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An Optimization Method for Monitoring Network Design in Multilayered Groundwater Flow Systems

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Multiple migration pathways and the need to consider several potential siting horizons render the problem of groundwater monitoring network design a difficult task for three-dimensional systems. While the application of quantitative simulation-based approaches for this problem is often impractical due to computational requirements, qualitative approaches can be ineffective because they are highly subjective and typically lack a set of well-defined criteria for locating sampling sites. An analytically based methodology developed in this paper integrates the practical implementation aspects of a qualitative approach with a quantitative analysis in deriving groundwater monitoring networks geared toward early detection of migrating contaminant. Monitoring wells are located among sets of candidate nodes in each of several hydrostratigraphic intervals on the basis of contamination susceptibility. Susceptibility is defined by nodal weights, which are derived on the basis of the locations of sites relative to contaminant source boundaries and potential contaminant plumes. As is illustrated from the results of a case study application, derived monitoring network configurations exhibit two key characteristics: (1) clustering of wells around outlets at contaminant source boundaries and (2) coverage of vacant areas in multiple siting horizons that are susceptible to contamination.

1. INTRODUCTION

Groundwater quality monitoring is an important task in aquifer protection and groundwater management. Accurate and timely information on the spatial distribution of the chemical properties of groundwater is essential in the formulation of corrective action plans and environmental management strategies for aquifers. The successful attainment of this key information is highly dependent on the groundwater monitoring configuration from which samples are collected. Approaches to groundwater monitoring network design can be classified as qualitative or quantitative. Qualitative approaches are practical and by far the most widely implemented, but require, by definition, a high degree of subjective analysis and may lack well-defined locating criteria. Comprehensive quantitative approaches to network design address uncertainty in potential migration pathways and resulting contaminant distributions, and typically involve an objective analysis, but are computationally impractical for many problems. This paper presents a network design methodology which integrates qualitative and quantitative analyses. The objective is to develop a practical, simple, and effective methodology for network design applicable to multilayered groundwater flow systems.

2. BACKGROUND

In this paper, we define the problem of primary groundwater quality monitoring network design as determining the locations of a set of monitoring wells in an uncontaminated aquifer at risk of contamination from an overlying waste facility. Early detection of a contaminant release is the key objective. We distinguish this problem from the problem of secondary network design in which wells are located within an existing contamination field. Numerous approaches have been developed to address the second problem (e.g., variance-based approaches of *Rouhani* [1985], *Rouhani and Hall* [1988], *Loaiciga* [1989], and *Graham and McLaughlin* [1989*a*, *b*]; transfer function approach of *Andricevic and Foufoula-Georgiou* [1991], and location modeling approach of *Hudak and Loaiciga* [1992]). Comparatively fewer approaches have been developed to address the primary network design problem. Approaches which have been developed for the primary problem can be classified as "simulation-based" or "qualitative."

Examples of simulation approaches for groundwater quality monitoring network design include *Massman and Freeze* [1987], *Meyer and Brill* [1988], and *Ahlfeld and Pinder* [1988]. In the simulation approach, the hydraulic conductivity of a porous medium is modeled as a random field. By generating multiple synthetic distributions of hydraulic conductivity, for each of which there is a corresponding contaminant distribution, it is possible to determine the statistical properties of mass transport in an aquifer and the detection capability of a monitoring network.

The simulation approach is computationally intensive. In addition to the generation of multiple hydraulic conductivity distributions, mass transport models are needed to generate contaminant distributions. Numerical modeling of contaminant transport, especially in three dimensions, is considerably more difficult than simulation of groundwater flow. Transport modeling not only is more vulnerable to numerical errors such as numerical dispersion and artificial oscillation, but also requires much more computer memory and execution time, making it impractical for many field applications. The generation of multiple contaminant distributions, each

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corresponding to some realization of a set of statistical aquifer parameters, is impractical for many problems.

In the qualitative approach [e.g., *Everett*, 1980; *Loaiciga et al.*, 1992], the groundwater monitoring network is designed on the basis of calculations and judgments made by the hydrogeologist without the use of quantitative mathematical methods. The locations of sampling sites are determined by the hydrogeologic conditions near the source of contamination. The ultimate configuration of monitoring wells is subject to the investigator's understanding of (1) the key properties of the groundwater flow system, (2) how these properties influence the movement of contaminant and resulting contaminant distributions, and (3) what constitutes an "optimal" monitoring well configuration given probable contaminant migration pathways. Due to ease of implementation, the qualitative approach is widely used in practice. However, in a qualitative approach, there is no provision for quantifying the relative value of numerous potential sampling sites. In this paper, we develop and apply a quantitative, analytically based hydrogeologic approach for monitoring network design in multilayered groundwater flow systems. The approach combines the implementation merits of a qualitative approach with a quantitative analysis.

3. Development of Methodology

The network design methodology includes three primary steps: (1) definition and discretization of a model domain, (2) derivation of weights for candidate monitoring sites, and (3) selection of an optimal monitoring well network configuration.

3.1. *Domain Definition and Discretization*

The model domain defines the area within which monitoring sites are selected. It includes the contaminant source and a surrounding area that could be impacted by a contaminant release. Where feasible, model boundaries are extended to local hydrogeologic boundaries. The domain is partitioned into a field of candidate monitoring sites (nodes), arranged in a regular geometric pattern. Node spacing is based on the width of potential contaminant migration outlets ("hydrogeologic outlets") at the source boundaries (explained below) and computational considerations. The spacing between nodes should be small enough to preclude the possibility of a plume migrating undetected at the narrowest outlet. For practical and computational purposes, it is necessary to restrict the total number of possible well sites to a few hundred.

Advection envelopes, defining zones which encompass probable contaminant migration pathways, are extended from hydrogeologic outlets on the basis of known or inferred hydraulic head contours (Figure 1). The boundaries of advection envelopes can be defined by constructing two flow lines, each originating from a separate end of a hydrogeologic outlet. Flow lines are constructed such that they intersect known or inferred hydraulic head contours at right angles. If detailed hydraulic head data are available, the head values can be entered into a particle tracking program such as GWPATH [*Shafer*, 1990], and the advection envelope boundaries can be determined by generating forward path lines from the ends of the hydrogeologic outlet. The analysis does not account for the effects of hydrodynamic dispersion which may result in contaminant spreading beyond advection envelope boundaries. However, nodes located outside such boundaries are treated as potential monitoring sites. As is discussed in the following section, the monitoring value of these nodes is weighed according to distance from the advection envelope, which is consistent with a general tendency for progressive outward spreading.

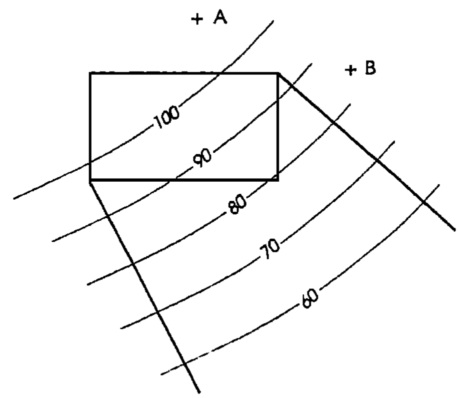

Fig. 1. Advection envelope extended from hypothetical contaminant source (rectangular area); plusses represent nodes referenced in text; contours in units of length.

The example in Figure 1 represents an advection envelope constructed for an upper interval in the zone of saturation for which the water table exerts the controlling influence on the movement of groundwater and migrating contaminants. Similar advection envelopes can be constructed for deeper hydrostratigraphic intervals (HSIs). (A HSI is defined as a layer within which hydraulic conductivity is relatively uniform.) For deeper HSIs, advection envelopes are extended laterally from areas of downward vertical flux within facility boundaries. The direction of vertical flux can be assessed directly from hydraulic head measurements in piezometer nests, or indirectly on the basis of topography or the shape of the water table (i.e., higher elevations are generally associated with downward gradients). Where vertical flux is directed downward, contaminants can travel into deeper HSIs and subsequently move horizontally beyond facility boundaries in response to hydraulic gradients characteristic of piezometric surfaces for lower elevation horizons.

In many regional scale flow systems, the horizontal hydraulic head distribution at successively greater depth intervals tends to become increasingly more uniform. At deeper intervals, a contaminant plume may spread over greater areal distances because (1) contaminant has to travel downward prior to reaching greater depths, incurring some degree of spreading in the process, and (2) there may be fewer troughs in deeper hydraulic head distributions to constrain migration pathways. Wider plumes at deeper intervals warrant a larger node spacing.

Within the field of candidate well sites, both "background" and "detection" monitoring wells are located. Detection wells are positioned to facilitate early detection of a contaminant release. Background monitoring wells, used to establish background water quality, are located within a specified upgradient zone of the model domain. The Resource Conservation and Recovery Act (RCRA) guidelines for groundwater monitoring [*U.S. Environmental Protection Agency (EPA)*, 1986] specify that upgradient wells must be (1) located beyond the upgradient extent of potential con-

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tamination from the hazardous waste management unit to provide samples representative of background water quality, (2) screened at the same stratigraphic horizon(s) as the downgradient wells to ensure comparability of data, and (3) of sufficient number to account for heterogeneity in background groundwater quality. These criteria dictate that background monitoring wells should be located in each HSI of a multilayered system. Furthermore, within a given HSI, background wells should be located beyond the area of potential contaminant spreading. There is a degree of qualitative judgment in defining the upgradient zone within which background wells are located. However, certain rules can facilitate the adequate definition of candidate background monitoring sites. They should not be located (1) within or adjacent to any advection envelope, (2) at points that could be intersected by an outward normal extended from an advection envelope, or (3) adjacent to the contaminant source.

3.2. *Nodal Weights*

Each candidate site is assigned a weight quantifying monitoring value. In the problem addressed herein, a monitoring configuration is derived for an uncontaminated aquifer at risk of contamination. Therefore nodal weights cannot be determined from existing contaminant concentrations. Instead, weights are derived by evaluating the location of a site relative to the contaminant source and likely contaminant migration pathways. For each candidate monitoring site, the nodal weight is calculated as

$$W_{jk} = \frac{1}{D(s)_{jk} D(e)_{jk}} \tag{1}$$

where

- j areal index of potential well site;
- k HSI index;
- W_{jk} weight for node j in HSI k;
- $D(s)_{jk}$ horizontal distance from node j in HSI k to contaminant source boundary;
- $D(e)_{jk}$ perpendicular distance from node j in HSI k to closest advection envelope boundary (equals $D(e)(\max)$ if node j cannot be intersected by a perpendicular from an advection envelope);
- $D(e)(\max)$ maximum $D(e)_{jk}$ value among nodes for which the quantity can be measured (see discussion below).

The nodal weight is inversely proportional to each of two distance variables defining the proximity of a node to locations of high contamination susceptibility. Nodes that are close to areas susceptible to contamination have high value as monitoring sites. To facilitate distance computations, the boundaries of the contaminant source and advection envelopes are approximated by nodes within the discrete lattice of candidate monitoring sites. The distance between a candidate monitoring site and a given boundary (contaminant source or advection envelope) is taken as the shortest of the distance values between the node and all nodes along the boundary. To avoid division by zero, nodes within an advection envelope are assigned a $D(e)_{jk}$ value equal to the minimum value calculated for all other nodes in layer k.

The quantity $D(e)_{jk}$ is calculated only for nodes which can be intersected by a perpendicular line extended outward from an advection envelope. Such nodes are considered to be susceptible to contamination from plume spreading. Nodes which cannot be intersected by an outward normal from an advection envelope are assigned the maximum $D(e)_{jk}$ value calculated for all other nodes. For example, consider nodes A and B in Figure 1. The nodes are the same distance from the advection envelope, but node B has a greater potential for becoming contaminated due its location relative to the boundary of the advection envelope. Node A is upgradient of the area of probable contaminant spreading and should be assigned a high value for $D(e)_{jk}$. According to the procedure outlined above, this node is assigned a $D(e)_{jk}$ value equal to $D(e)(\max)$. Given an advection envelope and a field of several nodes, $D(e)(\max)$ is equal to the distance from the envelope to the furthest node that can be intersected by an outward normal as described above.

The weighting scheme is consistent with qualitative hydrogeologic guidelines used in practice. For example, RCRA guidelines [*EPA*, 1986] specify that the placement of downgradient detection wells must consider (1) the distance to the contaminant source and the direction of groundwater flow, (2) the likelihood of intercepting potential pathways of contaminant migration, and (3) the characteristics of the contaminant source controlling the movement and distribution of contamination in the aquifer. Distance to the contaminant source is quantified by the variable $D(s)_{jk}$. The direction of groundwater flow is considered in the definition of advection envelopes. In a general sense, the likelihood of intercepting potential contamination may be related to the distance between a node and high susceptibility areas. Finally, the characteristics of the contaminant source boundaries are considered in the definition of hydrogeologic outlets, from which advection envelopes are extended. In effect, the methodology outlined herein attempts to quantify the qualitative guidelines listed above. Quantification facilitates a ranking of the relative value of a set of potential monitoring sites.

3.3. *Selecting Monitoring Sites*

Given a field of candidate nodes and associated weights, a mathematical programming model selects an optimal configuration of monitoring sites. The primary objective of the selection process is preferential location of monitoring wells at points of high contamination susceptibility. In multilayered systems, it is necessary to impose constraints requiring a minimum number of wells in each HSI. Additional constraints include upgradient monitoring and a specified total number of wells. The model formulation is

$$\text{Max } Z = \sum_{k \varepsilon K} \sum_{j \varepsilon J_{k-uk}} W_{jk} x_{jk} - \sum_{k \varepsilon K} \sum_{j \varepsilon J_{uk}} W_{jk} x_{jk} \tag{2}$$

subject to

$$\sum_{j \varepsilon J_k} x_{jk} \geq P_k(\min) \qquad \text{for each } k \varepsilon K \tag{3}$$

$$\sum_{j \varepsilon J_{uk}} x_{jk} = P_{uk} \qquad \text{for each } k \varepsilon K \tag{4}$$

$$\sum_{k \varepsilon K} \sum_{j \varepsilon J_k} x_{jk} = P \tag{5}$$

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$$x_{jk} = (0, 1) \qquad \text{for each } j \varepsilon J_k,\ k \varepsilon K \tag{6}$$

where

j areal index of potential well site;
J_k set of potential well sites in HSI k;
J_{k-uk} set of potential well sites, excluding sites in upgradient zone, in HSI k;
J_{uk} set of potential well sites in upgradient zone in HSI k;
k HSI index;
K set of HSIs;
W_{jk} weight for node j in HSI k;
x_{jk} 1 if a well is installed at node j in HSI k, 0 otherwise;
$P_k(\min)$ minimum number of wells to be located in HSI k;
P_{uk} number of wells to be located in upgradient zone in HSI k;
P total number of wells to be located.

The first term on the right-hand side of (2) is the sum of the weights for all nodes, excluding those in the upgradient zone specified for background monitoring, in each HSI of a multilayered system. The second term is the negative sum of the weights for nodes in the upgradient zone. If a well is sited at a node, it adds (term 1) or subtracts (term 2) the corresponding weight value to/from the objective function. The form of the second term ensures that the nodes with the lowest weights in the upgradient zone will be selected for background monitoring. Constraint (3) ensures that a minimum number of wells are located in each HSI, and (4) is a zonal constraint [*Church*, 1990]. In the present application, the zonal constraint ensures that a specified number of wells are allocated to the upgradient zone in each HSI. Constraint (5) establishes the total number of wells to be located throughout the model domain, and constraint (6) requires that the decision variable x_{jk} be a binary integer.

If a specified number of wells are assigned to each HSI, the general formulation can be decomposed to a series of separate HSI applications. The form of the HSI-decomposed model is, for each HSI k,

$$\text{Max } Z = \sum_{j \varepsilon J_{k-uk}} W_{jk} x_{jk} - \sum_{j \varepsilon J_{uk}} W_{jk} x_{jk} \tag{7}$$

subject to

$$\sum_{j \varepsilon J_k} x_{jk} = P_k \tag{8}$$

$$\sum_{j \varepsilon J_{uk}} x_{jk} = P_{uk} \tag{9}$$

$$x_{jk} = (0, 1) \qquad \text{for each } j \varepsilon J_k \tag{10}$$

The quantity P_k is the number of wells to be located in HSI k (other variables retain earlier definitions). The layer-decomposed model has fewer decision variables and constraints and is easier to implement and solve.

Each of the general and layer-decomposed model formulations requires specification of a total number of wells to be located. This number could be determined by regulatory requirements or budget constraints. Budgetary considerations could be incorporated directly by including a constraint of the form (using general model format)

$$\sum_{k \varepsilon K} \sum_{j \varepsilon J_k} C_{jk} x_{jk} \leq R \tag{11}$$

where C_{jk} is the cost of constructing a well at site j in HSI k and R is the total available monetary resources. However, with constraint (11), a low value of R may cause an inappropriately sparse detection network. Furthermore, the form of the constraint decreases the likelihood of model solution via relaxed linear programming and thus increases computational requirements.

The HSI-decomposed model given by (7) through (10) can be further reduced to two smaller, zonal problems.

Problem 1: Detection

For each HSI k:

$$\text{Max } Z_1 = \sum_{j \varepsilon J_{k-uk}} W_{jk} x_{jk} \tag{12}$$

subject to

$$\sum_{j \varepsilon J_{k-uk}} x_{jk} = P_{1k} \tag{13}$$

$$x_{jk} = (0, 1) \qquad \text{for each } j \varepsilon J_{k-uk} \tag{14}$$

Problem 2: Background

For each HSI k:

$$\text{Min } Z_2 = \sum_{j \varepsilon J_{uk}} W_{jk} x_{jk} \tag{15}$$

subject to

$$\sum_{j \varepsilon J_{uk}} x_{jk} = P_{2k} \tag{16}$$

$$x_{jk} = (0, 1) \qquad \text{for each } j \varepsilon J_{uk} \tag{17}$$

Integer programming techniques such as branch and bound [*Land and Doig*, 1960] can be used to solve the general formulation given by (2) through (6) as well as the various decomposed formulations. Problems 1 and 2 are single constraint 0–1 "knapsack" problems [*Dantzig*, 1963] with cost coefficients equal to one. Solutions to these problems are obtained by selecting the nodes with the P_{1k} highest and P_{2k} lowest weight values among the sets J_{k-uk} and J_k, respectively. Problems 1 and 2 can be solved with "sort functions" available in statistical software packages. Solutions to problems 1 and 2 can be combined to solve the formulation given by (7) through (10). The value of the objective function Z in (7) is equal to the sum of Z_1 in (12) and Z_2 in (15). Optimal well sites for problem 2 define the locations of the P_{uk} upgradient wells in constraint (9), and the combined well sites from problems 1 and 2 define the locations of the P_k wells in constraint (8).

4. APPLICATION

4.1. *Site Description and Background*

The network design methodology was applied to the Casmalia Resources hazardous waste facility in northern Santa Bar-

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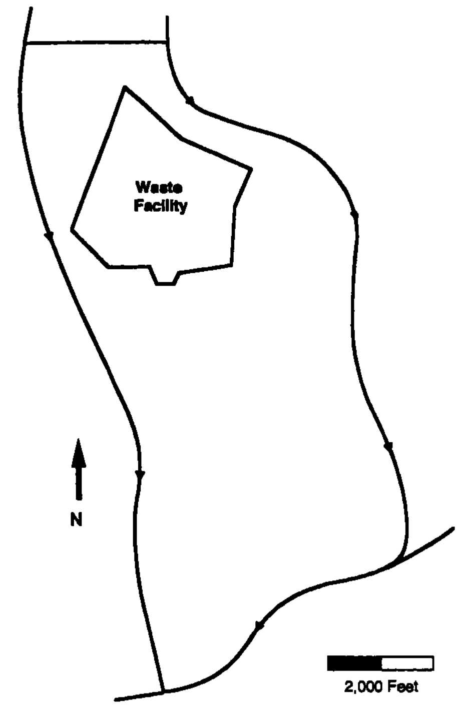

Fig. 2. Model boundaries; arrows indicate flow directions in ephemeral streams (1 foot = 0.3048 m).

bara County, California (Figure 2). Liquid and solid waste was disposed to unlined ponds and landfills located throughout the facility during the period from 1972 to 1989. In 1987, tests required by the California Regional Water Quality Control Board confirmed widespread contamination of groundwater beneath the site. Groundwater quality data reported by *Woodward-Clyde* [1987] and *Woodward-Clyde and Canonie* [1988] suggests the extent of off-site contamination may be limited to areas adjacent to the southern margin of the waste facility.

4.2. *Hydrogeology*

The claystone bedrock formation which underlies the site can be divided into two distinct hydrostratigraphic units: an upper, weathered unit and a lower, unweathered unit. The weathered unit varies in thickness from about 30 to 60 feet (9 to 18 m), and the unweathered unit extends to a depth of approximately 2000 feet (600 m) to underlying shale. The geometric mean hydraulic conductivity values estimated from pumping tests and piezometer tests for the upper and lower units are approximately 0.19 feet/day (6.7×10^{-7} m/s) and 0.0042 feet/day (1.5×10^{-8} m/s), respectively [*Woodward-Clyde*, 1987].

Figure 3 illustrates water table contours in the vicinity of the facility. The water table configuration controls offsite migration in upper hydrostratigraphic intervals (HSIs). Advection enve-

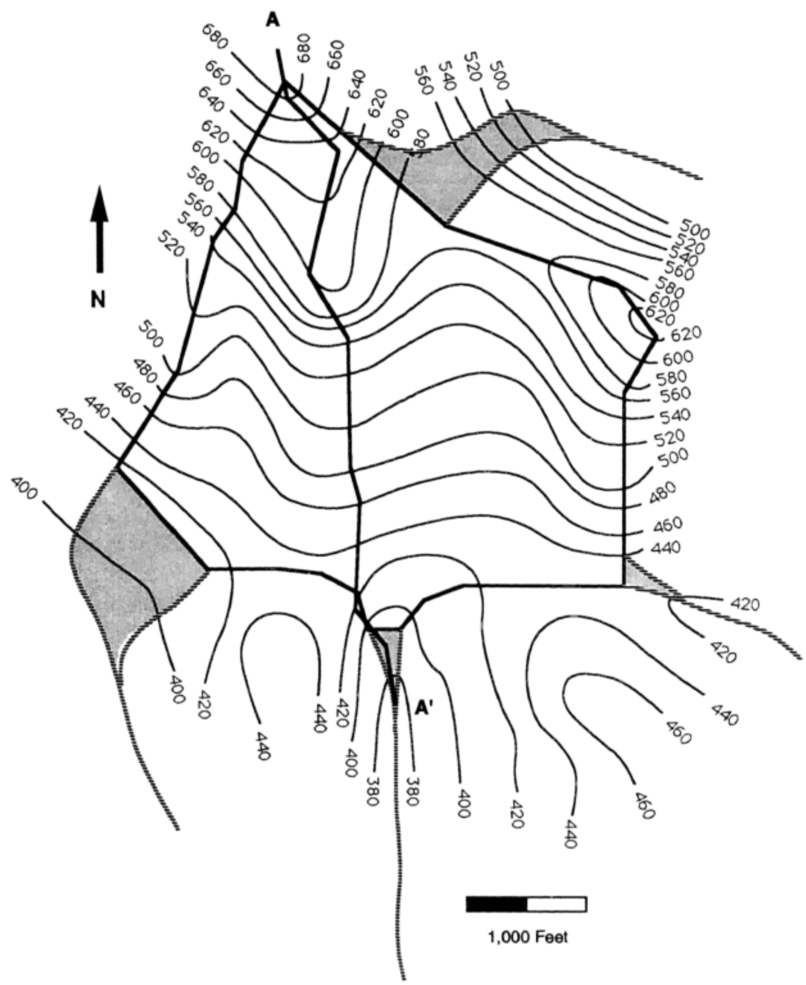

Fig. 3. Water table contours (feet above mean sea level), advection envelopes (shaded areas), and location of cross section A-A′ (1 foot = 0.3048 m) [modified after *Woodward-Clyde*, 1987].

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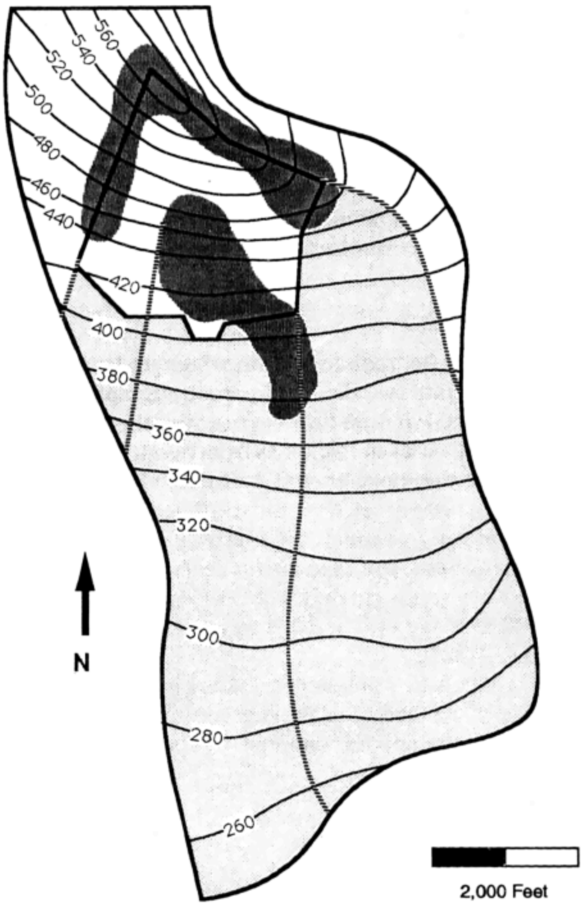

Fig. 4. Inferred hydraulic head distribution in HSI 3 (contours in feet above mean sea level; depth interval is 200–300 feet above mean sea level) and advection envelopes (light shaded area) extended from areas of downward flux (dark shaded areas) within facility boundaries (1 foot = 0.3048 m).

lopes extended from hydrogeologic outlets at facility boundaries converge along the axes of ephemeral streams downgradient from the source. The potential distribution of contamination in deep HSIs is determined by (1) the location of areas of downward flux within facility boundaries, and (2) the potentiometric surface for deeper intervals (Figure 4).

4.3. *Model Boundaries and Discretization*

The continuous boundary encompassing the facility boundary in Figure 2 defines the lateral extent of the analytical model domain. The eastern, western, and southern boundaries generally coincide with ephemeral stream channels, which represent local groundwater discharge areas. The northern model boundary does not coincide continuously with a natural hydrogeologic boundary. Groundwater flows laterally from this boundary into the model area. Vertically, the model domain is separated into three (HSIs): an upper unit extending from land surface to the weathered/unweathered claystone bedrock contact (HSI 1); an intermediate unit extending from the weathered/unweathered bedrock contact to 40 feet (12 m) below the contact (HSI 2); and a deep unit at 40+ feet (12+ m) below the contact (HSI 3) (Figure 5).

A triangular grid was used for domain discretization. In a triangular grid, adjacent rows are "offset" by half a grid unit. This property is effective for mitigating the potential for a contaminant plume migrating undetected between adjacent columns. The analytical model domains for HSIs 1 through 3 are illustrated in Figure 6. The node spacing for upper intervals is based on the size of hydrogeologic outlets at the perimeter of the facility and computational considerations. The smallest hydrogeologic outlet, at the southern margin of the waste facility (Figure 3), is approximately 350 feet (107 m) wide. Grid spacing for HSIs 1 and 2 is 350 feet (107 m), and rows are oriented approximately perpendicular to the mean direction of groundwater flow inferred from hydraulic head contours (Figures 3 and 4). There are fewer nodes in

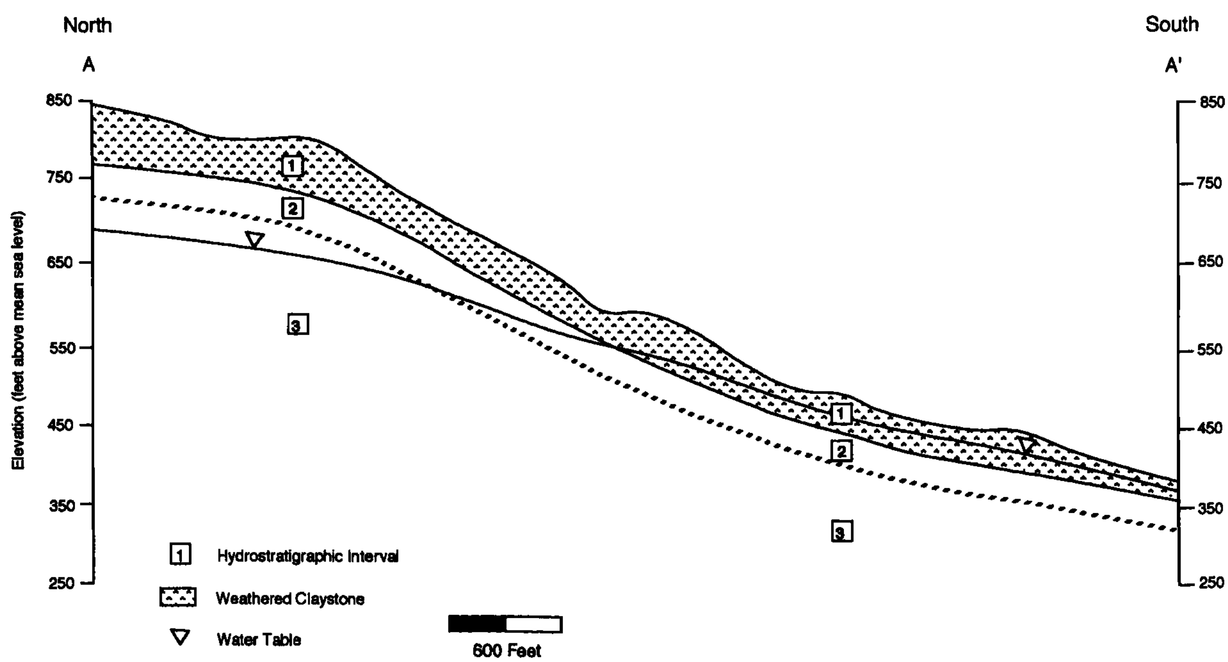

Fig. 5. Schematic cross section along A-A′ (1 foot = 0.3048 m) [modified after *Woodward-Clyde*, 1987].

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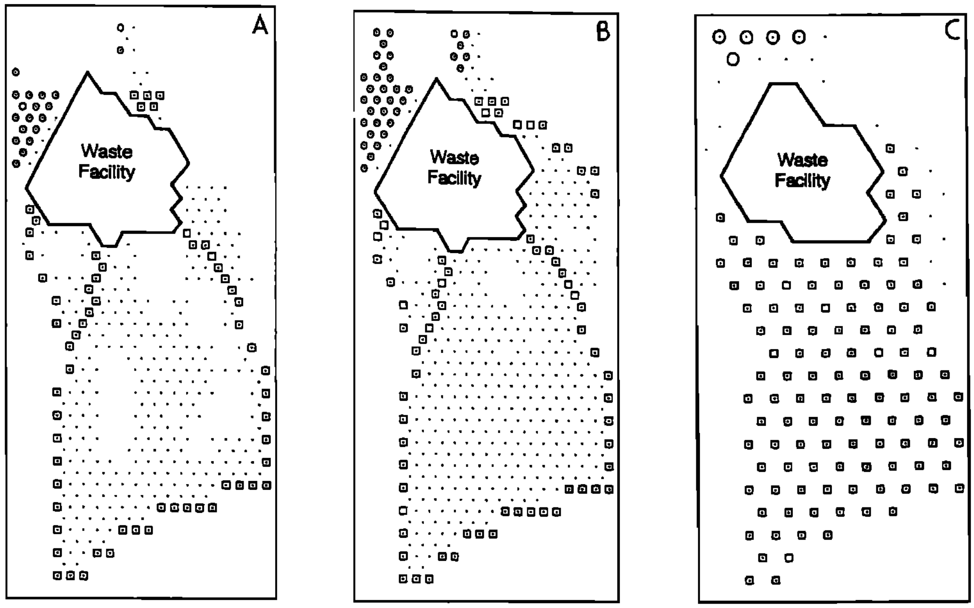

Fig. 6. Model domains for (*a*) HSI 1, (*b*) HSI 2, and (*c*) HSI 3; dots represent nodes; circles designate background nodes; squares designate advection envelope nodes; distance between nodes = 350 feet (107 m) (*a*) and (*b*) and 700 feet (213 m) (*c*).

the model domain for HSI 3. Nodes in HSI 3 are located at every other node in the networks for HSIs 1 and 2. In general, fewer wells are required in deeper HSIs in a multilayered system. To facilitate adequate areal coverage, where fewer wells are sited, the number of potential well sites is set to a correspondingly lower value. A large number of potential well sites could lead to a condition of "well clustering," where sited wells are grouped around a relatively local area within the overall domain. This condition is inappropriate for deeper HSIs, especially where the hydraulic head distribution is relatively uniform and, together with the distribution of areas of downward vertical flux within the source, dictate the possibility of relatively wide migrating contaminant plumes. Overall, the model domain consists of 373 nodes in HSI 1, 495 nodes in HSI 2, and 138 nodes in HSI 3.

Beneath some topographically higher areas, the water table is below HSI 1 and is located within HSIs 2 or 3. Figure 6*a* shows voids with no nodes where HSI 1 is dry, thereby precluding the siting of monitoring wells at these locations. There are similar voids for HSI 2, but the void areas are not as extensive (Figure 6*b*). HSI 2 is deeper than HSI 1 and, as a result, there are fewer areas where the water table is below the base of HSI 2. In a few locations, both HSIs 1 and 2 are dry (i.e., the water table is below HSI 2), which accounts for the void areas in Figure 6*b*.

The water table configuration and related hydrogeologic outlets are important factors governing the potential distribution of contaminants in HSIs 1 and 2. As a result, the overall analytical model representations for HSIs 1 and 2 are fairly similar (Figures 6*a* and 6*b*). HSI 3 differs from HSIs 1 and 2 with regard to the nature of the hydraulic head distribution controlling horizontal groundwater flow (Figures 3 and 4). For HSI 3, advection envelopes have been constructed from areas of downward flux documented from head measurements in piezometer nests [*Woodward-Clyde*, 1987]. Wider advection envelopes for HSI 3 result from (1) the relatively extensive areas of downward vertical flux within facility boundaries and (2) the more uniform distribution of hydraulic head governing flow in the deeper HSI.

4.4. *Network Configurations*

The existing ("original") groundwater monitoring configuration beyond facility boundaries consists of 83 wells, including 31 in HSI 1 (6 designated for background monitoring), 35 in HSI 2 (7 background), and 17 in HSI 3 (2 background). The locations of these wells, assigned to the nearest node in the analytical model domain for comparative purposes, are illustrated in Figure 7. The wells in Figure 7 were sited to detect potential contamination beyond facility boundaries. In this regard, the original network constitutes a primary network design, as is defined in section 2.

The general formulation given by (2) through (6) was solved for 83 wells with the parameter P_{uk} equal to 6, 7, and 2 for HSIs 1, 2, and 3, respectively, as in the original network. The model was not constrained, however, to site a specified total number of wells in each HSI. Instead, a minimum number of wells equal to five percent of the total number of nodes in a given layer was specified for each HSI (P_k(min) = 19, 25, and 7 for HSIs 1, 2, and 3, respectively). Specification of P_{uk} and a percentage number for P_k(min) for each HSI requires user judgment. The number of upgradient wells may be specified as some fraction of the total number of wells sited. For example, guidelines in the *Groundwater Monitoring Technical Enforcement Guidance Document* [*EPA*, 1986] which establish minimum requirements for groundwater monitoring suggest that one fourth of the total number of wells be used for background monitoring.

Utilization of a percentage number for P_k(min) ensures that a minimum area of the model domain in each layer is covered by monitoring wells. In some cases, P_k(min) may be defined by regulatory requirements established for a site. If

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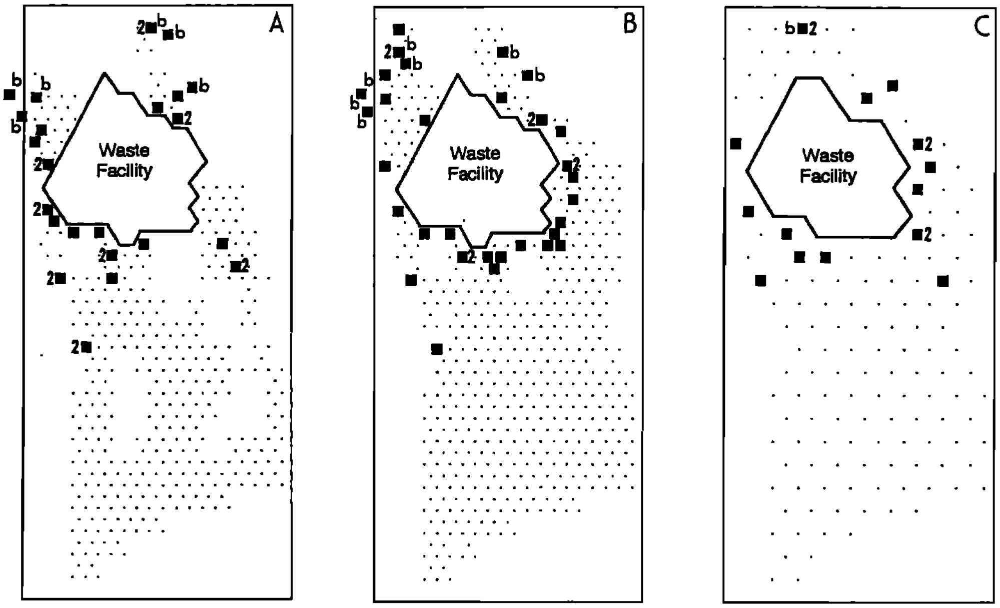

Fig. 7. Locations of monitoring wells (squares) in original network for (*a*) HSI 1, (*b*) HSI 2, and (*c*) HSI 3; "2" indicates two wells at corresponding node; "b" designates a background monitoring well.

a minimum number of monitoring wells is defined for one layer, the percentage number can be calculated and used to determine $P_k(\min)$ for other layers. For example, specification of 7 for layer 3 leads to 7/138, or 5%. Multiplying this percentage by the total numbers of nodes in layers 1 and 2 yields $P_k(\min)$ for these layers. If a percentage number is used to establish $P_k(\min)$ for all layers (i.e., $P_k(\min)$ is not expressed a priori), it should be high enough to ensure an adequate minimum coverage in each layer but sufficiently low that the sum of $P_k(\min)$ values for all layers is less than P. This condition is advantageous because, after satisfying minimum layer constraints, the model then has the capability for siting $[P - \sum_{k \varepsilon K} P_k(\min)]$ wells to void areas near the contaminant source and along potential contamination outlets in each layer.

The solution to the general formulation given by (2) through (6) is illustrated in Figure 8. The solution consists of 29 wells in HSI 1, 37 wells in HSI 2, and 17 wells in HSI 3. The total number of wells sited in each HSI is similar to the original network. However, the locations of wells sites differ markedly from those in the original network (Figure 7). In the model-derived network, background monitoring wells are generally further from areas susceptible to contaminant spreading, including the source, than are wells designated

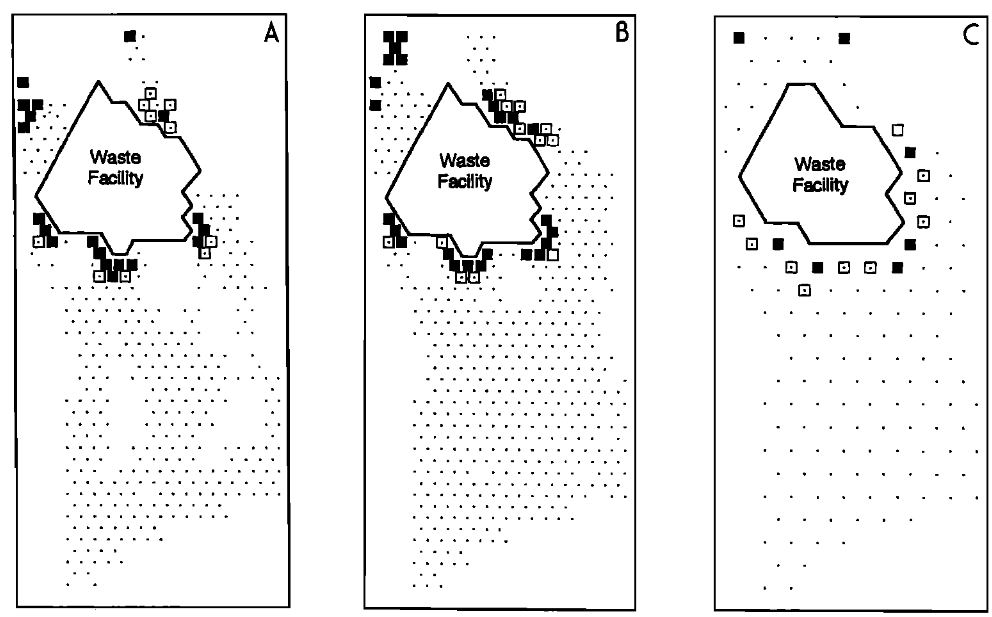

Fig. 8. Locations of model-derived monitoring wells in (*a*) HSI 1, (*b*) HSI 2, and (*c*) HSI 3; solid squares designate wells sited to satisfy constraints (3) and (4) in general formulation; open squares designate additional wells sited to satisfy constraint (5).

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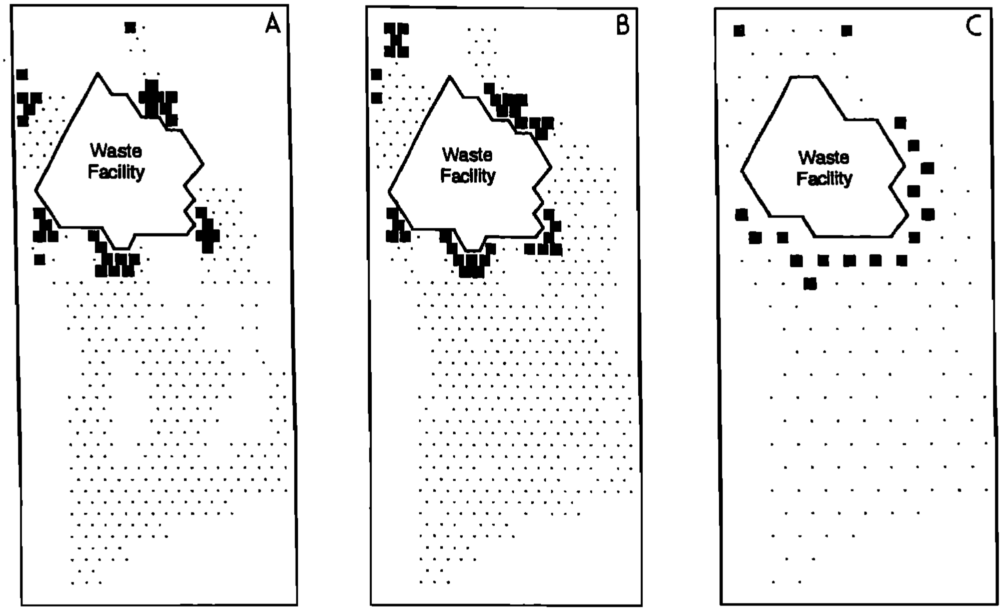

Fig. 9. Locations of model-derived monitoring wells in (*a*) HSI 1, (*b*) HSI 2, and (*c*) HSI 3 for decomposed version of general formulation.

for background monitoring in the original network. As a result, background wells in the model-derived network may be less likely to become contaminated than wells in the original network. Model-determined detection well sites are clustered near the source and around hydrogeologic outlets. Collectively, these wells cover an area exceeding the width of advection envelopes extending from the source. This property is a result of the inclusion of both distance variables, $D(s)_{jk}$ and $D(e)_{jk}$, in the derivation of the nodal weights. If $D(e)_{jk}$ was the only variable utilized, wells would be sited exclusively within advection envelope boundaries. This result would be inappropriate because the advection envelope is a mere approximation, or "best guess" of a zone which encompasses probable contaminant migration pathways. Including the $D(s)_{jk}$ distance variable in addition to $D(e)_{jk}$ leads to the siting of monitoring wells near both the contaminant source and advection envelopes.

Comparison of Figures 7 and 8 suggests the model-derived network is better suited to early detection of a potential contaminant release. The well sites in the model-derived network cover the predominant contamination outlets and extend downgradient along likely migration zones. Upgradient detection wells and detection wells located several node intervals downgradient from the source in the original network are relatively ineffective for early contaminant release detection. The detection wells furthest downgradient in the original network are particularly ineffective, given the low hydraulic conductivity characteristic of the saturated zone underlying the site.

In the model-derived network, sited wells within each HSI can be classified as (1) wells sited to satisfy the constraint which requires a minimum number of wells in each HSI and the upgradient constraint (i.e., constraints (3) and (4) in the general formulation), and (2) additional wells sited to satisfy the constraint requiring a specified total number of wells throughout the model domain (i.e., constraint (5) in the general formulation). In the present application, after satisfying constraints (3) and (4), 32 wells remain to be allocated. Of the 32 remaining wells, 10, 12, and 10 are allocated to HSIs 1, 2, and 3, respectively. These numbers represent 53, 48, and 143% of the number of wells sited in each HSI to satisfy constraints (3) and (4). The higher percentage for HSI 3 is the result of more extensive voids in high susceptibility areas in this HSI relative to the other HSIs after the first set of wells are sited. The preferential allocation of additional wells to vacant susceptible areas in a multilayered system is an important property of the general formulation. This property is effectively lost in formulations specifying a total number of wells to be sited in each HSI of a multilayered system.

The decomposed formulation given by (7) through (17), was also solved for P_{uk} equal to 6, 7, and 2 (for HSIs 1, 2, and 3, respectively). For comparative purposes, the numbers of wells to be allocated to each layer was set equal to the values defined in the original network. The solution to the decomposed formulation is illustrated in Figure 9. The configuration in Figure 9 is similar to the solution to the general formulation (Figure 8). This similarity results from a choice of P_k values (for the decomposed formulation) which are nearly identical to the numbers of wells in corresponding HSIs for the general solution. While the decomposed formulation requires that the user define a number of wells for each HSI, the general formulation allows the model to allocate wells to priority sites throughout the model domain. A potential problem with the decomposed formulation is specification of an inadequately low number of wells for one or more layers. The general formulation ensures coverage of priority sites in all layers with wells available after minimum layer constraints have been satisfied. Thus it is better suited than the decomposed formulation to detection monitoring in three dimensions.

5. SUMMARY AND CONCLUSIONS

The methodology presented in this paper is a viable approach to detection-based groundwater quality monitoring

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network design in multilayered groundwater flow systems. Susceptibility to contamination at points throughout a model domain is quantified by weight values. The weights are utilized in a mathematical programming model, which selects monitoring sites in each of several hydrostratigraphic intervals comprising an overall model domain. The approach is tailored to the early detection of a contaminant release as opposed to the characterization of a contaminant plume. A typical solution will exhibit a clustering of well sites within areas of high contamination susceptibility, but an absence of well sites within low-priority areas. Therefore in the event of a contaminant release, more than one well in a model-derived network may detect contamination within a relatively small portion of the model domain. Although the clustering strategy is ineffective for characterizing contaminant levels throughout a large area, it is effective for ensuring that a release is verified early, by at least one well.

After satisfying constraints requiring a minimum number of wells in each HSI and background monitoring, remaining wells are allocated to unoccupied nodes in high susceptibility areas throughout the model domain. This property facilitates adequate detection monitoring throughout a multilayered system. A decomposed formulation can be obtained by specifying a number of wells for each HSI in a multilayered system. This alternative formulation is easily solved, but can lead to poor results if an inadequately low number of wells is specified for one or more HSIs.

To our knowledge, this is the first published methodology which is directly applicable to detection-based groundwater monitoring network design in multilayered systems. The presented approach is based on a subjective analysis, most notably in the definition of nodal weights. However, the weights are intuitively logical and consistent with objectives reflected in existing regulatory guidelines. The weighting approach facilitates an assessment of the relative value of numerous monitoring sites. User judgment is required to define a node field, minimum numbers of wells to be allocated to each HSI, and numbers of upgradient wells to be allocated to each HSI. The general approach to multilayered network design presented herein may provide a basis for future work in the area of three-dimensional groundwater monitoring. Practical aspects of the developed methodology which facilitate its use by practitioners such as groundwater hydrologists include relative ease of implementation and solution, applicability to multilayered systems, and capability for including established regulatory policy such as background monitoring.

NOTATION

C_{jk} cost of constructing a well at site j in HSI k.
$D(e)_{jk}$ perpendicular distance from node j in HSI k to advection envelope boundary.
$D(e)(\max)$ maximum $D(e)_{jk}$ value among nodes for which the quantity is defined.
$D(s)_{jk}$ horizontal distance from node j in HSI k to contaminant source boundary.
J_k set of potential well sites in HSI k.
J_{k-uk} set of potential well sites, excluding sites in upgradient zone in HSI k.
J_{uk} set of potential well sites in upgradient zone z in HSI k.
j areal index of potential well site.
K set of HSIs.
k HSI index.
P total number of wells to be located.
P_k total number of wells to be located in HSI k.
$P_k(\min)$ minimum number of wells to be located in HSI k.
P_{uk} number of wells to be located in upgradient zone in HSI k.
P_{1k} total number of wells to be located outside upgradient zone in HSI k.
P_{2k} total number of wells to be located in upgradient zone in HSI k.
R total available monetary resources.
W_{jk} weight for node j, in HSI k.
x_{jk} 1 if a well is sited at node j in HSI k, 0 otherwise.

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