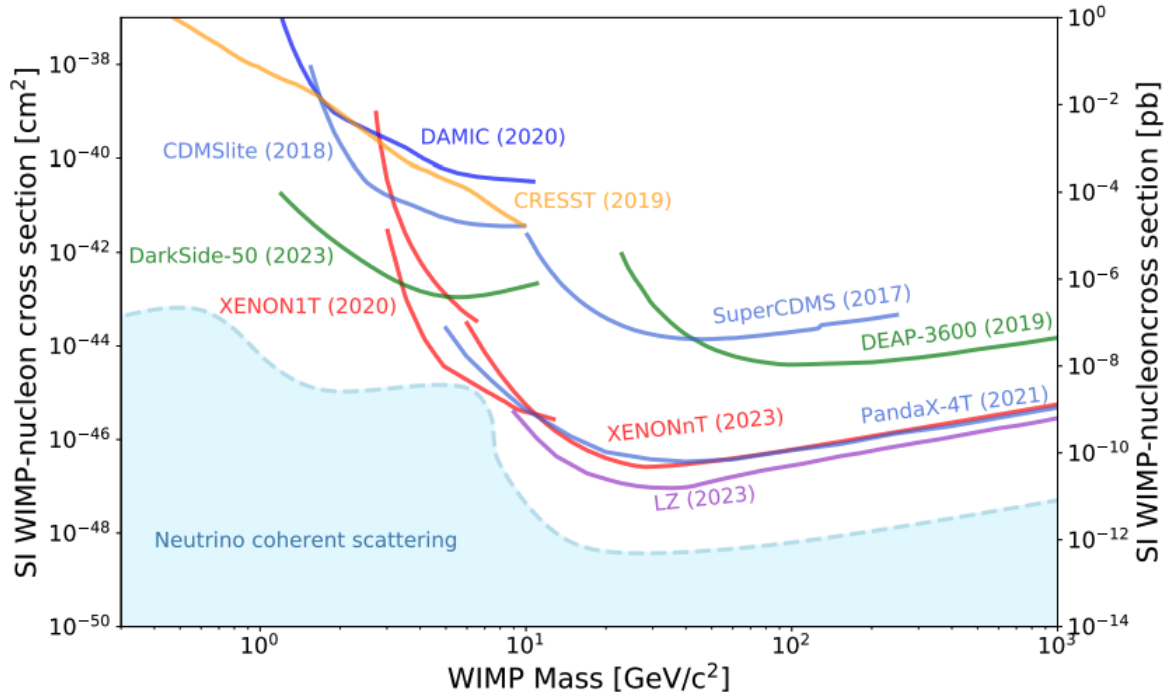


Figure 1.21: 2023 exclusion limits on spin-independent (SI), WIMP-nucleon cross section vs. mass. This figure was published by the particle data group in 2024, but the most recent limit is 2023. Figure reproduced from [9].

Figure 1.21 provides a big-picture view; however, recently, a few more limits have been published. These are shown in Fig 1.22. Planned experiments will push these limits lower in the coming years [7, 76].

The lack of direct detection of a WIMP (not to mention the lack of supersymmetry at accelerators) raises the possibility that natural WIMPs may have been an appealing but

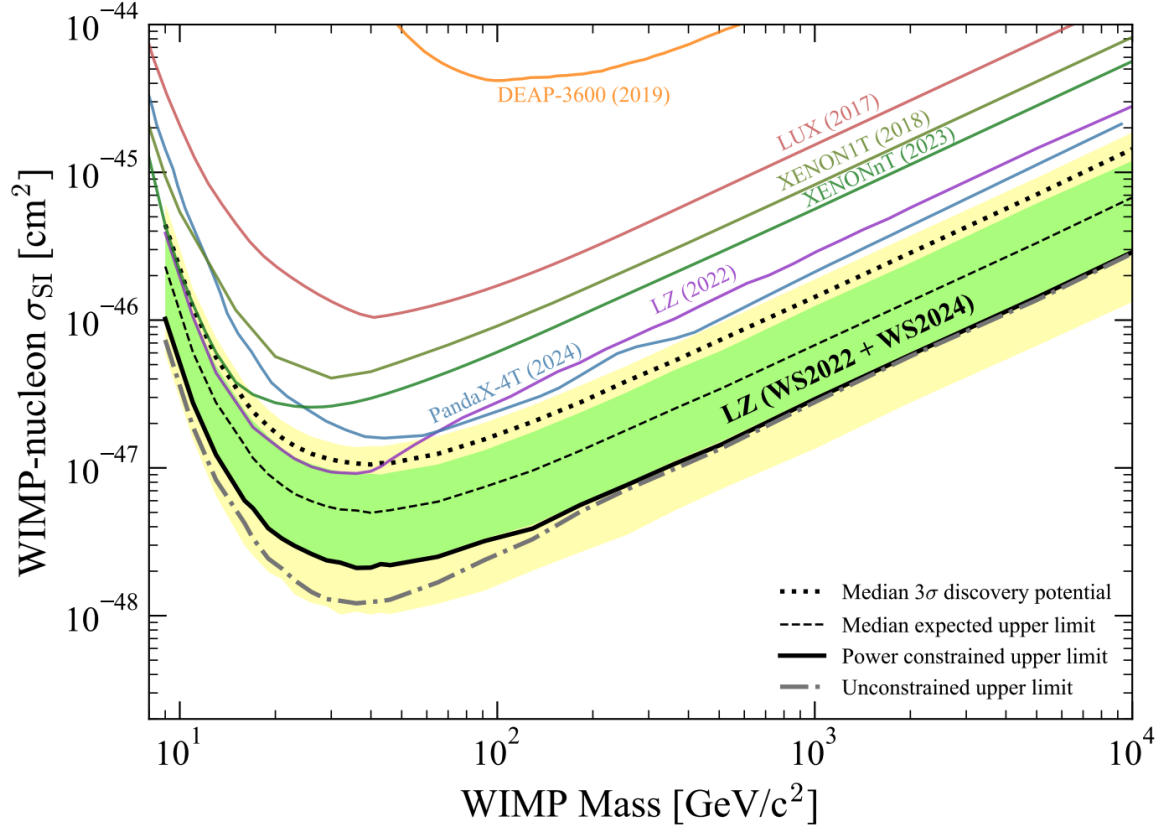


Figure 1.22: 2024 exclusion limits on spin-independent (SI), WIMP-nucleon cross section vs. mass. This figure shows a zoomed-in section of Fig. 1.21, and is a few years more recent, showing the improved LZ, PandaX-4T and XENON1T limits. Figure reproduced from [75]

ultimately misleading hypothesis. A 2010 review [77] of the field predicted an absence of evidence for WIMPs would lead to them falling out of favor within the next 5-10 years, which appears to be the current state. As experiments continue to close in on the neutrino fog, the chances of directly detecting a WIMP continue to fall.

1.4.2 WISPs

Weakly interacting slim particles (WISPs) are weakly interacting, massive AND particles. Yet WISPs are distinct from WIMPs because of their significantly lower mass and their non-thermal production.

The three main classes of WISPs are axions, axion-like-particles (ALPs) and dark photons (also known as hidden photons). This subsection will focus on axions and dark photons. ALPs are a generalization and will be briefly described in the axion section.

1.4.2.1 Axions

Axions were originally proposed by Peccei and Quinn in 1977 as a solution to the strong Charge-Parity (CP) problem of quantum chromodynamics (QCD) [78]. In the early 1980s, Axions were shown to be a nice solution to the dark matter problem [79–81].

Strong CP problem A gentle overview of the strong CP problem by way of analogy is given by Pierre Sikivie in [82]. In this analogy, a pool player who lived his life on the pool table and believes the symmetry of pool balls to have no preferred side of the table represents a universal symmetry S . One day, our hero jumps off the table only to learn the floor is not horizontal. He is shocked to find that S is broken! But why should the pool table be flat (respect S) when the rest of the world does not? After measuring the pool table’s angle to one part in 10^9 , he finds the table’s angle is 0 to within the precision of his experiment. The situation here is analogous to the strong CP problem. It was shocking in 1964 to learn that CP symmetry was not obeyed [83]. If it’s not obeyed, why is the neutron’s electric dipole

moment constrained to be $< 10^{-9}$? The analogy continues, inventing a pendulum-based mechanism to level the pool table, whose oscillations can be measured (analogous to the axion) that was set in motion by the pool table’s deceleration during its delivery to Mars on a rocket (analogous to the switch-on of non-perturbative QCD effects when the temperature of the Universe was $\sim \text{GeV}$).

A more mathematically rigorous (yet approachable) explanation is provided in Anson Hook’s lecture notes [84], and the reader is encouraged to consult them to connect the pieces provided below (which follow [85]).

The QCD Lagrangian $\mathcal{L}_{\text{SM}}$ contains a term which violates CP symmetry such that

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{SM}} + \frac{\theta g_s^2}{32\pi^2} G\bar{G}, \quad (1.40)$$

where $\mathcal{L}_{\text{SM}}$ is the standard model Lagrangian, G is the gluon field strength tensor so that $G\bar{G} \equiv G_{\mu\nu}\bar{G}^{\mu\nu}$, with the dual $\bar{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$, and g_s is the strong coupling constant. Peccei and Quin’s solution [78] is to introduce a new $U(1)$ symmetry, making θ a dynamic parameter. The spontaneous breaking of this Peccei-Quinn (PQ) symmetry means that θ will naturally relax to 0. The particle associated with this breaking of PQ creates a new pseudo-Nambu-Goldstone boson, independently shown by both Weinberg [86] and Wilczek [87] in 1978. The name axion was coined by Wilczek [88]. Axion is the name of a popular dish soap, and since axion particles “clean up” both the DM problem and strong CP problem, the name stuck.

Axion Dark matter Axions nicely respect the requirements of cold, non-interacting DM. Astronomical bounds restrict their mass to $10^{-5} < m < 10^{-3} \text{ eV}$. They could have been produced in the early universe via the misalignment mechanism [89]. This is a non-thermal production mechanism, in contrast to the WIMP.

The axion has an electromagnetic interaction governed by the Lagrangian

$$\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}, \quad (1.41)$$

where a is the axion field, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, respectively of two photons and $g_{a\gamma\gamma}$ is the axion-photon-photon coupling (see Fig. 1.23). The mass and coupling are linearly related, and the constant of proportionality is either 0.38 or 0.14 for the KSVZ (Kim-Shifman-Vainshtein-Zakharov) [90, 91] and more conservative DFSZ (Dine-Fischler-Srednicki-Zhitnitskii) [92, 93] models respectively. In other words,

$$g_{a\gamma\gamma}^{KSVZ} \approx 0.38 \frac{m_a}{\text{GeV}^2} \quad (1.42)$$

$$g_{a\gamma\gamma}^{DFSZ} \approx 0.14 \frac{m_a}{\text{GeV}^2}. \quad (1.43)$$

Panel B of Fig. 1.23 hints that axions in our own galactic DM halo may be visible with a strong enough magnetic field. This class of experiment is called a haloscope. The predicted couplings of Eq. 1.42 and 1.43 provide a target for axion haloscope experiments, and appear as red lines surrounded by yellow in Fig. 1.24. These targets assume that axions are all of the DM.

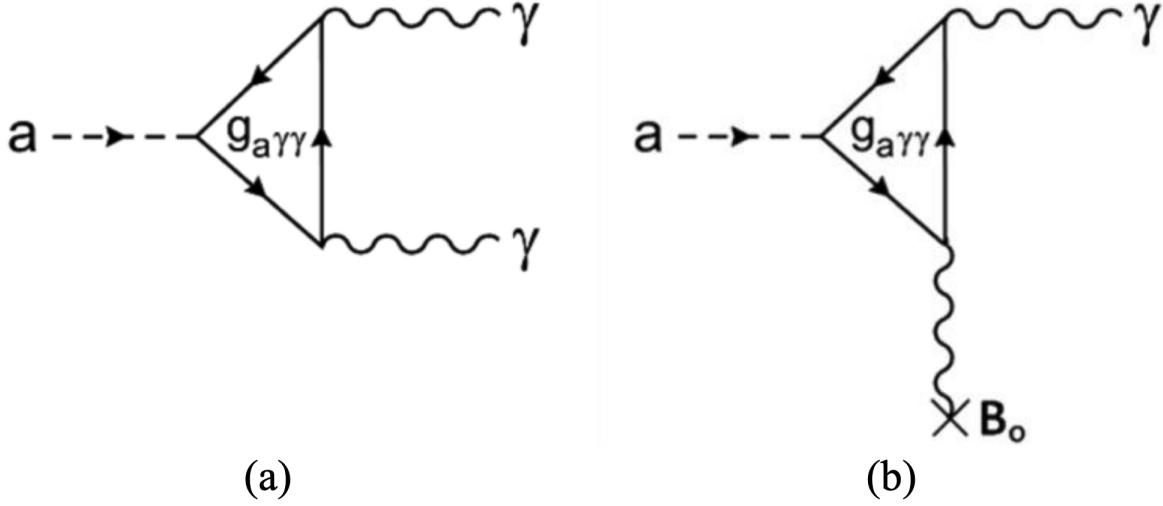


Figure 1.23: Feynman diagrams showing an axion decaying into two photons. Panel A shows the decay in vacuum, with the coupling $g_{a\gamma\gamma}$. Panel B shows the conversion in a static magnetic field $\mathbf{B}_0$. In the presence of $\mathbf{B}_0$, one of the photons is replaced by a virtual photon via the inverse Primakoff effect [94, 95]. Figure reproduced from [85].

ALPs ALPs are a generalization of the axion. Similar to axions, ALPs are pseudo-scalars and make interesting dark matter candidates. ALPs, however, do not help with the strong CP problem. While the axion-photon coupling $g_{a\gamma\gamma} \propto m_a$ (Eqs. 1.42 and 1.43), this coupling is relaxed for ALPs. This means mass and coupling are independent for ALPs, such that a search for an ALP is not constrained to a band in the mass-coupling plane (such as the KSVZ or DFSZ models, seen on Fig. 1.24)

1.4.3 Dark photons

Dark photons (DP) are another DM candidate first proposed in [97]. They could be detected using electromagnetic experiments similar to axions, though they do not require a magnetic field to convert. Instead, they convert to SM photons via a process known as kinetic mixing.

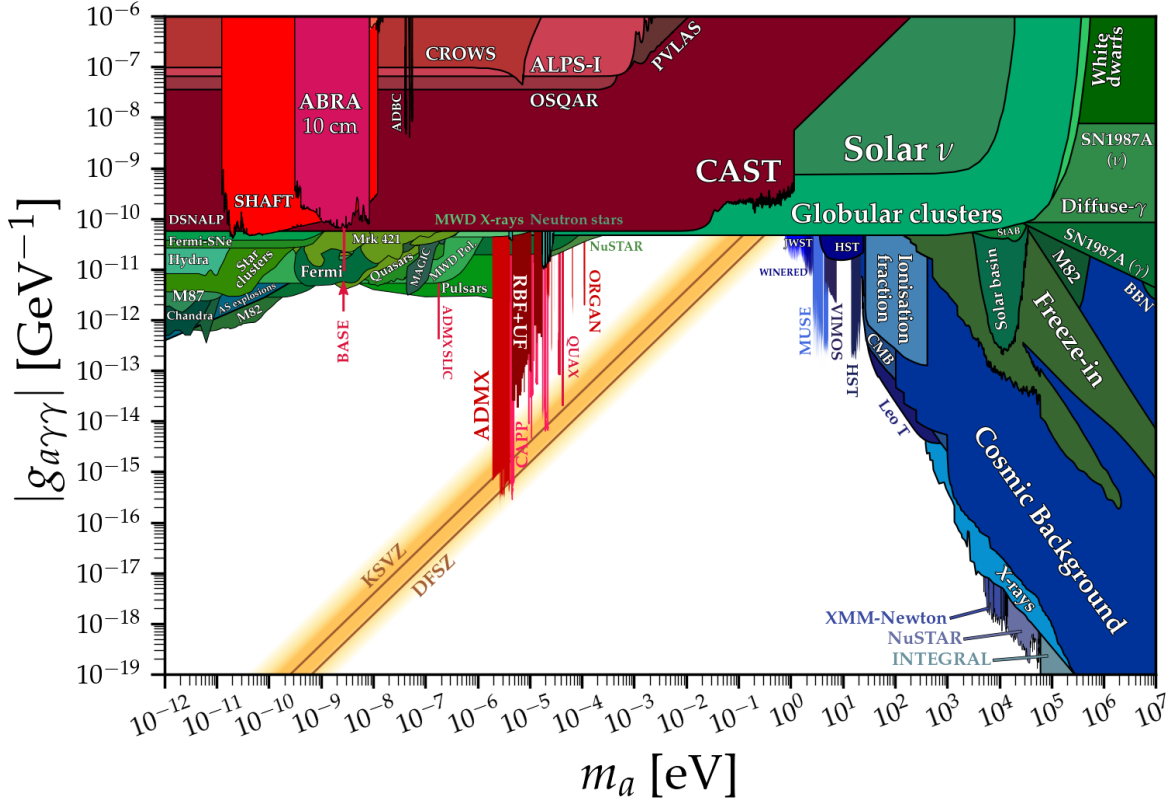


Figure 1.24: Exclusion limits on axion-photon coupling $g_{a\gamma\gamma}$. The haloscope limits (red, center. ADMX, ORCAN etc.) assume that axions are all of the DM and that the local DM density $\rho_{\text{DM}} = 0.45 \text{ GeV}/\text{cm}^3$. Figure reproduced from [96]

They are an example of the growing interest in a dark sector in order to solve the DM problem. There are many dark sector theories, but the common theme is a new sector of physics that interacts with the SM through gravity and hopefully through some “portal” such as kinetic mixing. This portal phenomenology is discussed, for example, in [98]. If there is only gravitational interaction, we would have little hope of directly observing the dark sector.

DPs are low-mass gauge bosons. Similar to axions, they could have been produced from the misalignment mechanism [97]. However, they also could have been produced through

quantum fluctuations of the inflaton field [99]. The latter case is interesting since the power spectrum of a vector is peaked at intermediate length scales (not scale-invariant like a scalar or tensor), and falls off at large scales, allowing agreement with the CMB measurements, namely a lack of large-scale correlations as in Fig. 1.12. As discussed in the introduction of Sec. 1.4, if the DP represented all of the local DM and had sufficiently low mass $\lesssim 1\text{eV}$, the local number density would be much greater than 1 per de Broglie wavelength, and the DP would oscillate coherently, behaving as a wave. The frequency of this field would be set by the mass, with a small dispersion due to the Maxwell-Boltzmann velocity distribution predicted by the standard halo model (see Sec. 1.19). Converting the free space DP energy density ρ_{DM} to the amplitude of an electric field is very similar to its classical counterpart, but with suppression by ε ,

$$|\mathbf{E}| \approx \varepsilon \sqrt{\frac{2}{\epsilon_0} \rho_{\text{DM}}}. \quad (1.44)$$

Where ε is the dimensionless coupling and ϵ_0 is the permittivity of free space.

This section closely follows Ben Godfrey’s dissertation [1]. I am grateful to Ben for leaving such an accessible summary of the field. A 2021 review of the DP, including production mechanisms and phenomenology, can be found in [100]. Finally, the notation used to denote the different bases is influenced by Peter Graham et al.[101, 102], specifically Appendix A of [101].

1.4.4 Dark photon physics

The DP arises from an extension of the SM whereby an additional $U(1)$ is introduced [103], denoted here as $U'(1)$. There are two main components to this. First, a Lagrangian representing the massive DP must be described in light of $U'(1)$. This is written in the so-called *kinetically mixed* basis. After this, a rotation of this Lagrangian into the *mass basis* is performed to diagonalize the mass term such that the interaction of the DP with SM charge-current J_μ^{SM} is clear.

For two Abelian gauge bosons associated with the gauge groups $U(1)$ and $U'(1)$, the most general form of the kinetic terms in the Lagrangian are [100, 103]

$$-4\mathcal{L}_{\text{K, kinetic}} = F_{\mu\nu}F^{\mu\nu} + F'_{\mu\nu}F'^{\mu\nu} + 2\varepsilon F_{\mu\nu}F'^{\mu\nu}, \quad (1.45)$$

where $F_{\text{K},\mu\nu}^{(\prime)} \equiv \partial_\mu A_\nu^{(\prime)} - \partial_\nu A_\mu^{(\prime)}$ is the field strength tensor of each boson in the kinetically mixed basis (indicated by the K subscript), and ε is the small ($\ll 10^{-3}$) dimensionless coupling between the fields. For clarity, the K subscript is suppressed on the right-hand side, here and wherever the basis is specified by the left-hand side.

If $U(1)$ is associated with the SM photon $A_{\text{K},\mu}$, we know that in the kinetically mixed basis, the interaction term of the Lagrangian is $eJ_\mu^{\text{EM}}A_K^\mu$, where e is the coupling constant, in this case, the electron charge. Therefore, it is natural that the Lagrangian contains an interaction term

$$\mathcal{L}_{\text{K, interaction}} = eJ_\mu^{\text{EM}}A^\mu + e'J'_\mu A'^\mu, \quad (1.46)$$

for the DP A'^μ , dark coupling e' and dark charge-current J'_μ .

Finally, a mass term can be added such that the DP acquires a mass m' by the Stueckelberg mechanism¹⁶

$$\begin{aligned}
-2\mathcal{L}_{\text{K, Stueckelberg}} &= m_\gamma^2 A_\mu A^\mu + m'^2 A'_\mu A'^\mu + 2m'_\gamma A_\mu A'^\mu \\
&= m'^2 A'_\mu A'^\mu.
\end{aligned} \tag{1.47}$$

In the second line, the terms containing the SM photon mass m_γ have been set to zero.

Now, an electromagnetic Lagrangian can be constructed as a sum of the kinetic, interaction and Stueckelberg mass terms (Eqs. 1.45 – 1.47),

$$\begin{aligned}
\mathcal{L}_{\text{K}} &= \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\varepsilon}{2}F_{\mu\nu}F'^{\mu\nu}}_{\text{kinetic}} \\
&\quad - \underbrace{\frac{1}{2}m'^2 A'_\mu A'^\mu}_{\text{mass}} \\
&\quad - \underbrace{eJ_{\text{EM}}^\mu A_\mu - e'J'^\mu A'_\mu}_{\text{interaction}}.
\end{aligned} \tag{1.48}$$

As written, $A_{\text{K},\mu}^{(\prime)}$ interact only with their own SM (dark) charge-currents. However, the kinetic terms contain a mixing term between the dark sector and SM fields $\varepsilon F_{\mu\nu}F'^{\mu\nu}$. For this reason, this basis is denoted as the kinetically mixed basis, again denoted by the K subscript. When considering an experiment to search for DPs, one needs to consider interactions between the DP and matter described by the SM. This is achieved by diagonalizing Eq. 1.48 into the so-called mass basis (also sometimes referred to as the vacuum or vacuum mass basis [101]). In this basis, $A_{\text{M},\mu}^{(\prime)}$ are the massless (massive) propagating eigenmodes in vacuum. Furthermore, linear combinations of these modes can be formed, which either

¹⁶A review of the subject can be found in [104]. Also see Sec. 1.3.2 of [1].

interact with J^{EM} or are sterile. These are not propagating eigenmodes, however, and will evolve into each other. This is discussed further in [102].

Now, $\mathcal{L}_K$ will be rotated into the mass basis $\mathcal{L}_M$. The rotation is reached by a non-unitary field redefinition^{17,18}

$$\begin{pmatrix} A_M \\ A'_M \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon \\ 0 & \sqrt{1-\varepsilon^2} \end{pmatrix} \begin{pmatrix} A_K \\ A'_K \end{pmatrix}. \quad (1.49)$$

Applying this transformation to $\mathcal{L}_K$ yields

$$\begin{aligned} \mathcal{L}_M = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} \\ & -\frac{m'^2}{2(1-\varepsilon^2)}A'_\mu A'^\mu \\ & -eJ_{\text{EM}}^\mu \left[A_\mu + \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A'_\mu \right] - e'J'^\mu \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}A'_\mu. \end{aligned} \quad (1.50)$$

or, neglecting $\mathcal{O}(\varepsilon^2)$ terms,

$$\begin{aligned} \mathcal{L}_M \approx & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} \\ & -\frac{1}{2}m'^2A'_\mu A'^\mu \\ & -eJ_{\text{EM}}^\mu [A_\mu + \varepsilon A'_\mu] - e'J'^\mu \varepsilon A'_\mu. \end{aligned} \quad (1.51)$$

From Eqs. 1.50 and 1.51, it is clear that the Lagrangian (written in the mass basis) contains a term that describes the interaction of $A'_{M,\mu}$ and J_μ^{EM} ,

$$\mathcal{L}_M \supset -\frac{e\varepsilon}{\sqrt{1-\varepsilon^2}}J_{\text{EM}}^\mu A'_\mu \approx -e\varepsilon J_{\text{EM}}^\mu A'_\mu. \quad (1.52)$$

¹⁷In the more general case of the massless DP, this involves a mixing angle θ , and there would be a rotation matrix $R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ applied to the A_K vector. By simultaneously diagonalizing the mass terms, θ is fixed to 0, so R is the identity matrix, and this rotation is neglected (see [1, 100, 101]).

¹⁸Note that the column vectors have the unprimed term on top. This follows [101], but is opposite of [1, 100].

From here the connection back to the experiment becomes more clear. The equations of motion follow from Eq. 1.50 [102]

$$(\partial_t^2 - \nabla^2)V = \rho_{\text{EM}}, \quad (\partial_t^2 - \nabla^2)\vec{A} = \vec{J}_{\text{EM}}, \quad \dot{V} + \nabla \cdot \vec{A} = 0, \quad (1.53)$$

$$(\partial_t^2 - \nabla^2 + m'^2)V' = \varepsilon\rho_{\text{EM}}, \quad (\partial_t^2 - \nabla^2 + m'^2)\vec{A}' = \varepsilon\vec{J}_{\text{EM}}, \quad \dot{V}' + \nabla \cdot \vec{A}' = 0. \quad (1.54)$$

Where Eqs. 1.53 are Maxwell's equations, and Eqs. 1.54 are Proca's equations in the Lorentz gauge with $c = 1$. Here, the scalar potential is V , vector potential $\vec{A}$, charge density ρ_{EM} and current density $\vec{J}_{\text{EM}}$. Note that the normal, classical definitions of $\vec{E}$ and $\vec{B}$ in the Lorentz gauge apply nicely to the dark fields and potentials,

$$\begin{aligned} \vec{E}^{(\prime)} &= -\nabla V^{(\prime)} - \dot{\vec{A}}^{(\prime)} \\ \vec{B}^{(\prime)} &= \nabla \times \vec{A}^{(\prime)} \end{aligned} \quad (1.55)$$

The Lorentz force can be modified simply to account for the dark fields,

$$\vec{F} = q \left[\left(\vec{E} + \varepsilon \vec{E}' \right) + \vec{v} \times \left(\vec{B} + \varepsilon \vec{B}' \right) \right]. \quad (1.56)$$

For a DP density ρ' , the free space electric field looks just like its classical counterpart for a photon density, but with ε suppression

$$|\mathbf{E}| = \frac{\varepsilon}{\sqrt{1 - \varepsilon^2}} \sqrt{\frac{2}{\epsilon_0} \rho'} \approx \varepsilon \sqrt{\frac{2}{\epsilon_0} \rho'}. \quad (1.57)$$

Note that ε is the dimensionless coupling introduced in Eq. 1.48, and ϵ_0 is the permittivity of free space.

1.4.5 Searching for dark photon dark matter

The relic density of dark photons with density ρ_{DM} can interact with charge-currents in an experiment as shown above. This is the principle behind the Dark E-field Radio experiment. The two unknowns m' and ε define a parameter space that can be probed by sensitive electromagnetic experiments. m' and ε are analogous to m and $g_{a\gamma\gamma}$ of the axion. However, in the axion case, they are related by Eqs. 1.42 and 1.43, whereas for the DP, they are both free parameters and the unconstrained m, ε parameter space must be probed. Current constraints on dark photon dark matter are shown in Fig. 1.25.

Figure 1.4.3 shows DP exclusion limits resulting from many sources, including direct detection and astrophysical observations. Various limits have been processed to appear on the same plot as discussed in [105]. Furthermore, uncertainty is not shown, leading to a slightly misleading view. However, this plot has become standard in the literature, so it is included here.

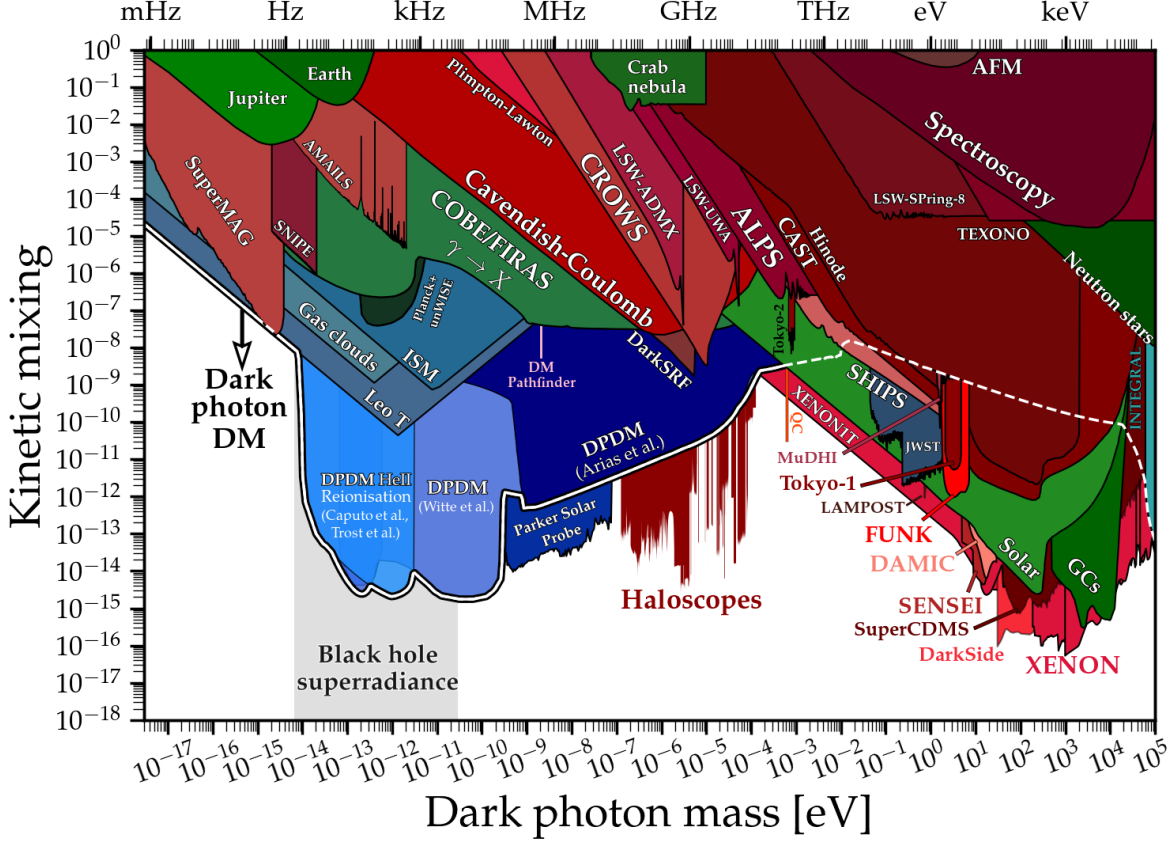


Figure 1.25: Exclusion limits on dark photon kinetic mixing parameter ϵ . The exclusion limit of Run 1A of the Dark E-field Radio experiment is included here as a component of the region labeled “Haloscopes”. These limits assume $\rho_{\text{DM}} = 0.45 \text{ GeV}/\text{cm}^3$, a 95% confidence limit and fixed DM polarization. See [105] for more details. This updated figure is reproduced from [96], but it first appeared in [105].

Chapter 2

Experiment Overview and Design

A month in the laboratory can often save an hour in the library.

Frank Westheimer

Veljko Radeka said of detectors, “One would imagine that in each particular case the best solution is arrived at by 1) the detector design to maximize the significant signal, 2) reduction of noise at its physical source, and 3) optimum filtering of signal and noise.” [106]. While he was referring to position-sensitive particle detectors, the same three principles apply to this experiment. Put more directly, the goal is to maximize the signal-to-noise ratio. The detector, in this case, is a low-noise, wide-band radio receiver system searching not for discrete instances of particle-like interactions but for coherent waves that are constant over long periods. The signal is a small RF-power excess on a noise background received by an antenna in a cavity. The dominant noise is due to the thermal background of the 300 K

cavity walls. This noise is nearly white, with small variations. See Sec. 2.1.1 for an overview and Sec. 2.3 for more detail. An overview of the entire experiment is shown in Fig. 2.1.

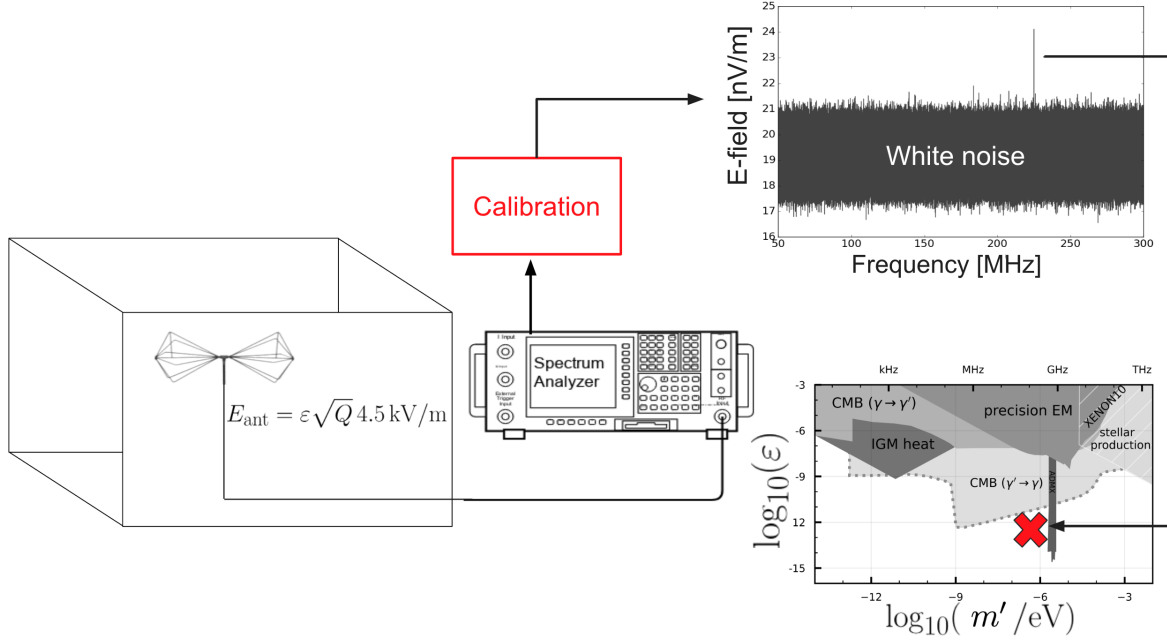


Figure 2.1: Overview of Dark E-field Radio experiment. The free space electric field strength was calculated in Sec. 1.4.4, and will be modified to include the Q of the cavity in Sec. 2.1.2. The design of the system (left and center) is outlined in this chapter, while its performance is measured in Ch. 3. The use of the system and its calibration (red box, and top right) are described in Ch. 4. Setting a limit on the dark photon’s coupling ε (lower right, red X) is discussed in Ch. 4.

This chapter begins with Sec. 2.1, devoted to exploring the sources of power in the measured spectrum. It walks through several back-of-the-envelope calculations to follow the signals and noises as they progress from fields in free space through a simplified detector. Section 2.2 takes the next step. It simulates these signals and noises and shows a framework for statistical data analysis. These two sections serve to build up intuition about the experiment which will explain design choices discussed in later chapters. Furthermore, this section

is useful because the actual experiment will report a null result, i.e. a lack of detection of a signal. By following a signal *forward* through the system and toy data analysis, it will be clearer how to infer an exclusion limit from a power spectrum and working *backward* through the experiment. In the following two sections, 2.3 and 2.4, effects will be introduced that were not apparent from the simplistic analysis of the first section. The final section, 2.5, details each piece of the system. Measurements of system performance are put off until Ch. 3.

A note on experiment “run numbers”. Run 1A is the 50-300 MHz run (defined, along with future planned runs in Table 6.1). There have been four attempts at collecting Run 1A data. The final, published data run [67] was collected between May 10 and May 19, 2023, and is called Run 1.4 or Run 1p4 throughout the file names in the referenced github. In this thesis, this run will be referred to as Run 1A or as Run 1A.4 if it is necessary to distinguish between specific runs.

2.1 Sources of Power in Measured Spectrum

This section will break down each term in the following equation for the input-referred power

$$P_i = \text{Thermal Noise} + \text{Dark Photon Signal} + \text{RFI} + \text{Amp Chain Noise} + \text{ADC Effects}/G, \quad (2.1)$$

where G is a gain factor. While convenient, this word equation is not rigorous and relies on the following subsections 2.1.1 - 2.1.5 for definition. Note that terms in Eq. 2.1 begin with

capital letters, and this is reflected throughout the section.

2.1.1 Thermal Noise

The primary source of power measured by the experiment is Thermal Noise. There is some subtlety involved, which is the topic of the following subsections.

2.1.1.1 Blackbody electric field density

This subsection estimates the noise-like¹ electric field in free space due to blackbody radiation. It assumes to be in *some* enclosure in that the ambient temperature is known to be 300 K and not, for example, the 3.6 K of the sky. The enclosure considered is assumed to be “black” in that all radiation impinging on its surface is absorbed. This assumption is introduced in Sec. 2.1.1.2 and investigated further in Sec. 2.3. A fascinating history of the physics involved here is given in [107].

Planck’s law gives the blackbody spectral energy density as

$$u_\nu(\nu, T) d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} d\nu \quad \left[\frac{J}{m^3} \right]. \quad (2.2)$$

This is frequently written in terms of spectral radiance,

$$B_\nu(\nu, T) d\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} d\nu d\Omega \quad \left[\frac{W}{m^2} \right]. \quad (2.3)$$

Integrating this isotropic radiance over a solid angle 4π sr as well as a small frequency band

$\Delta\nu$ gives the flux density $|\mathbf{S}|$ (AKA the magnitude of the Poynting vector),

¹“Noise-like” simply means that the power contained in a signal is proportional to the bandwidth measured. Noise-like signals are more conveniently described as a power spectral density (PSD) which we will describe with the symbol S . As we will see, coherent signals that have finite width in frequency space $\Delta\nu_{\text{sig}}$ can share this property if the measurement bandwidth $\Delta\nu_{\text{RF}} \lesssim \Delta\nu_{\text{sig}}$, even though one wouldn’t think of a coherent signal as being “noise-like”

$$\begin{aligned}
|\mathbf{S}| &= \int_0^{4\pi} \int_{\nu}^{\nu+\Delta\nu} B_{\nu}(\nu, T) d\nu d\Omega \\
&= \int_0^{4\pi} \int_{\nu}^{\nu+\Delta\nu} \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} d\nu d\Omega \quad \left[\frac{W}{m^2} \right] \\
&\approx \frac{8\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \Delta\nu
\end{aligned} \tag{2.4}$$

Poynting's theorem relates this flux density to the RMS electric field

$$|\mathbf{S}| = \frac{|E_{\text{rms}}|^2}{\eta} \quad \left[\frac{W}{m^2} \right], \tag{2.5}$$

where η is the impedance of free space. Equating Eqs. 2.4 and 2.5 and solving for the electric field gives

$$\frac{|E_{\text{rms}}|}{\sqrt{\Delta\nu}} = \sqrt{\eta \frac{8\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}} \quad \left[\frac{V}{m \cdot \sqrt{\text{Hz}}} \right], \tag{2.6}$$

and is plotted in Fig. 2.2.

Eq. 2.6 breaks up nicely into two regimes,

$$\frac{|E_{\text{rms}}|}{\sqrt{\Delta\nu}} = \begin{cases} \sqrt{\eta \frac{8\pi kT\nu^2}{c^2}} & \text{Rayleigh-Jeans regime } (h\nu \ll kT) \\ \sqrt{\eta \frac{8\pi h\nu^3}{c^2} e^{-h\nu/kT}} & \text{Wien approximation } (h\nu \gg kT) \end{cases} \quad \left[\frac{V}{m \cdot \sqrt{\text{Hz}}} \right]. \tag{2.7}$$

At frequencies and temperatures where the experiment is operated (< 300 MHz and 300 K), $h\nu/kT \lesssim 5 \times 10^{-5}$ suggesting the Rayleigh-Jeans approximation is valid. At 300 K, this yields electric field spectral densities of 1 and 6 nV/(m $\sqrt{\text{Hz}}$) at 50 and 300 MHz respectively.

It is interesting to note, however, that for cryogenic experiments operating at a few GHz and in the sub-K range (A common technique; see, for example, [108–110]), $h\nu/kT \approx 1$ and

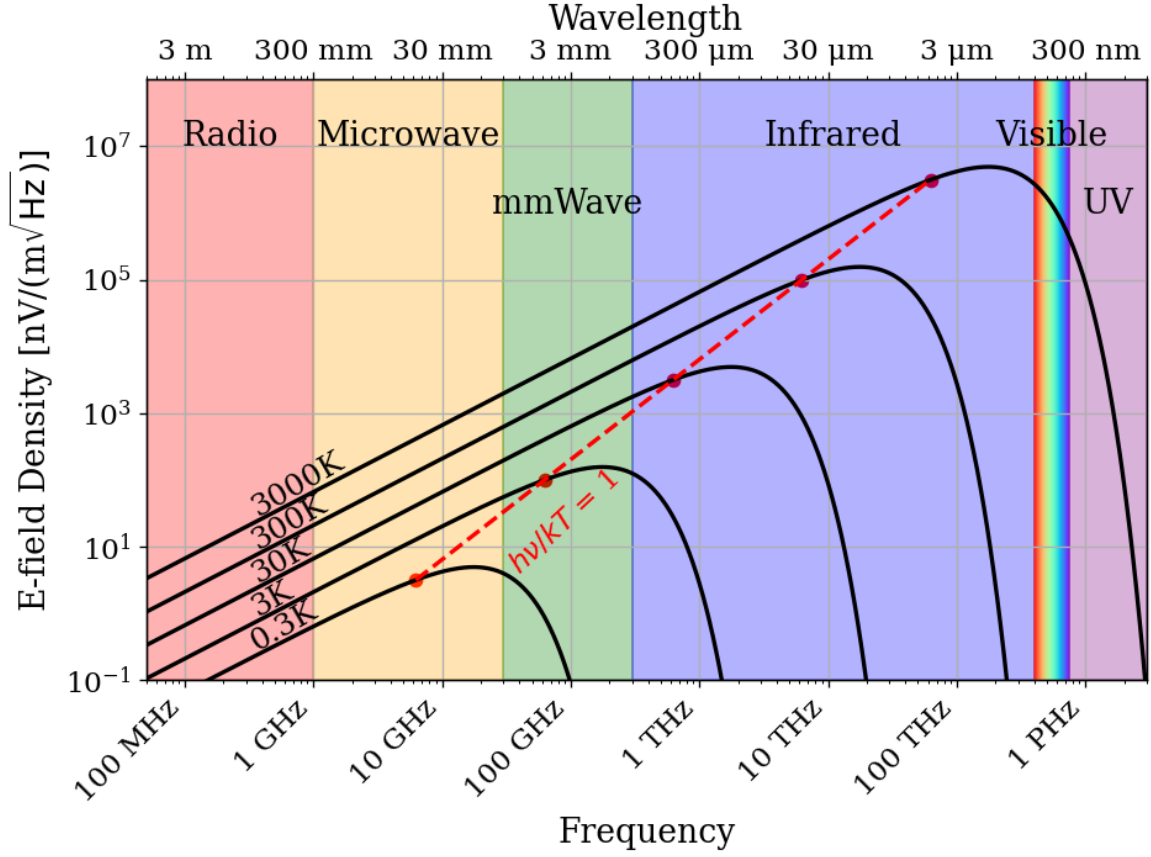


Figure 2.2: Blackbody electric field spectral density from radio to UV frequencies. A variety of temperatures are shown in black. The dashed red curve and red points indicate where along the black curves $h\nu = kT$, i.e. where Eq. 2.7 breaks up between the Rayleigh-Jeans regime to the Wien approximation. Note that these points are only a function of ν and T ; their vertical placement is chosen to lie on their respective curve but has no physical significance. The dark radio experiment is firmly in the Rayleigh-Jeans regime.

the full form of Eq. 2.6 must be used. This is shown at frequencies and temperatures to the right of the red dashed line in Fig. 2.2.

2.1.1.2 Antenna noise

An antenna's effective aperture, A_e [m²], represents the effective area that it has to collect power density or irradiance [W/m²] from an incident Poynting vector,

$$P_A = |\mathbf{S}|A_e, \quad (2.8)$$

Where $|\mathbf{S}|$ is the magnitude of the incident Poynting vector, and P_r is the power received at the antenna, which is available at its terminals.

A_e is a directional quantity that varies with the antenna's directivity $D(\Omega)$, where Ω represents the solid angle around the antenna. It varies with frequency ν , though it is generally discussed in terms of wavelength λ . Three matching parameters are introduced to describe how much actual power the antenna is able to deliver to a transmission line; p the polarization match of the wave to the antenna, m the impedance match of the antenna to the transmission line and η_a the efficiency of the antenna which represents how much power is absorbed compared to that lost to Joule heating of the antenna. p , m and η_a are all real, dimensionless and vary between 0 and 1.

$$A_e \equiv \frac{\lambda^2}{4\pi} D(\Omega) p m \eta_a. \quad (2.9)$$

This definition follows [111], though some authors do not include p in the definition [112, 113].

A simple derivation of the direction-averaged effective aperture based on thermodynamics will provide intuition. An isotropic antenna placed in a cavity at temperature T will be

illuminated by randomly polarized, isotropic radiation of the form given by the Rayleigh-Jeans limit of Eq. 2.4, $|\mathbf{S}| = 8\pi kT \Delta\nu \nu^2 / c^2$. The power received by the antenna can be found by Eq. 2.8,

$$\begin{aligned} P_A &= \langle A_e \rangle \frac{1}{2} |\mathbf{S}| \\ &= \langle A_e \rangle \frac{4\pi kT \nu^2}{c^2} \Delta\nu, \end{aligned} \quad (2.10)$$

where the factor of 1/2 is introduced to account for the random polarization and the $\langle \rangle$ indicate an average aperture over all angles around the antenna. If a resistor is placed in a second cavity, also at temperature T, it will deliver its Johnson-Nyquist noise power [114, 115]

$$P_R = kT \Delta\nu \quad (2.11)$$

into a matched transmission line. This is shown in Fig. 2.3. The second law of thermodynamics makes a very powerful statement here: the net power flow between the antenna and terminator must equal 0 if the two temperatures are indeed equivalent. This means $P_R = P_A$ or Eq. 2.10 = Eq. 2.11,

$$\langle A_e \rangle \frac{4\pi kT \nu^2}{c^2} \Delta\nu = kT \Delta\nu \quad (2.12)$$

or,

$$\begin{aligned} \langle A_e \rangle &= \frac{c^2}{4\pi \nu^2} \quad [\text{m}^2] \\ &= \frac{\lambda^2}{4\pi} \end{aligned} \quad (2.13)$$

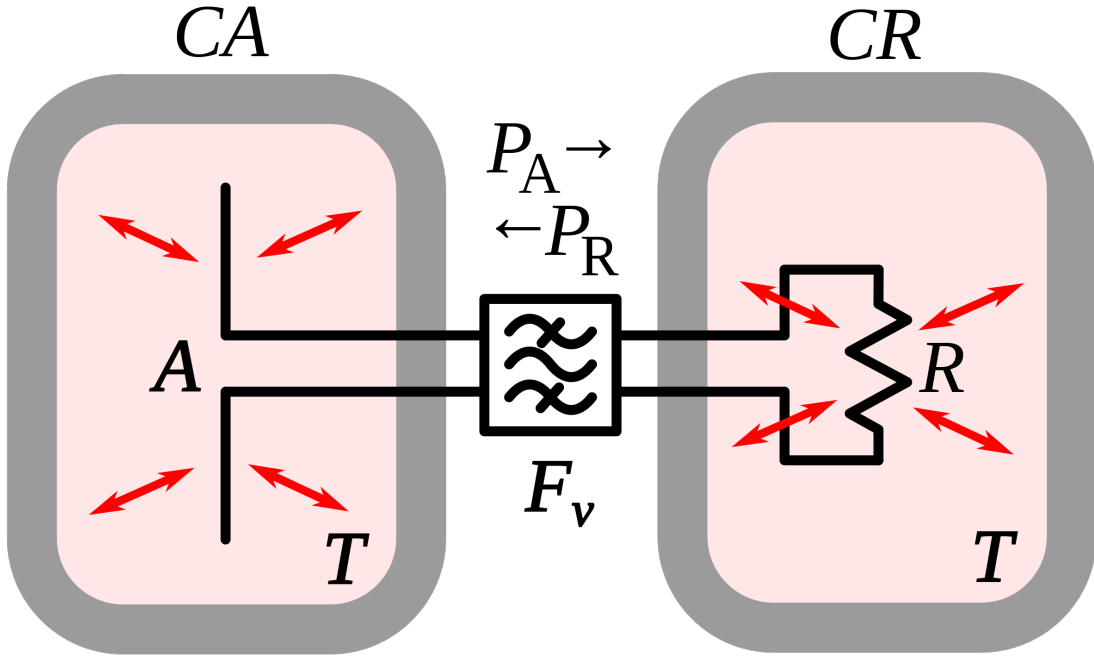


Figure 2.3: An antenna and matched resistor in cavities that are in thermal equilibrium. They are connected by a narrow filter permitting a narrow frequency band $\Delta\nu$. Image from Wikipedia, and is similar to the thought experiment proposed by Dicke [116].

This allows us to conclude that the power spectral density S_{ant} received by an antenna surrounded by an isotropic temperature is simply $kT\Delta\nu$ in the Rayleigh-Jeans limit of room temperatures and standard electronic frequencies. This result is independent of frequency, which can be understood by the reciprocal frequency dependence of the blackbody radiation (Eq. 2.4) and the average effective aperture (Eq. 2.13). The result is an antenna power which is equivalent to the well-known result for a resistor at 290 K,

$$S_{\text{ant}} = 3.9 \times 10^{-22} [\text{W/Hz}] = -174 [\text{dBm/Hz}]. \quad (2.14)$$

Note that S_{ant} indicates power spectral density and should not be confused with $\mathbf{S}$ which

indicates a Poynting vector.

2.1.1.3 Dicke radiometer equation

Equation 2.14 gave the mean of a power spectrum that is inherently noisy. We will now show the origin of this spectrum.

An enclosure whose electrically-lossy walls contain free charge carriers at a finite temperature will radiate incoherently by the fluctuation-dissipation theorem. This theorem is the underlying principle of phenomena such as Brownian motion [117] and Johnson-Nyquist noise [114, 115], but was not generally proven until 1951 by Callen and Wellton [118]. The random thermal fluctuation of the charge carriers will radiate a blackbody spectrum. Observing the electric field in the time domain, one can imagine the radiation arriving at a detector at a wide variety of random frequencies and phases. This is incoherent noise in that at each time domain sample is independent of the one proceeding it². The detector will produce a voltage that can be modeled as Gaussian with zero mean and standard deviation $\sigma = \sqrt{S_{\text{ant}} \Delta \nu_{\text{RF}} |Z|}$ where Z is the system impedance (here 50Ω). This is shown for a room temperature antenna (or equivalently a room temperature resistor, see Fig. 2.3) in Fig. 2.4.

²The hidden assumption here is that blackbody radiation is totally incoherent. It actually has a coherence time $\tau_c \approx 2 \times 10^{-14}$ s at 300 K [119]. For this statement to hold, the sample time $\tau_s \gg \tau_c$. For Run 1A, $\tau_s = 1/800 \text{ MHz} = 1.25^{-9}$ s, 5 orders of magnitude more than τ_c

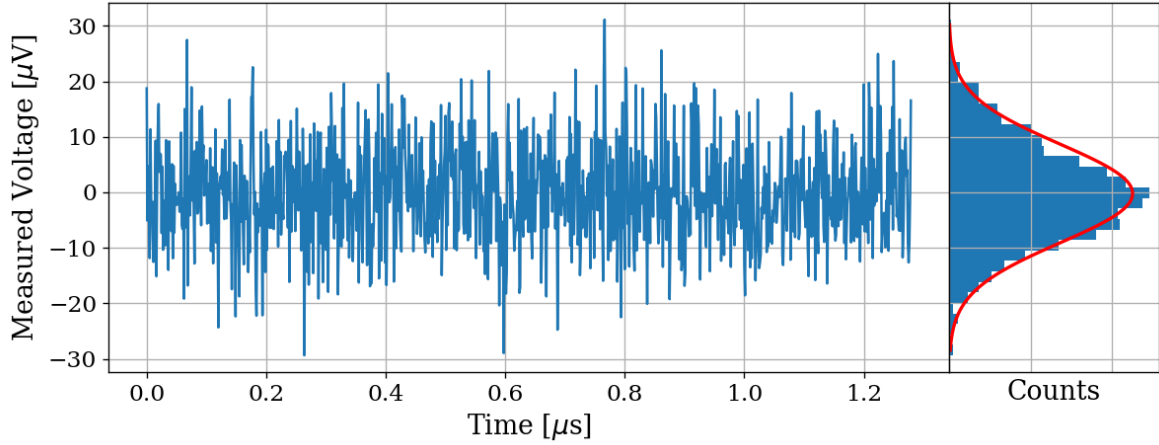


Figure 2.4: Simulated antenna noise voltage at room temperature in the time domain. Sample rate $\nu_s = 800\text{MHz}$ and number of samples $N = 2^{10} = 1024$. Bin width $\Delta\nu_{\text{RF}} = \nu_s/N \approx 800\text{ kHz}$. Data are binned and plotted as a histogram to the right. The best fit Gaussian is shown on the histogram in red with $\mu = -0.199\text{ }\mu\text{V}$ and $\sigma = 8.86\text{ }\mu\text{V}$. Counts have been normalized such that the bins add up to unity.

The next step is converting this time-domain voltage signal to a frequency-domain power spectral density (PSD). The first step is taking a discrete Fourier transform. This is usually implemented with an algorithm known as a fast Fourier transform (FFT), so that $\tilde{V} = \text{FFT}(V)$. In order to convert to a power spectrum, a non-trivial normalization prefactor must be included;

$$S = \frac{2}{N^2 |Z|} |\text{Re}(\tilde{V})[:N//2]|^2, \quad (2.15)$$

where N is the number of samples, Z is the system impedance, and $[:N//2]$ is Python notation for the second half of the samples contained in the $\tilde{V}$ array. Performing this operation on the data in Fig. 2.4 yields the PSD in Fig. 2.5

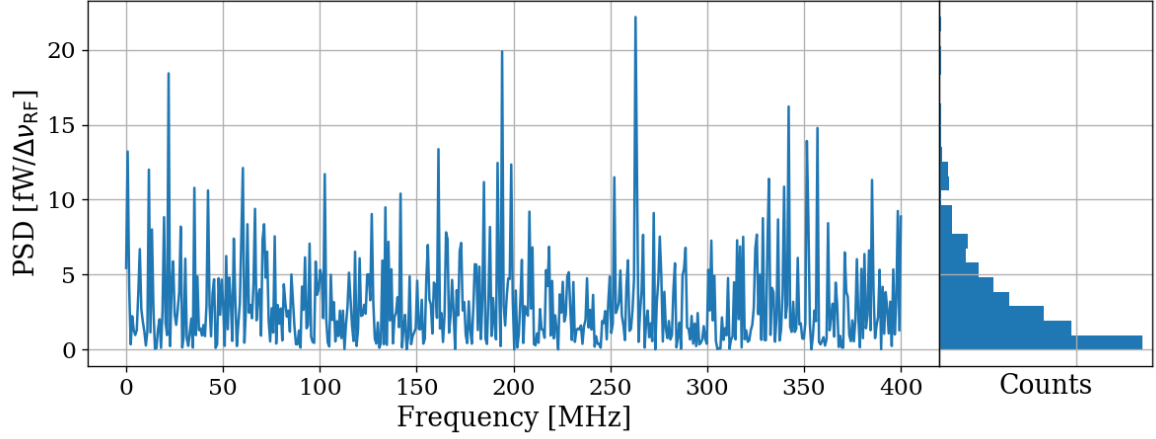


Figure 2.5: Simulated antenna noise power spectral density (PSD) at room temperature in the frequency domain. Sample rate $\nu_s = 800\text{MHz}$ and number of samples $N = 2^{10} = 1024$. Bin width $\Delta\nu_{\text{RF}} = \nu_s/N \approx 800\text{ kHz}$. Data are binned and plotted as a histogram to the right. Counts have been normalized such that the bins add up to unity.

The peculiar PDF of the histogram shown in Fig. 2.5 is known as a χ^2 distribution with 1 degree of freedom and comes about because power is a positive-definite quantity and the standard deviation of the PSD is greater than it's mean.

However, by averaging many of these power spectra together, the central limit theorem dictates that we can expect the resulting PDF to be Gaussian. The transition from χ^2 to Gaussian distributed spectra is shown in Fig. 2.6.

Finally, this averaged power spectrum can be modeled with the Dicke radiometer equation. The measured power (assuming only thermal noise) is given by

$$P_{\text{ant}} = kT\Delta\nu \left(1 \pm \frac{1}{\sqrt{\Delta\nu\tau}} \right) \quad [\text{W}]. \quad (2.16)$$

Here τ is the total acquisition time, and so $\Delta\nu\tau$ is equivalent to the number of spectra that

are averaged together. This can be nondimensionalized and written

$$\frac{P_{\text{ant}}}{kT\Delta\nu_{\text{RF}}} = 1 \pm \frac{1}{\sqrt{N}}, \quad (2.17)$$

which will become important during data analysis (Sec. 4.2).

For the remainder of this thesis, unless otherwise stated, it will be assumed enough spectra have been averaged together such that the PSD is Gaussian and scales with the square root of total acquisition time according to Eq. 2.16.

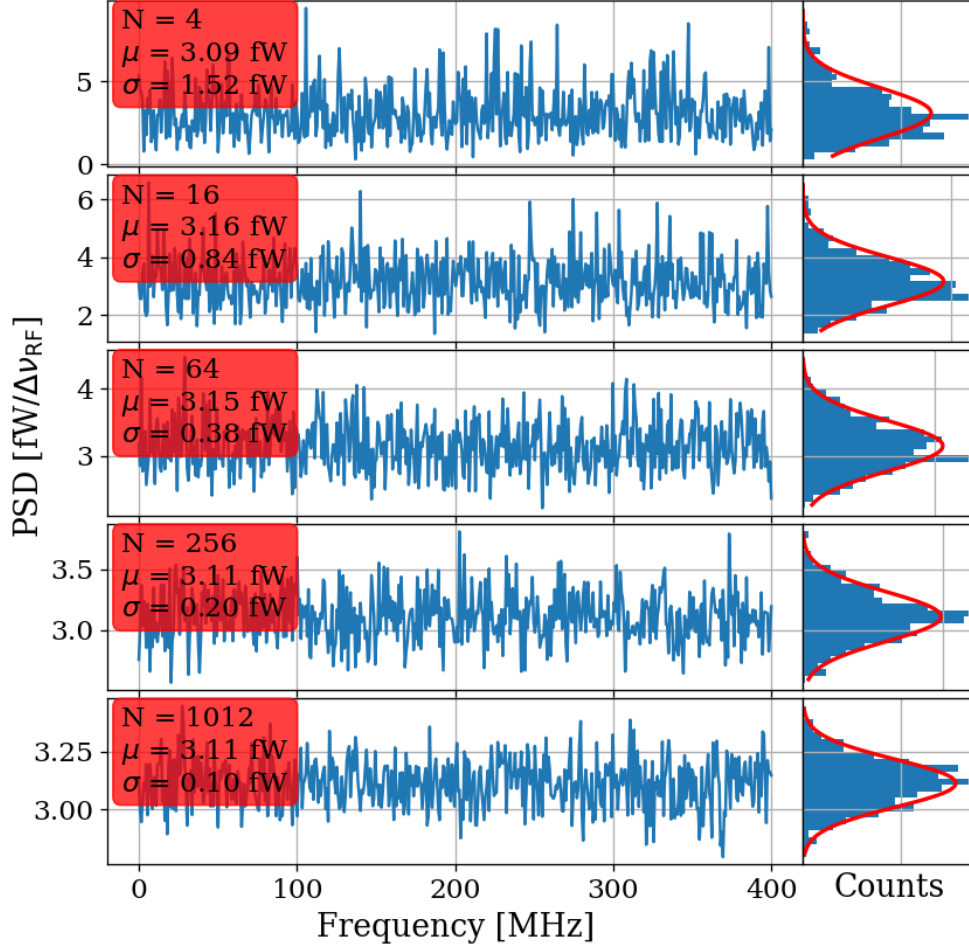


Figure 2.6: Effect of averaging on simulated antenna noise power spectral density (PSD) at room temperature in the frequency domain. Subplots show different numbers of averaged power spectra from $N = 4$ to 1012. Each subplot shows a factor of 4 times more averaging than the previous one. The Dicke radiometer equation predicts the standard deviation σ will scale like $N^{-1/2}$ once a large number of averages have been taken such that $\sigma \ll \mu$. Sample rate $\nu_s = 800\text{MHz}$ and number of samples $N = 2^{10} = 1024$. Bin width $\Delta\nu_{\text{RF}} = \nu_s/N \approx 800\text{ kHz}$. Data are binned and plotted as a histogram to the right with a best-fit Gaussian shown in red. Counts have been normalized such that the bins add up to unity. Note that the Y-axis scale changes between subplots to show the improvement of the Gaussian fit. The standard deviation is indeed decreasing between plots.

2.1.2 Dark Photon Signal

Thus far, the analysis has focused only on thermal noise, however, there are other sources of noise and interfering signals that must be considered, not to mention the actual dark photon signal. The electric field of a dark photon of mass m' in free space $\mathbf{E}^{\text{free space}}$ was shown in Eq. 1.57. The frequency is given by $\nu_{\text{DP}} = mc^2/h$. In a cavity, the E-field will be enhanced by the quality factor Q of the cavity. This Q must be measured or simulated, but for this toy analysis, we will assume it to be known³. The E-field inside the cavity is then

$$\mathbf{E}_{\text{ant}} = \mathbf{E}_{\text{ant}}^{\text{free space}} \sqrt{Q}, \quad (2.18)$$

since Q is proportional to power, i.e. $\mathbf{E}^2$. This E-field will then need to be converted from a wave in the cavity to a wave in a 50Ω transmission line by an antenna. Similar to the thermal noise of the previous section, this electric field will be converted via the effective aperture of the antenna. Similar to Q , the aperture will be assumed to be known.

The total received power from a coherent signal inside the room then is

$$P = \frac{\mathbf{E}_{\text{ant}}^{\text{free space}^2}}{\eta} Q < A_e >, \quad (2.19)$$

where η is the impedance of free space.

The observed spread of the frequencies of the dark photon is an important effect that determines system design. In the following paragraphs of this section, several sections of Gramolin et al. [121] are summarized. Also note that the original calculation for the predicted line shape was presented by Michael Turner in 1990 [122].

³Typical values are in the ballpark of 100. Some experiments have ultra-high Q cavities $\approx 10^{10}$ [120]

A simple model of the dark photon line assumes it is monochromatic, i.e. its line shape is a delta function in the frequency-domain,

$$\nu_{\text{obs}} = \delta(\nu - \nu_{\text{DP}}). \quad (2.20)$$

This is consistent with its production (see Sec. 1.4.3). However, when observed in a frame other than its rest frame, the frequency of a (single) dark photon will shift by an amount proportional to its kinetic energy

$$\nu_{\text{obs}} = \left(1 + \frac{v_n^2}{2c^2}\right) \nu_{\text{DP}}, \quad (2.21)$$

where ν_{obs} is the observed frequency of the n th dark photon, v_n is its velocity, c is the speed of light, and ν_{DP} is its rest frequency. The end result will be a signal that has some spread in frequency, $\nu_{\text{DP}}/(\Delta\nu) \equiv Q_{\text{DP}} \approx 10^6$, with a line shape which is now derived.

By summing over an infinity of dark photons of random phases and velocities (sampled from the relative Maxwell-Boltzmann velocity distribution of the dark matter halo), each with a frequency given by Eq. 2.21, one can construct a PSD of the dark photon signal S [W/Hz] as measured on earth.

When performing a measurement, one records the voltage V emerging from a detector for a period of time greater than the coherence of the dark photon $\tau_{\text{FFT}} \gg \tau_c$. The Fourier transform of $V(t)$ is denoted $\tilde{V}(\nu)$

The signal will have a total power

$$P_0 = \frac{1}{\tau_{\text{FFT}}} \int_0^{\tau_{\text{FFT}}} \frac{V(t)^2}{|Z|} dt = \int_0^{1/\tau_{\text{FFT}}} S(\nu)^2 d\nu, \quad (2.22)$$

which is a statement of Parseval's theorem.

The normalized line shape is defined by dividing by P_0 ; $\lambda(\nu) \equiv S(\nu)/P_0$. This then has the property of being normalized to unity,

$$\int_0^\infty \lambda(\nu) d\nu = 1. \quad (2.23)$$

Finally, the result for this normalized line shape is

$$\lambda(\nu) = \frac{2 c^2}{\sqrt{\pi} v_0 v_{\text{lab}} \nu_{\text{DP}}} \exp \left(-\frac{\beta^2 v_0^2}{4 v_{\text{lab}}^2} - \frac{v_{\text{lab}}^2}{v_0^2} \right) \sinh \beta \quad \left[\frac{1}{\text{Hz}} \right], \quad (2.24)$$

where $|v_0| \approx 220$ km/s is the circular rotation speed of the Galaxy at the radius of the sun (approximately 8 kpc), $v_{\text{lab}} \approx 233$ km/s is the relative velocity of the Sun to the rest frame of the Galaxy and

$$\beta \equiv \frac{2 c v_{\text{lab}}}{v_0^2} \sqrt{\frac{2(\nu - \nu_{\text{DP}})}{\nu_{\text{DP}}}}.$$

2.1.3 Radio frequency interference

Radio Frequency Interference (RFI) includes any coherent interfering signals which can be detected by the experiment. While noise is better described as a power spectral *density* [W/Hz] or electric field *density* [V/(m $\sqrt{\text{Hz}}$)], RFI is made up of more narrow lines and is discussed in terms of power [W] or electric field [V/m]. In this experiment, RFI is mitigated through the shielding effectiveness (SE) of the cavity. SE measurements and more details about local RFI are discussed further in Sec. 2.5.1 and Sec. 3.2.

The peak RFI spike in Run 1.4 is at 186 MHz and approximately $100 \mu\text{V}/\text{m}$, an energy density of roughly $10^{-11} \text{ W}/\text{m}^2$. This will be reduced by the SE of the room (roughly 120

dB at this frequency), but just like a coherent dark photon, it will be enhanced by the Q /effective aperture. This will be right on the edge of detection, but in the actual data run, it was not detected.

2.1.4 Amplifier Chain Noise⁴

Any amplifier will have some noise which it adds to an incoming signal that will degrade the signal-to-noise ratio (SNR) of the measurement⁵. A low noise amplifier (LNA) is an amplifier that has been specifically designed to minimize the noise contribution. This process is shown schematically in Fig. 2.7.

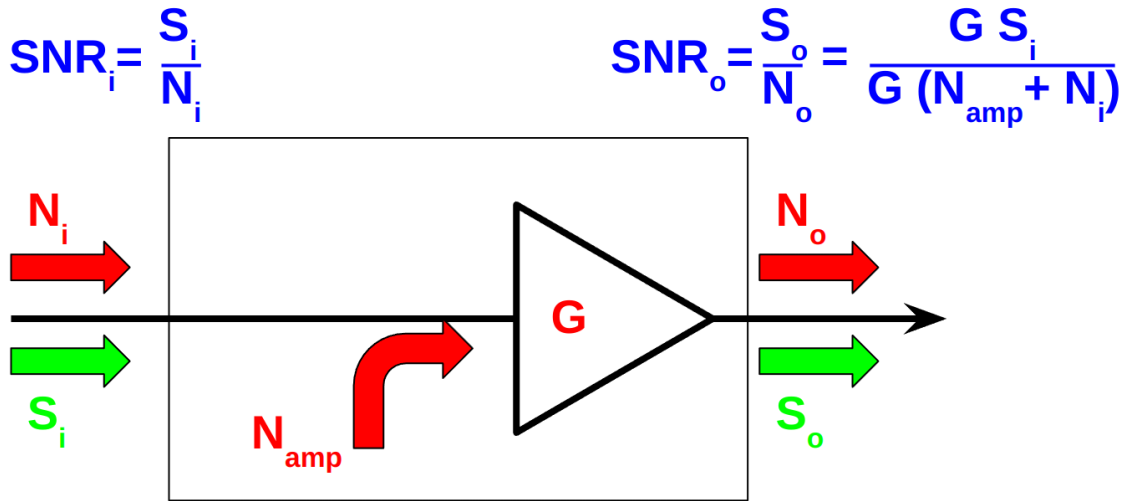


Figure 2.7: Schematic of LNA (with power gain G) adding input referred noise N_{amp} . Since N_{amp} is referred to the input, it can be directly added to the input noise which is itself input-referred. The ideal, noiseless amplifier is represented by the triangle, while the physical amplifier, including noise, is contained in the rectangle.

⁴Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/thesis/ch2/CH2.ipynb>

⁵A great lecture on the subject by Prof. Greg Durgin can be found at [123]

The performance of an LNA can be evaluated by its noise figure F ⁶ or equivalent temperature T_e . They are different ways of saying the same thing, namely the excess noise contribution of an amplifier (or attenuator) that is driven by a known source impedance. Here, we will assume everything is impedance-matched (as is the case for standard 50 Ω RF systems).

F is defined as the ratio of the SNR at the input of an LNA to that at its output,

$$\begin{aligned}
F &\equiv \frac{\text{SNR}_i}{\text{SNR}_o} \\
&= \frac{S_i/N_i}{[G S_i]/[G(N_{\text{amp}} + N_i)]} \\
&= \frac{1}{1/[1 + N_{\text{amp}}/N_i]} \\
&= 1 + \frac{N_{\text{amp}}}{N_i},
\end{aligned} \tag{2.25}$$

where S_i and N_i are the signal and noise [W] presented to the LNA respectively, N_{amp} is the input-referred noise added by the LNA, and G is the power gain. By factoring out the implicit $k \Delta\nu$ from $N_i = kT\Delta\nu$, we find

$$F = 1 + \frac{T_e}{T_0}, \tag{2.26}$$

where T_e is the equivalent amplifier noise temperature referred to the input and T_0 is the physical temperature of a resistor matched to the amplifier input (typically $T_0=290$ K).

Note that following the same derivation as Eq. 2.25, it is simple to show that the noise

⁶Sometimes denoted NF, but F will be used here. Also note NF is sometimes referred to as noise *factor* and means noise figure in decibels. Most textbooks are consistent, but online resources are not.

figure of an attenuator at physical temperature T with loss $L \equiv P_i/P_o$ is given by

$$F_{\text{att}} = 1 + \frac{(L - 1)T}{T_0}, \quad (2.27)$$

where T_0 is the reference temperature defined above. If $T = T_0$, Eq. 2.27 simplifies to $F_{\text{att}} = L$

In order to standardize device specifications across system applications, it is common to choose a reference temperature T_0 of 290K. If not specified, it is generally safe to assume this has been done.

Note that noise figure is frequently expressed in decibels,

$$F_{\text{dB}} \equiv 10 \log_{10}(F). \quad (2.28)$$

One important generalization is that of a cascaded series of amplifiers, shown schematically in Fig. 2.8.

Here the total noise figure of n amplifiers can be shown to be

$$F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}, \quad (2.29)$$

following the same derivation as Eq. 2.25. Since the noise temperature of a system depends on the noise temperature of a given amplifier divided by the gain that precedes it, a front-end LNA with modest gain ensures the total noise figure of the system is equal to its noise figure to very good approximation. This is shown in Fig. 2.9. We will use this approximation and assert

$$\text{Amp Chain Noise} = \text{LNA Noise}, \quad (2.30)$$

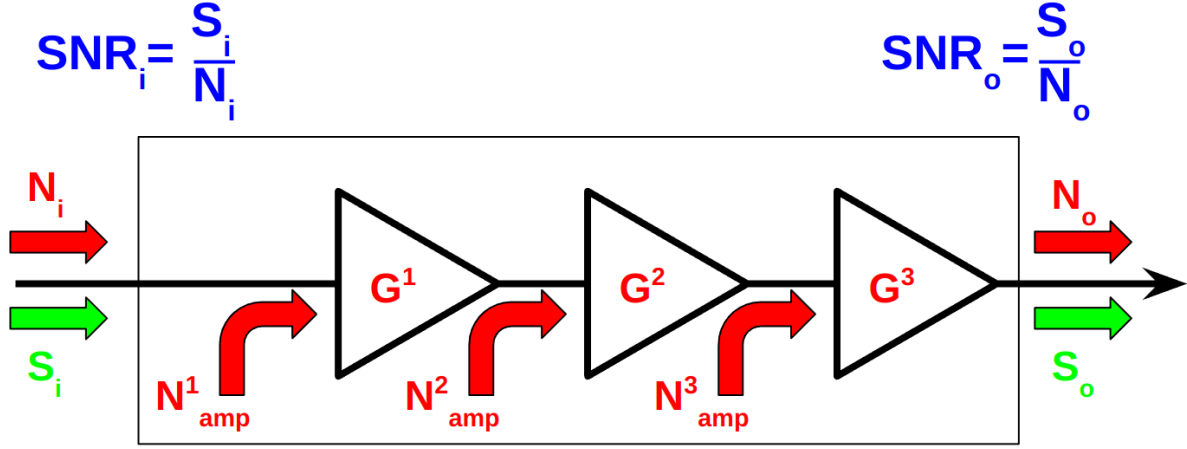


Figure 2.8: Schematic of cascade of $n = 3$ amplifiers and their added noise N_{amp}^n . Each amplifier has a gain of G^n . The SNR at the output is derived in Eq. 2.29. Note that superscripts in the figure and caption refer to the index of each component and are not exponents.

where LNA here is taken to mean the first gain stage in the amplifier chain.

Noise figures are typically frequency dependent, though they vary slowly over frequency and can be approximated as constant over narrow frequency bands. Noise figures are typically given on the data sheet of the LNA [124], but can also be measured. Measurement of LNA noise is covered in Sec. 3.1.1.

The power contributed by the LNA's noise is simply given by

$$P_{\text{LNA}} = kT_e \Delta\nu. \quad (2.31)$$

This is, again, the mean of a fluctuating power, just as Eq. 2.16.

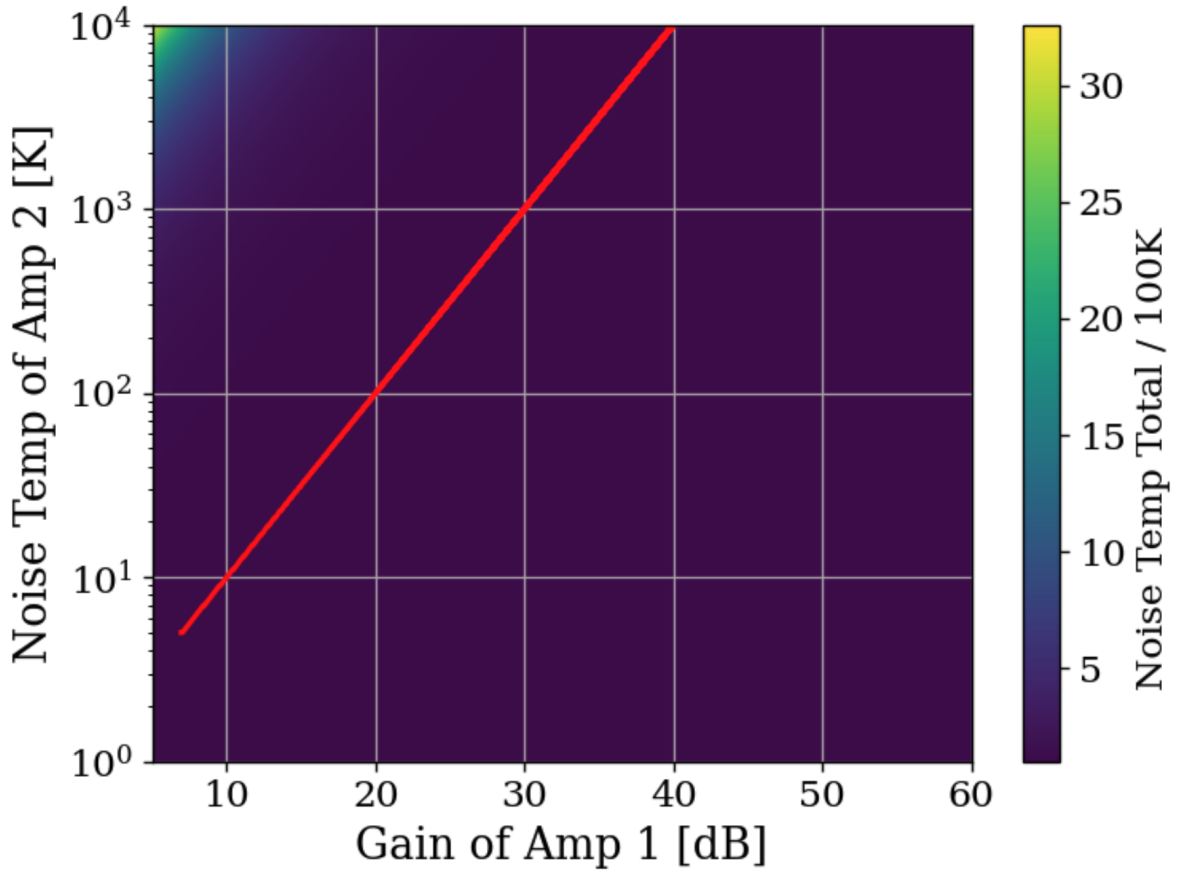


Figure 2.9: Cascaded noise temperature for a system with $n = 2$ amplifiers, normalized to noise temperature of amplifier 1 (the so-called LNA) = 100 K. Shown schematically (for $n = 3$) in Fig. 2.8. The first amplifier's noise temperature and gain (40 dB and 100 K, respectively) are roughly equivalent to the LNA used in phase 1A of the experiment[124]. The red curve shows where the system's noise temperature is 1% higher than LNA noise temperature. This shows that for a 40 dB, 100 K LNA, in order to change the system noise temperature by 1 K, a second-stage amplifier with a noise temperature of 10^4 would be required. Secondary amplifiers with noise temperatures closer to 500 K are realistic and inexpensive. Note that the red curve should continue but is cut off as a plotting artifact.

2.1.5 ADC Effects

“ADC Effects” is a catch-all term that refers to power introduced by an analog-to-digital converter. It contains two components, listed in order of importance;

$$\text{ADC Effects} = \text{Spurious Signals} + \text{ADC Noise Floor} \quad (2.32)$$

Equation 2.29 shows that (power) gain G introduced before a noisy element in the RF chain reduces the relative contribution of that noise (power) by a factor of G . The same idea applies to ADC Effects, though one must be careful with the language used to describe this; spurious *signals* are not noise, and the experiment’s output *is* mostly noise.

2.1.5.1 Spurious signals

Spurious signals (also known as spurs) are coherent signals that are introduced into the signal path at the ADC⁷. They are likely caused by RFI due to various clocks in the personal compute (PC) in close proximity to the ADC. The coherence of spurs means that, unlike true noise, their amplitude remains constant upon averaging. Spurs don’t degrade the SNR of the experiment in the same way a noisy amplifier chain would; instead, they produce false positive candidates, which must be excluded, similar to RFI discussed in Sec. 2.1.3. Similar to ADC noise, they can be measured easily by terminating the input of the ADC and scanning.

They are investigated in Sec. 3.3.2, where they are shown to be nearly negligible, having

⁷Note that the spurs described here are not the same as the spurs that are described by the ADC specification *spur free dynamic range* (SFDR). SFDR is measured in dBc, i.e. *relative* to a carrier. Since our “carrier” is broadband noise, each bin produces some spurs, which are -66 dBc [125] relative to itself. The aggregate of these spurs is also broadband and averages down with the experiment’s noise. The SFDR spurs are negligible for an ADC that measures at a noise-like background.

been mitigated by the gain of the system. There is a single spur detected after a few days in Run 1A (see Sec. 4.2.5), but for this simple analysis, we will assume spurs are negligible.

2.1.5.2 ADC noise

ADC noise can simply be measured by terminating the input and taking a scan. This is the same procedure used to measure spurs and can be seen in Fig. 3.15. The result is that ADC noise is $\approx -130\text{dBm}/47.7\text{Hz}$; a factor of 10^5 lower than the thermal noise of the experiment $\approx -81\text{dBm}/47.7\text{Hz}$, both output-referred⁸, which agrees with the ADC’s data sheet [125]. Since ADC noise follows the same scaling as the experiment’s thermal noise (Eq. 2.16), this factor of 10^5 is independent of averaging, and ADC noise is totally negligible.

2.2 Toy Analysis ⁹

With each of the terms of Eq. 2.1 defined in the previous section, we will now perform and view several simulations of a simplified dark photon signal on a simplified background. This “toy analysis” lays the conceptual framework for the additional complications that are introduced in the actual analysis of Sec. 4.2. This section should provide intuition about the process of detecting a weak, narrow signal on a background PSD of thermal noise. It is assumed the noise has been averaged a sufficient number of times such that its PDF is Gaussian (discussed in Sec. 2.1.1.3). Furthermore, following the discussion of Sec. 2.1,

⁸Technically it doesn’t matter where they are referred since they are taken in ratio. As long as they are referred to the same point!

⁹Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/thesis/ch2/toyAnalysis.ipynb>

Eq. 2.1 can be simplified by setting RFI and ADC Effects to zero and combining Thermal Noise (300 K) with Amp Chain Noise (100 K) into a single term which represents the total noise in the whole system, $S_{\text{sys}} = P_{\text{sys}}/\Delta\nu_{\text{RF}} = k(T_{\text{ant}} + T_{\text{LNA}})$. With these simplifications, the input-referred measured power of Eq. 2.1 reduces to

$$P_{\text{i}} = P_{\text{DP}} + kT_{\text{sys}}\Delta\nu_{\text{RF}} \left[1 \pm \frac{1}{\sqrt{N}} \right], \quad (2.33)$$

where $T_{\text{sys}} \equiv 400$ K, realistic for the experiment that is being simulated. Also, note that this equation assumes the dark photon's line shape is much more narrow than $\Delta\nu_{\text{RF}}$ such that the measured input-referred dark photon power is independent of $\Delta\nu_{\text{RF}}$. In the simulations, the line shape will be modeled as a delta function as in Eq. 2.20. The signal will be introduced simply by adding some power in a single bin to a Gaussian background in the frequency-domain. Performing an FFT on a perfect (discretized) sine wave can cause it's power to be split among adjacent bins depending on the ratio of the sample rate to the sine wave's frequency. This can be minimized by windowing the time-domain data as is discussed in Ch. 2 Sec. 2 of Ben Godfrey's dissertation [1]. This effect is avoided by working in the frequency domain and adding power to a single bin which is the method used here. A reminder that throughout this thesis, code is available at github and is linked in the footnote of each section title.

With a simple simulation framework in place, we can now begin to generate signal-containing-spectra. Figure 2.10 qualitatively shows the 400 K system noise (input-referred antenna plus LNA) averaging down, leaving a small signal visible. Further subsections in this section will quantify this.

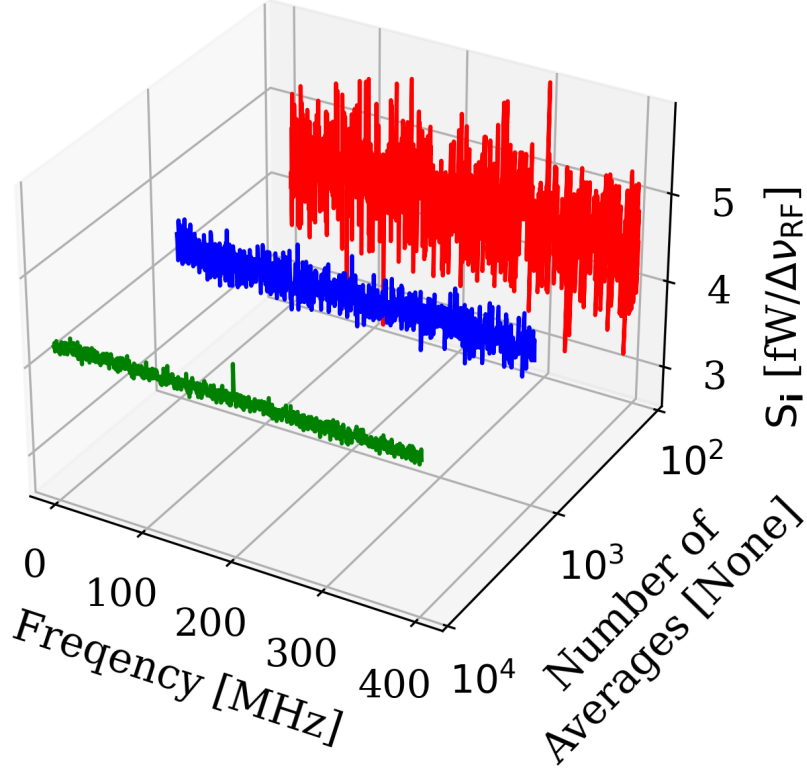


Figure 2.10: Input-referred power spectral density S_i from simplified simulation illustrating noise averaging down to reveal a persistent, software-injected, dark photon proxy signal. The red, blue and green spectra represent 100, 1,000 and 10,000 averages, respectively. The power of the signal P_{DP} is set to 0.4 fW and it only occupies a single bin. The mean of the noise is $\approx 4.3 \text{ fW}/\Delta\nu_{RF}$ where $\Delta\nu_{RF} \approx 0.78 \text{ MHz}$. The standard deviation ranges between $0.41 \text{ fW}/\Delta\nu_{RF}$ and $0.041 \text{ fW}/\Delta\nu_{RF}$ for 100 and 10,000 averages respectively. Note that this factor of 10 reduction in noise is predicted by the Dicke radiometer equation Eq. 2.16 for a factor of 100 times more averaging, as is shown in the red and green curves. Noise represents a 300 K antenna into a 100 K LNA for a total system temperature of 400 K. Signal is in a single bin at 200 MHz with a delta function line shape, defined in Eq. 2.20.

With a basic conceptual framework in place, it is now simpler to quantify signal detection and the uncertainty on how many averages are required to detect a signal and have that detection be significant in that it isn't a random fluctuation (false positive).

2.2.1 Signal significance

The problem of extracting a signal from noise is fundamentally a statistical one since, in general, both the signal and noise are random variables. A method for computing a *significance threshold* (ST) must be established, such that any bin containing more power than this threshold is X% significant. In this way, it is possible to have some known confidence a given signal was not just a random fluctuation.

The probability that all N bins are less than z standard deviations $z\sigma$ for a standard Gaussian distribution is given by

$$P(\max < z\sigma) = \left\{ \frac{1}{2} \left[1 + \operatorname{erf} \left(z/\sqrt{2} \right) \right] \right\}^N, \quad (2.34)$$

where P is the probability, $\operatorname{erf}(z)$ is the standard error function and z is real. Setting this equal to $100\% - X$ (where X is the *significance* or the desired probability a fluctuation crosses the $z\sigma$ threshold assuming no signal), and inverting $\operatorname{erf}(z)$ yields a significance threshold (ST). A convenient significance that was used in [67] is $X = 5\%$ corresponding to a 5% probability that an observed fluctuation above this ST is due to chance rather than a significant effect (i.e., a signal). A 5% ST for $2^{10}/2 = 512$ frequency bins¹⁰ works out to 3.9σ .

It should be noted that it is common in physics to discuss “ 5σ significance”. This means that a given experiment has a $1 - \operatorname{erf}(5/\sqrt{2})$ probability (about 1 in 3×10^6) of a false positive. The analysis of these normal spectra involves testing many independent frequency bins to see if any one of them exceeds some threshold. It is helpful to view these bins as “independent

¹⁰Note that a real FFT produces half the number of frequency bins as an output compared to the time domain sample it received, hence the factor of 2 established in Fig. 2.4

experiments”, each involving a random draw from the same parent Gaussian distribution. In this context, we discuss global significance (all of the bins) in contrast to local significance (a single bin). Setting a global 5% significance threshold is equivalent to setting a local threshold of 3.9σ given 512 bins.

One concept that can assist in choosing the significance is known as the *cost* of a decision. If an experiment requires a facility that charges by the hour and where the schedule is set years in advance, a false positive is quite expensive since it will lead to publicity and, ultimately, humiliation. A follow-up experiment will have to take out more expensive time to verify the results, and until that happens, theorists will spend their time rewriting physics to explain the result of a random fluctuation. In this case, the global significance should be quite low to avoid these high-cost outcomes, hence the 5σ discussed in the previous paragraph.

In the case of the dark radio experiment, a false positive is quite inexpensive. If a signal is detected, just repeat the experiment. For Run 1A, this is 9 days of averaging which is mostly passive and is little more than an annoyance. If a signal is detected at the same frequency, things become interesting. This concept of cost is discussed formally in Appendix 1 of Extraction of Signals From Noise by Wainstein and Zubakov [126].

For this reason, a significance of 5% (i.e. 5% chance of a false positive) is acceptable for this experiment, where it is certainly not acceptable at the LHC.

2.2.1.1 Computing an exclusion limit

With the significance threshold (ST) defined for a spectrum containing a signal in the previous subsection, we turn briefly to the concept of an exclusion limit. In the actual experiment, no signals were detected. In this case, the null result must be translated into an exclusion limit, as in Ch.. 5. While this section looks at small signals riding on noise spectra, it can be easy to lose sight of the fact that no signal was observed.

It is tempting to draw a line above the spectrum, claim no bins were observed above it, and cite that as the exclusion limit. This is the red dashed line in Fig. 2.11. However, this line is riding on the thermal background and is actually significantly higher than the true exclusion limit, which is shown in dashed green. The mean μ must be subtracted from this upper (red) line to compute the exclusion limit. In Fig. 2.12, the exclusion limit is pushed down with additional averaging, resulting in the detection of a small signal shown as an orange dot. The frequency-dependent exclusion limit from the actual Run 1A data run is shown in Sec. 4.2.

With the derivation of the significance threshold complete, the next subsection will focus on predicting the amount of time required to detect a signal.

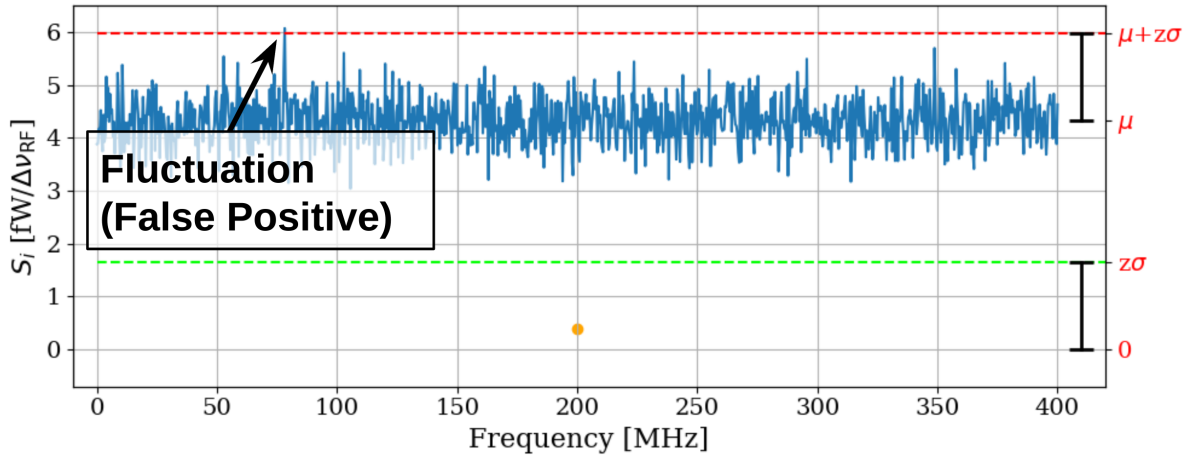


Figure 2.11: Simulated, input-referred noise spectrum containing small signal at 200MHz. The signal power = 0.4 fW, system temperature = 400 K and $N_{avg} = 100$; the same as is shown in the red spectrum of Fig. 2.10. The 0.4 fW signal is shown as an orange point. This power is added to the random background, so the measured power in the bin at 200 MHz is a Gaussian random variable given by Eq. 2.33; the mean is shifted up by the power contained in the signal. The dashed red line shows $\mu + z\sigma$, where z was derived in section 2.2.1. The detection threshold (dashed green line) is the red line minus the mean, $(\mu + z\sigma) - \mu = z\sigma$. This shows that the detection threshold is set by *fluctuations* of the measured power spectrum and not its mean. Note the detection at approximately 75 MHz; this is a random fluctuation and is expected to occur in 5 out of 100 simulations of these spectra since the significance used to calculate the ST was set at 5%.

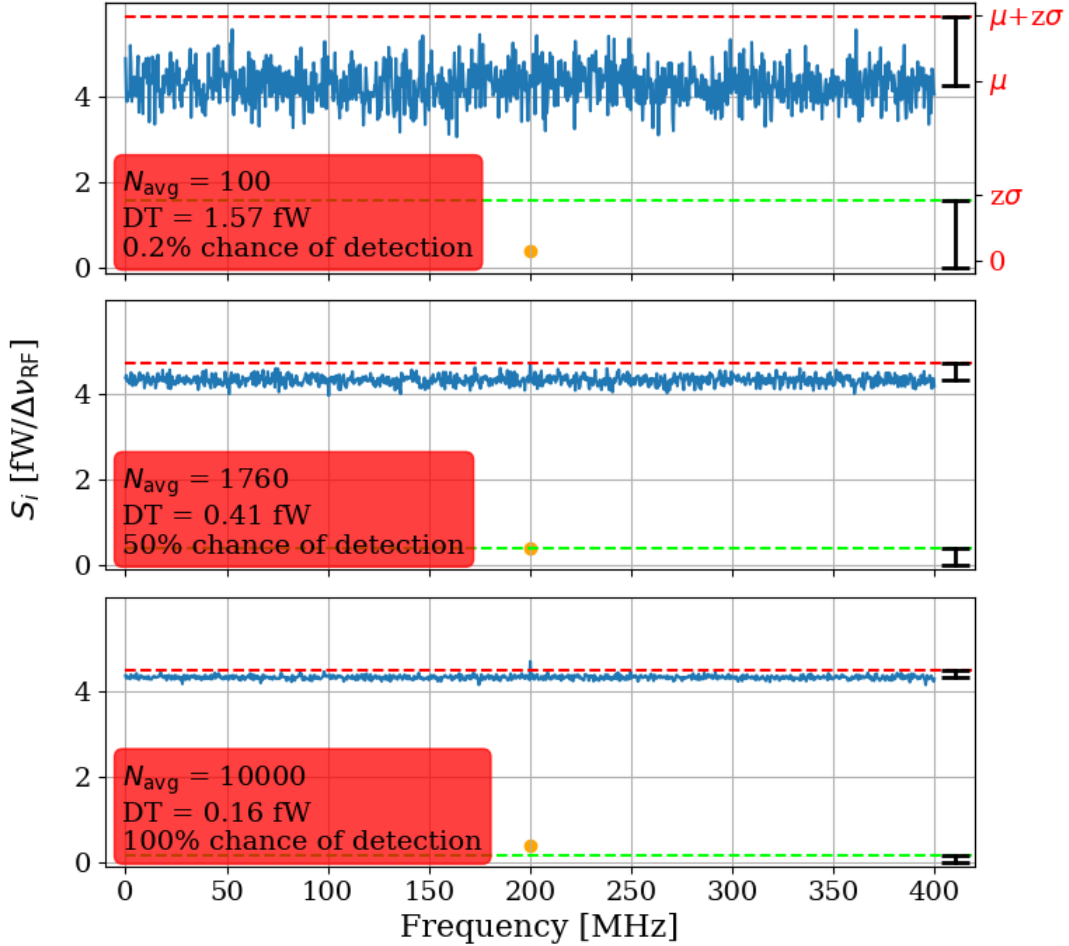


Figure 2.12: Simulated, input-referred noise spectrum containing small signal at 200MHz. The signal power = 0.4 fW, system temperature = 400 K and $N_{avg} = 100, 1,760$ and 10,000; the first and last are shown as the red and green spectra respectively in Fig. 2.10. The 0.4 fW signal is shown as an orange point. The chance of detection is computed via a simple Monte Carlo simulation where 100,000 similar spectra containing noise and signal are generated, and the number of times the measured power at 200 MHz is greater than $\mu + z\sigma$ (red dashed line). The full set of statistics from this Monte Carlo simulation is shown in Table 2.1. The detection threshold (DT, dashed green line) is $(\mu + z\sigma) - \mu = z\sigma$. Note that red labels on the right Y-axis are suppressed for the second and third subplots to prevent clutter, but they are the same as the first subplot.

2.2.2 Predicted time to detection

To conclude this subsection, Fig. 2.13 fills in the gaps between the three sub-plots shown in Fig. 2.12. The continuous detection threshold is shown to decrease following $1/\sqrt{N}$ shape given by Eq. 2.16. The intersections with this curve and the $1\text{-}\sigma$ uncertainty of the background give the $1\text{-}\sigma$ uncertainty on the number of averages required for a known signal and background. Table 2.1 summarizes the statistics for the probability of detection for a few numbers of averages.

Number of Averages	True Pos. [%]	False Neg. [%]	False Pos. [%]	True Neg. [%]
100	0.2	99.8	5.7	94.3
971	16.3	83.7	5.5	94.5
1,760	49.8	50.2	5.7	94.3
2,782	83.8	16.2	5.7	94.3
10,000	100.0	0.0	5.6	94.4

Table 2.1: Statistics of simple Monte Carlo simulation for the probability of signal detection in the toy analysis. Number of averages chosen based on Fig. 2.13. 100 and 10,000 averages show extreme cases, 1760 is a 50% chance of detection, and the two remaining values show the $\pm 1\sigma$ uncertainty band (shaded red region in Fig. 2.13).

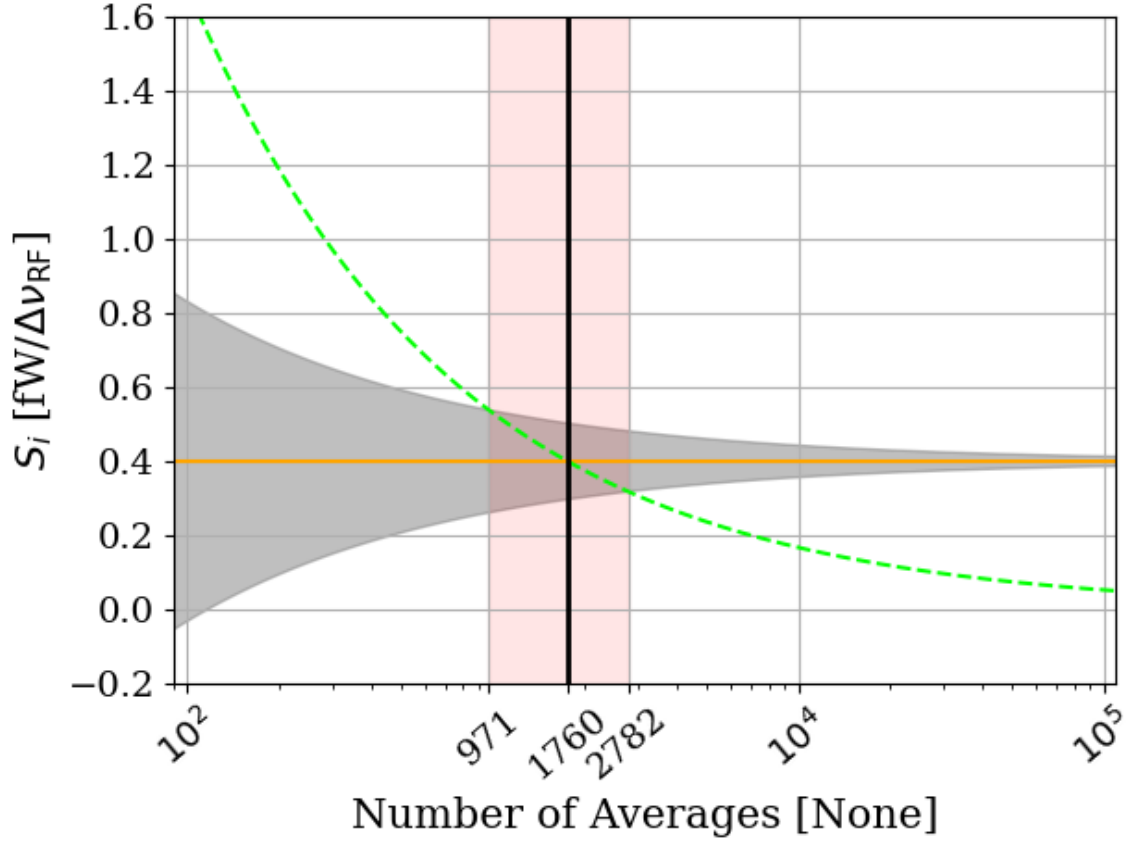


Figure 2.13: Predicted time to detection for the toy analysis spectra shown throughout this section. This is a companion to Fig. 2.12 and shows all numbers of averages between the three subplots shown in that figure, though only at a single frequency bin; 200 MHz in the case of the previous figures in this section. Orange line represents the known, injected signal, shown as orange points in Fig. 2.12. Gray shaded region shows the standard deviation of the noise σ , which is equivalent to the uncertainty on the measured power. It is given by Eq. 2.16, and scales with the square root of time. Dashed green curve is the exclusion limit defined in Sec. 2.2.1.1. It is also shown as a dashed green line in Fig. 2.12. Finally, the red shaded region shows the intersection of the exclusion limit (dashed green) with measurement uncertainty (shaded gray), which gives the $\pm 1\sigma$ uncertainty on the number of averages required to detect the signal. The detection statistics from a simple Monte Carlo simulation at a few points from this plot are shown in Table 2.1. This figure is a recreation of Fig. 3 in [127].

2.3 Thermal Noise in a cavity: Thermal Variations

This section is concerned with the topic of variations of thermal noise emerging from a cavity. Throughout this experiment, this was a challenging concept since the theory outlined in Sec. 2.3.1 predicts the experiment will measure a frequency-independent noise spectrum, and it doesn't. This theory will be expanded upon, the data that seem not to conform to this theory will be presented, the reasons for this disagreement will be discussed, and finally, an experiment that tests this understanding will be presented.

2.3.1 Theory of thermal radiation in a cavity

In his 1946 paper *The Measurement of Thermal Radiation at Microwave Frequencies* [116], Robert Dicke presented a thought experiment to derive the aperture of a matched antenna from thermodynamic arguments. This was previously discussed in Sec. 2.1.1.2.

To recap the argument; if an antenna (in a black cavity) and matched load are matched to a transmission line and in thermal equilibrium (the situation presented in Fig. 2.3), the net power flow in the line must be zero by the second law of thermodynamics. If there was a power flow, one of the environments would warm up, resulting in spontaneous pumping of heat. An interesting way to model this situation is by treating the antenna as an aperture in the cavity, as pointed out in [128].

A simpler model removes the antenna and load altogether. Although removal of the antenna also removes the ability to perform the measurement, this is a useful thought experiment. In *Fundamentals of Statistical and Thermal Physics* [129], F. Reif uses four examples

to explore the radiation field $f_{\alpha}^{(i)}(\boldsymbol{\kappa}, \mathbf{r})$ (wave vector $\boldsymbol{\kappa}$ at position $\mathbf{r}$ with polarization α inside of the i th cavity) within cavities connected by an aperture (see Fig. 2.14). All cavities and material contained within them are presumed to be in thermal equilibrium. As we shall see, this function is independent of position, direction, polarization and specifics of the cavity. It can be written simply as $f(|\boldsymbol{\kappa}|)$, where the dependence on wave number is simply given by Eq. 2.6.

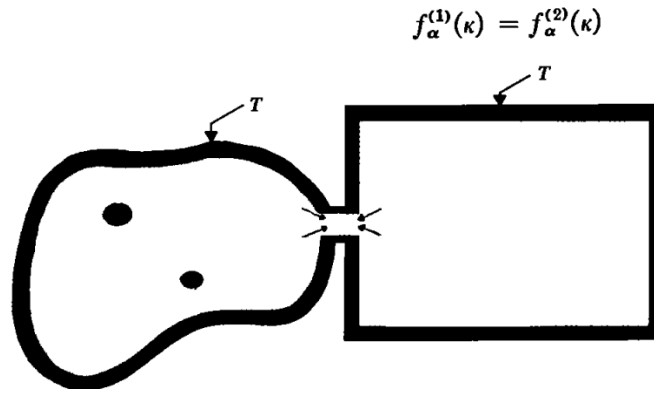


Figure 2.14: Two cavities in thermal equilibrium of arbitrary shape, material and physical contents contain equivalent radiation fields $f_{\alpha}^{(i)}(\boldsymbol{\kappa}, \mathbf{r})$. Figure reproduced from Reif [129].

1. **Independence of Position:** The radiation field inside an enclosure is homogeneous, meaning $f_{\alpha}(\boldsymbol{\kappa}, \mathbf{r}) = f_{\alpha}(\boldsymbol{\kappa})$, independent of position $\mathbf{r}$. If the radiation field depended on position, two identical bodies at temperature T placed at different points in the enclosure would absorb different amounts of radiation, leading to a temperature difference.
2. **Independence of Direction:** The radiation field is isotropic, meaning it depends

only on $|\boldsymbol{\kappa}|$ and not its direction. If this were not true, then bodies placed in the enclosure would absorb different amounts of energy based on orientation.

3. Independence of Polarization: The radiation field is unpolarized, meaning $f(|\boldsymbol{\kappa}|)$ is independent of the polarization index α . If it depended on polarization, bodies surrounded by polarizing filters would absorb different amounts of radiation depending on the orientation, leading to temperature differences.

4. Independence of Enclosure Shape and Contents: The function $f(|\boldsymbol{\kappa}|)$ is independent of the shape, volume, and material of the enclosure, as well as the bodies contained within. The argument is that if $f^{(1)}(|\boldsymbol{\kappa}|)$ and $f^{(2)}(|\boldsymbol{\kappa}|)$ were different for two enclosures at the same temperature T , then connecting them would result in unequal radiation transfer. Therefore, $f(|\boldsymbol{\kappa}|)$ must be the same across different enclosures.

To further flesh out argument four, picture a photon entering a metallic cavity (emissivity < 1) through a small hole as in Fig. 2.15. At each bounce, the photon has a probability of absorption given by the emissivity (a photon hitting a perfect blackbody has a 100% chance of being absorbed). As long as the geometries of the cavity and aperture allow a photon to bounce many times before it escapes, even a highly reflective cavity will behave like a blackbody. This is because when a photon enters, it is likely to be absorbed and a new photon will be emitted with a random wavelength drawn from the blackbody spectrum corresponding to the temperature of the walls. (Fig. 2.2).

The conclusion is rather surprising; a cavity with a small aperture will behave as a blackbody, and the spectrum emanating from a blackbody is a universal function that is

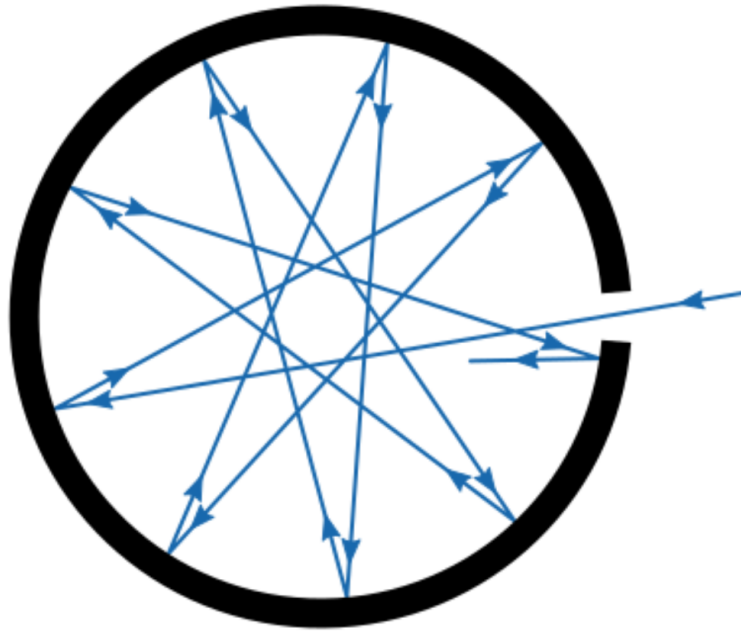


Figure 2.15: A reflective cavity with a small hole behaves like a blackbody as long as the probability of absorption $\ll 1$. The probability of absorption at each reflection is given by the emissivity, so the total probability of *reflection* is the emissivity raised to the power of the average number of bounces. Figure from Wikipedia[130].

independent of the cavity and its modes. Therefore, **a well-matched antenna in a cavity will not measure the modal structure of the cavity.** This seems to defy intuition. Cavities are resonators, and resonators... resonate?

The resolution to the seeming discrepancy is that this intuition only holds outside of equilibrium, a situation that is nearly ubiquitous in engineering contexts. A resonant cavity resonates because photons are pumped in faster than they can be absorbed by the walls and reemitted with a thermal distribution. In this out-of-equilibrium case, the photons interfere in such a way as to excite cavity modes. Turn the amplitude of the source down to $\approx kT$, and the modes vanish. This case, among others, will be investigated in the following subsections.

2.3.2 Inspection of thermal noise spectra ¹¹

The theory outlined above predicts the (input-referred) spectrum of an antenna in a cavity should not vary with frequency if there is thermal equilibrium between the cavity and the receiver system¹². A real measurement will take place after an amplifier (in other words, will be output-referred), and will vary due to the gain and noise figure of the amplifier. A simple way to correct for the amplifier is to compare the spectrum between an antenna and a terminator since both will have identical gain and amplifier noise contributions. Since we are more focused on the qualitative *shape* of the spectrum and not absolute input referred power, this method is acceptable. A comparison of antenna and terminator data from an identical amplifier chain is shown in Fig. 2.16.

The variations are unexpected in light of the theory presented in the previous subsection. In the following subsection, the thermal spectrum emerging from a simple resonator will be presented.

¹¹Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/thermalNoiseVsH/thermalWiggles_writeup.ipynb

¹²This assumes $h\nu \ll kT$ (Rayleigh-Jeans limit), an ideal antenna aperture $\propto \lambda^2$, and an impedance match between the antenna and amplifier. The final assumption will be relaxed in Sec. 2.3.8.

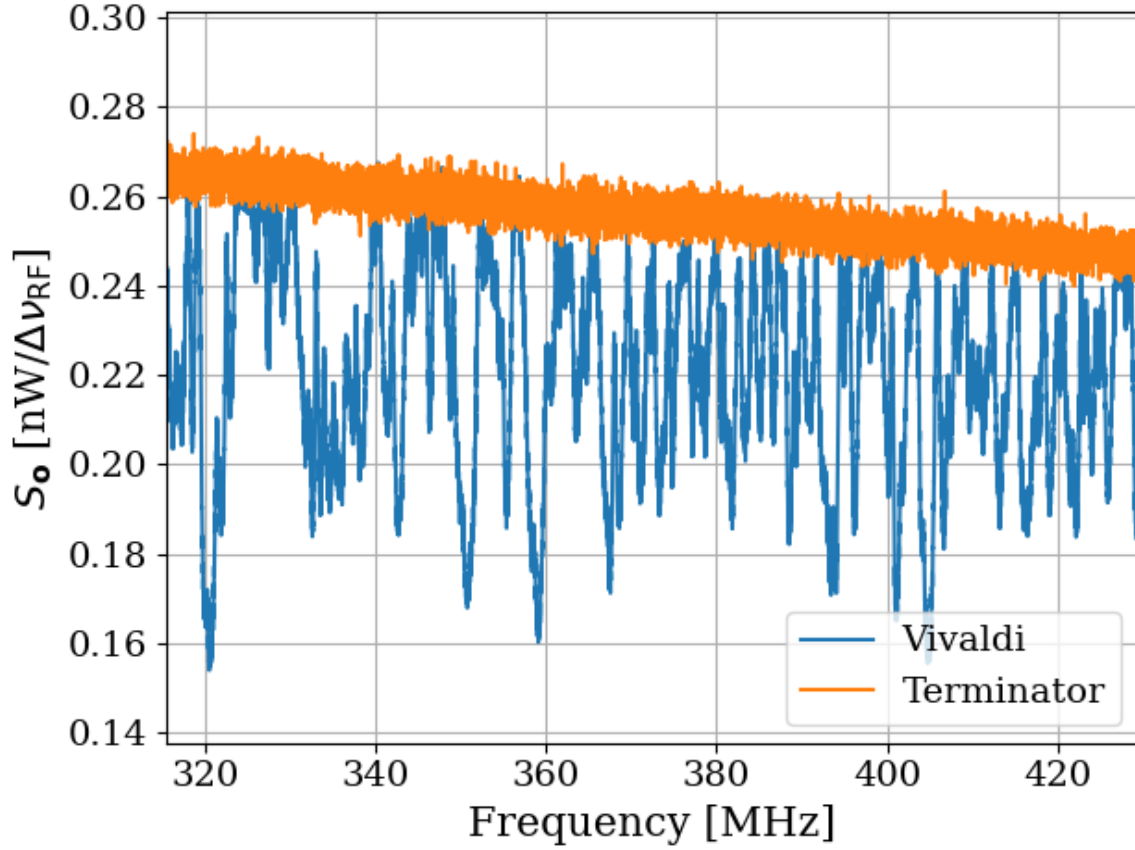


Figure 2.16: Output-referred Vivaldi antenna and terminator thermal spectra as measured through an amplifier chain ($G \sim 68$ dB, noise temperature ~ 120 K). The Vivaldi antenna is inside the shielded room. Both spectra represent about 1 second of data, with $\Delta\nu_{\text{RF}} = 9.5$ kHz. The Vivaldi antenna is in a single position throughout all averaging, in contrast to the technique where it moves, outlined in later chapters.

2.3.3 A simple resonator: shorted coax cable

A coax cable that is shorted on one end and matched to a measurement device¹³ on the other is a simple resonator. The short provides a boundary condition demanding the voltage goes to zero¹⁴ while the measurement device, being matched, absorbs the wave. A “closed-open”

¹³In the entirety of this section, a Pasternack PE15A-1012 will be used as the front end amplifier.

¹⁴Similarly, an open termination will force the *current* to zero, resulting in a similar resonator, though with a 180-degree phase shift.

resonator such as this will resonate with frequency

$$\nu = \frac{v(2n - 1)}{4L}, \quad (2.35)$$

for integer $n > 0$, where v is the wave speed in the cable and L is the length. For a 185 cm cable with velocity factor = 69.5% (i.e. $v = 0.695c$), this works out to a first mode at 28.2 MHz and following modes every 56.4 MHz. A schematic of the setup is shown in Fig. 2.17, and the output power spectrum is shown in ratio to a terminator in Fig. 2.18. This normalized spectrum is defined as $S_{\text{o}}^{\text{norm}}$. Also shown in Fig. 2.18 is an open termination at the end of the cable in place of the short. Fitting the peaks ¹⁵ reveals the average spacing between peaks is 55.2 ± 2.4 MHz, in good agreement with the prediction of 56.4 MHz of Eq. 2.35.

By the fluctuation-dissipation theorem, the loss of the cable will dissipate some thermal noise into the system, which can excite the cable. As discussed in Sec 2.3.1, if the resonator (cable) is in thermal equilibrium with the measurement device, a frequency-independent power spectrum is expected. The deviation from this expectation is surprising because the data seem to be in contradiction with thermodynamics. Now, a cable is not a particularly good resonator ($Q \lesssim 10$), so the effects are rather small here, but the simplicity of the system makes it one that is worth exploring.

One assumption which is critical to the above logic is that of equilibrium. The amplifier is a likely culprit for breaking this equilibrium, so a test that can remove the amplifier's

¹⁵Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/thermalNoiseVsH/thermalWiggles_writeup.ipynb

ability to interact with the resonator is worth pursuing. Fortunately, such a test is quite simple to implement using a circulator¹⁶.

¹⁶I got one for \$36 on Ebay!

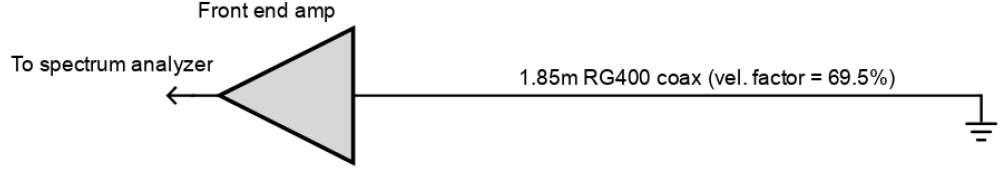


Figure 2.17: Schematic diagram of a coax cable with a short on the end and a matched measurement device on the other. This system behaves like a “closed-open” resonator. Replacing the short termination with an open (not shown here) produces a similar resonator, though with a 180-degree phase shift.

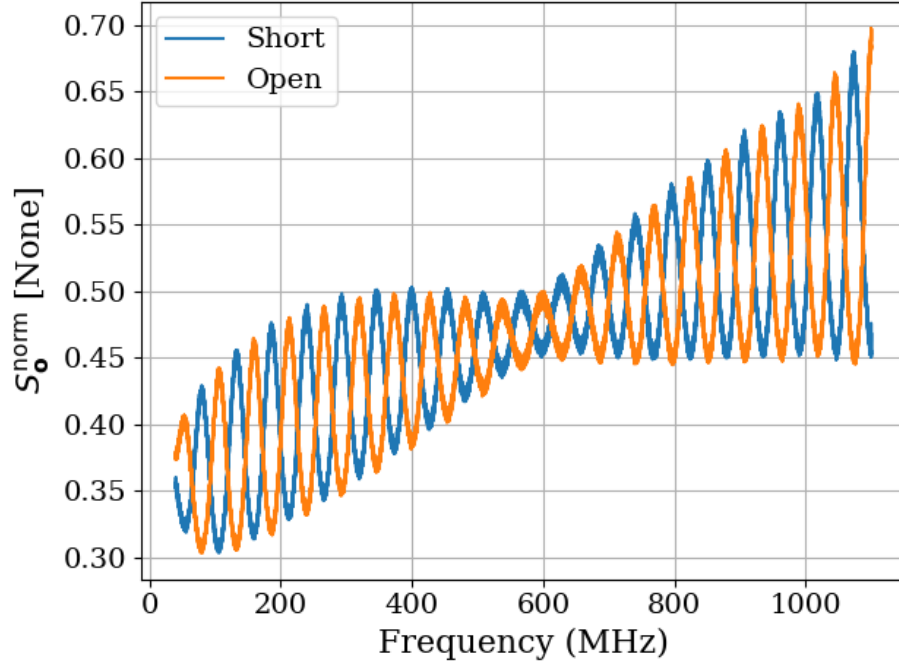


Figure 2.18: Power spectra of short (blue) and open (orange) terminations at the end of 1.85 m of RG400 coax cable (velocity factor = 69.5%). Schematic of this set-up is shown in Fig. 2.17. These spectra are normalized to the spectrum of a terminator, which was measured through the same amplifier chain. For example, when $S_o^{\text{norm}} = 0.5$, the power measured in this setup is half of the power measured by a terminator through the same amplifier chain. The average spacing between peaks is 55.2 ± 2.4 MHz, in good agreement with the prediction of 56.4 MHz of Eq. 2.35.

2.3.3.1 RF circulators ¹⁷

A circulator is a three-port, non-reciprocal device that allows power to flow only in specific ways between its ports¹⁸. A schematic symbol is shown in Fig. 2.19.

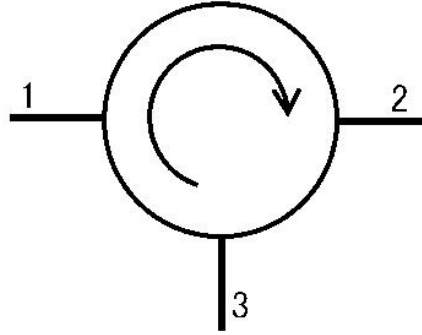


Figure 2.19: Schematic symbol of a circulator. Power can only flow from ports 1 to 2, 2 to 3 and 3 to 1. Image from Wikipedia.

Circulators can be understood in terms of their scattering parameters (known simply as S-parameters). For an N-port microwave device, the ratio between the voltage presented at port i and the voltage measured at port j is known as S_{ji} [131, 132]. The (linear) S-parameters [131, 132] of an ideal circulator are given by the matrix

$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (2.36)$$

¹⁷Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/thesis/ch2/circulatorData/SParameter_circulator/calc3portSPParam.ipynb

¹⁸See Ch. 9 of Pozar's *Microwave Engineering* for a detailed reference.

In other words, in an ideal circulator, $S_{12} = 0$, so power can not flow from port 2 to port 1, while $S_{21} = 1$, so power can flow from 1 to 2. Also of note, the diagonal elements $S_{ii} = 0$, meaning ports do not reflect power.

A Teledyne C-0S03A-3M RF circulator has an approximate bandwidth from 490-510 MHz and was available inexpensively on eBay, so it is used for testing. A photo of it is shown in Fig. 2.20, and its S-parameter data are shown in Fig. 2.21. Note that a circulator with one port terminated is sometimes known as an “isolator”, but I will refrain from using this term.



Figure 2.20: Photo of Teledyne C-0S03A-3M RF circulator. Lines in the background are collage-ruled lined paper ($\approx 7\text{mm}$ spacing) for scale.

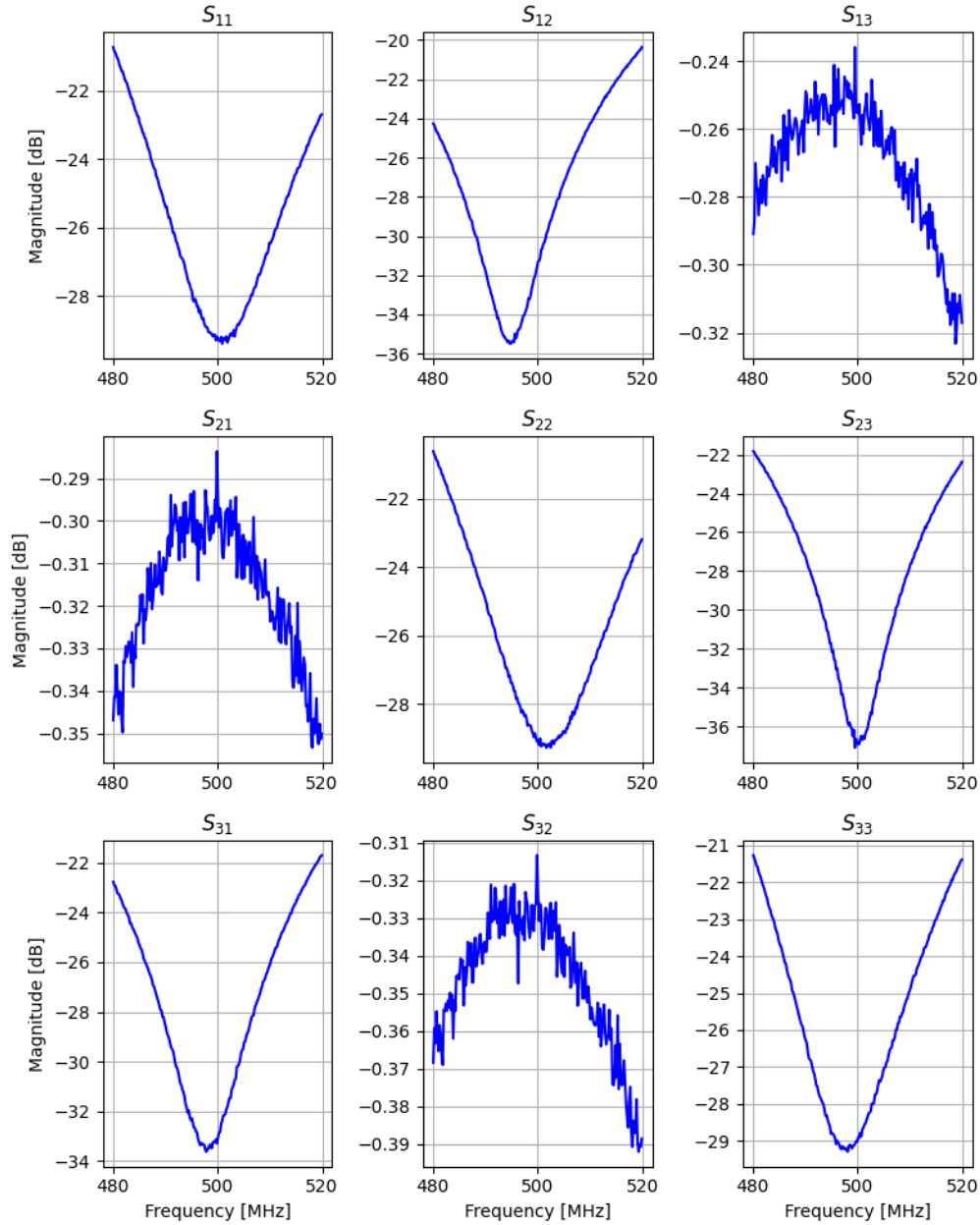


Figure 2.21: 3-port, frequency-dependent S-parameter data for Teledyne C-0S03A-3M circulator. The data were taken by Ben Godfrey and Andrea Lopez Arguello with 2-port VNA with the circulator's unused port terminated. This circulator is rated for use between 490 and 510 MHz. These data show good agreement with Eq. 2.36. Note that magnitude is in dB, so these data are proportional to power, i.e. the square of linear S-parameters.

2.3.4 A simpler resonator: shorted coax cable and circulator

In light of the variations observed in Fig. 2.18, a similar measurement was made using a circulator in order to isolate the coax resonator from any effects of the amplifier. The schematic of this setup is shown in Fig. 2.22, and resulting the spectrum is shown in Fig. 2.23. This is an interesting measurement because any net power flow from the amplifier will be absorbed by the terminator on port three since it presents a good impedance match. This net power flow has the potential to cause a temperature change in the terminator; however, it is in a thermally-conductive metal can that is exposed to the air, which serves as a heat bath and holds it very close to room temperature. The coax resonator will just see the Johnson noise of the room temperature terminator in equilibrium with the Johnson noise caused by its internal loss. The overall effect is a resonator that is in thermal equilibrium yet is able to be measured by an extremely sensitive spectrum analyzer.

The result of introducing the circulator and matched load on port 3 is the removal of the variations that were seen without the circulator (Fig. 2.23).

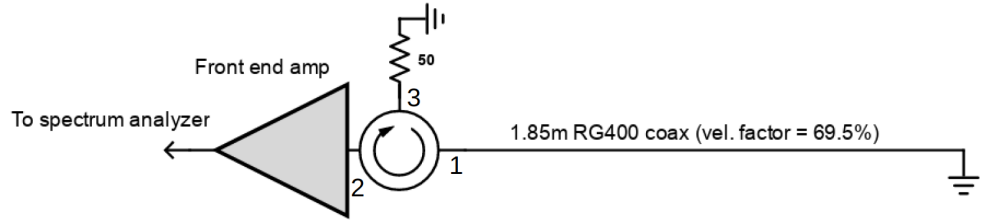


Figure 2.22: Schematic diagram of a coax cable with a short on one end and a circulator on the other. The circulator isolated the system from the amplifier.

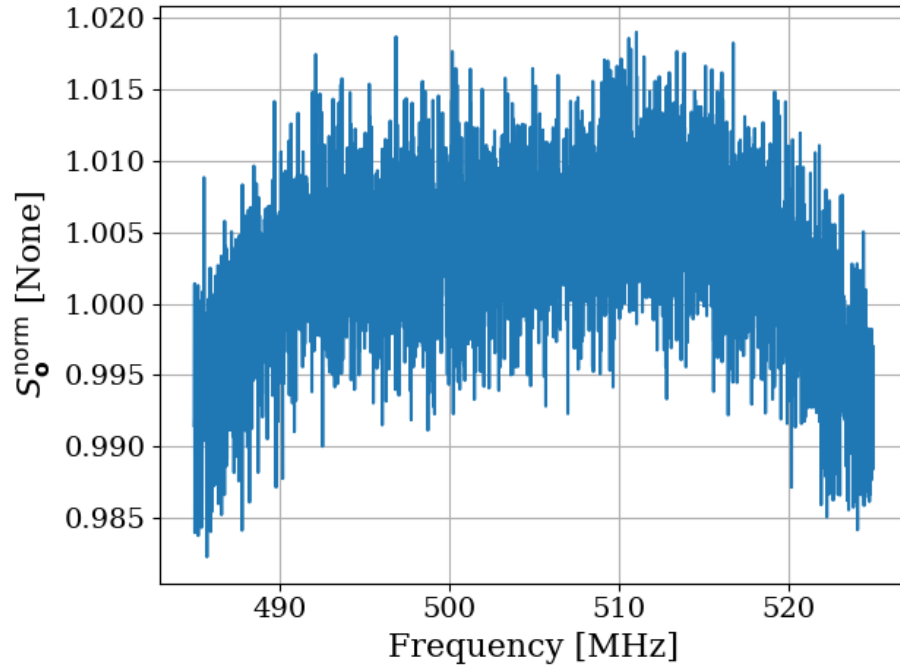


Figure 2.23: Normalized power spectrum of 185 cm coax resonator as measured through a circulator. Short termination at the end of the cable as depicted in Fig. 2.22. The amplitude of the variations in this spectrum is on the order of half a percent and hardly visible under the noise. They are much smaller than those at the $\sim 15\%$ level without a circulator (Fig. 2.18). Also, note this spectrum is normalized to a terminator through the same amplifier chain and very close to 1. The normalized spectrum without the circulator shown in Fig. 2.18 was significantly lower, around 0.5. Note that the peak-to-peak frequency variations of the cable without a circulator (Fig. 2.18) are ≈ 55 MHz, and would be visible in the ≈ 30 MHz span shown here. This span is limited by the circulator.

2.3.5 A more complex resonator: antenna in room

Similar to the coax resonator shown in Fig. 2.17, the antenna data presented in Fig. 2.16 are of a resonator (antenna-room system) which is being measured by an amplifier. The experiment in the previous section suggests that the amplifier seems to have an effect on the delicate thermal equilibrium, which can be mitigated by including a circulator. The experimental set-up for the antenna in the room is shown in Fig. 2.24, and the data from this setup is shown in Fig. 2.25.

The result is striking. The theory outlined early in the chapter predicted the noise power spectrum of an antenna in a cavity will look the same as a matched terminator; -174 dBm/Hz, independent of frequency. The Dark E-Field Radio Experiment measures just such a spectrum, but comparing a terminator and antenna in Fig. 2.16, they are remarkably different; the Vivaldi antenna has large variations. However, introducing a circulator to the set up such as in Fig. 2.24 removes these variations. Compare blue/orange curves in Fig. 2.25.

A nice test case would be to measure the thermal noise of an extremely high Q cavity after carefully ensuring the radiation that is allowed to enter has a blackbody spectrum of the same temperature of the cavity. This is nicely demonstrated in a few places, notably by Cervantes et. al with a cavity of $Q = 10^{10}$ (!!!) [120]. When care is taken to create equilibrium here, the resulting spectrum is flat, even on a frequency span that includes the resonance (Ref. [120], Fig. 2). When this equilibrium condition is broken, the cavity resonance becomes visible (Ref. [120], Fig. 10).

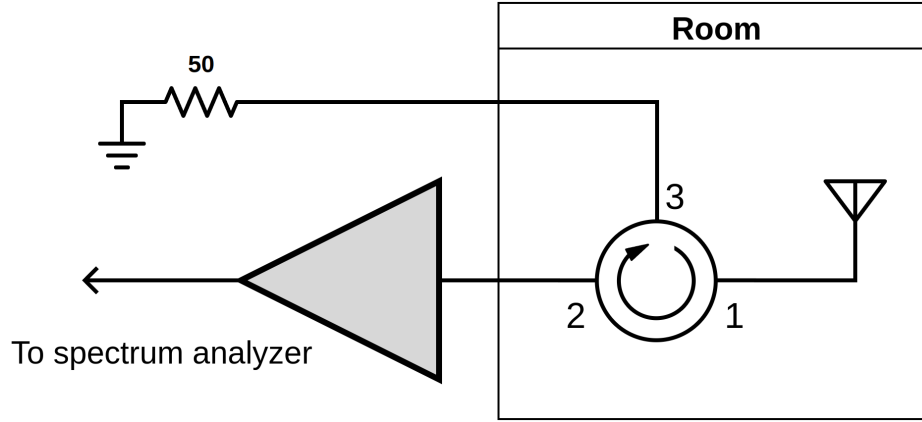


Figure 2.24: Schematic diagram of an experimental set-up using a circulator to isolate the antenna from amplifier effects.

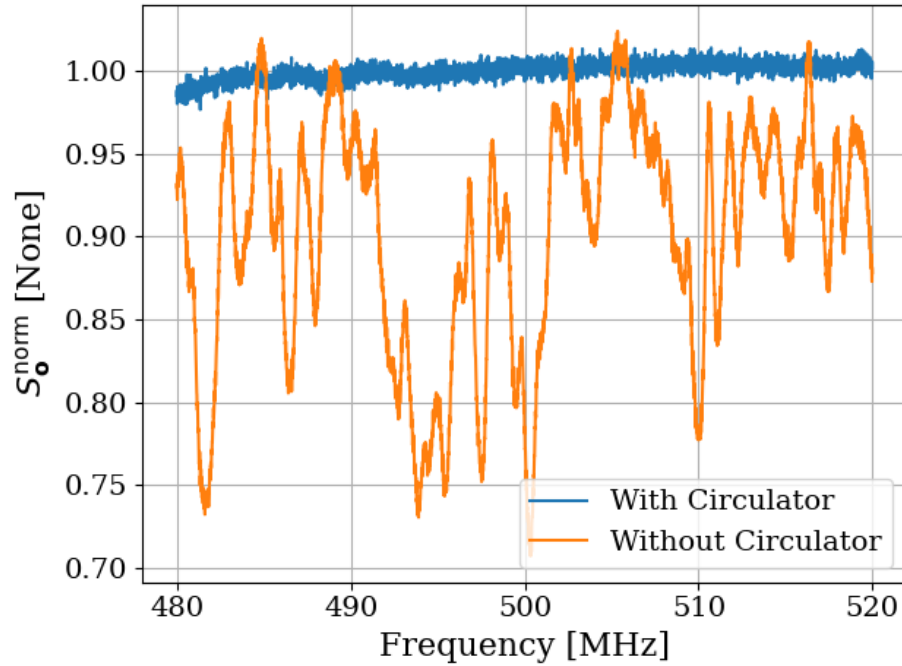


Figure 2.25: Normalized power spectrum for Vivaldi antenna in the room with/without circulator (blue/orange). This setup is shown schematically in Fig. 2.24. The normalized spectrum of the same antenna in the same position is shown with the circulator removed in orange for reference. This orange spectrum is the same as that shown in Fig. 2.16, but here it has been normalized to a terminator measured through the same amplifier chain. The terminator's spectrum is also shown in Fig. 2.16.

2.3.6 Effective temperature of amplifier

At this point, the seemingly obvious explanation is that the amplifier, being warm, is sourcing more power than it's absorbing. To test this theory, two amplifiers can be placed input-to-input as shown in Fig. 2.26. The noise power emerging *out* of the amp-under-test's input will be measured by the front-end amplifier. This amplifier has a noise temperature ~ 100 K, so it should be sensitive to very small variations in power. The data from the set-up are shown in Fig. 2.27. Also shown in this figure are the spectra of the amp-under-test replaced by both short and open terminations for reference.

An important number to keep in mind is the noise floor of this detector and what it looks like in the dimensionless units shown. This is set by the noise temperature of the front end amp, ~ 100 K. If the load-under-test were at 0 K, the power measured $S_{0\text{K}}^{\text{meas}}$ would be only that of the front end amp. Taken in ratio to a 300 K matched terminator measured by the same amp chain,

$$\frac{S_{0\text{K}}^{\text{meas}}}{S_{300\text{K}}^{\text{meas}}} \approx \frac{0 + 100\text{ K}}{300 + 100\text{ K}} \quad (2.37)$$

$$= 0.25, \quad (2.38)$$

where the factors of Boltzman's constant k and the measurement bandwidth $\Delta\nu_{\text{RF}}$ were suppressed since they cancel immediately.

Thus, anything with an apparent noise temperature $\ll 100$ K will appear with a dimensionless power spectral density of ~ 0.25 in Fig. 2.27.

This phenomenon actually has been discussed in the literature[106]¹⁹, which I will provide a brief summary of.

In this case, two amplifiers are placed back to back on either side of a transmission line. When a particle interacts with the transmission line, a pulse is detected at each of the amplifiers, and the difference in time provides a means to work out the position the particle came in along the line. It is advantageous in this case to minimize the noise emanating out of the inputs of these amplifiers. By tuning the reactance of the input of these amplifiers, they can absorb a net power, putting them at an “effective temperature” lower than their physical temperature.

In the case of off-the-shelf Pasternack RF amplifiers, this was likely not an intentional effect. However, the data presented here seem to agree with the idea that the amplifiers have an effective temperature $\ll 100$ K.

In the next subsection, I will demonstrate that by changing the temperature of the matched load outside the shielded room (shown in Fig. 2.24), the amplitude of the thermal variations can be controlled.

¹⁹I am very grateful to Greg Wright at Nokia Bell Labs for pointing this out.

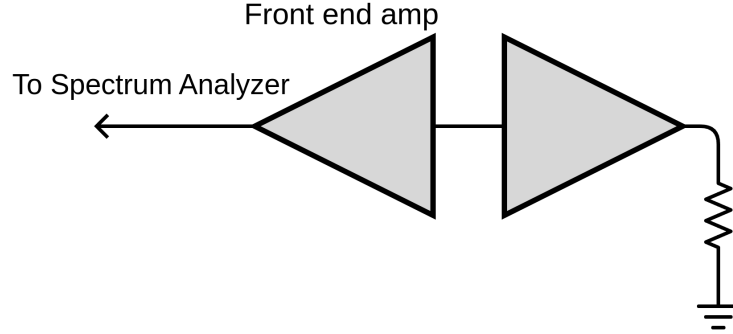


Figure 2.26: Schematic diagram of set-up to measure the noise emerging *out* of an amplifier's input.

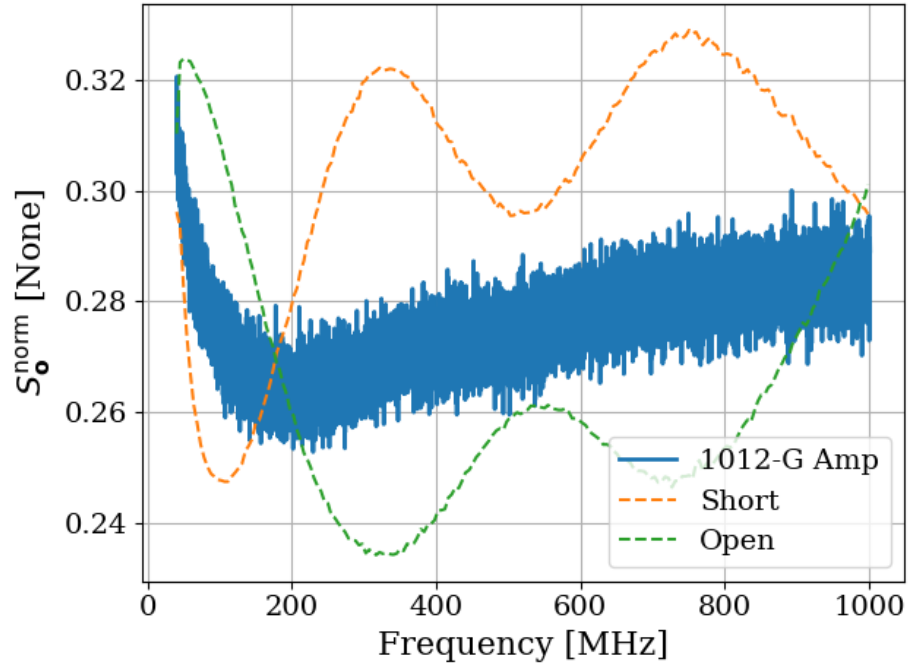


Figure 2.27: Normalized power spectrum of noise emerging *out* of the input of an amplifier's (Pasternack PE15A-1012-G) input, see Fig. 2.26. Normalized to a matched 50Ω terminator. Also shown as dashed curves are the spectra of a short and open termination. These spectra have a median fit applied to reduce visual clutter. Low-frequency behavior is due to the high noise figure of the amplifier at these frequencies.

2.3.7 Intentional breaking of thermal equilibrium

At this point, it has been demonstrated that an amplifier absorbs more thermal noise than it emits, likely due to the electronic cooling effect described by Radeka [106]. This causes the spectrum of a resonator, which is measured with such an amplifier, to exhibit variations, which disappear when a circulator is used to isolate the system from the amplifier. An interesting question naturally arises; what happens when the thermal equilibrium is disturbed by varying the temperature of the $50\,\Omega$ terminator on port 3 of the circulator (Fig. 2.24)? Since the terminator is outside the room, it is simple to conduct a highly controlled experiment where the terminator's temperature is varied without entering the room and disturbing sensitive boundary conditions.

The hot terminator is created by using a noise source (red device in the left panel of Fig. 2.28). The noise source has an effective noise ratio (ENR) of approximately 16.1 dB according to its calibration sheet. Noise temperature is related to ENR by the formula [133]

$$T_n = (10^{\text{ENR}/10} \times 290\,\text{K}) + 290\,\text{K}, \quad (2.39)$$

where ENR is measured in dB and a reference temperature of 290 K has been assumed. Therefore, the noise source has a noise temperature of around 12,100 K²⁰.

The cold terminator is a standard Pasternack $50\,\Omega$ (with the blue rubber case removed), and the cable is semi-rigid and rated for cryogenic temperatures. This assembly is shown immersed in liquid nitrogen in the right panel of Fig. 2.28.

²⁰This testing was performed before the Y-factor measurements of Sec. 3.1.1. The noise source worked correctly here but failed before the Y-factor measurements.

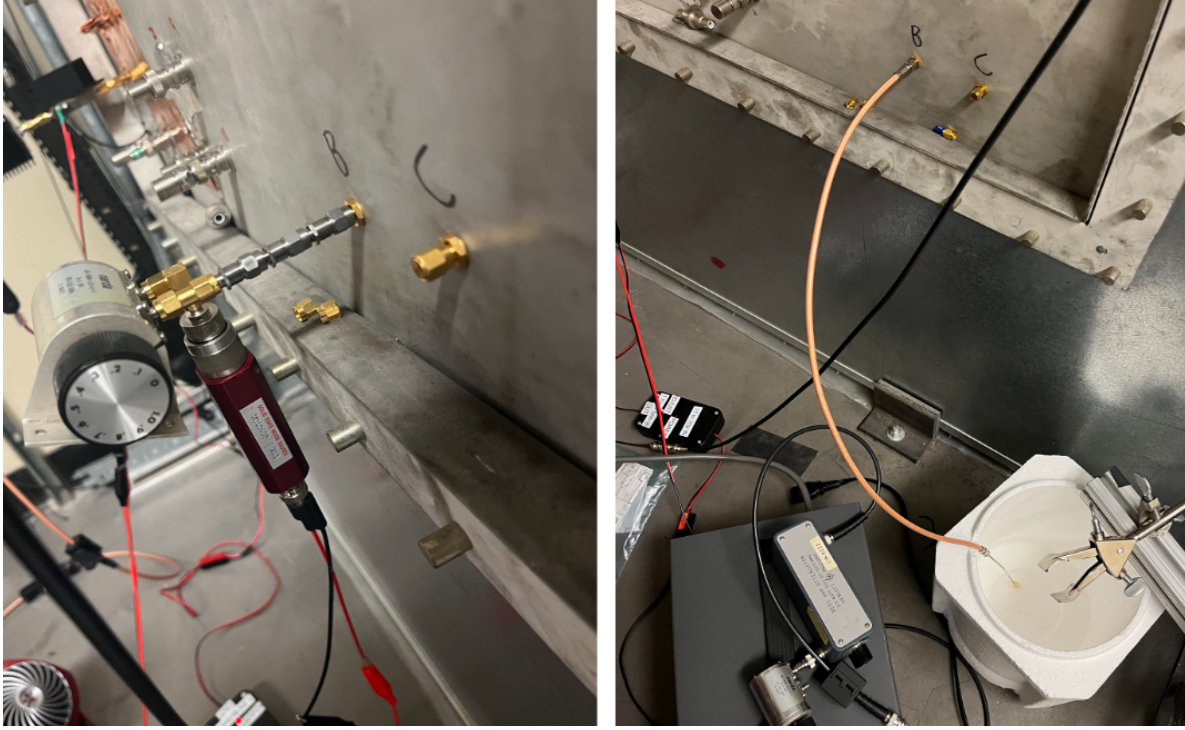


Figure 2.28: Setup to create hot and cold terminator. The hot/cold load is connected to port 3 of the circulator (as shown in Fig. 2.24) via the patch pannel of the shielded room, shown here. The circulator is in the room and not visiable in this photo. The left panel shows the noise source (16.1 dB ENR $\approx$ 12,000 K noise temperature) and attenuators allowing the specific control of the hot temperature (see Eq. 2.40). Course attenuation is controlled by adding fixed attenuators, while fine control (0.1 dB steps) is provided by the step attenuator. The right panel shows the semi-rigid cryogenic-capable cable in liquid nitrogen to create a cold load.

In both the hot and cold measurements, the loads were shown to have a good impedance match to the $50\,\Omega$ line using a VNA. The noise source is designed to have a good match, but the terminator is not rated for cryogenic temperatures, so this is an important test. The cryogenic test load (semi-rigid cable plus terminator) was measured to have $S_{11} < -35$ dB at both room temperature and at 77 K (by submerging in liquid nitrogen), confirming performance at cryogenic temperatures.

The noise source looks like a terminator, which is a factor of ~ 40 times hotter than room temperature, while the cryogenic terminator is a factor of ~ 4 colder than room temperature. To account for this, room temperature attenuators can be added to bring the effective temperature of the noise source down closer to room temperature. Therefore, the total noise temperature T_{out} of a terminator at physical temperature T_{in} in series with an attenuator at physical temperature T_{att} with (linear) loss $L \equiv P_i/P_o$ is a useful quantity. For brevity, the derivation outlined in the white paper by Whitham D. Reeve [134] is skipped, and the result is provided;

$$T_{\text{out}} = \frac{T_{\text{in}}}{L} + T_{\text{att}}\left(1 - \frac{1}{L}\right). \quad (2.40)$$

Returning to the set-up in Fig. 2.24, replacing the terminator on port 3 with either a noise source (including an attenuator chain) or a terminator in liquid nitrogen provides a means to break thermal equilibrium in both the hot and cold direction by applying a matched load at a very precise temperature to port 3 of the circulator. Equation 2.40 converts the attenuation and noise source temperature into an output temperature.

Figure 2.29 shows the resulting spectra from the set-up shown in Fig. 2.24 using a noise source/attenuator chain shown in the left panel of Fig. 2.28. The two panels show the spectra from the same set-up, but for different ranges and resolutions of T_{out} as defined by Eq. 2.40.

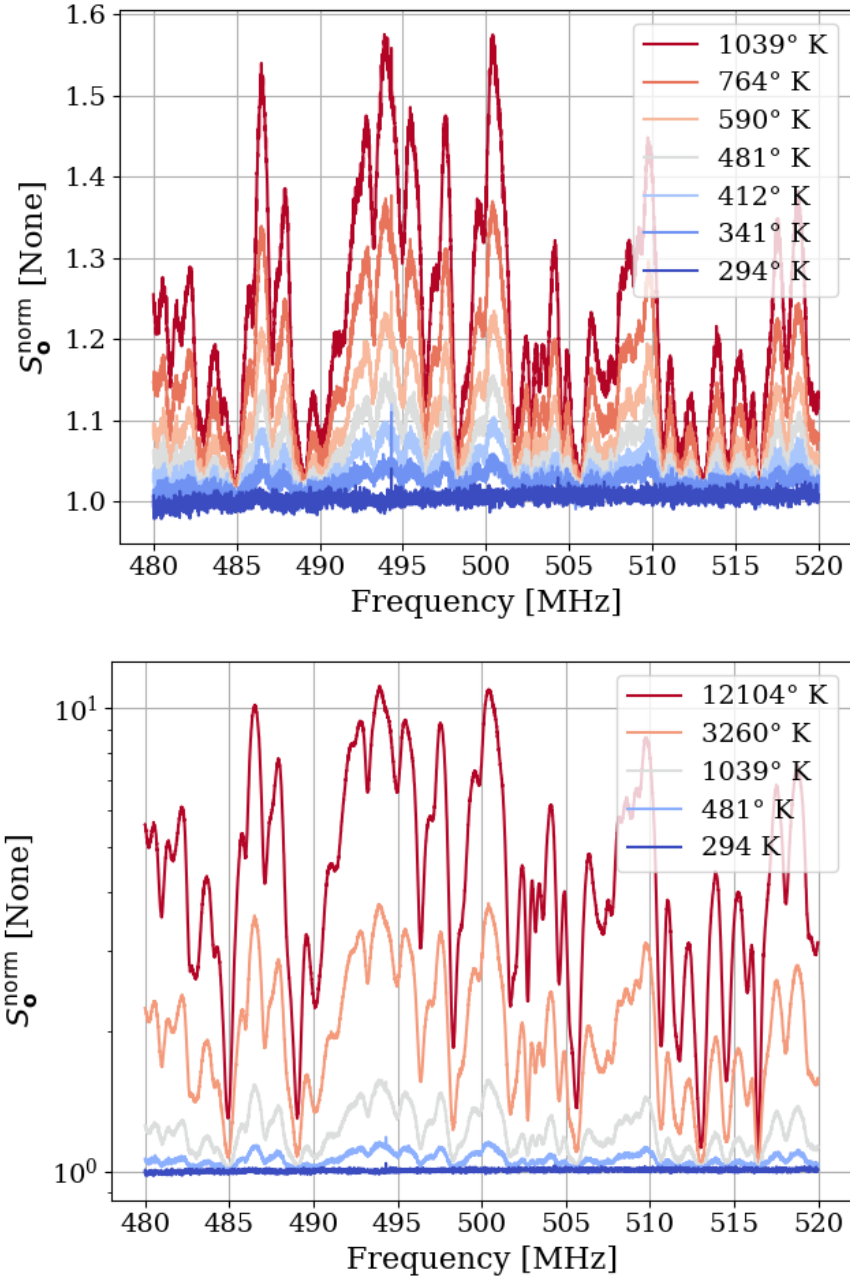


Figure 2.29: Spectra resulting from the antenna and circulator set up of Fig. 2.24 with the terminator on port 3 of the circulator at a variety of temperatures, as calculated by Eq. 2.40. Spectra are all normalized to a terminator through the same amplifier chain. The two panels show different ranges and resolutions of temperatures for clarity, but are the same setup. Also, note the top and bottom panels have linear/logarithmic scaling respectively.

Figure 2.29 showed spectra from the set-up shown in Fig. 2.24 for a hot terminator, but a cold terminator can also be used by dunking a terminator into liquid nitrogen (Fig. 2.28). Neglecting the small attenuation of the cable, the noise temperature is simply 77 K. An interesting test case is to set the hot temperature such that room temperature is halfway between T_{hot} and T_{cold} . In other words, let

$$T_{\text{hot}} = (T_{\text{room}} - T_{\text{cold}}) + T_{\text{room}} \quad (2.41)$$

$$\approx 511 \text{ K}. \quad (2.42)$$

For $T_{\text{hot}} = 12,100 \text{ K}$, $T_{\text{att}} = 294 \text{ K}$ and $T_{\text{out}} = 511 \text{ K}$, inverting Eq. 2.40 we expect L to be ~ 54.4 or 17.4 dB .

The two spectra with the terminator at 504 K (the closest temperature accessible with the 0.1dB step attenuator) and 77 K are shown in Fig. 2.30. Again, they have been normalized to the spectrum of a room-temperature terminator.

The two spectra shown in Fig. 2.30 appear to be mirrored about $S_{\text{o}}^{\text{norm}} = 1$. In a test of this reflection, these spectra are added, and the resulting sum spectrum is flat to about 1% with another 1.5% offset from the expected value of 2. This is shown in Fig. 2.31. The offset is likely due to the amplifier gain or ambient temperature drift between measurements.

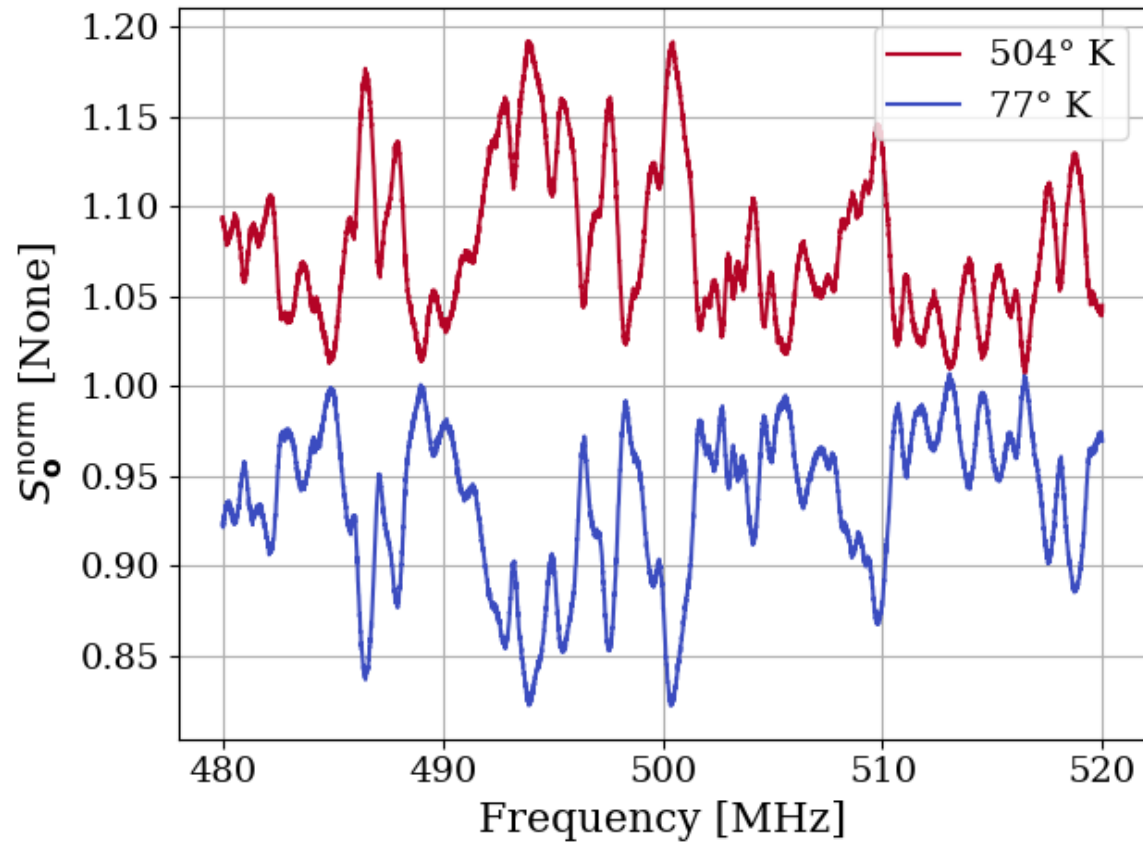


Figure 2.30: Spectra resulting from the antenna and circulator set up of Fig. 2.24 with the terminator on port 3 of the circulator ≈ 210 K above and below room temperature, as calculated by Eq. 2.40. Spectra are both normalized to a terminator through the same amplifier chain.

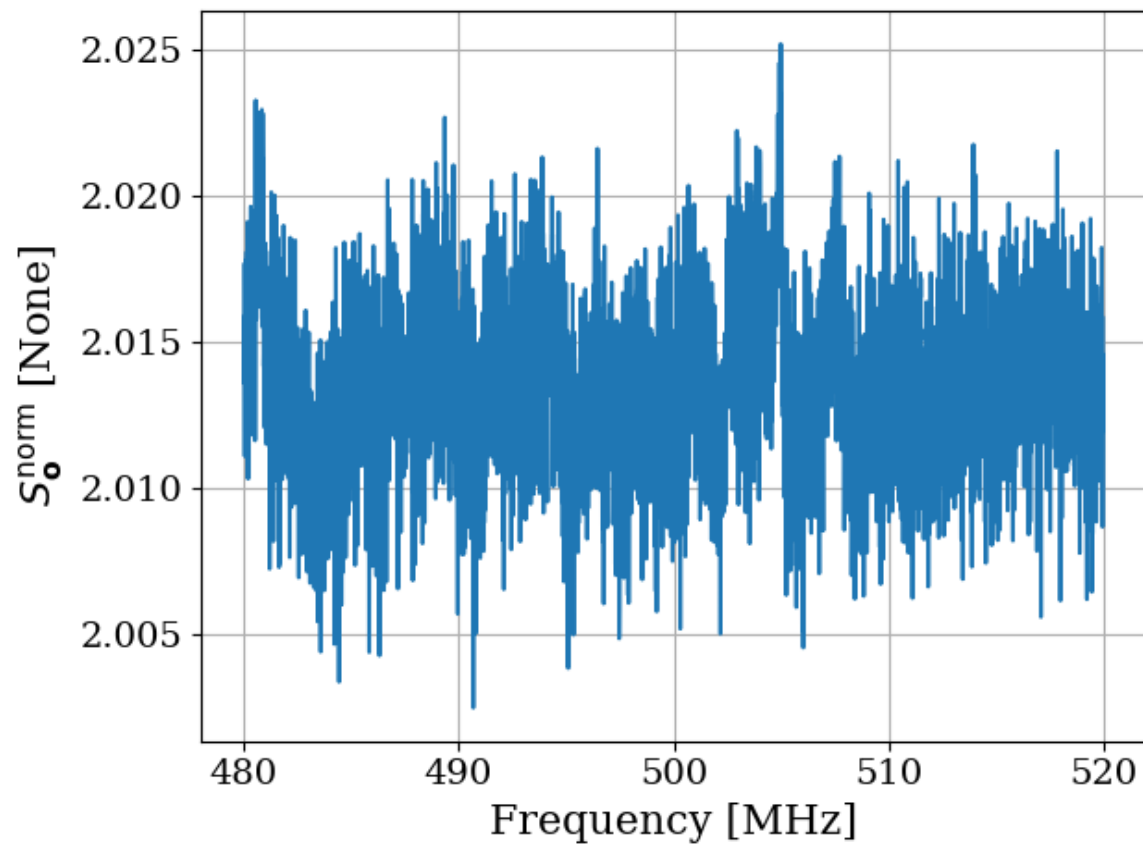


Figure 2.31: Sum of normalized spectra shown in Fig. 2.30. If the two normalized spectra in that figure were perfect reflections about 1, their sum would be a constant 2 in this figure. That is close to what is observed.

2.3.8 Relation of antenna S_{11} to thermal noise without a circulator

As pointed out in [128], the antenna impedance determines how much power is transferred from the cavity's electric field noise into the transmission line, and therefore what is measured by the amplifier²¹. Indeed, impedance is a useful tool in understanding the situation outside equilibrium. However, viewing from this perspective misses a subtle point when considering equilibrium; while an impedance mismatch will reduce the antenna's noise power from entering the transmission line, it will also reduce the noise in the line from leaving by the same amount! This power will reflect off of the mismatch at the antenna and be absorbed by the matched amplifier, resulting in a flat spectrum. This can be seen in the coax resonator when measured with the circulator in Sec. 2.3.4, specifically Figs. 2.22 and 2.23. Figure 2.32 shows a comparison of $1 - |S_{11}|^2$ and the noise spectrum of an antenna measured with a Pasternack low noise amplifier (LNA). There is no circulator in the setup, so variations are observed. Figure 2.33 shows the strong correlation between the curves in Fig. 2.32.

²¹The amplifier has a small impedance mismatch as well ($S_{11} \lesssim -15$ dB), but this is a small effect and is neglected.

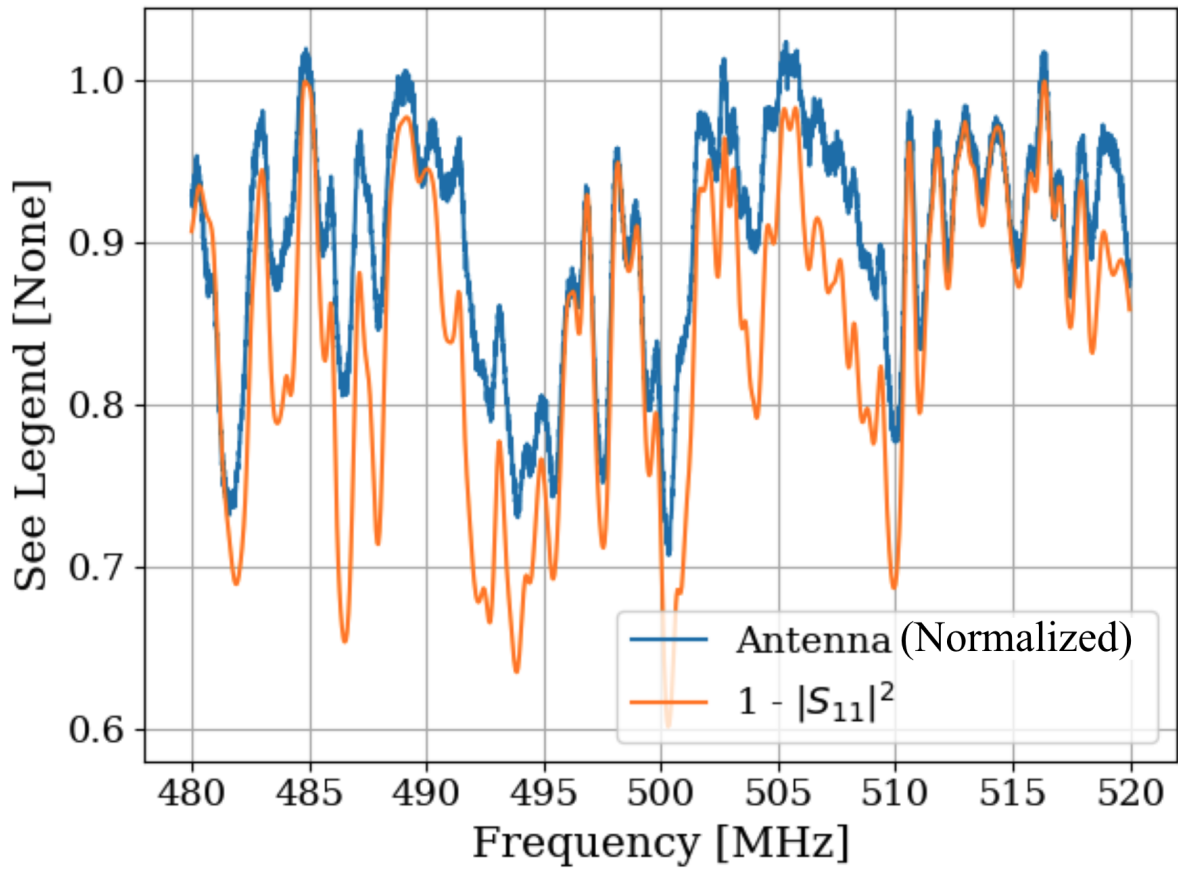


Figure 2.32: $1 - |S_{11}|^2$ and the normalized antenna spectrum without circulator. Shown at significant zoom to show detail. A much wider span is used to generate the correlation shown in Fig. 2.33.

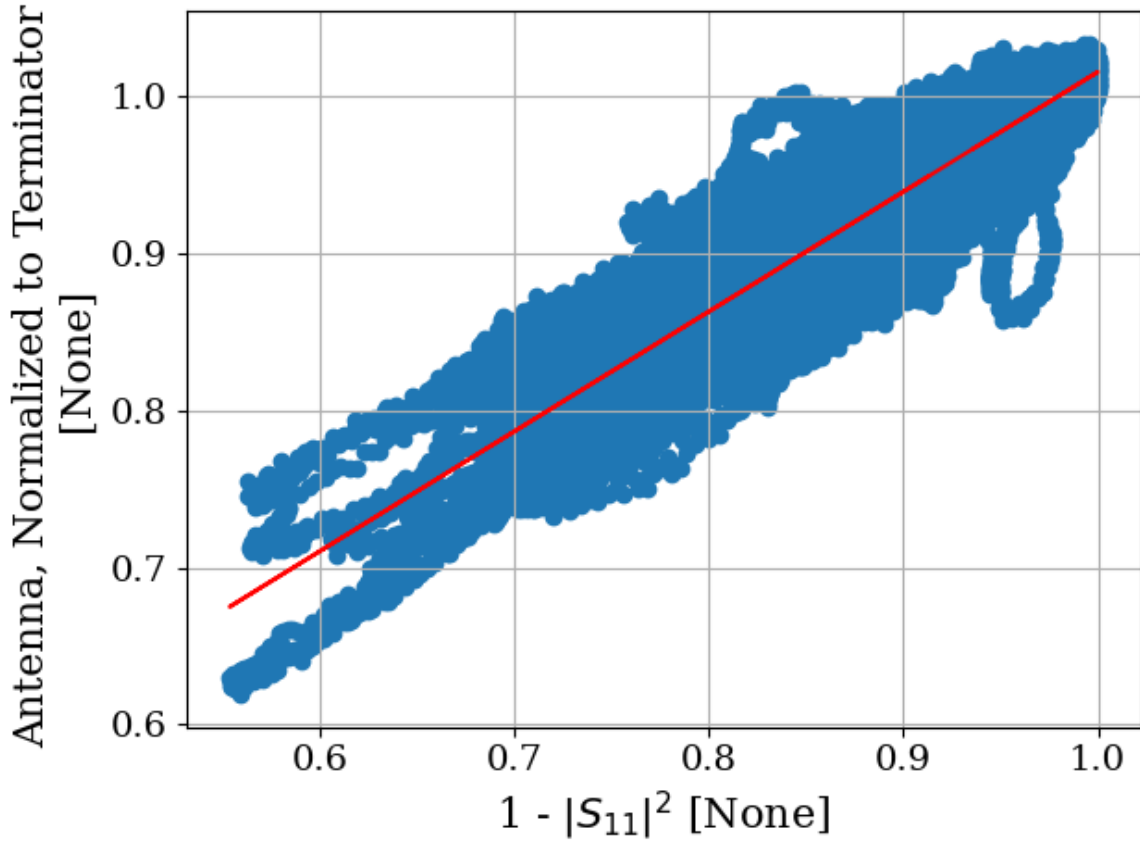


Figure 2.33: Correlation of $1 - |S_{11}|^2$ (Fig. 2.32, orange curve) and the normalized antenna spectrum without the circulator (Fig. 2.32, blue curve). The data are taken between 300 and 800 MHz, a much wider span than shown in Fig. 2.32. The antenna is connected directly to the low noise amplifier (LNA), i.e. with the circulator shown in Fig. 2.19 removed. The line of best fit is shown in red. The Pearson correlation coefficient is 0.92.

2.3.9 Discussion of thermal variations

Inspection of the output of the experiment (S_o , Fig. 2.16) reveals small variations in power spectral density PSD over spans of tens of kHz. Given an antenna in a cavity in thermal equilibrium with the input of an LNA, whose input is assumed to be real and matched, one would expect an output PSD which is constant with respect to frequency (up to small variations in system gain and noise temperature). The theory for this was outlined in Sec 2.3.1. These observed variations are not noise; for a given antenna position, the same shape is repeatedly measured (though the noise riding on these variations *is* random). The origin of the observed small variations lies in the effective temperature difference between the room and the LNA, causing a net power flow from the antenna into the LNA (Fig. 2.26). This effective temperature difference partially excites modes of the antenna/cavity system, causing the observed variations. This effect likely originates from a small reactive component of the LNA's input causing the electronic cooling described originally by Radeka [106]. This effect can be eliminated by adding a circulator between the antenna and LNA [135, 136] though for this experiment, it is impractical to get an isolator that covers such a wide band at relatively low frequency. Furthermore, the relatively wide (tens of kHz) variations can be handled by fitting to them, which is discussed in Sec. 4.2. Finally, a topic that was not described here but may be fruitful for future inquiry into this topic is the analysis of noise waves. See [137] or Sec. 9.12 of [138].

2.4 Reverberation Chambers and Statistical

Uniformity

The Dark E-Field Radio experiment consists of an antenna in a cavity. An averaged noise power spectrum is measured and an exclusion limit is set on the amount of power excess that would have been detected if it were there. In order to convert this limit on power into a limit on kinetic mixing ε , the antenna/cavity detector system must be calibrated.

In many situations, an electromagnetic cavity can be treated with a "spherical cow" approach; they contain one mode with a few simple properties that can be analytically computed. This works well enough for a smooth, empty cavity that resonates near its first mode, but for complex cavities, this treatment turns out to be insufficient. At high frequencies (where the wavelength is much smaller than the cavity), many modes, each with a finite spread in frequency, overlap. Each of these modes depends on very specific boundary conditions of the cavity and everything within it. The configuration of the cavity and its contents simply cannot be known to a level of precision that would allow for an analytic or simulated solution. Qualitatively, placing a small conducting object²² in a cavity greatly impacts the cavity's resonances as measured by its S-parameters (Fig. 2.36).

²²This is a point made by Hill [139], with the example of placing a soda can in a reverb chamber. Ben Godfrey and I independently discovered this by measuring S_{21} of an antenna in our shielding room with and without a small SMA terminator on the ground with surprisingly different results.

The problem is summarized nicely by Price et al. [140];

“The solution cannot depend in detail on such things as whether a small metallic can has been set down somewhere inside the test article, or the position of the pilot’s arms, or whether some mechanical widget has moved from position A to position B, changing the mode structure. If the answer did depend on those things, all of the measurements would be useless, defeated by the minutiae present in all systems.”

The answer to this dilemma is found in the study of mode-stirred reverberation chambers, such as those studied by Price, whose quote appears directly above. These are electromagnetic or acoustic cavities in which a large volume is occupied by an object that is highly reflective and designed to move, a so-called *mode stirrer*. Making the geometry more complex seems counter-intuitive. However, the payoff is in the transition from a deterministic theory to a statistical one. The fields in the cavity at any given configuration are complex and unknown, but the statistics of the fields subject to the stirring can be rather simple.

This section provides a mostly qualitative overview of the subject. The de facto reference is David Hill’s 2009 book [139] which consolidates his many papers spanning his ~30-year career. There is little I can do to explain the theory of electromagnetic reverberation chambers, which is not in this book, so I will focus on their application to the experiment and cite Hill where appropriate. The reader is encouraged to consult this book and its references for a more detailed exploration of the subject.

2.4.1 Deterministic solutions to electromagnetic waves in cavities

The electromagnetic fields within a cavity can be modeled by applying Maxwell's equations with the appropriate boundary conditions. In principle, with enough knowledge about the contents of the cavity, this treatment could calculate fields in any cavity. Unfortunately, it quickly becomes untenable for all but the simplest cases. Therefore, we will restrict ourselves to an empty cavity with perfectly conducting surfaces. The resulting fields have simple analytic solutions which vary sinusoidally in both space and time. They are derived in many places. See for example [139, 141]. While this treatment will not solve the problem at hand, it is a good starting point and will illustrate important features leading to the statistical treatment in the following subsection.

For a rectangular cavity of linear dimensions a , b and d , the frequencies of resonance are given by

$$\nu_{mnp} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}, \quad (2.43)$$

for integer mode numbers m , n and $p \geq 0$ and wave speed c . Resonances are denoted TE/TM for transverse electric/magnetic, with the mode numbers m, n, p in the subscript. The lowest resonance mode requires at least two non-zero mode numbers. Thus, for $a < b < d$ the resonance with the lowest frequency is at TE_{011} . For an ideal cavity with dimensions equal to that of the shielded room in this experiment ($8 \times 10 \times 12$ feet), the lowest frequency mode is 63.6 MHz. An important consideration is the degeneracy of electric and magnetic fields for any mode where all three mode numbers are non-zero. For example, TE_{111} and TM_{111} occur at the same frequency, and both of these modes must be counted in the following section

(see page 28 of Hill [139] for a discussion).

A quantity of interest is the functional form of the cumulative number of modes at frequencies below a given frequency $N_s(\nu)$. This is rather simple by brute force computer counting²³, though an analytic solution can be computed by looking at the volume enclosed in a sphere of k -space [142], where k is the wave vector²⁴. This analytic form is given by

$$N_s = \frac{8\pi}{3} abd \frac{\nu^3}{c^3} - (a + b + d) \frac{\nu}{c} + \frac{1}{2}. \quad (2.44)$$

Differentiation of Eq. 2.44 results in a functional form for the mode density, i.e. the number of modes contained in a frequency band,

$$\frac{dN_s}{d\nu} = 8\pi abd \frac{\nu^2}{c^3} - \frac{a + b + d}{c}. \quad (2.45)$$

Equations 2.44 and 2.45 are plotted in Fig. 2.34

This is useful because as the mode density becomes high, modes in a cavity of finite conductivity begin to overlap. The modification of modal structure by conductors is demonstrated in Figs. 2.35 and 2.36 by placing different conductors in the room and measuring S_{11} .

From these measurements it can be seen that the frequency of resonances can be pulled around, even by conductors occupying a small percentage of room volume. This is especially true for high frequencies. However, by using conductors that occupy a large volume of the room such as a mode-stirrer (simulated here with scrap metal), high-frequency modes are

²³As long as you don't forget the degeneracy!

²⁴This whole business of mode counting is directly analogous the calculation of density of states in statistical mechanics. It shouldn't come as a surprise that the calculation is carried out in the same way.

pulled around so much that they overlap. This is the intuition behind reverb chambers and the idea of statistical uniformity.

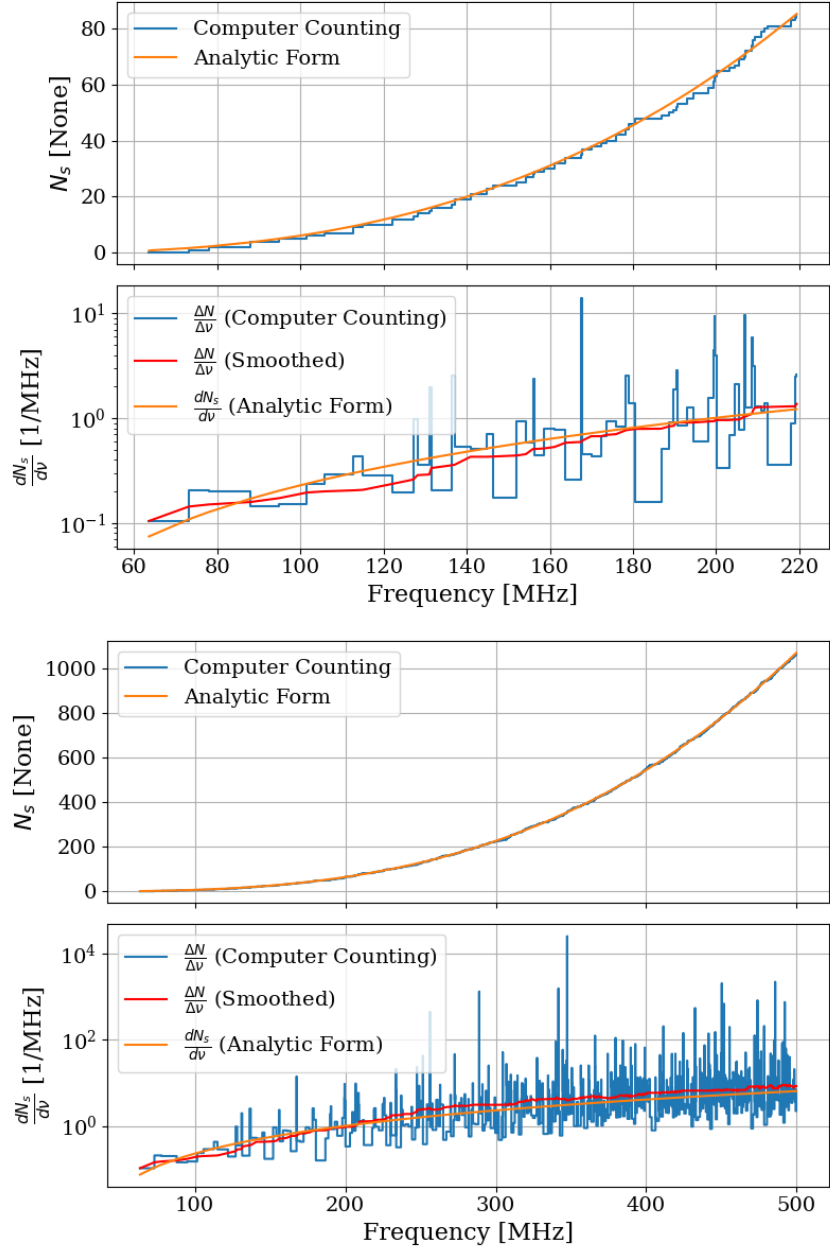


Figure 2.34: Modal density for an electromagnetic cavity with dimensions of the shielded room (nominally $8 \times 10 \times 12$ ft, see table 2.2). The upper sub-plot shows $N_s(\nu)$, the cumulative number of modes below a given frequency. The lower sub-plot shows the derivative of $N_s(\nu)$. The upper and lower panels show the same data at two different frequency spans. These plots are a recreation of Figs. 3, 4 and 5 from [142] for a cavity with dimensions of the shielded room. Note that large spikes in the blue curves (finite differences, $\Delta N/\Delta \nu$) should be interpreted as binning artifacts and are only shown for reference. The analytic derivative is more useful.



Figure 2.35: Pictures of conductor configurations in the shielded room. The antenna is in the same position between photos. The left panel shows a tin foil hat, and the right shows a random placement of scrap metal. Hat and scrap metal courtesy of Tyler Erjavec.

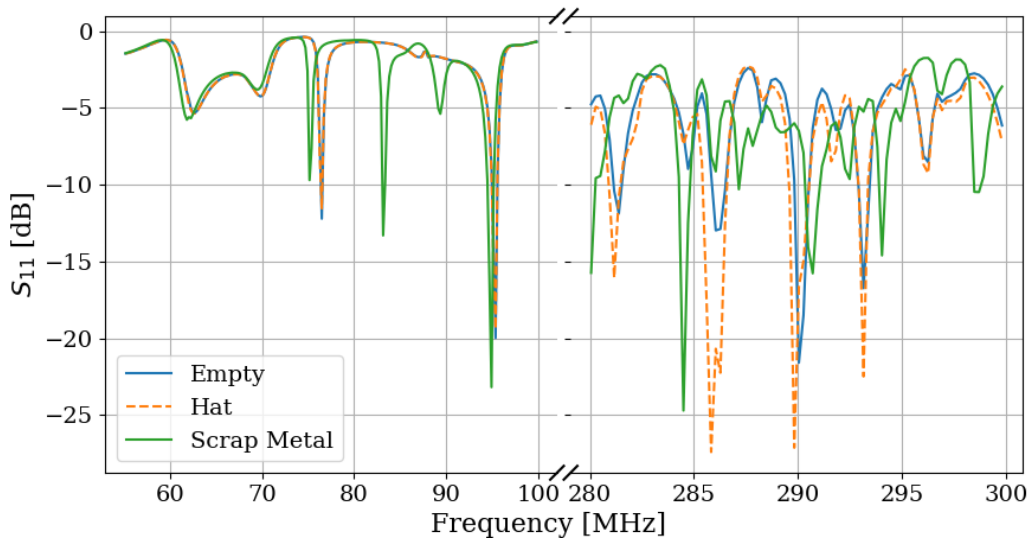


Figure 2.36: Measured S_{11} with different conductors in shielded room, as pictured in Fig. 2.35. Modes visible at low frequency agree nicely with the predictions of Eq. 2.43, though they are pulled around by the scrap metal. Note the frequency scaling changes slightly after the X-axis break.

2.4.2 Statistical approach to electromagnetic waves in cavities

By using a mode stirrer, modes are intentionally pulled around in frequency, and the average response of the cavity begins to converge to be relatively flat. The regime where this occurs is known as being well-stirred. It requires the stirrer to be large enough and obey some design principles and that the frequency to be above a threshold. This lowest usable frequency (LUF) is determined by a minimum modal density since the modes must be close enough together in order to overlap. For a room-sized reverberation chamber with a lowest resonance of around 60 MHz (~ 3 or 4 meters per side), the rule of thumb cited by Hill, among others, is the LUF is close to where the modal density is about 1 mode/MHz. Therefore, according to Fig. 2.34, the LUF of our shielded room is about 200 MHz.

A simple alternative to using a purpose-built mode stirrer is moving a receive antenna around in the room. Instead of moving the spatial structure of the modes around the antenna, the antenna is moved through the modes. While not as effective, it is simpler to implement, so this was the method used in Run 1A. Note that Run 1A does not rely on statistical uniformity. Simulations provide a means to calibrate the system as discussed in Sec. 4.3. However, using these statistical ideas provides a much more stable simulation with a much better agreement with measurement.

One final remark that is relevant to the experiment is the concept of composite Q . This parameter represents a resonant enhancement factor that corresponds to the antenna/room system's tendency to "ring up" in the same way any resonator will. It is referred to as *composite* Q and represented as $\tilde{Q}$. It is analogous to the standard quality factor of a resonator

with one important modification; the experiment is operated across a wide frequency range, so $\tilde{Q}$ is defined across the continuum of these resonances, not only on classical eigenmodes of the system.

2.5 System Design

This section outlines the subsystems which make up the experiment. While specifics and basic calculations are provided as they apply to design choices of subsystems, testing and characterization of the system as a whole is left to Ch. 3. A simplified schematic of the entire experiment is shown in Fig. 2.37 and a photo of the lab is shown in Fig. 2.38.

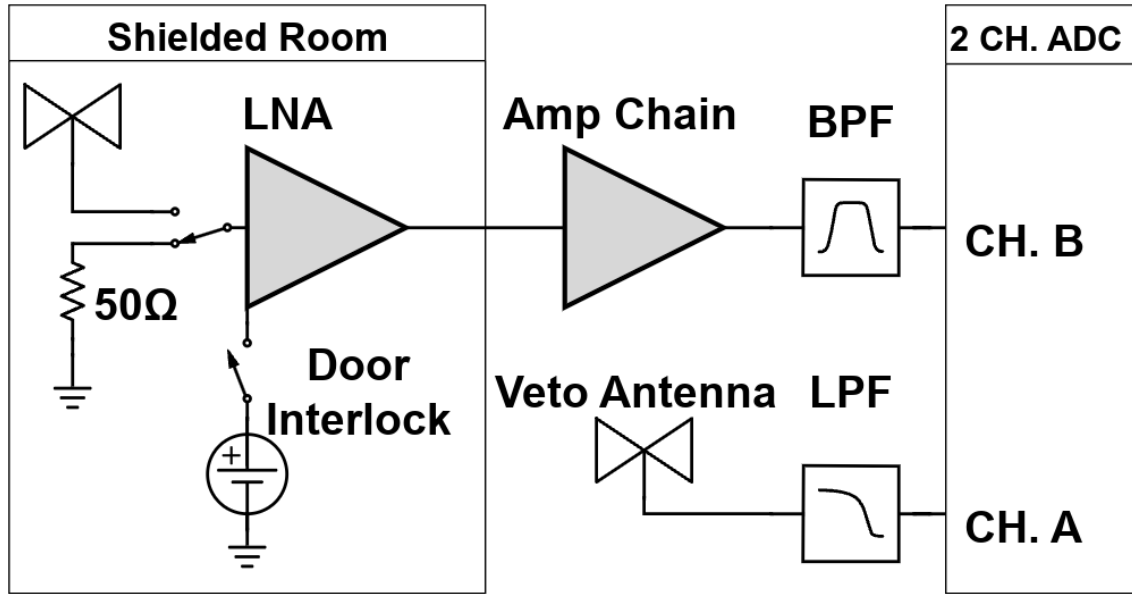


Figure 2.37: Schematic diagram of the RF receiver system. An RF coax switch allows the computer to control the source (antenna or terminator). The amplifier chain is shown with more detail in Fig. 2.42. The switch is controlled by optical fiber to maintain the isolation of the room. The LNA is interlocked (Fig. 2.43) to the door to protect the ADC from large signals when the door is open. The band pass filter (BPF) defines the experiment's bandwidth, $-3\text{ dB} \approx 40 - 320\text{ MHz}$ and is discussed in Sec. 2.5.4. The veto antenna is outside of the room and interference is not reduced by the $\approx 100\text{ dB}$ shielding effectiveness (SE) of the room, so no gain is required. The low pass filter (LPF) on the veto is for anti-aliasing.



Figure 2.38: Photo of dark radio lab. The shielded room contains the main antenna as well as the LNA and power supply (not visible). The veto antenna can be seen hanging outside of the shielded room. The photograph was taken looking south.

2.5.1 Shielded room

The shielded room [143] serves two purposes. The first is straightforward; to shield the antenna, keeping radio frequency interference (RFI) *out*. The second purpose is a bit more subtle; to keep any converted dark photons *in*. This second point is addressed further as an aspect of system calibration in Ch. 4, but roughly can be described by the loaded quality factor [144–146] of the antenna/room system. Namely, a more resonant system will be more sensitive to coherent signals. This subsection will focus on the first point, keeping RFI out.

Shielding effectiveness SE is a measurement of a shielding enclosure’s ability to attenuate electromagnetic waves from entering,

$$\text{SE} \equiv 10 \log_{10} \left(\frac{P_{\text{open}}}{P_{\text{closed}}} \right) = P_{\text{open}, \text{dB}} - P_{\text{closed}, \text{dB}}, \quad (2.46)$$

where $P_{\text{open}}/P_{\text{closed}}$ are powers received with the door open/closed. The ratio of powers allows all the specifics of antenna matching to cancel, allowing for a very simple differential measurement. The results of this are described in Sec. 3.2.

Another important measurement are the dimensions, shown in table 2.2. I carefully measured the room with a laser range finder. I checked the rangefinder against a measuring tape, and it gives good agreement to 1 mm²⁵. The room is out of square by a few mm, especially the height measured in the south-west corner compared to the height measured everywhere else. This is the most extreme deviation and is about 5 mm.

²⁵After a year, I dropped it and it now gives crazy readings which jump around by 10s of cm. Be careful!

Direction	Coordinate	Nominal Length [ft]	Measured Length [m]
West-East	x	10	3.070
Vertical	y	8	2.457
North-South	z	12	3.684

Table 2.2: Direction, coordinate, and length measurements of the shielded room in lab 314. Note that Fig. 2.38 is looking south, so x is right-left, and z is into the page. Note that these are the mean values of several measurements. The room was found to be about 5 mm out of square, so these should be interpreted as ± 5 mm.

2.5.2 Antenna

The antenna plays an important role in the experiment as the matching device between electromagnetic waves in the cavity and the 50Ω receiver system. For a broadband search such as the 50-300 MHz run (a 6:1 bandwidth), a broadband antenna must be used. The chosen antenna must provide a good impedance match and high efficiency since an inefficient antenna would convert a substantial amount of the converted dark photon’s power into heat within the antenna’s structure. The antenna is connected to the RF switch via a low loss (0.45dB at 300 MHz) 21 ft. LMR400 cable. This contributes about 31.6 K to the ≈ 400 K system noise (see Eq. 2.27). Note that the final limit on ε coupling scales with the square root of system temperature, so this is only a few percent degradation in the final limit after a 9 day run.

For the 50-300 MHz run, a ≈ 131 cm biconical antenna (bicon) was chosen. The selected model is manufactured by COMPOWER, model AB-900A [147]. In a phone call with the manufacturer as well as testing of the isolated balun, it was determined that the balun used in the antenna was 1:1. This allows for simple simulation of a free-space aperture which

agrees remarkably well with manufacturer data, Fig. 2.39. Using commercial finite element analysis software (COMSOL [148]), the lumped port option allows for a balanced drive of an antenna. A match to a $50\ \Omega$ transmission line through a 1:1 balun is simply modeled as a lumped port, a very simple object in COMSOL featured in nearly all of the antenna tutorials²⁶. Additionally, there is good agreement between simulated and measured antenna impedance. The complex impedance is shown in Fig. 2.40.

²⁶See for example the dipole antenna tutorial, available at <https://www.comsol.com/model/dipole-antenna-8715>

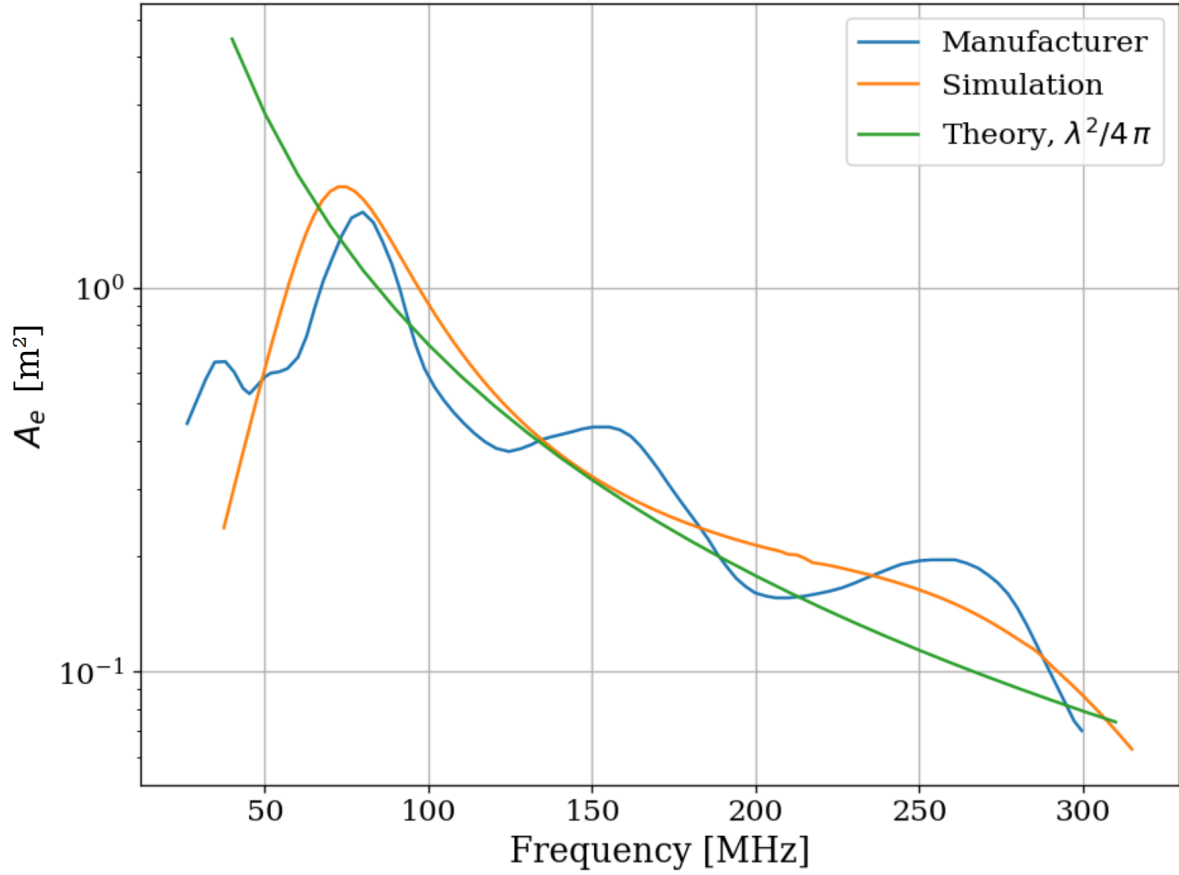


Figure 2.39: AB-900A biconical antenna effective aperture, simulated, measured and theoretical in free space. Simulation performed in COMSOL [148]. The measurement was provided by the manufacturer [147] as an antenna factor and was converted to aperture. Variations observed in the manufacturer’s measured data are the result of testing over a ground plane and are a known discrepancy between simulations and measurements of “free space” antenna factor (see for example [149]).

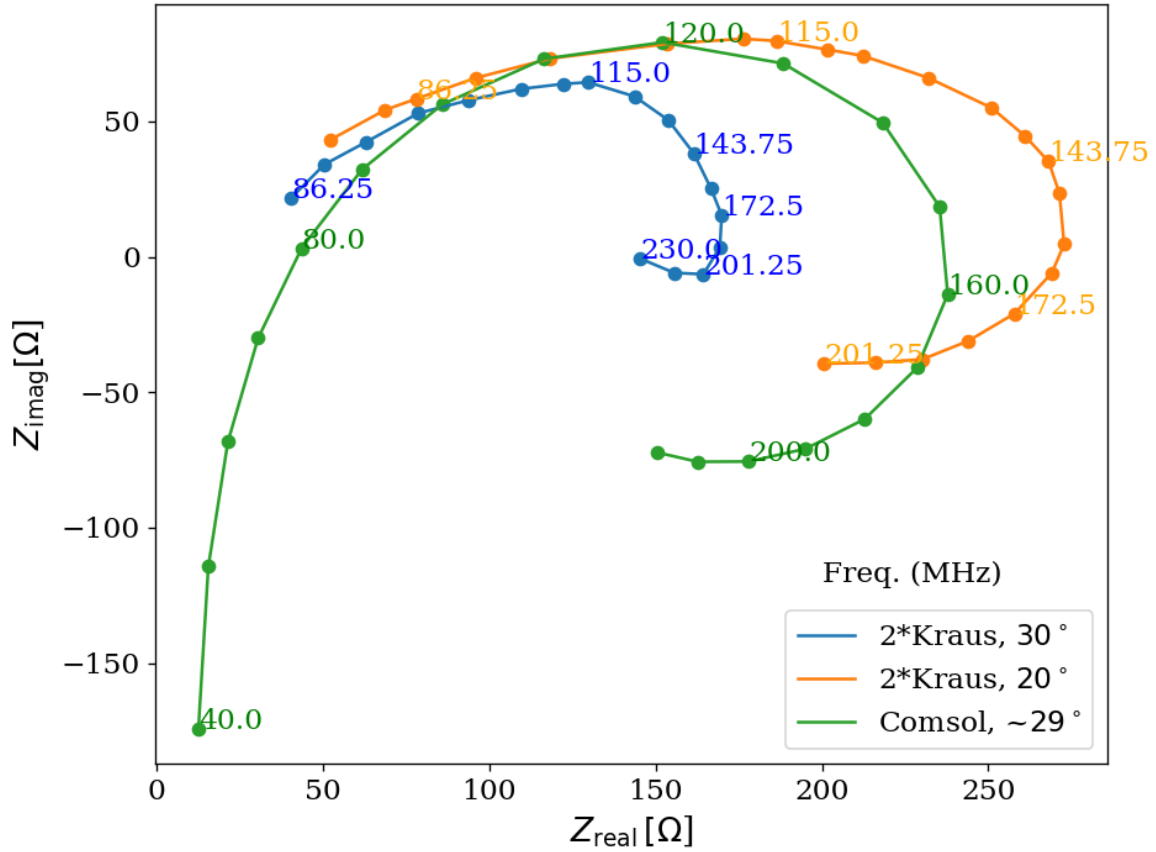


Figure 2.40: Simulated AB-900A biconical antenna free space complex input impedance shown in blue. The simulation was performed in COMSOL. The measurement is from Kraus' Antennas, second edition, Fig 8-13 [113]. Measured data are for a monoconical antenna and must be multiplied by 2 to compare to a biconical antenna, as discussed in Kraus. Numbers along the curve indicate the frequency in MHz.

2.5.3 Terminator and fiber-optic switch control

Experience has shown that it is advantageous to have a stable noise reference at the beginning of the signal chain to monitor system performance. One may think the antenna can provide this following Sec. 2.1.1.2, but in practice, the thermal variations described in Sec. 2.3 mean that the antenna noise is not that stable. Furthermore, while RFI was not significant in the actual run, it very well could have been, introducing some uncontrolled, coherent power into the spectrum, which would have had an unknown origin. A terminator will give a very constant noise power density $S_{\text{term}} = kT$ regardless of RFI and antenna position. For this reason, one is included. It is at the same temperature as the walls and is a nice passive solution. The RF switch shown in Fig. 2.37 is actuated periodically during the run to measure the terminator's noise power through the same amplifier chain as the antenna. The terminator is connected to the switch via a 21-foot cable (LMR 400) that is identical to the antenna's cable. The switch is controlled via a fiber-optic link in order to reduce RFI. The fiber optic is an extremely important feature that was overlooked for several years at the beginning of the experiment, leading to lots of RFI.

2.5.4 Signal conditioning

As discussed in Sec. 2.1.5, the ADC has internal signals that are mitigated by introducing gain before the ADC. The amount of gain must be carefully chosen since too much will cause the ADC to saturate. Additionally, high frequencies must be limited before digitizing to prevent aliasing. To accomplish this, several RF components must be selected to condition

the analog signal: an LNA, a secondary amplifier, band-pass filter and several attenuators.

2.5.4.1 Low noise amplifier

The important concepts of the low-noise-amplifier (LNA) have been introduced in Sec. 2.1.4. The key takeaway is that introducing gain from a LNA early in the signal path improves the overall signal-to-noise ratio, even if noisy components follow. The relevant specifications for the Pasternack PE15A1012 [124] are summarized in table 2.3.

Specification	Value	Uncertainty (50-300MHz)	Units
Frequency Range	50-1,000	-	MHz
Gain	40	± 1	dB
Noise Temperature	110	± 10	K
Input Return Loss	< -15	-	dB
Output Return Loss	< -15	-	dB
Price	500	-	USD
Voltage	9-15		V
Current	100	10	mA

Table 2.3: Specifications for the Pasternack PE15A1012-E low noise amplifier. The voltage is regulated internally, so the exact voltage supplied is not critical, though there is a slight gain dependence on voltage since a higher voltage causes the amp to run warmer, see Fig 3.8.

We have several identical amplifiers which are labeled with letters. At the time of writing, amplifiers A-D are out of commission. Amplifier E was used for Run 1A.4.

2.5.4.2 Secondary amplifier

As discussed in 2.1.4, the noise temperature of a secondary amplifier has negligible impact on the total system noise temperature. For this reason, a cheaper secondary amplifier is

used. The specifications for the Mini-Circuits ZKL-1R5+ [150] are outlined in Table 2.4. This amplifier has no internal regulator, so gain and noise temperature depend strongly on the voltage presented to its terminal. The voltage was set to 9.05 V for Run 1A using an external regulator (built around a TI LM317 [151]).

Specification	Value	Uncertainty (50-300MHz)	Units
Frequency Range	50-1,000	-	MHz
Gain	40.5	.3	dB
Noise Temperature	275	15	K
Input VSWR	<1.14	-	dB
Output VSWR	<1.37	-	dB
Price	235	-	USD
Voltage	9-15	-	V

Table 2.4: Specifications for the Mini-Circuits ZKL-1R5+ as measured with a supply voltage of 9.05 V. This amplifier contains no internal regulator, so the voltage is set using an external regulator (built around a TI LM317 [151]).

2.5.4.3 Band pass filter

The band-pass filter has two purposes. The first purpose is to minimize the bandwidth entering the ADC. This allows for introducing as much gain as possible without wasting power amplifying frequencies where the antenna doesn't offer a good match. This will be computed below in Sec. 2.5.4.4. The second purpose is to prevent aliasing²⁷. Aliasing occurs when the analog signal contains frequency components at frequencies greater than half the sample rate, in other words, when the signal and ADC don't obey a condition called the

²⁷Aliasing is a rather complex topic that is greatly simplified here. Wikipedia's aliasing page is an excellent reference. For a more rigorous treatment, see Ch. 7 Sec. 3 of the second edition of Signals and Systems by Oppenheim et. al [152]

Nyquist criterion, $\nu_s/2 \geq \nu$, where ν_s is the sampling frequency and ν is the frequency of the analog signal.

When this criterion is not met, higher frequencies are mapped back down to a lower frequency, described by the aliasing formula:

$$\nu_{\text{alias}} = |\nu_{\text{signal}} - n \times \nu_s|, \quad (2.47)$$

where ν_{alias} is the aliased frequency, ν_{signal} is the original signal frequency, ν_s is the sampling rate, and n is an integer which specifies the Nyquist zone. In the simple case with bandpass filters that roll off well within the first Nyquist zone, only $n = 1$ must be considered.

As an example, a $\nu_{\text{signal}} = 500$ MHz signal sampled at $\nu_s = 800$ MHz (the Run 1A sample rate), it would alias to $\nu_{\text{alias}} = 300$ MHz. This also means that 1,300 MHz (residing in the second Nyquist zone) would alias down to 300 MHz, but again this and higher frequencies won't be considered. This means that by choosing only to analyze frequencies less than 300 MHz, only frequencies greater than 500 MHz will alias into this analysis span.

The Mini-Circuits ZX75LP-288-S+ low pass filter[153] serves as the anti-aliasing filter in Run 1A. It has an insertion loss of 68 dB at 500 MHz, while being flat to within 2dB from 50 to 300 MHz. Additionally, the Mini-Circuits SHP-50+ [154] is used as the high pass filter. When connected in series, these two filters constitute the band-pass filter shown in Fig. 2.37.

2.5.4.4 Assembling a signal conditioning chain

The total gain required can be estimated by setting the output referred power of bandlimited, 400 K noise source times the gain equal to the maximum power the ADC can handle,

$V_{\text{RMS}}^2/Z = 0.63 \text{ mW}$ for $Z = 50 \Omega$. Assuming a perfect filter from 50-300MHz (a 250 MHz bandwidth), the gain required is

$$G = \frac{0.63 \text{ mW}}{k 400\text{K } 250 \text{ MHz}} = 4.5 \times 10^8 = 87\text{dB}. \quad (2.48)$$

Noise is a Gaussian random process, however. This much gain ensures 1σ of the time-domain samples are below clipping. Since many samples are collected ($2^{24} \approx 1.6 \times 10^7$ in Run 1A), and each has a probability of about 16% of clipping, many samples will clip with 87 dB of gain. However, it gives a good estimation for what to expect.

The band-pass filter is not a brick wall from 50 - 300 MHz as was assumed in 2.48. The effective bandwidth of a filter is defined here as the integral of the square magnitude of its gain,

$$B_{\text{eff}} \equiv \int_{-\infty}^{\infty} d\nu \ 10^{S_{21}/10}, \quad (2.49)$$

where S_{21} is measured in dB, and we are interested in integrating a quantity that is proportional to linear power, so it is divided by 10 rather than 20. The linear S-parameters of components connected in series multiply [131], but since dB are logarithmic, this is equivalent to adding their S-parameters (in dB). Also note that $|S_{21}| \equiv \text{IL}$, the insertion loss. Insertion loss is frequently given on data sheets.

With all this in mind, we can compute $B_{\text{eff}} = 237.06 \text{ MHz}$ for the actual filters (Mini-Circuits SLP-50+ high pass filter and Mini-Circuits ZX75LP-288-S+ low pass filter) from their data sheets. Curves of S_{21} for the real band-pass filter and an ideal brick wall filter are

shown in Fig. 2.41. Note that it is actually slightly less than the ideal 250 MHz bandwidth since there is some loss in-band.

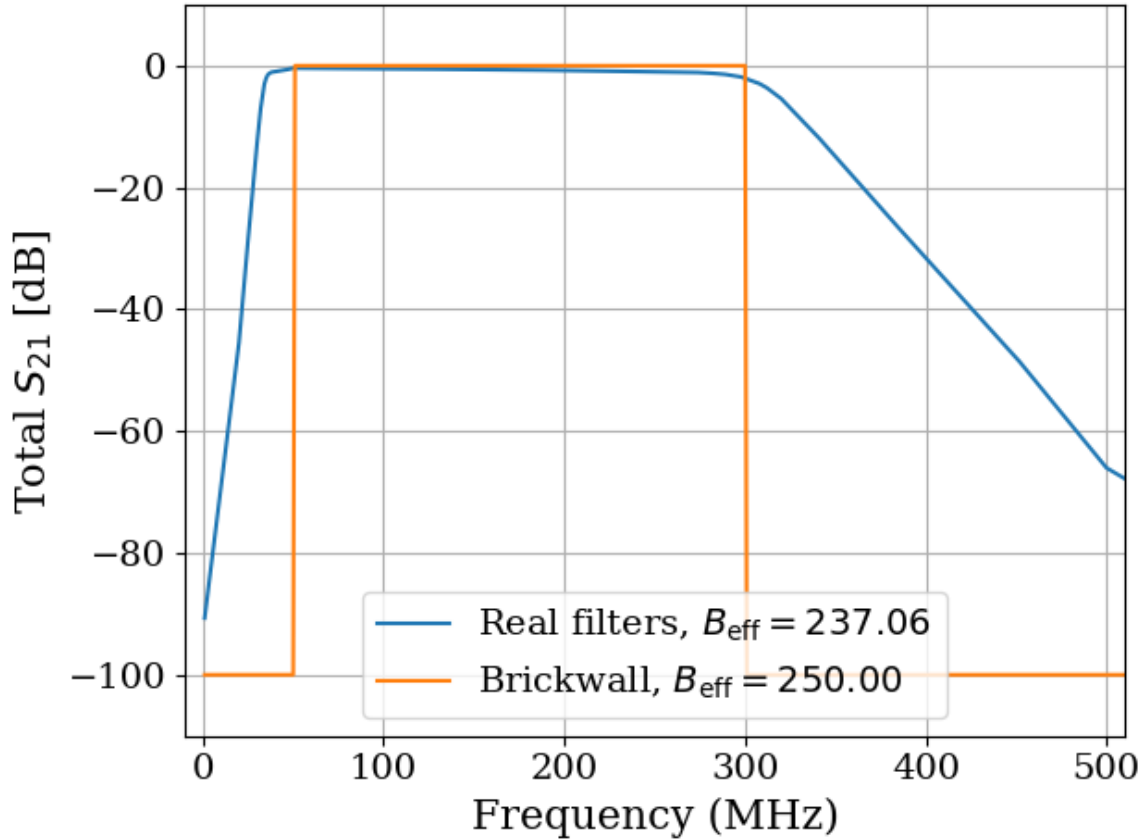


Figure 2.41: Measured gain (S_{21}) of the Run 1A band-pass filter. Shown in dB, however B_{eff} is computed with linear S_{21} as in Eq. 2.49. The response of an ideal brickwall filter is shown for reference.

Setting the gain is ultimately done by trial and error; taking a scan, adding some attenuation if it clips, taking a scan, etc. Experience and preliminary simulations have shown that a few dB of clipping for a noise-dominated signal actually doesn't matter much, but this was not fully explored. If future runs are to scan for longer, they may have significant contamination from ADC spurs (see Sec. 2.1.5.1), so additional gain driving into clipping

may be fruitfully explored. Every dB of gain added allows for $10^{1\text{dB}/5} \approx 44\%$ more averaging before ADC effects are at the same relative level²⁸, so a little extra gain goes a long way.

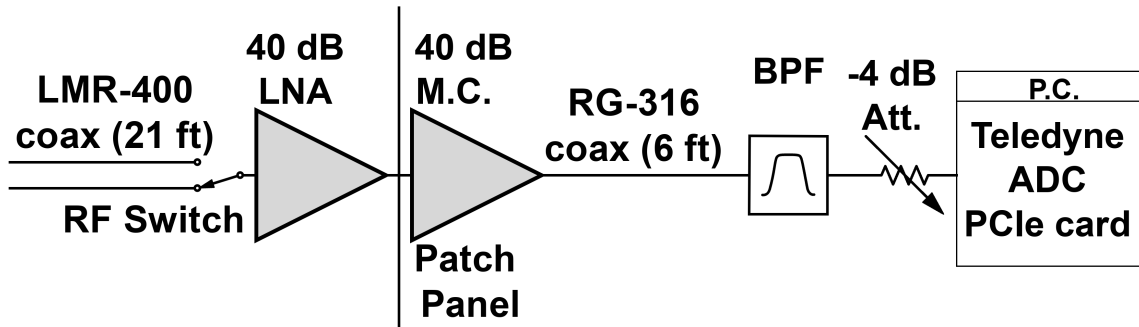


Figure 2.42: Schematic diagram of Run 1A amplifier chain. Antenna (Sec. 2.5.2) and terminator (Sec. 2.5.3) are not part of the amplifier chain and, therefore, are not shown, but they would be at the far left of the schematic. All RF connectors are SMA, except the connection between the bicon antenna and its cable which is N type (though that cable has SMA on the switch-side). RF switch is Teledyne CR-33S8D-T. 40 dB LNA is Pasternack PE15A1012-E (Table 2.3). The patch panel uses an SMA bulkhead connector labeled “A” (far left-hand side when standing outside the shielding room, as shown in Fig. 2.38). 40 dB Mini-Circuits (M.C.) is Mini-Circuits ZKL-1R5+ (Table 2.4). Band-pass filter (BPF) is made up of Mini-Circuits SLP-50+ high pass filter and Mini-Circuitis ZX75LP-288-S+ low pass filter. Note that filters and attenuator are directly connected to the SMA port of the Teledyne ADC (Sec. 2.5.7) to mitigate RFI received in the RG-316 cable run.

2.5.5 Veto antenna

The veto antenna is an identical Compower AB900 bicon antenna [155] as is used in the shielded room. It is connected to channel A of the ADC with no amplification. However, it has a Mini-Circuits ZX75LP-288-S+ low pass filter [153] to prevent aliasing. This is the same model low pass filter that is used for the main channel. The antenna is hung a few

²⁸Calculations in dB like this are handy once they are understood, but can seemingly come out of no where. This can be worked out by a careful reading of section 2.2, using properties of logarithms and definition of the dB.

feet in front of the door from some pipes on the ceiling. The proximity to these pipes likely gives the antenna a strange response, but its purpose is simply to look for large RFI signals, so this isn't that important.

2.5.6 12 V power system

The LNA and switch are active components that require power to operate. The experiment is incredibly sensitive to RFI, so while there is 120 VAC in the room, it is simpler to provide the power from a 12 V battery than to use an AC/DC regulator. Originally a 12 V lithium-ion (LiFePO₄) battery was used. Lithium-ion batteries contain several cells and a controller to regulate charge/discharge between the cells. In order to remove the possibility of this controller emitting RFI in the room (which would create candidates that would be extremely difficult to veto), the lithium-ion was replaced with a 12 V lead acid golf cart battery²⁹.

The LNA is interlocked to the door such that power is cut when the door is opened. This prevents the large radio signals from being amplified once they enter the room and protects the ADC. The circuit that controls this is shown in Fig. 2.43. Note that it takes a few seconds for the slow turn on circuit to discharge, so the door should be opened slowly

²⁹This was done between run 1A.2 and 1A.3 due to interference resulting in many candidate detections. Most of these were likely external RFI, which were eliminated by cleaning the door (discussed in Sec. 3.2). Two variables were changed (cleaning of the door and replacement of the battery) so it's unclear if a lead acid is necessary. It works, however, and if it ain't broke, don't fix it.

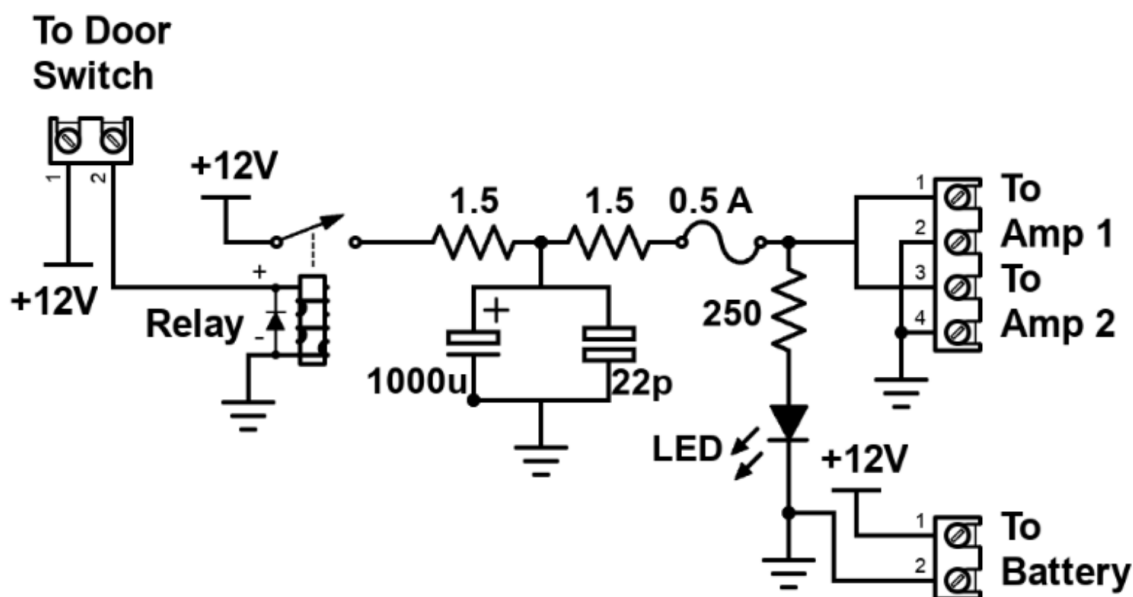


Figure 2.43: Schematic diagram of Interlock board. No voltage regulation is provided because it is designed to work with amplifiers containing internal regulation (Pasternack PE15A1012 [124]). Not shown is a simple “slow turn on circuit” consisting of a 0.68 F capacitor and an 8 Ω , 10 W resistor (time constant = 5.4 seconds) which was installed to protect the amplifier from transient voltages when the door is closed. This circuit can be seen in Fig. 2.44. Experience has shown the liberal use of fuses to be prudent when working with car batteries in a metal room.

2.5.7 GPU-Based Real-Time Spectrum Analyzer ³⁰

The use of commercial Spectrum Analyzers (SAs) that feature so-called real-time spectrum analyzer (RTSA) mode comes with several restrictions that limit the efficiency with which they are able to perform wide-band scans with narrow frequency resolution, as pointed out the dark radio pilot run [127]. The number of frequency bins output by a real discrete Fourier transform (DFT) is equal to half of the number of time domain samples, while the bandwidth is given by half of the sample rate. Furthermore, the ability to acquire data in real-time requires a DFT algorithm (generally implemented as a fast Fourier transform, FFT) and computational resources that can operate on time domain data at least as fast as it is acquired. In practice, real-time DFTs with high-frequency resolution and wide bandwidth require modest memory, transfer rates and processing resources. Commercial “real-time spectrum analyzers” tend to cheat a bit to reduce hardware requirements. Frequency mixers reduce the required sample rate (and therefore span), limits on FFT lengths reduce either span or resolution, and limits on the rate of scans make it so that these SAs aren’t real-time (at least in the sense that we require them to be).

For this reason, I have constructed a custom SA based on the Teledyne ADQ32 PCIE digitizer[125], which is wide-bandwidth (up to 1.25 GHz frequency span), high resolution (2^{24} point FFT), and nearly 100% real-time (see Fig. 3.16). I have been unable to find a commercial SA with comparable capabilities. Specifications are shown in table 2.5.

³⁰Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/teledyne>

Bit depth	12 bits
Sample rate	800MHz
DFT input length	2^{24} samples
FFT compute time	2 ms
Channel count	2
Efficiency	99.765%

Table 2.5: Specifications for the custom, real-time spectrum analyzer used for Run 1A.

After passing through an amplifier and filter chain outlined in Sec. 2.5.4, both the main and veto antenna’s RF signals are digitized by the ADQ32’s two independent ADCs. This raw, digitized time series is sampled at the digitizer’s clock rate. Since the discrete Fourier transform (DFT) of a perfect sinusoid sampled by an unstable clock will have a finite spectral width, clock stability must be better than the expected spectral width of candidate signals, which in our case is set by the expected $Q_{\text{DP}} \approx 10^6$. To achieve the required stability, we synchronize the sample clock (Valon 5009 RF synthesizer) of our ADC to a 10 MHz rubidium frequency standard (Stanford Research Systems FS725), which is further steered by the one pulse-per-second (PPS) signal from a GPS receiver. Clock performance is discussed in Sec. 3.3.3.

This system utilizes a GPU direct write in order to minimize CPU-GPU copies tends to be significantly slower than the FFT itself. This GPU direct write is implemented by the digitizer’s C++ API which is called from python. This happens in the `teledyneTemplate/drDaq.py` script (within the directory linked as a footnote at the beginning of this subsection). The GPU is a Nvidia A5000. The process is shown graphically in Fig. 2.45

First, several buffers are allocated in GPU memory. A record is acquired (2^{24} time

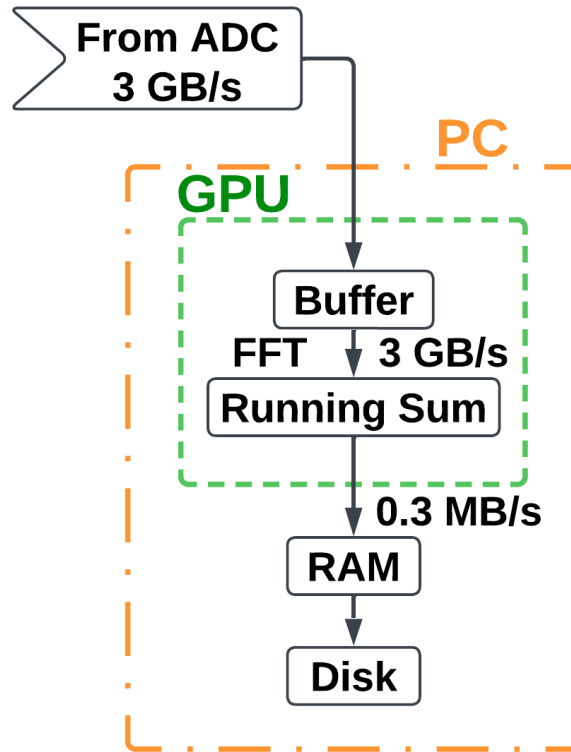


Figure 2.45: Real time DAQ data stream. Approximately 10^4 time series records (about 3 minutes of real-time data) are written from the ADC directly to GPU buffers. FFTs are performed on these records, resulting in a pre-averaged spectrum that can be saved to disk. This setup is duplicated for channels A and B, though the data rates indicate the sum of both channels.

domain samples for Run 1A). This record is written to a pre-allocated, time-domain buffer in the GPU’s memory. Next, an FFT is performed using Pytorch, which I found to be the fastest algorithm, at least on a Nvidia GPU. 8,600 FFTs are performed and added to a cumulative sum on the GPU (representing about 3 minutes of real-time data). Dividing by the number of FFTs provides an averaged spectrum that is saved for offline processing. This *pre-averaging* reduces the raw ≈ 1.5 GB/s/channel data stream to ≈ 0.15 MB/s/channel, which greatly reduces storage requirements. However, this comes at the cost of temporal

resolution of transient candidates. Since we are interested in constant (or at least very slowly varying) signals, this is not a problem, but in other applications, this step should be avoided. Pre-averaging is set using the `NOF_BUFFERS_TO_RECEIVE` variable, see Appendix A.

Chapter 3

System Characterization and Data Acquisition System

When you can measure what you are speaking about and express it in numbers you know something about it; but when you cannot measure it, when you cannot express it in numbers your knowledge is of meagre and unsatisfactory kind; it may be the beginning of knowledge but you have scarcely progressed in your thoughts to the stage of science whatever the matter may be.

Lord Kelvin

This chapter covers the characterization of the system as a whole, including the shielded room (Introduced in Sec. 2.5.1), amplifier chain (Sec. 2.5.4) and real-time spectrum analyzer system (RTSA, Sec. 2.5.7). This section provides data and information about how they were collected from tests that were performed, but that distract from the narratives of the preceding and following chapters. This chapter can be skimmed and referenced with further care during the reading of Ch. 4.

3.1 Measurement of Amplifier Chain Performance

The amplifier chain conditions the analog signal (described in Sec. 2.5.4). The following data show some measurements that are useful in confirming the system is operating as expected. Similar data should be collected and inspected before, during and after a new data run so that any performance degradation can be monitored. At the very least, one should take note of the power spectrum from a room temperature terminator (Fig. 3.10) and monitor it throughout the run for any changes. The heads-up display (Fig. A.1) makes this very easy.

3.1.1 Y-factor method¹

The Y-factor method [156, 157] is a common technique for measuring the gain and noise temperature of an amplifier chain (details in Sec. 2.1.4). A matched termination is connected to one end of a transmission line. The other end of this line is connected to the input of the amplifier chain under test. The output of the amplifier chain is measured with the

¹Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/yFactorTest/yFactorTest.ipynb>

terminator at two different temperatures². These temperatures must be known, and the larger the difference the better the measurement because it is easier to extrapolate the slope and intercept from these data (see Fig. 3.1).

There are a few different ways to handle the algebra, but the simplest is to fit a line of the form

$$T_{\text{out}}(\nu) = G T_{\text{term}} + B, \quad (3.1)$$

where T_{term} is the temperature of the terminator, and all terms are dependent on frequency. The x-intercept (and therefore the negative amplifier temperature) is simply G/B . This is shown as a cartoon for a single frequency in Fig. 3.1.

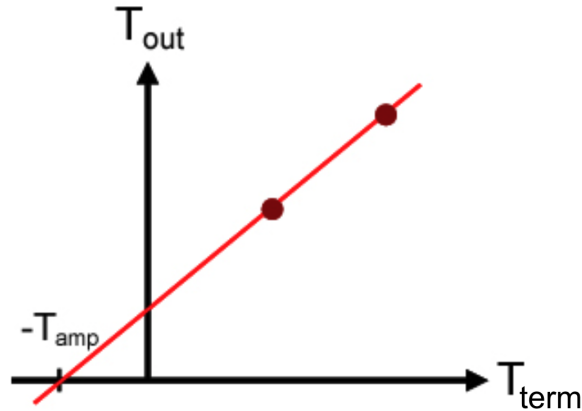


Figure 3.1: Cartoon showing Y-factor data. The (negative of) amplifier temperature is read off of the x-intercept. The gain is given by the slope. Figure from Wikipedia [158].

To make this measurement, I wanted to use liquid nitrogen for the cold temperature (77 K) and a calibrated noise source for the hot temperature ($\sim 12,000$ K). The test was

²Alternatively, a calibrated noise source can be used to simulate a very hot terminator, usually thousands of degrees Kelvin.

conducted using the noise source and a 30 dB attenuator placed directly before the ADC. This attenuation was later accounted for, yielding a calibrated measurement. However, while the test was set up to use this noise source, it gave unreliable results. I was able to confirm it became uncalibrated using a spectrum analyzer, so I did not use that data point. Additionally, I made the measurements with a much wider band-pass filter (0-1 GHz) compared to the span of Run 1A (0-0.3 GHz) so that the same data would also give the gain and noise temperature up to 1 GHz (though this is not shown in this thesis). Both the 30 dB attenuator and the 0-1 GHz band-pass filter will be calibrated out. As a check that this was done correctly, the gain was measured again using a tracking generator with the actual setup, avoiding corrections. This is shown in Fig 3.7. The two gain curves agree quite well.

The test setup is nearly identical to the Run 1A amplifier chain (see Fig. 2.42). The differences are the 30 dB attenuator and band-pass filter (discussed above) and the short, semi-rigid, cryogenic-capable cable between the terminator and LMR 400 cable. The cryogenic setup is the same as shown in the right panel of Fig. 2.28. The raw data (including only liquid nitrogen and room temperature) are shown in Fig. 3.2.

The spectra of Fig. 3.2 contain hot and cold measurements at 2^{18} frequency points. At each point, a fit is performed according to Eq. 3.1. To demonstrate, this is shown for a single frequency (625 MHz) in Fig. 3.3.

Finally, fitting the raw spectra shown in Fig. 3.2 with Eq. 3.1 at each of the frequency points, the frequency-dependent gain and noise temperature of the amplifier chain is extracted. These are shown (after correcting for the 30 dB attenuator) in Figs. 3.4 and 3.5.

As a final check, I took the Run 1A terminator data and corrected it for the gain and noise

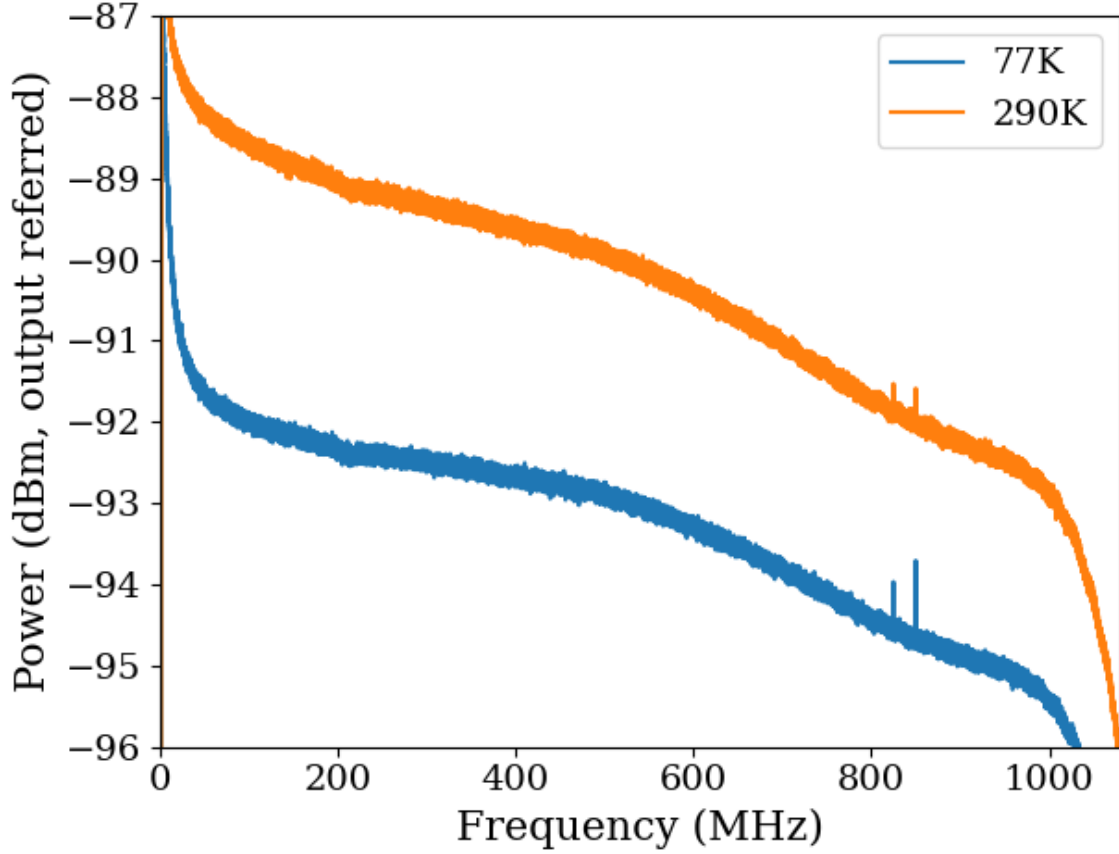


Figure 3.2: Raw output power spectra used to compute the gain and noise temperature for the Run 1A amplifier chain using the Y-factor method. The bandwidth $\Delta\nu_{\text{RF}} \approx 9.5\text{ kHz}$, and 10,000 averages were taken. These data are only used to characterize the system between 50 and 300 MHz, so the low frequency ADC effects and RFI around 800 MHz do not affect the calculation. Note that the amplifier chain under test has a 30 dB attenuator directly before the ADC which is discussed in the text. This had an impact on the measured gain, and it is corrected for in the following analysis. It has negligible impact on the noise figure however, as shown in Eq. 2.29.

figure found using the Y-factor method of this section, expecting to recover the -174 dBm/Hz as predicted by Johnson’s formula for a terminator at room temperature. The agreement is excellent, as shown in Fig. 3.6.

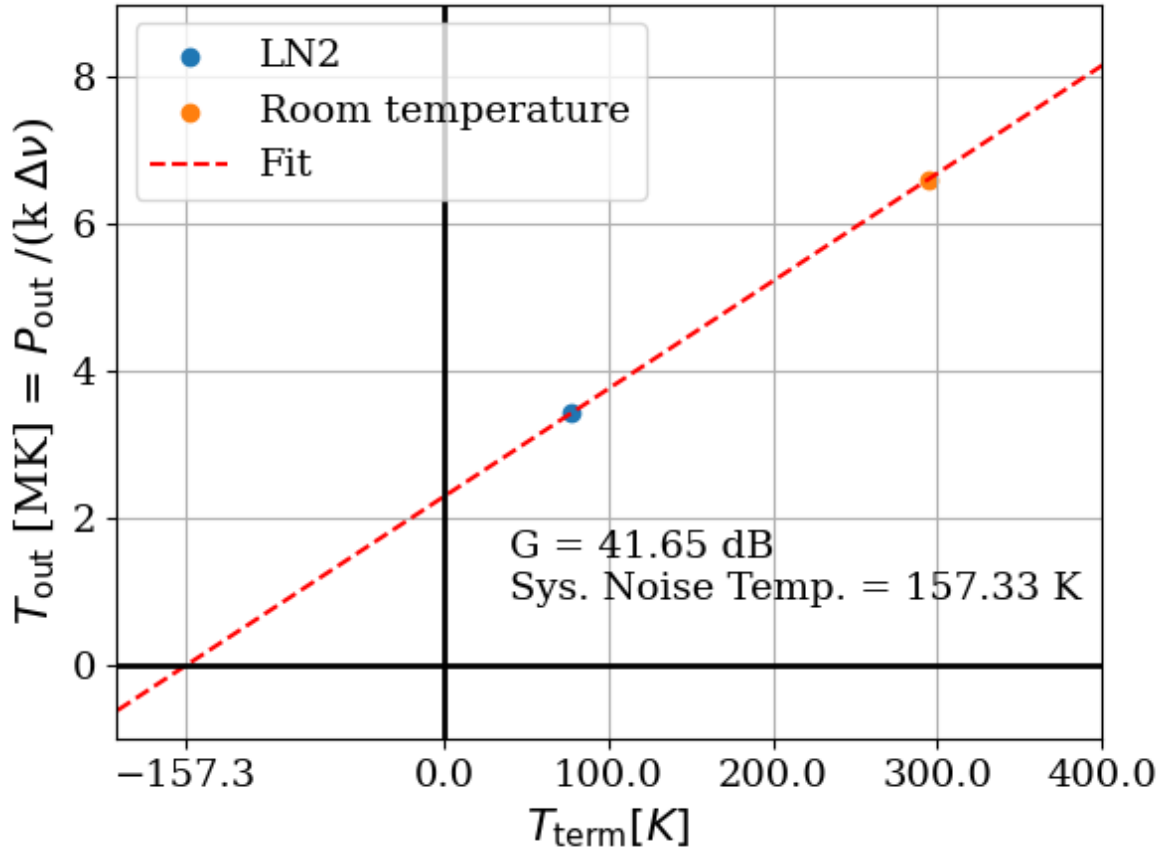


Figure 3.3: Y-factor data from the Run 1A amplifier chain at a single frequency (625 MHz). Note that T_{out} is in MK or millions of degrees Kelvin. This is expected since there is approximately 41 dB of gain on an input temperature of a few hundred Kelvin. Reminder, there is a 30 dB attenuator which has not yet been corrected for.

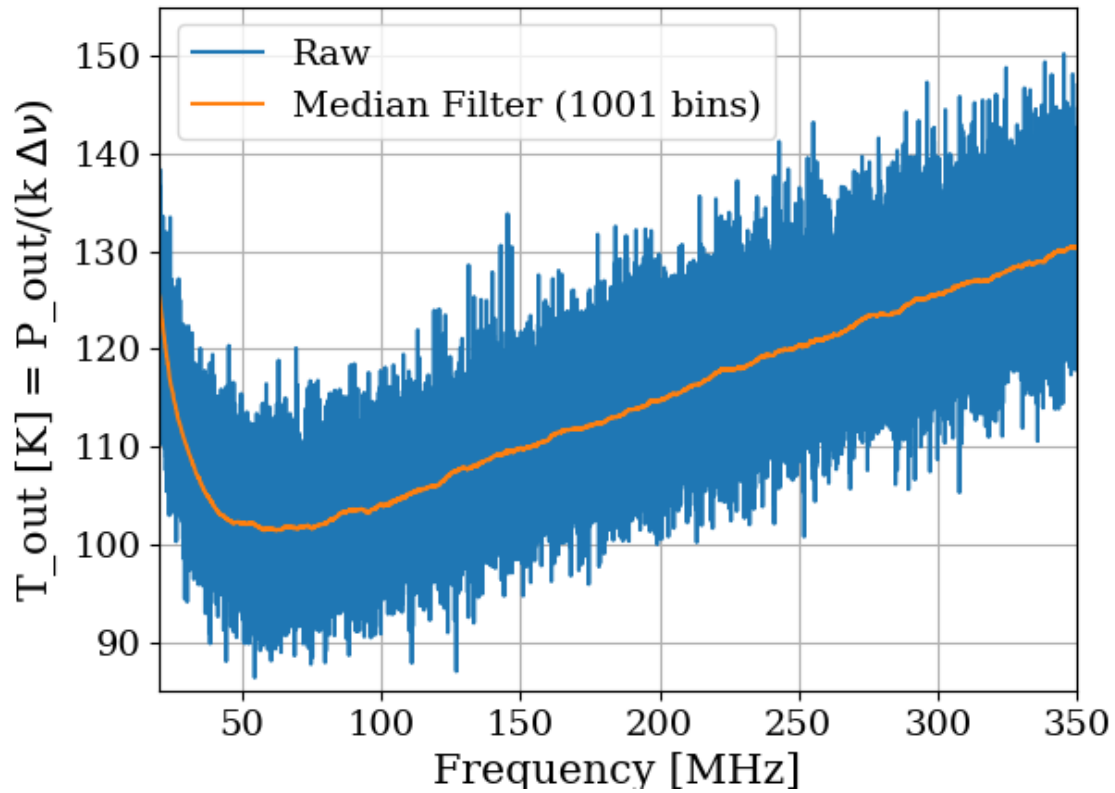


Figure 3.4: Frequency-dependent noise temperature of the Run 1A amplifier chain, measured using the Y-factor method. Low-frequency behavior is consistent with the data sheet of the LNA [124]. The increase at high frequency is due to the attenuation of the cable before the LNA. Future runs may place the LNA directly on the antenna to minimize this issue, though other complications will be introduced. The median filter is a simple rolling median to smooth out the noisy measurements.

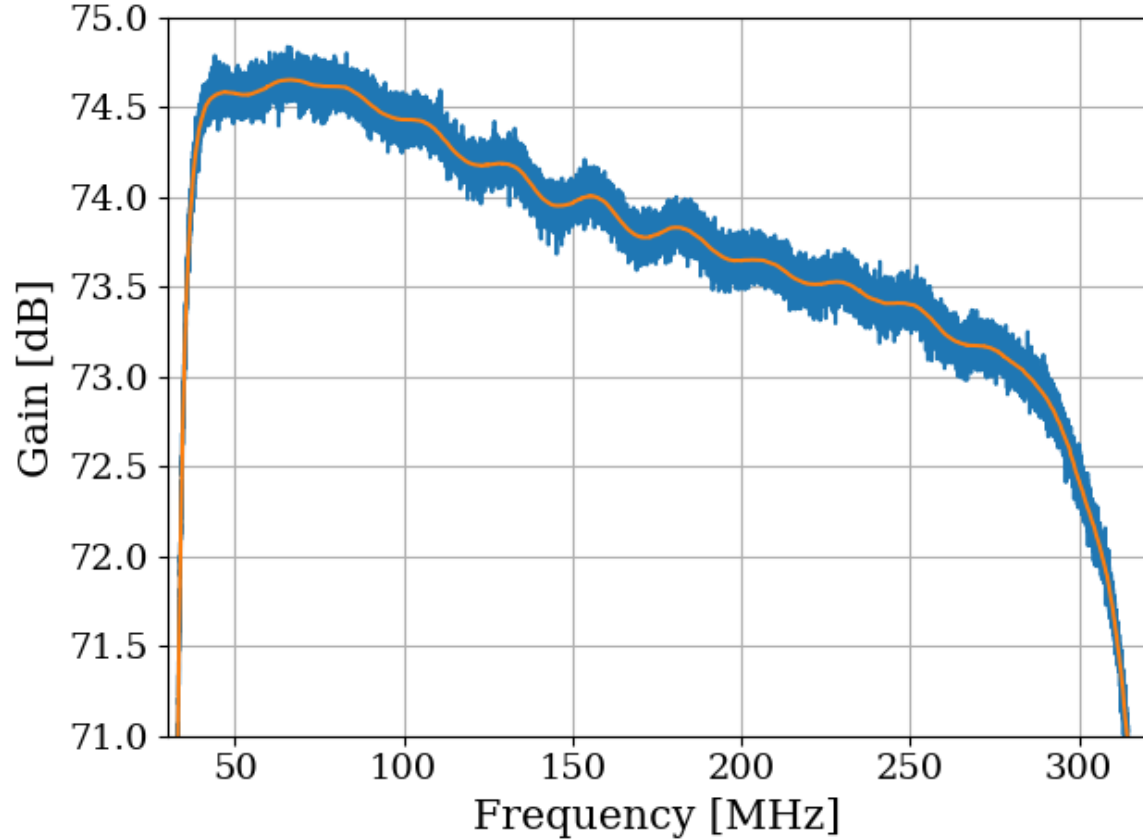


Figure 3.5: Frequency-dependent gain of the Run 1A amplifier chain, measured using the Y-factor method. The 30 dB attenuator and band-pass filter have been corrected for in this plot. The slow decrease of gain with frequency is consistent with the datasheets for the LNA [124] and secondary amplifier [150]. Roll-off at high and low frequencies is due to the band-pass filter (see 2.5.4.3). Bumpy behavior is due to the small impedance mismatch of the terminator to the 50Ω system.

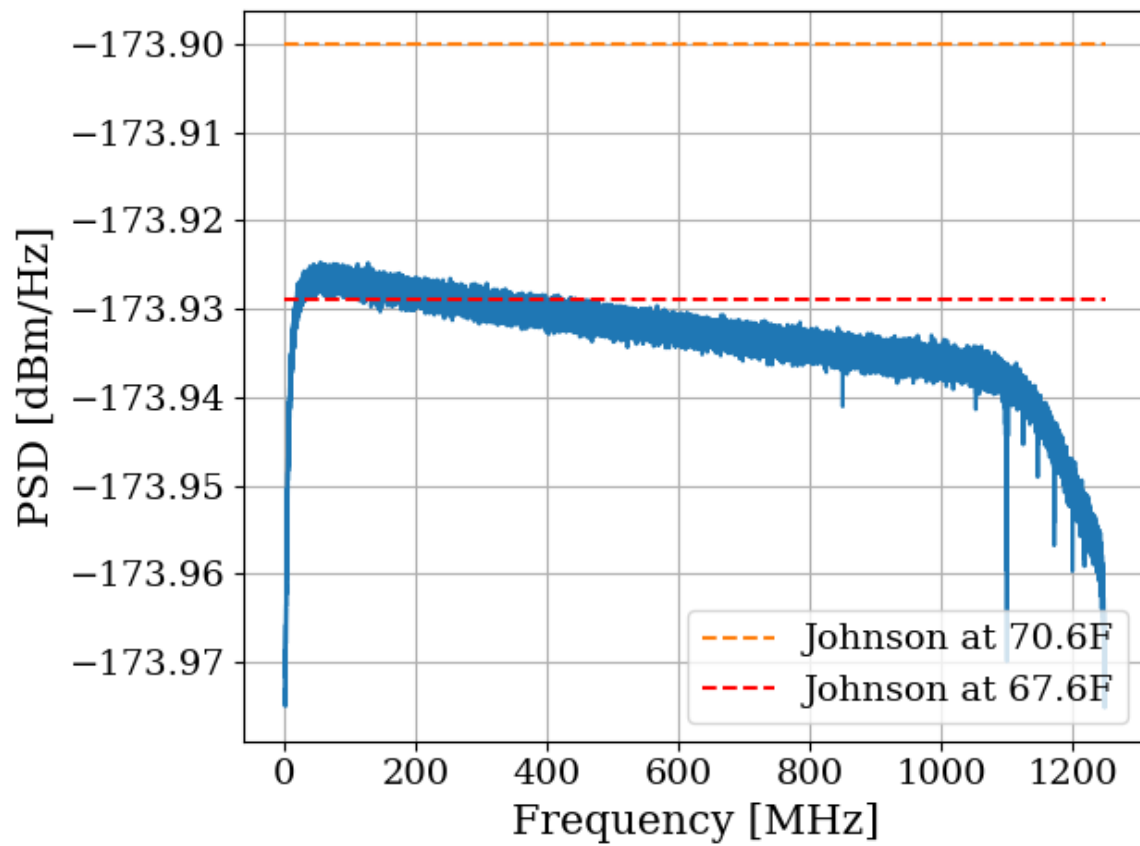


Figure 3.6: Run 1A terminator, input-referred power spectral density. Excellent agreement with the predicted value from the formula for Johnson noise at room temperature. Two reference temperatures are given with the dashed lines.

3.1.2 Confirming gain with tracking generator measurement

As a sanity check for the Y-factor method, the system gain was measured again using the tracking generator on the Rigol RSA-5065-TG [159], with good agreement. This measurement was useful since there were some corrections involved in the Y-factor measurement (the 30 dB attenuator and band-pass filter).

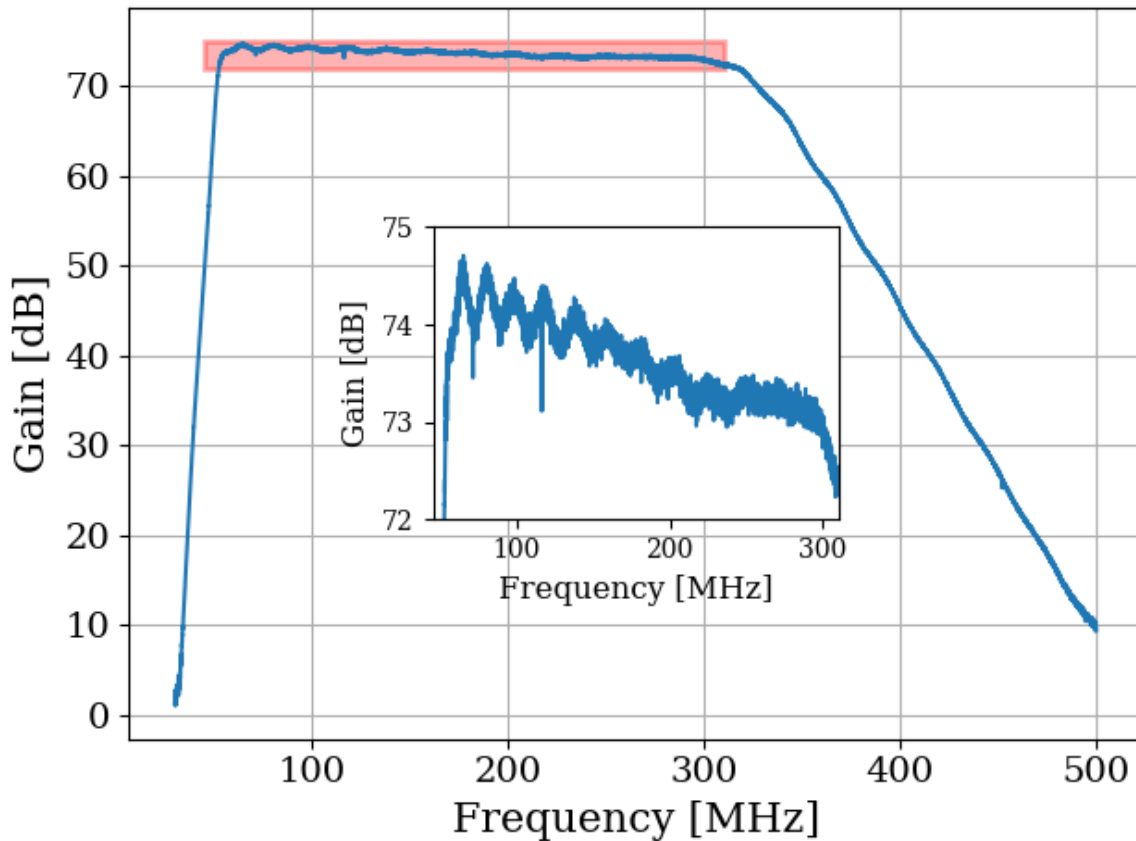


Figure 3.7: Gain vs. frequency of the Run 1A amplifier chain, including LNA, secondary amplifier, band-pass filter and attenuators. The inset shows the zoom of the region enclosed in the red box. This was measured using the tracking generator on Rigol RSA-5065-TG [159]. Wiggles visible in the inset are a known artifact due to an impedance mismatch between the Rigol’s tracking generator and the system under test. The gain was confirmed by comparing it to the gain found with the Y-factor method (Sec. 3.1.1)

3.1.3 System stability over a run period ³

The system's gain will vary slightly over the course of a run. An obvious culprit is the battery voltage decreasing with time⁴. At first glance, one may conclude that since the LNA is equipped with an internal voltage regulator, it is insensitive to the battery voltage as long as it is above some minimum value. However, at higher voltage, the regulator must dissipate more power which will make the LNA warmer. Temperature does have an effect on the gain of the LNA. Tests of gain vs. LNA voltage show that this is a minor issue. This is shown in Fig. 3.8.

Knowing the gain vs. voltage is only useful if it is understood how the battery voltage will behave over the course of the run. These data are shown in Fig. 3.9.

As a final check of system performance during the run, Fig. 3.10 shows the output-referred power (averaged from 50-300 MHz) over the course of the 9-day Run 1A. The gain varies by about 0.7%, an insignificant amount given the other uncertainties involved (especially that of simulation and calibration, Sec. 4.3).

³Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/run1p4_analysis/run1p4_timeDependence.ipynb
https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/gainAndNf/gainAndNF_analysis.ipynb

⁴The LNA is powered by a battery. The secondary amplifier is outside of the room and powered with a voltage regulator.

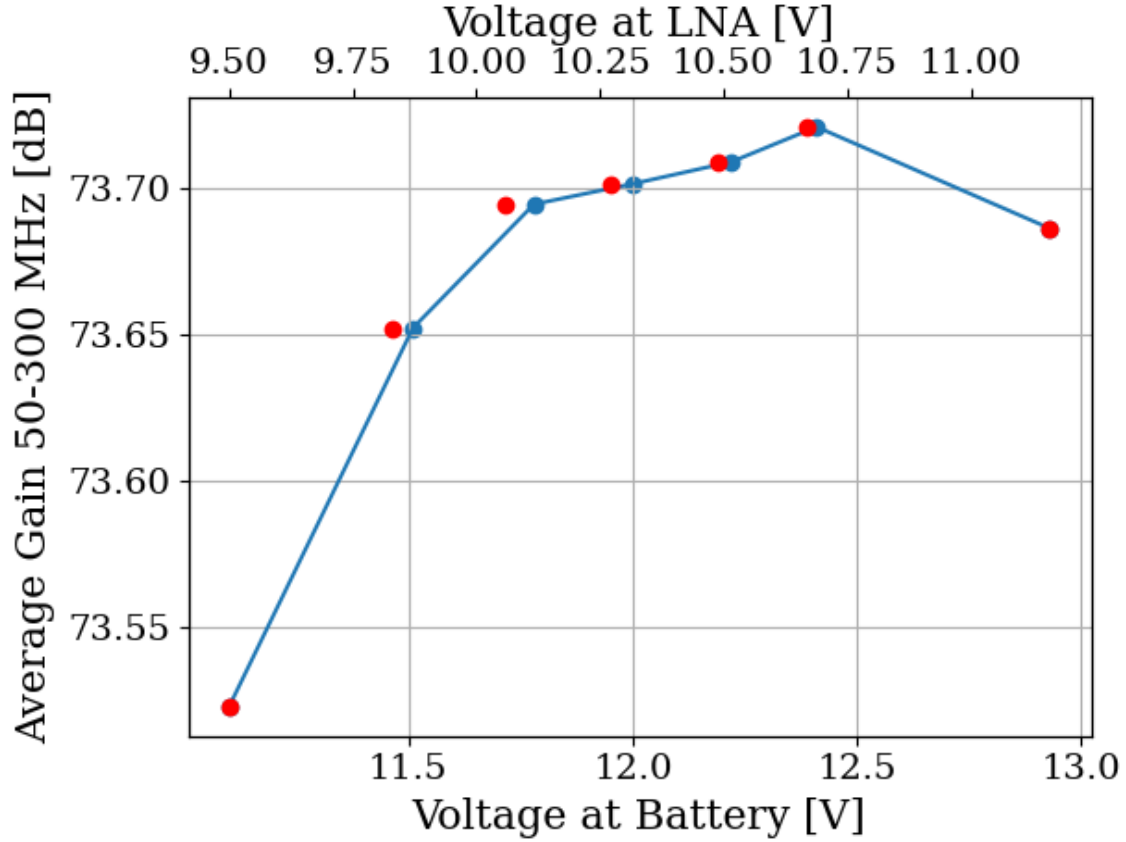


Figure 3.8: Gain vs. LNA voltage of the Run 1A amplifier chain, including LNA, secondary amplifier, band-pass filter and attenuators. This was measured using the tracking generator on Rigol RSA 5065TG [159]. Gain is frequency dependent; its average value is between 50 and 300 MHz is shown here. The voltage at the amplifier was measured directly at its terminals after a significant voltage drop due to interlock and slow turn-on circuits (described in Sec. 2.5.6). Gain is in good agreement with the previous two subsections, 3.1.1 and 3.1.2. The operating voltage range specified on LNA data sheet [124] is 10-15 V. The voltage at the LNA (red) does not exactly line up with voltage at battery (blue), so there is a slight offset. The scale of the x-axes scale has been set so that the maximum and minimum voltages line up.

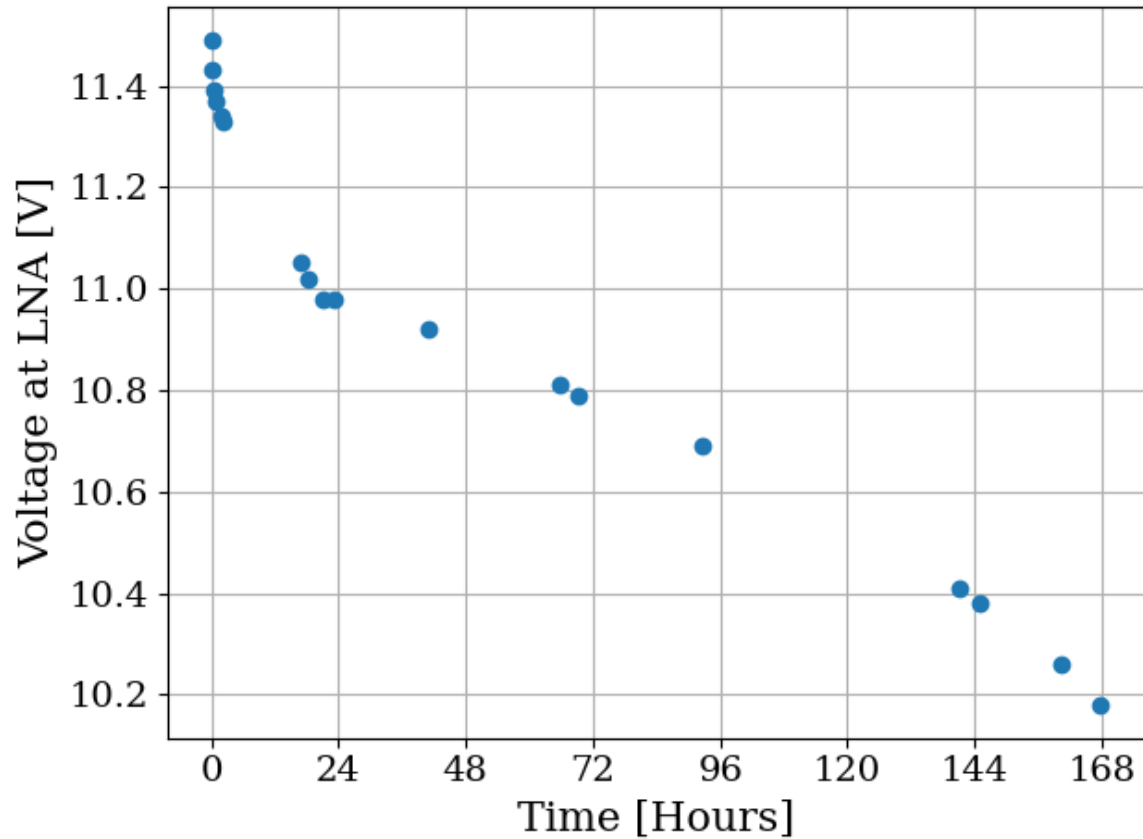


Figure 3.9: Voltage at LNA vs. time with Run 1A setup. These data were acquired 7/15/2024 through 7/22/2024 using battery number 2 immediately after being charged. The system was set up as it was for Run 1A, including the switching cycle (on for 3 minutes, off for 45 minutes). The battery supplies 177 mA when the switch is off and 370 mA when the switch is on. There is an additional data point (not shown) around 196 hours where the voltage has dropped to 6.2 V, well below the minimum required voltage. The battery had approximately 20 charge cycles at the time of this test and was about 14 months old. A repeat of this test may be required to use the battery for longer periods between charges.

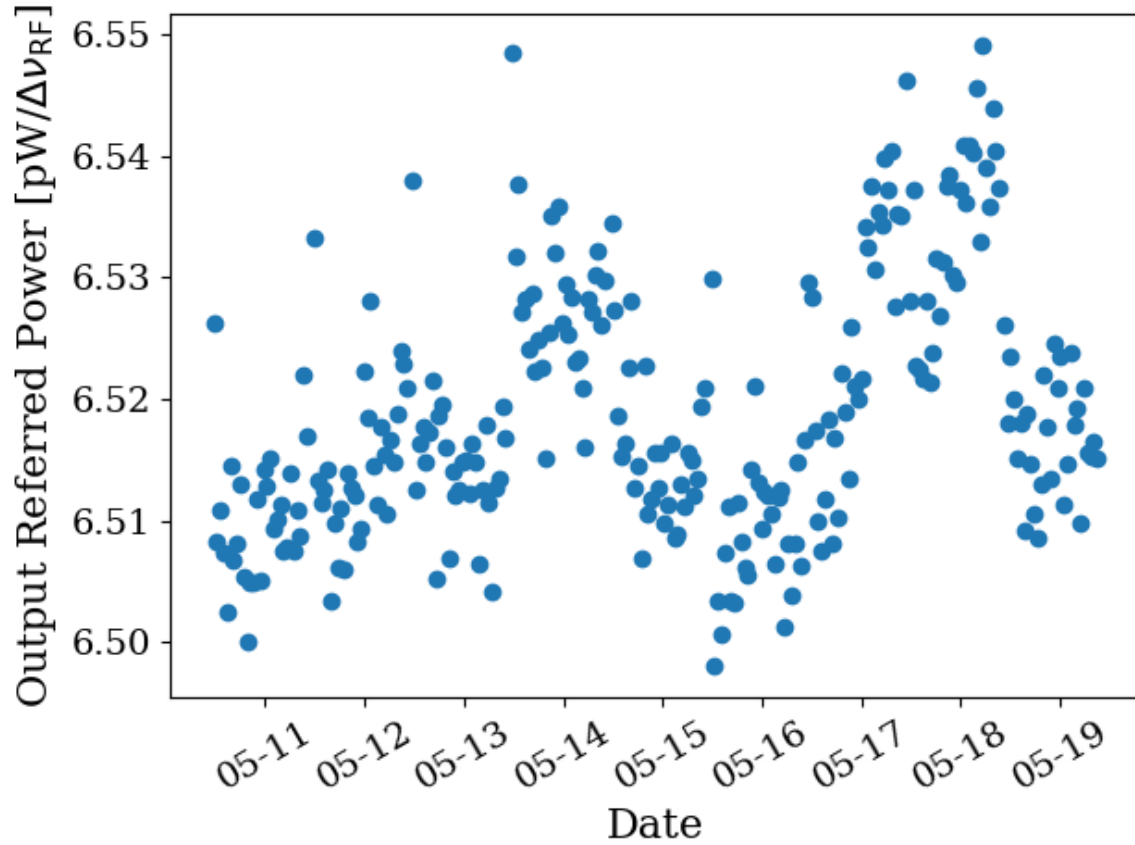


Figure 3.10: Frequency-averaged power spectral density vs. time of the terminator from Run 1A (schematically shown in Fig. 2.37). Each data point represents the mean power between 50 and 300 MHz of the terminator. In Run 1A, a ~ 3 minute long scan of the terminator took place after ~ 45 minutes of antenna scans. The dates shown are all May 2023 (month-day format, year suppressed), and the x-axis ticks are placed at midnight.

3.2 Measurement of Shielding Effectiveness and Radio Frequency Interference

Shielding effectiveness (SE) and radio frequency interference (RFI) were introduced in Secs. 2.1.3 and 2.5.1. Their measurement is introduced here.

Similar to the warning given in the previous section, SE should be measured before starting a data run or after entering a new frequency range/modifying anything on the patch panel. In addition, it is best practice to clean the door's RF seal before beginning a run (the procedure is described in the text of this section).

Due to reciprocity between antennas, measuring the shielded room's ability to keep radiation *out* is equivalent to measuring how well the room can keep it *in*. The latter method is simpler and was performed. The IEEE standard prescribing a very detailed SE measurement procedure [160] was used as a rough guideline, but the simple results presented here should be viewed as an estimate. While simplified, the results do tend to agree with predictions of when RFI should become detectable, based on comparisons to the veto antenna spectrum⁵.

The SE was measured by placing a Rigol DSG830 signal generator and 25 Watt RF power amplifier inside the room, both powered by the filtered 120 VAC Edison outlets inside the room, shown in Fig. 3.11. An antenna was connected to the amplifier. Outside of the room, an identical antenna was connected to a Rigol RSA-5065-TG spectrum analyzer [159]. The spectrum is scanned to find a frequency without local interference, which would confuse the

⁵Or at least they did agree before cleaning the door. With the excellent isolation values, no RFI was detected in Run 1A.

results. The spectrum analyzer was set to attenuate its input to prevent clipping⁶, and the signal generator was set to output a sine wave near the maximum power of the RF amplifier (20 W = 43 dBm). The power measured on the spectrum analyzer is P_{open} in Eq. 2.46. The door was closed, attenuation removed and P_{closed} was measured. In some cases, the SE was so high a signal was buried below the noise floor of the spectrum analyzer. These data are shown in red in Figs. 3.12 3.13 and should be interpreted as an upper limit on isolation, since the true value is lower (better).

SE can drop off to less than 80 dB if the RF gasket around the door is not clean. Previous day-long test runs detected several hundred signals that originated from RFI emitted from the PC and several local radio stations. Cleaning was performed by scrubbing the copper finger stock and steel mating surfaces with red Scotch-Brite using denatured alcohol as a lubricant⁷. After two passes of polishing were complete, a layer of DeoxIT D100L liquid was added. Maintenance cleanings were performed using only DeoxIT D5 spray. These signals were not detected after the gasket was cleaned, which is consistent with calculations of Sec. 2.1.3.

The primary RFI in the 50-300 MHz span are from local radio broadcasts. The electric field of the strongest signals is $\sim 100 \mu\text{V}/\text{m}$ when measured in the lab. There are also many lower level peaks which span the entire frequency range, though are more pronounced in a few frequency bands (60-75 MHz, 130-140 MHz, and 270-290 MHz). These peaks come from

⁶The spectrum analyzer automatically calibrates the displayed spectrum to its internal attenuator and pre-amplifier. If external gain/attenuation are used, they must be accounted for manually. Since this is a relative measurement (i.e., a ratio of $P_{\text{open}}/P_{\text{closed}}$), this calibration does not matter much.

⁷Alcohol fumes are nasty. Be careful to open the hallway door and use a fan to blow in air from the hall. Take frequent breaks. Don't get dizzy while standing on a chair!



Figure 3.11: Photo showing setup to measure SE of 314. Photo taken using Vivaldi antennas for data shown in Fig. 3.13. The same setup was used with bicon antennas for the 50-300 MHz data shown in Fig. 3.12. The bicon antenna shown in the background of this photo was not used for this test.

the PC which is an integral part of the spectrum analyzer system, housing the ADC, so it can not be removed. Before the door was cleaned, many of these signals were detectable after a few hours of integration, but cleaning the door resulted in a clean spectrum for the 9-day data Run 1A. The spectrum from the veto antenna (discussed in Sec. 2.5.5) is shown in Fig. 3.14

314 Shielding Effectiveness

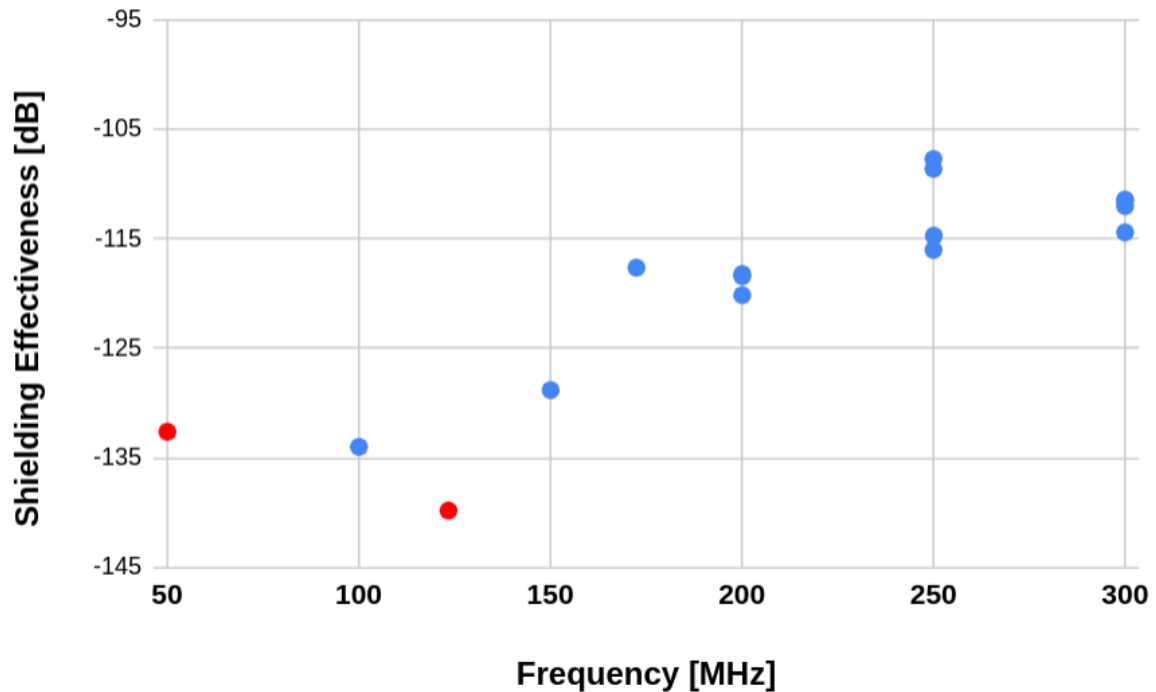


Figure 3.12: Shielding effectiveness of shielded room measured with bicon antenna. Red points indicate measurements limited by the noise floor of the spectrum analyzer located outside the shielded room and are an upper limit. The true SE is lower (better). To get the true SE for these points, measurement would require a higher gain amplifier inside the room and/or an LNA before the spectrum analyzer. SE measurements are all better than required for Run 1A, and no RFI candidates were detected. At a few frequencies, the antenna inside the room was moved to get a sense of the uncertainty of the measurement.

314 Shielding Effectiveness

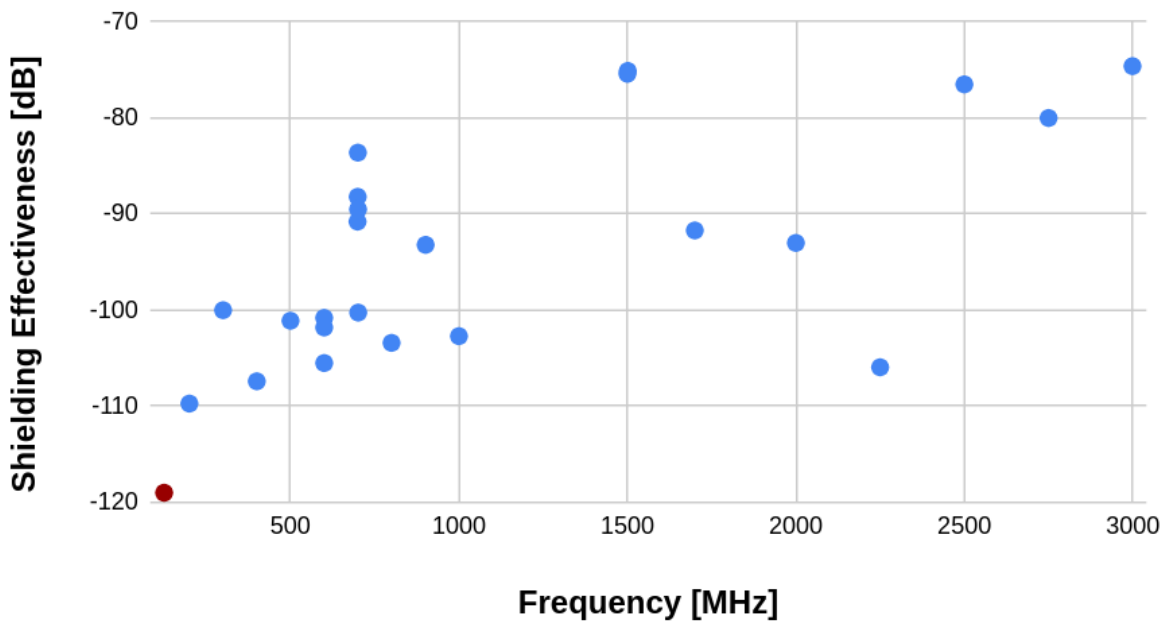


Figure 3.13: Shielding effectiveness of shielded room measured with Vivaldi antenna. Red points indicate measurements limited by the noise floor of the spectrum analyzer located outside and are an upper limit; true SE is lower (better). Measurement would require a higher gain amplifier inside the room. At a few frequencies, the antenna inside the room was moved to get a sense of the uncertainty of the measurement.

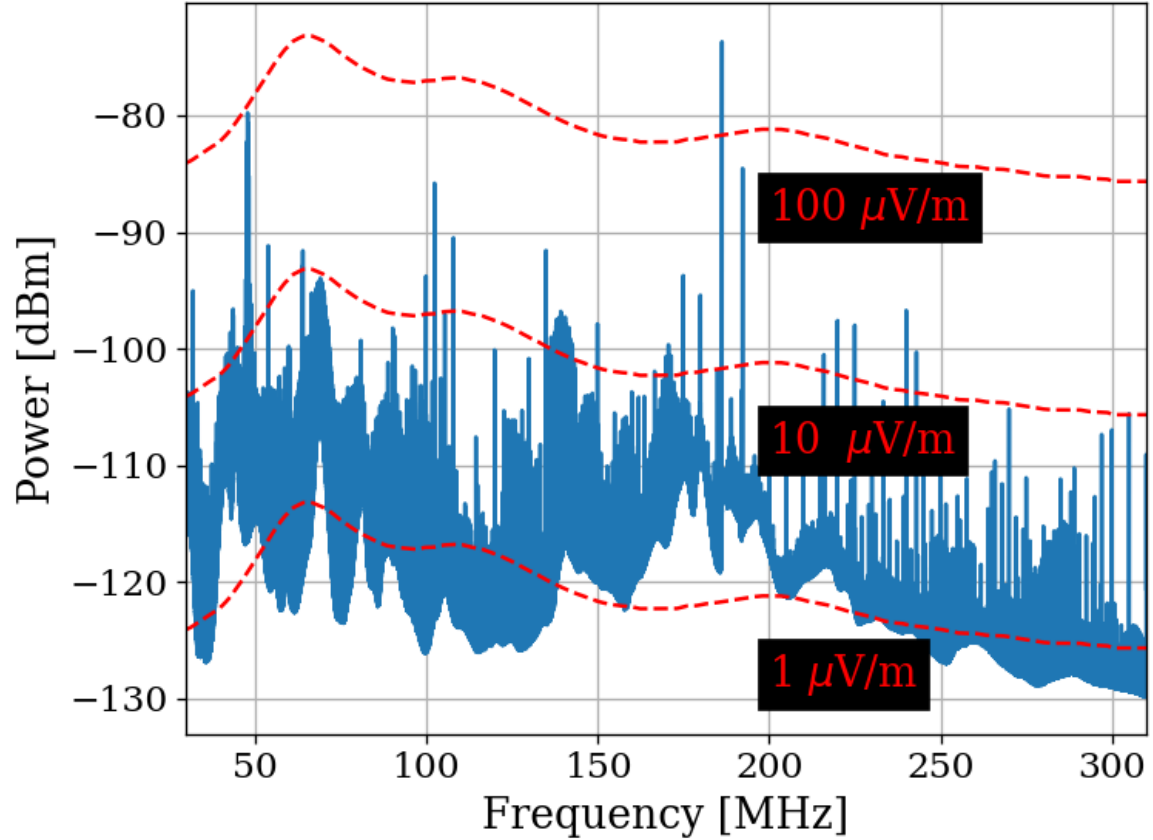


Figure 3.14: Spectrum from veto antenna during the 300 MHz data run. Spectrum plotted as measured power in dBm. Red curves indicate the equivalent field strength. These curves track the free-space antenna factor with frequency for the Com-Power AB900 [147] bicon. Two identical AB900-A antennas are used, one to search for dark photons in the shielded room, and a second to monitor the local RFI background.

3.3 Spectrum Analyzer Characterization

This section outlines several tests performed to ensure the GPU-based real-time spectrum analyzer (RTSA) system performs as expected. Background information that relates to this system is described in Sec. 2.5.7.

3.3.1 Spectrum analyzer calibration

The first step in determining if the RTSA system is working adequately is to make sure it is able to correctly identify known, injected signals and noise. This calibration step is important since the FFT is normalized manually (see Eq. 2.15), and it is rather easy to mess up. The test signals were generated with a Rigol DSG830 signal generator. The time-domain peak-to-peak voltage measured by the RTSA system agreed with the injected voltage to about 2 %. The signal generator's output frequency and power were verified in both the frequency and time domain using a separate oscilloscope and spectrum analyzer. This is an important step since cable losses are significant at this level of precision. They can be taken into account by consulting the data sheet for the cable⁸.

Next, it is important to understand how the system handles noise spectra, since this is the primary use of the system. This is tested by amplifying the Johnson noise of a terminator (see Sec. 2.1.1.2) and measuring the power spectral density. Knowing the gain and noise figure of the amplifier chain allows for calibration using the procedure outlined in detail earlier in this chapter, see Sec. 3.1, specifically Fig. 3.6. These results were confirmed several times

⁸There are several cheap black SMA cables which are great for testing, but not for calibration. They have more than 5 dB of loss above 1 GHz. Use a nice cable for these tests, and blow out the SMA connector with compressed air while you are at it.

preceding the data run. Note that due to windowing, other spectrum analyzers will disagree with the theoretical calculation by a few dB if the equivalent noise bandwidth (ENBW) of the window function is not included. Also remember to take cable effects into account, as discussed in the previous paragraph.

One final test on a system that integrates noise is to confirm the standard deviation σ of the noise averages down as predicted by Eq. 2.16. This was confirmed, and the results are shown in Figs. 4.16 and 4.17.

3.3.2 Spurious signal performance of the ADC

Any practical spectrum analyzer will have small, coherent signals that sneak into the analog signal path and, after being digitized and Fourier transformed, manifesting as spurious signals or *spurs* (see Sec. 2.1.5.1). These spurs likely come from clock signals within the same enclosure as the ADC. They can also come from local oscillators bleeding through mixers in a mixed, high-frequency system such as the Rigol RSA 5065TG [159] or the future, upgraded dark radio system discussed in Ch. 6.

Regardless of their origin, the spurs must be characterized such that if a known spur is detected, it can be classified as such and rejected quickly. Ideally, the system has enough gain that any spurs are subdominant to the amplified thermal noise, as discussed in Eq. 2.1. However, since there is a maximum amount of gain that can be added before clipping the ADC (see Sec. 2.5.4.3), this is not always possible.

Experience with the Teledyne RTSA system has shown that spurs are rather simple to

classify because they do not change much with time in either frequency or amplitude. Their frequency drifts by ~ 1 kHz and their amplitude changes by a few dB.

To classify them, the input of the Teledyne ADC is terminated and a scan is performed for a few minutes using the exact same system configuration as Run 1A (with the possible exception of the number of averages). See Appendix A for details. The averaged spectra of the terminated inputs are shown in Fig. 3.15.

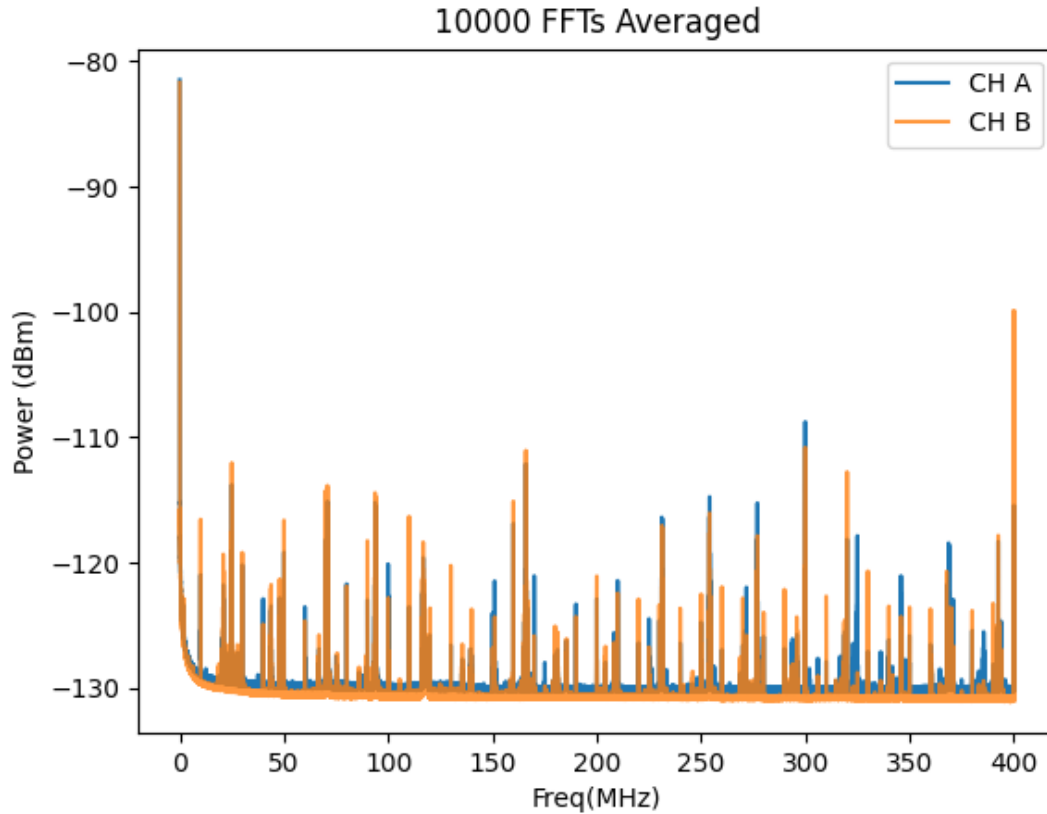


Figure 3.15: Scan of terminated input of Teledyne spectrum analyzer system to measure spur performance. The spectra were acquired with run 1A parameters ($\Delta\nu_{\text{RF}} = 47.7$ Hz) with 10,000 averages (~ 3.5 minutes). This plot is taken directly from test mode of the system (not post-processed in any way). The largest spur in the 50-300 MHz range is at 299.97 MHz and is also the single significant signal in Run 1A (see Sec. 4.2.5).

Once a test scan is acquired, comparing the largest spurs (≈ -110 dB) to the expected output-referred background ($[\text{Johnson noise} + \text{LNA noise}] * \text{Gain} \approx -81 \text{ dBm}/\Delta\nu_{\text{RF}}$) allows the calculation of the number of averages which are required before the spurs are significant. Working this out is a good test of understanding of Sec. 2.2. This spur should be detectable with 95% confidence after about 4 days, which is exactly what happened in Run 1A. The false positive candidate is described in Sec. 4.2.5.

3.3.3 ADC clock performance

Any ADC requires a clock that dictates the precise time at which a sample should be acquired. This subsection deals with the evaluation of the performance of that clock.

Determining the performance of a clock generally relies on having a better (read: more expensive) clock known as a *standard* and measuring the clock under test against the standard. The Valon 5009a [161] RF synthesizer was tested against a rubidium frequency standard and was found to be just on the threshold of stable enough to work (this threshold is explained in Sec. 2.5.7). To ensure the measurement is not limited by clock instability, the sample clock of the ADC is synchronized to a 10 MHz rubidium frequency standard (Stanford Research Systems FS725) which is further steered by the one pulse-per-second (pps) signal from a GPS receiver. This system has medium and long term fractional frequency stability (Allan deviation [162, 163]) of $\sigma_y(\tau) < 3 \times 10^{-12}$ (where τ is the averaging time) and phase noise of less than -65 dBc/Hz at offset frequencies > 50 Hz from the carrier [164]. This means that over the course of a single acquisition, the power contained in a bin will spread to an

adjacent bin by less than 1 part in 10^6 which is more than sufficient for our experiment.

This is a rare example of something in this experiment where we trust the datasheet [164]. We do not have access to a clock that is better than this, so confirming its performance is difficult. Furthermore, even if the specifications are an order of magnitude worse than advertised, there is still more than enough stability for the experiment. This statement holds at higher frequencies. Since the Q of the dark photon signal is presumed to be fixed, higher frequencies have wider bins, and so the relevant offset frequency at which the phase noise is measured increases with frequency. The dimensionless Allan deviation is also not affected.

3.3.4 Real-time data collection efficiency

As a final test, one would like to know how efficiently data scans can be acquired and averaged. We call a 100% efficient system *real-time* as described in Sec. 2.5.7. Figure 3.16 illustrates this efficiency. For Run 1A, it is $\sim 99.765\%$. However, in practice, run logistics such as moving the antenna prevent the full efficiency of the system from being realized.

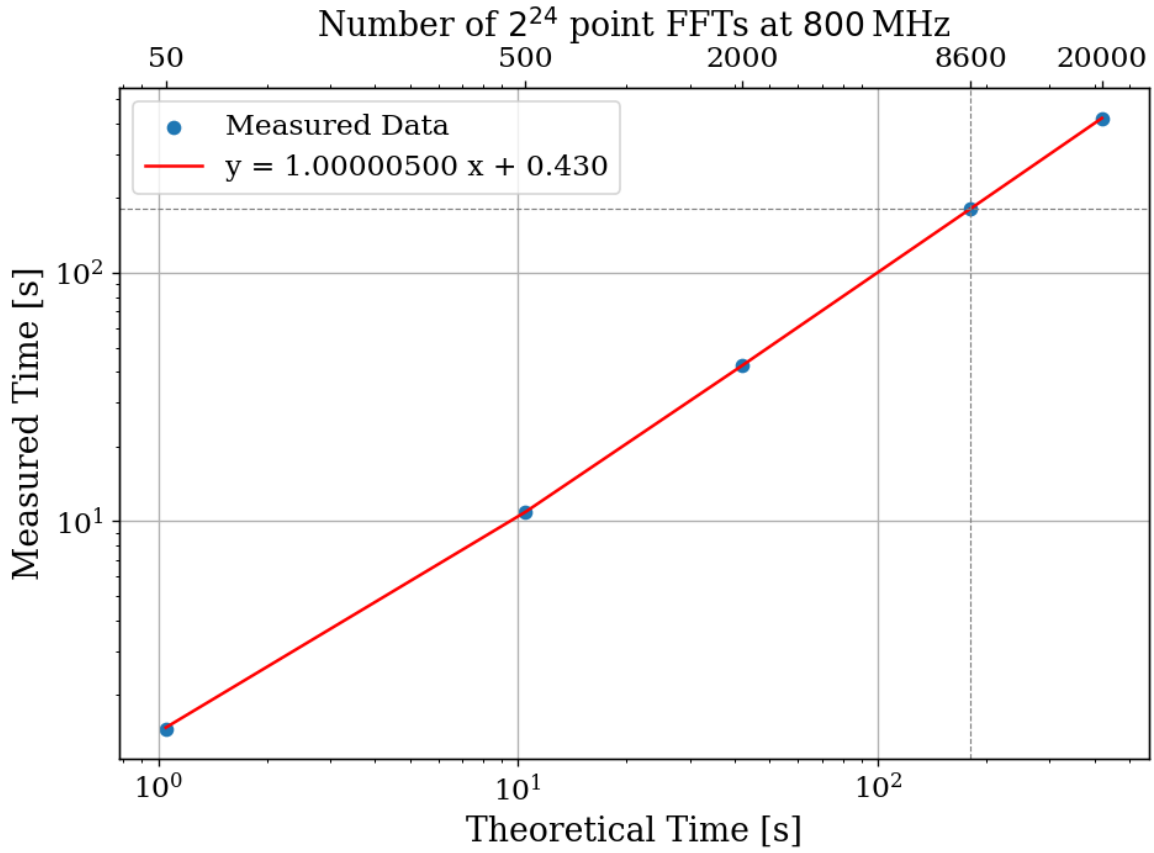


Figure 3.16: Acquisition efficiency for GPU-based real-time spectrum analyzer computed from measured vs. theoretical times. The constant offset of 0.43 s corresponds to a small set-up period when starting an acquisition containing, in the case of Run 1A, 8600 FFTs. In the limit of an infinite length acquisition, the system's efficiency may be read off from the slope and is 99.9995%. The more realistic efficiency is a function of the number of FFTs per acquisition and for Run 1A $= 180.3551 \text{ [s]} / 180.7782 \text{ [s]} = 99.765\%$.

Chapter 4

Data Acquisition, Data Analysis and Calibration

Data is like garbage. You'd better know what
you are going to do with it before you collect it.

attributed to Mark Twain

Thus far, this dissertation has built a foundation for the dark radio technique as well as the design and testing of the experiment and its sub-systems. This chapter outlines the process of acquiring actual data, analyzing that data to search for a small power excess, and, in the absence of that excess, converting the null result into an exclusion limit on the dark photon mass/ ε parameter space. Section 4.4 describes an injection test that detects a realistic, hardware-injected, dark photon proxy-signal. This chapter borrows heavily from *New Limit on Dark Photon Kinetic Mixing in the 0.2-1.2 μeV Mass Range From the Dark*

E-Field Radio Experiment [67], which was published by the DER group in August 2024.

The cleanest way to model the contents of this chapter is by compartmentalizing the steps outlined above. For this reason, the chapter is organized as follows. In Sec. 4.1, the procedure for a 9-day data run is outlined. Where appropriate, references to previous sections are provided which are helpful in understanding how different procedures were developed. This section concludes with the frequency-dependent, output-referred power spectral density $S_{\mathbf{o}}$, which must be searched for a power excess. We call this search *analysis*, and it is covered in Sec. 4.2. The task of analysis is to extract a dark photon signal from this spectrum if it is present. Otherwise, in its absence, we set a limit on the amount of output-referred power that would have been detectable *most of the time* were a narrow signal to be present in this averaged dataset. We quantify the meaning of “most of the time” by conducting a series of Monte Carlo *pseudo-experiments* on artificial signal-containing spectra for synthetic signals of varying powers and frequencies. With a limit on dark photon power extracted, Sec. 4.3 works back through the system to determine a limit on ε above which we have some confidence we would have observed a signal. This produces the exclusion limit, which is ultimately the deliverable of this experiment. This limit is shown in Ch. 5.

4.1 Data Acquisition

Data were collected during a 9-day run from May 10 to May 19, 2023. As discussed in the introduction to Ch. 2, this data run is referred to Run 1.4 in the code base since it was the fourth attempt at taking these data. It was used to publish the limit of Run 1A [67], so it

is referred to as Run 1A throughout this dissertation or as Run 1A.4 if it is necessary to distinguish between previous attempts. The details of this run are shown in Tab. 4.1.

Specification	Value
Analysis span	50 - 300 MHz
-3 dB span of analog filters	34 - 208 MHz
Antenna	AB-900A biconical antenna
RTSA	Teledyne GPU system (Sec. 2.5.7)
Frequency resolution ($\Delta\nu_{\text{RF}}$)	47.7 Hz
Length of record	2^{24} samples
Sample rate	800 MHz
Window type	None (rectangular)
Acquisition time per spectrum	20.96 ms
Run start time	2023-05-10 11:29:48
Total time of run	8 days, 21.13 hours
Efficiency (Time spent scanning antenna/total time)	92.03%

Table 4.1: Run 1A details. Many specifications are related and can be computed from each other but are listed for reference. The efficiency differs from that calculated in Fig. 3.16 mainly because of switching to a terminator and brief daily pauses to move the antenna.

Each day was subdivided into data-collection (23 hours 15 minutes) and setup (45 minutes) periods. The setup period includes moving the antenna, changing a 12 V battery for the LNA (Sec. 2.5.6), file management and documentation. In order to reduce the data rate and storage requirements, all data were pre-averaged into 3-minute chunks and then saved. This pre-averaging is shown in Fig. 2.45. Additionally, an RF switch (see Fig. 2.37) is actuated for a 3-minute scan for every 15 antenna scans in order to monitor the status of the amplifier chain. For the data analysis, all 9 days of data were averaged together to create a single spectrum S_{O} (Fig. 4.1). If candidates were found, their time dependence can be observed by looking at the 3-minute pre-averages. All further analysis was performed on

the full 9-day S_o spectrum, described below (Sec. 4.2).

4.1.1 Raw data, S_o

All 9 days of pre-averaged data from the run were averaged together. The stability of the sample clock (Sec. 3.3.3) ensures that this is a simple process. Frequency bins ($\Delta\nu_{\text{RF}} = 47.7 \text{ Hz}$) corresponding to a given frequency were added and normalized by the total number of pre-averaged spectra. This process produces the raw spectrum, S_o (Fig. 4.1), on which we will perform a search for power excess.

Inspection of S_o reveals small power variations over spans of tens of kHz. The origin of these variations is explored in Sec. 2.3, but it is summarized here. Given an antenna in a cavity in thermal equilibrium with the input of an amplifier, whose input is assumed to be real and matched, one would expect an output PSD which is constant with respect to frequency up to small variations in system gain. The theory for this is outlined by Dicke in [165]. These variations are not noise; for a given antenna position, we repeatedly measure the same shape (though the noise riding on these variations *is* random). The origin of the observed small variations lies in the effective temperature difference between the room and LNA causing a net power flow from the antenna into the LNA. This effective temperature difference partially excites modes of the antenna/cavity system, causing the observed variations. We suspect this effect originates from a small reactive component of the LNA's input causing the electronic cooling described originally by Radeka [106]. This effect can be eliminated by adding an isolator between the antenna and LNA [120, 166] though for our experiment, it

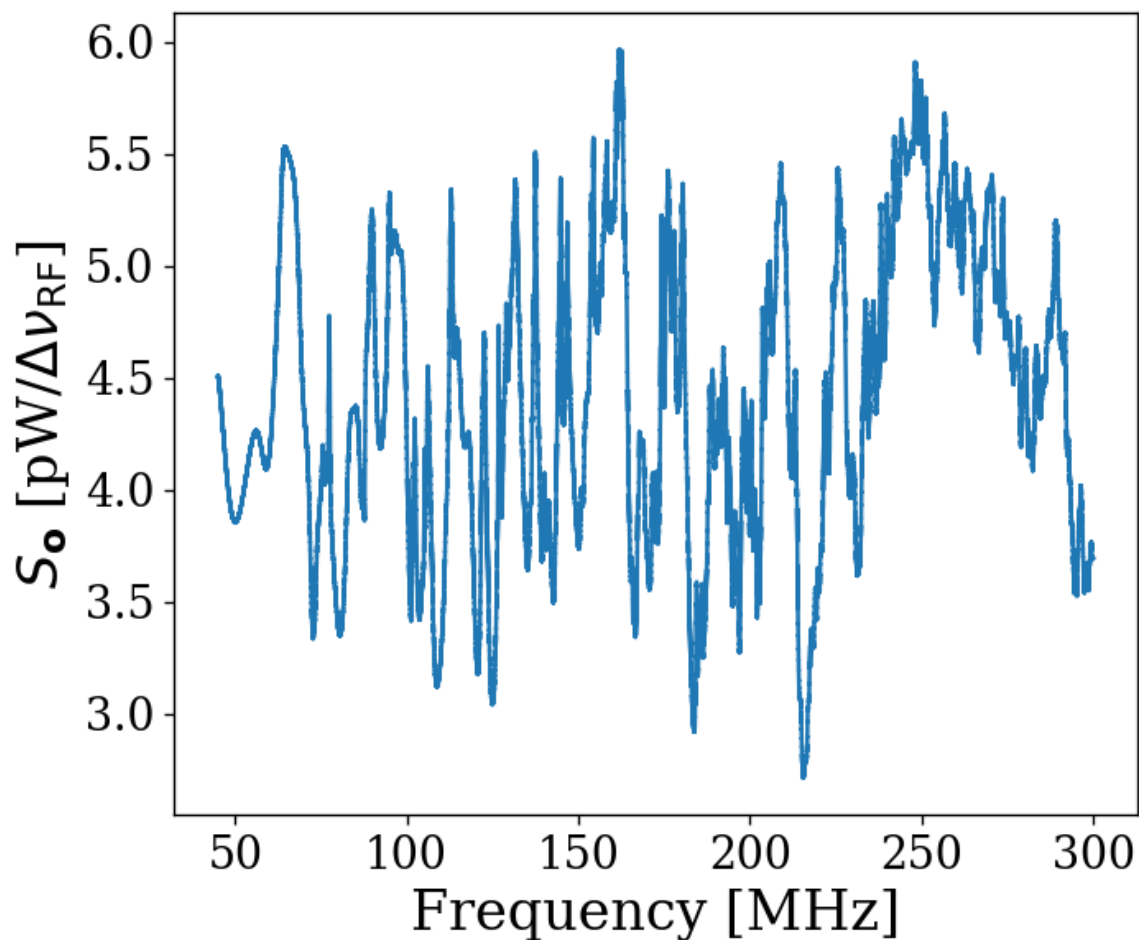


Figure 4.1: Run 1A averaged, output-referred antenna power spectrum S_o . Data were taken over a 9-day period at 9 antenna positions. For context, Fig. 4.2 shows these data plotted alongside the power spectrum of the terminator. The narrow variations are mainly due to the effective temperature difference between the room and LNA (Sec. 2.3), though there is a small contribution due to amplifier gain and noise temperature variations (Sec. 4.1.1). The variations seen here are not noise; their shape is repeatable for a given antenna position. The noise on this background is not visible at this level of zoom, but is seen in Fig. 4.4, which shows a zoomed-in view of the spectrum at 240 MHz. The noise is also seen nicely in Fig. 4.18

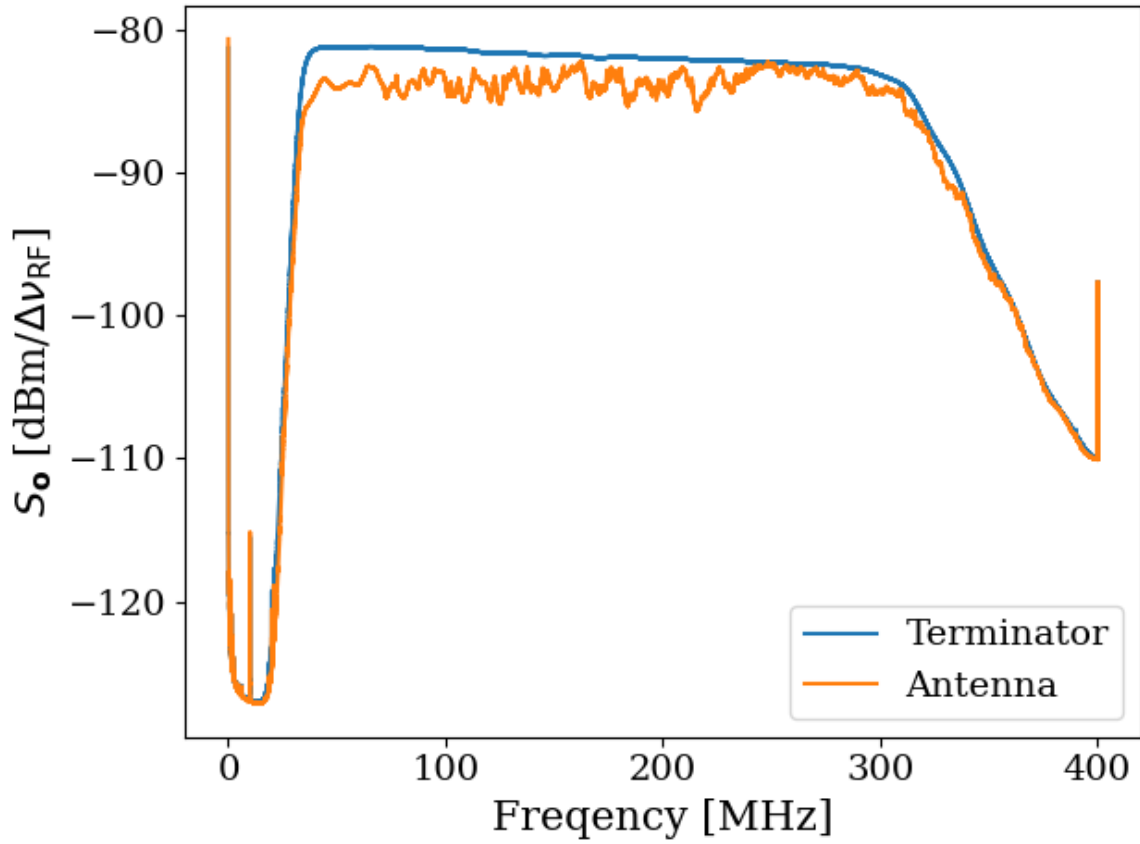


Figure 4.2: Logarithmic scaled Run 1A averaged, output-referred, antenna and terminator power spectra. Bandwidth is defined by hardware band-pass filters (Fig. 2.37). Spurious signals at 10 MHz, as well as low and high-frequency behavior, are ADC effects and are visible in Fig. 3.15.

is impractical to get an isolator that covers such a wide band at relatively low frequency. Furthermore, the relatively wide (tens of kHz) variations are handled by fitting to them, which is discussed in Sec. 4.1.

4.2 Data Analysis

At this point, we have compiled a single, averaged, output-referred power spectrum, S_o (Fig. 4.1). The task of *analysis* is to extract a dark photon signal from this spectrum if it is present. Otherwise, in its absence, we would like to set a limit on the amount of output-referred power we would be able to detect *most of the time* were a narrow signal to be present in this averaged dataset. We quantify the meaning of “most of the time” by conducting a series of Monte Carlo “pseudo-experiments” on artificial, signal-containing spectra for synthetic signals of varying powers and frequencies. The following subsections are organized as follows:

4.2.1: Fit S_o to extract an estimate of the background B (which we call $\hat{B}$) whose origin was discussed in Sec. 4.1.1. See Fig. 4.4.

4.2.2 Divide the spectrum by $\hat{B}$ to generate the *normalized spectrum*, which very nearly follows a Gaussian distribution. Discuss statistics of the normalized spectrum and choose a global significance level and its associated *significance threshold*. See Fig. 4.5. Also note this was derived in Sec. 2.2.

4.2.3 Apply a matched filter to the normalized spectrum and establish a significance threshold on its output using the same method defined in the previous section. See Fig. 4.6. The previous three steps comprise our *detection algorithm* which is shown in Fig. 4.3.

4.2.4 Perform a Monte Carlo analysis to simulate the required power of a signal that can be detected above the significance threshold 95% of the time. We use this to recover a

95% exclusion limit on the output referred power spectrum.

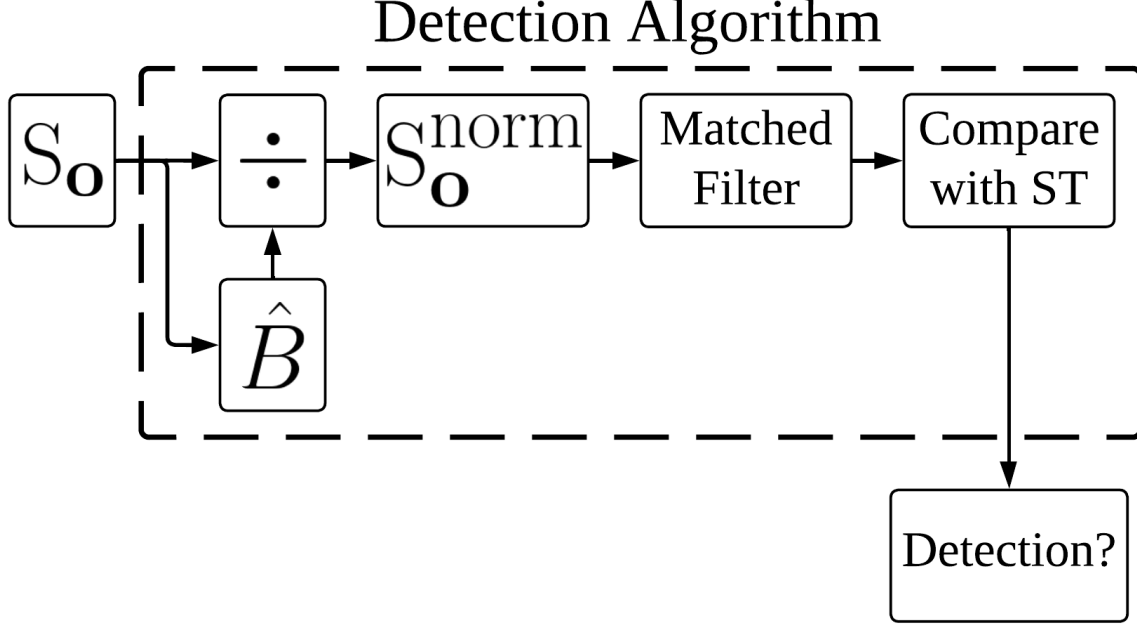


Figure 4.3: Flow chart outlining the logic of signal processing in the detection algorithm of sections 4.2.1 through 4.2.3. $\hat{B}$ is the smoothed fit to S_o generated by low pass filtering. The output, *Detection?*, is a Boolean array that signifies a detection or lack thereof at each frequency bin. We detect a candidate if a bin contains more power than a significance threshold (ST) (Sec. 4.2.2).

In Sec. 4.3 we convert this threshold on S_o into an actual limit on ε .

Throughout the figures of this section, we will follow a relatively large (40 fW, output-referred) software-injected, synthetic dark photon signal at 240 MHz to illustrate what a candidate would look like as it passes through the analysis procedure. This signal is added to S_o . For clarity, a single interfering candidate has been removed. This is discussed in Sec. 4.2.5.

As a final note, at this point, if you are unfamiliar with the DER data analysis, it would

be advantageous to spend a few minutes looking through the figures located at the end of this chapter to get a sense of the task at hand. It can be disorienting to deal with a spectrum that contains 8 million frequency points. Zooming in on a signal is very informative.

4.2.1 Fit background, $\hat{B}(\nu)$ ¹

As shown in Fig. 4.1, the measured power spectrum looks like flat thermal noise *multiplied* by some frequency-dependent background, $B(\nu)$ ². However, for this section, we will not concern ourselves with the origin of B or any details of the experiment aside from two assumptions:

1. The measured background is the product of a normally distributed spectrum and some background. This is enforced by the central limit theorem due to the large number of averaged spectra, independent of any experimental specifics.
2. The line shape of the signal is known and the width of this signal is much narrower than the width of features on the background, viz. $\Delta\nu_{\text{DP}} \ll \Delta\nu_{\text{B}}$

The first assumption (1) implies that if we were able to extract the background, dividing S_o by this extracted background would yield a *dimensionless*, normally distributed power spectral density on which a search for a dimensionless signal is performed. The second assumption (2) will be critical in both performing the fit to the background (this section), and performing matched filtering (Sec. 4.2.3).

¹Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/run1p4_analysis/analysisClass.ipynb

²When I was first working on this, I used $H(\nu)$ to represent this background. This naming convention persists in the analysis code.

In light of these assumptions, we attempt to fit the background power spectrum. Since this fit estimates B , we use the symbol $\hat{B}$ to refer to it. As discussed in [108], a particularly effective fitting technique that can discriminate between the wide bumps of S_{o} and a narrow signal is to use a low pass filter. We implement this filter in two stages:

1. A median pre-filter (51 bins or about 2.4 kHz wide) attenuates any very narrow, very large excursions which would interfere with any following filters, causing them to “ring”³.
2. A 6th-order Butterworth low pass filter (corner frequency of 210 bins or 10 kHz).

These bin widths/frequencies should be interpreted as the width of spectral features on S_{o} that are attenuated and will, therefore, not show up in the background fit. A narrow zoom of this fit with a synthetic signal is shown in orange in Fig. 4.4.

³I recommend that this pre-filtering step be omitted if the spectrum analyzer in use does not produce large spurs. It is the slowest part of the analysis chain. It also causes the filtered spectrum to deviate slightly from Gaussian.

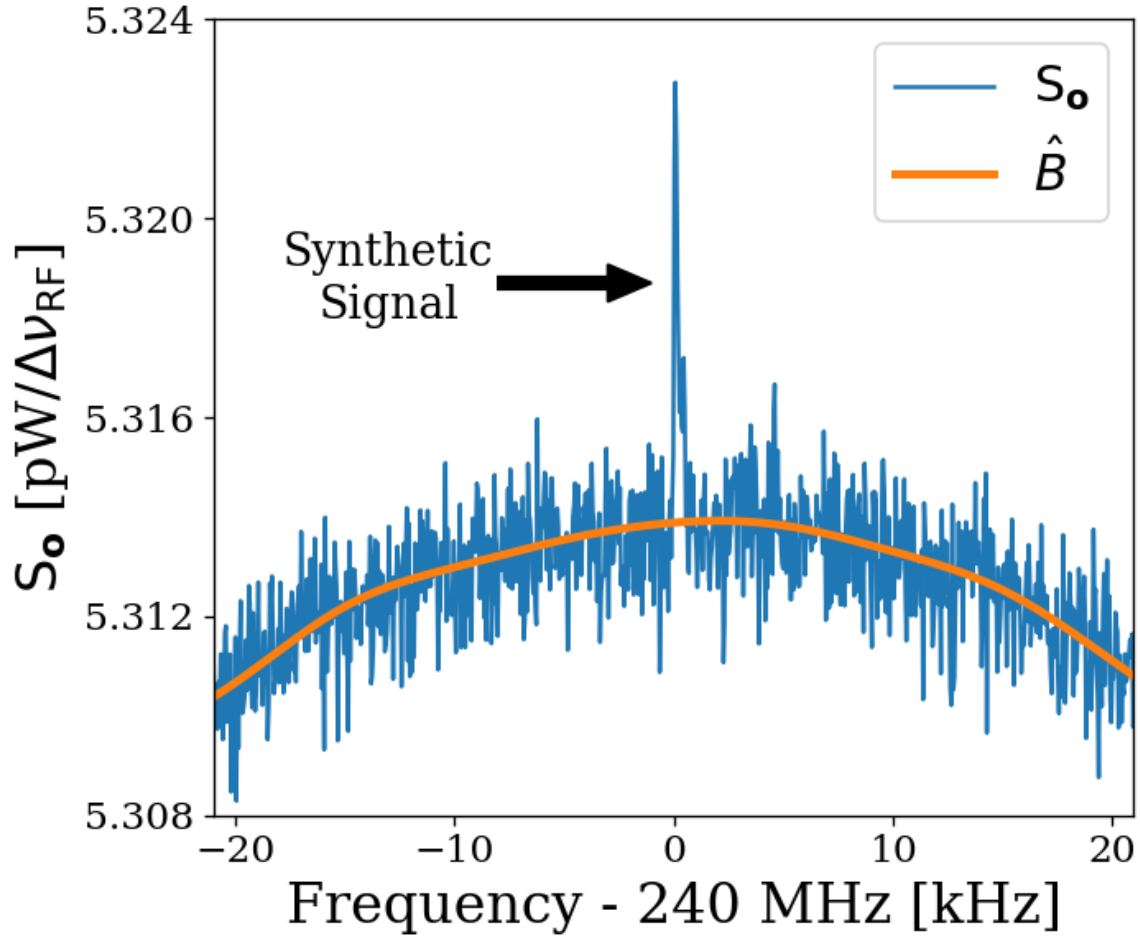


Figure 4.4: Fitting background $\hat{B}$ in the presence of a synthetic signal injected at 240 MHz. Starting from the averaged, output-referred spectrum (S_o), we fit the background using a series of filters (section 4.2.1, and Fig. 4.3). This figure is a highly zoomed-in view ($240 \text{ MHz} \pm 20 \text{ kHz}$) in order to show the noisy Rayleigh signal shape.

4.2.2 Normalized spectrum, S_o^{norm}

Once we have a fit to the background, $\hat{B}$, division of S_o by this fit yields a dimensionless,

Gaussian distributed spectrum

$$S_o^{\text{norm}} \equiv \frac{S_o}{\hat{B}}. \quad (4.1)$$

As discussed in Sec. 2.1.1.3, this normalized spectrum (Fig. 4.5) should have a mean $\mu_{\text{norm}}=1$ and a standard deviation given by the Dicke radiometer equation $\sigma_{\text{norm}} = (\tau \Delta\nu_{\text{RF}})^{-1/2}$ where τ is the total integration time (≈ 9 days) and $\Delta\nu_{\text{RF}}$ is the width of a bin (47.7 Hz). This works out to a predicted σ_{norm} of 1.727×10^{-4} . μ_{norm} and σ_{norm} calculated from the data are $1 - 1.2 \times 10^{-5}$ and 1.741×10^{-4} respectively, which agree with the predicted values to better than 1%. Knowing the statistics of the background allows us to set a threshold above which we have some confidence that a candidate is not a random fluctuation. This significance threshold (ST) was derived in Sec. 2.2.1. As a reminder, the probability that all N bins are less than z standard deviations, $z\sigma$, for a standard normal distribution is given by $\left\{ \frac{1}{2} [1 + \text{erf}(z/\sqrt{2})] \right\}^N$, where $\text{erf}(z)$ is the standard error function and z is real. A 5% ST for 5.2×10^6 bins (our 50-300 MHz analysis span) works out to 5.6σ . This is shown in Fig. 4.5.

It is possible to set a simple limit using this significance threshold on the normalized spectrum, which was our method in [127]. However, knowing the line shape of the dark photon signal provides additional information that improves sensitivity (up to a factor of ≈ 2) at the higher frequency end of the spectrum, as shown in the following section.

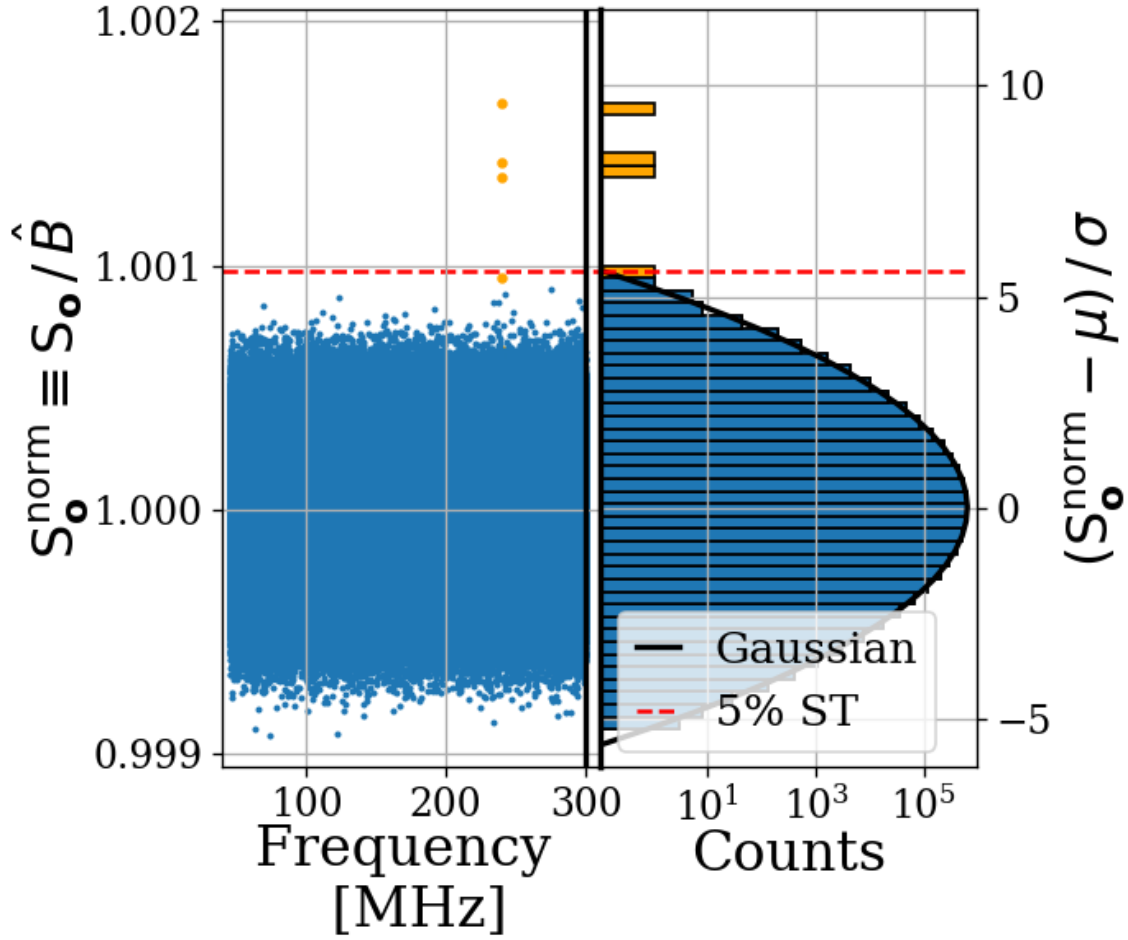


Figure 4.5: Dividing S_o by $\hat{B}$ yields a dimensionless, normally distributed power spectrum that we define as S_o^{norm} . We show S_o^{norm} in two ways: a normalized power/frequency spectrum (*left*) and rescaled into Z-score units and collapsed into a histogram (*right*). The histogram shows power excess and Gaussian fit, but frequency information is lost. We compute a 5% significance threshold ST (*dashed red*), above which we will detect a candidate by chance 5% of the time. Bins adjacent to the 240 MHz synthetic signal show up in orange on both plots. A single interfering signal has been removed for clarity. We discuss this further in Sec. 4.2.5

4.2.3 Signal-matched filter

As discussed in 4.2.2, one simple method to set a limit is to look for single-bin excursions above some threshold. However, galactic dynamics impart a dark photon candidate with a Rayleigh-distributed, spectral signature, which has a dimensionless width $Q_{\text{DP}} \approx 10^6$ [121]. This means that the expected width of a candidate signal over our analysis span (50-300 MHz) varies between 50-300 Hz. We set $\Delta\nu_{\text{RF}} = 47.7$ Hz to maximize SNR for the lowest expected signal width. However, this divides signal power between adjacent bins, an effect that becomes more pronounced at higher frequencies, leading to a decrease in sensitivity. By using a signal processing technique known as *signal-matched filtering* [167, 168], we restore some of the sensitivity lost due to the splitting of signal between the fixed-width frequency bins of the FFT. A similar “optimal weighting” procedure has been well established in axion haloscope experiments, notably by [108, 169]. Below, the signal-matched filter is referred to simply as a *matched filter*.

For a known signal shape, the detection technique that optimizes SNR is the matched filter. This is implemented on the normalized power spectrum using the Rayleigh-distributed spectral line shape of [121] as a template. Since we have a constant $\Delta\nu_{\text{RF}}$ and expect the width of the signal to vary across our span, we must calculate several templates of varying width to match the expected line shape. Every 10% of fractional frequency change, a new template is generated and used to search that small subspan of the normalized spectrum, each of which is also normally distributed though with its own standard deviation. This results in 20 subspans (50-55 MHz, 55-60.5 MHz etc.). The normalized spectra of all 20

subspans and the histogram of the 227-250 MHz subspan are shown in Fig. 4.6.

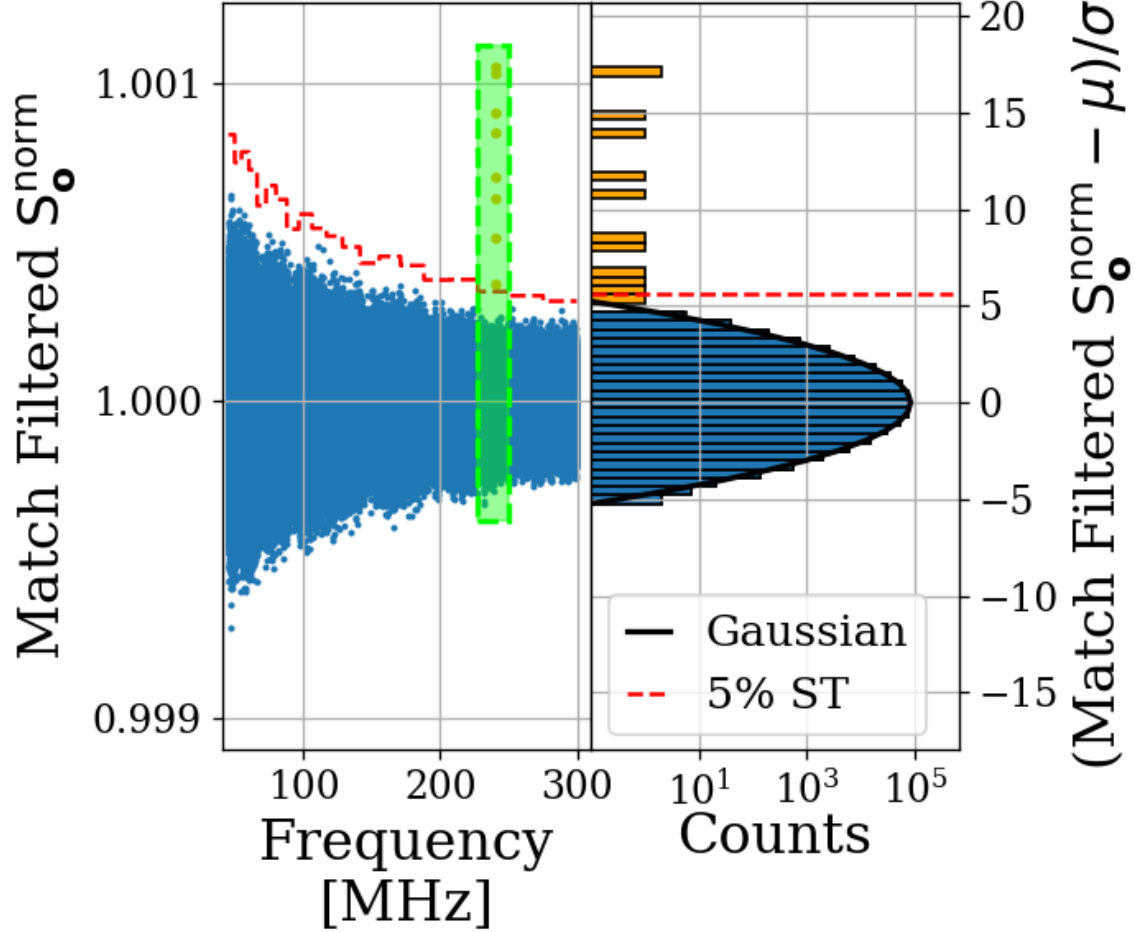


Figure 4.6: S_o^{norm} after it has been passed through a matched filter. The template varies in width throughout the frequency span, resulting in 20 subspans, each with a constant 5% significance threshold ST (*dashed red*). The histogram only includes 227-250 MHz subspan (enclosed in the green box). The signal-to-threshold ratio of the synthetic signal (orange) improves by a factor of about 1.8 as compared to Fig. 4.5 without a matched filter. The frequency dependence of this effect is shown in Fig. 4.8. A single interfering signal has been removed for clarity.

As the width of the templates increases, the standard deviation of the output decreases, resulting in the $\nu^{-1/2}$ shape of the 5% significance threshold shown in Fig. 4.6. It should be

noted that since the total number of bins remains 5.2 million, the 5% significance threshold still corresponds to 5.6σ ; the shaping in Fig. 4.6 is due to the variation in σ for different templates, not a change in the $z = 5.6$ pre-factor.

4.2.4 Monte Carlo: pseudo experiments

The previous three sub-sections outline the procedure for detecting the presence of a signal of known spectral line shape embedded in wide-band noise. We refer to this procedure as a *detection algorithm* (see Fig. 4.3), which we now calibrate through a Monte Carlo method. Synthetic background spectra are generated, and known signals are added to this background to create a *test spectrum*. This test spectrum is passed to the detection algorithm, which attempts to detect the signal. This process is repeated many thousands of times, and the statistics of the detectability of a signal (as a function of its frequency and power) provide insight into how much power could have been detected in the experiment.

A synthetic, background spectrum is constructed by multiplying some background $B(\nu)$ by randomly generated Gaussian white noise characterized by μ_{norm} and σ_{norm} , as discussed in section 4.2.2. A Rayleigh-distributed signal of known, total integrated, output-referred power and frequency, $\lambda(p, \nu)$, can now be added to this spectrum to create a *test spectrum* that can be passed through the detection algorithm (see Sec. 2.1.2 for a discussion of signal line-shape). The frequencies of the synthetic signals are evenly spaced (approximately every 10 MHz). However, because the signal spans a small number of bins (one to six), the shape of the discretized signal is very sensitive to the frequency at which its peak lands relative to

the bins. To compensate for the unknown dark photon frequency relative to a given bin, the frequency of the synthetic signal is randomly jittered by $\pm\Delta\nu_{\text{RF}}/2$, which is drawn from a uniform probability distribution at each iteration of the Monte Carlo. By repeatedly running these synthetic, signal-containing, test spectra through the detection algorithm, statistics are built up about how much total integrated power is required to detect a signal as a function of frequency *most of the time*. We quantify this as the statistical power of the detection algorithm and denote it $100\% - Y = 95\%$ following the standard convention of hypothesis testing.

	Only Noise	Noise + Signal
Detection	X	$100\% - Y$
No Detection	$100\% - X$	Y

Table 4.2: Threshold parameters that are part of the detection algorithm and Monte Carlo. X is the significance of the analysis. It is a parameter passed to the detection algorithm which specifies the significance threshold. The quantity $100\% - Y$ is the statistical power of the analysis. It is a parameter in the Monte Carlo, which specifies a threshold on signal power where a given signal is detected in $100\% - Y$ of the Monte Carlo iterations. We choose both X and $Y = 5\%$.

This Monte Carlo allows us to treat the detection algorithm as a black box that can be calibrated by passing it a known input (a test spectrum, i.e. a synthetic $S_{\mathbf{o}}$ containing a synthetic signal, both software-generated) and looking at its output; a Boolean array of frequency bins representing signal detection. These data, along with a simple linear fit to the 95% confidence interval, are shown in Fig. 4.7.

The Monte Carlo’s output $\text{MC}_{\text{lim}}(\nu, Y)$, is a normalized limit on signal power required for detection. It is a function of frequency ν and statistical power $1-Y$. MC_{lim} can simply be

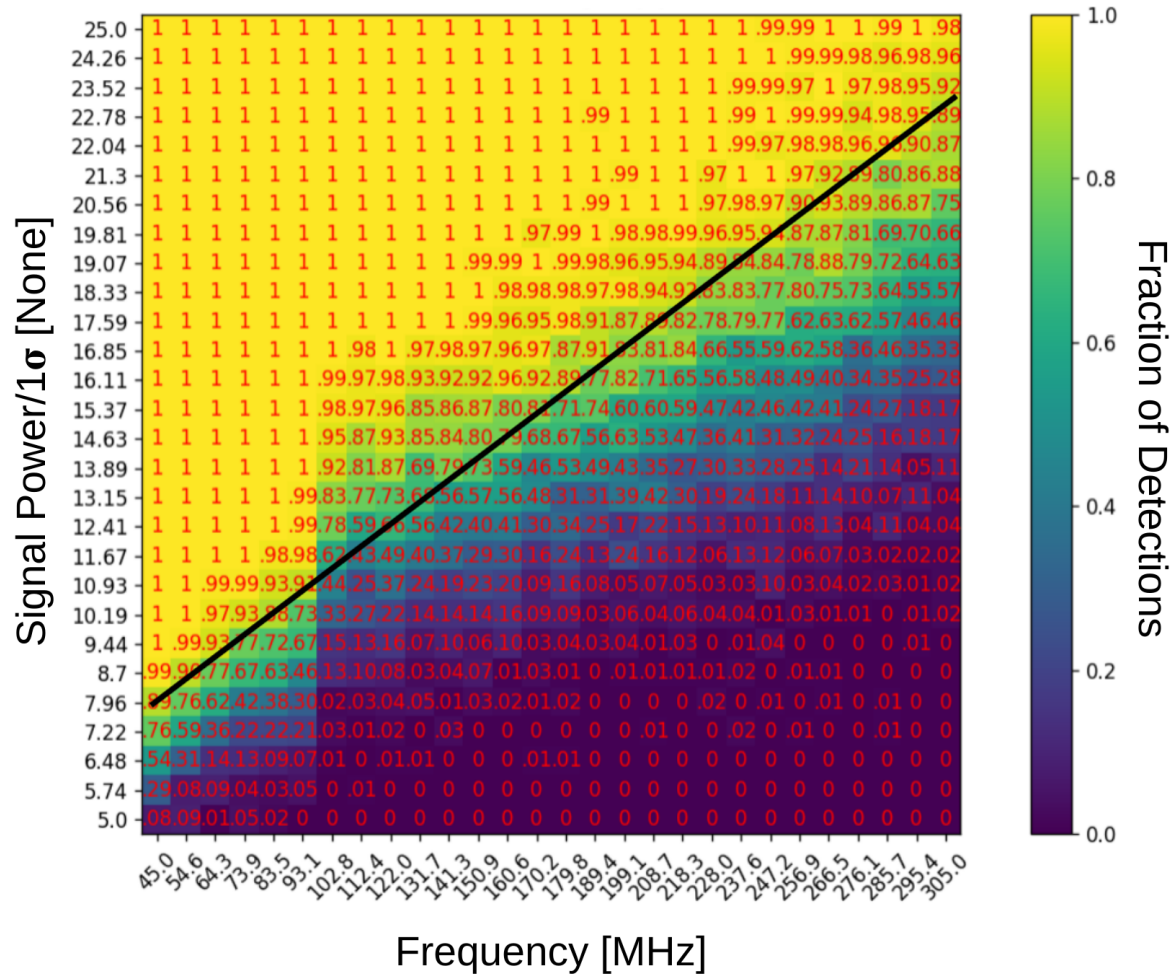


Figure 4.7: Result of Monte Carlo pseudo experiments on signal detection. Color and red numbering show the fraction of detections for each frequency/injected signal power of the 784 combinations tested. The signal power is shown in units of noise standard deviation σ . MC_{lim} is shown as a black line. This line was inserted by eye and gives an approximate fit to the frequency vs. injected signal power which results in a detection 95% of the time. The approximate form of this line is $MC_{lim}(\nu, Y = 5\%) = 0.0686[P_{norm}/\text{MHz}] + 2.411[P_{norm}]$ where P_{norm} is the normalized injected signal power shown on the Y-axis.

converted back to the limit on the total output-referred power contained in injected signals which can be detected 95% of the time,

$$P_{\mathbf{o}}^{\text{lim}} = MC_{\text{lim}} \sigma_{\text{norm}} \hat{B}. \quad (4.2)$$

$P_{\mathbf{o}}^{\text{lim}}$ is shown in Fig. 4.8 in blue. Also shown in Fig. 4.8 is a limit that does not include any matched filtering (orange) to highlight the frequency-dependent improvement of the matched filter. This limit is only for illustration and not used in the following sections.

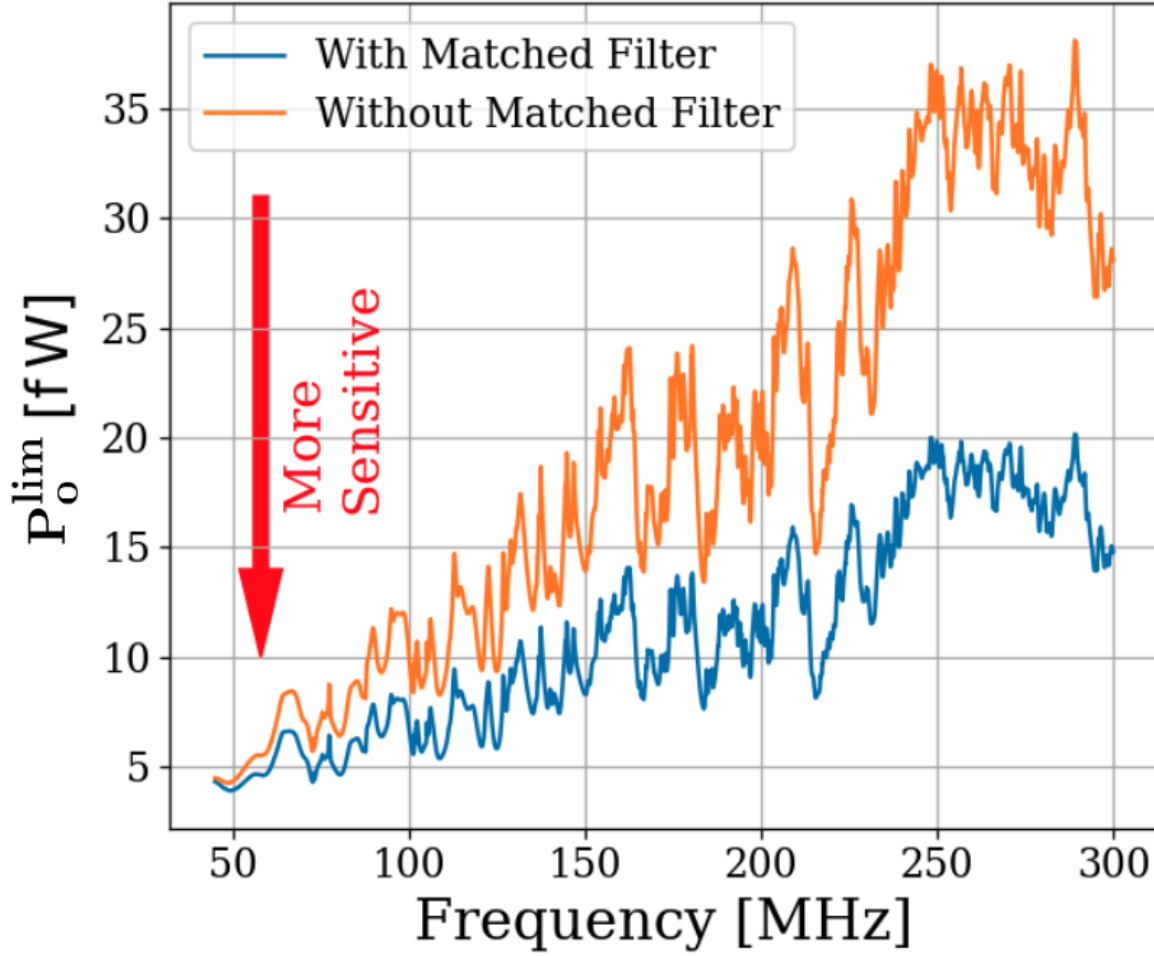


Figure 4.8: Limit on output-referred total integrated signal power, P_o^{lim} . Limits computed with (*blue*) and without (*orange*) a matched filter (Sec. 4.2.3). The limits are similar at lower frequencies, but the matched filter improves sensitivity at higher frequencies where the signal power is split among more bins. The blue curve is used in the following sections.

4.2.5 Rejection of a single candidate

Passing S_o through the detection algorithm diagrammed in Fig. 4.3 yields a single candidate at 299.97 MHz which is approximately 1 kHz wide. This candidate first became detectable above the noise after about 4 days of averaging, indicating it is just on the threshold of

detectability. Four factors allow for the conclusion that the candidate is an interfering signal originating from within the PC or ADC, and not a signal in the shielded room (either a dark photon or RFI):

- The candidate is present not only in the main spectrum, but also in the veto and terminator spectrum.
- Inspection of the time evolution of this signal shows a narrow signal (about two bins, or ~ 100 Hz wide) which seems to wander in frequency periodically over the course of a day and, therefore, with temperature. This is expected behavior for a quartz oscillator.
- Reducing the gain of the system causes the SNR of the candidate to *increase*, indicating it enters the signal path after the gain stages.
- Changing the clock rate causes the frequency of the candidate to change.

The limit set in this section is referred to the output of the amplifier chain. A single significant candidate was found, and the method of determining that it was not due to a dark photon was outlined above. The topic of the next section will be to work back through the amplifier chain, to an E-field limit in the cavity and ultimately to a limit on ε .

4.3 Calibration

In this section, we describe the calibration of the experiment and estimate its uncertainty. The previous section concluded with a frequency-dependent limit on the output-referred power P_{ϕ}^{lim} (Fig. 4.8), which we now must convert into a frequency-dependent limit on ε .

We begin by inverting Eq. 1.57,

$$\varepsilon(\nu) < \sqrt{\frac{|\mathbf{E}_{\text{ant}}^{\text{lim}}|^2 \epsilon_0}{2 \rho_{DM}}}, \quad (4.3)$$

where the *lim* superscript indicates a limit, below which a detectable electric field may be hiding.

The first step of calibration is to convert from output-referred power to *antenna-referred power*. This represents the signal power presented to the LNA by the antenna via a matched transmission line and is given by

$$P_{\text{ant}}(\nu) = \frac{P_{\circ}}{G} - T_{\text{amp}} k_B \Delta\nu_{\text{RF}}, \quad (4.4)$$

where G and T_{amp} are the frequency-dependent system gain and noise temperature (74–75 dB and 100–120 K respectively, measured via the Y-factor method, see Sec. 3.1.1) and k_B is Boltzmann’s constant.

Ultimately, the exclusion limit is set by fluctuations on this baseline described by

$$\begin{aligned} P_{\text{ant}}^{\text{lim}}(\nu) &= \frac{P_{\circ}^{\text{lim}}}{G} - \left(\frac{2}{n}\right)^{1/2} T_{\text{amp}} k_B \Delta\nu_{\text{RF}} \\ &= \frac{P_{\circ}^{\text{lim}}}{G} - \left(\frac{2 \Delta\nu_{\text{RF}}}{\tau}\right)^{1/2} T_{\text{amp}} k_B, \end{aligned} \quad (4.5)$$

where the *lim* superscript indicates an exclusion limit, n is the total number of spectra averaged together, and τ is the total integration time. In the second line, we have used $n = \Delta\nu_{\text{RF}} \tau$. In practice, the LNA correction is small; the first term divided by the second varies with frequency between 7 and 50. The $\tau^{-1/2}$ dependence of $P_{\circ}^{\text{lim}}$ is implicit because it

was calculated from S_o which is itself an averaged spectrum. As mentioned above, this $\tau^{-1/2}$ dependence implies that the limit on ε scales as $\tau^{-1/4}$.

In the remainder of this section we explore the relationship between $P_{\text{ant}}^{\text{lim}}$ and $|\mathbf{E}_{\text{ant}}^{\text{lim}}|$ so that we can use our experimental data to set a constraining limit on ε by employing Eq. 4.3.

4.3.1 Average effective aperture, $\langle A_e(\nu) \rangle$

An antenna's effective aperture, $A_e [\text{m}^2]$, represents the effective area that it has to collect power density or irradiance $[\text{W}/\text{m}^2]$ from an incident Poynting vector. It was defined in Eq. 2.9. Notably, it assumes a polarization match between the wave and the antenna (see Sec 2.1.1.2).

A_e is useful for an antenna in free space. However, some modifications must be made to construct an analogous quantity for an antenna in a cavity.

The first modification is to perform an ensemble average, namely to average over many configurations of the system. The background for this is given in Sec. 2.4. As discussed, we denote this averaging with $\langle \rangle$ so that the average, effective aperture is denoted $\langle A_e \rangle$. It is interesting to note that by performing an ensemble average over configurations (namely antenna direction), $\langle A_e \rangle$ simplifies since $\langle D(\Omega) \rangle = 1$ by construction [111].

The second modification is to introduce a resonant enhancement factor that corresponds to the system's tendency to “ring up” in the same way any resonator will. We refer to this as *composite* Q and represent it as $\tilde{Q}$. It is analogous to the standard quality factor of a resonator with one important modification; we operate our experiment across a wide

frequency range so we define $\tilde{Q}$ across the continuum of these resonances, not only on classical eigenmodes of the system.

These modifications provide a relationship between an observable E-field ($\mathbf{E}_{\text{ant}}$ in Eq. 4.3) and the power available at the port of an antenna for a given aperture

$$\langle P_{\text{ant}} \rangle = \frac{|\mathbf{E}_{\text{ant}}|^2}{\eta_0} \langle \tilde{Q} A_e \rangle, \quad (4.6)$$

where η_0 is the impedance of free space. With this in mind, we perform an RF simulation to compute $\langle \tilde{Q} A_e \rangle$.

4.3.2 Simulation of $\langle \tilde{Q} A_e \rangle$

It is difficult to make claims about statistical uniformity in the “undermoded” regime where modes are not sufficiently mixed [170], so we have employed a commercial, electromagnetic, finite-element modeling software package (COMSOL Multiphysics RF module [148]). Within the simulation, a model of the antenna (with a 50Ω feed) is placed in a simplified room with wall features removed. Spot testing at various frequencies has shown that averaging results from various antenna positions using this simplified simulation behaves very similarly to one with the room features included at a fraction of computational complexity.

Two similar simulations are run; driving an E-field while measuring the antenna’s response and driving a second small monopole antenna and measuring the response of the primary antenna.

In the first simulation, we drive currents on the walls which correspond to a surface E-

field magnitude of 1 V/m (made up of equal components in the x, y and z directions) using COMSOL's source electric field option. This field takes the place of $\mathbf{E}_{\text{ant}}$ in Eq. 4.6. The antenna/cavity system resonates and causes an enhancement by $\tilde{Q}$. The power received at the antenna's port is measured, allowing the calculation of $\tilde{Q} A_e$, again from Eq. 4.6. By repeating this simulation for several positions, averaging allows us to compute $\langle \tilde{Q} A_e \rangle$.

The second simulation shares the same geometry, but is used to compute a correction factor to account for differences between simulation and measurement and to estimate uncertainty on the first simulation through comparison to physical measurement. Rather than driving the system through currents on the walls, power is injected into the system with a 40 cm monopole. From this simulation, two-port scattering parameters (S-parameters, defined in 4.3.3) are computed. A similar test is performed on the physical system using a vector network analyzer (VNA) which provides a physical measurement of the S-parameters to compare with the simulation. The processing of the simulated and measured S-parameter datasets is discussed in the following sub-section. A screenshot of the COMSOL model GUI is shown in Fig. 4.9. The resulting S-parameters from the simulation are plotted against the measured S-parameters in Fig. 4.10

Both simulations are run at the same 18 positions; 9 of which are approximately equivalent to the physical antenna positions while the other 9 are different in order to estimate how many positions are required for decent convergence of $\langle \tilde{Q} A_e \rangle$. Repeatedly averaging 9 different, random positions (with replacement) results in about 20% variation on their averaged $\langle S_{21} \rangle$ coefficients at each frequency, allowing us to conclude 9 positions and polarizations provide acceptable convergence. This method is greatly inspired by the statistical

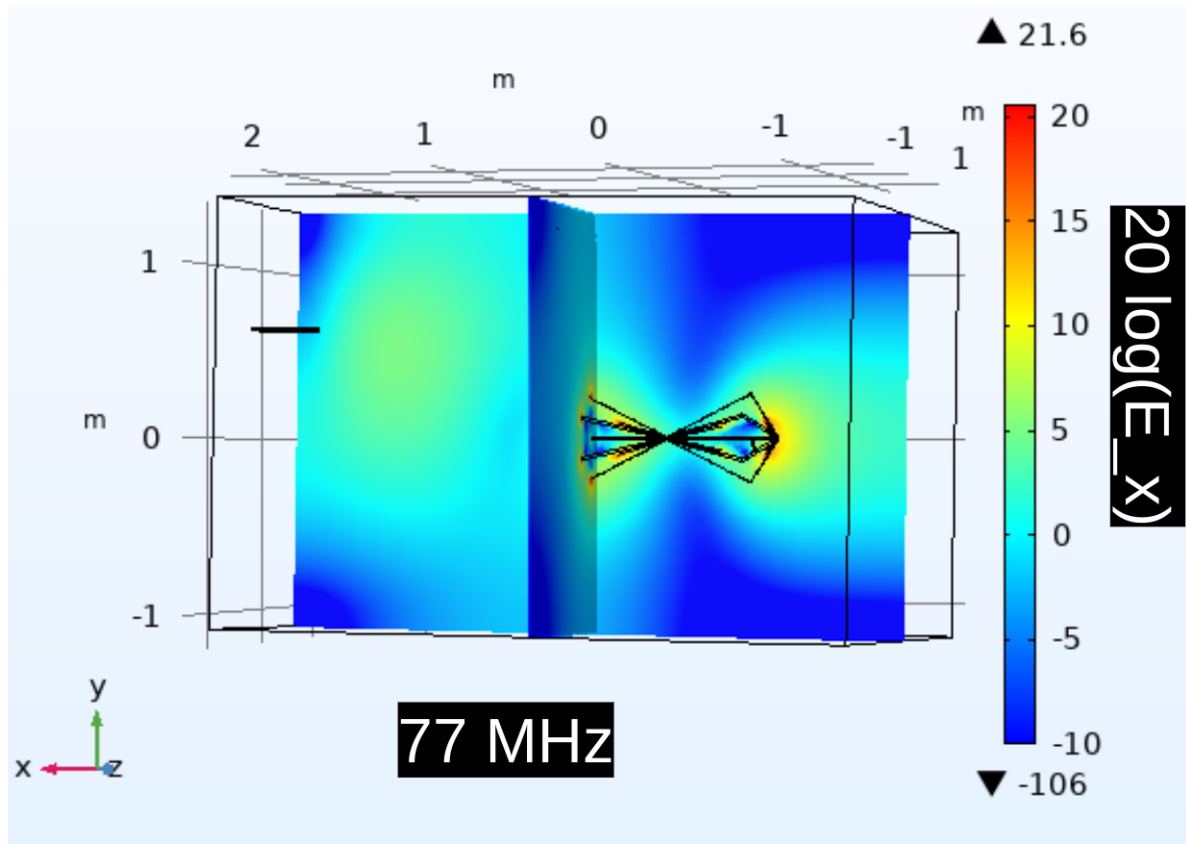


Figure 4.9: Screenshot of COMSOL simulation GUI for two-antenna validation. Shown only at a single position and single frequency (77 MHz). There are 18 antenna positions, 3 E-field components, and ~ 1000 frequency points, so there are many similar figures to this one. The X-component of the electric field is shown in color compared to 1 V/m.

bootstrap [171].

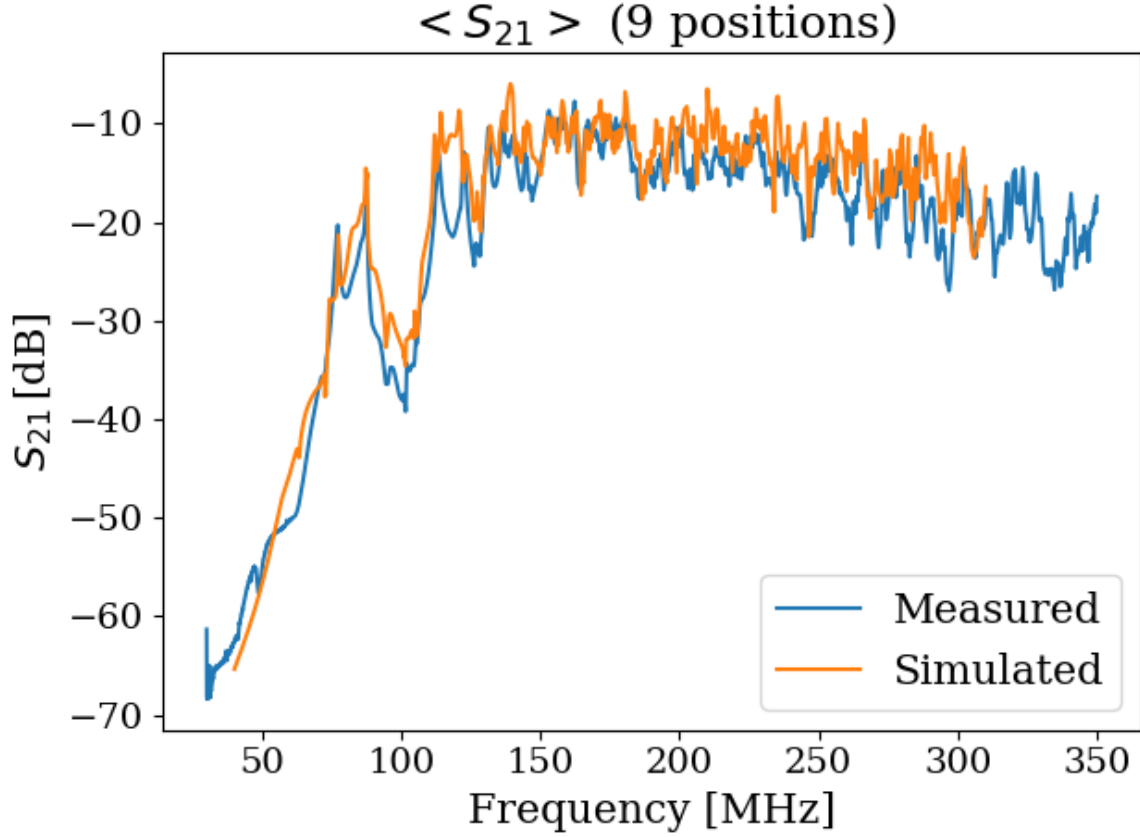


Figure 4.10: Simulated vs. measured $\langle S_{21} \rangle$. Averaged over 9 antenna positions. The procedures for generating these data are described in the text of this section.

4.3.3 Correction and uncertainty of $\langle \tilde{Q} A_e \rangle$

As outlined above, we approximate the uncertainty of the simulation by injecting power into the system via a second antenna and comparing the results to simulation.

For a two-port microwave device, the ratio between the voltage presented at port one and the voltage measured at port two is known as S_{21} [131, 132]. For our system, S_{21} is a measurable quantity that is similar to a dark photon detection in that it requires the antenna to convert an electric field (which has interacted with the room) into a port voltage.

Having frequency-dependent measurements of S_{21} for simulation and measurement gives us a correction to the simulation (to account for discrepancies in geometry) and estimate the uncertainty on $\langle \tilde{Q} A_e \rangle$.

The difference between the measured and simulated values of $\langle |S_{21}| \rangle$ is described by

$$\langle |S_{21}^{\text{meas}}|^2 \rangle = \alpha \langle |S_{21}^{\text{sim}}|^2 \rangle, \quad (4.7)$$

where meas/sim indicates measured/simulated and the average is over all 18 measured/simulated positions and orientations of the antenna and α is a frequency-dependent, multiplicative correction factor which results in a corrected $\langle |S_{21}^{\text{sim}}|^2 \rangle$. We have taken the square since we are interested in the aperture, which is proportional to the square of the voltage. We find α to have a mean of 0.6, a minimum of 0.1 and a maximum of 2.

To determine uncertainty on effective aperture, we define the following test statistic

$$\Delta = \frac{\langle |S_{21,n}^{\text{meas}}|^2 \rangle - \alpha \langle |S_{21,n}^{\text{sim}}|^2 \rangle}{\langle |S_{21}^{\text{meas}}|^2 \rangle}, \quad (4.8)$$

where n refers to the subset of n measured/simulated positions sampled randomly with replacement. Δ defines the fractional difference between corrected, simulated S_{21} and measured S_{21} . The test statistic, Δ , is calculated 1000 times, providing a distribution of frequency-dependent Δ s. The curves bounding 63% of these curves are taken to be the uncertainty on Δ . Thus we calculate the corrected $\langle \tilde{Q} A_e \rangle$ as well as its uncertainty. This is shown as a function of frequency in Fig. 4.11. The uncertainty on it is shown in gray and is simply

$$\delta \langle \tilde{Q} A_e \rangle = \langle \tilde{Q} A_e \rangle \delta \Delta. \quad (4.9)$$

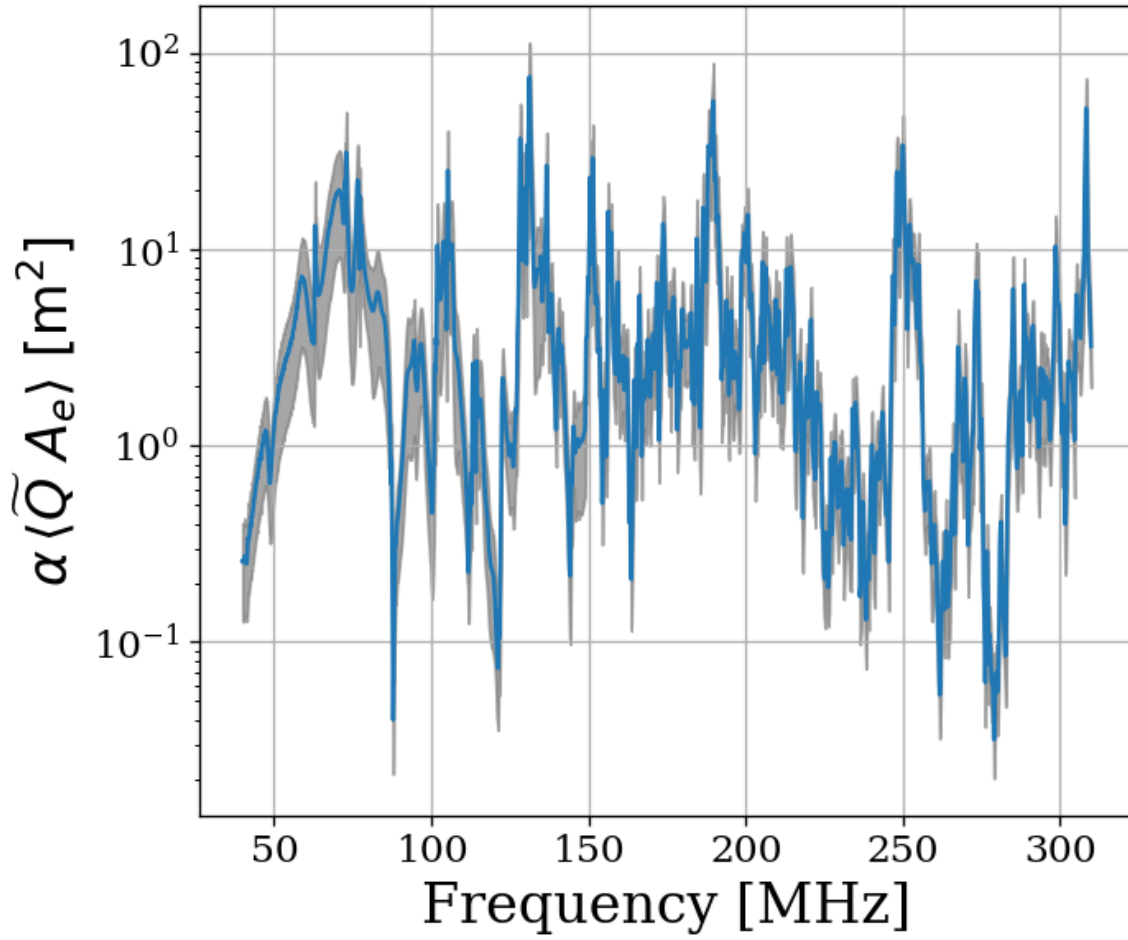


Figure 4.11: Corrected average effective aperture. Calculated with COMSOL RF. The aperture correction α (Eq. 4.7) and its uncertainty (*gray*) are estimated by comparing simulations to measured S parameters.

A brief recap of the system's aperture is in order. In free space, an antenna's ability to couple an incoming wave's power density into a transmission line is given by its effective aperture, Eq. 2.9. An antenna in a cavity acts as a coupled, resonant system that exhibits very complex resonances above the first few modes (~ 100 MHz for our system). Attempts to simulate an aperture for the antenna-cavity system are difficult because of the system's

extreme dependence on the placement of any conductor in the room, especially the antenna. Averaging over system configurations (antenna positions and polarizations in our case) allows for a significantly more repeatable *statistical* treatment of the aperture/quality factor, which we call $\langle \tilde{Q} A_e \rangle$. Comparison of simulated and measured S_{21} gives a small, dimensionless correction factor α , Eq. 4.7.

Armed with $\alpha \langle \tilde{Q} A_e \rangle$, we now compute a limit on ε using the measured and simulated quantities via Eqs. 4.5 and 4.6,

$$\varepsilon(\nu) < \sqrt{\frac{1}{2c\rho_{\text{DM}}} \frac{P_{\text{ant}}^{\text{lim}}}{\alpha \langle \tilde{Q} A_e \rangle}}, \quad (4.10)$$

where c is the speed of light, ρ_{DM} is the local dark matter density and $P_{\text{ant}}^{\text{lim}}$ is defined in Eq. 4.5. We have separated the equation into constants (or, in the case of ρ_{DM} , values that we fix) and values that we measure or simulate.

In order to validate our entire detection system, we inject sub-threshold signals into the shielded room to verify we are able to detect them.

4.4 Hardware Injection Test ⁴

To validate the detection methodology, a separate proof-of-concept run with a proxy dark photon signal injected into the shielded room was performed. Apart from the injection antenna (a 40 cm monopole, see Sec. 4.3.2), the setup was equivalent to Run 1A, including the data analysis. The proxy dark photon signal (detailed in Sec. 4.4.1.2) was injected at a

⁴Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/injectionTesting/injectionTesting.ipynb>

frequency set by a colleague and was unknown to me at the time of analysis, constituting a blind analysis.

4.4.1 Injection test prerequisites

4.4.1.1 Determination of required injected power

To accomplish the test, a minimum detectable power required for injection P_{inject} must be computed. P_{inject} should correspond to a signal that can be detected in a predictable amount of time (with some uncertainty, discussed in detail in Sec. 2.2.2). A simple way to begin is to read off the detectable, total integrated, power from Fig. 4.8. In other words the power contained in a dark photon line, integrated over the few bins spanned by the line ($Q_{\text{DP}} \approx 10^6$, discussed in Sec. 2.1.2). This gives the amount of output-referred power that would be detectable 95% of the time after 9 days of integration. Since we do not want to wait 9 days for this test, it is simple to convert this detection limit into one that would be produced in a shorter time using the Dicke radiometer equation, Eq. 2.16. Namely, the limit on power scales like the square root of time⁵, so a one-hour integration will require a factor $\sqrt{9 \text{ days}/1 \text{ hour}} = 14.7$ more power than is shown in Fig. 4.8.

At this point, the simplest way to proceed is to measure the average power transfer of the monopole to the bicon in several antenna positions. This power is proportional to $\langle |S_{21}|^2 \rangle$.

This is the same as the set up described in Sec. 4.3.2. The bicon was moved to 9 positions

⁵It is important to point out that one needs to test whether or not the system in question actually behaves as predicted by the Dicke equation for the amount of averaging in question. After lots of averaging, one may encounter non-thermal backgrounds that do not scale properly. It is shown in Fig. 4.17 that the dark radio system follows the Dicke radiometer equation at least for 9 days.

and the resulting S-parameters were measured at the reference planes shown in Fig. 4.12. They are shown in Fig. 4.13 after being averaged together.

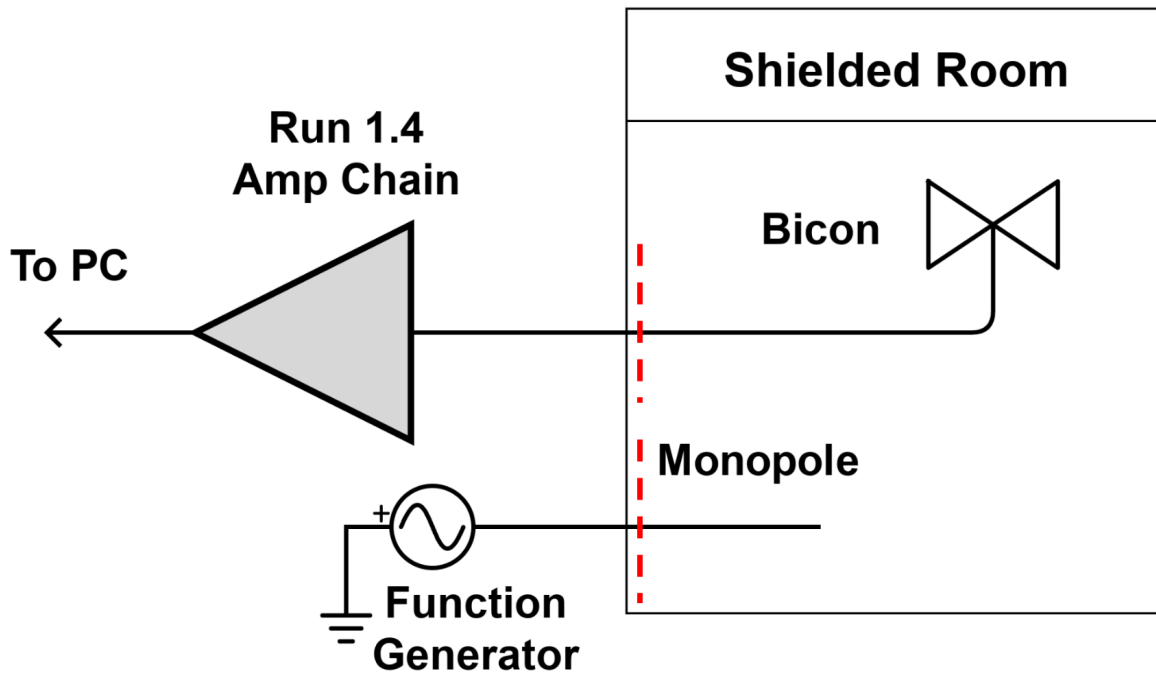


Figure 4.12: Schematic of hardware injection test. Dashed red lines indicate reference planes used to measure S-parameters, shown in Fig. 4.13. “Amp Chain” includes amplifiers, attenuators and filters as described in Sec. 2.5.4.

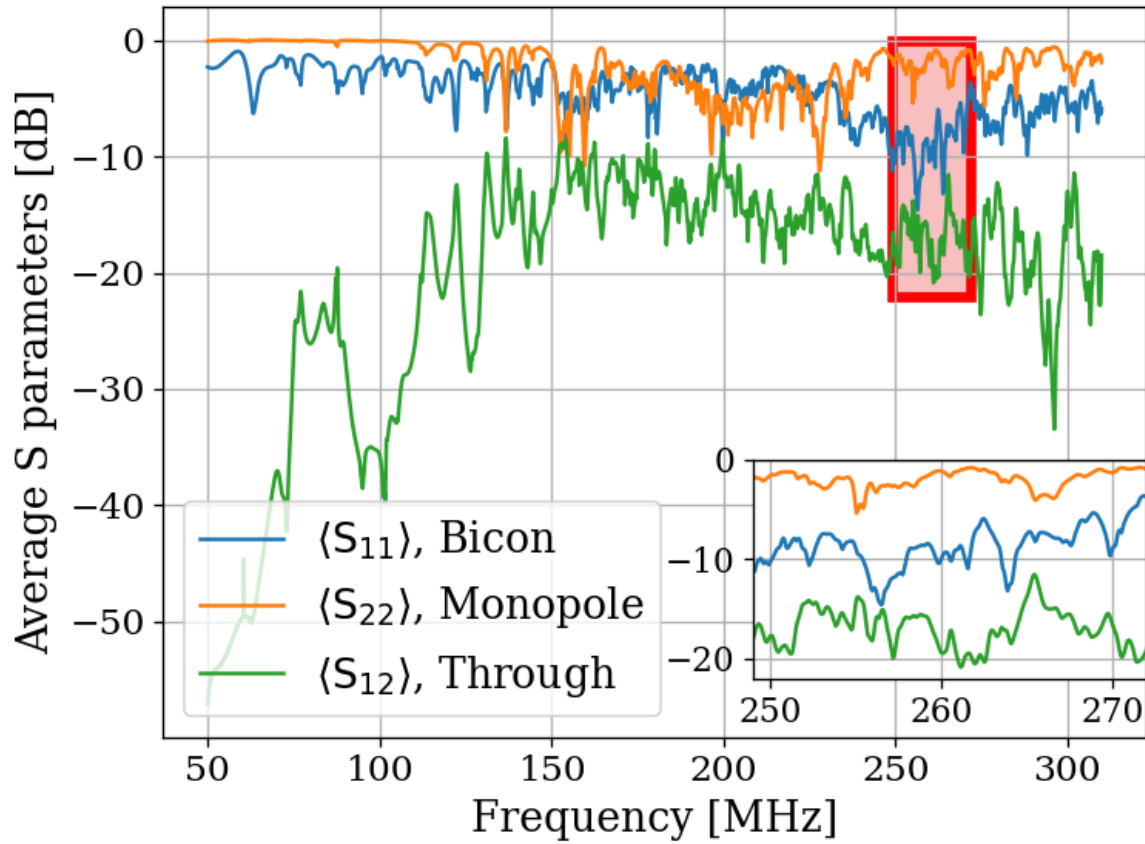


Figure 4.13: Average S-parameters of hardware injection test. Taken at 9 positions of the bicon antenna and averaged together (linearly). Ports 1 and 2 are defined to be for the bicon and monopole respectively. Note that $\langle S_{12} \rangle = \langle S_{21} \rangle$ so only $\langle S_{12} \rangle$ is shown. The inset shows a zoom-in of the area enclosed by the red box. Also note that dB is implicitly proportional to power.

Knowing the output-referred power limit, the system's average $\langle |S_{21}|^2 \rangle$, and the gain G , allows for a calculation of the required signal strength (as a function of frequency) which will become detectable after a known amount of time. In order to simplify the test, a smaller 1 MHz band is chosen between 268 and 269 MHz for the blind injection, where $\langle |S_{21}|^2 \rangle$ is constant to about 1 dB. At first glance, this seems like cheating. However, there are still $\sim 21,000$ frequency bins in this span, so a detection is very unlikely to be random.

Furthermore, the entire 50-300 MHz span is sent to the detection algorithm (Discussed in Sec. 4.2 and illustrated in Fig. 4.3), which produces an output without knowing about this frequency restriction.

With all this in mind, the signal power required is simply

$$P_{\text{inject}} = \frac{P_{\text{o}}^{\text{lim}}}{G \langle |S_{21}|^2 \rangle},$$

where each term is a function of frequency. For the parameters described in this section (including the increase in the power limit $\sqrt{9 \text{ days}/1 \text{ hour}} = 14.7$), this works out to $\approx 6 \times 10^{-19} \text{ W}$ or -152.5 dBm. The Rigol DSG830 signal generator is not calibrated to such low levels, so this was achieved through attenuation⁶.

4.4.1.2 Proxy dark photon signal injection

Now that the power for signal injection has been established, the finite-width proxy-signal ($Q_{\text{DP}} \approx 10^6$) is generated. The signal's line-shape is discussed in Sec. 2.1.2. An intuitive way to accomplish this would be with an arbitrary waveform generator injecting a time domain signal which is the Fourier transform of the the expected Rayleigh line shape, Eq. 2.24. This is the method employed by the ADMX experiment (see, for example, [172]). Another option would be to frequency-modulate a sine wave such that it slowly sweeps out the line shape, spending an amount of time at each frequency weighted by Eq. 2.24. While intuitive, I was unable to get this to work. Zhu et al.'s method of frequency hopping [173] is the discretized version of this, and it was very simple to implement. The signal generator is set to change

⁶Experience with this signal generator has shown it exhibits the best performance is when it is set around -30 dBm. Higher than this, large non-harmonic distortions appear, contaminating the run. Smaller, and the signal-to-spurious-noise-floor of the generator is poor, also contaminating the run.

frequencies at some interval (discussed below). The frequency is randomly drawn from the PDF of Eq. 2.24.

There are three considerations that determine the frequency hop period τ_{FH} that the frequency is changed⁷. First, τ_{FH} should be much longer than the acquisition time of a single buffer τ_{FFT} ⁸. On the Rigol signal generator, the power is briefly shut off while the frequency is changed. $\tau_{\text{FFT}} \ll \tau_{\text{FH}}$ ensures that most FFTs of data do not contain a frequency-hop. Second, τ_{FH} should be small compared to the total time of integration τ , so that there are many frequencies represented in the entire run. In the limiting case, $\tau_{\text{FH}} = \tau$ will yield an averaged spectrum containing a single injected frequency. In this case, the proxy-signal will be a delta function in the frequency domain. Third, τ_{FFT} should be longer than the ring-up time of the room, $\tau_{\text{FFT}} \gg Q/\nu$. For $Q = 100$ and $\nu = 260$ MHz, the right-hand side of this inequality is $\approx 0.3 \mu\text{s}$.

Testing has shown that $\tau_{\text{FH}}/\tau_{\text{FFT}} \approx 10$ is more than adequate to address the first consideration. For Run 1A (and therefore, this test, which shares settings with Run 1A), $\tau_{\text{FFT}} = 2^{24}/800 \text{ MHz} = 21 \text{ ms}$, so τ_{FH} was set to 250 ms. This means that over 1 hour, the frequency will be set to $\approx 1.4 \times 10^4$ values, which addresses the second and third concerns. A histogram of this signal is shown in Fig. 4.14.

⁷Zhu et al. randomized this period to prevent any unintentional periodic signals entering. I did not find this to be necessary.

⁸One can likely bypass this restriction by coordinating the signal generator and ADC such that there is some dead time between collection of buffers, in which the frequency is set. Testing has shown that this restriction is adequate to avoid this extra programming step

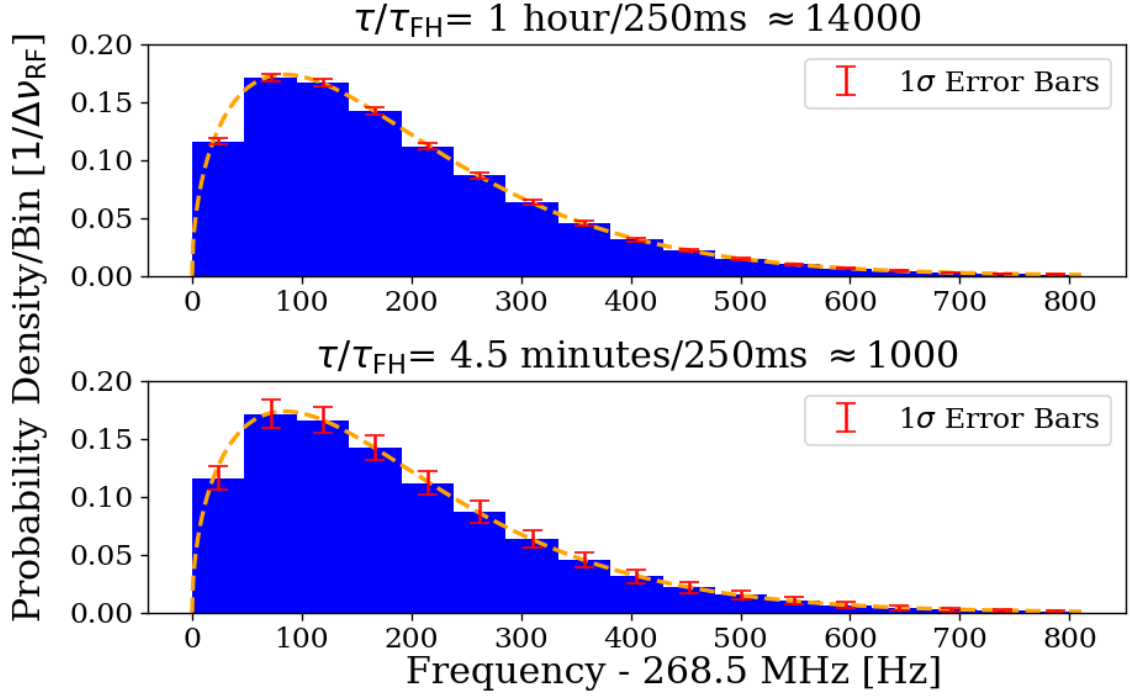


Figure 4.14: Histogram of frequencies used for hardware injection test, with realistic $\Delta\nu_{\text{RF}} = 47.7 \text{ Hz}$. The Orange dashed curve is the expected line shape from Eq. 2.24, and is the PDF frequencies are drawn from. The blue histogram and error bars are generated from the Monte Carlo simulation. They show the mean value per bin, with 1σ error bars in red. This involves generating 1000 lists of random frequencies (each of length τ/τ_{FH}), binning the data and calculating the standard deviation of each bin. τ is the total acquisition time and τ_{FH} is the amount of time spent on each frequency before “hopping” to the next. Their ratio, τ/τ_{FH} , is the number of frequencies that are injected in a given injection test, and was approximately 1.4×10^4 for the one-hour test outlined in this section. Two plots give an idea of how error scales with τ/τ_{FH} .

4.4.2 Performing the injection test

Due to the uncertainties involved, more data were taken than the required 1 hour. 3.6 hr of data were collected and saved in 30 s pre-averages so that progressively more data could be averaged if the signal was not detected at the predicted time. As mentioned above, the signal was injected at a relatively high frequency within the span so that it would be split up into

5 or 6 bins, testing the matched filter's effectiveness. The bicon was moved to 9 positions. Spectra resulting from 9 antenna positions and 30 seconds of pre-averaging at each position were averaged together giving 4.5 m time resolution. Three of these spectra are shown in Fig. 4.15. The standard deviation of these spectra average down with the square root of time, closely following the Dicke radiometer equation (discussed in Sec. 2.1.1.3). This scaling is shown in Figs. 4.16 and 4.17.

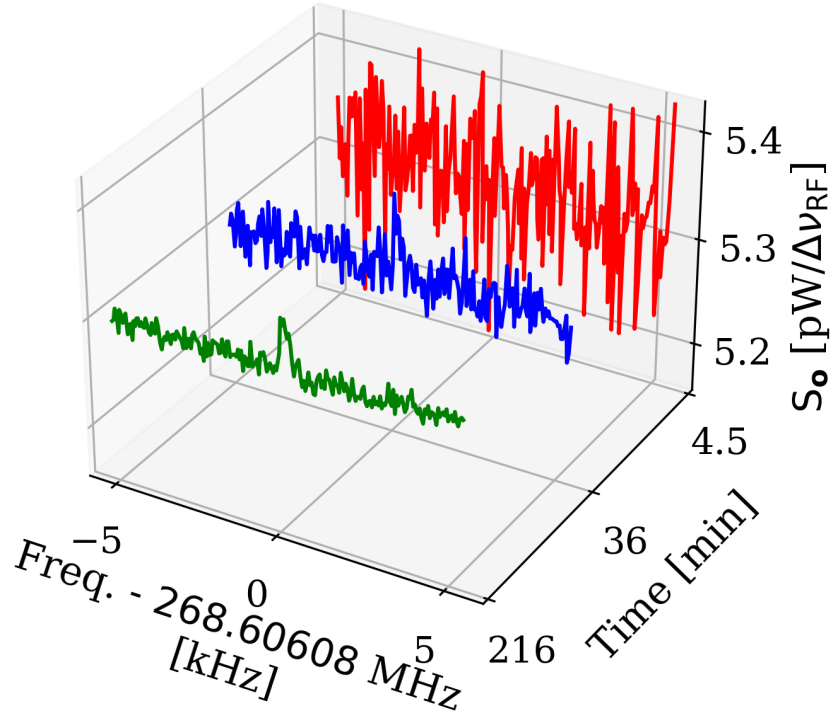


Figure 4.15: Output-referred power spectral density from the hardware injection test illustrating noise averaging down to reveal a persistent, hardware-injected, dark photon proxy signal. Spectra shown are highly zoomed around the injected frequency, 268.60608 MHz. The red, blue and green spectra represent 4.5, 36 and 216 minutes of integration time, respectively. The standard deviation of these spectra (excluding the bins containing the injected signal) averages down with the square root of time as expected. The blue spectrum shows the amount of averaging required for the signal to be detected by the detection algorithm (including the matched filter) at 5% significance. The tight zoom shown here is less than 1 part in 10^4 of the full 50-300 MHz spectrum analyzed.

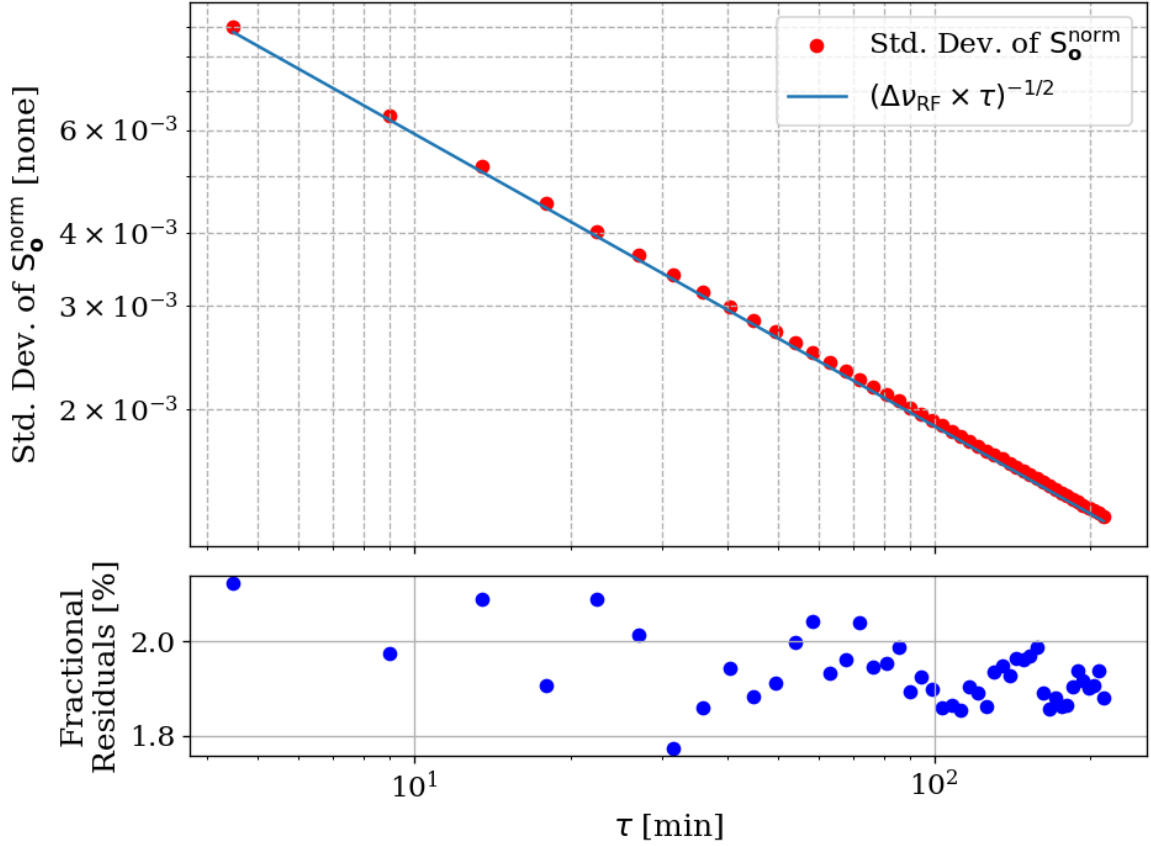


Figure 4.16: Standard deviation of normalized, output-referred power spectral density from the hardware injection test. The blue curve is the predicted standard deviation from the Dicke radiometer equation, Eq. 2.16. Each point represents the 9 antenna positions with an additional 4.5 minutes of data averaged (see Sec. 4.4.2). Correlation in residuals are observed when computing the standard deviation directly on S_0^{norm} as an artifact of fitting $\hat{B}$, as described in Sec. 4.2.1. A better fit is achieved through the median average deviation, Shown in Fig. 4.17.

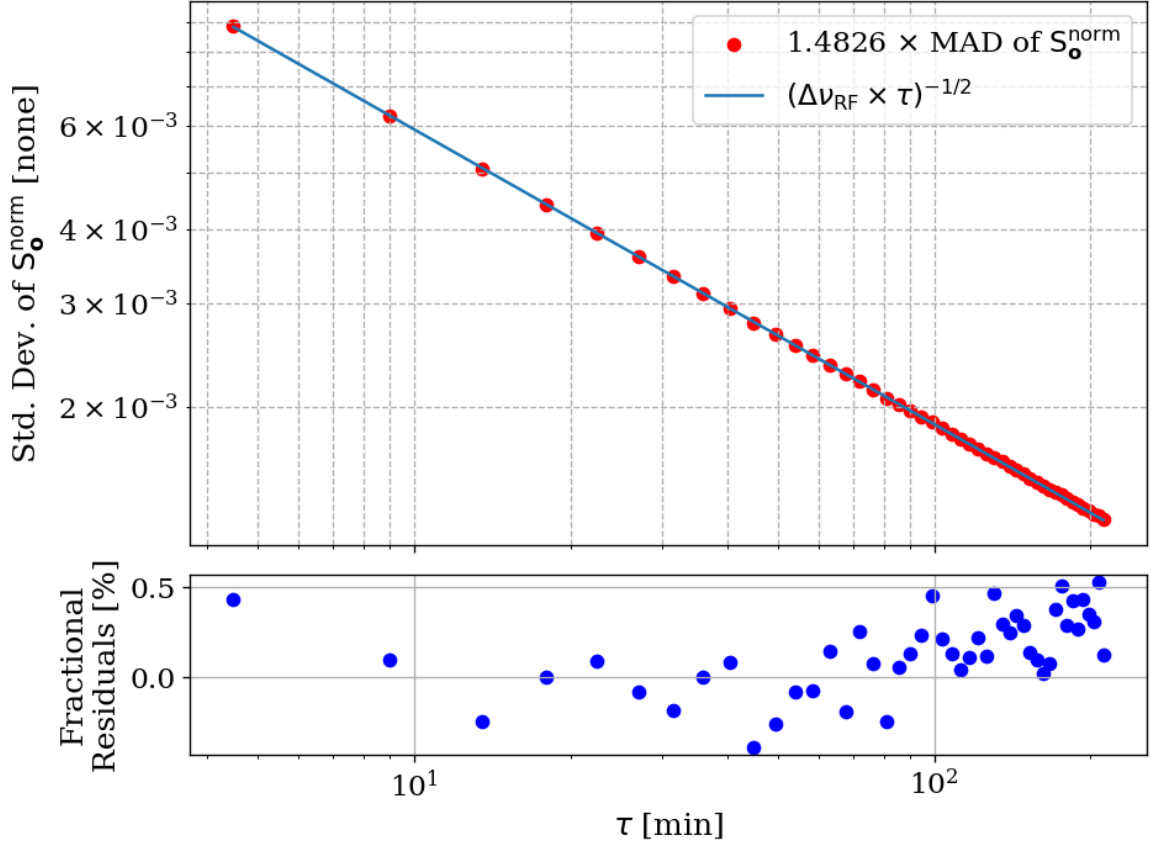


Figure 4.17: Standard deviation of normalized, output-referred power spectral density from the hardware injection test, computed with median absolute deviation (MAD). The blue curve represents the predicted standard deviation from the Dicke radiometer equation, Eq. 2.16. Each point corresponds to 9 antenna positions with an additional 4.5 minutes of data averaged (see Sec. 4.4.2). MAD provides a more robust measure of variability, reducing the influence of outliers and offering a better fit to the normalized spectra than direct standard deviation calculations. Note that the factor of 1.4826 is required to convert between MAD and standard deviation and assumes normally distributed data. It is derived in Sec. 2.3 of [174].

These spectra were generated one at a time and passed through the detection algorithm.

The first spectrum where a signal was detected was at 36 minutes, shown in blue in Fig. 4.15.

Although hardly detectable to the eye, the matched filter detects the signal with 5% signifi-

cance. At the point the signal was detected (i.e. before all data were averaged together), the injection frequency was confirmed to have been correctly identified, resulting in a successful, blind, hardware injection test. Only after this confirmation were all the data averaged together to make Fig. 4.15.

4.4.3 Inspection of Data

This final subsection simply contains some full-page figures which show data from the injection test. They are all from the same 34 minutes of data but at different stages of processing, closely following the three main steps of Analysis, Sec. 4.2. They are meant to simulate the experience of inspecting a 2^{24} -point FFT's power spectrum in a matplotlib widget window. The zoom is seemingly unending, a feature that is difficult to appreciate in a printed document. These figures should provide some context for how futile a manual search of unfiltered data would be. Try and pick out the signal in the top left panel of Fig. 4.18!

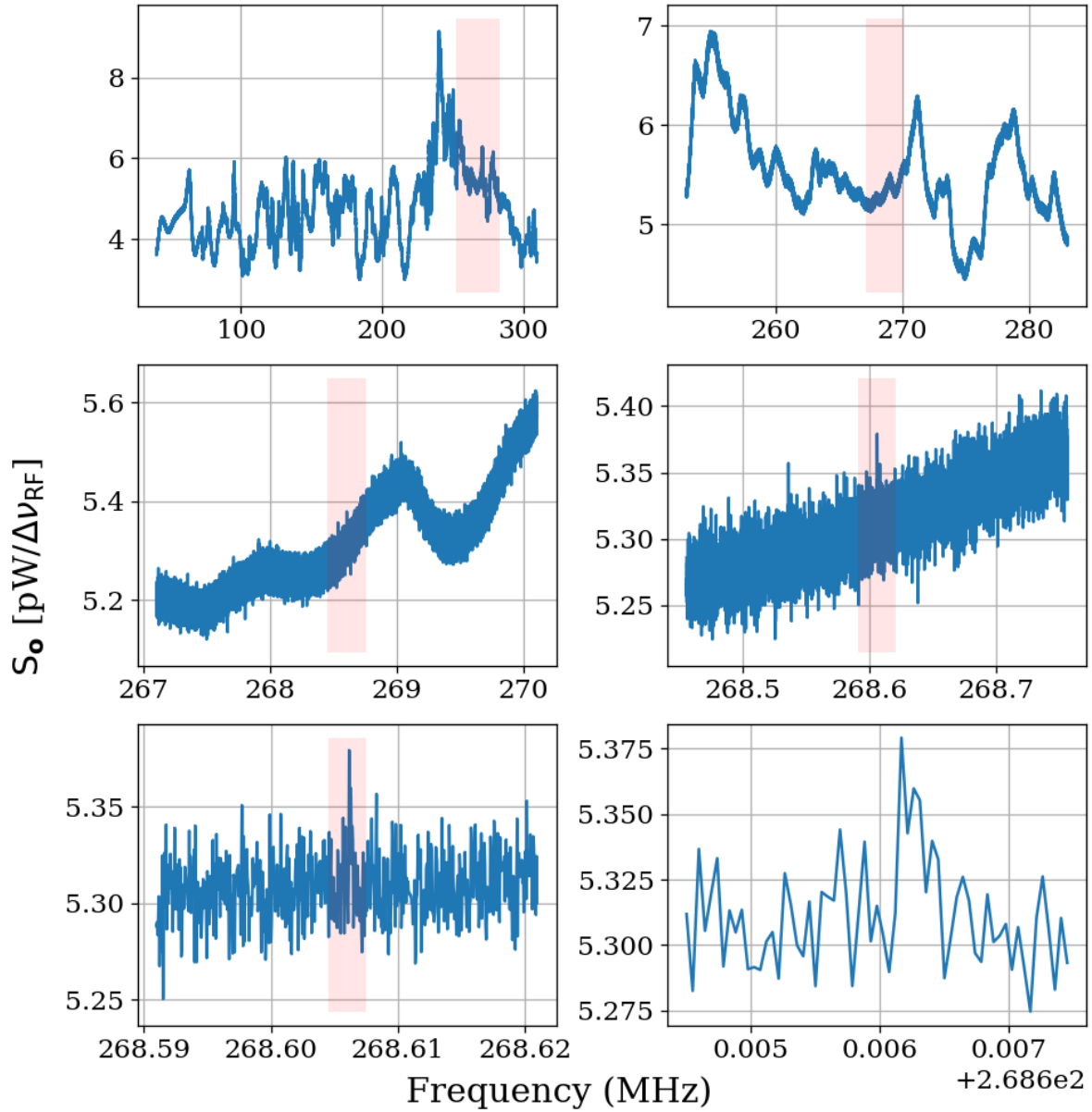


Figure 4.18: Output-referred power spectrum from hardware injection test. Injected signal at 268.60608 MHz. All spectra correspond to a total of 36 minutes of averaging, split evenly between 9 antenna positions. The full 50-300 MHz span contains $\approx 5.2 \times 10^6$ bins. Light pink boxes show the zoom range on the following plot.

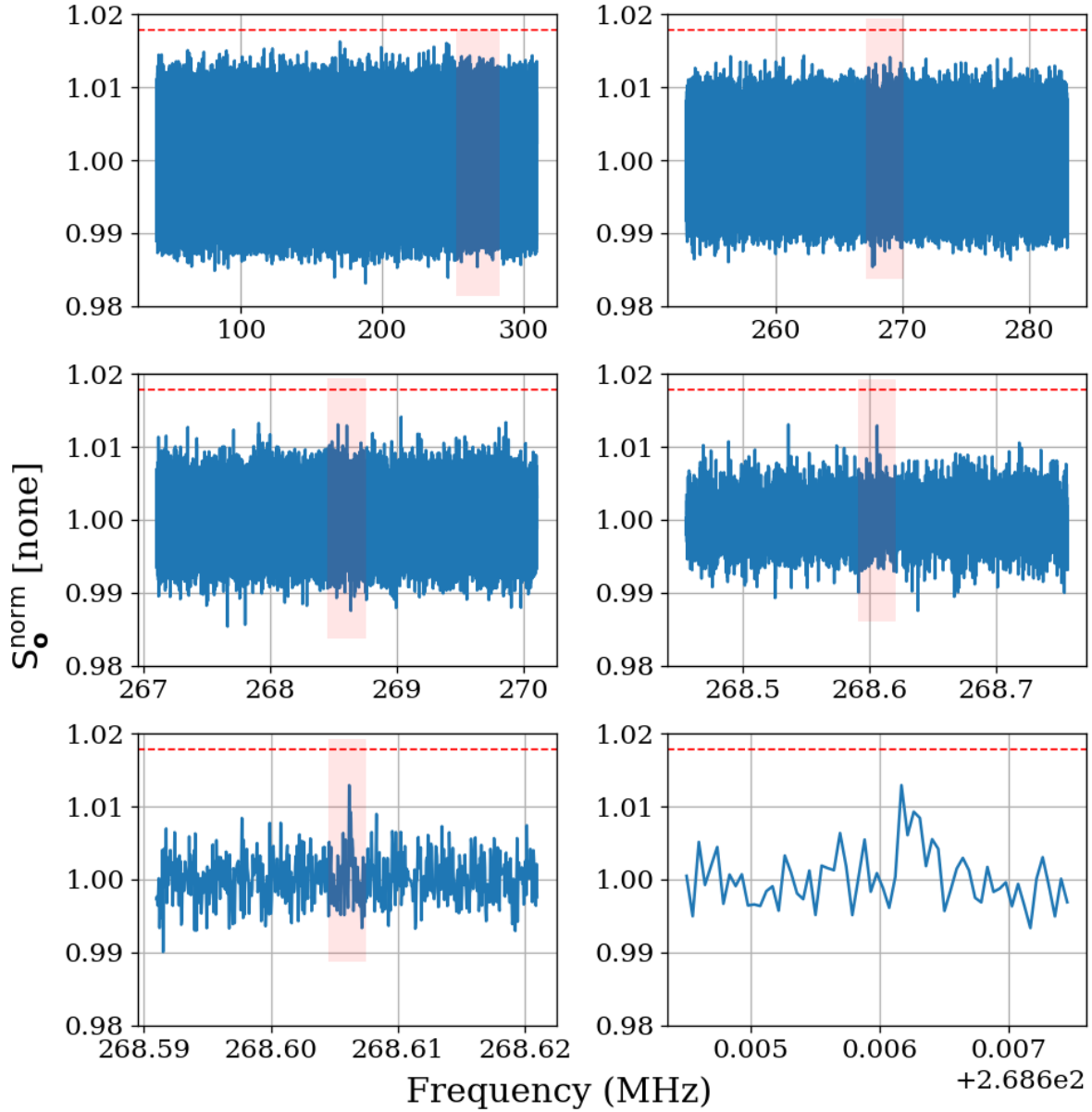


Figure 4.19: Normalized, output-referred power spectrum from hardware injection test. Injected signal at 268.60608MHz. Light pink boxes show the zoom range on the following plot. The red dashed line indicates the 5% significance threshold, derived in Sec. 2.2.1. A signal was not detectable above this threshold.

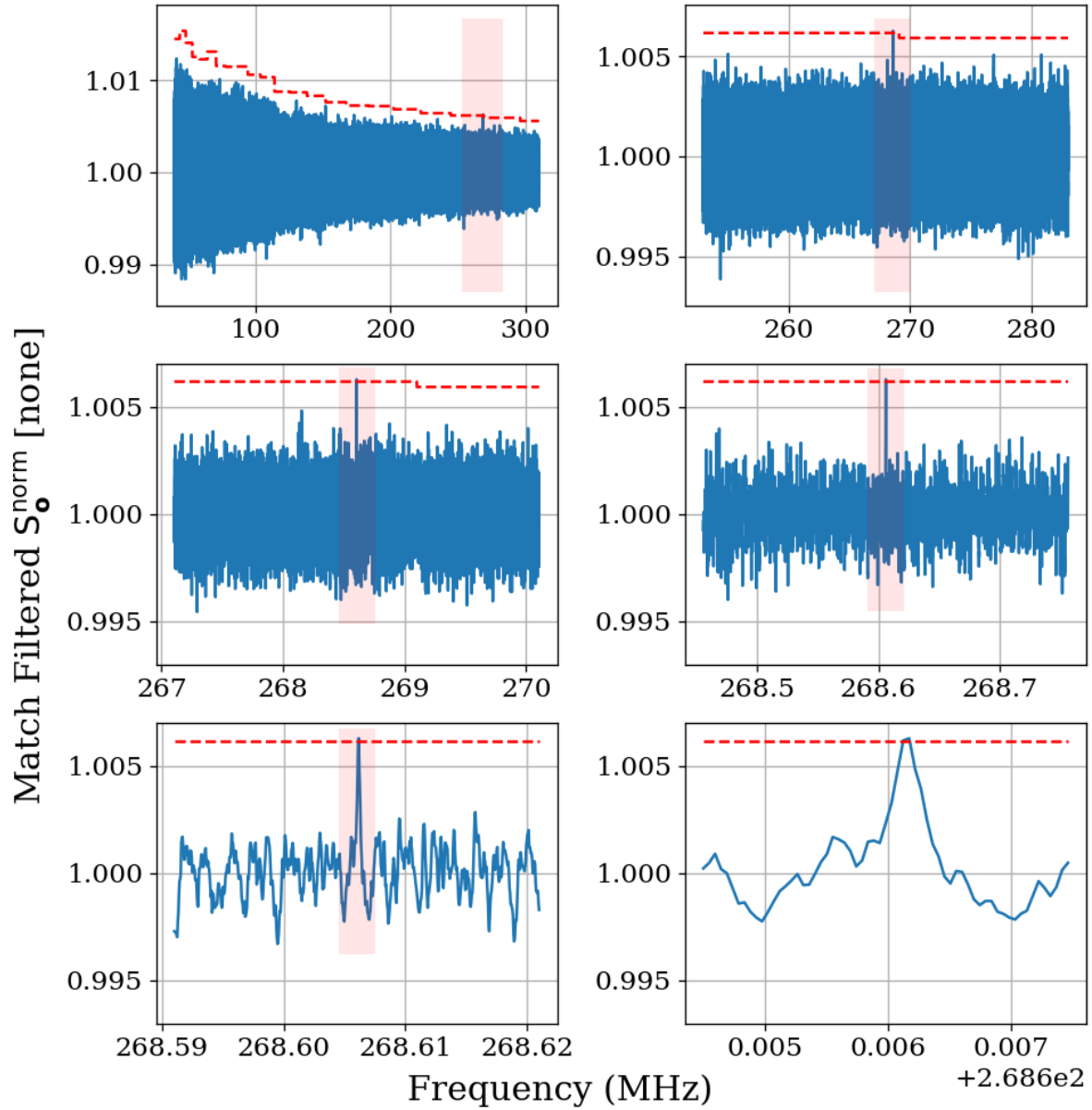


Figure 4.20: Matched filtered, output-referred power spectrum from hardware injection test. Injected signal at 268.60608MHz. Light pink boxes show zoom level on following plot. Red dashed line indicates the 5% significance threshold, derived in Sec. 2.2.1. Introducing the matched filter pushed signal above detectable above threshold compared to Fig. 4.19.

Chapter 5

Results

I always believe when someone tells me they
haven't discovered something.

Harry Nelson

In this chapter, we report a 95%, mass-dependent, exclusion limit on the kinetic mixing strength ε of the dark photon. We discuss the uncertainties of the measured data. Finally, we display our results in context by plotting these new limits on top of an aggregation of existing limits. Future runs of this experiment from 0.3-14 GHz in similar room-temperature RF enclosures and using 100 K noise temperature LNAs are indicated (the foundation for such a system is outlined in Sec. 6.2). We have only indicated planned runs, however at microwave frequencies, highly resonant cryogenic cavities and cryogenic LNAs as well as sub-THz instrumentation are feasible and could result in an order of magnitude improvement in the limit over the indicated mass range and beyond. The design principles of such a system

are outlined in Sec. 6.3.

5.1 Discussion of Uncertainties

The systematic uncertainty in this experiment comes primarily from three sources, listed in order of their contribution from greatest to least:

1. Fractional uncertainty on the simulated antenna aperture, which is discussed in Sec. 4.3.3, $\approx 60\%$
2. Fractional uncertainty on the first-stage amplifier noise temperature, $\approx 10\%$
3. Fractional uncertainty on the gain of the amplifier chain, $\approx 5\%$

The uncertainty on the simulated antenna aperture is significantly larger than the other two, and so we neglect them in the reported uncertainty on the ε limit.

We follow the convention of similar experiments where we fix the value of ρ_{DM} and solve for an ε limit given this value. Therefore, we treat $\rho_{\text{DM}} = 0.45 \text{ GeV}/\text{cm}^3$ as a known constant with no uncertainty.

5.2 Exclusion Limit¹

This section incorporates the exclusion limit on output-referred power (Fig. 4.8) and Eq. 4.10 to report a 95%, frequency-dependent, exclusion limit on the kinetic mixing strength ε of

¹Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/Computelimit>

the massive dark photon. The Lagrangian was introduced in Sec. 1.4.4, and varies from the Standard Model (SM) Lagrangian as

$$\mathcal{L}_K = \mathcal{L}_{\text{SM}} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\varepsilon}{2}F_{\mu\nu}F'^{\mu\nu} - \frac{m'^2}{2}A'_\mu A'^\mu - e' J'^\mu A'_\mu, \quad (5.1)$$

where $A_\mu^{(\prime)}$ are the SM (dark) photon fields, $F_{\mu\nu}^{(\prime)}$ are the SM (dark) photon field strengths and m' is the dark photon mass, e' is the dark coupling and J'_μ is the dark charge-current. The K subscript indicates all terms are written in the kinetically mixed basis (see Sec. 1.4.4).

This limit is presented as a function of m' in Fig. 5.1. In addition, the limit is shown in context by plotting it with other similar experiments in Fig. 5.2. Here, limits are projected for planned runs that will take place in the next $\sim$ year. They are described in Ch. 6.

These limits have been published in our 2024 paper [67]. Our previous 2021 limits in the same mass range [127] are included in the particle data group's (PDG) 2024 update under Hidden Photons: Kinetic Mixing Parameter Limits [175]. Our 2024 limits will be included in the 2025 PRD review.

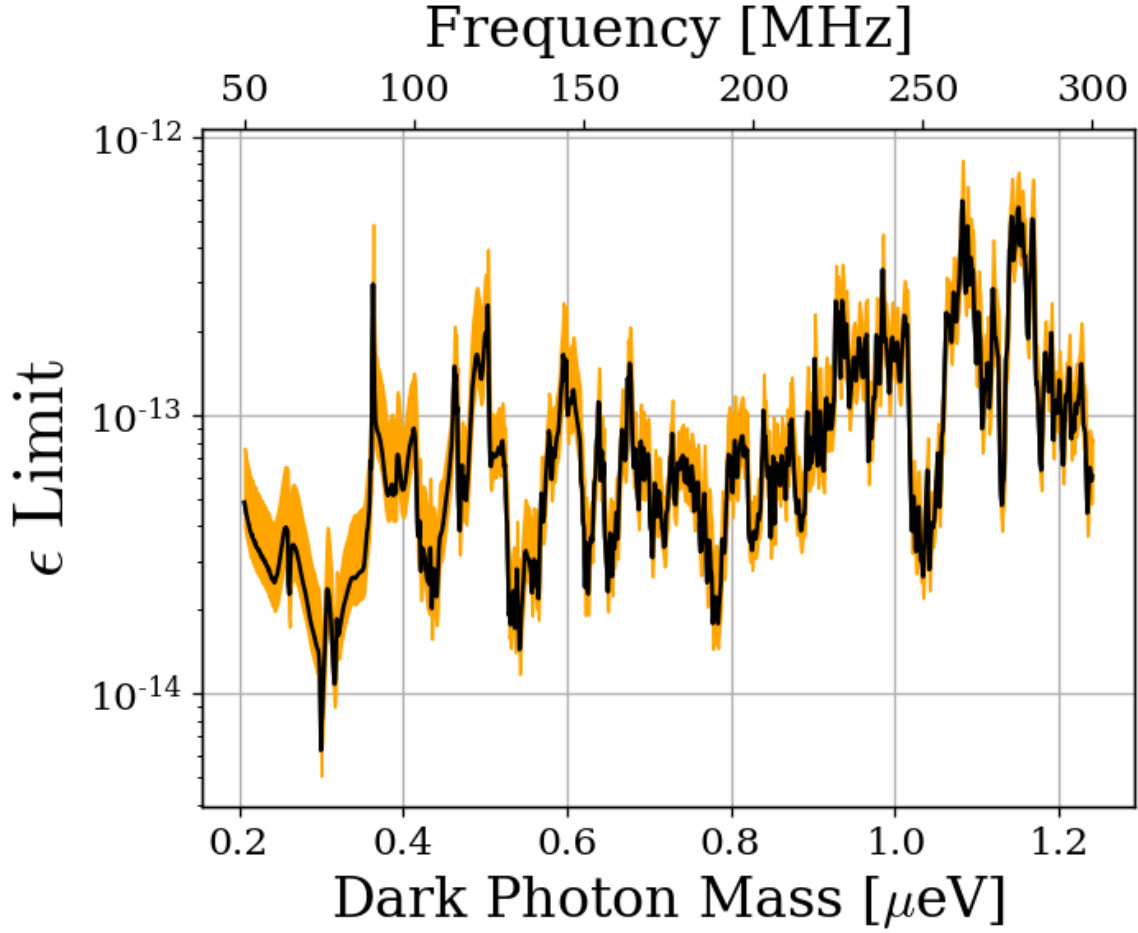


Figure 5.1: Zoom of 95 % exclusion limit on ϵ from Dark E-field Radio Run 1A. The uncertainty is shown in the orange-shaded region. This is based on a local dark matter density of $\rho_{\text{DM}} = 0.45 \text{ GeV}/\text{cm}^3$. The error estimate does not take the comparatively small gain and amplifier noise temperature errors into account.

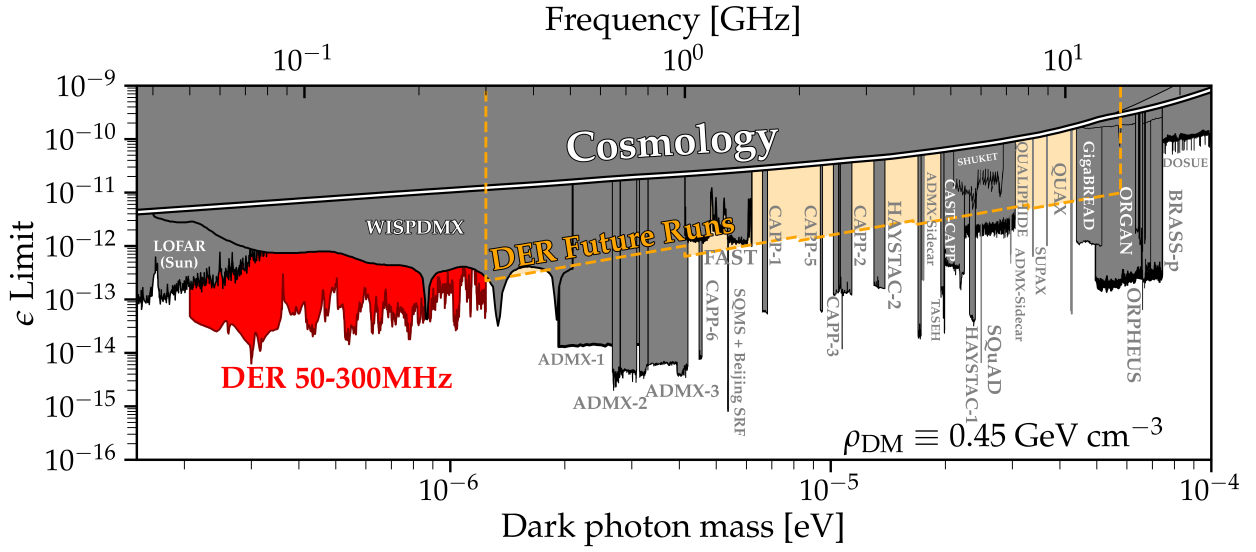


Figure 5.2: Dark photon limits of various experiments circa August 2024, with this work shown in red. The ragged lower bound is due to the complex structure of the resonant modes of the shielded room. Plot adapted by Ben Godfrey from [105] using code found at [96] and includes limit projections of various axion experiments. Astrophysical limits such as CMB interactions with the dark photon are in the region labeled *Cosmology*. Planned wideband extensions of our experiment searching from 0.3-14 GHz in similar room temperature RF enclosures are indicated (*orange*). These planned runs are summarized in Table 6.1.

Chapter 6

Beyond 300 MHz

To infinity and beyond!

Buzz Lightyear

As discussed in previous chapters, there is a wide mass/ ε parameter space available for exploration. This chapter details how the Dark E-Field Radio Experiment (DER) is poised to probe additional parameter space.

Since the parameter space consists of two variables, namely mass (aka frequency ν) and kinetic mixing ε , the “area” of this space must be considered. The advantage of this experiment is not its absolute sensitivity; many experiments are orders of magnitude more sensitive to kinetic coupling due to extremely high- Q superconducting cavities [120, 135, 176] or lumped-element circuits [177, 178]¹. Rather, the advantage lies in the ability to probe

¹Note that these are axion experiments. The conversion of axion limits to dark photon limits is discussed in Sec. 6.4.

large frequency spans efficiently. The aforementioned high- Q experiments are only capable of searching a single, narrow frequency span at a given time and must slowly step through the total required frequency span, waiting for a period of time $\approx Q/\nu$ at each step for the system to ring up. Limitations on tuning require swapping out hardware throughout the run, slowing down the process. See, for example, DMRadio m³ [178].

The four completed or planned phases of DER are outlined in Table 6.1. These phases are projected to probe the orange-shaded region shown in Fig. 5.2. This region spans from 0.3-14 GHz, though Run 2B may push slightly higher (16 GHz without much trouble).

Section 6.1 covers progress made toward Run 1B. Section 6.2 covers the technique of frequency mixing, which extends the frequency range of the existing 1 GHz spectrum analyzer (Sec. 2.5.7) up to ~ 14 GHz.

Beyond this, the experiment becomes more difficult. Section 6.3 describes work required to extend the DER experiment into new regions of phase space, namely beyond ~ 14 GHz. This section is less detailed than the previous sections, providing more general guidance as opposed to specific designs and testing procedures.

Finally, Sec. 6.4 introduces a few considerations for axion experiments that would improve their sensitivity to dark photons. Axion experiments represent a significant investment of resources and are extremely sensitive, so this avenue is an extremely efficient one to explore the dark photon parameter space.

Run Name	Frequency Range	Antenna	LNA	Mixer	Status	Statistical Uniformity	Comments
1A	50-300 MHz	Com-Power Bicon, 50-300 MHz	Pasternack PE15A-1012	N/A	Complete, published	N/A. Below lowest usable frequency. Move antenna and simulate aperture	N/A
1B	300-1000 MHz	RFSpace UWB-5 Vivaldi, 0.3-6 GHz	Pasternack PE15A-1012	N/A	Preliminary run complete by moving antenna. 21 candidates detected.	Mode stirrer	50% dead time to allow for 2 GHz sampling rate on Teledyne
2A	1-8 GHz	Small Vivaldi, model TBD	Low noise factory LNC0.3_14B	Miteq TB0440LW1	Waiting on mode stirrer and mixer system commissioning	Mode stirrer	N/A
2B	8-14 GHz (possibly ending at 16 GHz)	TBD	Low noise factory LNC0.3_14B	Miteq TB0440LW1	Waiting on mode stirrer and mixer system commissioning	Mode stirrer	N/A

Table 6.1: Overview of planned future runs including hardware (antenna, low noise amplifier (LNA), mixer), status, how/if statistical uniformity will be ensured, and any comments

6.1 A First Attempt at Run 1B²

A preliminary attempt at Run 1B (300-1000 MHz) has been completed. This attempt had a few issues, which are discussed here and will be addressed in the future, but are beyond the scope of this thesis. In this section, upgrades and data acquisition are discussed. The projected ε limit from this run (along with Runs 2A and 2B) is shown in Yellow in Fig. 5.2. However, this projection is independent of the data shown here.

6.1.1 Run 1B upgrades

There are four main upgrades to consider before beginning Run 1B: software upgrades of the GPU-based spectrum analyzer, modifying the analog signal conditioning chain, modifying the current antenna stand to accept a Vivaldi antenna, and building a mode stirrer to ensure statistical uniformity. The remainder of this subsection will detail these upgrades.

6.1.1.1 Run 1B spectrum analyzer

The GPU-based spectrum analyzer (Sec. 2.5.7) is capable of sampling at up to 2.5 GHz, easily allowing for a run up to 1 GHz. However, the PCIe-based ADC has a maximum data transfer rate of 7 GB/s. To accommodate two channels (main experiment and veto), the system was configured with 50% dead time to allow for data to transfer. The limit on ε scales with the quarter root of time, so this is only a 16% reduction in sensitivity (for a given acquisition length). This is purely a software upgrade and has been completed.

²Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1B/analysis>

6.1.1.2 Run 1B signal conditioning

The signal conditioning chain (defined in Sec. 2.5.4) of Run 1B is very similar to that of Run 1A, Fig. 2.42. The amplifiers are the same as defined in Tables 2.3 and 2.4. The band-pass filter was changed from Run 1A to allow the required 300-1000 MHz span of Run 1B. The Run 1B band-pass filter is constructed from a Mini-Circuits ZX75LP-1050-S+, 1090 MHz low-pass filter³ and a Mini-Circuits ZX75HP-250-S+, 250 MHz high pass filter. An additional 1 dB attenuator was added to account for the larger integrated noise power of the wider frequency span.

6.1.1.3 Vivaldi antenna stand

The non-conducting Com-Power bicon antenna stand was modified⁴ to accommodate the RFSpace UWB-5 Vivaldi antenna. This was completed and pictures of the modifications are shown in Figs. 6.1 and 6.2.

³The attenuation of this filter is greater than 50 dB above 1400 MHz. This is the relevant aliasing frequency for a 1 GHz analysis span and 2.4 GHz sample rate. Note there are higher frequencies that will alias, but they are reduced by the filter by more than 50 dB so they are neglected.

⁴Thanks to Dave Hemmer in the UC Davis machine shop for his assistance



Figure 6.1: Picture of modified Com-Power antenna stand holding Vivaldi antenna for Run 1B. This antenna stand was designed for electromagnetic compliance applications and is non-conducting with the exception of a few pieces of metallic hardware.

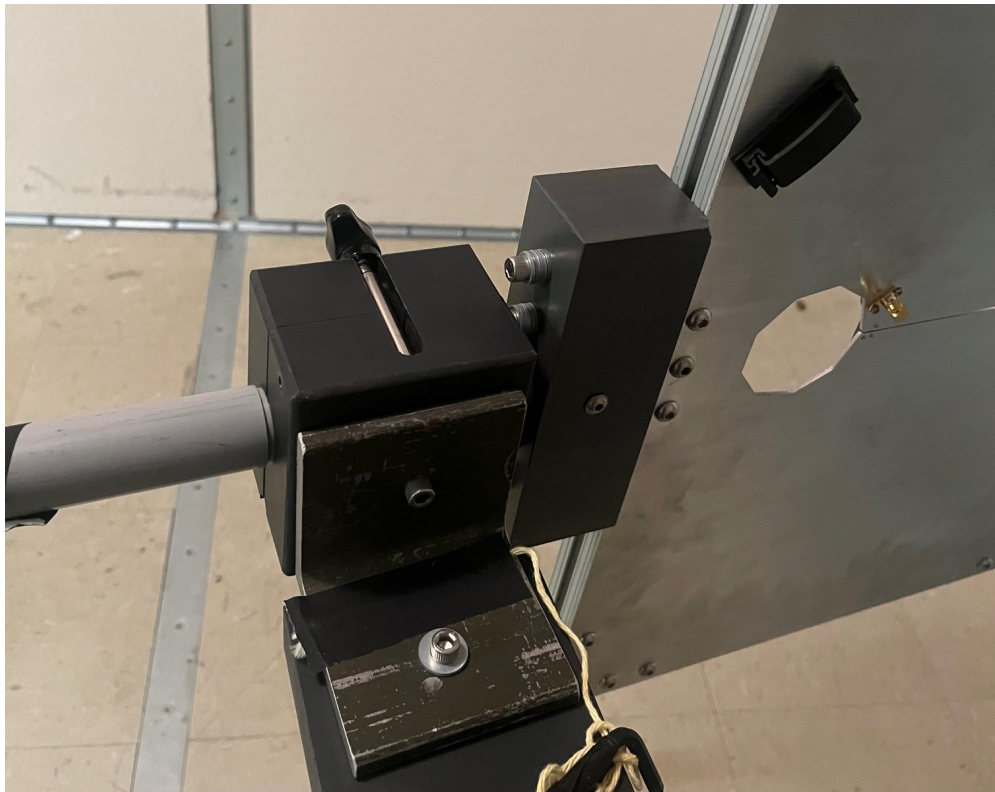


Figure 6.2: Closeup of modified Com-Power antenna stand holding the Vivaldi antenna for Run 1B

6.1.1.4 Mode stirrer and statistical uniformity

Run 1A relied on moving the antenna to improve the statistical uniformity (Sec. 2.4) but still required simulation to calibrate it (Sec. 4.3.2). Run 1B, however, benefits from its frequency range beginning above the lowest usable frequency of the shielded room (~ 200 MHz, see Sec. 2.34). To take advantage of this, a proper mode stirrer (Sec. 2.4) must be employed. A lack of mode stirrer is the main limitation of this attempt at Run 1B. At the time of writing, work is ongoing to construct a mode stirrer. The hardware is mostly assembled, but it lacks control software. A photo of the current state is shown in Fig. 6.3. The base is constructed from extruded aluminum, and the blades are thick cardboard covered in industrial aluminum foil that is about $100\text{ }\mu\text{m}$ thick (> 20 skin-depths). The design chosen is known as a Z-Fold mode stirrer. Its design replicates the mode stirrer in the large reverberation chamber at the Universität Magdeburg and is described in [179]⁵.

There are a few interesting design choices that improve performance. The primary factor that influences performance is the occupied volume. A larger stirrer will offer better performance but reduce the working volume of the reverberation chamber. The stirrer implemented here occupies 10.5% of the chamber volume, significantly more than the 2.0% occupied by the stirrer at Magdeburg⁶, which has demonstrated adequate performance at low frequencies. In addition to the volume, there are two geometric considerations. First, the top and bottom blades are not parallel, reducing correlations of the fields reflected off of

⁵Thank you to Mathias Magdowski at the Universität Magdeburg for providing additional information about the design and construction.

⁶These numbers are approximate and assume both stirrers are simple cylinders that span from floor to ceiling with a radius given by half of the plate width. While they are both simplified estimates, they are calculated in the same way, so it is a fair comparison

these blades. Second, the angles between adjacent blades are not 90° (one is $\sim 100^\circ$ and the other is $\sim 80^\circ$). A 90° corner will reflect an incident wave in the same direction it originated from. Choosing these angles improves the statistical isotropy of the fields.



Figure 6.3: Current state of mode stirrer construction. It was not in place during the Run 1B data acquisition described in the following section. The physical structure is currently complete, but it requires control software before it can be utilized in Run 1B and beyond. It is shown here in two positions of its 360° rotation.

6.1.2 Run 1B: data run and analysis

The first attempt at Run 1B, (Run 1B.1) was taken between September 13 and 22, 2023.

Run details are shown in Tab. 6.2.

Specification	Value
Analysis span	300-1000 MHz
-3 dB span of analog filters	250-1090 MHz
Antenna	RFSpace UWB-5 Vivaldi antenna
RTSA	Teledyne GPU system (Sec. 2.5.7)
Frequency resolution ($\Delta\nu_{\text{RF}}$)	286 Hz
Length of record	2^{23} samples
Sample rate	2.4 GHz
Window type	None (rectangular)
Acquisition time per spectrum	3.5 ms
Run start time	2023-09-14 00:32:41.33 UTC
Total time of run	8 days, 23.52 hours
Efficiency (Time spent scanning antenna/total time)	44.90%

Table 6.2: Run 1B details. Many specifications are related and can be computed from each other but are listed for reference. The efficiency differs from that calculated in Fig. 3.16 mostly because of the required dead time to allow for data transfer from the ADC to the GPU. There is a further, smaller, degradation to the efficiency due to the switching to a terminator and brief daily pauses to move the antenna.

At the time of writing, Run 1B.1 has been the only attempt at this run, so it will be referred to simply as Run 1B. The raw data are shown in Fig. 6.4. The normalized standard deviation σ_{norm} (defined in Sec. 4.2.2 from the Dicke radiometer equation) is predicted to be 0.9885×10^{-3} , and is calculated from the data to be 1.001×10^{-3} . The sample rate for this run was set at 2.4 GHz. This is slightly higher than the required 2 GHz to prevent aliasing (described in Sec. 2.5.4.3).

Processing these data through the detection algorithm (Fig. 4.3) of Run 1A, yields the normalized, signal-matched-filtered spectrum and 5% significance threshold which are shown in Fig. 6.5.

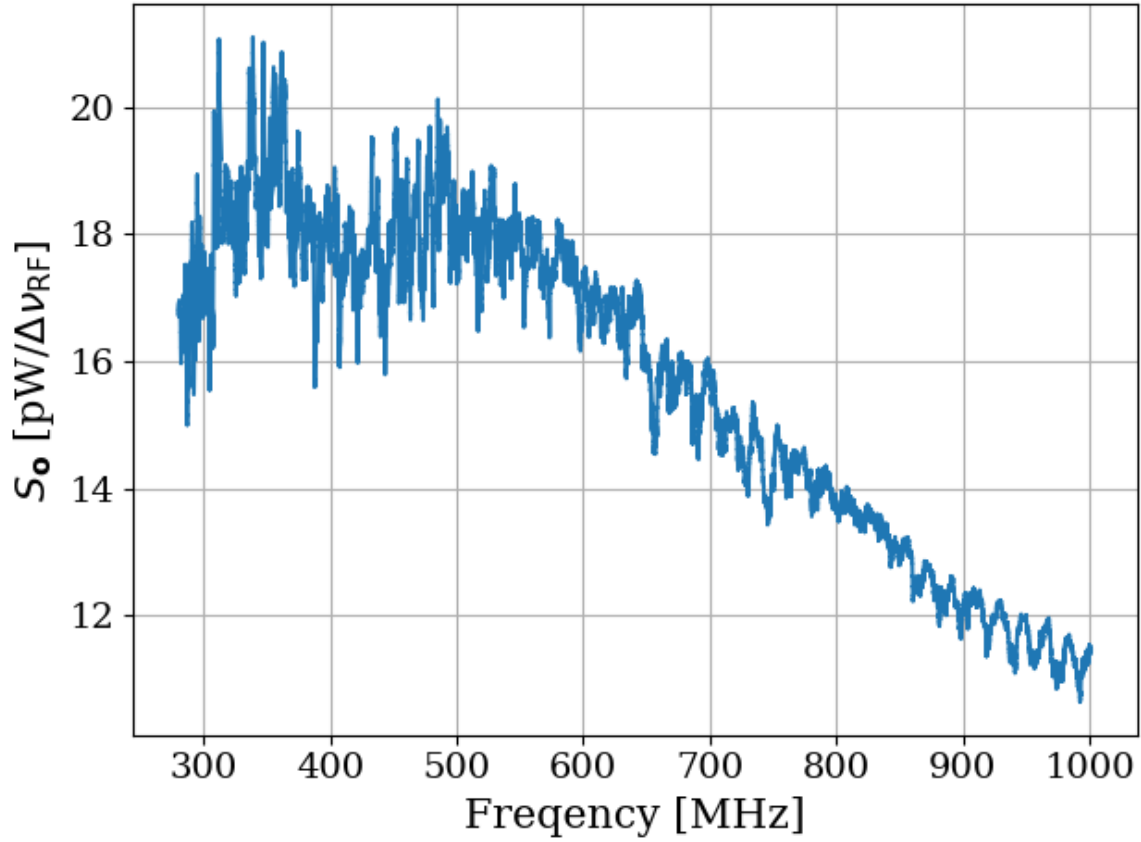


Figure 6.4: Run 1B averaged, output-referred antenna power spectrum S_o . Data were taken over a 9-day period at 9 antenna positions. The narrow variations are mainly due to the effective temperature difference between the room and low noise amplifier (LNA) (Sec. 2.3), though there is a contribution due to amplifier gain and noise temperature variations (Sec. 4.1.1). The variations seen here are not noise; their shape is repeatable for a given antenna position. The noise on this background is not visible at this level of zoom.

Comparison of the spectrum and threshold pictured in Fig. 6.5 reveal 21 candidates⁷.

Furthermore, 13 of these are clustered between 758 and 768 MHz. This frequency span is reserved for the First Responder Network Authority (FirstNet) and is likely in use locally

⁷There are a total of 60 bins over the threshold, but many bins are adjacent. Combining adjacent bins results in 21 independent candidates

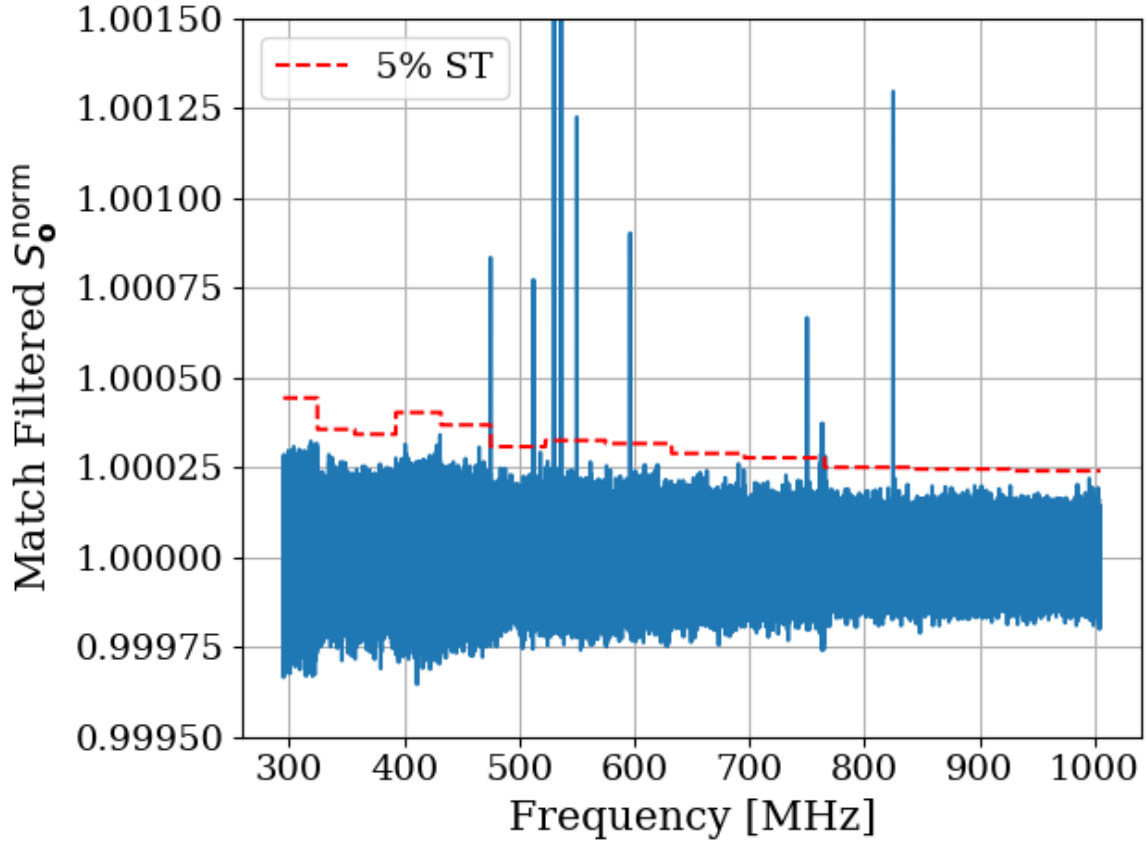


Figure 6.5: Run 1B S_o^{norm} after it has been passed through a matched filter. The template varies in width throughout the frequency span resulting in 13 subspans (a new subspan for every 10% increase in frequency.), each with a constant 5% significance threshold ST (*dashed red*), see Secs. 2.2.1 and 4.2.3. This plot is to Run 1B as Fig. 4.6 was to Run 1A. In contrast to Fig. 4.6, however, the narrow signals are not injected. They are almost certainly RFI with enough power to overcome the isolation of the shielded room (Fig. 3.13). These signals are visible in the veto spectrum, but further investigation is required to conclusively exclude them as dark photons. The vertical scale has been set such that the tops of two of the signals are cut off.

by emergency services.

All of these candidates are almost certainly RFI with enough energy (i.e. signal power integrated over the 9-day run) to overcome the isolation of the shielded room (Fig. 3.13).

These signals are visible in the veto spectrum (Fig. 6.6), but further investigation is required

to conclusively exclude them as dark photons.

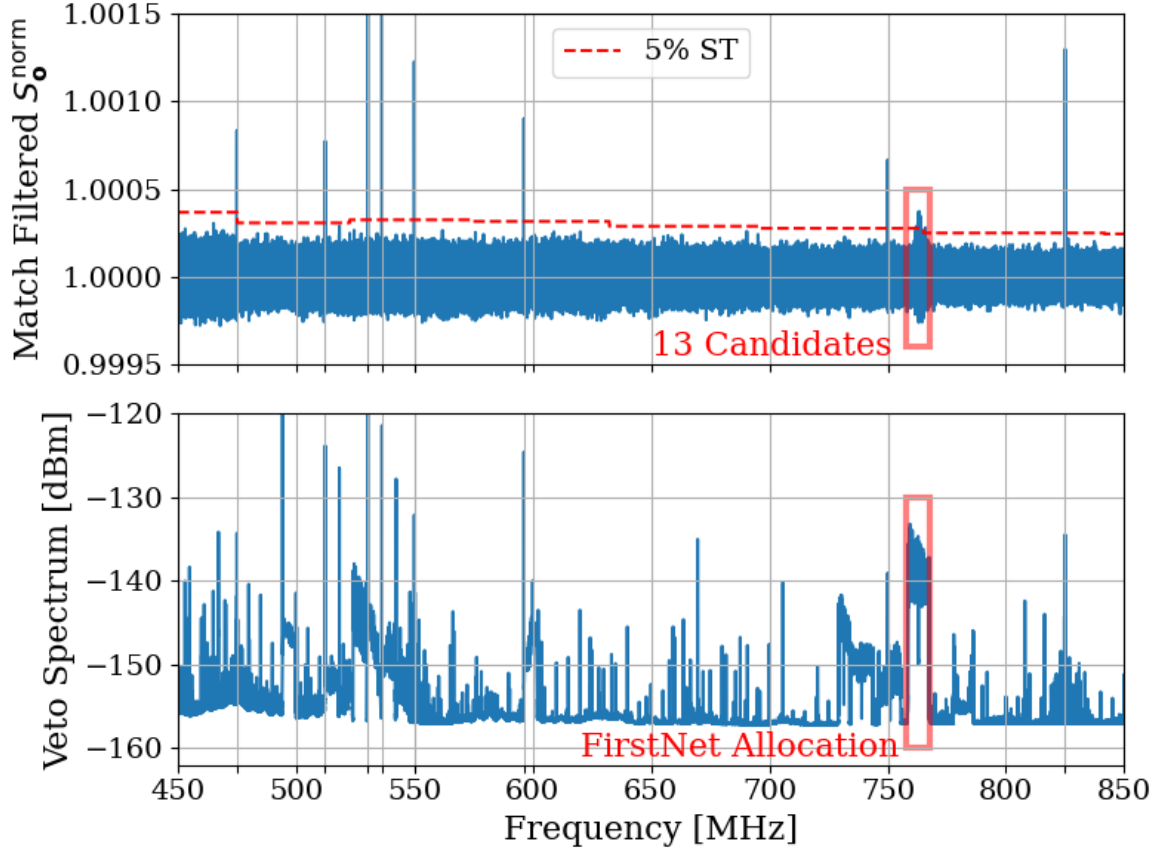


Figure 6.6: 9-day averaged veto spectrum (bottom) and Run 1B S_0^{norm} after it has been passed through a matched filter (top). The 5% significance threshold ST (Secs. 2.2.1 and 4.2.3) is shown as a dashed red line. There are 21 independent candidates, however 13 of them are found in the FirstNet frequency span (758-768 MHz) which is discussed in the text of this subsection. This frequency span is shown in both the veto and experimental spectra with a red box. The remaining 8 candidates have ticks added to the x-axis to guide the eye between candidates in the upper panel and RFI in the lower panel. The vertical scale has been set such that the tops of two of the signals are cut off.

Since simulations of this run are extremely difficult to calculate (wavelength at 1GHz $\ll$ room dimensions), it will have to be revisited when statistical uniformity can be established, i.e. with a mode stirrer. For this reason, further time domain analysis of these candidates is

not explored here.

6.2 Mixer System: Run 2

Run 1 relied on an ADC with a sample rate up to 2.5 GHz, allowing for direct RF sampling of Run 1A/B. This system is a flexible platform, allowing for future upgrades to extend its maximum frequency through the use of a mixer system which is described in this chapter.

6.2.1 Frequency mixing

Frequency mixing is a technique in RF and microwave systems used to shift the frequency of a signal to another frequency range which is more useful for a given task. For example, electronics are simpler to design at kHz frequencies, but antennas operate at MHz. After processing a kHz signal for transmission, it can be mixed up (“upconverted”) to MHz for transmission over the air. Once it is received, it can be mixed back down (“downconverted”) to kHz. Mixer engineering is a rather complex topic, and many details will be omitted here. The reader is encouraged to reference the Marki Microwave mixer primer [180] for a tutorial on mixers and Ch. 13 of Pozar’s *Microwave Engineering* [181] for a more thorough treatment.

In a mixer system, an input signal (RF) is combined with another signal, typically from a local oscillator (LO), such that the two frequencies multiply or “mix”, producing an intermediate frequency (IF). In other words, the RF signal of amplitude A_{RF} , $A_{\text{RF}} \cos(2\pi f_{\text{RF}} t)$ is

multiplied by a local oscillator signal $A_{\text{LO}} \cos(2\pi f_{\text{LO}} t)$ ⁸. Using the trigonometric identity

$$\cos(A) \cos(B) = \frac{1}{2} (\cos(A + B) + \cos(A - B)), \quad (6.1)$$

this multiplication produces two new components: one at the sum frequency $f_{\text{RF}} + f_{\text{LO}}$ and one at the difference frequency $|f_{\text{RF}} - f_{\text{LO}}|$. Generally, either the sum or the difference frequency will be desired, while the other is an artifact that should be filtered. This undesired artifact is known as the image.

A mixer is a 3-port device that has two inputs and one output. The LO port is almost always an input, while the RF and IF ports can be used as either an input or an output (as long as they are opposite with respect to being an input/output)⁹. Such a system can be used for upconversion/downconversion, where the RF is shifted to a higher/lower frequency, as shown in Fig. 6.7. The situation when the signal above (below) the LO is desired, and the signal below (above) the LO is the image is known as low-(high-)side injection downconversion. In other words,

$$\text{Low-Side Injection: } f_{\text{LO}} < f_{\text{RF}} \text{ (desired signal is above the LO)} \quad (6.2)$$

$$\text{High-Side Injection: } f_{\text{LO}} > f_{\text{RF}} \text{ (desired signal is below the LO).}$$

The remainder of this subsection will focus on downconversion, but both downconversion and upconversion are important in the system, which is presented later in this section.

⁸The multiplication of CW signals outlined here is a simplification. In reality, the LO should have higher harmonics and approximate a square wave to overcome the transition region of the voltage-current curve of a real diode. See the Marki Microwave mixer primer for more details [180].

⁹Again, this is simplified. All three ports actually behave as both a load and a source. See the first footnote of the Marki Microwave mixer primer [180].

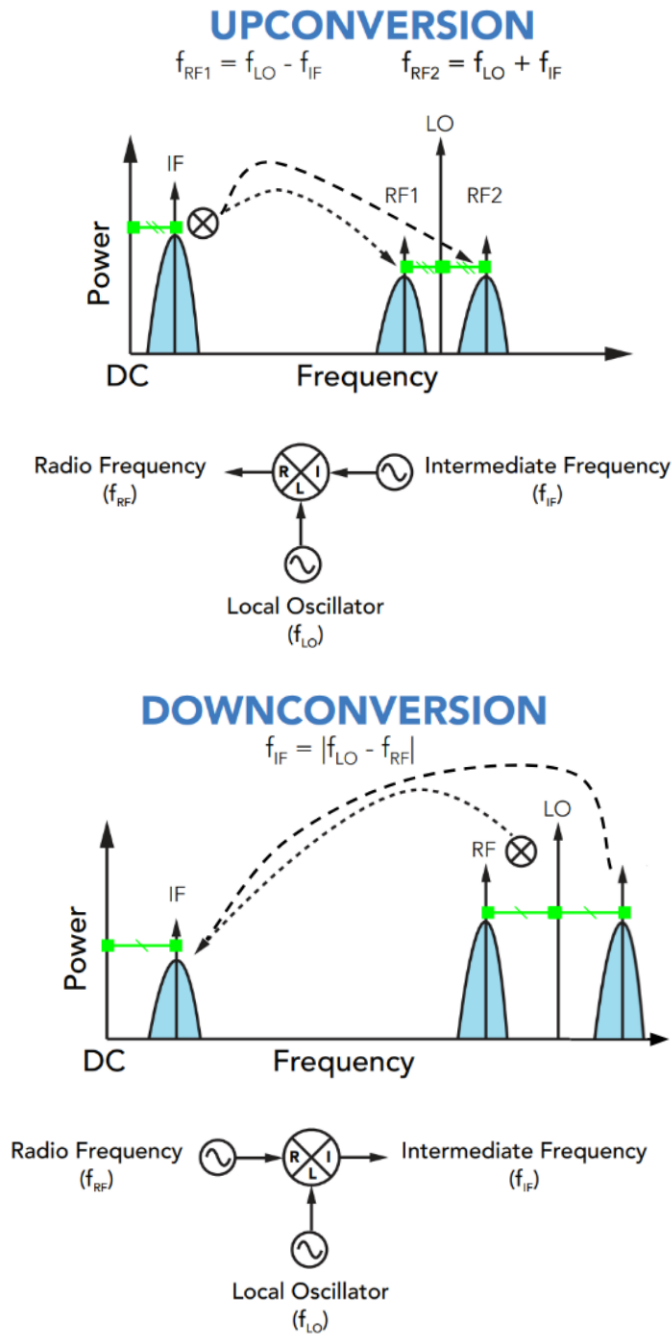


Figure 6.7: Overview of basic mixer operation in both upconversion and downconversion mode. The green lines with hash marks represent the IF frequency. Figure modified from Marki Microwave's mixer primer [180].

6.2.1.1 The superheterodyne receiver

The presence of both sum and difference frequencies in Eq. 6.1 creates a challenge when employing mixers. A down-converting mixer will present both the sum-frequency and difference-frequency at its IF port (see Eq. 6.1). Usually, one of these signals will be desirable and will not. The latter is known as an image. Two power spectra illustrating this situation are shown in Fig. 6.8.

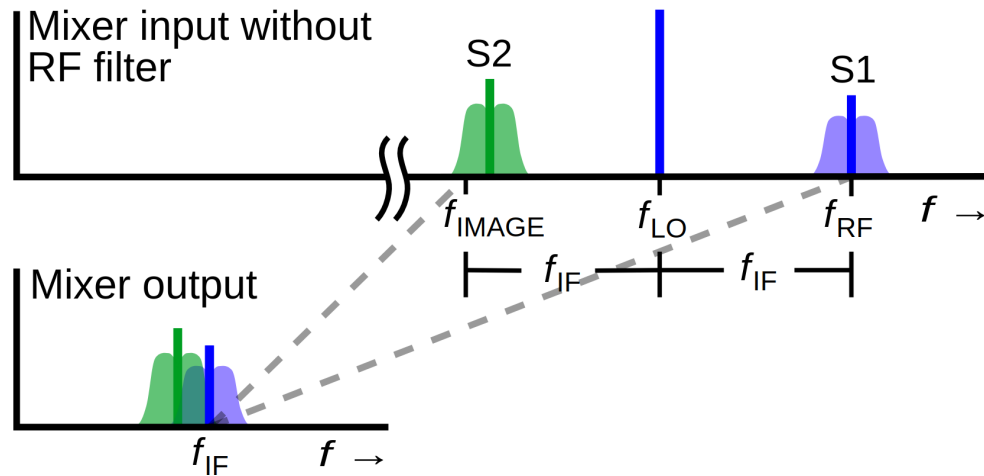


Figure 6.8: Illustration of the problem of images in frequency mixing. Both signals, S1 and S2, are overlapping at the mixer's output as predicted by Eq. 6.1. The image must be filtered *before* mixing to avoid this problem. Figure from Wikipedia [182, 183]

The solution to this image problem is to filter the image out of the RF before mixing. Furthermore, in a realistic mixer, the LO will bleed through to the IF port and will also need to be filtered. These points motivate including both an RF and IF filter in the design of a mixer-based receiver system (sometimes known as a superheterodyne receiver). The schematic of this type of system is shown in Fig. 6.9. Figure 6.10 shows the spectra after filtering the RF and IF; the signal S1 is available for further processing at the desired

frequency.

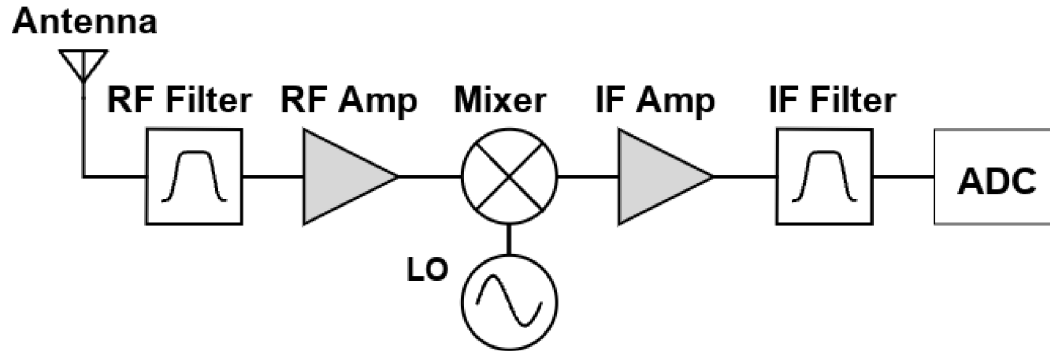


Figure 6.9: Schematic of a basic superheterodyne receiver system. The introduction of the RF filter before the mixer removes the image. Power spectra at various points are shown in Fig. 6.10. This figure is based on one from Wikipedia [182]

The RF filter required for the superheterodyne receiver may be quite difficult to construct if the IF frequency is low since the RF and image will be very close together. To overcome this problem, superheterodyne receivers frequently utilize two mixers, first performing up-conversion so that the high frequency of the first IF ensures image/RF separation. After filtering, the second mixer down-converts to the final IF frequency. This setup is known as a dual conversion superheterodyne receiver (DCSHR) have frequently been employed since the 1950s. They will be discussed further in Sec. 6.2.3.

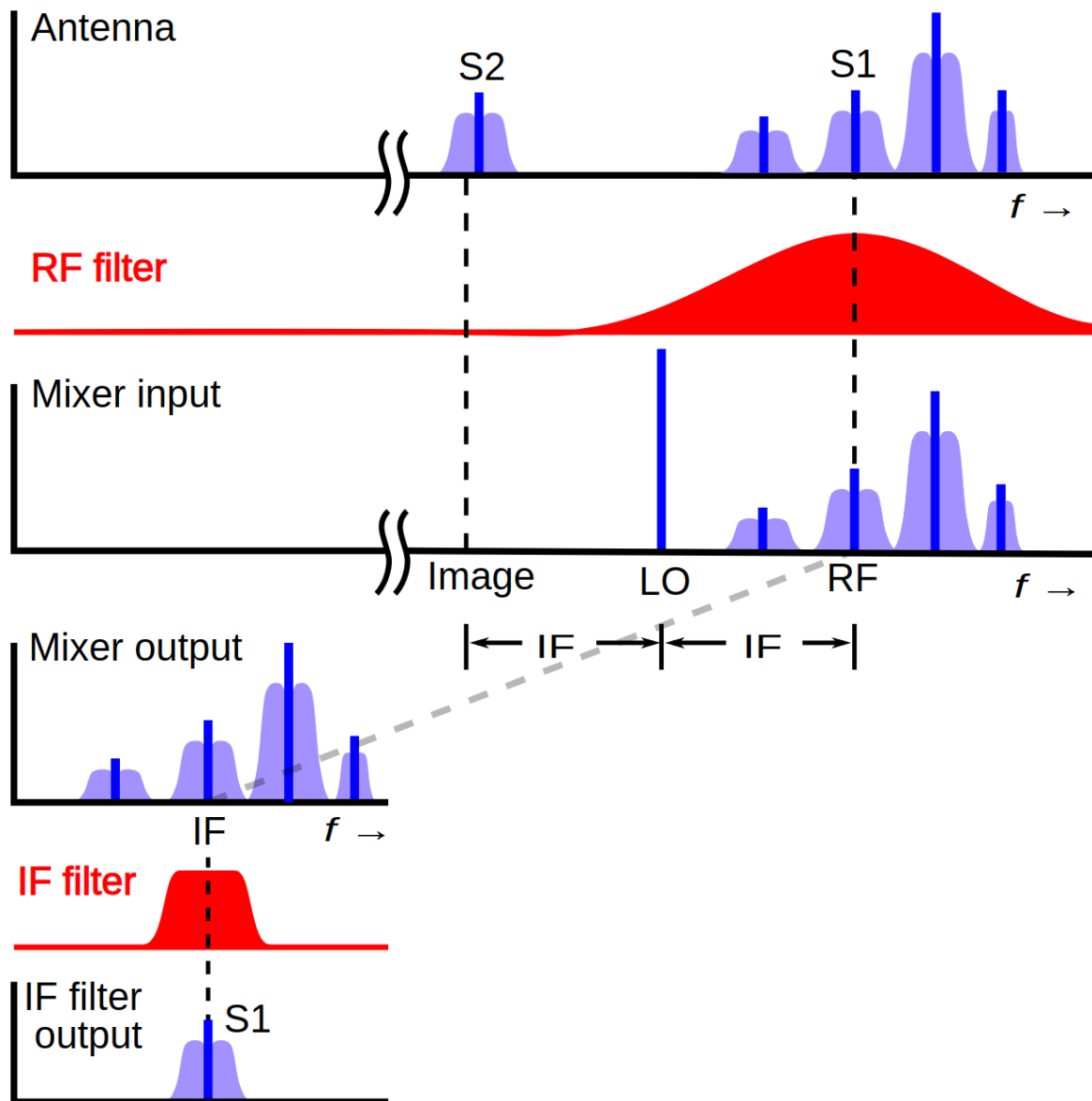


Figure 6.10: Power spectra along various points of a superheterodyne receiver, shown schematically in Fig. 6.9. The introduction of the RF filter before the mixer removes the image. The IF filter separates interfering signals that were near $S1$. This is an example of low-side injection down conversion, as described in the text of this subsection. Image from Wikipedia [182, 184]

6.2.2 Design requirements

The maximum sampling rate of the ADC (2.5 GHz) sets a limit as to how high of frequencies it can directly sample. Introducing a mixer system between the front end of the experiment (antenna/low noise amplifier) and the DAQ (ADC/GPU/PC) allows for previously inaccessible frequencies to be mixed down into the first Nyquist zone (aka, the *baseband*) of the ADC, $\lesssim 1$ GHz. The block diagram of such a system is shown in Fig. 6.11.

The ability to control the mixer system via software greatly boosts efficiency since it means the experiment can be set up and stepped through a large frequency span (several GHz) without human intervention. The span of the wideband RF spectrum will be limited by the hardware of the front end, e.g. antenna and low noise amplifier (LNA). The bandwidth of the baseband (1.25 GHz) sets the maximum span that may be scanned at each step, though to reduce the data transfer rate, a smaller span closer to 500 MHz will likely be used for Run 2.

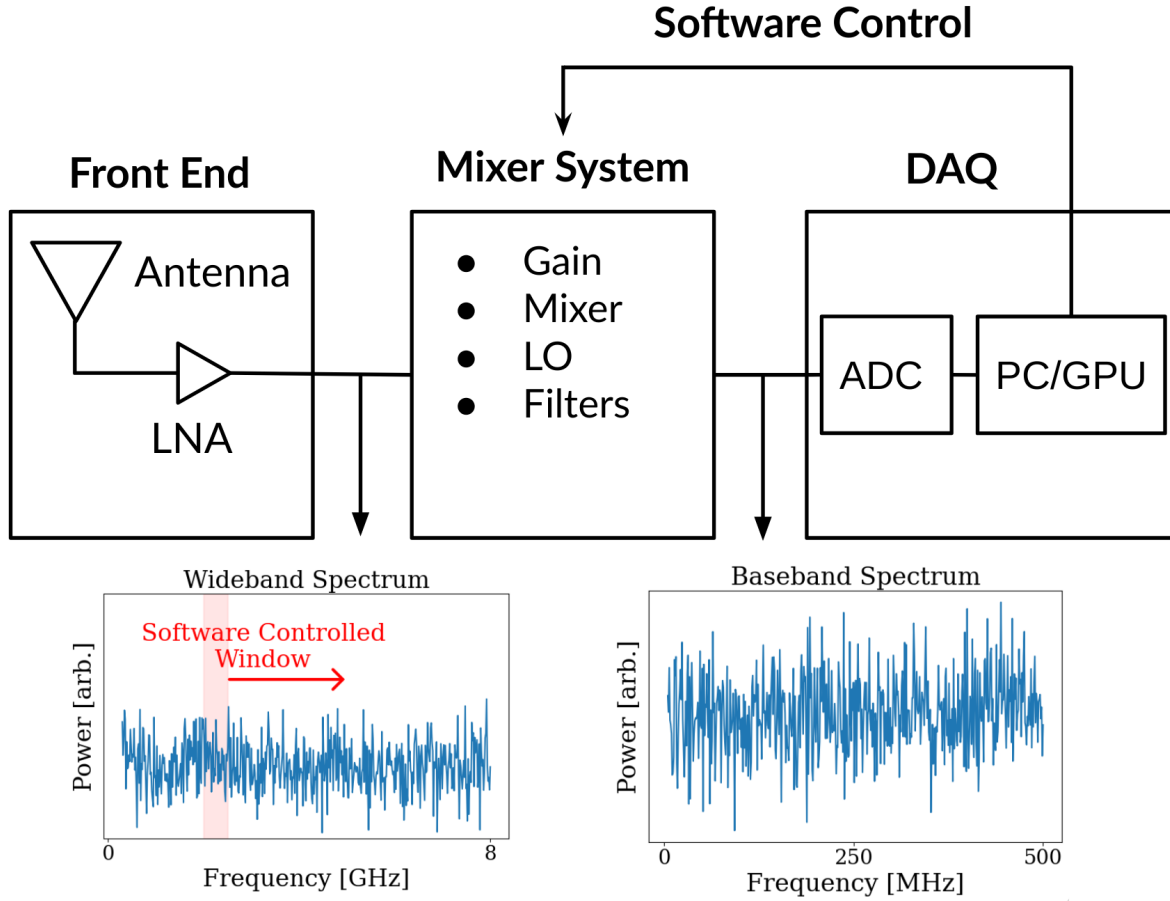


Figure 6.11: Block diagram of the mixer system to be used in Run 2. The PC provides control of the mixer system to sweep the frequency span (shown as a red box on the wideband spectrum) that is mixed down to the baseband and digitized. Frequencies are based on Run 2A, but are nominal and shown only for illustration. The two plots in this figure are only for illustration, but are analogous to the first and sixth subplots respectively in Figs. 6.14 and 6.15.

6.2.3 The Dual-conversion superheterodyne receiver ¹⁰

Allowing wideband (~ 8 GHz) RF into the mixer means the IF will be a sum of two frequency bands (see Eq. 6.1 and Fig. 6.8). This can be avoided with by using a narrow RF filter.

¹⁰Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/mixerSystem/simFreqResponse/simFreqResponse.ipynb>

However, this filter would need to be changed for each ~ 500 MHz span. The requirement that the system be software-controlled is non-trivial in light of the image problem.

One simple solution is to use a bank of RF filters and a pair of coax RF switches. Assuming a 500 MHz baseband, a 1-8 GHz run would require 14 RF filters. This is impractical due to the high cost of high channel-count switches (two of which would be required) and the limited frequency span of these switches. A 16-channel switch is $\sim \$3,000$, and has a maximum frequency of 8 GHz, limiting the utility of this setup. Additionally, many of these filters would need to be custom-made, further adding to the cost.

An elegant solution¹¹ is the dual conversion superheterodyne receiver (DCSHR). In this configuration, the RF frequency band is mixed twice before being output as a final intermediate frequency to be digitized (IF2). The first mixer (M1) mixes the RF to a fixed IF frequency span (IF1), set by a fixed IF1 filter¹². This mixer is controlled by an LO (LO1) with a software-controllable frequency. LO1 is set such that the desired RF frequency span will be mixed to the pass band of the IF1 filter. After this, the rest of the system looks just like the superheterodyne receiver of Sec. 6.2.1.1. The fixed, band-limited IF1 is simply mixed to IF2 (also known as baseband). In this case, ~ 100 -600 MHz, set by the ADC. This second conversion step can be either low-side or high-side injection.

A schematic of a simple DCSHR is shown in Fig. 6.12. A cartoon illustrating the operation of the first mixer stage of a DCSHR is shown in Fig. 6.13. Simulations¹³ of the system

¹¹Thank you to Greg Wright of Nokia Bell Labs for suggesting this.

¹²This first mixing stage can be either an upconverter, a downconverter, or even a nonconverter. The only requirement is to mix RF such it lands in the span of the IF1 band-pass filter. A simple way to conceptualize the first mixer stage is that it simply moves where the IF1 filter's span lands on RF.

¹³Code for this section can be found at: <https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/mixerSystem/simFreqResponse/simFreqResponse.ipynb>

response at various points along the signal path are shown in Figs. 6.14 and 6.15. These simulations assume the filters specified for Run 2A. This system will be specified in more detail in Sec. 6.2.3.1 and in Tab. 6.3.

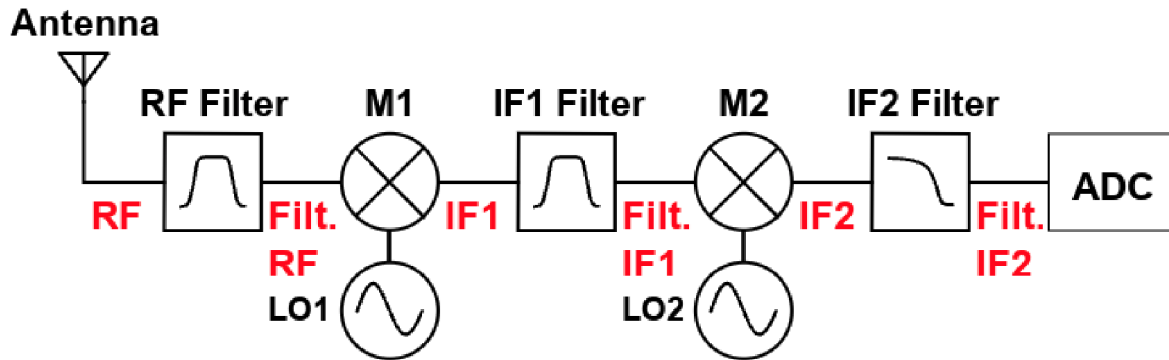


Figure 6.12: Schematic of a simple dual conversion superheterodyne receiver (DCSHR). Text in red indicates the location of each subplot in Figs. 6.14 and 6.15.

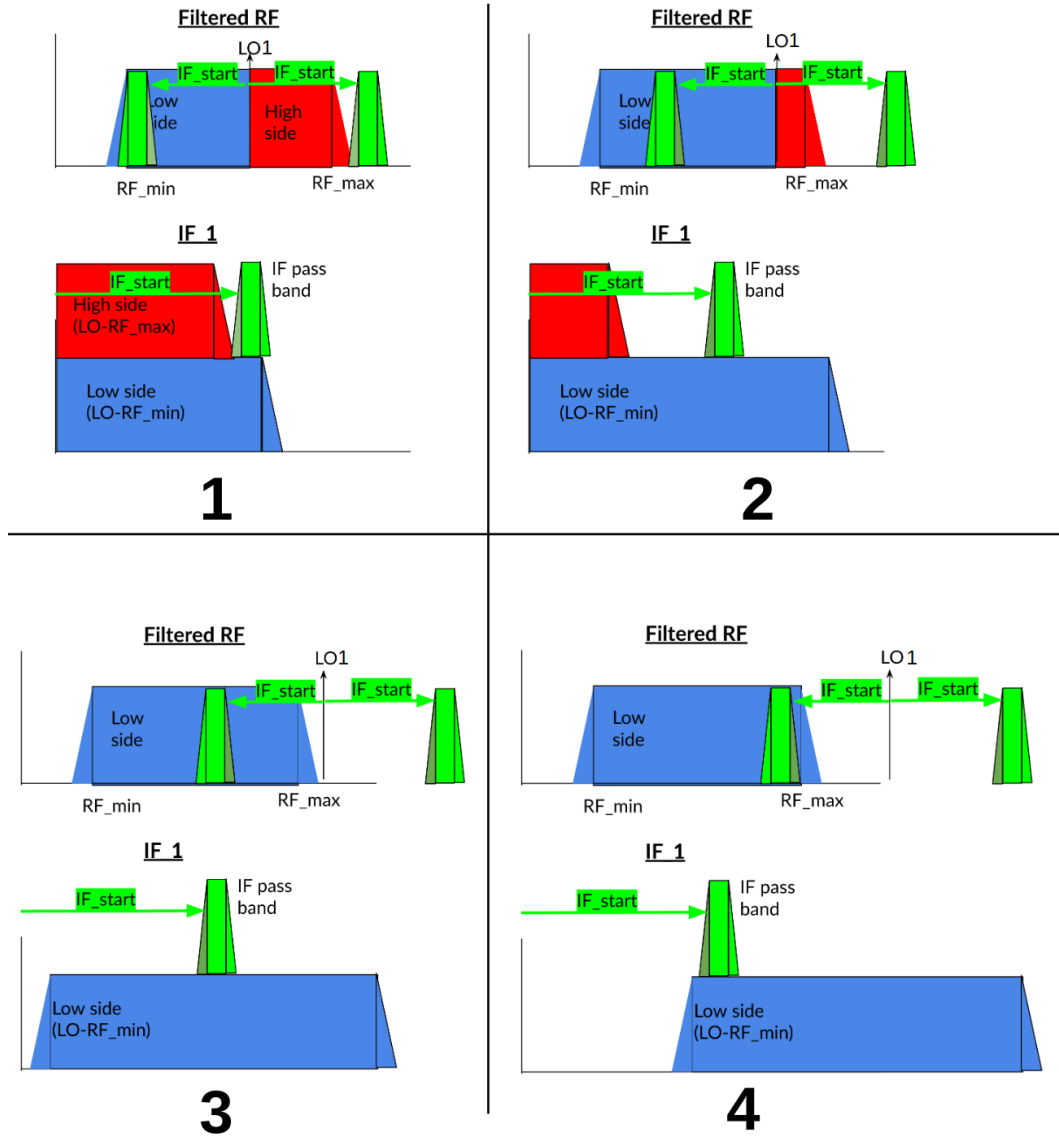


Figure 6.13: Cartoon illustrating the first mixer stage of dual conversion superheterodyne receiver system. Figures 6.14 and 6.15 are more detailed simulations than the cartoon shown here. The four panels represent four different frequency steps of LO1. Panel 1 corresponds to the first step of this LO1 (Fig. 6.14) and panel 3 to the last step (Fig. 6.15). Similar to Figs. 6.14 and 6.15, the low and high side RF are shown separately in the IF1 band (red and blue, respectively). Note the dark green on the low-frequency side of the IF filter response. This is not a physical effect and is shown only to illustrate how frequencies appear to mirror when using high-side injection.

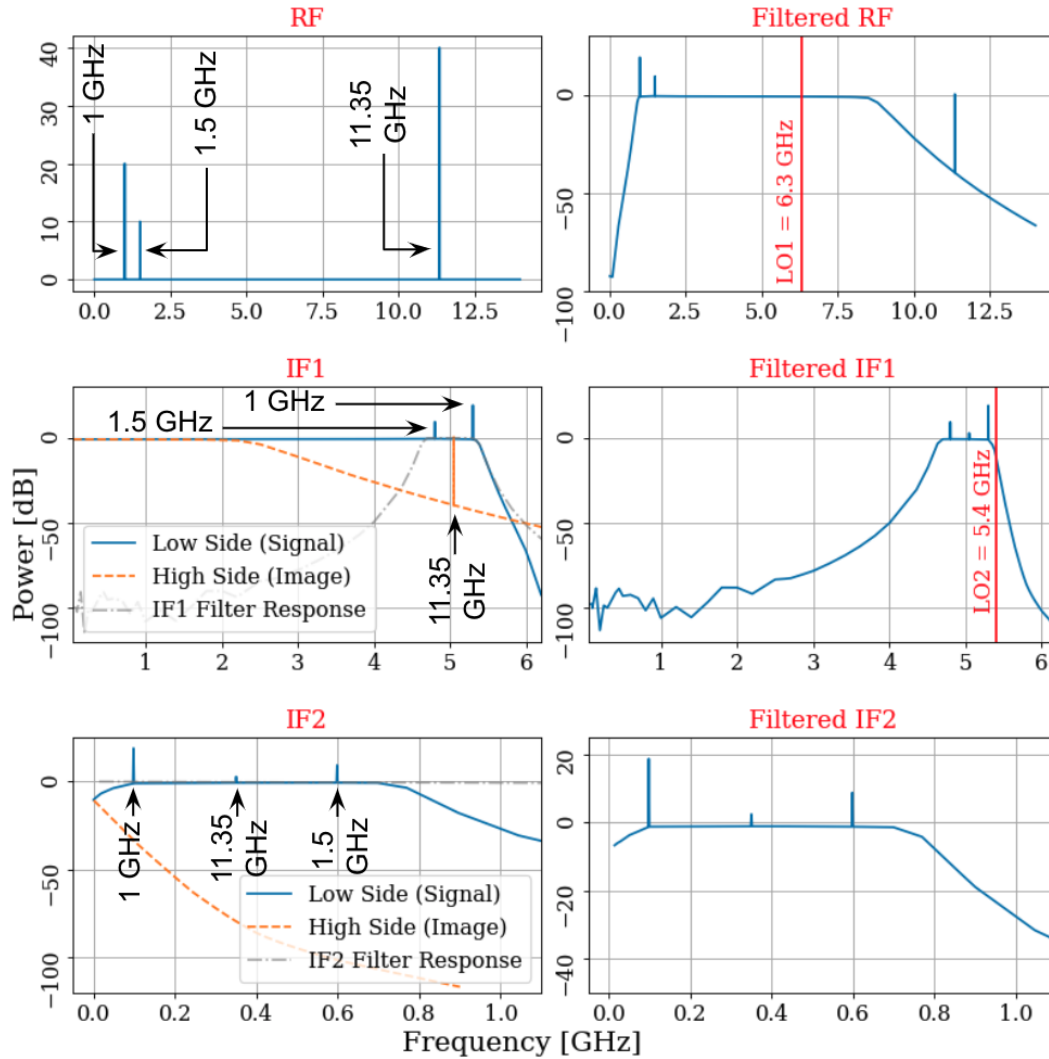


Figure 6.14: Simulated frequency response of dual conversion superheterodyne receiver (DCSHR) with signals around 1 GHz, the lowest frequencies of Run 2A. The input RF is the mean of broadband noise (truncated at 14 GHz), plus three coherent signals; two signals at 1 and 1.5 GHz simulate real signals, while one at 11.35 GHz simulates an image. All three of these signals will mix to the 4.7-5.3 GHz pass band of the IF1 filter in the Run 2A DCSHR. Each panel shows the spectrum at the points indicated in the red text in Fig. 6.12. LO1 is set to 6.3 GHz, the approximate lowest setting that will be used in Run 2A. The labels and arrows indicate the original frequency of the signal before mixing. The RF and IF1 filter response functions are taken from manufacturer data sheets for the filters in the Run 2A system described in Tab. 6.3. For other frequency steps in this run, LO2 remains fixed at 5.4 GHz while LO1 steps in frequency.

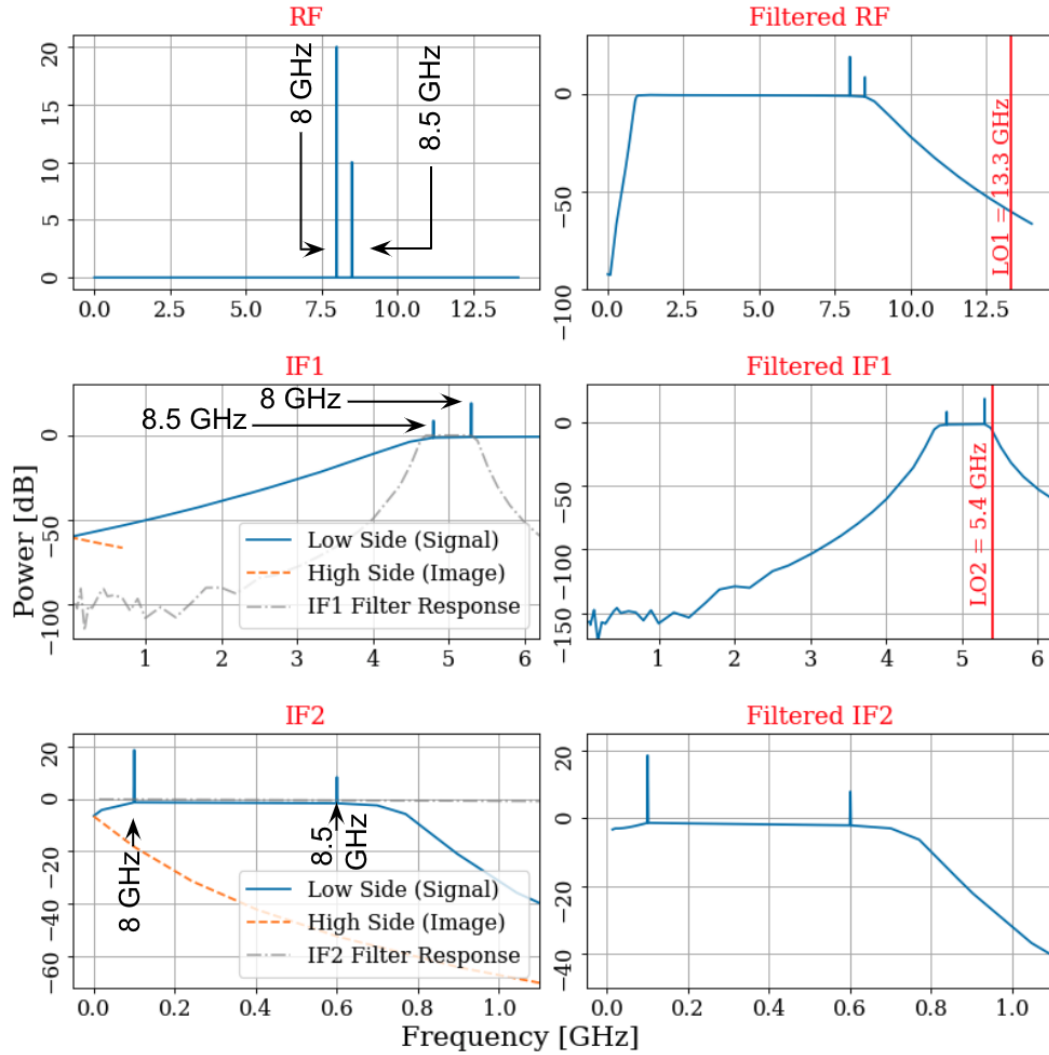


Figure 6.15: Simulated frequency response of dual conversion superheterodyne receiver (DCSHR) with signals around 8 GHz, the highest frequencies of Run 2A. The input RF is the mean of broadband noise (truncated at 14 GHz), plus two coherent signals at 8 and 8.5 GHz. The image signal would have been around 21.5 GHz, but is not shown here since it is so far out of band of the RF filter and LNA. Both of these signals will mix to the 4.7-5.3 GHz pass band of the IF1 filter in the Run 2A DCSHR. Each panel shows the spectrum at the points indicated in the red text along Fig. 6.12. LO1 is set to 13.3 GHz, the approximate highest setting that will be used in Run 2A. The labels and arrows indicate the original frequency of each signal before mixing. The RF and IF1 filter response functions are taken from manufacturer data sheets from the filters in the Run 2A system described in Tab. 6.3. For other frequency steps in this run, LO2 remains fixed at 5.4 GHz while LO1 steps in frequency.

6.2.3.1 Run 2A mixer system design

The Run 2A DCSHR was constructed and tested using the schematic shown in Fig. 6.16. The testing demonstrated proof of concept but was not comprehensive. An 18 GHz spectrum analyzer was used to monitor the signal path as various signals progressed through the system¹⁴. LO bleed (after both mixers) and inter-modulation products (after M2) were measured at expected frequencies and amplitudes. Further testing is required before using the system for a science run. An important test will be to terminate the input of the system in order to integrate several days of noise using the Teledyne GPU-based spectrum analyzer (see Sec. 2.5.7). In this test, LO1 should step through the list of frequencies that it will use for Run 2A. This will confirm that the standard deviation of the thermal noise averages down following the Dicke radiometer equation, Eq. 2.16. This is the test depicted in Fig. 4.16. In addition, these spectra should be searched for candidates using the Run 2A analysis pipeline. Furthermore, by adding some attenuation with loss L (in dB) before M1, any signals introduced after this attenuation will have a higher signal-to-noise ratio and will be detected in less real-time than would be required otherwise.

¹⁴Both the spectrum analyzer and signal generator were loaned. Thank you to Paul Stucky in the UC Davis Chemistry department.

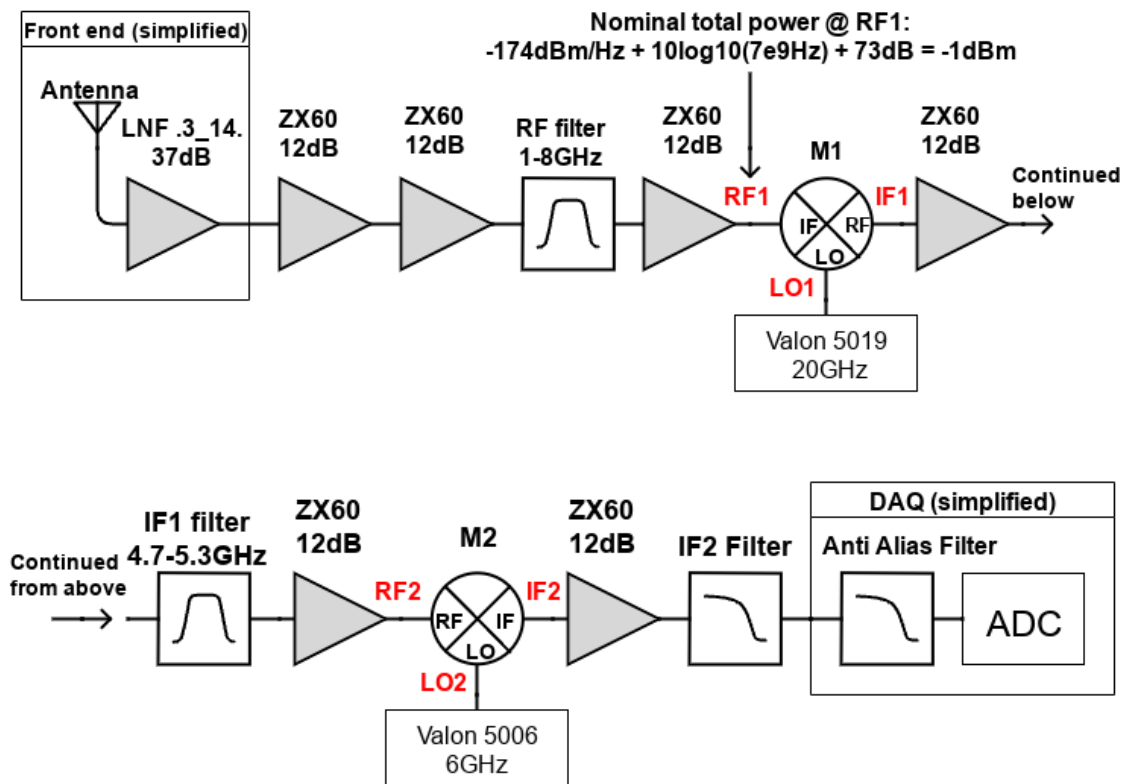


Figure 6.16: Schematic of Run 2A dual conversion superheterodyne receiver (DCSHR). See Tab. 6.3 for specific part numbers. The system was tested as shown here, but additional tweaks (such as adding a few small attenuators) may be necessary. The text in red labels the signal at each point so it can be related to other figures in this section. Note, however, that M1 is intentionally connected “backwards”, such that RF1 enters the IF port.

Function	Part name	Passband [GHz]	Comments
General amp, 12dB	MC ZX60-14012L+	0.003 – 14.0	
RF filter, LPF	MC ZLSS-8G-S+	< 8.0	
RF filter, HPF	MC SHP-1000+	> 1.0	Only rated up to 3 GHz. Tested up to 6 GHz. Requires test at 8 GHz.
Amp before M1, 17-23dB	ZX60-83LN12+	0.5 – 8.0	Not used, but we have one. Could replace two ZX60-14012L+.
Mixer 1	Miteq TB0440LW1	0.5 – 20.0	RF1 signal enters IF port. See schematic 6.16. Borrow from Paul Stucky.
LO 1	Valon 5019	0 – 20.0	
IF1 filter	MC ZVBP-5G-S+	4.7 – 5.3	
IF1 isolator	Teledyne T-5S83T-1C	4.7 – 12.3	Not used, but we have one. It would go directly before M2.
Mixer 2	Marki M1-0218LP	2.0 – 18.0	We have an extra.
LO 1	Valon 5009A		
IF2 filter	MC VLFX-1350+	< 1.35	
Digital step attenuator	ZSAT-31R5+	0.01–1.0	Optional. Compensate for gain variations upstream of DAQ.
Limiter	MC VLM-52-S+	0.01 – 0.5	Optional. Protection for the ADC. Untested.
Antialias filter	MC ZX75LP-470-S+	DC – 0.510	Extra rejection of high frequencies.
Power split	MC ZFRSC-183-S+	DC – 18	Used only for test setup.
Noise source	NoiseCom NC4563	0.01 – 12.4	Used only for test setup.

Table 6.3: Dual Conversion Mixer System Components, as shown in schematic, Fig. 6.16. Mini-Circuits is abbreviated “MC”.

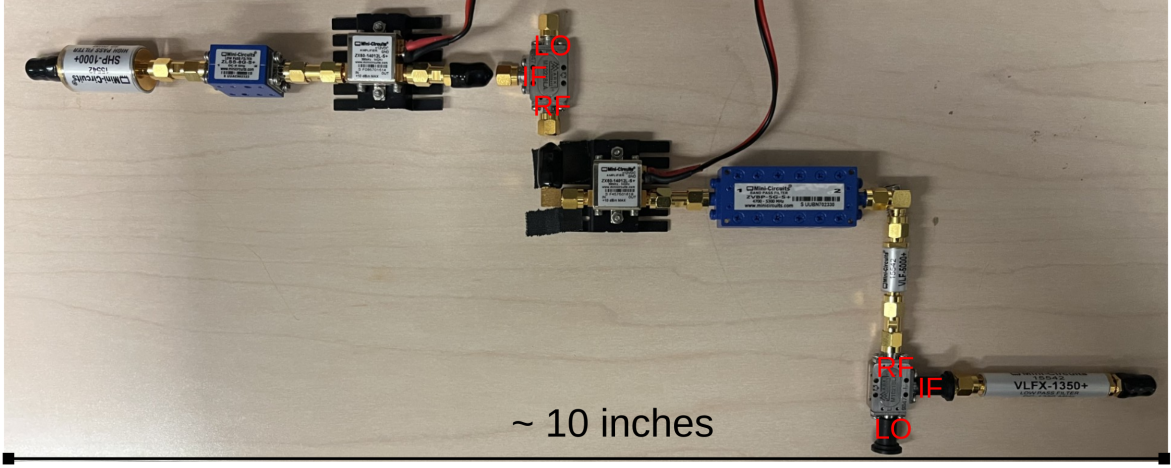


Figure 6.17: Picture of the (partially assembled) dual-conversion mixer system. Mixer 1 (M1) is a Miteq TB0440LW1, but is on loan¹⁵, so an extra Marki M1-0218LP is used as a placeholder. Note that M1 is connected “backwards”. The RF goes into the IF port, and the IF emerges from the RF port. This configuration lets M1 to accept frequencies down to 500 MHz for the first conversion step.

6.2.3.2 Additional considerations

This sub-subsection lists a few design points considered during the design phase of the mixer system. Hopefully it clarifies some design choices.

RF port as a source of the $2L \times 1R$ image spur The Marki mixer primer [180] notes that the RF port of a real mixer will source a $2L \times 1R$ image spur (where $f_{\text{spur}} = nf_{\text{LO}} \pm mf_{\text{RF}}$). If this spur is out of the pass band of the RF filter, it will reflect off the filter, mix with the LO and downconvert to the desired difference frequency. To mitigate this, amplifiers were placed on either side of M1 and an isolator was selected for the input of M2. This spur was not measured, so the isolator is labeled “optional” in Tab. 6.3.

Integrated noise power When dealing with wideband noise, the total power in a given bandwidth must be considered to prevent saturation at each stage of a signal chain (see Sec. 2.5.4.4). On the other hand, the ~ 13 dBm LO power, along with the finite LO-IF¹⁶ isolation of M1 means that a healthy signal level is required to keep the signal-to-noise ratio high¹⁷. This motivates adding gain throughout the system¹⁸. The amount of required gain changes drastically when going through the IF1 filter since the bandwidth is reduced from ~ 7 GHz to ~ 0.6 GHz. Assuming a flat power spectral density, this requires $10\log_{10}(7\text{ GHz}/0.6\text{ GHz}) \sim 10.7$ dB of make-up gain to achieve the same total-integrated power as before the filter.

Extra IF1 filter A secondary low-pass filter in the IF1 filter section may be required because the IF1 band-pass filter is not rated above 12 GHz, and testing it would require borrowed equipment.

RF filter The RF low-pass filter (Mini-Circuits SHP-1000) is only rated to 3 GHz. It was unit-tested up to 6 GHz, and 8 GHz signals made it through the assembled DCSHR with minimal attenuation, but this filter should be unit-tested up to 8 GHz when high-frequency test equipment is available.

paragraphIntermodulation spurs Intermodulation spurs occur because of the non-linear

¹⁶Remember, M1 is “backward”, so the relevant specification here is labeled LO-RF isolation, given on the datasheet from 6-13 GHz as between 20 and 30 dB [185].

¹⁷If we are being literal, we actually want a high noise-to-signal ratio. The experiment gives noise (which may contain a small, buried signal), and it is the coherent LO that is contaminating this noise. One person’s noise is another’s signal.

¹⁸In practice, gain comes quantized in 12 dB amplifiers, so attenuators may be needed to get precisely the correct amount.

behavior of the mixer [180]. They can be either single-tone or multi-tone. In the case of the former, a single tone enters the mixer and the spurs are computed using the power and frequencies of this tone along with that of the LO. In the case of the latter, multiple signals enter and all of them, along with the LO, are used to compute the spurs. Balanced mixers attempt to reduce the impacts of these effects, but they still occur to some degree. In the case of the Run 2 DCSHR, both single- and multi-tone intermodulation spurs must be considered. Neither appears to pose a major issue as will be discussed in the following paragraphs.

The spectrum that the system is designed to mix is mostly broadband noise. This noise can be viewed as many tones, all of which will cause multi-tone intermodulation spurs with each other. Since there is a dense sea of tones, the spurs will be equally dense, creating a new broadband noise floor. However, all of these tones are of random amplitudes and phases, so their spurs are as well. This noise floor will average down in the same way that the amplified thermal noise of the experiment will. The effect is another noise floor, which is 10s of dB below the primary thermal noise and is irrelevant. This is a similar consideration to the ADC noise of Sec. 2.1.5.2.

Single-tone intermodulation distortion must be considered with a bit more care. These spurs are denoted as $nL \times mR$ for $f_{\text{spur}} = nf_{\text{LO}} \pm mf_{\text{RF}}$. This is necessary to consider because LO1 will be present in IF1 and create a spur in M2.

In the first frequency step of Run 2A, LO1 will be set to 6.3 GHz. The IF1 filter has a response of -60 dB at 6.3 GHz. M1 has an LO-RF isolation of -20 dB at 6.3 GHz and requires an LO power around +10 dBm. After including the two 12 dB amplifiers, LO1

will be present at the input of M2 with an amplitude of about -46 dBm (see Fig. 6.16). This signal will create single-tone intermodulation spurs in M2 with LO2. These spurs can easily be computed. Marki Microwave has a very nice graphic calculator, which was used to generate the figure shown in Fig. 6.18. It turns out the only “intermodulation spur” that is significant is the $-1L \times 1R$ spur. In other words, the normal image! At the first frequency step ($LO1 = 6.3$ GHz), this image will appear at .9 GHz in IF2¹⁹. Assuming the ADC samples at 1,250 MHz, this 900 MHz image will alias to 350 MHz; right in the analysis span. An anti-aliasing filter will be required to reduce the 900 MHz image. The anti-aliasing filter specified in Tab. 6.3 attenuates 900 MHz by 63 dB.

Referring to Fig. 6.16, the LO1 image will be -106 dBm at the input of the ADC. The thermal noise density will be -55 dBm per 1 kHz²⁰. According to Eq. 2.16, this will be detectable in $\sim 10^{10}$ averages, or about 153 days (for 1.25 GHz sample rate and a 2^{20} point FFT). Again, reducing the noise power introduced into the DCSHR through attenuation will cause this signal to become detectable by averaging for less real time, and will be a useful troubleshooting step. In this test, the image should appear as predicted without waiting two months.

The image introduced here is the most prevalent. Other spurs are both lower in amplitude (see Fig. 6.18) and will be reduced by nearly 100 dB by the IF1 filter.

DCSHR for veto channel In order to use a veto channel in future runs, it will require a similar DCSHR system. Ideally, it should be an identical system to the main channel.

¹⁹Remember, LO2 is always 5.4 GHz.

²⁰This resolution bandwidth is chosen to match the expected linewidth of the DP at 1 GHz, see Sec. 2.1.2.

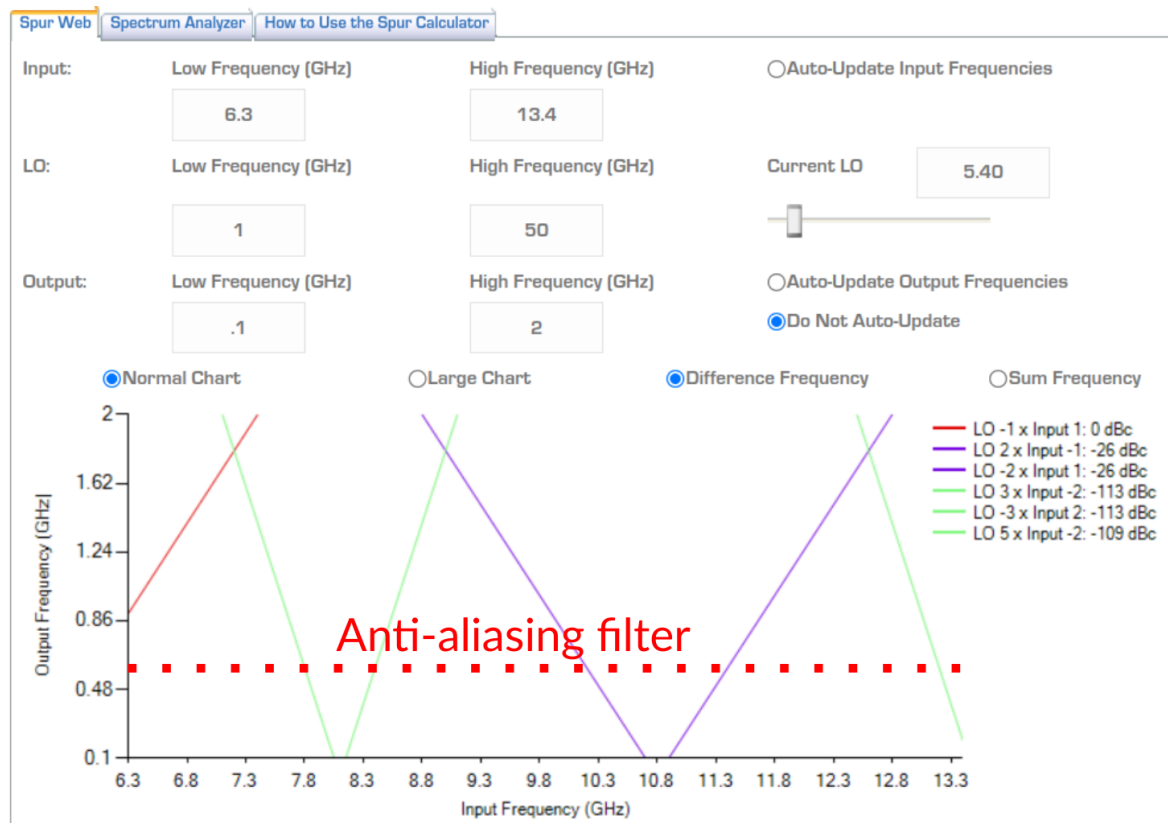


Figure 6.18: Spur web, calculated using Marki Microwave’s online calculator [186]. This figure shows the input vs. output frequency of a mixer for a fixed LO. In this case, $LO = 5.4$ GHz to match LO2. The curves show where various spurs will appear at the output as a function of input frequency. Each spur is denoted as $nL \times mR$ for $f_{spur} = nf_{LO} \pm mf_{RF}$, and the colors appear to the right of the plot. This shows that the only relevant intermodulation spur is -1×1 , in other words, the normal mixer image. This spur is easily handled by the anti-aliasing filter following M2, which will filter everything above the dashed red line.

This may not be possible due to budget, or the unavailability of sourcing a second Miteq TB0440LW1 mixer. One possible workaround is to use a coax switch and alternate between scans of the veto and experiment. This has the disadvantage that we lose the ability to correlate the experiment to possible RFI on the veto channel. However, RFI tends to be persistent, and this is likely not a major issue. Another solution is choosing another mixer.

Marki T3H-20G mixers are a good choice. However, they have large unit-to-unit variation and may not offer enough LO-RF isolation.

6.2.3.3 Comments on Run 2B

While the exact system for the 8-14 GHz Run 2B will not be specified here, a careful reading of Sec. 6.2 should be adequate preparation for the design of this system.

Selection of RF filter A new RF filter will be required with a pass band of 8-14 GHz. Note that this 14 GHz restriction is set by the low noise amplifier (LNA) [187]. This LNA is rated up to 16 GHz, though the gain drops by 6 dB between 14 and 16 GHz. In other words, this 14 GHz limit is soft, and Run 2B could push up to 16 GHz with some care.

Change first mixer to low-side injection The LO1 source is a Valon 5019 synthesizer [188], with a maximum output frequency of 20 GHz. The final LO1 setting of Run 2A will be around 13.3 GHz. By continuing with high-side injection (defined in Eq. 6.2), the highest RF frequency that can be mixed to the IF1 filter's passband (4.7-5.3 GHz) by a 20 GHz LO1 is 15.3 GHz. If Run 2B (or a run beyond this that uses a similar DCSHR) pushes beyond an RF frequency of 15.3 GHz, M1 will need to operate in low-side injection mode.

Send RF to the RF port of M1 Run 2A sends the RF to the IF port of M1 (see Fig. 6.16). M1 [185] has about 5 dB better LO-IF isolation than LO-RF isolation. Run 2B begins around 8 GHz, which will allow the RF to enter the RF port of M1. This configuration will offer better isolation between LO1 and the following signal path stages.

6.3 Possible Future Upgrades

The previous sections in this chapter addressed the progress towards the planned Run 1B (Sec. 6.1) and Runs 2A/B (Sec. 6.2). This section gives an outline for experiments beyond this. This section will be rather vague compared to the previous sections of this chapter. If those sections were a road map, this section is verbal instructions providing general directions.

6.3.1 Detector design principles to maximize the signal-to-noise ratio

Chapter 2 began with a quote by Veljko Radeka [106] that claimed an optimal detector must have three components:

1. The detector design must maximize the signal
2. Noise must be reduced at its physical source
3. The signal and noise must be optimally filtered

The common thread is the improvement of the signal-to-noise ratio. The considerations are as true today as they were in 1974.

6.3.1.1 Maximize the signal

There are a few ways to ensure that the signal emerging from the detector is maximized. The traditional approach has been to use a highly resonant cavity. See, for example ADMX

[109] or HAYSTAC [135]. To push to lower masses with wavelengths larger than practical cavities, lumped element detectors use electronic resonant circuits in place of resonant cavities to achieve a similar effect. See DMRadio [110, 178, 189] and ABRACADABRA [177].

While these experiments have very high Q , they are only sensitive at a single frequency which must be scanned. The dark E-field radio (DER) technique differs in that it is broadband; many frequencies are scanned simultaneously by taking a Fourier transform. At higher masses, a smaller reverb chamber can be employed, as long as the frequency of interest remains above the lowest usable frequency (defined in Sec. 2.4.2).

A careful analysis of scan rates to reach a given sensitivity of ε for the DER technique compared to single-resonance experiments should be performed before attempting to overcome the difficulties of a DER-like experiment above ~ 20 GHz.

6.3.1.2 Reduce noise

The DER experiment operates at room temperature, using a low noise amplifier (LNA). Compared to cryogenic experiments, this represents an advantage in terms of reduced experimental complexity, but it means the dominant background is classical thermal noise (~ 300 K, Sec. 2.1.1), followed closely by amplifier noise (~ 100 K, Sec. 2.1.4). These noises add to form the total system temperature $T_s \approx 400$ K. However, the limit in sensitivity scales with $T_s^{-1/2}$ (at least in the classical regime, $h\nu \ll k_b T$). For this reason, most haloscope experiments choose to forgo experimental simplicity and operate at extremely cold temperatures (100s of mK, sometimes lower). At these temperatures, quantum effects begin to dominate the noise budget and simple linear amplifiers must be replaced by read-out tech-

nologies such as superconducting quantum interference devices (SQUIDs) or single photon counting. If a cold detector is used, more complex amplification must also be employed or the noise budget will be dominated by the readout.

6.3.1.3 Filtering

Section 4.2.3 introduced signal-matched filtering as the optimal filtering technique for a dark photon of known lineshape, discussed in Sec. 2.1.2. This will be an important consideration going forward, but will not change much.

However, there is an additional filter that has the potential to improve the sensitivity of Run 1A as well as any future runs. Assuming that the dark photons are uniformly polarized, the power received by a linearly polarized antenna will be modulated by the rotation of the earth. The time series²¹ of each frequency bin can be searched for modulation by $\alpha \cos^2(12\text{hr} \times t)$, where α is a dimensionless parameter that depends on the geometry of the antenna relative to the rotation of the earth. Many useful ideas for this calculation are presented in [105], however, the idea in this paper is simply to set up the experiment to maximize the signal, not search for modulations on existing data. As far as I know, this analysis idea is original and was independently arrived at by both myself and Brian Kolner in the UC Davis Electrical Engineering Department.

²¹The time series will have temporal resolution set by the pre-averaging defined in Sec. 2.5.7; several minutes. Remember, the RF time series samples are not saved.

6.4 Leveraging Axion Experiments for Dark Photon Searches

Axions (introduced in Sec. 1.4.2.1), are an ultra-light dark matter candidate similar to dark photons. They require a magnetic field to convert into Standard Model photons, but they are detected using techniques very similar to dark photons. Axion experiments represent a significant investment of resources [190, 191] and frequently do not take full advantage of their sensitivity to dark photons. This section lists a few methods that can leverage this investment to probe the dark photon parameter space.

6.4.1 Reanalysis of axion haloscope data

A highly efficient dark photon search can be conducted by recognizing that many axion searches would have been sensitive to dark photons if not for vetoes which rely on the state of an applied magnetic field. SHAF'T [192], for example, has two toroids that can generate fields in opposite directions²². While the magnetic flux through the pick-up coil from an axion would flip polarity as the magnetic fields are reversed, the flux from a dark photon would not (see Fig. 1 of [192]). This differential technique gives a convenient way to veto radio frequency interference. However, careful inspection of the time dependence of candidates may allow these signals to be reduced without relying on the magnetic field veto and ruling out dark photons²³. This analysis was performed in 2021 [193], but should be revisited.

²²Actually, the field directions flip to all four possible combinations, and they have a clever analysis chain to maximize sensitivity.

²³This is just an example, as DMRadio will not rely on a veto and have much better sensitivity to dark photons in the SHAF'T frequency range.

6.4.2 Design and operation of axion experiments for dark photons

By recognizing that an axion experiment is almost a dark photon experiment, the experiment can be designed from the beginning to maximize its sensitivity to dark photons. The main modification is to not veto based on the state of the magnetic field, as pointed out in Sec. 6.4.1. This comes with the caveat that interfering signals must be mitigated in a different way. This is likely harder than it sounds. One possible solution is to simply conduct the experiment in a shielded room, similar to the Dark E-Field Radio Experiment.

Appendix A

Overview of RTSA code base

A basic overview of the code that is used to acquire and process data is outlined here.

The version control is very simple, and each experiment has its own directory containing several key files. The general usage template from which other experiments can be developed is in the `teledyneTemplate` directory. The important files, in order of importance, are

- `settings.py`: Settings are controlled from this file. If the DAQ code isn't to be modified, this is all the user must interact with in normal usage. While it is a python script, it functions more like a text file. Descriptions and notes about allowed values are included as comments. Read them carefully; settings can conflict.
- `drDaq*.py`: Main script which calls all the helper functions. To take data, run this script after modifying (and saving) `settings.py`. There is usually a suffix indicating the date and information about version.
- `avgFftModule.py`: This is modified code from Teledyne. It handles all the heavy

lifting: Interfacing with the C++ API, pinning GPU memory, transferring data from PCIE card to the GPU and computation of the FFT on the GPU. This is all wrapped in a class called `avgFft`. An instance of this class is called `avgSpec` and is the workhorse of `drDaq.py`

- `daqHelpers.py`: Lots of helper functions which are separated here to keep other code clean. It is imported as a module in other files. Lots of useful code lives in here, including the code that converts time series to power spectra (normalization is non-trivial, see Eq. 2.15), writes info to `database.txt`, saves the pre-averaged spectra including metadata in an HDF5 file.
- `plotTesting.py`: This uses `dash` to host a web app which allows interactive `plotly` graphs for simple visualisation of run data. This is extremely useful as it can tell you if amplifiers die. Without this, all data would have to be averaged and more fully analyzed, but this is a good lightweight option. There is some creative use of data down sampling so it runs quickly without removing any candidates. An example window is shown in Fig. A.1.
- `backup.sh`: Simple shell script that backs up run data to locations of your choosing. In the `teledyneTemplate` directory, it is set up to back up to the secondary hard disk drive in the DR2 machine and to peloton, but this can be easily modified. When taking real data, this script should not be run at the same time as `drDaq.py`. **`backup.sh`** should be run first, so it completes while the antenna is moved and the batteries are changed. This ensures there is not a heavy load on the hard drive due to back ups

while data acquisition is ongoing. `valonInit.py`: sets up the valon signal generator to work as a clock. This should probably be a function inside of `daqHelpers.py`, but there may have been a reason I kept it separate.

- `gdrapi.py` Comes from Teledyne. Defines functions for the API. I have not modified it at all.
- `helperCupy.py` Comes from Teledyne. Defines functions for the GPU. I have not modified it at all.
- `streamingHelpers.py` comes from Teledyne. Defines functions for streaming from PCIE card to GPU. I have not modified it at all.

A Tips for Using the RTSA System

- As of September 2024, you must boot into kernel version 5.15 (Ubuntu 22.04)
- Make sure to run `insmod.sh` in the `teledyneInstall/gdrcopy` directory after restarting the machine.
 - look at `teledyneInstall/installGuide.txt` on how to handle common errors involving this process.
- Read through the `settings.txt` file carefully! Some settings will conflict with others, but they are mostly noted. There are also some notes about possible upgrades that could be made, some of which without much effort.

B Data Structure and Processing

Figure 2.45 shows the handling of data as it comes in as an RF time series and is converted to pre-averaged spectra. While significantly less cumbersome than the raw data, processing these spectra still represents a challenge. This subsection outlines how I have attempted to handle it. While it is a little convoluted, this is the third iteration of how to handle this data processing and is likely simpler than it seems on first blush. In other words, there is probably a better way to do this, but don't knock it till you try it.

B.1 Writing data

Once a pre-averaged spectrum is computed by dividing the running sum by `NOF_BUFFERS_TO_RECEIVE`, this can be written to an HDF5 file. Whether or not it is, can be controlled by the `SAVE_H5` Boolean variable¹. Each pre-averaged spectrum is uniquely specified by `ACQ_NUM` in a given data run. In order to simplify backups and avoid placing all our eggs in one HDF5 basket, `NUM_SPEC_PER_FILE`² pre-averaged spectra are saved into a single HDF5 file, before starting a new file. These files are simply named as a zero-indexed number followed by their `.hdf5` extension. These files are saved in `SAVE_DIRECTORY`³. I usually make this save directory in a secondary SSD named `drBiggerBoy` in order to preserve the main

¹This variable exists because when testing things or taking miscellaneous measurements, you will frequently want to acquire a single spectrum without engaging the complex machinery of the HDF5 saving procedure.

²I have kept this around 16 and not experimented much outside of this range, but it's probably fine. This keeps the files around 1 GB. For Run 1A, this is about 45 minutes of antenna data and 3 minutes of terminator data per file, so if a file is corrupted, it's not a big deal. I have never had a problem; this is paranoia inherited from Ben.

³A reminder that this is specified, like all other variables, in `settings.py`. Note you must create this directory ahead of time and include a sub-directory called `data`. It says this in the comments of `settings.py` which you are reading, right?

drive. I think the stress of continuous reads and writes will probably kill this drive, so I prefer to keep it separate from the main boot drive, `drBigBoy`.

When `SAVE_H5 == 1`, a `database.txt` file is created in `SAVE_DIRECTORY`, shown in Table A.1.

Attribute	Pre-averaged spectrum 0	Pre-averaged spectrum 1
ACQ NUM	0	1
DATETIME	2023-05-10 11:32:48.365	2023-05-10 11:35:49.193
SWITCH POS	0	1
ANT POS IDX	0	0
TEMP	295.64	295.54
LEN FFT LOG2	24	24
SAMPLE RATE MHZ	800.0	800.0
NOF BUFFERS	8600	8600
AMP1	1012_E_PbAcid	1012_E_PbAcid
AMP2	ZKL_9p05VReg	ZKL_9p05VReg
LPF	HSP50+	HSP50+
HPF	288S+	288S+
ATTENUATOR	4dB_FIXED	4dB_FIXED
ADC	ADQ32	ADQ32
CLOCK	SRS_VIA_VALON	SRS_VIA_VALON
File Number	0	0

Table A.1: Example database file from Run 1A. In this run, `ACQ_NUM` counts up to 4175, and these spectra are saved between 261 HDF5 files. Some of the values are auto-generated (`temp`, `DATETIME`, etc.) while others are manually entered into `settings.txt` (`AMP1`, `LPF`, etc.) Note that this table has been transposed to fit on the page.

When setting up a data run, you must test that the data are saved how you expect. I have had success by reducing `NOF_BUFFERS_TO_RECEIVE` to a small number⁴, and taking a simulated data run. The process of switching is hacked together and can give you unpredictable

⁴`NOF_BUFFERS_TO_RECEIVE` times the time per buffer must be larger than around 2 seconds or it will crash

results. Did I mention to read the comments in `settings.py`?

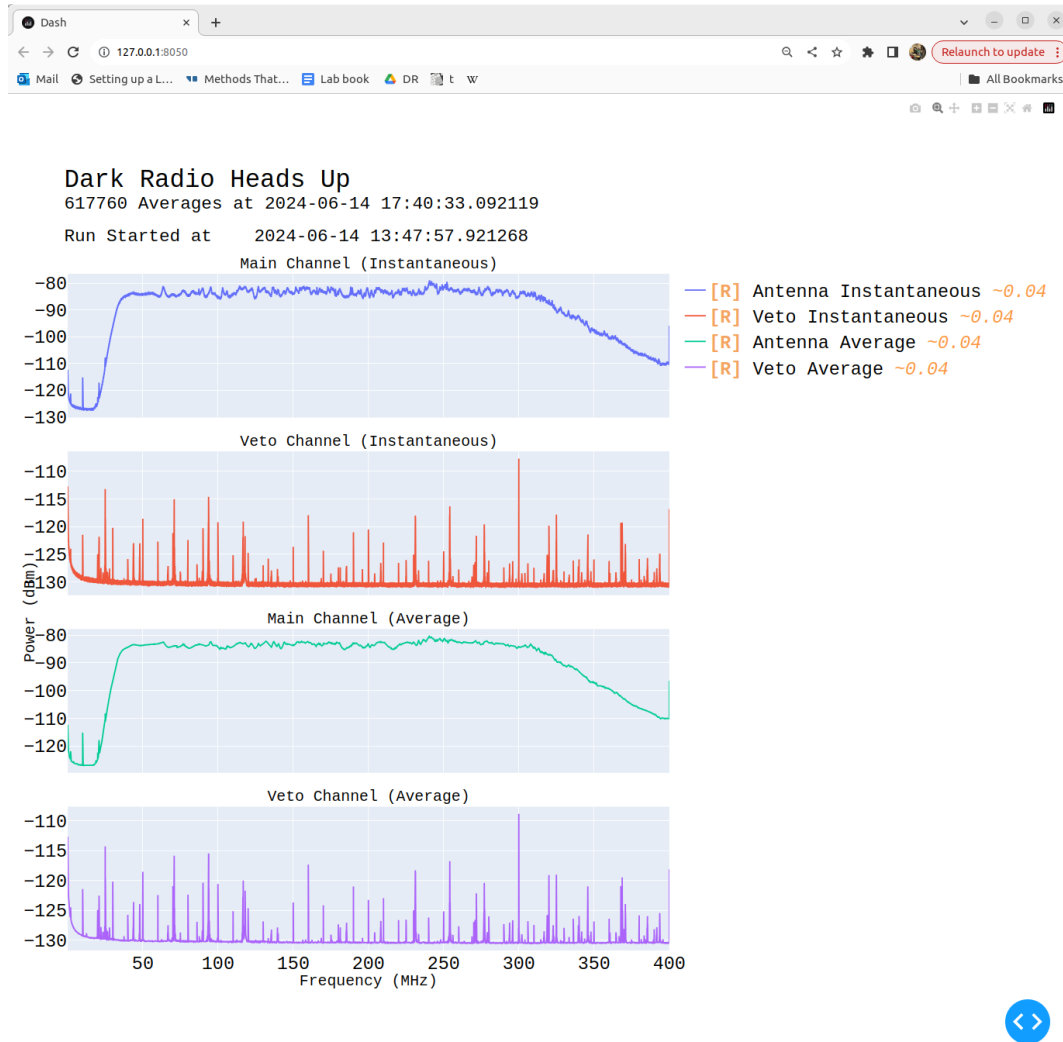


Figure A.1: Real-time heads-up window for data acquisition. Built with Plotly and Dash, and displays in a web browser, in this case, Google Chrome. Spectra contain 2^{23} frequency bins, so are down-sampled so as to display 10^4 bins at any given level of zoom. This allows for interaction with the plot in real-time with very little lag. The resolution currently displayed (in MHz) is shown in yellow next to the legend (right of the first plot). This down-sampling is “smart” in that it will show any excursions and is implemented with the `plotlyResampler` package in `MinMaxAggregator` mode. Mousing over a curve causes a “Tooltip” to appear, indicating the frequency and power at that point in the curve. The frequency axes are all linked, so zooming in on one plot will cause all plots to display the same frequency range. Icons on the upper right allow for navigation of the plot. Frustratingly, there is no back button, so be careful when zooming on a very narrow feature; a wrong move means you have to go all the way back out and start over.

B.2 Reading and averaging data⁵

After taking a data run, you should have a `SAVE_DIRECTORY` containing a directory full of many HDF5 files and a `database.txt` file (and possibly a directory of plotting spectra if that is chosen in `settings.txt`). The basic idea is to “pre-process” the HDF5 files into a single, large HDF5 file, which can be more quickly accessed to compute a single, averaged S_o spectrum for further analysis (see next chapter). The `database.txt` file is loaded into python as a pandas dataframe in order to find specific spectra in this large data structure.

I tried (for a while) to keep the files separate to allow for multi-processing to speed up the averaging, but I ran into issues. The most serious issue is that all the files live on the same drive, so you can’t get much of a speed-up since the drive is read-limited to around 500 MB/s. I found multiprocessing is more effort than it’s worth. It takes about 20 minutes to pre-average run 1.4 (9 days of data, 3 minute pre-averages yields 4175 spectra, which take about 280 GB), and you only have to do this once after a run.

Following the example of `run1p4_packAvgAllAnalysis.ipynb`, the first step is to load `database.txt` as a pandas dataframe which can be used to index the pre-processed file. Next, a list of which `ACQ_NUMs` correspond to an antenna and terminator must be computed, after removing any which are known to be contaminated⁶. The list of indices that are to be averaged together is called either `antIdx` or `termIdx`. Note that you can devise any number of ways to generate these lists or even hard code them. The code given is just an example.

⁵Code for this section can be found at: https://github.com/josephmlev/darkRadio/tree/master/daqAnalysisAndExperiments/run1p4/run1p4_analysis/run1p4_packAvgAllAnalysis.ipynb

⁶In run 1.4, the first antenna and terminator spectra are thrown out, since the computer monitor was on to verify the run started successfully. This was out of paranoia and probably didn’t do anything.

Next, info in the dataframe (no power spectra data yet) are analyzed as a sanity check. These are not super important but a good check.

The cell beginning with the comment `#pack pre proc dataset` is where the heavy lifting happens. A large HDF5 file is initialized. It contains two datasets, one for each channel (main experiment and veto in run 1.4). Note that in this example, it is hardcoded to expect 8388609 frequency bins ($2^{23} + 1$), and this must be changed if doing something other than a 2^{24} -point FFT. The HDF5 sub-files are iterated over, their data are read and they are written into the pre-processed HDF5 file. Make sure not to remove the error handling for closing the HDF5 file, or you will corrupt it and have to rerun the pre-processing.

Now that the pre-processed HDF5 file has been packed with data and a list of indices to be averaged has been generated, averaging is trivial (though it still takes a few minutes, so maybe test it with a subset of `antIdx`. The code is provided below:

```

1 def avgSpecFromPacked(avgIdxs , specStr):
2     sum = np.zeros(2**23+1)
3     for i in avgIdxs:
4         sum += f[specStr][:,i]
5     avgSpec = sum/(len(avgIdxs))
6     return avgSpec

```

Listing A.1: Python function for averaging spectra from pre-processed data

Note that this assumes a single HDF5 file, `f`, is in scope. `specStr` is a string that specifies which channel to average, for example, `spec_W_chA`. The output of this function is S_o that can be passed to the following functions for further analysis.

Bibliography

- [1] Benjamin Godfrey. “A Novel Search for Dark Photons”. Ph.D. dissertation. UC Davis, 2022. URL: <https://escholarship.org/uc/item/72b776v4>.
- [2] Billy Boxer. “LUX-ZEPLIN Dark Matter Searches: Development of Optical and Source Based Calibration Systems”. PhD thesis. PhD thesis. University of Liverpool, 2021. URL: <https://livrepository.liverpool.ac.uk/3135532/>.
- [3] Noah Alexander Kurinsky. “The Low-Mass Limit: Dark Matter Detectors with eV-Scale Energy Resolution”. PhD thesis. Stanford U., Phys. Dept., 2018. DOI: 10.2172/1472104.
- [4] Marco Cirelli, Alessandro Strumia, and Jure Zupan. *Dark Matter*. 2024. arXiv: 2406.01705 [hep-ph]. URL: <https://arxiv.org/abs/2406.01705>.
- [5] Gianfranco Bertone and Dan Hooper. “History of dark matter”. In: *Reviews of Modern Physics* 90.4 (2018), p. 045002.
- [6] Virginia Trimble. “History of Dark Matter in Galaxies”. In: *Planets, Stars and Stellar Systems: Volume 5: Galactic Structure*. Ed. by Gerard Gilmore. New York, NY: Springer, 2013, pp. 1093–1116.

- [7] Marcin Misiaszek and Nicola Rossi. “Direct Detection of Dark Matter: A Critical Review”. In: *Symmetry* 16.2 (2024), p. 201.
- [8] M. Battaglieri et al. “US Cosmic Visions: New Ideas in Dark Matter 2017: Community Report”. In: 2017. arXiv: 1707.04591. URL: <https://arxiv.org/abs/1707.04591>.
- [9] S. Navas et al. “Review of particle physics”. In: *Phys. Rev. D* 110.3 (2024), p. 030001. DOI: 10.1103/PhysRevD.110.030001.
- [10] Andrea Cimatti, Filippo Fraternali, and Carlo Nipoti. *Introduction to Galaxy Formation and Evolution: From Primordial Gas to Present-Day Galaxies*. Cambridge University Press, 2019.
- [11] Barbara Ryden. *Introduction to cosmology*. Cambridge University Press, 2017.
- [12] E. E. Barnard and A. C. Ranyard. “Structure of the Milky Way”. In: *Knowledge: An Illustrated Magazine of Science* 17 (Nov. 1894), p. 253.
- [13] Kelvin, William Thomson, Baron. *Baltimore Lectures on Molecular Dynamics and the Wave Theory of Light*. English. Collection: University of California Libraries. London, Baltimore: C. J. Clay and sons; Publication agency of the Johns Hopkins University, 1904, pp. xxi, 703. URL: <https://archive.org/details/baltimorelecture00kelviala/page/n5/mode/2up>.
- [14] Fritz Zwicky et al. “The redshift of extragalactic nebulae”. In: *Helv. Phys. Acta* 6.110 (1933), p. 138.

- [15] STEPHEN M. MAURER. “The Idea Man”. In: *SLAC Beam Line* 31.1 (2001). Accessed: 2024-06-23, pp. 21–27. URL: <https://www.slac.stanford.edu/pubs/beamline/31/1/31-1-maurer.pdf>.
- [16] Edwin Hubble and Milton L Humason. “The velocity-distance relation among extragalactic nebulae”. In: *Astrophysical Journal*, vol. 74, p. 43 74 (1931), p. 43.
- [17] P. J. E. Peebles. *Principles of Physical Cosmology*. Princeton University Press, 1993. DOI: 10.1515/9780691206721.
- [18] Edwin Hubble. “A relation between distance and radial velocity among extra-galactic nebulae”. In: *Proceedings of the national academy of sciences* 15.3 (1929), pp. 168–173.
- [19] Herbert Goldstein, Charles P. Poole, and John L. Safko. *Classical Mechanics*. 3rd. San Francisco: Addison-Wesley, 2002. ISBN: 9780201657029.
- [20] R. Clausius. “XVI. On a mechanical theorem applicable to heat”. In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 40.265 (1870), pp. 122–127. DOI: 10.1080/14786447008640370. eprint: <https://doi.org/10.1080/14786447008640370>. URL: <https://doi.org/10.1080/14786447008640370>.
- [21] A. S. Eddington. “The Kinetic Energy of a Star Clusters”. In: *Monthly Notices of the Royal Astronomical Society* 76.6 (Apr. 1916), pp. 525–528. ISSN: 0035-8711. DOI: 10.1093/mnras/76.6.525. eprint: <https://academic.oup.com/mnras/article-pdf/76/6/525/18494199/mnras76-0525.pdf>. URL: <https://doi.org/10.1093/mnras/76.6.525>.

- [22] Fritz Zwicky. “On the Masses of Nebulae and of Clusters of Nebulae”. In: *The Astrophysical Journal* 86 (1937), p. 217.
- [23] F. D. Kahn and L. Woltjer. “Intergalactic Matter and the Galaxy.” In: *American Journal of Physics* 130 (Nov. 1959), p. 705. DOI: 10.1086/146762.
- [24] Hugo van Woerden and Richard G. Strom. “The beginnings of radio astronomy in the Netherlands”. In: *Journal of Astronomical History and Heritage* 9.1 (June 2006), pp. 3–20.
- [25] James Binney and Scott Tremaine. *Galactic dynamics*. Vol. 13. Princeton university press, 2011.
- [26] Jose Luis Sersic. *Atlas de Galaxias Australes*. Observatorio Astronomico de Cordoba, 1968. URL: <https://ui.adsabs.harvard.edu/abs/1968adga.book.....S>.
- [27] Vera C Rubin and W Kent Ford Jr. “Rotation of the Andromeda nebula from a spectroscopic survey of emission regions”. In: *The Astrophysical Journal* 159 (1970), p. 379.
- [28] Wikimedia Commons contributors. *Rotation curve of spiral galaxy Messier 33 (Triangulum)*. Accessed: 2025-02-12. 2021. URL: [https://commons.wikimedia.org/wiki/File:Rotation_curve_of_spiral_galaxy_Messier_33_\(Triangulum\).png](https://commons.wikimedia.org/wiki/File:Rotation_curve_of_spiral_galaxy_Messier_33_(Triangulum).png).
- [29] A. Bosma. “The distribution and kinematics of neutral hydrogen in spiral galaxies of various morphological types”. PhD thesis. University of Groningen, Netherlands, Mar. 1978.

- [30] Yongjun Jiao et al. “Detection of the Keplerian decline in the Milky Way rotation curve”. In: *Astronomy & Astrophysics* 678 (Oct. 2023), A208. ISSN: 1432-0746. DOI: 10.1051/0004-6361/202347513. URL: <http://dx.doi.org/10.1051/0004-6361/202347513>.
- [31] Y. Sofue et al. “Central Rotation Curves of Spiral Galaxies”. In: *The Astrophysical Journal* 523.1 (Sept. 1999), pp. 136–146. ISSN: 1538-4357. DOI: 10.1086/307731. URL: <http://dx.doi.org/10.1086/307731>.
- [32] Federico Lelli, Stacy S. McGaugh, and James M. Schombert. “SPARC: MASS MODELS FOR 175 DISK GALAXIES WITH SPITZER PHOTOMETRY AND ACCURATE ROTATION CURVES”. In: *The Astronomical Journal* 152.6 (Nov. 2016), p. 157. ISSN: 1538-3881. DOI: 10.3847/0004-6256/152/6/157. URL: <http://dx.doi.org/10.3847/0004-6256/152/6/157>.
- [33] S. M. Faber and J. S. Gallagher. “Masses and Mass-To-Light Ratios of Galaxies”. In: *Annual Review of Astronomy and Astrophysics* 17. Volume 17, 1979 (1979), pp. 135–187. ISSN: 1545-4282. DOI: <https://doi.org/10.1146/annurev.aa.17.090179.001031>. URL: <https://www.annualreviews.org/content/journals/10.1146/annurev.aa.17.090179.001031>.
- [34] A. Arbey and F. Mahmoudi. “Dark matter and the early Universe: A review”. In: *Progress in Particle and Nuclear Physics* 119 (2021), p. 103865. ISSN: 0146-6410. DOI: <https://doi.org/10.1016/j.pnpnp.2021.103865>. URL: <https://www.sciencedirect.com/science/article/pii/S0146641021000193>.

- [35] Jim Gunn and Jill Knapp. *Lecture VII: The Early Universe, Fluctuations, and the Development of Structures*. Lecture notes, Princeton University. 25 February 2008. URL: <https://www.astro.princeton.edu/~gk/A204/lecture7.pdf>.
- [36] Alexander Friedmann. “Über die Krümmung des Raumes”. German. In: *Zeitschrift für Physik* 10 (1922), pp. 377–386. DOI: 10.1007/BF01332580. URL: <https://doi.org/10.1007/BF01332580>.
- [37] Albert Einstein. “Die Grundlage der allgemeinen Relativitätstheorie”. German. In: *Annalen der Physik* 354.7 (1916), pp. 769–822. DOI: 10.1002/andp.19163540702. URL: <https://doi.org/10.1002/andp.19163540702>.
- [38] Galina Weinstein. *Einstein’s 1916 derivation of the Field Equations*. 2013. arXiv: 1310.6541 [physics.hist-ph]. URL: <https://arxiv.org/abs/1310.6541>.
- [39] Gianfranco Bertone, Dan Hooper, and Joseph Silk. “Particle Dark Matter: Evidence, Candidates and Constraints”. In: *Physics Reports* 405.5–6 (Jan. 2005). arXiv:hep-ph/0404175, pp. 279–390. ISSN: 03701573. DOI: 10.1016/j.physrep.2004.08.031.
- [40] Steven Weinberg. “The cosmological constant problem”. In: *Rev. Mod. Phys.* 61 (1 Jan. 1989), pp. 1–23. DOI: 10.1103/RevModPhys.61.1. URL: <https://link.aps.org/doi/10.1103/RevModPhys.61.1>.
- [41] Darren J. Croton. “Damn You, Little h! (Or, Real-World Applications of the Hubble Constant Using Observed and Simulated Data)”. In: *Publications of the Astronomical Society of Australia* 30 (2013). ISSN: 1448-6083. DOI: 10.1017/pasa.2013.31. URL: <http://dx.doi.org/10.1017/pasa.2013.31>.

- [42] N. Aghanim et al. “Planck 2018 results”. In: *Astronomy & Astrophysics* 641 (Sept. 2020), A6. ISSN: 1432-0746. DOI: 10.1051/0004-6361/201833910. URL: <http://dx.doi.org/10.1051/0004-6361/201833910>.
- [43] Michael S. Turner. “The Road to Precision Cosmology”. In: *Annual Review of Nuclear and Particle Science* 72 (2022), pp. 1–35. ISSN: 1545-4134. DOI: <https://doi.org/10.1146/annurev-nucl-111119-041046>. URL: <https://www.annualreviews.org/content/journals/10.1146/annurev-nucl-111119-041046>.
- [44] Alan H. Guth. “Inflationary universe: A possible solution to the horizon and flatness problems”. In: *Phys. Rev. D* 23 (2 Jan. 1981), pp. 347–356. DOI: 10.1103/PhysRevD.23.347. URL: <https://link.aps.org/doi/10.1103/PhysRevD.23.347>.
- [45] A.D. Linde. “A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems”. In: *Physics Letters B* 108.6 (1982), pp. 389–393. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(82\)91219-9](https://doi.org/10.1016/0370-2693(82)91219-9). URL: <https://www.sciencedirect.com/science/article/pii/0370269382912199>.
- [46] S. Dodelson. *Modern Cosmology*. Elsevier Science, 2003. ISBN: 978-0-12-219141-1. URL: <https://books.google.com/books?id=3oPRxdXJexcC>.
- [47] Andrew R. Liddle. “Acceleration of the Universe”. In: *New Astronomy Reviews* 45.4–5 (Mar. 2001), pp. 235–253. ISSN: 1387-6473. DOI: 10.1016/S1387-6473(00)00146-9. URL: [http://dx.doi.org/10.1016/S1387-6473\(00\)00146-9](http://dx.doi.org/10.1016/S1387-6473(00)00146-9).

- [48] A. Arbey et al. “AlterBBN v2: A public code for calculating Big-Bang nucleosynthesis constraints in alternative cosmologies”. In: arXiv:1806.11095 (Apr. 2019). DOI: 10.48550/arXiv.1806.11095. URL: <http://arxiv.org/abs/1806.11095>.
- [49] Lloyd Knox. *Physics 156 - A Cosmology Workbook*. Accessed: 2025-01-07. LibreTexts, 2017. URL: https://phys.libretexts.org/Courses/University_of_California_Davis/UCD%3A_Physics_156_-_A_Cosmology_Workbook.
- [50] Brian D. Fields et al. “Big-Bang Nucleosynthesis after Planck”. en. In: *Journal of Cosmology and Astroparticle Physics* 2020.03 (Mar. 2020), p. 010. ISSN: 1475-7516. DOI: 10.1088/1475-7516/2020/03/010.
- [51] D. J. Fixsen. “THE TEMPERATURE OF THE COSMIC MICROWAVE BACKGROUND”. In: *The Astrophysical Journal* 707.2 (Nov. 2009), p. 916. DOI: 10.1088/0004-637X/707/2/916. URL: <https://dx.doi.org/10.1088/0004-637X/707/2/916>.
- [52] Planck Collaboration. “Planck 2015 results - XI. CMB power spectra, likelihoods, and robustness of parameters”. In: *A&A* 594 (2016), A11. DOI: 10.1051/0004-6361/201526926. URL: <https://doi.org/10.1051/0004-6361/201526926>.
- [53] A. A. Penzias and R. W. Wilson. “A Measurement of Excess Antenna Temperature at 4080 Mc/s”. In: *The Astrophysical Journal* 142 (1965), pp. 419–421. DOI: 10.1086/148307. URL: <https://doi.org/10.1086/148307>.

- [54] G. F. Smoot et al. “Structure in the COBE Differential Microwave Radiometer First-Year Maps”. In: *The Astrophysical Journal* 396 (1992), pp. L1–L5. DOI: 10.1086/186504. URL: <https://doi.org/10.1086/186504>.
- [55] G. F. Smoot, M. V. Gorenstein, and R. A. Muller. “Detection of Anisotropy in the Cosmic Blackbody Radiation”. In: *Physical Review Letters* 39.14 (1977), pp. 898–901. DOI: 10.1103/PhysRevLett.39.898. URL: <https://doi.org/10.1103/PhysRevLett.39.898>.
- [56] Wayne Hu and Scott Dodelson. “Cosmic Microwave Background Anisotropies”. In: *Annual Review of Astronomy and Astrophysics* 40. Volume 40, 2002 (2002), pp. 171–216. ISSN: 1545-4282. DOI: <https://doi.org/10.1146/annurev.astro.40.060401.093926>. URL: <https://www.annualreviews.org/content/journals/10.1146/annurev.astro.40.060401.093926>.
- [57] R. K. Sachs and A. M. Wolfe. “Perturbations of a cosmological model and angular variations of the microwave background”. In: *Astrophys. J.* 147 (1967), pp. 73–90. DOI: 10.1007/s10714-007-0448-9.
- [58] F. W. Dyson, A. S. Eddington, and C. Davidson. “A Determination of the Deflection of Light by the Sun’s Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919”. In: *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character* 220 (1920), pp. 291–333. ISSN: 02643952. URL: <http://www.jstor.org/stable/91137> (visited on 01/16/2025).

- [59] Anthony Tyson. “Mapping Dark Matter with Gravitational Lenses”. In: *Physics Today* 45.6 (June 1992), pp. 24–32. ISSN: 0031-9228. DOI: 10.1063/1.881309. eprint: <https://pubs.aip.org/physicstoday/article/45/6/24/407200/Mapping-Dark-Matter-with-Gravitational-LensesMost>. URL: <https://doi.org/10.1063/1.881309>.
- [60] J Anthony Tyson, Greg P Kochanski, and Ian P Dell’Antonio. “Detailed mass map of CL 0024+ 1654 from strong lensing”. In: *The Astrophysical Journal* 498.2 (1998), p. L107. DOI: 10.1086/311314. URL: <https://dx.doi.org/10.1086/311314>.
- [61] Wesley N. Colley, J. Anthony Tyson, and Edwin L. Turner. “Unlensing Multiple Arcs in 0024+1654: Reconstruction of the Source Image”. In: *The Astrophysical Journal* 461.2 (Apr. 1996), p. L83. DOI: 10.1086/310015. URL: <https://dx.doi.org/10.1086/310015>.
- [62] Patrick L. Kelly et al. “Multiple images of a highly magnified supernova formed by an early-type cluster galaxy lens”. In: *Science* 347.6226 (2015), pp. 1123–1126. DOI: 10.1126/science.aaa3350. URL: <https://www.science.org/doi/10.1126/science.aaa3350>.
- [63] P. L. Kelly et al. “DEJA VU ALL OVER AGAIN: THE REAPPEARANCE OF SUPERNOVA REFSDAL”. In: *The Astrophysical Journal Letters* 819.1 (Feb. 2016), p. L8. DOI: 10.3847/2041-8205/819/1/L8. URL: <https://dx.doi.org/10.3847/2041-8205/819/1/L8>.

- [64] Douglas Clowe et al. “A Direct Empirical Proof of the Existence of Dark Matter*”. In: *The Astrophysical Journal* 648.2 (Aug. 2006), p. L109. DOI: 10.1086/508162. URL: <https://dx.doi.org/10.1086/508162>.
- [65] Indranil Banik et al. “Strong constraints on the gravitational law from Gaia DR3 wide binaries”. In: *Monthly Notices of the Royal Astronomical Society* 527.3 (Nov. 2023), pp. 4573–4615. ISSN: 0035-8711. DOI: 10.1093/mnras/stad3393. eprint: <https://academic.oup.com/mnras/article-pdf/527/3/4573/53896686/stad3393.pdf>. URL: <https://doi.org/10.1093/mnras/stad3393>.
- [66] Alexander Yelland. *AYelland/AstroWiki_2.0: AstroWiki_2.0 (v2.0)*. 2025. DOI: 10.5281/zenodo.14755789. URL: <https://doi.org/10.5281/zenodo.14755789>.
- [67] Joseph Levine et al. *New Limit on Dark Photon Kinetic Mixing in the 0.2-1.2 μeV Mass Range From the Dark E-Field Radio Experiment*. 2024. arXiv: 2405.20444 [hep-ex]. URL: <https://arxiv.org/abs/2405.20444>.
- [68] J.I. Read. “The Local Dark Matter Density”. In: *J. Phys.* G41 (2014), p. 063101. URL: <https://arxiv.org/abs/1404.1938>.
- [69] S Sivertsson et al. “Estimating the local dark matter density in a non-axisymmetric wobbling disc”. In: *Monthly Notices of the Royal Astronomical Society* 511.2 (Apr. 2022), pp. 1977–1991. ISSN: 0035-8711. DOI: 10.1093/mnras/stac094.
- [70] P. Arias et al. “WISPy cold dark matter”. In: *J. Cosmol. Astropart. Phys.* 2012.06 (June 2012), p. 013. URL: <https://arxiv.org/abs/1201.5902>.

- [71] Tim Zimmermann et al. *Dwarf galaxies imply dark matter is heavier than 2.2×10^{-21} eV*. 2024. arXiv: 2405.20374 [astro-ph.CO]. URL: <https://arxiv.org/abs/2405.20374>.
- [72] Alex Drlica-Wagner et al. *Probing the Fundamental Nature of Dark Matter with the Large Synoptic Survey Telescope*. 2019. arXiv: 1902.01055 [astro-ph.CO]. URL: <https://arxiv.org/abs/1902.01055>.
- [73] Gianfranco Bertone and Tim M. P. Tait. “A new era in the search for dark matter”. In: *Nature* 562 (2018), pp. 51–56. DOI: 10.1038/s41586-018-0542-z. URL: <https://doi.org/10.1038/s41586-018-0542-z>.
- [74] André de Gouvêa, Daniel Hernández, and Tim M. P. Tait. “Criteria for natural hierarchies”. In: *Phys. Rev. D* 89 (11 June 2014), p. 115005. DOI: 10.1103/PhysRevD.89.115005. URL: <https://link.aps.org/doi/10.1103/PhysRevD.89.115005>.
- [75] J. Aalbers et al. *Dark Matter Search Results from 4.2 Tonne-Years of Exposure of the LUX-ZEPLIN (LZ) Experiment*. 2024. arXiv: 2410.17036 [hep-ex]. URL: <https://arxiv.org/abs/2410.17036>.
- [76] Ioannis Manthos. “DarkSide-20k: Next generation Direct Dark Matter searches with liquid Argon”. In: *Proceedings of The European Physical Society Conference on High Energy Physics — PoS(EPS-HEP2023)*. EPS-HEP2023. Sissa Medialab, Feb. 2024, p. 113. DOI: 10.22323/1.449.0113. URL: <http://dx.doi.org/10.22323/1.449.0113>.

- [77] Gianfranco Bertone. “The moment of truth for WIMP dark matter”. In: *Nature* 468.7322 (Nov. 2010), pp. 389–393. ISSN: 1476-4687. DOI: 10.1038/nature09509.
- [78] R. D. Peccei and Helen R. Quinn. “CP Conservation in the Presence of Pseudoparticles”. In: *Phys. Rev. Lett.* 38 (25 June 1977), pp. 1440–1443. DOI: 10.1103/PhysRevLett.38.1440. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.38.1440>.
- [79] Michael Dine and Willy Fischler. “The not-so-harmless axion”. In: *Physics Letters B* 120.1 (1983), pp. 137–141. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(83\)90639-1](https://doi.org/10.1016/0370-2693(83)90639-1). URL: <https://www.sciencedirect.com/science/article/pii/0370269383906391>.
- [80] John Preskill, Mark B. Wise, and Frank Wilczek. “Cosmology of the invisible axion”. In: *Physics Letters B* 120.1 (1983), pp. 127–132. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(83\)90637-8](https://doi.org/10.1016/0370-2693(83)90637-8). URL: <https://www.sciencedirect.com/science/article/pii/0370269383906378>.
- [81] J R Primack, D Seckel, and B Sadoulet. “Detection of Cosmic Dark Matter”. In: *Annual Review of Nuclear and Particle Science* 38. Volume 38, 1988 (1988), pp. 751–807. ISSN: 1545-4134. DOI: <https://doi.org/10.1146/annurev.ns.38.120188.003535>. URL: <https://www.annualreviews.org/content/journals/10.1146/annurev.ns.38.120188.003535>.

- [82] Pierre Sikivie. “The Pool-Table Analogy with Axion Physics”. In: *Physics Today* 49.12 (Dec. 1996), pp. 22–27. ISSN: 1945-0699. DOI: 10.1063/1.881573. URL: <http://dx.doi.org/10.1063/1.881573>.
- [83] J. H. Christenson et al. “Evidence for the 2π Decay of the K_2^0 Meson”. In: *Phys. Rev. Lett.* 13 (4 July 1964), pp. 138–140. DOI: 10.1103/PhysRevLett.13.138. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.13.138>.
- [84] Anson Hook. *TASI Lectures on the Strong CP Problem and Axions*. 2023. arXiv: 1812.02669 [hep-ph]. URL: <https://arxiv.org/abs/1812.02669>.
- [85] I. Stern. *ADMX Status*. 2017. arXiv: 1612.08296 [physics.ins-det]. URL: <https://arxiv.org/abs/1612.08296>.
- [86] Steven Weinberg. “A New Light Boson?” In: *Phys. Rev. Lett.* 40 (4 Jan. 1978), pp. 223–226. DOI: 10.1103/PhysRevLett.40.223. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.40.223>.
- [87] F. Wilczek. “Problem of Strong P and T Invariance in the Presence of Instantons”. In: *Phys. Rev. Lett.* 40 (5 Jan. 1978), pp. 279–282. DOI: 10.1103/PhysRevLett.40.279. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.40.279>.
- [88] ASU News. *How did Axions get their name? Frank Wilczek: Arizona State University (ASU)*. Accessed: 2025-02-11. April 30, 2021. URL: <https://www.youtube.com/watch?v=C7KFBo6jV5w>.

- [89] Ann E. Nelson and Jakub Scholtz. “Dark light, dark matter, and the misalignment mechanism”. In: *Physical Review D* 84.10 (Nov. 2011). ISSN: 1550-2368. DOI: 10.1103/PhysRevD.84.103501. URL: <http://dx.doi.org/10.1103/PhysRevD.84.103501>.
- [90] Jihn E. Kim. “Weak-Interaction Singlet and Strong CP Invariance”. In: *Phys. Rev. Lett.* 43 (2 July 1979), pp. 103–107. DOI: 10.1103/PhysRevLett.43.103. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.43.103>.
- [91] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov. “Can confinement ensure natural CP invariance of strong interactions?” In: *Nuclear Physics B* 166.3 (1980), pp. 493–506. ISSN: 0550-3213. DOI: [https://doi.org/10.1016/0550-3213\(80\)90209-6](https://doi.org/10.1016/0550-3213(80)90209-6). URL: <https://www.sciencedirect.com/science/article/pii/0550321380902096>.
- [92] Michael Dine, Willy Fischler, and Mark Srednicki. “A simple solution to the strong CP problem with a harmless axion”. In: *Physics Letters B* 104.3 (1981), pp. 199–202. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(81\)90590-6](https://doi.org/10.1016/0370-2693(81)90590-6). URL: <https://www.sciencedirect.com/science/article/pii/0370269381905906>.
- [93] A. R. Zhitnitsky. “On Possible Suppression of the Axion Hadron Interactions. (In Russian)”. In: *Sov. J. Nucl. Phys.* 31 (1980), p. 260.
- [94] H. Primakoff. “Photo-Production of Neutral Mesons in Nuclear Electric Fields and the Mean Life of the Neutral Meson”. In: *Phys. Rev.* 81 (5 Mar. 1951), pp. 899–899. DOI: 10.1103/PhysRev.81.899. URL: <https://link.aps.org/doi/10.1103/PhysRev.81.899>.

- [95] P. Sikivie. “Experimental Tests of the ”Invisible” Axion”. In: *Phys. Rev. Lett.* 51 (16 Oct. 1983), pp. 1415–1417. DOI: 10.1103/PhysRevLett.51.1415. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.51.1415>.
- [96] Ciaran O’Hare. *cajohare/AxionLimits: AxionLimits*. <https://cajohare.github.io/AxionLimits/>. Version v1.0. July 2020. DOI: 10.5281/zenodo.3932430.
- [97] Ann E. Nelson and Jakub Scholtz. “Dark light, dark matter, and the misalignment mechanism”. In: *Phys. Rev. D* 84 (10 Nov. 2011), p. 103501. DOI: 10.1103/PhysRevD.84.103501. URL: <https://link.aps.org/doi/10.1103/PhysRevD.84.103501>.
- [98] Peter W. Graham et al. “Dark matter direct detection with accelerometers”. In: *Phys. Rev. D* 93 (7 Apr. 2016), p. 075029. DOI: 10.1103/PhysRevD.93.075029. URL: <https://link.aps.org/doi/10.1103/PhysRevD.93.075029>.
- [99] P.W. Graham, J. Mardon, and S. Rajendran. “Vector Dark Matter from Inflationary Fluctuations”. In: *Phys. Rev. D* 93 (2016), p. 103520. URL: <https://arxiv.org/abs/1504.02102>.
- [100] Marco Fabbrichesi, Emidio Gabrielli, and Gaia Lanfranchi. *The Physics of the Dark Photon: A Primer*. Springer International Publishing, 2021. ISBN: 9783030625191. DOI: 10.1007/978-3-030-62519-1. URL: <http://dx.doi.org/10.1007/978-3-030-62519-1>.
- [101] Michael A. Fedderke et al. “Earth as a transducer for dark-photon dark-matter detection”. In: *Phys. Rev. D* 104 (7 Oct. 2021), p. 075023. DOI: 10.1103/PhysRevD.104.075023. URL: <https://link.aps.org/doi/10.1103/PhysRevD.104.075023>.

- [102] P. W. Graham et al. “Parametrically enhanced hidden photon search”. In: *Physical Review D* 90.7 (Oct. 2014). ISSN: 1550-2368. DOI: 10.1103/physrevd.90.075017. URL: <http://dx.doi.org/10.1103/PhysRevD.90.075017>.
- [103] B. Holdom. “Two U(1)’s and ϵ charge shifts”. In: *Phys. Lett. B* 166.2 (Jan. 1986), p. 196. DOI: 10.1016/0370-2693(86)91377-8.
- [104] HENRI RUEGG and MARTÍ RUIZ-ALTABA. “THE STUECKELBERG FIELD”. In: *International Journal of Modern Physics A* 19.20 (2004), pp. 3265–3347. DOI: 10.1142/S0217751X04019755. eprint: <https://doi.org/10.1142/S0217751X04019755>. URL: <https://doi.org/10.1142/S0217751X04019755>.
- [105] Andrea Caputo et al. “Dark photon limits: A handbook”. In: *Phys. Rev. D* 104 (9 Nov. 2021), p. 095029. DOI: 10.1103/PhysRevD.104.095029. URL: <https://link.aps.org/doi/10.1103/PhysRevD.104.095029>.
- [106] V. Radeka. “Signal, Noise and Resolution in Position-Sensitive Detectors”. In: *IEEE Transactions on Nuclear Science* 21.1 (1974), pp. 51–64. DOI: 10.1109/TNS.1974.4327444.
- [107] Don S. Lemons, William R. Shanahan, and Louis J. Buchholtz. *On the Trail of Blackbody Radiation: Max Planck and the Physics of his Era*. The MIT Press, 2022. ISBN: 9780262047043.
- [108] B. M. Brubaker et al. “HAYSTAC axion search analysis procedure”. In: *Physical Review D* 96.12 (Dec. 2017). DOI: 10.1103/physrevd.96.123008. URL: <https://doi.org/10.1103/physrevd.96.123008>.

- [109] C. Boutan et al. *Axion Dark Matter eXperiment: Run 1A Analysis Details*. 2023. arXiv: 2312.16668.
- [110] Maximiliano Silva-Feaver et al. “Design Overview of DM Radio Pathfinder Experiment”. In: *IEEE Transactions on Applied Superconductivity* 27.4, 2631425 (June 2017), p. 2631425. DOI: 10.1109/TASC.2016.2631425.
- [111] D.A. Hill. “Plane wave integral representation for fields in reverberation chambers”. In: *IEEE Transactions on Electromagnetic Compatibility* 40.3 (1998), pp. 209–217. DOI: 10.1109/15.709418.
- [112] W. L. Stutzman and G. A. Thiele. *Antenna Theory and Design*. John Wiley Sons, 1998. ISBN: 978-0-479-57664-9.
- [113] John D. Kraus. *Antennas*. 2nd. New York: McGraw-Hill, 1988. ISBN: 0-07-463219-1.
- [114] H. Nyquist. “Thermal Agitation of Electric Charge in Conductors”. In: *Phys. Rev.* 32 (1 July 1928), pp. 110–113. DOI: 10.1103/PhysRev.32.110. URL: <https://link.aps.org/doi/10.1103/PhysRev.32.110>.
- [115] J. B. Johnson. “Thermal Agitation of Electricity in Conductors”. In: *Phys. Rev.* 32 (1 July 1928), pp. 97–109. DOI: 10.1103/PhysRev.32.97. URL: <https://link.aps.org/doi/10.1103/PhysRev.32.97>.
- [116] R. H. Dicke. “The Measurement of Thermal Radiation at Microwave Frequencies”. In: *Review of Scientific Instruments* 17.7 (July 1946), pp. 268–275. ISSN: 0034-6748. DOI: 10.1063/1.1770483. URL: <https://doi.org/10.1063/1.1770483>.

- [117] Albert Einstein. “Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen”. In: *Annalen der Physik* 322.8 (1905), pp. 549–560.
- [118] Herbert B. Callen and Theodore A. Welton. “Irreversibility and Generalized Noise”. In: *Phys. Rev.* 83 (1 July 1951), pp. 34–40. DOI: 10.1103/PhysRev.83.34. URL: <https://link.aps.org/doi/10.1103/PhysRev.83.34>.
- [119] C. L. Mehta. “Coherence-time and effective bandwidth of blackbody radiation”. In: *Il Nuovo Cimento (1955-1965)* 28.2 (1963), pp. 401–408. DOI: 10.1007/BF02828589. URL: <https://doi.org/10.1007/BF02828589>.
- [120] Raphael Cervantes et al. *Deepest Sensitivity to Wavelike Dark Photon Dark Matter with SRF Cavities*. 2022. arXiv: 2208.03183 [hep-ex].
- [121] Alexander V. Gramolin et al. “Spectral signatures of axionlike dark matter”. In: *Physical Review D* 105.3 (Feb. 2022). ISSN: 2470-0029. DOI: 10.1103/physrevd.105.035029. URL: <http://dx.doi.org/10.1103/PhysRevD.105.035029>.
- [122] Michael S. Turner. “Periodic signatures for the detection of cosmic axions”. In: *Phys. Rev. D* 42 (10 Nov. 1990), pp. 3572–3575. DOI: 10.1103/PhysRevD.42.3572. URL: <https://link.aps.org/doi/10.1103/PhysRevD.42.3572>.
- [123] Greg Durgin. *Lecture on Noise Figure*. Accessed: 2024-06-23. 2017. URL: https://www.youtube.com/watch?v=M_SRUF4TQgA.

- [124] Pasternack Enterprises. *PE15A1012: 1 Watt Amplifier Operating From 0.01 GHz to 6 GHz*. Accessed: 2024-05-17. 2024. URL: <https://www.pasternack.com/images/ProductPDF/PE15A1012.pdf>.
- [125] SP Devices. *ADQ32 Datasheet*. Accessed: 2024-06-23. 2024. URL: https://www.spdevices.com/en-us/Products_/Documents/ADQ32/20-2378%20ADQ32%20datasheet.pdf.
- [126] L.A. Wainstein and V.D. Zubakov. *Extraction of Signals from Noise*. Englewood Cliffs, NJ: Prentice-Hall, 1962. ISBN: 9780132981330.
- [127] Benjamin Godfrey et al. “Search for dark photon dark matter: Dark E field radio pilot experiment”. In: *Phys. Rev. D* 104 (1 July 2021), p. 012013. DOI: 10.1103/PhysRevD.104.012013. URL: <https://link.aps.org/doi/10.1103/PhysRevD.104.012013>.
- [128] Andy Marvin and Simon Jonathan Bale. “Thermal Noise Measurements in a Reverberation Chamber”. English. In: *IEEE Transactions on Electromagnetic Compatibility* 64.3 (June 2022). © 2022 IEEE. This is an author-produced version of the published paper. Uploaded in accordance with the publisher’s self-archiving policy. Further copying may not be permitted; contact the publisher for details, pp. 893–896. ISSN: 0018-9375. DOI: 10.1109/TEM.2022.3142937.
- [129] F. Reif. *Fundamentals of Statistical and Thermal Physics*. Long Grove, IL: Waveland Press, 2009. ISBN: 9781577666127.

- [130] AG Caesar. *Black body realization*. Accessed: 2024-08-14. 2018. URL: https://commons.wikimedia.org/wiki/File:Black_body_realization.svg.
- [131] R. H. Dicke. “A Computational Method Applicable to Microwave Networks”. In: *Journal of Applied Physics* 18.10 (Oct. 1947), pp. 873–878. ISSN: 0021-8979. DOI: 10.1063/1.1697561. URL: <https://doi.org/10.1063/1.1697561>.
- [132] Hewlett-Packard. *S-Parameter Design*. Application Note 154. 1990. URL: https://www.hpmemoryproject.org/an/pdf/an_154.pdf.
- [133] NoiseWave. *NoiseWave FAQ Document*. Accessed: 2024-06-23. 2024. URL: <https://noisewave.com/faq.pdf>.
- [134] Whitham D. Reeve. *Noise Tutorial Part III: Attenuator and Amplifier Noise*. Revision 0.1, Updated: July 7, 2014. Reeve Engineers. 2014. URL: https://www.reeve.com/Documents/Noise/Reeve_Noise_3_AttenAmpNoise.pdf.
- [135] S. Al Kenany et al. “Design and operational experience of a microwave cavity axion detector for the 20-100 μeV range”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 854 (May 2017), pp. 11–24. ISSN: 0168-9002. DOI: 10.1016/j.nima.2017.02.012. URL: <http://dx.doi.org/10.1016/j.nima.2017.02.012>.
- [136] Raphael Cervantes et al. *Deepest Sensitivity to Wavelike Dark Photon Dark Matter with SRF Cavities*. 2022. arXiv: 2208.03183 [hep-ex].
- [137] Mauro B. Grando, Christian R. Boutan, and Jihee Yang. *A Multiport Approach to Thermal Noise and Scattering Parameter Simulation of Cryogenic Experiments*. 2022.

- arXiv: 2209.04008 [physics.ins-det]. URL: <https://arxiv.org/abs/2209.04008>
- 8.
- [138] G.H. Bryant. *Principles of Microwave Measurements*. London, United Kingdom: Peter Peregrinus Ltd., on behalf of the Institution of Electrical Engineers, 1993. ISBN: 0863412963.
 - [139] David A. Hill. “Wall Losses and Cavity Q”. In: *Electromagnetic fields in cavities: Deterministic and statistical theories*. John Wiley & Sons Inc., 2009, pp. 31–33.
 - [140] R.H. Price, H.T. Davis, and E.P. Wenaas. “Determination of the statistical distribution of electromagnetic-field amplitudes in complex cavities”. In: *Physical Review E* 48 (1993), pp. 4716–4729. DOI: 10.1103/PhysRevE.48.4716.
 - [141] John David Jackson. *Classical Electrodynamics*. 3rd. New York: Wiley, 1998. ISBN: 9780471309321.
 - [142] Bing-Hope Liu, David C. Chang, and Mark T. Ma. *Eigenmodes and the Composite Q Factor of a Reverberating Chamber*. Tech. rep. Aug. 1983.
 - [143] *Series 83 RF Shielded Enclosure*. Lindgren R.F. Enclosures, Inc. 2021. URL: <https://www.ets-lindgren.com/datasheet/shielding/rf-shielding-and-accessories/11003/1100307>.
 - [144] David A. Hill. *Electromagnetic Theory of Reverberation Chambers*. Technical Note 1506. Gaithersburg, MD: National Institute of Standards and Technology, 1998. URL: https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=24427.

- [145] D.A. Hill. “A reflection coefficient derivation for the Q of a reverberation chamber”. In: *IEEE Transactions on Electromagnetic Compatibility* 38.4 (1996), pp. 591–592. DOI: 10.1109/15.544314.
- [146] Lawrence Krauss et al. “Calculations for cosmic axion detection”. In: *Physical Review Letters* 55.17 (1985), pp. 1797–1800. DOI: 10.1103/PhysRevLett.55.1797. URL: <https://doi.org/10.1103/PhysRevLett.55.1797>.
- [147] *Biconical Antenna AB-900A*. Rev. D05.19. Com-Power Corporation. May 2019. URL: <https://documentation.com-power.com/pdf/AB-900A-1.pdf>.
- [148] *COMSOL Multiphysics*. 5.4. Available at <https://www.comsol.com/>. COMSOL, Inc. One First Street, Suite 4 Los Altos, CA 94022.
- [149] Zhong Chen and Michael D. Foegelle. “A numeric investigation of ground plane effects on biconical antenna factor”. In: *Electromagnetic Compatibility, 1998. 1998 IEEE International Symposium on*. Vol. 2. IEEE. IEEE, 1998, pp. 802–806. DOI: 10.1109/ISEMC.1998.750302. URL: <https://ieeexplore.ieee.org/document/750302>.
- [150] Mini-Circuits. *ZKL-1R5: 50Ω Gain Block Amplifier*. Accessed: 2024-06-23. 2024. URL: <https://www.minicircuits.com/pdfs/ZKL-1R5.pdf>.
- [151] Texas Instruments. *LM317: 3-Terminal Adjustable Regulator*. Accessed: 2024-06-23. 2024. URL: <https://www.ti.com/lit/ds/symlink/lm317.pdf?ts=1720749307506>.
- [152] Alan V. Oppenheim, Alan S. Willsky, and S. Hamid Nawab. *Signals and Systems*. 2nd. Upper Saddle River, NJ: Prentice Hall, 1996. ISBN: 978-0138147570.

- [153] Mini-Circuits. *ZX75LP-288-S+: Low Pass Filter*. Accessed: 2024-06-23. 2024. URL: <https://www.minicircuits.com/pdfs/ZX75LP-288-S+.pdf>.
- [154] Mini-Circuits. *SHP-50+: High Pass Filter*. Accessed: 2024-06-23. 2024. URL: <https://www.minicircuits.com/pdfs/SHP-50+.pdf>.
- [155] *Biconical Antenna*. AB-900A. Rev. D. Com-Power Corporation. May 2019.
- [156] *The Y Factor Technique For Noise Figure Measurements*. 1MA178. Version 5e. Rhode & Schwarz. Oct. 2021.
- [157] *Noise Figure Measurement Accuracy: The Y-Factor Method*. 1MA178. Version 5e. Keysight Technologies. Oct. 2021.
- [158] Wikipedia contributors. *Y-factor Sketch*. Accessed: 2024-06-23. 2024. URL: https://en.wikipedia.org/wiki/Y-factor#/media/File:Y-factor_sketch.jpg.
- [159] Rigol Technologies. *RSA5000 Series: Real-Time Spectrum Analyzer Datasheet*. Accessed: 2024-06-23. 2024. URL: https://beyondmeasure.rigoltech.com/acton/attachment/1579/f-0816/1/-/-/-/-/RSA5_datasheet.pdf.
- [160] “IEEE Standard Method for Measuring the Effectiveness of Electromagnetic Shielding Enclosures”. In: *IEEE Std 299-1997* (1998), pp. 1–48. DOI: 10.1109/IEEESTD.1998.88115.
- [161] Valon Technology, LLC. *5009a Dual Frequency Synthesizer Module Operations Manual*. Version 1.39. Updated September 8, 2023. Northern California, USA, 2023. URL: https://www.valontechnology.com/5009users/Valon_5009_opman.pdf.

- [162] David W. Allan. “Statistics of atomic frequency standards”. In: *Proceedings of the IEEE* 54.2 (1966), pp. 221–230.
- [163] Keysight Technologies. *Analyzing Frequency Stability in the Frequency and Time Domains*. Tech. rep. 5991-4797EN, August 11, 2014. USA: Keysight Technologies, 2014.
- [164] Stanford Research Systems. *FS725 Rubidium Frequency Standard Manual*. Accessed: 2024-06-23. 2024. URL: <https://www.thinksrs.com/downloads/PDFs/Manuals/FS725m.pdf>.
- [165] R. H. Dicke. “The Measurement of Thermal Radiation at Microwave Frequencies”. In: *Rev. Sci. Instrum.* 17 (1946), p. 268. DOI: 10.1063/1.1770483. URL: <https://aip.scitation.org/doi/10.1063/1.1770483>.
- [166] S. Al Kenany et al. “Design and operational experience of a microwave cavity axion detector for the 20-100 micro-eV Range”. In: *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 854 (May 2017), pp. 11–24. ISSN: 0168-9002. DOI: 10.1016/j.nima.2017.02.012. URL: <http://dx.doi.org/10.1016/j.nima.2017.02.012>.
- [167] G. Turin. “An introduction to matched filters”. In: *IRE Transactions on Information Theory* 6.3 (1960), pp. 311–329. DOI: 10.1109/TIT.1960.1057571.
- [168] Robert N McDonough and Anthony D Whalen. *Detection of signals in noise*. 2nd. Academic Press, 1995. ISBN: 0127448500.

- [169] S. Asztalos et al. “Large-scale microwave cavity search for dark-matter axions”. In: *Phys. Rev. D* 64 (9 Oct. 2001), p. 092003. DOI: 10.1103/PhysRevD.64.092003. URL: <https://link.aps.org/doi/10.1103/PhysRevD.64.092003>.
- [170] G. Orjubin et al. “Statistical model of an undermoded reverberation chamber”. In: *IEEE Transactions on Electromagnetic Compatibility* 48.1 (2006), pp. 248–251. DOI: 10.1109/TEM.2006.870705.
- [171] B. Efron. “Bootstrap Methods: Another Look at the Jackknife”. In: *The Annals of Statistics* 7.1 (1979), pp. 1–26. DOI: 10.1214/aos/1176344552. URL: <https://doi.org/10.1214/aos/1176344552>.
- [172] R. Khatiwada et al. “Axion Dark Matter Experiment: Detailed design and operations”. In: *Review of Scientific Instruments* 92.12 (Dec. 2021), p. 124502. ISSN: 0034-6748. DOI: 10.1063/5.0037857. eprint: https://pubs.aip.org/aip/rsi/article-pdf/doi/10.1063/5.0037857/16136265/124502\1_online.pdf. URL: <https://doi.org/10.1063/5.0037857>.
- [173] Yuqi Zhu et al. “An improved synthetic signal injection routine for the Haloscope At Yale Sensitive To Axion Cold dark matter (HAYSTAC)”. In: *Review of Scientific Instruments* 94.5 (May 2023). ISSN: 1089-7623. DOI: 10.1063/5.0137870. URL: <http://dx.doi.org/10.1063/5.0137870>.
- [174] Frank Hampel et al. *Robust Statistics: The Approach Based on Influence Functions*. John Wiley & Sons, Mar. 1986. ISBN: 9780471735779. DOI: 10.1002/9781118186435.

- [175] S. Navas et al. “Review of particle physics”. In: *Phys. Rev. D* 110.3 (2024), p. 030001. DOI: 10.1103/PhysRevD.110.030001.
- [176] N. Du et al. “Search for Invisible Axion Dark Matter with the Axion Dark Matter Experiment”. In: *Phys. Rev. Lett.* 120 (15 Apr. 2018), p. 151301. URL: <https://arxiv.org/abs/1804.05750>.
- [177] Jonathan L. Ouellet et al. “Design and implementation of the ABRACADABRA-10 cm axion dark matter search”. In: *Phys. Rev. D* 99 (5 Mar. 2019), p. 052012. DOI: 10.1103/PhysRevD.99.052012. URL: <https://link.aps.org/doi/10.1103/PhysRevD.99.052012>.
- [178] DMRadio Collaboration et al. *Electromagnetic modeling and science reach of DMRadio- m^3* . 2023. arXiv: 2302.14084 [hep-ex]. URL: <https://arxiv.org/abs/2302.14084>.
- [179] Igor Agafonov. “Redesign of a Mode Stirrer for the Large Reverberation Chamber in Magdeburg”. Master’s thesis. Otto von Guericke Universität Magdeburg, May 2020.
- [180] Ferenc Marki and Christopher Marki. *Mixer Basics Primer: A Tutorial for RF & Microwave Mixers*. Marki Microwave, Inc. Morgan Hill, CA, 2010. URL: https://markimicrowave.com/assets/c2c4688b-15c7-4421-a703-254cb238f9fb/Mixer_Basics_Primer.pdf.
- [181] David M. Pozar. *Microwave Engineering*. 4th ed. Hoboken, NJ: Wiley, 2011. ISBN: 978-0470631553.
- [182] Wikipedia contributors. *Superheterodyne receiver*. Accessed: 2024-06-23. 2024. URL: https://en.wikipedia.org/wiki/Superheterodyne_receiver.

- [183] Chetvorno. *Superheterodyne image problem*. Accessed: 2024-11-14. 2024. URL: https://commons.wikimedia.org/wiki/File:Superheterodyne_image_problem.svg.
- [184] Chetvorno. *How superheterodyne receiver works*. Accessed: 2024-11-14. 2024. URL: https://commons.wikimedia.org/wiki/File:How_superheterodyne_receiver_works.svg.
- [185] Narda-MITEQ. *TB0440LW1 4 to 40 GHz Triple-Balanced Mixer*. Technical Bulletin TB-0440, Revised: April 24, 2012. 2012. URL: <https://nardamiteq.com/docs/MITEQ-TB0440.PDF>.
- [186] Marki Microwave. *Spur Calculator*. Accessed: 2024-06-23. 2024. URL: <https://markimicrowave.com/technical-resources/tools/spur-calculator>.
- [187] Low Noise Factory. *LNF-LNC0.3-14B: 0.3-14 GHz Cryogenic Low Noise Amplifier*. Datasheet, Revised: 2023-02-24. Low Noise Factory AB. Nellickevägen 24, 412 63 Göteborg, Sweden, 2023. URL: https://lownoisefactory.com/wp-content/uploads/2023/03/lnf-lnc0-3_14b.pdf.
- [188] Valon Technology, LLC. *5015/5019 Frequency Synthesizer Module Operations Manual*. Version 1.21. Updated February 17, 2023. Redwood City, CA, USA, 2023. URL: https://valontechnology.com/5015users/Valon_5015-5019_opman.pdf.
- [189] Arran Phipps et al. “Exclusion limits on hidden-photon dark matter near 2 neV from a fixed-frequency superconducting lumped-element resonator”. In: *Microwave Cavities and Detectors for Axion Research*. Springer, 2020, pp. 139–145.

- [190] Shoji Asai et al. *Exploring the Quantum Universe: Pathways to Innovation and Discovery in Particle Physics. Pathways to Innovation and Discovery in Particle Physics. Report of the 2023 Particle Physics Project Prioritization Panel (P5)*. Tech. rep. 2023. DOI: 10.2172/2368847. arXiv: 2407.19176. URL: <https://cds.cern.ch/record/2905898>.
- [191] C. B. Adams et al. *Axion Dark Matter*. 2023. arXiv: 2203.14923 [hep-ex]. URL: <https://arxiv.org/abs/2203.14923>.
- [192] Alexander V. Gramolin et al. “Search for axion-like dark matter with ferromagnets”. en. In: *Nature Physics* 17.1 (Jan. 2021), pp. 79–84. ISSN: 1745-2473, 1745-2481. DOI: 10.1038/s41567-020-1006-6.
- [193] Sumita Ghosh et al. “Searching for dark photons with existing haloscope data”. In: *Physical Review D* 104.9 (Nov. 2021), p. 092016. DOI: 10.1103/PhysRevD.104.092016.