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Authors

Vucetic, M

Mortezaie, A

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Cyclic secant shear modulus versus pore water pressure in sands at small cyclic strains

Mladen Vucetic^{a,*}, Ahmadreza Mortezaie^b

^a Civil and Environmental Engineering Department, University of California, Los Angeles, CA 90095-1593, United States

^b Exponent, 5401 McConnell Avenue, Los Angeles, CA 90066, United States

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ABSTRACT

Cyclic strain-controlled behavior of fully saturated sands in undrained condition is analyzed at small cyclic shear strain amplitudes, γ_c , around the threshold shear strain for cyclic pore water pressure buildup, $\gamma_{tp} \approx 0.01\%$. The cyclic triaxial and simple shear test results obtained in the past by different researchers and the results of new cyclic simple shear tests reveal that: (i) at very small γ_c below γ_{tp} where there is no buildup of cyclic pore water pressure, Δu_N , with the number of cycles, N , the cyclic secant shear modulus, G_{SN} , initially increases with N for 10–20% of its initial value G_{S1} and then levels off or just slightly decreases, (ii) at small γ_c between $\gamma_{tp} \approx 0.01\%$ and 0.10–0.15%, Δu_N continuously increases with N while the modulus G_{SN} first increases for up to 10% of G_{S1} and then gradually decreases, and (iii) at γ_c larger than approximately 0.15%, relatively large Δu_N develops with N while the modulus G_{SN} constantly and significantly decreases. This means that at γ_c between γ_{tp} and 0.10–0.15% the sand stiffness initially increases with N in spite of the reduction of effective stresses caused by the cyclic pore water pressures buildup. In this range of γ_c , the pore water pressure Δu_N can reach up to 40% of the initial effective confining stress before G_{SN} drops below G_{S1} . The microstructural mechanisms believed to be responsible for such a complex behavior are discussed. It is suggested that during cyclic loading the changes at mineral-to-mineral junctions of grain contacts can cause soil stiffening while, at the same time, the buildup of cyclic pore water pressure causes the softening.

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1. Introduction

When fully saturated soils are subjected to cyclic loading in undrained conditions involving moderate to large cyclic shear strain amplitudes, γ_c , their stiffness and strength decrease with the number of cycles, N . Such a cyclic degradation is one of the most damaging and important phenomenon in soil dynamics. Fully saturated sands are most susceptible to cyclic degradation. Under cyclic loading, saturated sands can lose a large percentage of the initial stiffness and strength, and in the extreme case they can completely lose the stiffness and strength and liquefy. On the other end of the spectrum are fully saturated clays of high plasticity that under significant cyclic loading can lose only a fraction of their original stiffness and strength. The difference between the cyclic degradation of clays and sands is analyzed explicitly by Vucetic in [1] and is illustrated here in Figs. 1 and 2. In these two figures, the stress-strain-pore water pressure results of two cyclic simple shear strain-controlled tests, one on clay and one

on sand, conducted with constant cyclic shear strain amplitude $\gamma_c \approx 0.5\%$ are presented. In Fig. 1, γ =shear strain, τ =shear stress, and σ'_{VC} =effective vertical consolidation stress. The tests were conducted in the standard Norwegian Geotechnical Institute (NGI) type of direct simple shear device (DSS) adapted for cyclic testing. The clay is normally consolidated kaolinite clay with plasticity index, $PI=28$, while the sand is uniform Nevada sand (see grain size distribution in Fig. 6) that has been used in various soil dynamics investigations (see [2]). The decreasing slopes of the cyclic stress-strain loops in Fig. 1 demonstrate how the secant shear modulus at cycle N , $G_{SN}=\tau_{CN}/\gamma_c$, decreases with N . Here, τ_{CN} =cyclic shear stress amplitude at cycle N . For convenience, the moduli G_{S1} and G_{S5} normalized to σ'_{VC} pertaining to cycles 1 and 5 are indicated on the upper plot for clay.

The reduction of G_{SN} with N in the cyclic strain-controlled tests with constant γ_c is typically quantified by the degradation index, δ_N :

$$\delta_N = \frac{G_{SN}}{G_{S1}} = \frac{\tau_{CN}/\gamma_c}{\tau_{C1}/\gamma_c} = \frac{\tau_{CN}}{\tau_{C1}} \quad (1)$$

The variation of δ_N with N is presented in a log-log format in Fig. 2a. It can be observed that while this relationship is practically a straight line for the kaolinite clay, it is slightly curved

* Corresponding author. Tel.: +1 310 206 6260.

E-mail addresses: vucetic@ucla.edu (M. Vucetic), armortezaie@ucla.edu (A. Mortezaie).

Fig. 1. Typical stress–strain behavior of fully saturated normally consolidated clay and sand in the constant-volume equivalent-undrained cyclic strain-controlled simple shear test.

downwards for the Nevada sand. The index δ_N was introduced by Idriss et al. [3] in the context of the evaluation of the cyclic degradation of marine clay deposits underlying offshore structures for oil explorations during ocean wave storms. Idriss et al. [3] found that for normally consolidated clays the δ_N versus N relationship in a log–log format is typically a straight line. Idriss et al. named the slope of this line the degradation parameter, t :

$$t = -\frac{\log \delta_N}{\log N} \quad (2)$$

Subsequently, it was found that for overconsolidated clays the δ_N versus N relationship in the log–log format is also approximately a straight line [4]. The parameter t can therefore be used to conveniently describe, with a single number, the rate of cyclic degradation of clay with N sheared at a certain constant γ_c . For sands, however, the characterization of degradation with t parameter can be done only approximately because the δ_N versus N relationship in a log–log format is typically curved, just as shown in Fig. 2a. For example, the curved relationships obtained in the cyclic triaxial strain-controlled tests on sand can be found in Dobry et al. [5], while the curved relationships obtained in the cyclic

simple shear tests on sands can be derived from the results presented by Vucetic and Dobry [6].

The buildup of the cyclic pore water pressure, Δu_N , with N is presented in Fig. 2b. The pressure Δu_N is the pore water pressure measured at the end of each strain cycle, and it is presented in Fig. 2b in the normalized form with respect to σ'_{vc} , $\Delta u_N^* = \Delta u_N / \sigma'_{vc}$. The figure reveals that in sand, Δu_N^* increases with N much more than in normally consolidated clay, indicating that larger degradation in sand is accompanied by larger increase in the cyclic pore water pressures and associated decrease in effective stresses. Such a behavior of normally consolidated clay and sand is therefore in general compliance with the effective stress principal.

However, soil is a complex granular material the behavior of which can be quite complicated and at times surprising. An example of a behavior that one would not anticipate is the undrained cyclic behavior of overconsolidated clays. The overconsolidated clays always cyclically degrade, even though, depending on the combination of γ_c and the overconsolidation ratio, OCR, the cyclic pore water pressure may at the same time decrease and effective stresses increase [7–9]. The changes of clay structure that are responsible for such a counterintuitive behavior are discussed by Dobry and Vucetic [10]. Other counterintuitive results

Fig. 2. Comparison between the typical cyclic degradation and pore water pressure change in fully saturated normally consolidated clay and sand in the cyclic strain-controlled simple shear test.

are those on the effect of the frequency of cyclic loading on the cyclic degradation of normally consolidated kaolinite clay published recently by the writers [11]. Those results reveal that a higher frequency of cyclic loading causes simultaneously larger degradation and smaller cyclic pore water pressures and accompanying higher effective stresses. Such results indicate that besides the cyclic pore water pressure buildup there are other significant contributors to the cyclic degradation of clays.

This paper deals with yet another complex cyclic soil behavior: the behavior of fully saturated sand at small cyclic strains under which the sand stiffness increases while the effective stress decreases. The results of simple shear cyclic strain-controlled test in Figs. 3 and 4 present an example of such a behavior. These results are obtained on uniform Nevada sand at relatively small $\gamma_c = 0.08\%$. As shown in Fig. 3, the cyclic pore water pressure consistently increased with N , while the stress amplitude τ_{cN} increased in cycles 2 and 3, and only then, in cycle 4, it started to decrease. This is illustrated further in Fig. 4, where the variations of the normalized cyclic pore water pressure, Δu_N^* , and the cyclic degradation index, δ_N , with N are plotted along with their relationship. Fig. 4 reveals that in the third cycle sand is 7% stiffer ($\delta_N = 1.07$) in spite of the development of $\Delta u_N^* = 0.19$ that corresponds to a 19% reduction of the initial vertical effective stress. Furthermore, δ_N dropped back to 1.0 at $N=10$ when $\Delta u_N^* = 0.35$, corresponding to a 35% reduction of the vertical effective stress. Such a behavior demonstrates that, in this case, the cyclic pore water pressure buildup cannot be employed in the context of the effective stress principal as the sole parameter of sand softening due to cyclic loading. Furthermore, because in such a case the secant shear modulus, G_{SN} , does not monotonically decrease with N , the index δ_N cannot be called the degradation index. Instead, throughout the rest of this paper, the index δ_N is called the stiffness index, i.e., the index describing the relative change of G_{SN} with N irrespective of its trend.

It should be noted that the magnitude of the cyclic pore water pressure buildup in fully saturated sands has been typically used as the sole parameter to assess the degree of cyclic degradation and the consequent effects on the response of sandy deposits to earthquake loading, including their eventual liquefaction (see e.g. [12,13]). For example, the nonlinear liquefaction site response computer code DESRA-2 [14] and its modifications DESRAMOD [6] and D-MOD [15] include the models for the cyclic degradation of sand shear stiffness and strength that are based exclusively on the cyclic pore water pressure increase. The results in Figs. 3 and 4, and those presented through the rest of this paper, suggest that such sand degradation models are not as accurate as previously thought and can be improved.

2. Small-strain cyclic behavior of sands under various boundary and loading conditions

This section examines whether the behavior of sand at small γ_c described above has been obtained for different sands in different laboratories, different testing devices and different modes of cyclic loading, i.e., whether such a behavior is a common property of sands cyclically sheared under various boundary and loading conditions. The results of several well-documented recent and past cyclic testing investigations on different sands conducted in different laboratories at different cyclic shear strains under triaxial and simple shear loading conditions are inspected.

In Fig. 5, the results of three simple shear cyclic strain-controlled tests conducted at the soil dynamics laboratory of the University of California, Los Angeles (UCLA), on three different sands at very small γ_c ranging between 0.0038% and 0.0045% are presented. These levels of γ_c are smaller than the threshold shear strain for cyclic pore water pressure, γ_{tp} . Below γ_{tp} the cyclic pore water pressure, Δu_N , practically does not develop even after many

Fig. 3. Cyclic strain-controlled behavior of sand in undrained conditions in the simple shear test at relatively small cyclic strain $\gamma_c=0.08\%$.

cycles of straining, whereas above it, a significant Δu_N consistently develops [5]. For sands, γ_{tp} is around 0.01%. The grain size distributions of the three sands in Fig. 5 and of two more sands discussed later in this paper are presented in Fig. 6. Fig. 5 reveals that at γ_c smaller than γ_{tp} the stiffness index, δ_N , also increases with N . In two of the three tests, δ_N increases in the first 8 cycles and then it slightly decreases or levels off. The increase of δ_N is smaller than 20%, roughly between 8% and 18%. It should be noted,

however, that these observations are based on the results of just three tests with a limited number of cycles. More small-strain tests with $\gamma_c < \gamma_{tp}$ on more soils and with a larger number of cycles are needed to establish definite patterns of the above small-strain cyclic behavior.

To examine whether the same behavior occurs in the cyclic stress-controlled mode of shearing, a cyclic stress-controlled simple shear test was conducted on Nevada sand at constant

Fig. 4. Change of the cyclic pore water pressure, Δu_N^* , and cyclic stiffness index, δ_N , with the number of cycles, N , at relatively small cyclic strain $\gamma_c=0.08\%$ in simple shear test and their relationship.

Fig. 5. Cyclic strain-controlled behavior of sand in simple shear test at very small cyclic shear strains, γ_c , smaller than the cyclic threshold shear strain for pore water pressure $\gamma_{tp} \approx 0.01\%$.

Fig. 6. Grain size distribution curves of five sands treated in this paper.

$\tau^* = \tau_c / \sigma'_{vc} = 0.068$. The results of the test are presented in Figs. 7 and 8. Fig. 7 reveals that the cyclic pore water pressure continuously increased with N while the double cyclic shear strain amplitude, $2\gamma_{cN}$, initially decreased and then remained approximately constant. The resulting increase and stabilization of stiffness index $\delta_N = G_{SN} / G_{S1} = (2\gamma_{c1}) / (2\gamma_{cN})$ with N are presented in Fig. 8. It is apparent that the phenomenon of stiffening at small

cyclic strains, while the cyclic pore water pressure increases, also occurs in the cyclic stress-controlled mode.

Dobry et al. [5] conducted an extensive cyclic triaxial strain-controlled testing program on Monterey no. 0 clean sand. The goal of the investigation was to develop a new approach for the prediction of pore water pressure buildup and liquefaction of sands during earthquakes based on cyclic strain. The tests were conducted more than 30 years ago in the laboratory of the geotechnical firm

Fig. 7. Cyclic stress-controlled behavior of Nevada sand in simple shear test at small cyclic strains.

Woodward-Clyde Consultants in Clifton, New Jersey. A key chart from the report by Dobry et al. [5] that shows the relationship between $\delta_N = G_{SN}/G_{S1}$ and the change of the cyclic pore water pressure expressed in terms of $(1 - \Delta u^*) = 1 - \Delta u_N/\sigma'_3$ is presented in Fig. 9. Here, σ'_3 is the initial effective isotropic confining pressure in triaxial test. The cyclic shear strain amplitudes, γ_c , listed in the table that is inserted in the figure, are calculated from the cyclic axial strain amplitudes, ε_c , using the equation $\gamma_c = 1.5\varepsilon_c$. This equation is based on the assumptions that for saturated sand Poisson's ratio is 0.5 and τ_c acts at the 45° plane. The amplitude γ_c in the 11 tests included in the chart ranged from 0.03% to 0.3%, i.e., from just

slightly above γ_{tp} to well above it. In Fig. 10, the data points from Fig. 9, plus the additional points corresponding to the first cycle of loading (corresponding to $G_{SN}/G_{S1} = 1$) that are not included in Fig. 9 but are available in the report [5], are presented in the $\Delta u^* = \Delta u_N/\sigma'_3$ versus $\delta_N = G_{SN}/G_{S1}$ format. Furthermore, the data points are connected with smooth lines, such that each line corresponds to one test. Accordingly, the format of Fig. 10 is equivalent to the format of the chart in Fig. 4c.

Figs. 9 and 10 reveal that few initial data points of some tests with $\gamma_c = 0.03\%$ and 0.1% plot above the $G_{SN}/G_{S1} = 1$ line. This means that at the corresponding cycles of straining the sand has stiffened

Fig. 8. Change of the stiffness index, δ_N , with the number of cycles, N , in the stress-controlled cyclic simple shear test on Nevada sand presented in Fig. 7.

Fig. 10. Relationship between the normalized cyclic pore water pressure, $\Delta u_N/\sigma'_3$, and the stiffness index, $\delta_N = G_{SN}/G_{S1}$, obtained in the triaxial cyclic strain-controlled tests on reconstituted specimens of Monterey no. 0 sand (derived from [5]).

Fig. 9. Relationship between the cyclic pore water pressure change, Δu_N , in terms of $(1 - u^*) = (1 - \Delta u_N/\sigma'_3)$ and the secant shear modulus reduction, G_{SN}/G_{S1} , with N obtained in the triaxial cyclic strain-controlled tests on reconstituted specimens of Monterey no. 0 sand (reproduced from [5]).

in spite of the cyclic pore water pressure increase. For convenience, in Fig. 11 the curves for all four tests conducted at $\gamma_c = 0.1\%$ and one test at $\gamma_c = 0.03\%$ are plotted in a larger scale. This figure clearly shows that in four of these five tests the stiffness index $\delta_N = G_{SN}/G_{S1}$ first increased above 1.0 and then decreased, while the cyclic pore water pressure continuously increased. The increase of δ_N was no larger than 10%. This trend is in excellent agreement with the relationship between δ_N and Δu_N^* for Nevada sand presented in Fig. 4c.

A similarly extensive cyclic strain-controlled testing program was conducted at the soil dynamics laboratory of the Rensselaer Polytechnic Institute (RPI) in Troy, New York, by Vucetic and Dobry [6,16]. In that investigation, both cyclic triaxial and cyclic simple shear tests were conducted on liquefiable silty sand from the Wildlife Site in California that liquefied in the past (see [17]). Both intact and reconstituted specimens of the natural silty sand from Wildlife Site were tested in the range of γ_c between 0.03% and 1.0%. The results of the tests derived from the data in the original RPI report [6] are presented in Fig. 12. It is evident from the figure that at small γ_c , between 0.03% and 0.1%, the stiffness index δ_N first increased and then decreased, while the pore water pressure continuously increased. In these tests with $\gamma_{tp} < \gamma_c \leq 0.1\%$, the increase of G_{SN} was also no larger than 10%. In the tests with larger $\gamma_c = 0.3\%$ and 1.0%, the stiffness index, δ_N , only decreased with N , just like for the Nevada sand at $\gamma_c = 0.47\%$ presented in Fig. 2 and the Monterey no. 0 sand at $\gamma_c = 0.3\%$ presented in Fig. 10 (Tests 4, 7 and 9).

To the above studies should be added a recent study by Omarov [18] conducted at the University of Alaska Fairbanks, using the cyclic triaxial device. In that study, the results of cyclic strain-controlled tests conducted at γ_c between 0.03% and 0.1% also show in the first few cycles a pattern of increase and then decrease of δ_N with N (see pages 127–129 of the document).

The results obtained in different investigations presented above show that fully saturated sands exhibit the following behavior during the cyclic straining in undrained conditions:

1. At very small cyclic shear strain amplitudes, γ_c , below the threshold shear strain for cyclic pore water pressure buildup, $\gamma_{tp} \approx 0.01\%$, where there is no change of the cyclic pore water pressure, Δu_N , the cyclic secant shear modulus G_{SN} initially increases with the number of cycles, N , and then after certain N essentially levels off or just slightly decreases.
2. At small γ_c above $\gamma_{tp} \approx 0.01\%$ and below 0.3%, modulus G_{SN} first increases and then decreases with N , while the cyclic pore water pressure, Δu_N , continuously and consistently increases with N .

Fig. 11. Relationship between the normalized cyclic pore water pressure, $\Delta u_N/\sigma'_3$, and the stiffness index, $\delta_N = G_{SN}/G_{S1}$, obtained in the triaxial cyclic strain-controlled tests on reconstituted specimens of Monterey no. 0 sand at smaller $\gamma_c = 0.03\%$ and 0.1% —extracted from Fig. 10 (derived from [5]).

Fig. 12. Variation of the stiffness index, δ_N , with the number of cycles, N , and the relationship between δ_N and the cyclic pore water pressure, Δu_N^* , obtained in the triaxial (labeled TRX) and simple shear (labeled DSS) cyclic strain-controlled tests on reconstituted and intact specimens of liquefiable silty sand from Wildlife Site (derived from the data provided in [6]).

3. At $\gamma_c = 0.3\%$ and larger, modulus G_{SN} only decreases with N while the buildup of substantial cyclic pore water pressure takes place.

Such a complex cyclic behavior was obtained for clean sands and silty sands under simple shear and triaxial loading conditions. The same behavior was also obtained on naturally structured intact sand specimens and specimens reconstituted from fully disturbed sand. It also occurred in the cyclic strain-controlled and cyclic stress-controlled mode of shearing. Furthermore, this behavior was obtained independently in several geotechnical laboratories by different research teams at different times. Such overwhelming evidence shows that this kind of behavior is not a consequence of laboratory procedure, specimen boundary conditions, or the errors by experimentalists. In conclusion, the above undrained cyclic behavior is intrinsic to fully saturated sands, i.e.,

such behavior is a common property of fully saturated sands cyclically sheared under different undrained loading conditions.

3. Cyclic strain-controlled testing and behavior of Nevada sand

To investigate the above cyclic sand behavior in more depth and further clarify its patterns, a series of 7 cyclic strain-controlled simple shear constant-volume equivalent-undrained tests were conducted on Nevada sand in the NGI-DSS device at UCLA. The testing program is summarized in Table 1.

The NGI-DSS device was originally introduced by Bjerrum and Landva [19] for the monotonic loading testing and was later modified in a number of laboratories for the cyclic testing. The NGI-DSS specimen is a short cylinder of soil sandwiched between the top and bottom porous stones and surrounded by the wire-reinforced

Table 1
Program of cyclic simple shear strain-controlled tests on Nevada sand.

Test no.	Void ratio, e	Vertical consolidation stress, σ'_{vc} (kPa)	Cyclic shear strain amplitude, γ_c (%)	Frequency, f (Hz)	Number of cycles, N
1	0.60	154.9	0.0045	0.01	5
2	0.66	153.4	0.08	0.01	20
3	0.63	145.1	0.08	0.01	10
4	0.66	155.4	0.12	0.01	20
5	0.65	154.1	0.15	0.01	10
6	0.64	158.1	0.24	0.01	10
7	0.66	157.1	0.47	0.01	10

rubber membrane. During consolidation and shearing the reinforced membrane almost completely prevents radial strains, which means that the specimen is consolidated to essentially K_0 condition and cyclically sheared under practically no-lateral-strains condition. In the current testing the specimens were prepared by the method of pluviation of air dry sand. The bottom cap with the built-in porous stone was first fastened to a pedestal and then the bottom part of the membrane was pulled on it with the help of a vacuum membrane stretcher. An appropriate amount of sand was then rained through a custom-made sieve into the membrane. After the completion of pluviation, the top cap with the porous stone was placed on the sand surface and the vacuum was released so that the membrane edges tightly surrounded both caps with porous stones. The whole setup was then mounted in the NGI-DSS device loading frame where the specimen was first consolidated and then cyclically sheared. At the end of the consolidation and prior to cyclic shearing, the height of the specimen was around 19 mm and the diameter 67 mm. Other details of the testing procedure and data processing are available in Mortezaie [20].

The NGI-DSS device used in this study is equipped with a computer-controlled closed-loop servo-hydraulic loading system and a modern data acquisition system. In all tests the constant cyclic shear strain amplitude, γ_c , was applied in sinusoidal mode, while the variations of the shear stress, τ , and the equivalent pore water pressures, Δu , were recorded with time. The number of cycles applied was 10 or 20, except in Test 1 where it was 5. The results of three tests have already been presented in Figs. 1–5 (Tests 1, 2 and 7).

It can be seen in Table 1 that the void ratio, e , and the vertical effective consolidation stress, σ'_{vc} , varied relatively little from test to test. The frequency of cyclic straining was $f=0.01$ Hz in all tests. Such a relatively low frequency was applied to better facilitate the application and recording of test parameters during sensitive small-strain cyclic testing, with the understanding that in many practical problems the effects of frequency on sand behavior are relatively small.

During the cyclic shearing, the low frequency was particularly helpful in maintaining the specimen volume practically constant by keeping the specimen height constant. This was facilitated with the original NGI-DSS device closed-loop system for maintaining the constant specimen height which is driven by electrical motor that has a relatively slow response. In the NGI-DSS constant-volume equivalent-undrained test, the change of vertical stress required to maintain the specimen height constant is practically equivalent to the pore water pressure change that would have developed in a true undrained test. That is why the pore water pressure derived from such a test is called the equivalent pore water pressure. The concept of the constant-volume equivalent-undrained NGI-DSS testing was introduced around 50 years ago [19]. In the meantime, the concept has been fully validated and became well established [21].

As shown in Table 1, the range of γ_c was between 0.0045% and 0.47%, i.e., it encompassed a very small cyclic strain below γ_{tp} and several small to moderate cyclic strains above it. This range of cyclic shear strain amplitudes was selected to obtain a pattern of gradual transition from small to medium cyclic strain behavior. The processed test results are presented in Fig. 13, while their details and the data processing methods are available in Mortezaie [20]. Because the standard NGI-DSS device is not originally designed for the small-strain testing, the processing of raw test data also included elimination of false loads and deformations such as described by Mortezaie and Vucetic [22].

The results in Fig. 13 are in agreement with the previous studies described above and reveal the following specific cyclic behavior. In Test 1, with very small $\gamma_c=0.0045\%$ below γ_{tp} , in which the cyclic pore water pressure, Δu_N , did not develop, the stiffness index δ_N essentially just increased, at least in the 5 cycles of loading applied (see detail record in Fig. 5). In Tests 2, 3 and 4, conducted at small $\gamma_{tp} < \gamma_c < 0.15\%$, the stiffness index δ_N first increased and then decreased with N , while the cyclic pore water pressure monotonically built up. The increase of δ_N in these tests is not larger than 10%. In Test 5, at $\gamma_c=0.15\%$, the index δ_N remained 1.0 in the 2nd cycle and then decreased with N , while at all times the cyclic pore water pressure increased. In tests 6 and 7, at larger γ_c above 0.15%, the index δ_N only decreased with N while the significant cyclic pore water pressure buildup took place. In the present case of Nevada sand, it seems that $\gamma_c=0.15\%$ divides the two trends of the cyclic stiffness change. At $\gamma_{tp} < \gamma_c < 0.15\%$, the index δ_N first increases and then decreases with N , while above $\gamma_c=0.15\%$, the index δ_N only decreases with N .

In Fig. 14, the results from Figs. 12 and 13 obtained on the Wildlife Site sand and Nevada sand are synthesized to examine further the following aspects of the cyclic behavior: (i) the magnitude of the above-described transitional γ_c , (ii) the percentage of δ_N increase above 1.0, and (iii) the magnitude of Δu_N^* at the moment when δ_N starts to drop below 1.0. For clarity, Test 1 on Nevada sand with $\gamma_c=0.0045\%$ is not included in Fig. 14. The results of the tests that exhibit an increase and then a decrease of δ_N with N are presented in Fig. 14a. The results of the tests that exhibit no change of δ_N in the second cycle and then a decrease in the subsequent cycles are presented in Fig. 14b. And the results of the tests that exhibit only a decrease of δ_N with N are displayed in Fig. 14c. Fig. 14b reveals that the transitional γ_c is approximately between 0.1% and 0.15%. This means that it can be somewhat lower than 0.1% and higher than 0.15%, depending most likely on the type of sand, and for the same sand on its fabric, void ratio, confining stress, and specimen boundary conditions (e.g., triaxial versus simple shear conditions).

Fig. 14a reveals that due to the cyclic shearing with $\gamma_c > \gamma_{tp}$, the index δ_N generally does not exceed 1.10, i.e., δ_N does not increase for more than 10%. According to Fig. 5, on the other hand, for $\gamma_c < \gamma_{tp}$ the index δ_N can be somewhat higher, but probably not higher than 1.20. Fig. 14a also reveals that the largest increment of δ_N is always recorded in the 2nd cycle. As explained in the next section, this is most likely because the changes at the sand particle contacts due to cyclic loading, which are responsible for the stiffness increase, are probably largest in the first two cycles. Furthermore, Fig. 14a reveals that the normalized cyclic pore water pressure, Δu_N^* , may reach the value of approximately 0.3–0.4 before δ_N starts to drop below 1.0. Ratio $\Delta u_N^*=0.3$ –0.4 corresponds to the cyclic pore water pressure of 30–40% of σ'_{vc} and the same percentage of the drop of the initial effective stress. Accordingly, at γ_c larger than γ_{tp} and smaller than 0.1–0.15%, the effective stress may decrease by 30–40% before the modulus G_{SN} starts to degrade with N with respect to its original value in the first cycle, G_{S1} . In other words, sand may not lose any of its initial stiffness even though the effective stress is reduced by 30–40%, which is a large effective stress reduction indeed.

Fig. 13. Variation of the stiffness index, δ_N , with the number of cycles, N , and its relationship with the cyclic pore water pressure, Δu_N^* , obtained in the simple shear cyclic strain-controlled tests on Nevada sand.

4. Sand structure mechanisms most likely responsible for the observed cyclic behavior

Sand is a granular material and the external load is transmitted in sand via grain contacts. The cyclic stress–strain behavior of sand therefore depends on the chemical, physical and geometric characteristics of the grain contacts prior and during the cyclic shearing, and on the magnitude and direction of the contact forces.

The microscopic aspects of the sand structure and how its behavior depends on the properties of grain contacts and their changes due to the external loading is a complex subject which is summarized to different extents in a number of geotechnical textbooks. Among them are books by Lambe and Whitman [23] and Mitchell and Soga [24] that describe the subject systematically and in considerable depth. Based on the synthesis of relevant studies, including the pioneering work by Terzaghi [25,26], these two books offer a description of sand microstructure that can be summarized as follows. The surface of sand grains is rough, with successions of nanoscale asperities and depressions. When the surfaces of adjacent grains are brought together by force, their contact is established through these asperities, which are subjected to very high stresses. The compressed asperities form the actual solid-to-solid contacts the cumulative area of which is just a small fraction of the total grain-to-grain contact surface. Due to the extreme stresses, the mineral material at and around the solid-to-solid contacts yields and undergoes plastic deformation which results in the chemical bond formation at the points of contacts. The cohesive forces at such plastic junctions are the source of sand shear stiffness and frictional resistance, the magnitudes of which depend on the molecular structure and composition of contacting asperities. In addition to that, the grain surfaces are covered with a thin layer of adsorbed material from adjacent phases (water and/or air in the voids), but under high pressure the grain mineral at

asperities penetrates through this film and forms the mineral solid-to-solid contacts.

It is easy to comprehend that in such a sand structure, even at very small cyclic strains, the cyclic deformations and rubbing at and around the solid-to-solid contacts must in some manner modify the properties of the contacts, and that such changes must change the stiffness and moduli of the sand. In fact, this phenomenon has been recognized before by Drnevich and Richart [27] in one of their studies on the cyclic behavior of sand under small cyclic strains. Drnevich and Richart investigated the effects of cyclic prestraining on the increase of G_{SN} and damping in dry sand that was allowed to change in volume (densify) during cyclic loading. In their resonant column-torsional shear cyclic tests on hollow cylinder specimens they first measured G_{SN} and damping at a very small γ_c . Then, they applied a large number of cycles of larger $\gamma_c = 0.06\%$. Following such a prestraining at $\gamma_c = 0.06\%$, they again measured G_{SN} and damping at the same previous very small γ_c . What they observed was a relatively large increase of G_{SN} that cannot be attributed to the recorded very small decrease in void ratio. In their explanation of the significant increase of G_{SN} due to the cyclic prestraining, they stated: “... However, the motions were not large enough to cause gross particle reorientation (densification or dilation) to occur. Hence, the points on the surface of a particle that were in contact with neighboring particles essentially remained in contact throughout the prestraining. The prestraining applied abrasive action and caused the nature of these points of contact to wear. Original contacts were composed of the minute asperities of each particle touching. The actual contact areas were quite small. Relative particle motion probably flattened these asperities, increased contact areas, and allowed additional contacts to form. Because these changes could occur without significantly changing the pore volume, the void ratio and density did not change perceptibly. ... An increase of the contact areas and the number of contacts would account for the observed modulus and damping increases.”

Fig. 14. Relationships between the stiffness index, δ_N , and cyclic pore water pressure, Δu_N^* , for the Wildlife Site silty sand and Nevada clean sand.

The phenomenon of the cyclic stiffening and associated increase of G_{SN} in the present study can be explained more or less along the lines of the conclusions reached by Drnevich and Richart [27]. This is particularly true for the small-strain cyclic tests with $\gamma_c < \gamma_{tp}$, in which the cyclic deformations at particle contacts essentially did not change from cycle to cycle and sand fabric remained essentially the same. Under such conditions, the increase of the soil's secant shear modulus G_{SN} can be attributed only to the changes at the contacts, i. e., to the strengthening of the contacts. How and why the stiffening of the contacts occurs during uniform cyclic straining and how it influences the sand behavior are speculated in the following paragraphs, first for $\gamma_c < \gamma_{tp}$, then for $\gamma_c > \gamma_{tp}$ but smaller than approximately 0.1–0.15%, and then for $\gamma_c > 0.15\%$.

During the uniform cyclic straining with constant $\gamma_c < \gamma_{tp}$, the same cyclic strains are imposed on the sand element in each cycle. This means that the cyclic relative displacements between the sand grains (relative displacements between the centers of the grains) are essentially the same in each cycle. If, during such uniform cyclic straining, the stiffness of the contacts changes due to certain changes of the contact properties, the cyclic forces at the contacts must also change to accommodate the imposed uniform cyclic relative displacements. If the contacts stiffen, the cyclic forces at the contacts have to increase, which means that the secant shear modulus $G_{SN} = \tau_c / \gamma_c$ must also increase. If the contacts would soften during cyclic loading the situation would be opposite, i. e., to maintain a constant γ_c the cyclic forces at the contacts would have to decrease, which would result in a reduction of G_{SN} .

To understand how and why the grain-to-grain contacts may stiffen and not soften during cyclic straining with $\gamma_c < \gamma_{tp}$, the

changes at a single contact between two grains should be considered. At the small solid-to-solid contact surfaces of a single grain contact, and in the material surrounding them, the imposed cyclic deformations generate large cyclic stresses and strains. In the centers of the solid-to-solid contacts, these cyclic stresses and resulting strains are generally the largest and decrease towards the edges. Along the edges, the surfaces of adjacent grains are rubbing against each other, while just outside the edges they are cyclically touching and separating. Such cyclic actions must change the solid-to-solid contact surfaces. One of the logical changes is the increase of the area of these contacting surfaces, just as concluded by Drnevich and Richart [27]. As the area of a single solid-to-solid contact increases, the volume of the grain material surrounding it that is engaged in concentrated shear deformation also increases. To generate the same cyclic shear deformation over the increased volume of the grain material during the uniform cyclic strain-controlled loading, the amplitude of lateral cyclic force at the solid-to-solid contact must be larger. This explains the increase in stiffness of three different sands displayed in Fig. 5 which were cyclically sheared at $\gamma_c < \gamma_{tp}$. The figure also shows that this increase is largest in the beginning of cyclic loading, that it diminishes after several cycles, and that it is not larger than 20%. The figure also reveals that after a larger number of cycles the gained stiffness stabilizes or may start slightly to decrease. This indicates that the growth of the solid-to-solid contact surfaces during cyclic strain-controlled shearing is limited.

At small γ_c above γ_{tp} and below approximately 0.1–0.15%, besides compressing and shearing in the middle and rubbing against each other along the edges of the solid-to-solid contacts, the grains are

also slipping relative to each other. In undrained, constant volume conditions, such slippage causes the tendency towards densification of the sand which translates into the pore water pressure buildup and reduction of the effective stresses. During the two-way cyclic straining at $\gamma_c > \gamma_{tp}$, the adjacent grains are cyclically distorted and displaced back and forth. In a single cycle, there are periods when the grains are distorted but do not slide with respect to each other, and moments when they slide. When the grains do not slide, the mineral contacts are subjected to stresses and strains and rubbing along their edges, similar to what is described above for the cyclic straining at $\gamma_c < \gamma_{tp}$. When the grains slide the contacting asperities are dragged to new positions and subjected to additional abrasion. In the process some contacts are lost while new contacts are formed, but because of the additional scratching the area of the solid-to-solid contacts most likely increases and the entire grain-to-grain contact becomes stiffer. As such cyclic stiffening is taking place, the cyclic pore water pressure builds up and the effective stresses are reduced. The reduction of the effective stresses essentially means the reduction of predominantly normal forces at grain contacts. The smaller normal contact forces are associated with smaller areas of the solid-to-solid contacts and thus their smaller stiffness, which is the cause of the softening of the sand structure and reduction of G_{SN} and associated index δ_N .

Accordingly, at this moderate level of γ_c between γ_{tp} and approximately 0.1–0.15%, two major mechanisms that affect the soil stiffness are taking place simultaneously. One is stiffening due to the increase of the solid-to-solid contact areas caused by their cyclic deformations, scratching and rubbing. The other is softening due to the decrease of the solid-to-solid contact areas caused by the reduction of effective stresses due to the cyclic pore water pressure buildup. The first mechanism dominates the sand behavior initially, but after a few cycles, it slows down and eventually comes to an end because the growth of the contact areas is limited. The second mechanism of sand softening is continuous because the cyclic pore water pressure continuously builds up. Accordingly, after a few cycles the softening of sand skeleton overrides its stiffening and starts to dominate the sand behavior, after which point G_{SN} and associated δ_N start to decrease. During subsequent cyclic straining, G_{SN} and associated δ_N continue to decrease due to the continuous buildup of the cyclic pore water pressure. This kind of behavior is displayed in Figs. 4, 11 and 14a.

Fig. 15 shows the conceptual model of how at γ_c between γ_{tp} and 0.1–0.15% these two mechanisms yield the observed variation

Fig. 15. Conceptual model of the variation of the stiffness index, δ_N , in fully saturated sands during the undrained cyclic strain-controlled loading with relatively small cyclic shear strain amplitude, γ_c , greater than the threshold shear strain γ_{tp} .

of δ_N with N . The upper broken line represents the increase of δ_N due to the stiffening of particle contacts alone, $(\Delta\delta_N)_{STIFF}$, while the lower broken line represents the decrease of δ_N (degradation) due to the cyclic pore water pressure buildup alone, $(\Delta\delta_N)_{DEGR}$. When the ordinates of these two lines are subtracted, as shown in the figure, the behavior due to their combined effect is obtained. This behavior is depicted by the solid line.

At γ_c larger than 0.1–0.15%, from the very beginning of cyclic shearing the effective stress reduction due to the cyclic pore water pressure buildup is large enough to cancel any stiffness increase due to the cyclic straining and rubbing at the grain contacts. During subsequent cycles, the cyclic pore water pressure buildup continues, causing continuous reduction of G_{SN} and associated δ_N . The corresponding behavior is exhibited in Fig. 14c.

5. Conclusions and discussion

The results of cyclic tests on fully saturated sands conducted in undrained conditions and cyclic tests conducted in the constant-volume equivalent-undrained conditions in the cyclic strain-controlled mode, with the constant cyclic shear strain amplitude, γ_c , are presented and analyzed. The results reveal that the variation of the secant shear modulus $G_{SN} = \tau_{cN}/\gamma_c$ and associated stiffness index $\delta_N = G_{SN}/G_{S1}$ with the number of cycles N is distinctly different at different levels of γ_c . Here, τ_{cN} = cyclic shear stress amplitude at cycle N that changes during the cyclic strain-controlled loading while γ_c is constant, and G_{SN} = secant shear modulus at cycle N . The corresponding trends at different ranges of γ_c are:

1. At very small γ_c between approximately 0.003% and the threshold shear strain for cyclic pore water pressure change, $\gamma_{tp} \approx 0.01\%$, when there is no residual cyclic pore water pressure buildup, G_{SN} and associated δ_N initially increase with N and then stabilize or slightly decrease. For the three different sands tested, the increase of G_{SN} and associated δ_N is smaller than 20%, roughly between 8% and 18%. These conclusions are based on the results of only three tests and are therefore preliminary. To reach some definite general conclusions, more small strain tests at $\gamma_c < \gamma_{tp}$ on more soils and with the larger number of cycles need to be conducted. Furthermore, the behavior at very small γ_c below 0.003% has not been investigated in the above context.
2. At small γ_c between $\gamma_{tp} \approx 0.01\%$ and approximately 0.1–0.15%, the modulus G_{SN} and associated δ_N first increase with N and then decrease, while the cyclic pore water pressure, Δu_N , continuously increases. Here, Δu_N = pore water pressure increase measured at the end of strain cycle N . The results of tests on three different sands show that the increase of G_{SN} and associated δ_N is no larger than 10%, and that the largest increments of δ_N always occur at the very beginning of cyclic straining. At these cyclic shear strain levels, the cyclic pore water pressure may reach 30–40% of the effective confining consolidation stress in triaxial test or the effective consolidation vertical stress in simple shear test before δ_N drops to its original value of 1.0 after initially increasing. This means that the effective stress may decrease 30–40% before the modulus G_{SN} starts to degrade with N with respect to its original value G_{S1} .
3. At γ_c larger than 0.1–0.15%, the modulus G_{SN} and associated δ_N just decrease with N while significant Δu_N builds up. Only in this range of γ_c , the relationship between the $\log \delta_N$ and $\log N$ of sand, which is typically slightly curved, can be roughly approximated with a straight line. The slope of this line is the degradation parameter, t , which quantifies the rate of cyclic degradation.

The parameter t approximated this way can be employed to compare the cyclic degradation between different sands and between sands and clays.

Two mechanisms that act simultaneously are believed to govern such a complex cyclic behavior of sand. They are (1) the stiffening of the sand structure due to the changes at particle contacts caused by the cyclic deformations and abrasion of their solid-to-solid mineral contact surfaces, and (2) softening of the sand structure due to the reduction of effective stresses caused by the cyclic pore water pressure buildup. At $\gamma_c > \gamma_{tp}$ but smaller than 0.1–0.15%, for example, at the beginning of cyclic loading the cyclic stiffening due to the changes at particle contacts prevails over the effects of the pore water pressure buildup and G_{SN} increases with N . After a certain number of cycles, when the contact areas are largely worn off and cannot change much further, the cyclic pore water pressure, which keeps increasing from the very beginning, starts to dominate the change of G_{SN} which consequently starts to decrease. During the cyclic straining with γ_c considerably larger than 0.15%, the cyclic pore water pressure can increase and the effective stress drop by the end of the first and second cycle so much to completely override the effect of the stiffening at particle contacts. In such a case, G_{SN} just decreases from the very beginning of cyclic loading.

From the effective stress principle prospective, the above cyclic behavior seems counterintuitive because it involves simultaneous effective stress reduction and cyclic stiffening. However, sand is a complex granular material. Besides the reduction of effective stresses, during the cyclic straining of fully saturated sand there are also changes at particle contacts which must affect the sand stress–strain behavior. If these changes are significant, they can significantly affect the behavior. As shown above, the changes at particle contacts can cause substantial sand stiffening. The reason for such stiffening is believed to be the strengthening of the solid-to-solid particle contacts due to the increase of their surfaces caused by the intensive cyclic straining, rubbing and abrasion.

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