Title

Earthquake Stress Analysis in Earth Dams

Authors

Clough, Ray

Chopra, Anil

Publication Date

1965-07-01

REPORT NO.
65-8

STRUCTURES AND MATERIALS RESEARCH
DEPARTMENT OF CIVIL ENGINEERING

EARTHQUAKE STRESS ANALYSIS IN EARTH DAMS

BY

RAY W. CLOUGH

ANIL K. CHOPRA

State of California
Department of Water Resources
Standard Agreement No. 352984

JULY, 1965

STRUCTURAL ENGINEERING LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY CALIFORNIA

Structures and Materials Research
Department of Civil Engineering
Report No. 65-8

EARTHQUAKE STRESS ANALYSIS IN EARTH DAMS

by

Ray W. Clough

Anil K. Chopra

Faculty Investigators: H. B. Seed
Ray W. Clough
Joseph Penzien

Prepared under the sponsorship of
State of California
Department of Water Resources

Standard Agreement No. 352984

University of California
Berkeley, California

July 1965

EARTHQUAKE STRESS ANALYSIS IN EARTH DAMS

by

Ray W. Clough,[1] F.ASCE, and Anil K. Chopra[2]

SYNOPSIS

The finite element method, which has been used extensively in the static analysis of elastic continua, is here extended to the dynamic analysis of two-dimensional stress systems. The method involves calculation of vibration mode shapes and frequencies, making use of a stiffness matrix evaluated by the finite element method and a mass matrix formed by lumping the mass at the finite element nodal points. The dynamic response is then calculated by the mode-superposition method, the response of each mode being obtained by step-by-step integration.

The method is demonstrated by the earthquake analysis of a triangular earth-dam cross section treated as an elastic plane-strain problem. Vertical and horizontal acceleration components of the 1940 El Centro earthquake are applied; results are presented in the form of plots showing the time history of stresses at selected points in the cross section and also plots of stress contours at selected instants of time.

INTRODUCTION

An earth dam is a three-dimensional continuum composed of anisotropic, non-homogeneous, non-elastic materials, and as such presents a formidable

---

1. Professor of Civil Engineering, University of California, Berkeley
2. Graduate Student, University of California, Berkeley

problem to the analyst who seeks to evaluate even its static stress distribution. The complete analysis of the dynamic response of such structures to earthquake excitation appears to be beyond the present capabilities of even the best digital computers. However, the earthquake behavior of earth dams is an extremely important subject; many important dams are being built in regions of high seismicity at the present time, and others will be built in the future. Thus, it is essential to obtain some theoretical understanding of the response of such structures to earthquake excitation so that improved methods may be developed for assessing their factors of safety against rupture.

In order to render the dynamic response problem mathematically tractable, it is necessary to make many sweeping assumptions regarding the geometry and material properties of the earth dam structure. In all analyses to date, including the work described herein, the material has been assumed to be linearly elastic, and the true three-dimensional nature of the geometry has been ignored.

The first mathematical treatments of the dynamic response of earth dams reduced the problem to a one-dimensional form. The structure was assumed to be of prismatic wedge-shaped form, of infinite length, and loaded uniformly along the length so as to produce plane strain behavior of the cross section. Furthermore, displacements within the cross sections were assumed to involve only horizontal shear. Thus, the system was reduced to a vertical shear beam with linearly varying width. The vibratory properties of such a system were first discussed by Mononobe, et.al.[3] Further studies of the earthquake behavior of the wedge-shaped shear beam were published by Hatanaka.[4,5] Ambraseys[6,7] extended the analyses to include the effects of end constraint, assuming that shear distortions could develop along vertical sections as well as horizontal.

All of these analyses took account only of shearing distortions in the earth dam material, a very limited approximation of the true behavior. The first attempt to include the complete two-dimensional nature of the cross section deformations in a dynamic analysis was made by Ishizaki and Hatakeyama.[8] They treated the dynamic plane strain problem using finite differences to solve the Navier equations of equilibrium at discrete intervals of time. Comparison of these results with the shear wedge type analysis demonstrated: (1) that the assumption of pure shear deformation is reasonable near the vertical axis of the cross section, but is in significant error near the upstream and downstream faces, and (2) that horizontal displacements near the faces of the dam may be significantly different

---

3. Mononobe, N., Takata, A. and Matumura, M., "Seismic Stability of an Earth Dam", Trans. Second Congress on Large Dams, Vol. IV, Washington, 1936.
4. Hatanaka, M., "Fundamental Considerations on the Earthquake Resistant Properties of the Earth Dam", Disaster Prevention Research Institute, Kyoto University, Bulletin No. 11, December 1955.
5. Hatanaka, M., "Fundamental Study on the Earthquake Resistant Design of Gravity Type Dams", Memoirs of the Faculty of Engineering, Kobe University, No. 8, March 1961.
6. Ambraseys, N. N., "On the Shear Response of a Two-Dimensional Truncated Wedge Subjected to an Arbitrary Disturbance", Bull. Seis. Soc. Am., Vol. 50, No. 7, January 1960, pp. 45-60.
7. Ambraseys, N. N., "The Seismic Stability of Earth Dams", Proc. Second World Conference on Earthquake Engineering, Vol. 2, Japan, 1960.
8. Ishizaki, H. and Hatakeyama, N., "Considerations on the Vibrational Behaviors of Earth Dams", Disaster Prevention Research Institute, Kyoto University, Bulletin No. 52, February 1962.

from the centerline displacements given by the shear wedge theory. Because the design criteria for earth dams are based largely on slope stability which is controlled by conditions near the faces of the dam, it is clear that the shear wedge analysis does not give an adequate measure of the dam behavior.

The finite difference method, however, does not appear to be the most advantageous method for studying the two-dimensional dynamic response problem. The arbitrary geometry and non-homogeneity of a typical earth dam cross section must inevitably lead to difficulties in the finite difference formulation. On the other hand, the finite element method[9,10,11] automatically takes account of any arbitrary geometry or material property variations, thus, it is ideally suited to earth dam analysis. The purpose of this paper is to describe the application of the finite element method to the earthquake stress analysis of an earth dam cross section. The example considered is a simple triangular section with homogeneous elastic properties, subjected to the vertical and horizontal acceleration history recorded at the El Centro earthquake of May 18, 1940.

---

9. Clough, R. W., "The Finite Element Method in Plane Stress Analysis", Second Conference on Electronic Computation, ASCE, Pittsburgh, Pa., September 1960.
10. Wilson, E. L., "Finite Element Analysis of Two-Dimensional Structures", Institute of Engineering Research, Report No. 63-2, University of California, Berkeley, California, June 1963.
11. Sims, F. W., Rhodes, James, A., and Clough, Ray W., "Cracking in Norfork Dam", Journal of the American Concrete Institute, Vol. 61, No. 3, March 1964, p. 265.

THE FINITE ELEMENT METHOD

The finite element method has been fully described in many publications and need not be discussed in detail here. Only the essential features of the method will be outlined, with particular reference to the dynamic response analysis.

The basic concept of the finite element procedure is the idealization of an actual elastic continuum as an assemblage of discrete elements interconnected at their nodal points. For the analysis of two-dimensional stress fields it has been found convenient to employ triangular plate elements in the idealization. In order to maintain compatibility between the edges of adjacent elements, it is assumed that the deformations within each element vary linearly in the x and y directions. On the basis of this assumption, it is possible to calculate the stiffness properties of the elements, i.e., the nodal force-deflection relationships. Finally the stiffness of the complete structural assemblage is obtained by merely superposing the appropriate stiffness coefficients of the individual elements connecting to each nodal point.

If the vector of all nodal point displacements in the complete assemblage is designated $\{r\}$ and the vector of the corresponding nodal forces is $\{R\}$, the structure stiffness matrix $[K]$ (which is obtained by superposing the finite element stiffnesses) expresses the relationship between these quantities as follows:

$$\{R\} = [K]\{r\} \qquad (1)$$

The order of these matrices is 2N if there are N nodal points in the structural idealization; that is, each nodal displacement may include both x and y components in general. Where boundary conditions impose displacement constraints on any of the nodal points, the matrices may be reduced by eliminating the corresponding rows and columns.

In the standard static finite element analysis, these linear equations of equilibrium are solved for the nodal displacements resulting from the given nodal forces. Then the stresses in all of the elements $\{\sigma\}$ are obtained from the nodal displacements by the matrix transformation

$$\{\sigma\} = [S] \{r\} \tag{2}$$

The stress transformation matrix $[S]$ in this equation takes account of the assumed linear displacement patterns in the elements, as well as their given material properties.

Concerning the finite element plane strain analysis procedure in general, it may be noted that: (1) compatibility is satisfied everywhere in the system, (2) equilibrium is satisfied within each element, and (3) equilibrium of stresses is not satisfied along the element boundaries, in general, but the nodal force resultants are in equilibrium. This local discrepancy in stress equilibrium represents the type and extent of the approximation involved in the finite element method of analysis.

ANALYSIS OF DYNAMIC RESPONSE

The equations of motion of the nodal points in the finite element system may be expressed in matrix form as follows:

$$\left[M\right]\left\{\ddot{r}\right\} + \left[C\right]\left\{\dot{r}\right\} + \left[K\right]\left\{r\right\} = \left\{R(t)\right\} \qquad (3)$$

wherein $\left[K\right]$ is the nodal stiffness matrix obtained by the finite element procedure described above, $\left[M\right]$ is the mass matrix associated with the inertia forces in the system, and $\left[C\right]$ is a viscous damping matrix. The dots represent differentiation with respect to time. The mass matrix may be defined in various ways, including the consistent mass matrix procedure described by Archer.[12] However, in the present study it was convenient merely to lump one-third of the mass of each element at each of its nodal points, and thus to include only diagonal terms in the mass matrix.

The load vector $\left\{R(t)\right\}$ in Eq. 3 is a listing of the horizontal and vertical force components applied at each nodal point at time "t". It may be shown[13] that the effective force induced in a structure by an earthquake acceleration $\ddot{v}_g(t)$ applied at the base is equal to the lumped mass at each point multiplied by the ground acceleration; it acts in the direction opposite to the ground acceleration. Thus, horizontal ground accelerations produce only horizontal effective forces and vertical ground accelerations produce vertical forces. The load vector associated

---

12. Archer, J. S., "Consistent Mass Matrix for Distributed Mass Systems", Proceedings, ASCE, Vol. 89, No. ST-4, August 1963.
13. Clough, R. W., "Dynamic Effects of Earthquakes", Trans., ASCE, Vol. 126, Part II, 1961, pp. 847-876.

with the earthquake acceleration of an earth dam therefore may be written.

$$\left\{R(t)\right\} = -\left\{E^x\right\} \ddot{v}_g^{\,x}(t) - \left\{E^y\right\} \ddot{v}_g^{\,y}(t) \tag{4}$$

in which

$$\left\{R(t)\right\} = \begin{Bmatrix} R_1^{\,x}(t) \\ R_1^{\,y}(t) \\ R_2^{\,x}(t) \\ R_2^{\,y}(t) \\ \vdots \\ Rn^x(t) \\ Rn^y(t) \end{Bmatrix} ; \quad \left\{E^x\right\} = \begin{Bmatrix} M_1 \\ 0 \\ M_2 \\ 0 \\ \vdots \\ Mn \\ 0 \end{Bmatrix} ; \quad \left\{E^y\right\} = \begin{Bmatrix} 0 \\ M_1 \\ 0 \\ M_2 \\ \vdots \\ 0 \\ Mn \end{Bmatrix} \tag{5}$$

and where $\ddot{v}_g^{\,x}(t)$ and $\ddot{v}_g^{\,y}(t)$ represent the horizontal and vertical components of the ground accelerations. In this analysis, it is assumed that the entire base section under the dam moves as a rigid body, and thus is subjected to the same accelerations at all points. Introducing Eq. 4 into Eq. 3 leads to the following expression for the equations of motion.

$$\left[M\right] \left\{\ddot{r}\right\} + \left[C\right] \left\{\dot{r}\right\} + \left[K\right] \left\{r\right\} = -\left\{E^x\right\} \ddot{v}_g^{\,x}(t) - \left\{E^y\right\} \ddot{v}_g^{\,y}(t) \tag{6}$$

In the present study, the dynamic response of the structure was evaluated by the mode-superposition method. To carry out the analysis it was necessary first to solve the characteristic value problem

$$\left[K\right] \left\{\phi_n\right\} = \omega_n^{\,2} \left[M\right] \left\{\phi_n\right\} \tag{7}$$

for the undamped free vibration mode shapes $\left[\phi\right]$ and frequencies $\left\{\omega\right\}$. These mode shapes have the following orthogonality properties

$$\left\{\phi_m\right\}^T \left[M\right] \left\{\phi_n\right\} = 0 \qquad m \neq n \qquad (8)$$
$$\left\{\phi_m\right\}^T \left[K\right] \left\{\phi_n\right\} = 0$$

and it is assumed in this study that the damping matrix of the finite element system satisfies the equivalent orthogonality condition

$$\left\{\phi_m\right\}^T \left[C\right] \left\{\phi_n\right\} = 0 \ ; \ m \neq n \qquad (9)$$

In this case the damped system has the same free vibration mode shapes as the undamped system.

If the nodal co-ordinates are transformed to the mode shape or "normal" co-ordinates as follows:

$$\left\{r\right\} = \left[\phi\right] \left\{Y\right\} \qquad (10)$$

in which $\left\{Y\right\}$ is the modal amplitude vector, the coupled equations of motion (Eq. 6) can be reduced to a set of uncoupled normal equations by virtue of the orthogonality properties of Eqs. 8 and 9. Each normal response equation has the form:

$$\ddot{Y}_n + 2\lambda_n \omega_n \dot{Y}_n + \omega_n^2 Y_n = \frac{P_n^*(t)}{M_n^*} \qquad (11)$$

using the notation:

$$\begin{aligned} \{\phi_n\}^T [M] \{\phi_n\} &= M_n^* \\ \{\phi_n\}^T [K] \{\phi_n\} &= \omega_n^2 M_n^* \\ \{\phi_n\}^T [C] \{\phi_n\} &= 2\lambda_n \omega_n M_n^* \end{aligned} \tag{12}$$

The generalized earthquake force in Eq. 11 is given by

$$P_n^*(t) = -\{\phi_n\}^T \{E^x\} \ddot{v}_g^x(t) - \{\phi_n\}^T \{E^y\} \ddot{v}_g^y(t) \tag{13}$$

It should be noted that either of the ground acceleration components can be considered separately, or they can be combined to give the total effective force.

DIGITAL COMPUTER PROGRAM

The analysis of the dynamic response of any significant plane strain system by the finite element method involves such voluminous calculations that it is practicable only when done by a digital computer. The computer program used in the present study consists of four parts:

(1) Analysis of the element stiffnesses and assembling to form the stiffness matrix of the structural idealization, formation of the mass matrix, and reduction of both matrices by eliminating fixed support components. This phase of the program was taken, with minor modifications, from an existing plane strain analysis program.

(2) Solution of the eigenvalue problem (of order 2N reduced by the number of support constraints) for the mode shapes and frequencies of the system.

(3) Solution of the normal equations (Eq. 11) for the response of each mode, using the linear acceleration method of step-by-step integration[14], and superposition of the modal responses by means of Eq. 10 to obtain the time history of nodal displacements $\left\{ r(t) \right\}$.

(4) Analysis of element stresses from nodal displacements for each instant of time, using Eq. 2.

The input information to the program consists of the geometric description of the structure (x and y co-ordinates of each nodal point), the material property definition (elastic properties and unit weight of each element), the load data (x and y components of the ground accelerations listed at equal increments of time), and the damping ratio assumed for each mode.

The output of the program includes the nodal displacement components and the element stresses, listed at equal increments of time during the earthquake. The direct output of the program is stored on magnetic tape. Plotting routines then automatically plot the output in the form of time histories of stresses at any specified points in the structure, or in the form of stress contours for the entire system at any specified instants of time.

---

14. Wilson, E. L., and Clough, R. W., "Dynamic Response by Step-by-Step Matrix Analysis", Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, October 1962.

EXAMPLE ANALYSIS

As an example of the type of results which may be obtained automatically by this procedure, the earthquake analysis of the 300 ft. high triangular dam section shown in Fig. 1 will be described. This structure has side slopes of 1-1/2: 1 ; the material is homogeneous, isotropic, and linearly elastic with a modulus E = 81,300 psi, Poisson's ratio $\mu = 0.45$, and a unit weight $\gamma = 130$ pcf. (These properties are associated with a shear wave propagation velocity of 1000 fps.) Damping was assumed to be 20 per cent of critical in each mode. It is important to note, however, that the simple geometry and homogeneity considered here is not essential to the procedure. Arbitrary geometry and material property variations could have been treated with equal ease.

The structural idealization consisted of 100 finite elements with 66 nodal points, as shown in the figure. Of these nodal points, 11 were assumed fixed to the base, thus the remaining 55 provided the structure with 110 degrees of freedom. The first 15 vibration mode shapes and frequencies of the system, computed by a standard eigenvalue program, are shown in Figs. 2 and 3. It is of interest to note that only the first mode resembles a pure shear distortion, of the type assumed in previous shear wedge analyses. Vertical motions, rocking, etc., are clearly involved in all other modes.

This structure was subjected simultaneously to two components of the ground acceleration history recorded at the El Centro earthquake of

May 18, 1940: the north-south and vertical components as shown in Fig. 4. The static stresses computed by an independent static finite element analysis also were considered in the analysis because the static stress in an earth dam represents a major part of the total stress state during an earthquake. Thus, the dynamic stresses computed in this analysis are changes of stress from the initial static condition.

The time history of stresses plotted for four selected nodal points of the structure are presented in Figs. 5 and 6. Each graph shows the variation at the specified nodal points of both principal normal stresses, the principal shear stress and of the shear stress on a horizontal surface. The nodal point stresses were obtained by averaging the stresses in the individual finite elements associated with each nodal point. The relative importance of the initial static stress at each point is clearly evident.

The distributions of stresses in the entire cross section at various instants of time are illustrated by the stress contour plots in Figs. 7, 8, and 9, showing the maximum tensile (or least compressive), the maximum compressive, and the horizontal shear stresses, respectively. The successive sketches in each figure show the initial static state of stress (at time $t = 0$) and the stress state at $t = 2.0$ seconds and at $t = 2.25$ seconds. It will be noted in Fig. 5 that these times are associated with a nearly maximum oscillation of stress conditions in the upper central portion of the cross section. The corresponding shift of the shear stress contours from right to left is clearly evident in Fig. 9. The fact that vertical as well as horizontal accelerations have been applied should be kept in mind, however. The system is not only subject to a lateral oscillation, even though a major part of the response appears to be associated with this type of motion.

CONCLUSIONS

It has been shown in this study that the finite element method, which previously had demonstrated its effectiveness in static analysis of plane stress or plane strain problems, provides an equally powerful tool for the dynamic analysis of such systems. The advantages of the method with regard to treatment of arbitrary geometry or material property variations are just as significant in dynamic analysis as in static cases. Although the technique has been described here with reference to the earthquake analysis of dams, it is equally applicable to the analysis of any elastic system subjected to any type of dynamic loading.

The earthquake analysis presented here, which includes the true two-dimensional deformation behavior of the dam cross section and which provides the time history of all stress components, clearly demonstrates the inadequacies of the shear wedge approach. Only by evaluating the complete state of stress near the faces of the dam will it be possible to estimate factors of safety of these surfaces against sliding. Moreover, in practical cases the variation of material properties in cross sections including impervious cores is an important factor which cannot be treated rationally by the shear wedge theory. Another factor of possible significance is that the finite element method can be adapted to take account of non-uniform base motions; that is, of the effects of seismic waves passing under the dam in which the wave lengths are short compared with the cross sectional width of the dam base.

In spite of the relative refinement of the solution presented here, however, it must be recognized that the assumed linearly elastic material property is only a crude approximation of the actual material behavior. Techniques are available for considering nonlinear materials under static conditions[10], and it is intended to extend the present investigation to include certain idealized nonlinear properties in the dynamic analysis.

ACKNOWLEDGMENTS

The research described herein is part of a program supported by a research grant from the California Department of Water Resources. This particular phase of the work was initiated by Professor J. Penzien and has been carried out by the authors. The entire investigation on seismic effects on earth dams is under the general supervision of Professor H. B. Seed.

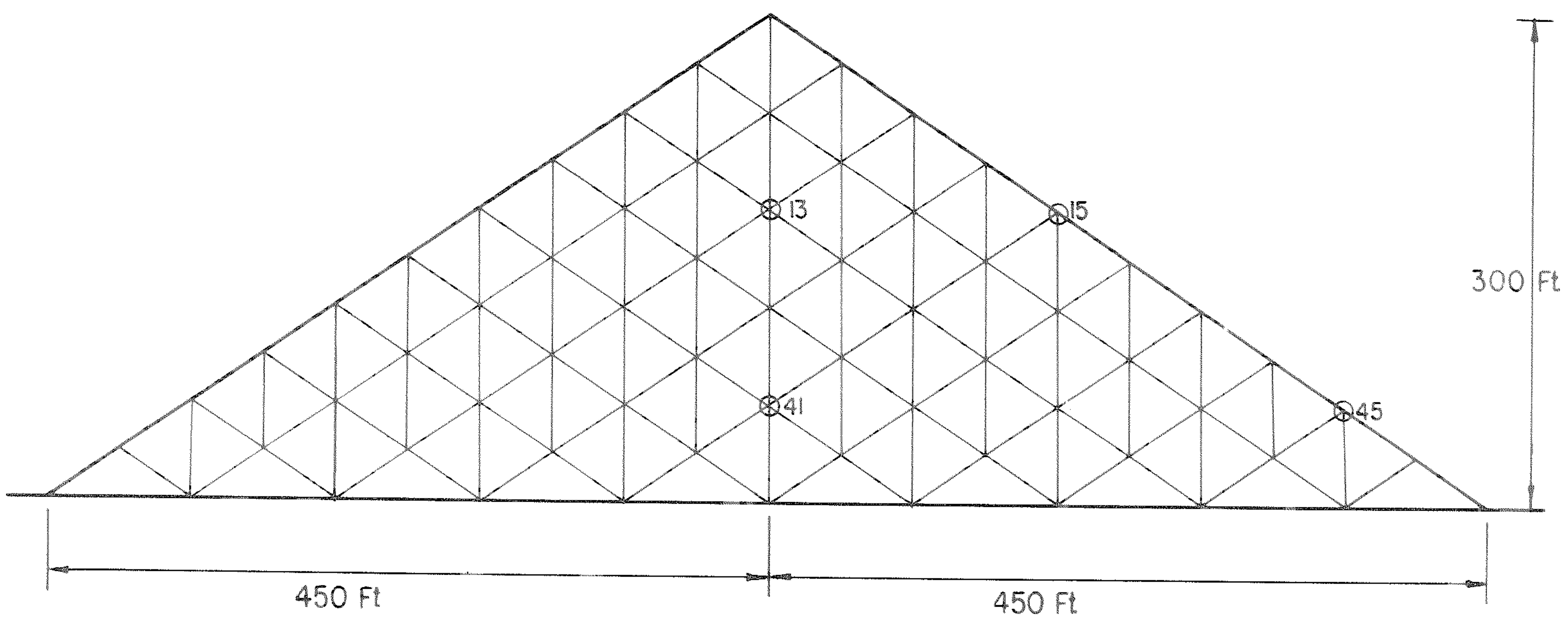


Fig. 1- FINITE ELEMENT IDEALIZATION OF EXAMPLE EARTH DAM

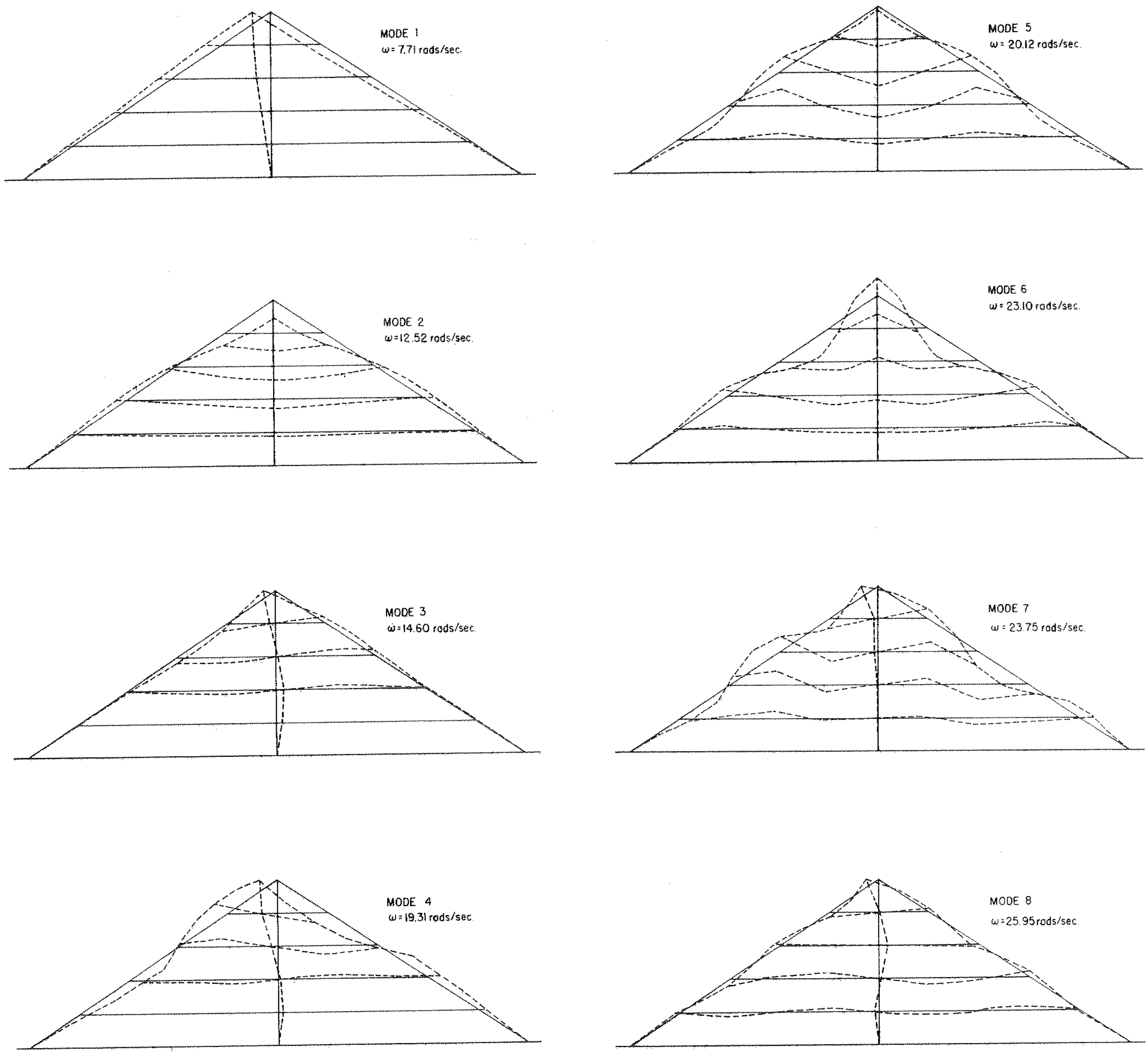


Fig. 2- FREE VIBRATION MODE SHAPES AND FREQUENCIES
(MODES 1-8)

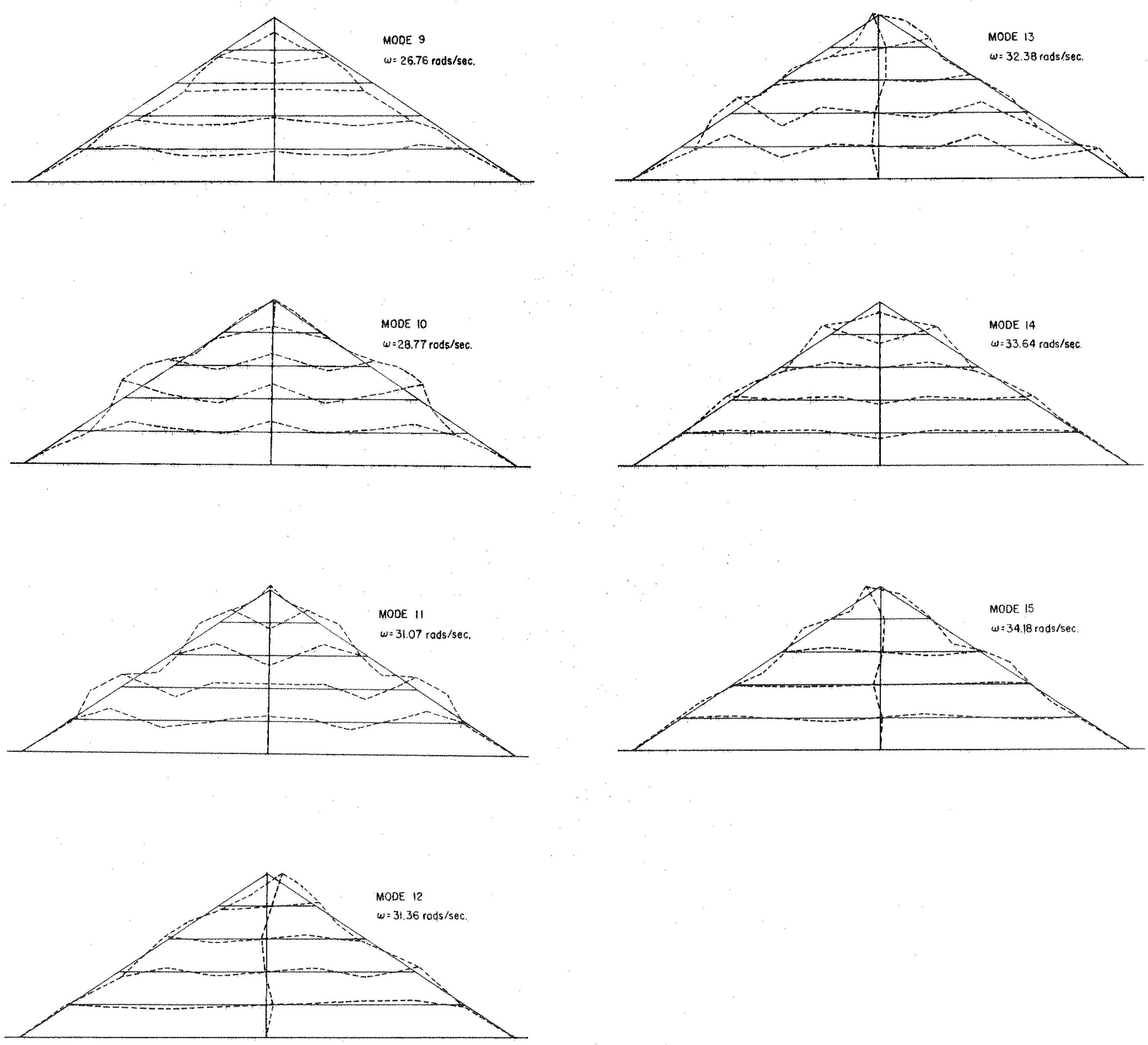


Fig.3- FREE VIBRATION MODE SHAPES AND FREQUENCIES
(MODES 9-15)

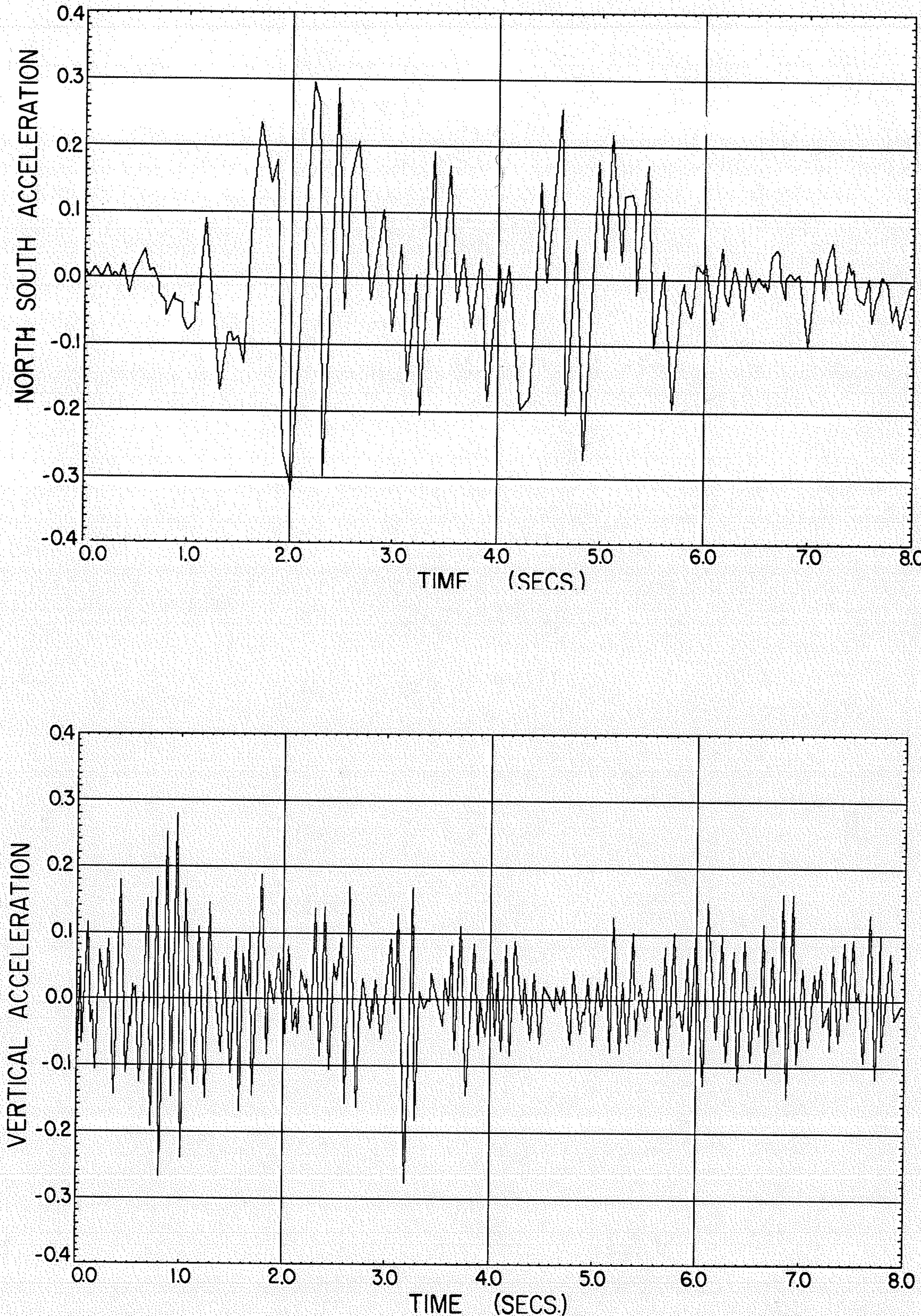
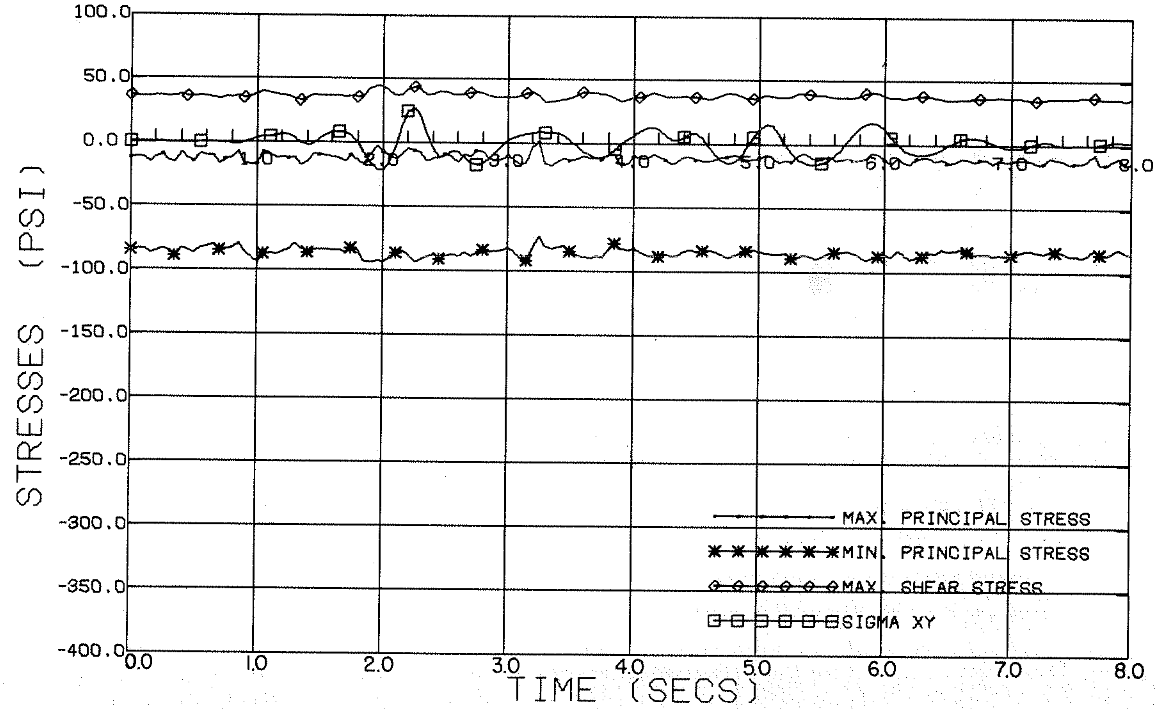
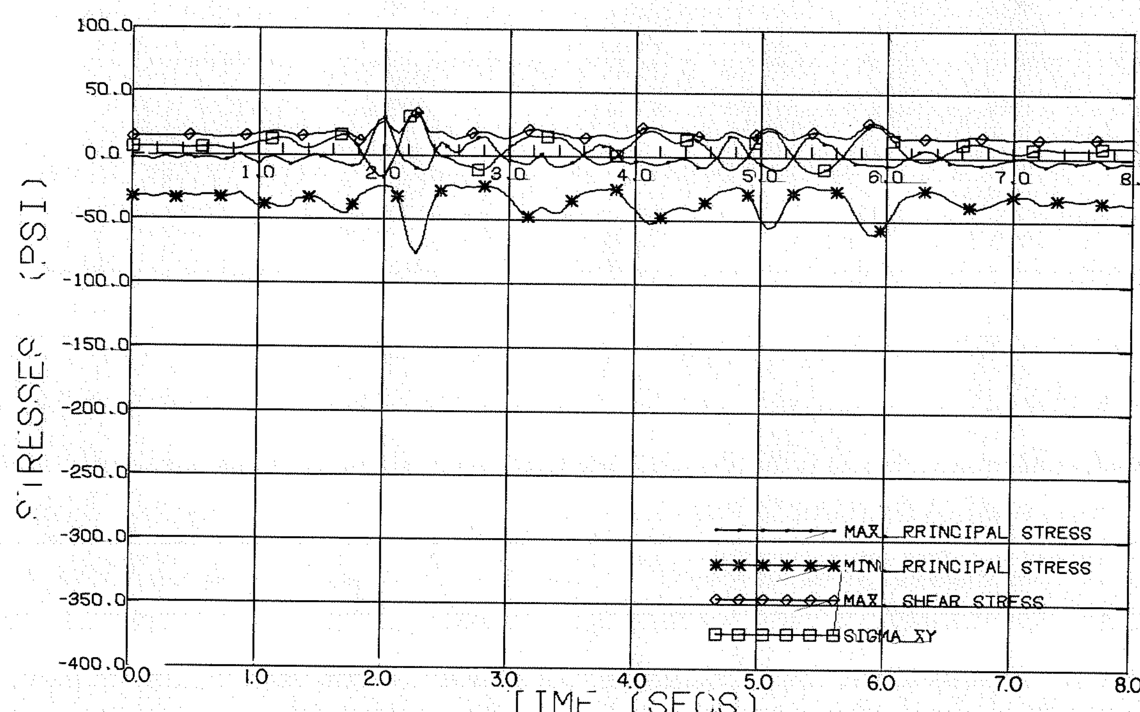


Fig. 4 - GROUND ACCELERATIONS
EL CENTRO EARTHQUAKE, MAY 18, 1940

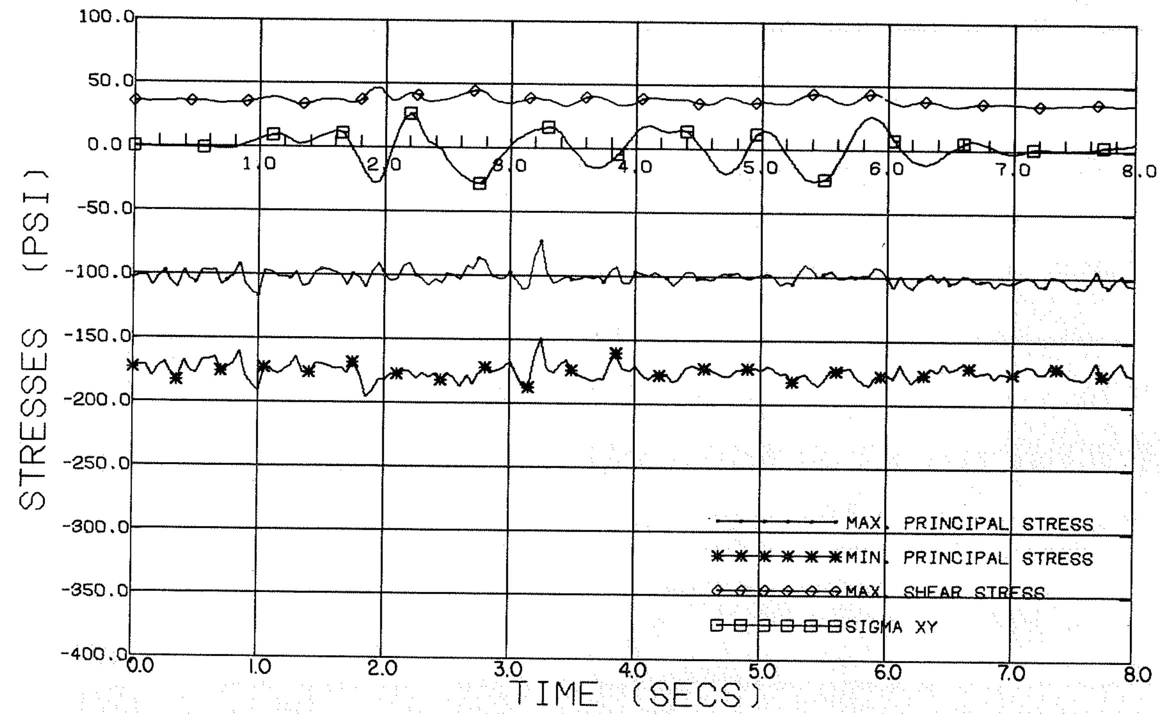


a. CENTERLINE: NODAL POINT 13

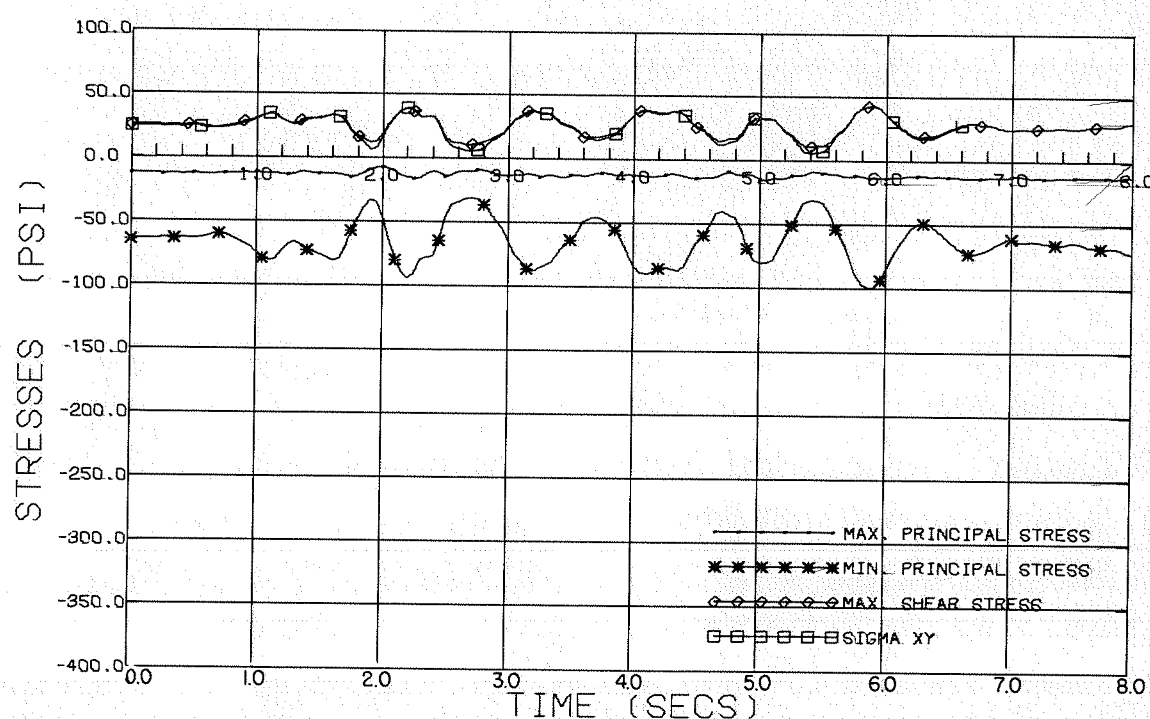


b. FACE: NODAL POINT 15

Fig. 5 - TIME HISTORY OF STRESSES AT THE 180 Ft. LEVEL

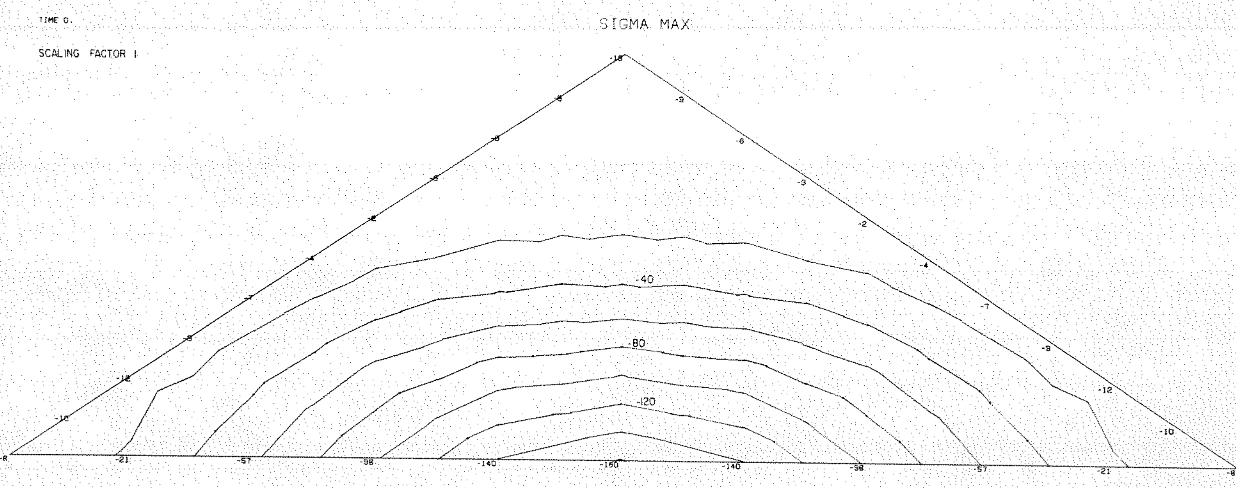


a. CENTERLINE: NODAL POINT 41

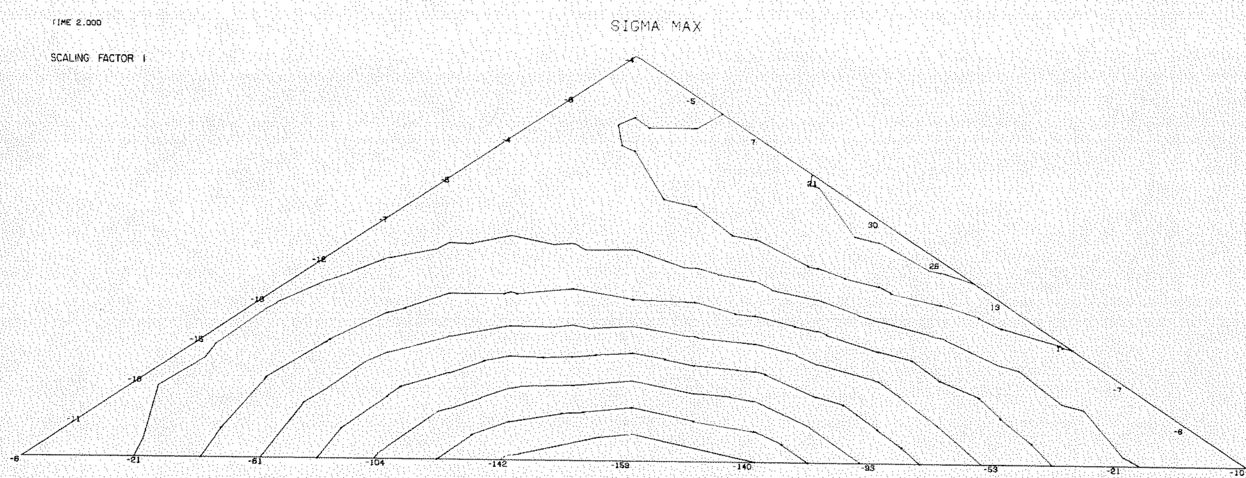


b. FACE: NODAL POINT 45

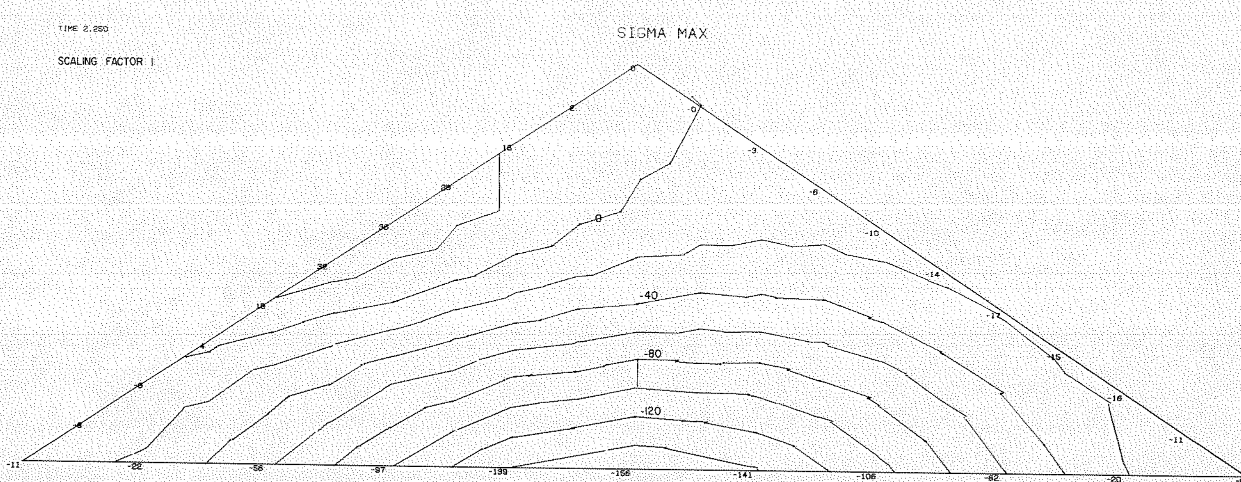
Fig.6- TIME HISTORY OF STRESSES
AT THE 60Ft. LEVEL



a. STATIC STRESSES, t=0.00 SECS.

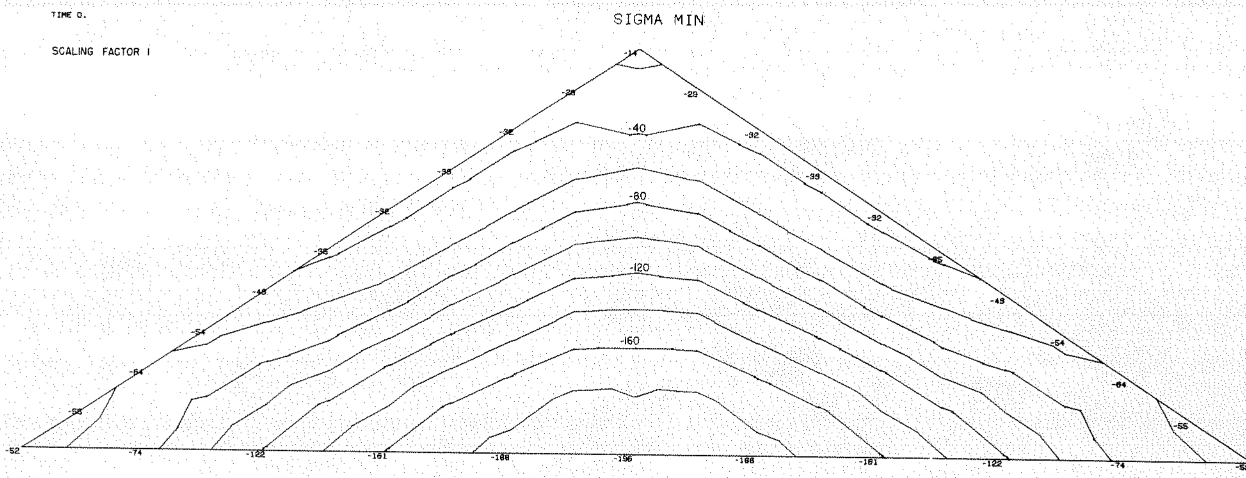


b. STRESSES, t=2.00 SECS.

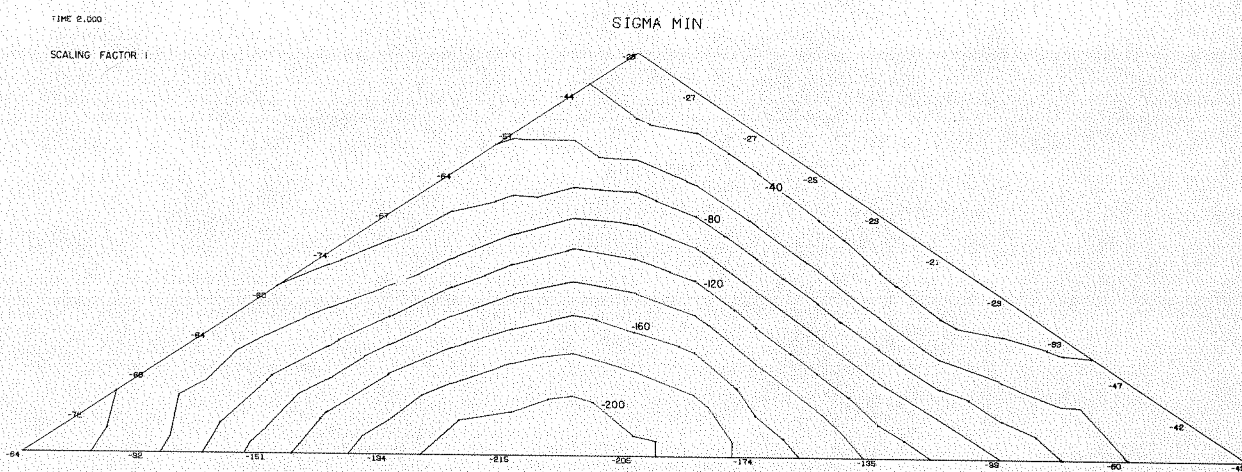


c. STRESSES, t=2.25 SECS.

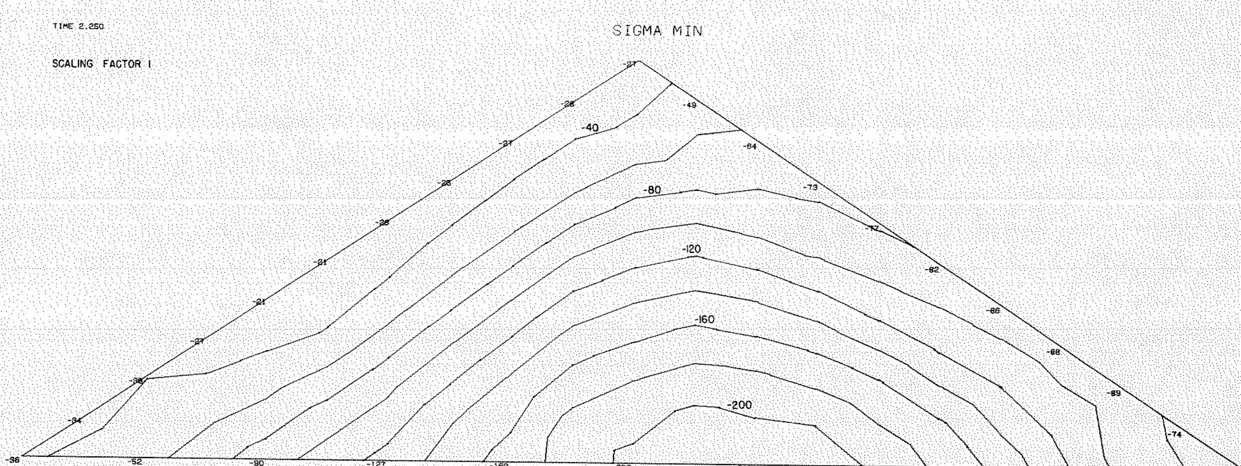
Fig. 7- CONTOURS OF MAJOR PRINCIPAL STRESS, σ_1



a. STATIC STRESSES, t=0.00 SECS.

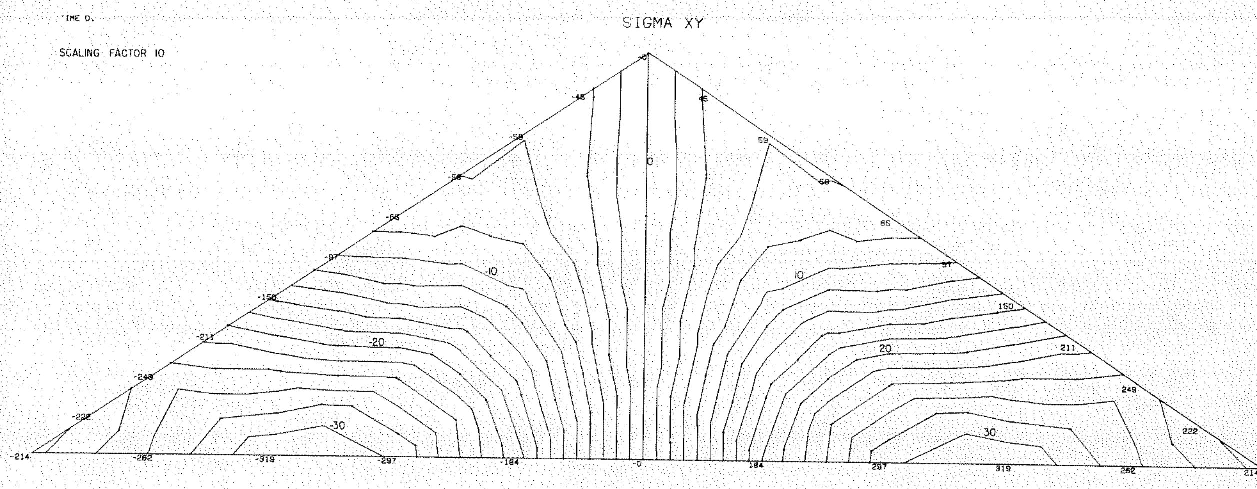


b. STRESSES, t=2.00 SECS.

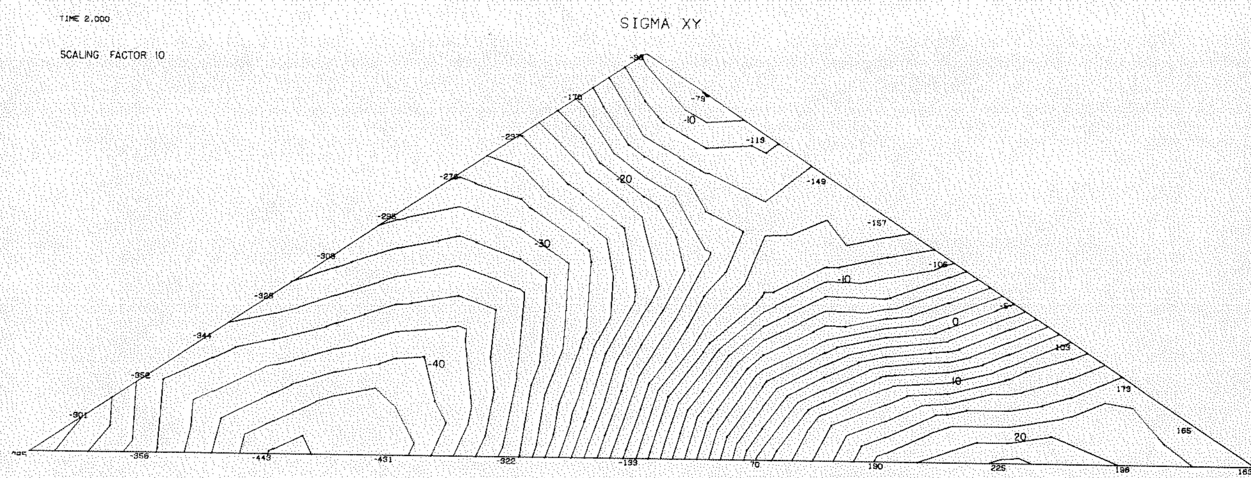


c. STRESSES, t=2.25 SECS.

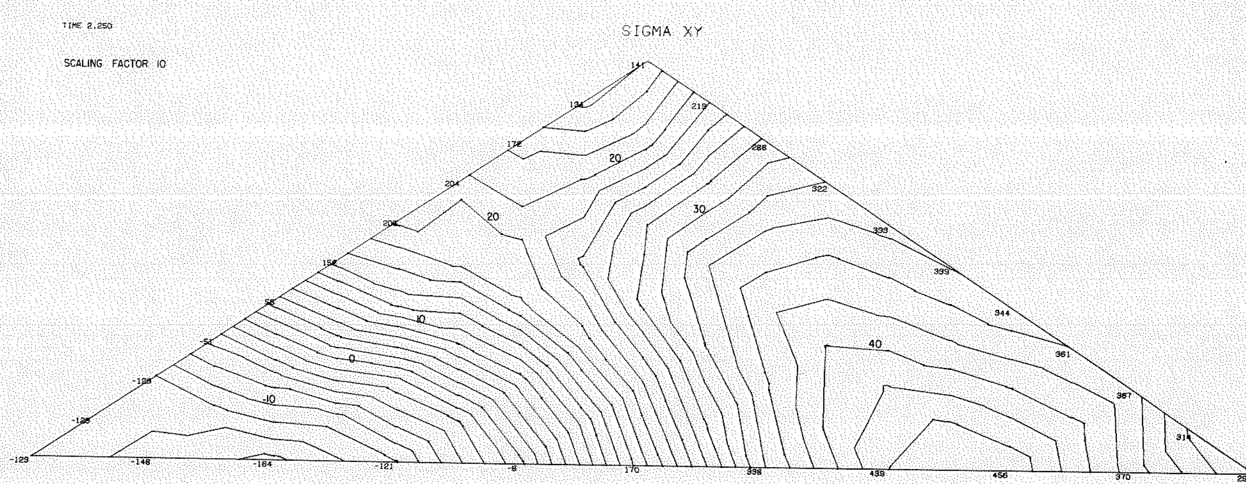
Fig.8- CONTOURS OF MINOR PRINCIPAL STRESS, σ_2



a. STATIC STRESSES, t= 0.00 SECS.



b. STRESSES, t=2.00 SECS.



c. STRESSES, t=2.25 SECS.

Fig.9- CONTOURS OF SHEAR STRESS ON HORIZONTAL PLANES, τ_{xy}