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ANOTHER LOOK AT THE GAUGED WESS-ZUMINO EFFECTIVE ACTION

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ABSTRACT

A compact form for the gauged Wess-Zumino effective action is found. The brevity of the result is due to the use of vector and axial fields, rather than left– and right–handed fields. The method used is in the spirit of Zumino's differential geometric approach.

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I. Introduction

Some years ago, Wess and Zumino [1] derived a low energy effective action for pions in the presence of an external gauge field. The result was given in terms of a certain five-dimensional integral involving the Bardeen anomaly [2]. It was observed that the integral is non-vanishing, even when the gauge fields vanish.

Witten [3] has given a simple intuitive reason for the importance of the Wess-Zumino action, namely, that it removes a certain symmetry from the non-linear sigma model. This symmetry, $\Pi^i \mapsto -\Pi^i$, where Π^i represents the pion fields, is not a symmetry of QCD, and is therefore undesirable. The term which breaks this symmetry (with the fewest number of derivatives) is the Wess-Zumino action with gauge fields set equal to zero. This term is a five-dimensional integral involving pion fields alone.

Witten showed that this term can be written more symmetrically as an integral over a five-disk. Written in this way, it is seen that the action depends on the particular five-disk of integration. This leads to an ambiguity in the action proportional to the winding number for the pion configuration, which implies that the overall coefficient is quantised.

Witten also showed that, when gauge fields are taken into account, the effective action becomes the sum of this five-space integral and a four-space integral involving both gauge fields and pions. This four-space integral, formula (24) of ref.[3], is extremely messy, containing 24 terms. Indeed, it contained a few minor errors, which has triggered several papers [4], all of which are in agreement. (See also [5].) However, the basic problem remains—the formula still covers half a page and is very tedious to check.

The purpose of this paper is to find a reasonably simple expression for the gauged Wess-Zumino action, using a minimum of mathematical machinery. A glance at the final result, formula (3.37), will verify that the first of these two goals has been achieved. The machinery needed to arrive at this result is 1) the language of differential forms, 2) Stokes' theorem and 3) the "homotopy operator".

This paper is organized as follows. Section 2 summarises our conventions. Section 3 outlines some rather pedestrian manipulations of Wess and Zumino's formula to obtain the final result. Section 4 contains a brief conclusion.

II. Conventions

We will work in Euclidean space, with $g^{\mu\nu} = \delta^{\mu\nu}$. It is customary to define

$$\xi \equiv \frac{\Pi^i X^i}{F_\pi} \tag{2.1}$$

where $F_\pi \approx 93$ MeV, Π^i are the pion fields, and X^i are the generators of U(N). N is the number of light quark flavors, N_c is the number of colors. The generators X^i satisfy

$$tr(X^i X^j) = \frac{1}{2}\delta^{ij}. \tag{2.2}$$

Many other authors [3,4,5] define

$$\mathcal{U}(\xi) \equiv e^{2i\xi} \tag{2.3}$$

which transforms under $U(N)_L \times U(N)_R$ by

$$(A, B) : \mathcal{U} \mapsto A\mathcal{U}B^{-1}. \tag{2.4}$$

Instead of this representation, we will use the Callan, Coleman, Wess and Zumino formalism [6], which will be defined below.

We are interested in the interaction of pions with vector and axial fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$. Under vector transformations, $\mathcal{V}_\mu$ transforms like a gauge field and $\mathcal{A}_\mu$ transforms in the adjoint representation. The reverse is true for axial transformations. The fields are anti-hermitian:

$$\mathcal{V}_\mu = -i\mathcal{V}^i_\mu X^i \tag{2.5a}$$

$$\mathcal{A}_\mu = -i\mathcal{A}^i_\mu X^i. \tag{2.5b}$$

For simplicity in the calculations, differential forms are essential. Define

$$\mathcal{V} \equiv \mathcal{V}_\mu dx^\mu \tag{2.6a}$$

$$\mathcal{A} \equiv \mathcal{A}_\mu dx^\mu. \tag{2.6b}$$

The usual field strength tensors

$$\mathcal{V}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu + [\mathcal{V}_\mu, \mathcal{V}_\nu] + [\mathcal{A}_\mu, \mathcal{A}_\nu] \tag{2.7a}$$

$$\mathcal{A}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{V}_\mu, \mathcal{A}_\nu] + [\mathcal{A}_\mu, \mathcal{V}_\nu] \tag{2.7b}$$

will be denoted by the forms

$$\hat{\mathcal{V}} \equiv \frac{1}{2}\mathcal{V}_{\mu\nu} dx^\mu dx^\nu \tag{2.8a}$$

$$\hat{\mathcal{A}} \equiv \frac{1}{2}\mathcal{A}_{\mu\nu} dx^\mu dx^\nu \tag{2.8b}$$

where we do not explicitly show the wedge product symbol.

In translating the Wess-Zumino formula into our notation, it is expedient to define

$$\mathcal{V}_t + \mathcal{A}_t\gamma_5 \equiv e^{-it\xi\gamma_5}(d + \mathcal{V} + \mathcal{A}\gamma_5)e^{it\xi\gamma_5} \tag{2.9a}$$

$$\hat{\mathcal{V}}_t + \hat{\mathcal{A}}_t\gamma_5 \equiv e^{-it\xi\gamma_5}(\hat{\mathcal{V}} + \hat{\mathcal{A}}\gamma_5)e^{it\xi\gamma_5}. \tag{2.9b}$$

$\mathcal{V}_t$ and $\mathcal{A}_t$ are axial gauge transforms of $\mathcal{V}$ and $\mathcal{A}$, with gauge parameter $-t\xi$. Note that $\mathcal{V}_0 = \mathcal{V}$, $\mathcal{A}_0 = \mathcal{A}$, $\hat{\mathcal{V}}_0 = \hat{\mathcal{V}}$ and $\hat{\mathcal{A}}_0 = \hat{\mathcal{A}}$.

For any value of t,

$$\hat{\mathcal{V}}_t = d\mathcal{V}_t + \mathcal{V}_t^2 + \mathcal{A}_t^2 \tag{2.10a}$$

$$\hat{\mathcal{A}}_t = d\mathcal{A}_t + \{\mathcal{V}_t, \mathcal{A}_t\}. \tag{2.10b}$$

We will find it convenient to define

$$\mathcal{G}_t \equiv \mathcal{V}_t + \mathcal{A}_t\gamma_5 \tag{2.11a}$$

$$\hat{\mathcal{G}}_t \equiv \hat{\mathcal{V}}_t + \hat{\mathcal{A}}_t\gamma_5. \tag{2.11b}$$

Finally, we define a covariant derivative $\mathcal{D}_t$ whose action on p-forms ω_p is

$$\mathcal{D}_t\omega_p \equiv d\omega_p + [\mathcal{V}_t, \omega_p]_\pm \tag{2.12}$$

where $+$ should be taken for p odd and $-$ for p even. Note that $\mathcal{D}_t$ is an anti-derivation, i.e., it satisfies the same Leibniz rule as the exterior derivative d.

III. Calculations

The Wess-Zumino action in Euclidean space is given by

$$\Gamma = -\frac{iN_c}{48\pi^2}\int_0^1 dt \int_{R^4} \mathrm{tr}[\xi \mathcal{B}_t] \tag{3.1}$$

where N_c is the number of colors and

$$\mathcal{B}_t \equiv 12\hat{\mathcal{V}}_t^2 + 4\hat{\mathcal{A}}_t^2 + 32\mathcal{A}_t^4 - 16(\mathcal{A}_t^2\hat{\mathcal{V}}_t + \mathcal{A}_t\hat{\mathcal{V}}_t\mathcal{A}_t + \hat{\mathcal{V}}_t\mathcal{A}_t^2). \tag{3.2}$$

The fields approach constants at infinity, so from now on, we shall compactify R^4 to S^4. We define the normalisation constant

$$\mathcal{K} \equiv -\frac{iN_c}{48\pi^2}. \tag{3.3}$$

Let D^5 be a five-dimensional disk whose boundary $\partial D^5 = S^4$. Then we can rewrite (3.1) via Stokes' theorem:

$$\begin{aligned}\Gamma &= \mathcal{K}\int_0^1 dt \int_{S^4} \mathrm{tr}[\xi\mathcal{B}_t] \\ &= \mathcal{K}\int_0^1 dt \int_{D^5} d(\mathrm{tr}[\xi\mathcal{B}_t]) \\ &= \mathcal{K}\int_0^1 dt \int_{D^5} \mathcal{D}_t(\mathrm{tr}[\xi\mathcal{B}_t]).\end{aligned} \tag{3.4}$$

The last step follows from the fact that the trace of anything is a singlet.

From the definitions in Section 2, it is easy to check that

$$\mathcal{D}_t\mathcal{V}_t = \hat{\mathcal{V}}_t + \mathcal{V}_t^2 - \mathcal{A}_t^2 \tag{3.5a}$$

$$\mathcal{D}_t\mathcal{A}_t = \hat{\mathcal{A}}_t \tag{3.5b}$$

$$\mathcal{D}_t\hat{\mathcal{V}}_t = [\hat{\mathcal{A}}_t, \mathcal{A}_t] \tag{3.5c}$$

$$\mathcal{D}_t\hat{\mathcal{A}}_t = [\hat{\mathcal{V}}_t, \mathcal{A}_t]. \tag{3.5d}$$

Using these formulas, a straightforward calculation will verify that

$$\mathcal{D}_t\mathrm{tr}[\xi\mathcal{B}_t] =$$

$$\mathrm{tr}\Big[\big(\mathcal{D}_t\xi\big)\mathcal{B}_t + 12\mathcal{A}_t\{\hat{\mathcal{V}}_t, [\hat{\mathcal{A}}_t, \xi]\} + 4\mathcal{A}_t\{\hat{\mathcal{A}}_t, [\hat{\mathcal{V}}_t, \xi]\} - 16\mathcal{A}_t^3[\hat{\mathcal{A}}_t, \xi]\Big] \tag{3.6}$$

From the definitions (2.9), it is easy to check that

$$\frac{d}{dt}\mathcal{V}_t = i[\mathcal{A}_t, \xi] \qquad (3.7a)$$

$$\frac{d}{dt}\mathcal{A}_t = i\left(\mathcal{D}_t \xi\right) \qquad (3.7b)$$

$$\frac{d}{dt}\hat{\mathcal{V}}_t = i[\hat{\mathcal{A}}_t, \xi] \qquad (3.7c)$$

$$\frac{d}{dt}\hat{\mathcal{A}}_t = i[\hat{\mathcal{V}}_t, \xi]. \qquad (3.7d)$$

Using these rules, it quickly follows that

$$\mathcal{D}_t \mathrm{tr}(\xi \mathcal{B}_t) = \frac{d}{dt}\Omega_t \qquad (3.8)$$

where

$$\Omega_t \equiv -i\mathrm{tr}\left[12\mathcal{A}_t\hat{\mathcal{V}}_t^2 + 4\mathcal{A}_t\hat{\mathcal{A}}_t^2 + \frac{32}{5}\mathcal{A}_t^5 - 16\mathcal{A}_t^3\hat{\mathcal{V}}_t\right]. \qquad (3.9)$$

Combining (3.8) and (3.4), we find

$$\Gamma = \mathcal{K}\int_{D^5}(\Omega_1 - \Omega_0). \qquad (3.10)$$

We would now like to convert this to an integral over S^4 by using Stokes' theorem again. The appropriate technology to apply is the homotopy operator formalism. Zumino [6] has given a very clear presentation of this method, so we simply quote the procedure (suitably modified to our more complicated situation).

For each value of t, there exists an operator k_t such that

$$k_t d + d k_t = 1 \qquad (3.11)$$

when acting on homogeneous polynomials of the fields $\mathcal{V}_t, \mathcal{A}_t, \hat{\mathcal{V}}_t$ and $\hat{\mathcal{A}}_t$. k_t is an anti-derivation and $k_t^2 = 0$. Its action on an arbitrary

polynomial $\mathcal{P}(\mathcal{V}_t, \mathcal{A}_t, \hat{\mathcal{V}}_t, \hat{\mathcal{A}}_t)$ is defined in terms of a one-parameter family of anti-derivations ℓ_t^s by the formula

$$k_t\mathcal{P}\left(\mathcal{V}_t, \mathcal{A}_t, \hat{\mathcal{V}}_t, \hat{\mathcal{A}}_t\right) = \int_0^1 \ell_t^s \mathcal{P}\left(\mathcal{V}_t^s, \mathcal{A}_t^s, \hat{\mathcal{V}}_t^s, \hat{\mathcal{A}}_t^s\right). \qquad (3.12)$$

Here ℓ_t^s acts on the "interpolating fields"

$$\mathcal{V}_t^s = s\mathcal{V}_t \qquad (3.13a)$$

$$\mathcal{A}_t^s = s\mathcal{A}_t \qquad (3.13b)$$

$$\hat{\mathcal{V}}_t^s = s\hat{\mathcal{V}}_t - s(1-s)(\mathcal{V}_t^2 + \mathcal{A}_t^2) \qquad (3.13c)$$

$$\hat{\mathcal{A}}_t^s = s\hat{\mathcal{A}}_t - s(1-s)\{\mathcal{V}_t, \mathcal{A}_t\} \qquad (3.13d)$$

by the formula

$$\ell_t^s \mathcal{V}_t^s = 0 \qquad (3.14a)$$

$$\ell_t^s \mathcal{A}_t^s = 0 \qquad (3.14b)$$

$$\ell_t^s \hat{\mathcal{V}}_t^s = \mathcal{V}_t ds \qquad (3.14c)$$

$$\ell_t^s \hat{\mathcal{A}}_t^s = \mathcal{A}_t ds. \qquad (3.14d)$$

Applying (3.11) to (3.10), we have

$$\begin{aligned}\Gamma &= \mathcal{K}\int_{D^5}[(k_1 d + dk_1)\Omega_1 - (k_0 d + dk_0)\Omega_0] \\ &= \mathcal{K}\int_{S^4}(k_1\Omega_1 - k_0\Omega_0) + \mathcal{K}\int_{D^5}(k_1 d\Omega_1 - k_0 d\Omega_0)\end{aligned} \qquad (3.15)$$

where Stokes' theorem has been used.

More progress can be made by defining

$$\mathcal{T} \equiv [k_1 d\Omega_1 - k_0 d\Omega_0]_{\mathcal{A}_0 = \mathcal{V}_0 = 0} \qquad (3.16)$$

Then the polynomial

$$\mathcal{Q} \equiv k_1 d\Omega_1 - k_0 d\Omega_0 - \mathcal{T} \tag{3.17}$$

is necessarily homogeneous in the fields $\mathcal{V}_0$, $\mathcal{A}_0$, $\hat{\mathcal{V}}_0$ and $\hat{\mathcal{A}}_0$, so (3.11) can be applied: $(k_0 d + dk_0)\mathcal{Q} = \mathcal{Q}$. This leads to the messy formula

$$k_1 d\Omega_1 - k_0 d\Omega_0 = \mathcal{T} + (k_0 d + dk_0)(k_1 d\Omega_1 - k_0 d\Omega_0 - \mathcal{T}). \tag{3.18}$$

Our goal now is to simplify this. In fact, only two terms in the expansion of the RHS are non-zero. Using formulas (3.5) on (3.9), it is routine to show that

$$\begin{aligned} d\Omega_t &= \mathcal{D}_t \Omega_t \\ &= -i\mathrm{tr}\left[12\hat{\mathcal{A}}_t \hat{\mathcal{V}}_t^2 + 4\hat{\mathcal{A}}_t^3\right] \\ &= -i\mathrm{Tr}\left[\gamma_5 \hat{\mathcal{G}}_t^3\right] \end{aligned} \tag{3.19}$$

where Tr denotes trace over both Dirac and flavor indices. (Ω_t is analogous to ω_{2n-1}^0 in Zumino's lectures. The notation here is intended to suggest this parallel, while keeping in mind the differences.)

Noting that

$$\hat{\mathcal{G}}_t = e^{-it\xi\gamma_5} \hat{\mathcal{G}}_0 e^{it\xi\gamma_5} \tag{3.20}$$

it is obvious from (3.19) that

$$d\Omega_1 = d\Omega_0. \tag{3.21}$$

But using (3.11),

$$\begin{aligned} d(k_1 d\Omega_1 - k_0 d\Omega_0) &= d\big((1 - dk_1)\Omega_1 - (1 - dk_0)\Omega_0\big) \\ &= d\Omega_1 - d\Omega_0 \\ &= 0. \end{aligned} \tag{3.22}$$

From the definition of $\mathcal{T}$ and ℓ_t^s, it is clear that

$$k_0 \mathcal{T} = 0 \tag{3.23a}$$

and that (3.22) implies

$$d\mathcal{T} = 0. \tag{3.23b}$$

Finally, noting $k_0^2 = 0$, (3.18) becomes

$$k_1 d\Omega_1 - k_0 d\Omega_0 = \mathcal{T} + dk_0 k_1 d\Omega. \tag{3.24}$$

Substituting this into (3.15) and using Stokes' theorem again,

$$\Gamma = \mathcal{K}\int_{S^4}(k_1\Omega_1 - k_0\Omega_0 + k_0 k_1 d\Omega_1) + \mathcal{K}\int_{D_5}\mathcal{T}. \tag{3.25}$$

Some explicit calculations are now in order.

$$\begin{aligned} k_t\Omega_t &= -i\mathrm{tr}\int_0^1 \ell_t^s\left[12\mathcal{A}_t^s\left(\hat{\mathcal{V}}_t^s\right)^2 + 4\mathcal{A}_t^s\left(\hat{\mathcal{A}}_t^s\right)^2 + \frac{32}{5}(\mathcal{A}_t^s)^5 - 16(\mathcal{A}_t^s)^3\hat{\mathcal{V}}_t^s\right] \\ &= i\mathrm{tr}\int_0^1 ds\left[12\mathcal{A}_t^s\{\mathcal{V}_t, \hat{\mathcal{V}}_t^s\} + 4\mathcal{A}_t^s\{\mathcal{A}_t, \hat{\mathcal{A}}_t^s\} - 16(\mathcal{A}_t^s)^3\mathcal{V}_t\right] \end{aligned} \tag{3.26}$$

and inserting (3.13), the integration over s is trivial,

$$k_t\Omega_t = i\mathrm{tr}\left[4\mathcal{A}_t\{\mathcal{V}_t, \hat{\mathcal{V}}_t\} + 2\mathcal{V}_t^3\mathcal{A}_t + 6\mathcal{V}_t\mathcal{A}_t^3\right]. \tag{3.27}$$

An entirely similar calculation leads to

$$k_t d\Omega_t = -i\mathrm{Tr}\left[\gamma_5\left(\mathcal{G}_t\hat{\mathcal{G}}_t^2 - \frac{1}{2}\mathcal{G}_t^3\hat{\mathcal{G}}_t + \frac{1}{10}\mathcal{G}_t^5\right)\right]. \tag{3.28}$$

Using the definition (3.16), this implies

$$\mathcal{T} = -\frac{i}{10}\mathrm{Tr}(\gamma_5\alpha^5) \tag{3.29}$$

where

$$\alpha \equiv g^{-1}dg \tag{3.30a}$$

$$g \equiv e^{i\xi\gamma_5}. \tag{3.30b}$$

Note that $\mathcal{T}$ looks very similar to the pion term given by Witten. It will be seen later that $\mathcal{T}$ differs by an exact form from Witten's expression.

The computation of $k_0k_1d\Omega_1$ is only slightly more complicated. The reason is because k_0 is defined in terms of ℓ_0^s, which has a simple action on $\mathcal{V}_0$, $\mathcal{A}_0$, etc., but not on $\mathcal{V}_1$, $\mathcal{A}_1$, etc. The best way to proceed is to define

$$\mathrm{G} \equiv g^{-1}\mathcal{G}_0 g \tag{3.31a}$$

$$\hat{\mathrm{G}} \equiv \hat{\mathcal{G}}_1 = g^{-1}\hat{\mathcal{G}}_0 g. \tag{3.31b}$$

A short calculation based on (3.13) and (3.14) gives the formulas

$$\mathcal{G}_1^s = s\mathrm{G} + \alpha \tag{3.32a}$$
$$\hat{\mathcal{G}}_1^s = s\hat{\mathrm{G}} - s(1-s)\mathrm{G}^2 \tag{3.32b}$$
$$\ell_0^s\mathcal{G}_1^s = 0 \tag{3.32c}$$
$$\ell_0^s\hat{\mathcal{G}}_1^s = \mathrm{G}ds. \tag{3.32d}$$

Combining these with (3.28), we find that

$$k_0k_1d\Omega_1 = -i\mathrm{Tr}\gamma_5\int_0^1 \ell_0^s\Big[\mathcal{G}_1^s\big(\hat{\mathcal{G}}_1^s\big)^2 - \frac{1}{2}(\mathcal{G}_1^s)^3\hat{\mathcal{G}}_1^s + \frac{1}{10}(\mathcal{G}_1^s)^5\Big]$$

$$= i\mathrm{Tr}\gamma_5\int_0^1 ds\Big[(s\mathrm{G}+\alpha)\{\mathrm{G}, s\hat{\mathrm{G}} - s(1-s)\mathrm{G}^2\} - \frac{1}{2}(s\mathrm{G}+\alpha)^3\mathrm{G}\Big] \tag{3.33}$$

which, after integration, becomes

$$k_0k_1d\Omega_1 = i\mathrm{Tr}\gamma_5\Big[\frac{1}{2}\alpha\{\mathrm{G},\hat{\mathrm{G}}\} - \frac{1}{2}\alpha\mathrm{G}^3 + \frac{1}{4}(\mathrm{G}\alpha)^2 + \frac{1}{2}\mathrm{G}\alpha^3\Big]. \tag{3.34}$$

In view of (3.31), this can be rewritten as

$$k_0k_1d\Omega_1 = i\mathrm{Tr}\gamma_5\Big[\frac{1}{2}\beta\{\mathcal{G}_0,\hat{\mathcal{G}}_0\} - \frac{1}{2}\beta\mathcal{G}_0^3 + \frac{1}{4}(\mathcal{G}_0\beta)^2 + \frac{1}{2}\mathcal{G}_0\beta^3\Big] \tag{3.35}$$

where

$$\beta \equiv dgg^{-1}. \tag{3.36}$$

Combining (3.25), (3.26), (3.29) and (3.35), the final result is

$$\begin{aligned}\Gamma = iK\int_{S^4}\mathrm{tr}\Big[4\mathcal{A}_t\{\mathcal{V}_t,\hat{\mathcal{V}}_t\} + 2\mathcal{V}_t^3\mathcal{A}_t + 6\mathcal{V}_t\mathcal{A}_t^3\Big]_{t=0}^{t=1} \\ + iK\int_{S^4}\mathrm{Tr}\gamma_5\Big[\frac{1}{2}\beta\{\mathcal{G}_0,\hat{\mathcal{G}}_0\} - \frac{1}{2}\beta\mathcal{G}_0^3 + \frac{1}{4}(\mathcal{G}_0\beta)^2 + \frac{1}{2}\mathcal{G}_0\beta^3\Big] \\ - \frac{iK}{10}\int_{D^5}\mathrm{Tr}(\gamma_5\alpha^5).\end{aligned} \tag{3.37}$$

Equation (3.37) is the main result of this paper (in Euclidean space). To obtain the Minkowski space $(+---)$ result, one adds a factor of i. We now check to see that (3.37) agrees with known results for some special cases.

If one sets $\mathcal{V}_0 = \mathcal{A}_0 = 0$, then Γ reduces to a purely pionic action given by

$$\Gamma^{(\pi)} = iK\int_{S^4}\mathrm{tr}(2v^3a + 6va^3) - \frac{iK}{10}\int_{D^5}\mathrm{Tr}(\gamma_5\alpha^5) \tag{3.38}$$

where v and a are defined by

$$v + \gamma_5 a \equiv \alpha. \tag{3.39}$$

An application of Stokes' theorem to the first term in (3.38), combined with an expansion of the second term, leads to

$$\Gamma^{(\pi)} = -\frac{32iK}{5} \int_{D^5} \mathrm{tr}(a^5). \tag{3.40}$$

One can show from (3.30a) and (2.3) that

$$a = \frac{1}{2} \mathcal{U}^{-\frac{1}{2}} (d\mathcal{U}) \mathcal{U}^{-\frac{1}{2}} \tag{3.41}$$

so we find that

$$\Gamma^{(\pi)} = -\frac{N_c}{240\pi^2} \int_{D^5} \mathrm{tr}(\mathcal{U}^{-1} d\mathcal{U})^5 \tag{3.42}$$

which, of course, is Witten's expression (in Euclidean space).

Another check we could perform on (3.37) is to expand it as a power series in ξ. After a long calculation, one finds

$$\Gamma = K \int_{S^4} \mathrm{tr}(\xi B_0) + O(\xi^2) \tag{3.43}$$

which is obviously correct, in view of (3.1)–(3.3).

IV. Conclusion

Our result (3.37) is more compact than any other published formula that we know of. In spite of the more complicated form for the anomaly when using vector and axial fields, the effective action is apparently simpler than the corresponding formula expressed in terms of left-handed and right-handed fields. This is gratifying, in view of

recent work [7] that indicates that the effective action derived from Bardeen's form of the anomaly correctly reproduces the vector meson decay amplitudes. While our formula is not much more convenient for doing perturbative calculations, (because it must be expanded using (2.9) and (2.11)), we expect that Skyrme modellers [8] will find our formula less cumbersome than others that have been published. This, in fact, provided a motivation for this work.

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