Physics and Design Issues of Asymmetric Storage Ring Colliders as B-Factories

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1989-08-01

LBL-27670

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

Presented at the XIV International Conference
on High Energy Accelerators, Tsukuba, Japan,
August 22–26, 1989

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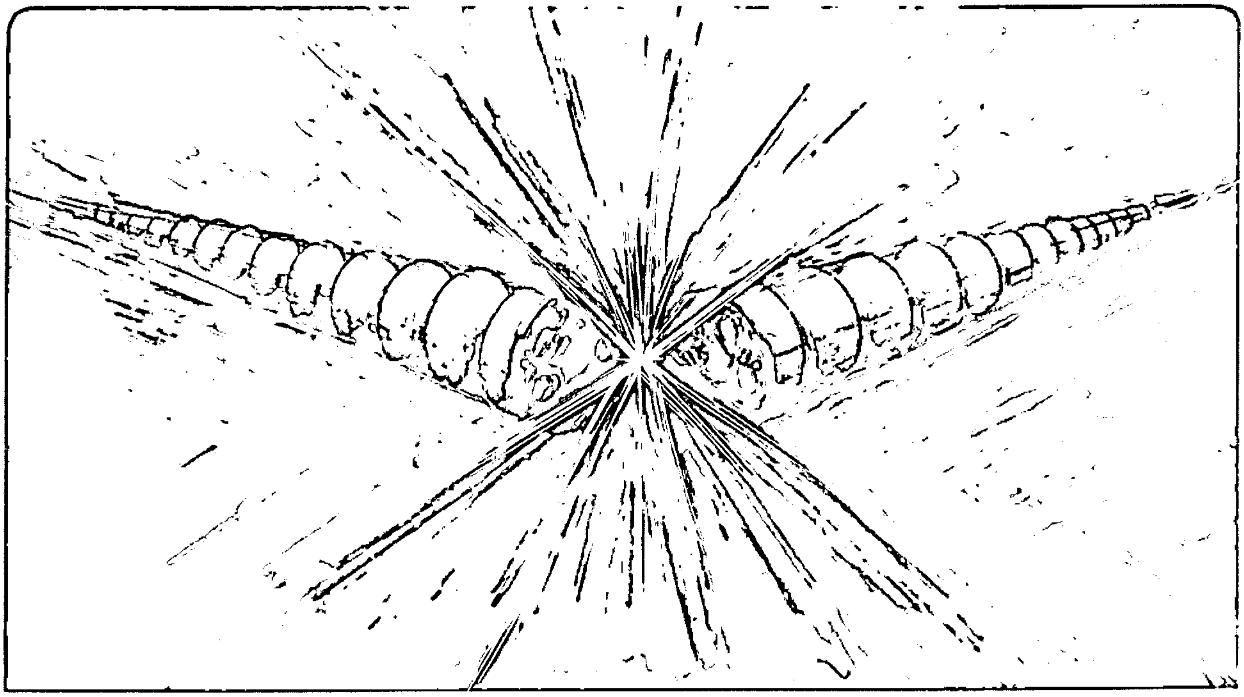

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

LBL-27670
ESG Note-75

PHYSICS AND DESIGN ISSUES OF ASYMMETRIC STORAGE RING COLLIDERS AS B-FACTORIES*

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INTRODUCTION

Enthusiasm for, and interest in, high luminosity electron-positron colliders as B-factories are felt today at many major laboratories worldwide and the topic has attained a truly international status. It is evident that the value of "rare" and CP-violating B-meson decays as fundamental probes of the Standard Model and the new physics beyond is unquestionable. One can anticipate that such interest in B-physics research and development, and even probable collider construction, will continue and be one of the major foci of high energy physics activities through the late 1990's and well into the twenty-first century.

These fundamental experiments can be done, in principle, at both hadron and e^+-e^- colliders, each having its own strengths and weaknesses. In hadron colliders, B-mesons are produced rather copiously and the challenge is in building a detector that can reject the overwhelming background of other hadronic channels, which dominate. In e^+-e^- colliders, the events are clean but luminosity is of highest concern. The challenge there is in building a high luminosity collider, with a luminosity well above 10^{33} cm^{-2} s^{-1}. The uncertainty[1,2,3] in the required luminosity for observing CP violation (estimated to be anywhere between 5×10^{32} and 8×10^{34} cm^{-2} s^{-1}) arises from a combination of the uncertainty in the weak decay parameters and the actual configuration of collider experiments (equal energy symmetric collider vs. unequal energy asymmetric collider, etc.).

Various approaches[4] to a collider exist at present, including linac-on-linac and linac-on-storage ring scenarios. These latter approaches are relatively more speculative, since the technology of linear colliders and high power, high current, high repetition rate linacs

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* This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

is still in its infancy at present. Moreover, they do not seem to offer any distinct advantages over the storage ring colliders. Consequently, serious design efforts (CESR (Cornell), PSI (Switzerland), KEK (Japan), INP (Novosibirsk), DESY (Germany), LBL/SLAC/Caltech (+ other California Universities), etc.) have focussed on rings.

This paper concentrates on generic R&D and design issues of asymmetric colliders via a specific example, namely a 9 GeV x 3 GeV collider based on PEP at SLAC. An asymmetric e^+-e^- collider at the $\Upsilon(4s)$ and with sufficiently high luminosity (10^{33}-10^{34} cm^{-2} s^{-1}) offers the possibility of studying mixing, rare decays, and CP violation in the $B\bar{B}$ meson system, as well as "beautiful" tau-charm physics, and has certain qualitative advantages from detection and machine design points of view. These include:[1,3,5] (1) the energy constraint; (2) clean environment (~25% B^+B^-, $B^o\bar{B}^o$); (3) large cross section (1 nb); (4) vertex reconstruction (from the time development of space-time separated B and $\bar{B}$ decays due to moving center-of-mass); (5) reduced backgrounds; (6) greatest sensitivity to CP violation in B → CP eigenstate; (7) the possibility of using higher collision frequencies, up to 100 MHz, in a head-on colliding mode using magnetic separation. It is estimated[1,2] that for $B \rightarrow \Psi K_S$, an asymmetric collider has an advantage equivalent to a factor of five in luminosity relative to a symmetric one. There are, however, questions with regard to the physics of the asymmetric beam-beam coulomb interaction that may limit the intrinsic luminosity and the possibility of realizing the small beam pipes (1-1.5 cm. radius) necessary to determine the vertices.

For a collider at the $\Upsilon(4s)$, the minimum acceptable asymmetry in energy between the beams is a factor of three, with a rather broad optimum between this low end (9 x 3 GeV) and the high end (12 x 2.3 GeV). The low end is more favorable for the ΨK_S-study (higher efficiency). There are two major aspects of these colliders that have to be addressed: (1) heteroenergetic colliding beams, peculiar to the asymmetric scenario; and (2) high luminosity, generic to all B-factories. The energy asymmetry poses complications from the physics of the beam-beam interaction, the choice of interaction point (IP) parameters, and the collision optics, etc. The required high luminosity implies high average and peak currents in the two rings. Issues of coherent stability of the beams, synchrotron radiation power into the vacuum chamber walls, vacuum degradation due to background gas pressure in the presence of high beam currents (leading to short beam lifetime), etc., become important. It is important to note that the highest luminosity achieved to date is about 10^{32} cm^{-2} s^{-1} in CESR at Cornell, with an expected upgrade to 5 x 10^{32} cm^{-2} s^{-1} in the near future. A viable B-factory thus requires an improvement by a factor of hundred or at least a factor of twenty in the most optimistic case. It is clear that a B-factory is a nontrivial challenge. We start with discussions on some general collider design issues with respect to luminosity.

LUMINOSITY

The general expression for luminosity in an asymmetric collider is cumbersome, involving various parameters of both low and high energy beams. To shed light on the general issues of high luminosity for B-factories, asymmetric or not, it is advantageous to express the luminosity in an energy-transparent way. The question is: can we arrange a situation where the luminosity can be expressed in terms of a common and single beam-beam interaction parameter, ξ, and a combination of other parameters that are taken from either the high energy (+) or the low energy (-) ring, irrespective of their energy? With parameters constrained to satisfy certain "scaling rules" (see Appendix), it is indeed possible to write the luminosity in the following simple and elegant energy-transparent form:

$$L = 2.167 \times 10^{34}\, \xi\, (1+r) \left(\frac{I \cdot E}{\beta_y^*} \right)^{(+,-)} \left[\mathrm{cm}^{-2} \mathrm{s}^{-1} \right] \qquad (1)$$

where ξ is the maximum saturated dimensionless beam-beam interaction parameter (the same for both beams and both planes, horizontal and vertical), r is the aspect ratio characterizing the shape of the beams (r=1 for round beams and r=0 for flat beams), I is the average circulating current in amperes, E is the energy in GeV and β_y^* is the value of the beta function, in cm, at the collision point in the ring. The combination in parenthesis is to be taken from *either* the high or the low energy ring.

What are the degrees of freedom in maximizing this luminosity? Energies, E: not a free parameter, constrained kinematically. Beam-beam interaction parameter, ξ: not really a free parameter. It is determined intrinsically by the nature of the beam-beam interaction. The range of maximum beam-beam tune-shifts achieved in various equal energy e^+-e^- colliders is $\xi_{max} \sim 0.03$-0.07. A choice of $\xi=0.05$ may be typical and conservatively optimistic. There is evidence[5] from computer simulations that ξ may depend intrinsically on the beam shape: $\xi \equiv \xi(r)$. This is a controversial issue, being debated at present.. But one obtains an enhancement in ξ by a factor of two for round beams, at best.[5] Aspect ratio, r: free to the extent that one can create round beams. The physics of the beam-beam interaction, however, is sensitive to the method used in creating round beams, e.g., coupling resonances, vertical wigglers. Maximum enhancement is by a geometric factor of two for round beams (r=1, 1+r = 2). Average beam current, I: relatively free parameter, however not absolutely. It is determined by various current-dependent coherent effects. The storage rings will have to accept the chosen current, given a certain impedance in the path of the beams. The low-beta, β_y^*: free and easy to vary down to a few centimeters, subject to the condition σ_ℓ (bunch length) $\lesssim \beta_y^*$.

It is thus clear that the luminosity is maximized for the highest currents with the lowest β_y^* and for round beams. What are the implications on parameters for a luminosity goal of 10^{34} $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$? Following the conservative route, one uses $\beta_y^* \sim$ few cm (typical low beta), $\xi \sim 0.05$ and round beams (r=1), implying an average current of I ~ few amperes to attain the desired luminosity. Typical stored currents achieved to date in high-energy electron storage rings are at most a few hundred milliamperes in multiple bunches. If intrinsic maximum tune-shift is truly enhanced for round beams, one expects another factor of two improvement.

Another speculative route could be to use extremely low-beta: $\beta_y^* \sim$ few mm implying an average circulating current I of hundreds of milliamperes. Hardware to produce a few-millimeter low beta is nontrivial. But, more importantly, one needs sub-millimeter bunch length as well, since luminosity degrades unless σ_ℓ (bunch length) $< \beta_y^*$. One way to produce ultrashort bunches is to use a zero momentum compaction ($\alpha \approx 0$) isochronous ring where particle path length is independent of energy.[7,8] Bunch length then is determined by injection conditions. However, one needs not only a high-precision 'zero' in the momentum compaction, but also a good control of the effects of higher order nonlinear momentum compaction coefficients.[8] To build such a ring is highly nontrivial and requires substantial technology R&D in controlling magnetic fields precisely. Studies along these lines are continuing at present. This paper is concerned with the more conventional former approach.

We now discuss the issues of beam-beam interaction and beam current-dependent effects. We start with a discussion of the peculiarities of the beam-beam effect for asymmetric colliders.

BEAM-BEAM TUNE-SHIFT

The attainable luminosity will be determined and limited by the physics of the beam-beam effect, aside from many other factors. Very little is known experimentally about the "beam-beam limit" under asymmetric energy conditions. Moreover, for the high luminosity situation, the beam-beam effect is expected to be in the "strong-strong" regime, which is quantitatively poorly understood at present. Design of an asymmetric collider must therefore be rationalized based on the only fact we know about the reality of the beam-beam effect under *symmetric* conditions, namely the beam-beam tune-shift limit, ξ, in equal energy e^+-e^- colliders. Consequently, one must allow for maximum possible flexibility and freedom in adjusting parameters that are anticipated to have an effect on the luminosity. Such parametric flexibility is essential in order to tune the collider to the highest tune-shift limit leading to the highest luminosity. Thus one may need to vary the beam emittances, sizes, shapes (aspect ratios) and damping decrement per collision to optimize luminosity. Numerical simulations[9] suggest that an asymmetric collider should probably have a parametric reach up to the "Asymmetric Energy Transparency Domain," where the following conditions are satisfied:

(1) Same linear beam-beam tune-shift parameter for both beams:

$$\frac{\beta^-}{\beta^+} \cdot \left(\frac{\gamma^+ N^+}{\gamma N^-}\right) = 1$$

(2) Same cross-sectional area of both beams at the IP:

$$\sigma^+ = \sigma^-$$

(and possibly equal emittances)

(3) Same radiation damping decrement per collision for both beams:

$$\lambda^+ = \lambda^-$$

(4) Same betatron phase modulation due to synchrotron motion:

$$\left(\frac{\sigma_\ell Q_s}{\beta_y^*}\right)^+ = \left(\frac{\sigma_\ell Q_s}{\beta_y^*}\right)^-$$

With parameters constrained as above, the two beams behave identically as far as the beam-beam effect in the transverse plane is concerned. The two beams of different energy evolve in a similar manner dynamically and saturate to the same ξ value. Otherwise, they settle quickly to a "weak-strong" situation. The above simulations argue for the idea of symmetrizing both the lattices and the beams of an asymmetric collider, that is, this regime should be essentially within the parametric reach of the design to ensure credible performance. The question arises whether one can relax such strong constraints by compensating one asymmetry (unequal damping decrements, say) by another (unequal beam intensities). The answer is not straightforward. While such a scenario may be plausible, we raise several concerns:

(a) There is a need to put more current in the low energy beam in the ratio of damping decrements. This is not desirable from a coherent stability point of view.

(b) There is evidence[10] that the stability of such a delicately compensated beam-beam mode is unpredictable. The situation is expected to be "touchy" and prone to bifurcation to a weak-strong situation rather easily at high tune-shifts.

(c) Beam intensity is not really a "knob" or freely adjustable parameter. The rings have to accept the desired currents.

We now turn to implications of the desired flexibility and domain of parametric reach, dictated by beam-beam considerations, for storage ring lattices.

LATTICE CONSIDERATIONS

It is clear that to achieve equal damping decrements per collision, the low energy ring design is severely constrained if it is based on bending magnets and focusing elements alone. The damping per collision goes as (E^3/ρ), where ρ is the bending radius. For an asymmetry of three in energy, one ends up with a very small radius, high-bending field ring. While the bending field for the low energy ring (reaching up to 1.8 for a PEP-based scenario) may be achievable, there are two severe limitations of such a design, one philosophical and one technological:

(1) For a pure bending magnet design, one gives up the crucial flexibility with regard to adjusting damping decrements and beam emittances, both of which are mainly fixed by the lattice as is.

(2) The synchrotron radiation power density in such a small ring could be a technological nightmare, reaching up to 10 kW/cm^2 along the path of the beam's radiation fan.[11] One also has to worry about the implied vacuum requirement and the reduced beam lifetime from beamstrahlung for such frequent collisions in the low energy ring.

Fortunately these conflicting requirements can be resolved by a simple and flexible solution: a wiggler lattice, where one can keep the low energy ring large but achieve extra damping, if necessary, via additional wiggler magnets. Since round beams are desirable, a respectable fraction of the wigglers could also be oriented vertically, with suitable lattice elements on both sides to create the requisite vertical dispersion. Beams made round intrinsically by emittance or temperature equipartition via noise-like excitations in the two planes (e.g., radiation in the wigglers) are expected to be more stable with respect to the beam-beam interaction than those made round via a coupling resonance. One would thus maintain tremendous flexibility in adjusting the lattice by distributing the radiation in each symmetry sector between bending and horizontal and vertical wiggler magnets and over larger circumferential lengths globally.

AN EXAMPLE: A PROPOSED PEP-BASED B-FACTORY

Parameters of a possible PEP-based asymmetric collider are shown in Table I.[12] In this design, beam-beam synchrobetatron resonances are avoided and ξ is maximized by using head-on collisions, zero-dispersion at the IP, and $\sigma_\ell < \beta_y^*$. Figure 1 illustrates the conceptual layout of magnetic elements in the interaction straight for zero crossing angle optics with small bunch spacing.[12] One would require a superconducting quadrupole triplet (pole tip field ≈ 5 T) to achieve the beta functions and a low-field (5 kG) bending magnet in the straight to separate the beams by $5\sigma_{x(y)}$ at a distance of one meter from the IP. Separation for small bunch spacing can also be obtained by a small crossing angle (~ 7 mrad). However, one would require a cavity to induce "crab-crossing" in order to avoid synchrobetatron resonances. In either case, one will require common two-in-one or three-in-one magnets for both beams in the IP region.

Table I Parameters for a proposed PEP-based B-Factory, APIARY-III

		High-Energy Beam	Low-Energy Beam
Energy	E(GeV)	9	3
Current	I (Amp)	2.8	2.8
Particles/bunch	N_B	1.2×10^{11}	1.2×10^{11}
Beta function at IP	$\beta^*_{x,y}$ (cm)	6	2
Bunch spacing	S_B(m)	2.3	2.3
Emittance	$\varepsilon_{x,y}$(nm)	33	100
Dispersion at IP	η^*(m)	0	0
Beam-beam tune-shifts	$\xi_{x,y}$	0.05	0.05
Aspect ratio	r	1(round)	1(round)
Circumference	$2\pi R$(m)	2200	600
Luminosity	L($cm^{-2}s^{-1}$)	1×10^{34}	

A prototype 3 GeV wiggler ring layout[12] is shown in Fig. 2(a) and the corresponding lattice functions[12] for one of the eight symmetry sectors are given in Fig. 2(b). Each sector has bending magnets, quadrupoles, horizontal wigglers and vertical wigglers. There are trim quadrupoles at both ends of the wigglers to match the Twiss parameters α_x

and α_y. The vertical and horizontal beta functions are matched to the intrinsic focussing of horizontal and vertical wigglers, respectively. The wigglers are ten meters long, with one meter period, and must be able to reach $B_0 = 1.6$ Tesla peak field on axis to match the required damping decrement per collision with PEP. Consequently, the wigglers probably would be of electromagnetic design. One expects such long-period wigglers to behave merely as extended linear focusing and drifts in the two transverse planes, interchangeably. No severe dynamic aperture restriction due to nonlinearities is expected (linear focusing $\propto B_0^2$, nonlinear effects $\propto (B_0/\lambda_w)^2$).

While the IP parameters and collision optics required for a 10^{34} cm^{-2} s^{-1} luminosity seem approachable, one still needs to sustain large currents in the two rings. We now turn to these coherent and incoherent current-dependent effects.

BEAM CURRENT DEPENDENT PHENOMENA

The collider would require hundreds of bunches with tens of milliamperes of current per bunch. Serious issues arise in connection with coherent instabilities and vacuum.

Studies[13,14] show that the most severe limitation stems from coupled multibunch instabilities driven by higher order sharp resonances of the RF cavities. The growth times are fractions of a millisecond. Radiation damping is of little help (tens of millisecond damping time). Powerful feedback systems would be required to counteract this growth. It is also generally true for the B-factories that very many bunches with less current per bunch is preferable to fewer bunches with higher current per bunch. This is because the former alternative helps avoid single bunch instabilities (while not affecting the multi-bunch instabilities, which are so strong that they totally disrespect the bunch pattern and are driven by *average* current predominantly). The single-bunch current will be limited by the transverse mode-coupling instability driven by the transverse impedance (generated mainly by the many RF cavities at high-β points in the ring). It may also be limited by the longitudinal microwave instability, which increases the bunch length and energy spread of the beam bunches.

Both these issues argue for a specially designed, better behaved, compact RF system. Such a system could be based on either superconducting RF cavities or specially designed room temperature RF cavities with low impedance. Advantages of a superconducting RF system are many:

(1) higher gradients;
(2) many fewer cells to produce the same voltage;
(3) less broadband impedance and higher order modes;
(4) large bore size, which reduces transverse impedance;
(5) compactness, which permits localizing the RF in low-beta regions in the ring.

However, superconducting RF becomes significantly attractive only if more power can be transmitted through the RF window than at present. One needs a scenario of single-cell RF cavities with gradients up to 7-9 MV/m, fundamental quality factor of $Q = 2 \times 10^9$ and loaded quality factors of higher order modes of $Q \approx 100$, and power fed to each cell through individual windows transmitting 400 kW, or more, CW.[15]

If power through the RF window turns out to be a significant barrier to the use of superconducting RF for B-factories, one may envisage an R&D program on windowless transmission of RF power through high quality, high vacuum waveguides (differentially pumped to isolate the cavity from the klystron) straight into the cavity.[16] The question of the high synchrotron radiation environment for the touchy superconducting RF system still remains.

Vacuum and other issues will limit the beam lifetime to a few hours and there remains the fundamental problem of retaining the peak instantaneous luminosity at a high average level. A fast and efficient injection system, possibly including continuous injection by trickling in charge, must be envisaged. The question of avoiding or improving fast switching of detectors during the injection process remains. The issue of trapped ions would also require special attention.

ISSUE OF EQUAL SIZED RINGS

There are certain advantages to having the low energy ring the same size as the high energy ring. These are:

(1) Luminosity lifetime from beamstrahlung is improved, since individual bunches collide less frequently.
(2) Vacuum chamber and vacuum issues are simplified since radiation is distributed over a larger circumferential length.
(3) Two IP's may be allowed.
(4) If gaps must be imposed on the bunch trains to avoid trapping of ions, they can be matched in both rings, so that anharmonic beam-beam effects are totally avoided.
(5) One does not need a proportionately larger number of wigglers, since the radiation would be dominated by wigglers in any scenario.

Possible disadvantages, of course, are that one needs to exercise special care in designing high quality, low-field bending magnets, and there might be additional cost considerations.

There is yet another reason why one may want to have the low energy ring as large as possible and maybe even the same size as the high energy ring. This has to do with the coherent dipole transverse beam-beam effect. The coherent tune-shift is expected to be close to $\Delta\nu_{coh} = \xi_{inc}$ (p/q) where (p/q) is the ratio of bunches in the high energy ring to the low energy ring. With $\xi_{inc} \sim 0.05$ -0.1, one can easily reach coherent tune shifts of the order of one to ten for small sized low energy rings. This purely reactive tune-shift may be harmful beyond a certain threshold value of current, when it develops a dissipative part leading to growth of coherent transverse motion. For equal sized rings, (p/q) = 1, and incoherent and coherent effects are comparable and small in magnitude.

OUTLOOK

It is apparent that wiggler lattices, high quality vacuum chambers of special design (to handle large doses of synchrotron radiation and to maintain good vacuum in the presence of large beam current), a superconducting or other specially designed RF system, feedback systems, efficient and fast injectors, a small beam pipe at the IP and radiation masking, etc., are all generic features of an asymmetric collider design. A collider with 10^{34} cm^{-2} s^{-1} luminosity is nontrivial but definitely approachable with significant R&D in the above areas. Such R&D is already in progress at various laboratories. It is also clear that a collider with 10^{33} cm^{-2} s^{-1} can already be built with state-of-the art technology and careful design. Finally, one notes that for the same luminosity, an asymmetric collider has an effective enhancement of luminosity over a symmetric one arising from detection efficiency.

ACKNOWLEDGMENTS

I am indebted to many people at LBL, SLAC, Caltech and Indiana U. who have contributed to the thoughts and materials presented in this paper. In particular, contributions from A.A. Garren on lattices, Y.H. Chin on beam-beam effects, and M. Zisman on collective effects are gratefully acknowledged. Finally, I acknowledge the tremendous motivation provided by P.J. Oddone in carrying out these studies.

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APPENDIX

A. Equal energy beams, complete overlap and $\Delta\nu_x = \Delta\nu_y$:

$$\beta_y/\beta_x = \varepsilon_y/\varepsilon_x = \sigma_y/\sigma_x = r \quad \text{(a constant)}$$

B. Unequal energy beams, complete overlap :

$$\beta_i^+/\beta_i^- = \varepsilon_i^-/\varepsilon_i^+ = b \quad \text{(a constant)} \qquad i=x,y$$

C. Unequal energy beams, complete overlap and all four tune shifts the same:

$$\Delta\nu_i^+ = \Delta\nu_i^- = \xi$$

$$\beta_i^+/\beta_i^- = \gamma^+N^+/\gamma N^- = b \quad \text{(a constant)} \qquad i=x,y$$

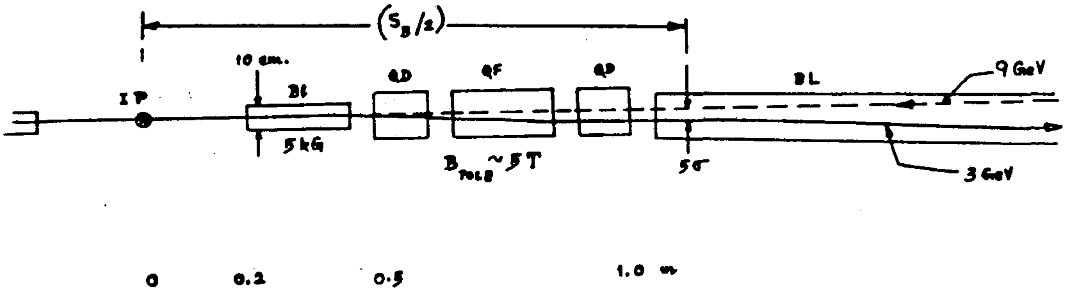

Fig. 1 Conceptual design for zero-crossing-angle optics for small bunch spacing.

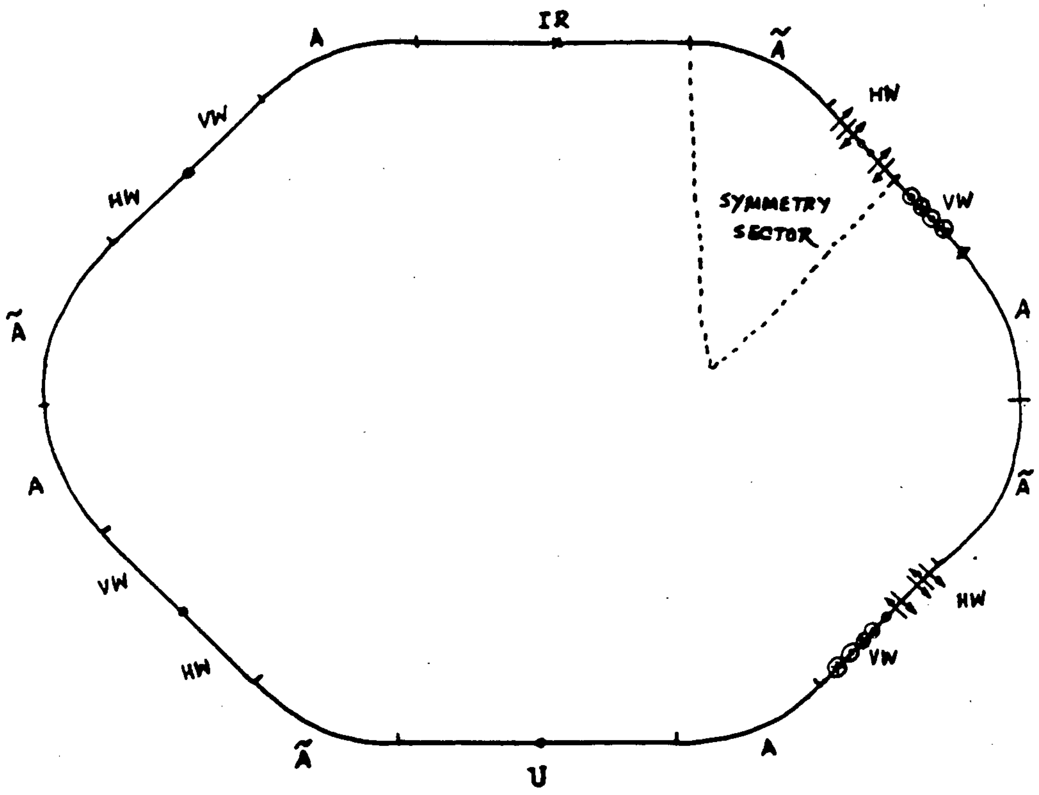

Fig.2a Prototype 3-GeV wiggler ring.

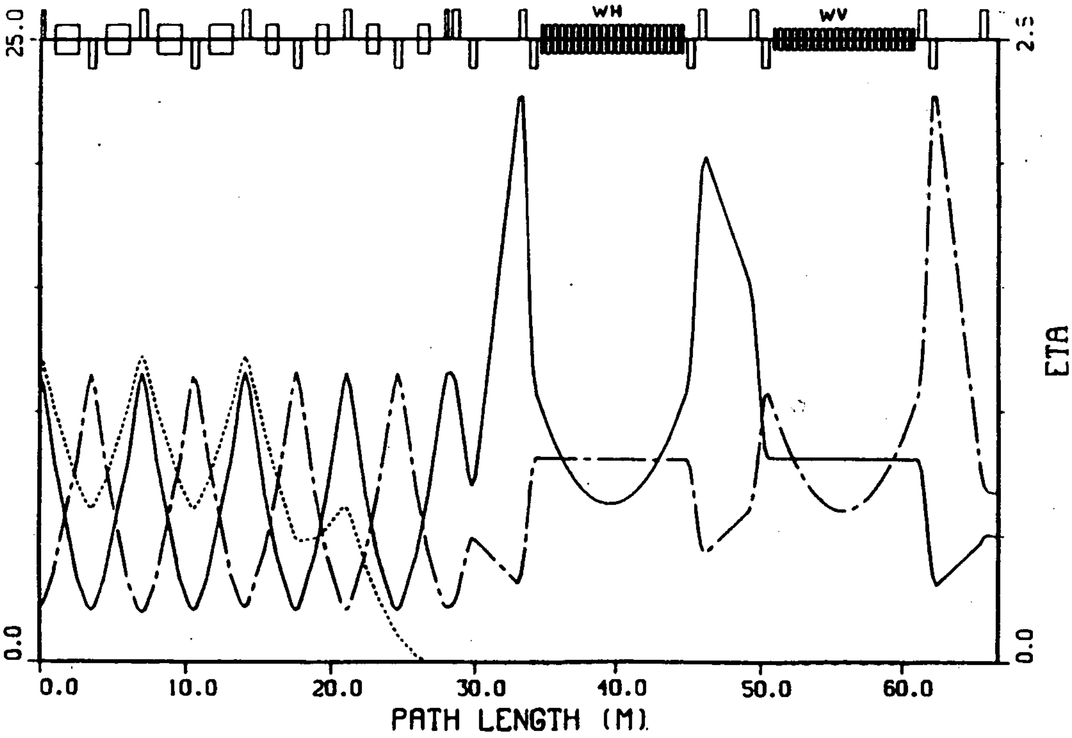

Fig. 2b Prototype 3-GeV wiggler lattice functions.

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