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Publication Date

1977

Submitted to Physics Letters

LBL-5597
Preprint C/

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January 1977

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

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LEL-5597

Preprint

Pion condensation and abnormal nuclear matter

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Abstract

We solve the sigma model in neutron matter with non-relativistic kinematics in mean-field approximation. The solution is characterized by a substantial reduction in the effective nucleon mass. In addition a pion condensate may or may not occur, depending on the value of the renormalized axial charge of the nucleon in the neutron matter.

⁺Work supported by ERDA

6004604717

The sigma model has been used recently to investigate the possible occurrence of two types of condensation in high-density nuclear matter: pion condensation in neutron stars, and "abnormal" nuclei with much larger binding energy and baryon number than normal nuclei. In these two areas of investigation the model has been solved with mutually exclusive assumptions. In the study of abnormal nuclei it was assumed that no pion condensate is present ⁽¹⁾; in the studies of pion condensation an assumption was introduced, to be described below, which guarantees that abnormal nuclear matter cannot be formed. ^{(2), (3)} Here we present the results of a semiclassical calculation in which the common framework of the sigma model is used to study the relation between the two types of condensation. In particular, we discuss the case of nonrelativistic neutron matter. The relativistic case and the extension to proton-neutron matter will be presented elsewhere.

We conclude from our investigation that the two types of condensation are not mutually exclusive: they may both be present when the model is solved with the same approximations and the same, physically reasonable choice of parameters. However, the presence of the pion condensate in the model depends more critically on the choice of parameters, in particular the magnitude of the renormalization of the axial-vector coupling constant g_A . In addition, the pion condensation may be more sensitive to physical effects not included in the simplest model: the exchange of heavier mesons responsible for the repulsive core of the nucleon-nucleon interaction, the effect of the $N^*(1236)$ resonance,

and corrections to the non-relativistic approximation for the nucleons.

The model Lagrangian is

$$\mathcal{L}(x) = \bar{\psi}(i\gamma - g(\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5))\psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{1}{2}m_0^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2)^2 + F_\pi m_\pi^2 \sigma \quad (1)$$

ψ , σ , and $\vec{\pi}$ are nucleon, sigma, and pion fields respectively, and F_π is fixed at 95 MeV. To facilitate a direct comparison we make the same choice of parameters as in ref. (3): $\lambda^2 = 50$ and $m_0^2 = 22 m_\pi^2$. The model is solved in the semiclassical approximation in which the meson fields are just classical fields or "condensates". Solving in the vacuum sector we have the usual results: $\langle \sigma \rangle_0 = F_\pi$, $\langle \vec{\pi} \rangle_0 = 0$, $m_\pi^2 = \lambda^2 F_\pi^2 - m_0^2$, and $m_\sigma^2 = 3\lambda^2 F_\pi^2 - m_0^2$. We parametrize the meson condensates in nuclear matter by

$$\begin{aligned} \langle \sigma \rangle &= A \cos \theta \\ \langle \vec{\pi} \rangle &= A \hat{\pi} \sin \theta \end{aligned} \quad (2)$$

where A , θ , and $\hat{\pi}$ are classical fields, $|\hat{\pi}| = 1$. As in refs. (2) and (3), we replace the mean fields by classical fields, parametrized as in eq. (2) with

$$\begin{aligned} \hat{\pi}_3 &= 0 \\ \hat{\pi}_\pm &= e^{\pm i\vec{k} \cdot \vec{x}} \end{aligned} \quad (3)$$

Including Lagrange multipliers (chemical potentials) to fix the baryon and charge densities, ρ_B and ρ_Q , the effective

Hamiltonian is

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & -i\bar{\psi}\gamma\cdot\nabla\psi + g\bar{\psi}\psi a + \frac{\lambda^2}{4}a^4 - \frac{1}{2}m_0^2 a^2 - v'\psi^+\psi \\ & + k_\mu [\bar{\psi}\gamma^\mu \frac{\tau_3}{2} \psi \cos \theta + g_A \bar{\psi}\gamma^\mu \gamma_5 \frac{\tau_2}{2} \psi \sin \theta] \\ & - \frac{1}{2}k_\mu k^\mu a^2 \sin^2 \theta - F_\pi m_\pi^2 a \cos \theta \end{aligned} \quad (4)$$

with $k^\mu = (\mu, \vec{k})$ where μ is the charge chemical potential and $v = v' + \frac{1}{2}\mu$ is the baryon chemical potential. As in ref. 3, the renormalized axial charge g_A is introduced into eq. (4) as a phenomenological device to incorporate partially the effect of quantum corrections to the semi-classical approximation. In vacuo, $g_A = 1.24$; in neutron matter, g_A is substantially reduced by the polarization of the nuclear medium.⁽⁴⁾ The contribution of the nucleon terms in (4) is evaluated by solving the Dirac equation for ψ which is implied by eq. (4), and then integrating over the Fermi sphere of bound nucleons. If the nucleon momenta are not too large compared to the effective nucleon mass ga , then the Dirac equation reduces to a Schrödinger equation. In this case the final expression for the energy is

$$\begin{aligned} E_{\text{eff}} &= \int d^3x \mathcal{H}_{\text{eff}} \\ &= \rho_B M_N a / F_\pi + \frac{3}{5} \frac{(3\pi^2)^{2/3} \rho_B^{5/3} F_\pi}{2M_N a} \\ &\quad + \frac{1}{2} \rho_B [\mu - \sqrt{\mu^2 \cos^2 \theta + g_A^2 k^2 \sin^2 \theta}] \\ &\quad + \frac{\lambda^2}{4} a^4 + \frac{1}{2} a^2 [(k^2 - \mu^2) \sin^2 \theta - m_0^2] - F_\pi m_\pi^2 a \cos \theta \end{aligned} \quad (5)$$

where $M_N = 938$ MeV and we have set $\rho_Q = 0$ (pure neutron matter) and have used $\partial E_{\text{eff}} / \partial v|_\mu = -\rho_B$ to eliminate v in favor of ρ_B . Only the Fermi sphere which corresponds to neutrons at $\theta = 0$ contributes to eq. (5), since for the range of parameters to be considered it can be shown that the proton-like eigenmodes are not bound. Eq. (5) reduces to eq. (3.41) of ref. (3) if we put $a = F_\pi$. For the details of the derivation the reader should consult ref. (3).

For a given value of ρ_B we wish to minimize E_{eff} . Requiring E_{eff} to be stationary with respect to variation of a , θ , k , and μ respectively we obtain the following equations:

$$0 = \rho_B M_N / F_\pi - \frac{3}{5} \frac{(3\pi^2)^{2/3} \rho_B^{5/3} F_\pi}{2M_N a^2} + \lambda^2 a^3 \quad (6)$$

$$\begin{aligned} &+ a [(k^2 - \mu^2) \sin^2 \theta - m_0^2] - F_\pi m_\pi^2 \cos \theta \\ 0 = & \frac{\rho_B (\mu^2 - g_A^2 k^2) \sin 2\theta}{4\sqrt{\mu^2 \cos^2 \theta + g_A^2 k^2 \sin^2 \theta}} + \frac{1}{2} a^2 (k^2 - \mu^2) \sin 2\theta \\ &+ F_\pi m_\pi^2 a \sin \theta \end{aligned} \quad (7)$$

$$0 = -k \sin^2 \theta \left\{ \frac{\rho_B g_A^2}{2\sqrt{\mu^2 \cos^2 \theta + g_A^2 k^2 \sin^2 \theta}} - a^2 \right\} \quad (8)$$

$$0 = \frac{1}{2} \rho_B - \frac{\rho_B \mu \cos^2 \theta}{2\sqrt{\mu^2 \cos^2 \theta + g_A^2 k^2 \sin^2 \theta}} - a^2 \mu \sin^2 \theta \quad (9)$$

Eliminating k and μ , we find

$$\begin{aligned} E_{\text{eff}}(a, \theta) = & \frac{1}{4} \lambda^2 a^4 - \frac{1}{2} m_0^2 a^2 - F_\pi m_\pi^2 a \cos \theta + \rho_B M_N a / F_\pi \\ & + \frac{3}{10} \frac{(3\pi^2)^{2/3} \rho_B^{5/3} F_\pi}{M_N a} - \frac{\rho_B^2 g_A^2}{8a^2} \left[1 - \frac{1}{\cos^2 \theta + g_A^2 \sin^2 \theta} \right] \end{aligned} \quad (10)$$

together with the conditions

$$0 = \lambda^2 \alpha^6 - m_0^2 \alpha^4 + \alpha^3 (\rho_B M_N / F_\pi - \frac{F_\pi m_\pi^2}{\cos \theta g_A^2 - 1} g_A^2) - \frac{3}{10} (3\pi^2)^{2/3} \rho_B^{5/3} F_\pi \alpha / M_N + \frac{1}{4} \rho_B^2 g_A^2 \quad (11)$$

$$\cos \theta = 1 \quad \text{or} \quad \frac{4m_\pi^2 F_\pi}{\rho_B^2} \alpha^3 \frac{g_A^2}{g_A^2 - 1} (1 - \cos^2 \theta \frac{g_A^2 - 1}{g_A^2})^2 \quad (12)$$

A minimum of E_{eff} with $\theta \neq 0$ corresponds to a pion-condensed phase.

We have obtained numerical solutions to eqs. (11) and (12) as a function of ρ_B , for various values of $g_A > 1$. Each solution has been checked to determine whether it represents a local minimum or a saddle point of E_{eff} in the 3 dimensional space (α, θ, k) . For small values of ρ_B , there is always only one solution, a local minimum for which $\theta=0$ and α is only slightly less than F_π , corresponding to normal neutron matter at the given density (in this case, since there is no pion condensate, k and μ are of course undetermined). As the density is increased somewhat beyond that of normal nuclear matter ($= 0.47 m_\pi^3$) the "normal" phase becomes unstable, and the model predicts one or more phase transitions to "pion-condensed" ($\theta = 0$) and or "abnormal" ($\alpha < F_\pi$) phases. The number and nature of the predicted phase transitions seems to depend critically on the value of g_A .

We consider first the case where g_A is fairly large (> 1.12 for the chosen values of the other parameters). As

ρ_B is increased beyond a critical density ρ_B^* (fig. 1), there is a second-order transition to a phase containing a pion condensate, $\theta = 0$. At densities slightly above the critical density, the $\theta = 0$ solution becomes a saddle point rather than a local minimum; instead, a local minimum is found at a value of θ which increases smoothly from zero as the density is increased above ρ_B^* . Surprisingly, this branch of the solution has an extremely ephemeral existence; it only exists for a range of densities a few percent greater than ρ_B^* . As the density is further increased past a second critical density ρ_B^{**} , the local minimum merges with a saddle point which approaches from larger θ and smaller α , and both stationary points disappear. Even though the pion-condensed phase is only found in a very narrow density interval, the condensate fraction $\sin^2 \theta$ can become quite substantial (fig. 2). In the pion condensate phase, the effective nucleon mass $M_N \alpha / F_\pi$ continues to decrease considerably (fig. 3). At densities greater than ρ_B^{**} , there is no local minimum of the energy in the non-relativistic regime.

For smaller values of g_A (< 1.11 for the case investigated here), only one phase transition is found. As in the case of larger g_A , the normal phase becomes unstable (turns into a saddle point) above the critical density. However, unlike the case discussed above, there is no local minimum corresponding to pion condensation. Inspection of the numerical solutions shows that in this case the saddle point responsible for the second phase transition in the case of large g_A develops

and, as the density increases, approaches the normal-phase minimum before the pion-condensed minimum appears. This is in contrast to the investigation of ref. 3, where Q was constrained at its vacuum value F_π , and the pion condensation was shown to persist - albeit at increasing densities - as g_A approaches unity from above. For $g_A < 1.11$ the pion condensation might be an artifact of constraining Q . We should remark that the considerations of ref. 4 imply that $g_A \lesssim 1.1$.

In summary, our study shows that the sigma model of neutron matter in the mean-field, non-relativistic approximation has for a physically plausible choice of parameters a phase embodying both a substantial reduction of the effective nucleon mass and also a pion condensate. But we also find with another, perhaps even more plausible choice of the parameters that the pion condensation may be entirely absent from the solutions. However it is important to appreciate the limitations imposed on the calculation by the nonrelativistic approximation. First, we do not know if the two phases discussed above are actually ground states of the system, since at any value of ρ_B there might be a more deeply bound state with $Q \ll F_\pi$ which could not be described by the nonrelativistic equations. Second, even for the range of densities and values of Q found for the pion-condensed phase, the error in the baryon Fermi energy due to nonrelativistic kinematics is larger than the condensation energy, so that a relativistic calculation could give quite different results. In particular, it is possible that the saddle point which "annihilates" the pion condensate at

ρ_B^{**} for $g_A > 1.12$ and which prevents the pion condensate from developing for $g_A < 1.11$ is itself an artifact of the artificial instability of the nonrelativistic formulae for very large k and very small Q ⁽⁵⁾. The question can only really be decided by solving the model with relativistic kinematics. The correct treatment of relativistic kinematics is possible, and under investigation.

It is clear that the simple model discussed here can certainly not provide detailed quantitative predictions of the properties of high-density matter. For example, the effect of repulsive internucleon forces will have to be treated very carefully, since the presence or absence of pion condensation in the model depends critically on the effect of the nuclear medium on both g_A and the effective nucleon mass. It will also be necessary to consider the effect of the $N^*(1236)$ resonance, which in the fixed $Q = F_\pi$ calculations ⁽⁴⁾ tends to decrease the sensitivity of the dependence on g_A .

The sigma model is not of interest for its phenomenological utility, which is not very great, but because it incorporates the low-energy theorems of chiral $SU(2)$ which provide a general and successful basis for understanding many aspects of low-energy pion-nucleon interactions. Applied to baryon matter, the approximate chiral symmetry implies that the energy of the many-body system depends very gently on the chiral angle θ . Thus it has been conjectured ⁽³⁾ that the development of a pion condensate may be a very general consequence of approximate chiral symmetry. Allowing both

a and θ to vary, our study of the sigma model leads us to reflect that this argument can be a two-edged sword. Precisely because the dependence of the energy on the chiral angle θ is weak, other physical effects - which might make an otherwise negligible contribution to the energy - can change the shape of the energy surface and thereby qualitatively modify the nature of the phase diagram.

One of the authors (M.C.) wishes to acknowledge the support and gracious hospitality of the Niels Bohr Institute.

Footnotes and references:

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Figure Captions:

Figure 1. The critical densities for phase transitions, plotted as a function of g_A . Other model parameters are given in the text. Densities in units of $m_\pi^3 = 0.36 \text{ fm}^{-3}$.

Figure 2. Maximum condensate fraction $\sin^2 \theta$ at the second phase transition, as a function of g_A .

Figure 3. Values of the effective nucleon mass Q/F_π , in units of $M_N = 938 \text{ MeV}$, at the phase transitions, for varying g_A .

This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.

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