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Branko Leskovar

August 1968

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ESSENTIAL NONLINEARITY OF PHASE-SENSITIVE DETECTOR CHARACTERISTICS

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ABSTRACT

The effect of essential nonlinearity of detector characteristics in a phase-sensitive detection system is studied and determined theoretically in detail over a wide dynamic range of optimum and nonoptimum operating conditions, assuming that the input signal is in the narrow-band Gaussian noise. Minimum, maximum, and limiting values of nonlinearities of detector characteristics are determined by means of computer-aided analysis.

I. INTRODUCTION

In many cases of practical interest, there is concern about the nonlinear behavior of phase-sensitive system characteristics over a wide dynamic range of operating conditions. For example, in wide-band Fourier-transform high resolution nuclear-magnetic-resonance and electron-spin-resonance spectrometry, total system nonlinearity is of prime importance. System nonlinearity is determined by the essential nonlinearity of the phase-sensitive detector used. Essential nonlinearities result from the inherent detector behavior in detection amplitude or phase of the input signal in the presence of noise. They do not involve nonlinearities resulting from the nonlinearity of the characteristics of the electronic components used.

In previous work [1] essential nonlinearity analysis of detector characteristics was presented for particular operating conditions. Based on this, further effort has been expanded by means of computer-aided analysis to include a very wide dynamic range of optimum and nonoptimum operating conditions.

According to Fig. 1, the normalized form of the phase-sensitive characteristics used for determination of detector nonlinearities as a function of the input signal-to-noise ratio $x = V_s/V_\sigma$, and the reference wave-to-input noise ratio $\mu = V_c/V_\sigma$ is given by

$$\frac{V_0}{V_\sigma} = \eta_d \left(\frac{\pi}{2} \right)^{1/2} \left\{ u[v(x)] - y[t(x)] \right\}, \quad (1)$$

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Fig. 1. Phase-sensitive detector.

where functions $u[v(x)]$ and $y[t(x)]$ are given by

$$u[v(x)] = {}_1F_1 \left(-\frac{1}{2}; 1; -\frac{\mu^2 + x^2 + 2\mu x \cos \psi}{2} \right) \quad (2)$$

$$y[t(x)] = {}_1F_1 \left(-\frac{1}{2}; 1; -\frac{\mu^2 + x^2 - 2\mu x \cos \psi}{2} \right) \quad (3)$$

By using a notation consistent with Ref. 1, V_0 is the detector output signal, V_σ is the root-mean-square value of the input narrow-band noise, η_d is the detection efficiency, V_s is the amplitude of the input sine signal, V_c is the amplitude of the reference wave, ψ is the phase angle between the input signal and the reference wave, and ${}_1F_1$ denotes the confluent hypergeometric function defined by

$${}_1F_1(a; b; \pm p) = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \cdot \frac{(\pm p)^n}{n!} \quad (4)$$

$$\text{where } (a)_n = a(a+1) \dots (a+n-1), \quad (a)_0 = 1, \quad (5)$$

$$(b)_n = b(b+1) \dots (b+n-1), \quad (b)_0 = 1.$$

Generally, three important cases of the nonlinearity of the phase-sensitive detector characteristics have been found. In the first case, the detector nonlinearity N_A has been determined as a function of x for the phase angle ψ between the input signal and the reference wave equal to $2n\pi$, where $n = 0, \pm 1, \pm 2, \dots$. In the second and the third cases, the detector nonlinearities N_B and N_C have been obtained as a function of the phase angle and x . In all cases, nonlinearities N_A , N_B , and N_C have been evaluated from the extent of their departure from the tangent drawn from the characteristic at points $x = 0$, $\psi_2 = (2n+1)\pi/2$, and $\psi_1 = 2n\pi$, respectively.

II. NONLINEARITY OF DETECTOR CHARACTERISTICS RELATING TO THE INPUT SIGNAL-TO-NOISE RATIO FOR PHASE ANGLE $\psi = 2n\pi$

For this case the essential nonlinearity N_A can be calculated over a wide range of operating conditions by means of

$$N_A = 1 - \frac{1}{x\mu} \frac{\gamma[f(x)] - \omega[\epsilon(x)]}{\chi(\mu)} \quad (6)$$

$$\text{where } \gamma[f(x)] = {}_1F_1 \left(-\frac{1}{2}; 1; -\frac{\mu^2 + x^2}{2} \right) \quad (7)$$

$$\omega[\epsilon(x)] = {}_1F_1 \left(-\frac{1}{2}; 1; -\frac{\mu^2 - x^2}{2} \right) \quad (8)$$

$$\chi(\mu) = {}_1F_1 \left(\frac{1}{2}; 2; -\frac{\mu^2}{2} \right) \quad (9)$$

By means of a digital computer, the nonlinearity N_A as a function of x with μ as parameter is calculated and plotted (Fig. 2). High-accuracy numerical calculations of the confluent hypergeometric function in Eq. (6) are performed in two different ways. Where the variable of the hypergeometric function is smaller than 20, the function is expressed and calculated in terms of the modified Bessel functions of the first kind according to the relations²

$${}_1F_1\left(-\frac{1}{2}; 1; -p\right) = \exp\left(-\frac{p}{2}\right) \left[(1+p) I_0\left(\frac{p}{2}\right) + x I_1\left(\frac{p}{2}\right) \right] \quad (10)$$

$${}_1F_1\left(\frac{1}{2}; 2; -p\right) = \exp\left(-\frac{p}{2}\right) \left[I_0\left(\frac{p}{2}\right) + I_1\left(\frac{p}{2}\right) \right] \quad (11)$$

For the hypergeometric function variable larger than 20, asymptotic expansion is used according to the following formula²

$${}_1F_1(a; b; -p) \sim \frac{\Gamma(b)}{\Gamma(b-a)} p^{-a} \sum_{n=0}^{\infty} \frac{(a)_n (a-b+1)_n}{n! p^n} \quad (12)$$

where $\text{Re } p > 0$, $(a)_n$ is defined by Eq. (5), and

$$(a-b+1)_n = (1+a-b)(2+a-b) \dots (n+a-b); (a-b+1)_0 = 1. \quad (13)$$

The total error of the calculated nonlinearity values $N_A \approx 5 \times 10^{-7}$ if one uses expression (4) and the first eight terms on the right-hand side of Eq. (12), including computer limitations.

Curves in Fig. 2 show that the nonlinearity N_A can be considerably decreased by increasing μ/x . For example, increasing μ/x for one order of magnitude will decrease the nonlinearity by three orders of magnitude, for $x = 1$. Furthermore, the nonlinearity is even more reduced for

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Fig. 2. Behavior of the nonlinearity N_A for the phase angle $\psi = 2n\pi$.

larger amounts of x . It can be seen that for $\mu = 10$ increasing μ by one order of magnitude decreases the nonlinearity more than four orders of magnitude. Consequently, the gain in the decrease of nonlinearity will be larger for larger x for the same increase of μ/x . However, for purposes of obtaining the largest output signal and a wide dynamic range in the wide-band phase-sensitive detection application by using solid-state components, x is close in value to μ . In such applications the essential nonlinearity can have an appreciable value, especially for small μ and x . With increasing μ and x the essential nonlinearity is gradually decreased, as can be concluded by a comparison of the curves given in Fig. 2. For $\mu = x$, approximation values of the nonlinearity N_A can be seen from Fig. 2: 8.5, 5.2, 0.51, and 0.035% for x equals 1, 10, 10^2 , and 10^3 , respectively. However, if both μ and x are greater than 10, μ always has to be slightly larger than x . If this condition is not satisfied the nonlinearity can reach an unacceptably high value; e.g., for $\mu = 100$ and $x = 80$ the nonlinearity $\approx 0.01\%$. On the other hand, for $\mu = 100$ and $x = 120$, nonlinearity is more than three orders of magnitude larger, more than approximately 10%. This amount is unacceptable for most practical applications. Generally, the nonlinearity deterioration will be larger for larger values of both μ and x under previously mentioned conditions. Consequently, wherever the essential nonlinearity is of prime importance, the maximum x value should be always smaller than μ , irrespective of other demands that can be imposed on the phase-sensitive detection system for other reasons. This is particularly important in applications in which x varies in a wide dynamic range. If x is only 10% smaller than μ the nonlinearity N_A can be reduced by more than 1.5 orders of magnitude; this can be concluded by inspection and comparison of curves (in Fig. 2) for $\mu = 10^2, 10^3$, and 10^4 .

III. NONLINEARITIES OF DETECTOR CHARACTERISTICS RELATING TO THE PHASE ANGLE BETWEEN THE INPUT SIGNAL AND THE REFERENCE WAVE

The essential nonlinearities N_B and N_C of the detector characteristics with respect to the phase angle between the input signal and the reference wave can be determined over a wide range of operating conditions (according to Ref. 1) by expressions

$$N_B = 1 - \frac{1}{x\mu} \frac{u[v(x)] - y[t(x)]}{w[f(x)] \left(\frac{\pi}{2} - \psi \right)} \quad (14)$$

and

$$N_C = 1 - \frac{u[v(x)] - y[t(x)]}{\phi[g(x)] - z[h(x)]} \quad (15)$$

where other functions are defined by relations (2) and (3) and by

$$w[f(x)] = {}_1F_1 \left(\frac{1}{2}; 2; -\frac{\mu^2 + x^2}{2} \right), \quad (16)$$

$$\phi[g(x)] = {}_1F_1 \left[-\frac{1}{2}; 1; -\frac{(\mu + x)^2}{2} \right], \quad (17)$$

$$z[h(x)] = {}_1F_1 \left[-\frac{1}{2}; 1; -\frac{(\mu - x)^2}{2} \right]. \quad (18)$$

Nonlinearities N_B and N_C have been calculated in relation to the phase characteristic operating points $\psi_2 = (2n+1)\pi/2$ and $\psi_1 = 2n\pi$, respectively. Both nonlinearities depend upon ψ , x , and μ . By using Eqs. (14) through (18), nonlinearities N_B and N_C are numerically

calculated, and shown in Fig. 3. The signal-to-noise and the reference wave-to-noise ratios of 1, 10^2 , 10^3 , and 10^4 are chosen as parameters.

The curves in the figure show that for $\mu = x = 1$, the nonlinearity N_B will be smaller than 1% if the phase-angle deviation is $\Delta\psi \leq 0.95 \pi/12$. The phase-angle deviation can be $\Delta\psi \leq 1.76 \pi/12$ for the same nonlinearity value if $\mu = x \geq 10$. The maximum nonlinearity $N_{B,\max}$ appears at the point $\psi_1 = 2n\pi$, but its value depends on x and μ . For the case $\mu = x$, the following maximum nonlinearities $N_{B,\max}$ can be calculated from Eq. (14): 35.41, 15.28, 10.52, 10.02, and 9.97% for x of 1, 10, 10^2 , 10^3 , and 10^4 . Generally, the maximum nonlinearity will be smaller for a larger μ and x . However, to obtain a better insight into nonlinearity behavior with x variation, the nonlinearity N_B is calculated and plotted in Figs. 4 and 5 with fixed values of $\mu = 1, 10^2, 10^3$, and 10^4 , and $\psi = 0, \pi/6, \pi/4, \pi/3, 5\pi/12, 11\pi/24, 47\pi/96$, and $99\pi/200$.

The minimum nonlinearity $N_{B,\min}$ is obtained with respect to x , for μ and ψ as parameters, if the following condition is satisfied:

$$w[f(x)] \left\{ \left(\frac{x^2}{2} + \frac{\mu \cos \psi}{2} x \right) s[v(x)] - \left(\frac{x^2}{2} - \frac{\mu \cos \psi}{2} x \right) m[t(x)] + y[t(x)] - u[v(x)] \right\} - \left(\frac{x}{2} \right)^2 K[f(x)] \left\{ y[t(x)] - u[v(x)] \right\} = 0, \quad (19)$$

where $w[f(x)]$, $y[t(x)]$, $u[v(x)]$ are given by relations (16), (3), and (2), respectively. The other functions are defined by

$$s[v(s)] = {}_1F_1 \left(\frac{1}{2}; 2; -\frac{\mu^2 + x^2 + 2\mu x \cos \psi}{2} \right), \quad (20)$$

$$m[t(x)] = {}_1F_1 \left(\frac{1}{2}; 2; -\frac{\mu^2 + x^2 - 2\mu x \cos \psi}{2} \right), \quad (21)$$

$$K[f(x)] = {}_1F_1 \left(\frac{3}{2}; 3; -\frac{\mu^2 + x^2}{2} \right). \quad (22)$$

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Fig. 3. Comparison of nonlinearities N_B and N_C .

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Fig. 4. Nonlinearity N_B as a function of the input signal-to-noise ratio with the phase angle ψ as parameter and the reference wave-to-input noise ratio $V_c/V_\sigma = 1, 10$.

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Fig. 5. Nonlinearity N_B as a function of the input signal-to-noise ratio with the phase angle ψ as parameter and the reference wave-to-input noise ratio $V_c/V_\sigma = 10^2, 10^3, 10^4$.

From curves in Fig. 4 it can be seen that for $\mu = 1$ the nonlinearity N_B will be practically independent of x . Curves have a small minimum at the point $x = 2.482$. However, the strong dependence of nonlinearity upon the phase angle is readily evident. Furthermore, for $\mu \geq 10$ the nonlinearity minimum value $N_{B,\min}$ strongly depends upon μ and x ratios, as well as on phase angle ψ . For any phase-angle value, the detector characteristics will have a nonlinearity minimum if μ is close to x . Approximate values of the minimum nonlinearity input signal-to-noise ratio can be found from the transcendental equation (19) for any amount of μ and ψ by a numerical method. The minimum nonlinearity $N_{B,\min}$ is approximately a half order of magnitude smaller than its average value for almost any value of ψ . Generally, $N_{B,\min}$ will be smaller for larger x and μ .

Figure 3 also illustrates the behavior of nonlinearity N_C for $\mu = x = 1, 10, 10^2, 10^3$, and 10^4 . Nonlinearity $N_C < 1\%$ if the phase angle deviation $\Delta\psi \leq 0.53 \pi/12$, for $x = \mu = 1$. The phase-angle deviation can be $\Delta\psi \leq 0.28 \pi/12$ for the same nonlinearity value if $\mu = x \geq 10$. Furthermore, for obtaining the optimum detector operating conditions, the nonlinearity N_C is calculated and plotted in Figs. 6 and 7 for $\mu = 1, 10, 10^2, 10^3$, and 10^4 ; and for $\psi = 47\pi/96, \pi/3, \pi/4, \pi/12, \pi/24, \pi/48, \pi/96$, and $\pi/300$. The maximum nonlinearity $N_{C,\max}$ is calculated with respect to x , for μ and ψ as parameters, by a numerical method from

$$\left\{ \phi[g(x)] - z[h(x)] \right\} \left\{ \left(\frac{x}{2} - \frac{\mu}{2} \cos\psi \right) m[t(x)] - \left(\frac{x}{2} + \frac{\mu}{2} \cos\psi \right) s[v(x)] \right\} + \left\{ \left(\frac{x}{2} + \frac{\mu}{2} \right) \rho[g(x)] - \left(\frac{x}{2} - \frac{\mu}{2} \right) \ell[h(x)] \right\} \left\{ u[v(x)] - y[t(x)] \right\} = 0, \quad (23)$$

where $u[v(x)]$, $y[t(x)]$, $\phi[g(x)]$, $z[h(x)]$, $s[v(x)]$, and $m[t(x)]$ are given by relations (2), (3), (17), (18), (20), and (21), respectively. Other functions are defined by

$$\rho[g(x)] = {}_1F_1 \left[\frac{1}{2}; 2; -\frac{(\mu+x)^2}{2} \right], \quad (24)$$

$$\ell[h(x)] = {}_1F_1 \left[\frac{1}{2}; 2; -\frac{(\mu-x)^2}{2} \right] \quad (25)$$

From curves in Figs. 5 and 6 it follows that nonlinearity N_C will be almost independent of x for $\mu = 1$. For any value of ψ , curves have a maximum for $x = 2.536$. The nonlinearity will strongly depend on the phase angle for any values of μ and x as well as upon x if $\mu \geq 10$. Generally, in these cases, the nonlinearity N_C will have a maximum value for a given ψ , if the ratios μ and x are approximately equal. A numerical solution of transcendental equation (23) gives x of the maximum nonlinearity $N_{C,\max}$. It is approximately one to two orders of magnitude larger than its average value for any value of ψ . The maximum nonlinearity will be larger than its average value for larger μ and x and for smaller ψ .

CONCLUSIONS AND COMPARISONS

A comparison of Eqs. (14) and (15) and of curves for N_B and N_C in Fig. 3 shows that the nonlinearity N_B is smaller than the nonlinearity N_C for the same phase-angle deviation $\Delta\psi$ around the operating point. In addition to the above curves (N_B and N_C versus ψ for constant x), another set of curves in Figs. 4 and 5 shows N_B and N_C versus x for constant μ and ψ . The curves of Figs. 4 and 5 show that for a fixed μ and ψ , the proper choice of x will minimize the nonlinearity N_B . The best value of the input signal-to-noise ratio can be found by solution of transcendental equation (19). For $\psi = \pi/4$ and $\mu = 1, 10, 10^2, 10^3$, and 10^4 , the nonlinearity

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Fig. 6. Nonlinearity N_C as a function of the input signal-to-noise ratio with the phase angle ψ as parameter and the reference wave-to-input noise ratio $V_c/V_\sigma = 1, 10$.

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Fig. 7. Nonlinearity N_C as a function of the input signal-to-noise ratio with the phase angle ψ as parameter and the reference wave-to-input noise ratio $V_c/V_\sigma = 10^2, 10^3, 10^4$.

N_B is minimized for $x = 2.48274, 1.00935, 1.00004, 1.00000$, and 1.00000 , respectively. Any other x value can be considered as a nonoptimum one, although the nonlinearity increase will not be significant for $\mu = 1$ and for small ψ . Furthermore, the proper value of x will also minimize the nonlinearity N_C , which can be concluded from Figs. 6 and 7. For this purpose it is important to avoid the region where curve N_C versus x has a maximum. This particularly nonoptimum x value can be found for any x and ψ by numerical solution of Eq. (23). If $\psi = \pi/24$ and $\mu = 1, 10, 10^2, 10^3$, and 10^4 , the nonlinearity N_C is maximized for $x = 2.53699, 1.01915, 1.00010, 1.00000$, and 1.00000 , respectively. From comparison of N_B and N_C curves it can be seen that nonoptimum values of x will have far larger influence on the nonlinearity N_C than on the nonlinearity N_B behavior. As such, the nonoptimum x values maximally increase the N_B for approximately a half order of magnitude in the worst case. However, N_C nonoptimum x values increase N_C for two orders of magnitude in the worst case. Consequently, one should choose, whenever possible, the phase angle $\psi_2 = (2n + 1)\pi/2$ as an operating point. In this case the larger permissible phase deviation for the same nonlinearity, and the significantly smaller nonlinearity for nonoptimum signal-to-noise ratio, will result in comparison with operating point $\psi = 2n\pi$ (case of N_C).

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