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By W. Michael Hanemann[1]

1. Introduction

A major accomplishment of econometric research in recent years has been the development of statistical models suitable for the analysis of discrete dependent variables. This has enabled economists to study behavioral relationships involving purely qualitatative variables which are not amenable to conventional regression techniques. These developments are reviewed and summarized by McFadden [13] and Amemiya [2]. Using the latter's terminology, the multi-response qualitative response (MRQR) model involves a dependent variable taking N distinct values, $\tilde{y} = 1, 2, \ldots,$ or N, which is related to a row vector of independent variables, W^*, and a column vector of unknown parameters, β^*, by some function of the general form[2]

$$(1.1) \qquad \pi_j \equiv \Pr\{\tilde{y} = j\} = H_j^*(W^*, \beta^*) \qquad\qquad j = 1, \ldots, N.$$

Moreover, in most applications this relation can be cast into the form

$$(1.2) \qquad \pi_j = H_j(W_1\beta_1, \ldots, W_N\beta_N) \qquad\qquad j = 1, \ldots, N.$$

Specific examples of (1.2) are the polychotomous probit model, [1], [5], [10],

$$(1.3) \qquad \pi_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{W_1\beta_1 - W_2\beta_2 + \varepsilon_1} \cdots \int_{-\infty}^{W_1\beta_1 - W_N\beta_N + \varepsilon_1} n(\varepsilon_1, \ldots, \varepsilon_N;\ 0, \Sigma)d\varepsilon_1, \ldots, d\varepsilon_N$$

where n is a multivariate normal density with zero mean and covariance matrix Σ, and the GEV model, [12],

$$(1.4) \qquad \pi_j = e^{W_j\beta_j} G_j(e^{W_1\beta_1}, \ldots, e^{W_N\beta_N})\, G(e^{W_1\beta_1}, \ldots, e^{W_N\beta_N})^{-1} \qquad\qquad j = 1, \ldots, N,$$

where G is a positive, linear homogeneous function, and G_j denotes the partial derivative with respect to the j^{th} argument. A special case of the latter is the independent logit model, [11], where $G(t_1, \ldots, t_N) = \Sigma t_j$ and

$$(1.5) \qquad \pi_j = e^{W_j\beta_j} [\sum_1^N e^{W_i\beta_i}]^{-1} \qquad\qquad j = 1, \ldots, N.$$

The statistical models (1.3)-(1.5) have been applied to a wide variety of economic topics. Aitchison and Bennett [1] and McFadden [11] have provided a theoretical derivation of these models which applies whenever the events whose probabilities are given by (1.2) represent the outcome of a decision by a maximizing agent. Suppose an agent is choosing among N courses of action and $\pi_j = \Pr\{j^{th}$ act chosen$\}$. Assume that the payoff or utility associated with the j^{th} act, $\tilde{u}_j$, is a random variable with mean $W_j\beta_j$. Equivalently, $\tilde{u}_j = W_j\beta_j + \tilde{\varepsilon}_j$ where $\tilde{\varepsilon}_j$ is a random variable with zero mean. The agent chooses that act which has the highest utility. This yields a MRQR model of the form (1.2):

$$(1.6) \qquad \pi_j = \Pr\{W_j\beta_j + \tilde{\varepsilon}_j \geq W_i\beta_i + \tilde{\varepsilon}_i, \text{ all } i\} = H_j(W_1\beta_1, \ldots, W_N\beta_N) \qquad j = 1, \ldots, N.$$

Let $\tilde{\eta}_{(j)} = (\tilde{\eta}_{1j}, \ldots, \tilde{\eta}_{j-1,j}, \tilde{\eta}_{j+1,j}, \ldots, \tilde{\eta}_{Nj})$ where $\tilde{\eta}_{ij} \equiv \tilde{\varepsilon}_i - \tilde{\varepsilon}_j$. It follows from (1.6) that:

$$(1.7) \qquad H_j(W_1\beta_1, \ldots, W_N\beta_N) = F_{(j)}(W_j\beta_j - W_1\beta_1, \ldots, W_j\beta_j - W_N\beta_N) \qquad j = 1, \ldots, N,$$

where $F_{(j)}$ is an (N-1) dimensional joint c.d.f. associated with the random vector $\tilde{\eta}_{(j)}$. As Daly and Zachary [6] have pointed out, the converse is also true: any MRQR model (1.2) in which the probability functions $H_j(\cdot)$ can be cast in the form of an (N-1) dimensional joint c.d.f. as in (1.7) can be derived from a utility maximization choice model like (1.6). For this reason a MRQR model satisfying (1.7) is said to be a random utility maximization (RUM) model.

This link between statistical models for discrete dependent variables and the economic concept of utility maximization is potentially very valuable because it raises the possibility of applying the apparatus of welfare theory to empirical models of purely qualitative choices. Suppose the statistical model satisfies (1.7) and some subset of the variables in W_j represent attributes of the j^{th} discrete choice. Can one derive from the fitted model an estimate of the effect on the agent's welfare of a change in these attributes analogous to the compensating and equivalent variation measures of conventional utility theory? In the literature on transportation mode choice this issue has been considered for some RUM models by Domencich and McFadden [7], Williams [16], Daly and Zachary [6] and Ben Akiva and Lerman [3], but it has received relatively little attention in other branches of applied economics.

More recently Small and Rosen [15] — henceforth SR — have investigated welfare measures in the context of what I will call a budget-constrained RUM model. By imposing three fairly strong conditions on the consumer's preferences they derive a closed-form expression for the compensating variation which can readily be calculated from the fitted MRQR model. I will show that one of these conditions implies another, and excludes the third. The key condition is a no income effects assumption which implies that the discrete choice probabilities (1.6) are independent of the consumer's income. I will show how to perform welfare evaluations when this stringent assumption is relaxed. In general the procedure requires the use of numerical techniques to solve an implicit equation for the compensating or equivalent variations, but I show that in some common logit and probit models approximate closed-form solutions can readily be obtained.

SR start with a budget-constrained deterministic utility maximization model containing an element of discreteness in the consumer's choice and then switch to a random utility setting, taking the expectation of certain relationships derived from the deterministic case. However, their deterministic utility model involves both qualitative and quantitative choices, while the MRQR model for which they develop the welfare formula involves a purely qualitative choice. This tends to obscure the comparison of the two models. I compare deterministic and random utility versions of a purely qualitative budget-constrained choice and show that there are both similarities and differences.[3] For example, in the deterministic case if a good is selected by the consumer and the price of that good falls, the compensating variation is equal to the change in the cost of buying the good. One might expect the random utility analog to be that if the price of a good falls the compensating variation is equal to the change in its cost times the probability of selecting it. I show that this does not hold as an exact result in any of the budget-constrained RUM models, but it is true within an order of magnitude in some of them.

Finally, it turns out that the budget-constrained RUM formulation imposes some restrictions on the regressors and their coefficients in (1.6) which are violated by many of the RUM models that have appeared in the empirical literature. Accordingly, I show how the welfare methodology can be extended to cover these models, and I develop some approximate closed-form solutions for the resulting welfare equations.

2. Budget-Constrained Choices

2.1. Deterministic Utility Models

The deterministic utility qualitative choice model is defined as follows. An individual consumer has a twice-differentiable, quasi-concave, increasing utility function u defined over the commodities $x_1, \ldots, x_N$, and z, where z is taken as the numeraire. In addition, the consumer's utility depends on some other variables, $q_1, \ldots, q_N$, which he takes as exogenous; these are, for example, quality attributes of the non-numeraire goods.[4] The consumer chooses (x, z) so as to maximize

(2.1) $$u = u(x_1, \ldots, x_N, q_1, \ldots, q_N, z)$$

subject to a budget constraint

(2.2) $$p_j x_j + z = y$$

and two other constraints which induce an element of discreteness into his choice. First, for some logical or institutional reason, the x_j's are mutually exclusive in consumption

(2.3) $$x_i x_j = 0 \qquad \text{all } i \neq j.$$

Secondly, the x_j's can be purchased only in fixed quantities

(2.4) $$x_j = \bar{x}_j \text{ or } 0 \qquad j = 1, \ldots, N.$$

An example would be where the x_j's are different brands of an indivisible durable good, and the consumer needs only one of these goods. Since the quantities of the x_j's are limited by (2.4), their selection is a qualitative choice. Moreover, although the numeraire is inherently a divisible good, once one of the x_j's has been selected the quantity of z is fixed because of the budget constraint (2.2).[5]

Thus, the model (2.1)-(2.4) represents a purely qualitative utility maximizing choice.

The reason for studying this model is that it provides an exact deterministic analog of the budget-constrained RUM to be introduced below. I will consider here the demand functions, duality relations, and welfare measures for changes in p or q associated with (2.1)-(2.4), in order to contrast them with the corresponding concepts in the RUM model. Since a deterministic utility model similar to (2.1)-(2.4) has already been studied by Mäler [14, pp. 131-36] and Small and Rosen [15, pp. 127-28], I simply sketch the main results, referring the reader to these sources for further details.

It is convenient, but not essential, to make the additional assumption about the utility function in (2.1) that

$$(2.5) \qquad x_j = 0 \rightarrow \partial u/\partial q_j = 0 \qquad j = 1, \ldots, N,$$

i.e., the attributes of a good do not matter unless that good is actually consumed. Now, suppose that the consumer has selected good j. His utility conditional on this decision will be denoted u_j. It follows from (2.2)-(2.5) that

$$(2.6) \qquad u_j = u(0, \ldots, 0, \bar{x}_j, 0, \ldots, 0, q_1, \ldots, q_N, y - p_j\bar{x}_j) \equiv v_j(q_j, y - p_j\bar{x}_j) \quad j = 1, \ldots, N,$$

where v_j is increasing in $(y - p_j\bar{x}_j)$. I will refer to $v_1, \ldots, v_N$ as conditional indirect utility functions. They play a central role in the analysis of the solution to the maximization problem (2.1)-(2.4). The consumer's decision can be represented by a set of binary functions $\delta_1, \ldots, \delta_N$, where $\delta_j \equiv 1$ if $x_j > 0$ and $\delta_j \equiv 0$ if $x_j = 0$. These discrete choice indices are related to the conditional indirect utility functions by

$$(2.7) \qquad \delta_j(p, q, y) = \{ \begin{matrix} 1 \text{ if } v_j(q_j, y - p_j\bar{x}_j) \geq v_i(q_i, y - p_i\bar{x}_i) \quad \text{all } i \\ 0 \text{ otherwise.} \end{matrix}$$

Accordingly, the unconditional ordinary demand functions associated with (2.1)-(2.4) can be expressed as $x_j(p, q, y) = \delta_j(p, q, y)\bar{x}_j$. Substitution of these demand functions into the utility function (2.1) yields the unconditional indirect utility function

$$(2.8) \qquad v(p, q, y) = \max[v_1(q_1, y - p_1\bar{x}_1), \ldots, v_N(q_N, y - p_N\bar{x}_N)] \,.$$

SR show that this satisfies Roy's Identity, $x_j = -(\partial v/\partial p_j)/(\partial v/\partial y)$.

The unconditional indirect utility function measures the utility achieved by the maximizing consumer when confronted with given prices, attributes and income. Accordingly, it can be employed to construct monetary measures of the welfare effects of a change in these variables. Suppose, for example, that the prices and attributes change from (p^0, q^0) to (p^1, q^1), while the consumer's income stays constant at y. By analogy with the standard welfare theory of price changes, the compensating and equivalent variations for this change, cv and ev, are defined by

$$(2.9) \qquad v(p^1, q^1, y + cv) = v(p^0, q^0, y)$$

$$(2.10) \qquad v(p^1, q^1, y) = v(p^0, q^0, y - ev).$$

Dual to the utility maximization problem is an expenditure minimization problem — minimize $\Sigma p_j x_j + z$ subject to (2.1), (2.3) and (2.4). Suppose that the consumer has selected good j. His expenditure conditional on this decision will be denoted e_j. It follows from (2.3)-(2.5) that

$$(2.11) \qquad e_j = g_j(q_j, u) + p_j\bar{x}_j \equiv e_j(p_j, q_j, u) \qquad j = 1, \ldots, N,$$

where g_j is the inverse of v_j, i.e., $g_j(q_j, v_j(q_j, z)) \equiv z$. The unconditional compensated demand functions associated with this expenditure minimization problem can be written $x_j(p, q, u) = \delta_j(p, q, u)\bar{x}_j$, where

$$(2.12) \qquad \delta_j(p, q, u) = \begin{cases} 1 \text{ if } e_j(p_j, q_j, u) \leq e_i(p_i, q_i, u), & \text{all } i \\ 0 \text{ otherwise.} \end{cases}$$

The unconditional expenditure function obtained by substituting unconditional compensated demand functions into the original minimand is

$$(2.13) \qquad e(p, q, u) = \min[e_1(p_1, q_1, u), \ldots, e_N(p_N, q_N, u)] \,.$$

SR point out that the expenditure function satisfies Shepherd's Lemma, $x_j(p, q, u) = \partial e/\partial p_j$, and the duality relationship $e(p, q, v(p, q, y)) \equiv y$. It follows that the welfare measures cv and ev can be expressed in terms of $e(\cdot)$ as

$$(2.14) \quad cv = e(p^1, q^1, u^0) - e(p^0, q^0, u^0)$$

$$(2.15) \quad ev = e(p^1, q^1, u^1) - e(p^0, q^0, u^1),$$

where $u^t = v(p^t, q^t, y)$, $t = 0, 1$. Maler uses (2.14) to examine the special case of a change in a single price — i.e., $q^0 = q^1$ and $p_j^0 = p_j^1$, $j \geq 2$. Suppose also that $x_1(p^0, q^0, y) = \bar{x}_1$. Maler shows that, if $p_1^1 < p_1^0$ and even, under some circumstances, if $p_1^1 > p_1^0$, then $cv = (p_1^1 - p_1^0)\bar{x}_1$ — i.e., the welfare gain is measured by the change in the cost of buying the good.[6]

In their discussion of welfare measurement SR invoke three special assumptions about the structure of consumer preferences. They apply these assumptions both to a utility model of the form (2.1)-(2.3) and also to the purely qualitative choice model (2.1)-(2.4) with which I am concerned here. The assumptions are: (A) *The conditional marginal utility of income, $\partial v_j/\partial y$, is approximately independent of p_j and q_j;* (B) *The discrete goods are sufficiently unimportant that income effects from quality changes are negligible; i.e., the unconditional compensated demand function $x_j(p, q, u)$ is adequately approximated by the unconditional ordinary demand function $x_j(p, q, y)$;* (C) *$\partial v_j/\partial q_j \to 0$ as $p_j \to \infty$.*

It can be seen from (2.6) that, for the purely qualitative choice model, Assumption A implies that the conditional indirect utility functions may be approximated by

$$(2.16a) \quad v_j(q_j, y - p_j\bar{x}_j) = h_j(q_j) + \gamma_j y - \gamma_j p_j \bar{x}_j \qquad j = 1, \ldots, N$$

where γ_j is a positive constant. This in turn implies that the direct utility function (2.1) may be approximated by

$$(2.16b) \quad u(x, q, z) = h(x, q) + \sum_1^N \theta_j \gamma_j z$$

for some function $h(\cdot)$ where, $\theta_j \equiv 1$ if $x_j > 0$ and $\theta_j \equiv 0$ otherwise.

It can also be shown[7] that Assumption B implies that the conditional indirect utility functions have the same form as in (2.16a) with the added restriction that $\gamma_j = \gamma$ all j, i.e., (2.17a) $v_j(q_j, y - p_j\bar{x}_j) = h_j(q_j) + \gamma y - \gamma p_j\bar{x}_j \qquad j = 1, \ldots, N,$

and the direct utility function (2.1) has the form

(2.17b) $u(x, q, z) = h(x, q) + \gamma z.$

It follows that Assumption B entails Assumption A. Both assumptions imply that $\partial v_j/\partial q_j$ is independent of p_j. Hence, if either holds, Assumption C cannot be satisfied.

I now turn to the random utility analog of the qualitative choice model (2.1)-(2.4).

2.2. Random Utility Models

A random utility model arises when one assumes that, although a consumer's utility function is deterministic for him, it contains some components which are unobservable to the econometric investigator and are treated by the investigator as random variables. This combines two notions which have a long history in economics — the idea of a variation in tastes among individuals in a population, and the idea of unobserved variables in econometric models. These components of the utility function will be denoted by the random vector $\tilde{\varepsilon}$, and the utility function will be written $\tilde{u} = u(x, q, z, \tilde{\varepsilon})$. In the present context it would be natural to postulate that

(2.1') $u(x, q, z, \tilde{\varepsilon}) = u(x, q, z) + \Sigma\xi_j\tilde{\varepsilon}_j$

where ξ_j is 1 if $x_j > 0$, and 0 otherwise. I will assume that the non-stochastic component $u(\cdot)$ satisfies (2.5). For the individual consumer $\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_N$ is a set of fixed constants (or functions), but for the investigator it is a set of random variables with some joint c.d.f. $F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N)$ which induces a distribution on $\tilde{u}$.

In the budget-constrained random utility qualitative choice model the consumer is assumed to maximize (2.1') subject to the constraints (2.2)-(2.4). This yields a set of ordinary demand functions and an indirect utility function which parallel those developed in the previous section, except that they involve a random component from the point of view of the econometric investigator.

Suppose that the consumer has selected good j. Conditional on this decision his utility is $\tilde{u}_j$ where, from (2.1')-(2.5),

$$(2.6') \qquad \tilde{u}_j = v_j(q_j,\ y - p_j\bar{x}_j) + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N,$$

with the non-stochastic component being identical to $v_j(\cdot)$ in (2.6). The ordinary demand functions are $\tilde{x}_j = x_j(p,\ q,\ y,\ \tilde{\varepsilon}) = \delta_j(p,\ q,\ y,\ \tilde{\varepsilon})\bar{x}_j$, where

$$(2.7') \qquad \tilde{\delta}_j = \delta_j(p,\ q,\ y,\ \tilde{\varepsilon}) = \begin{cases} 1 \text{ if } v_j(q_j,\ y - p_j\bar{x}_j) + \tilde{\varepsilon}_j \geq v_i(q_i,\ y - p_i\bar{x}_i) + \tilde{\varepsilon}_i & \text{all } i \\ 0 \text{ otherwise} \end{cases}$$

is a Bernoulli random variable with mean $E\{\tilde{\delta}_j\} \equiv \pi_j$ given by

$$(2.18) \qquad \begin{aligned} \pi_j &= \Pr\{v_j(q_j,\ y - p_j\bar{x}_j) + \tilde{\varepsilon}_j > v_i(q_i,\ y - p_i\bar{x}_i) + \varepsilon_i\ ,\ \text{all } i\} \\ &= F_{(j)}[v_j(q_j, y - p_j\bar{x}_j) - v_1(q_1,\ y - p_1\bar{x}_1), \ldots, v_j(q_j, y - p_j\bar{x}_j) - v_N(q_N, y - p_N\bar{x}_N)] \end{aligned}$$

where $F_{(j)}$ is the joint c.d.f. of the (N-1) differences $\tilde{\eta}_{ij} = \tilde{\varepsilon}_i - \tilde{\varepsilon}_j$. The expected quantity demanded is $E\{\tilde{x}_j\} \equiv X_j(p, q, y) = \pi_j\bar{x}_j$. Assuming that v_j can be cast in the form $v_j = W_j\beta_j$, (2.18) constitutes a random utility model as defined in (1.7). I refer to it as a budget-constrained RUM because of the restrictions on the regressors W_j and coefficients β_j implied by (2.6').

Substituting the ordinary demand functions into the utility function (2.1') yields the unconditional indirect utility function

$$(2.8') \qquad \tilde{v} = v(p, q, y, \tilde{\varepsilon}) = \max[v_1(q_1, y - p_1\bar{x}_1) + \tilde{\varepsilon}_1, \ldots, v_N(q_N, y - p_N\bar{x}_N) + \tilde{\varepsilon}_N].$$

Recall that $\tilde{v}$ is the utility attained by the individual maximizing consumer when confronted with the choice set (p, q, y). This is a known number for the consumer, but for the econometric investigator it is a random variable with a c.d.f. $F_v(w) \equiv \Pr\{v \leq w\}$ derivable from the assumed distribution F_ε by $F_v(w) = F_\varepsilon(w - v_1, \ldots, w - v_N)$. Accordingly, it might be natural for the investigator to focus on the mean of this distribution $E\{v(p, q, y, \tilde{\varepsilon})\} \equiv V(p, q, y)$ in evaluating the welfare effects of a change in the choice set. Formulas for $V(\cdot)$ are provided in Table I for the GEV model (1.4), the independent logit model (1.5), and two probit models (1.3).[8]

Suppose that the available prices and attributes change from (p^0, q^0) to (p^1, q^1). I propose to measure the effect of this change on the consumer's welfare by the quantities CV or EV defined by

(2.9') $V(p^1, q^1, y + CV) = V(p^0, q^0, y)$

(2.10') $V(p^1, q^1, y) = V(p^0, q^0, y - EV)$.

CV is the amount of money that one would have to give the consumer after the change in order to render him as well off as he was before it; EV is the amount that one would have to take from him before the change in order to render him as well off as he would be after it. In both cases, because the consumer's preferences are partially unobservable, the welfare comparison is based on the investigator's expectation of his utility.

As in the deterministic utility case, the RUM (2.1')-(2.4) has a dual problem, to minimize $\Sigma p_j x_j + z$ subject to (2.1'), (2.3), and (2.4). Suppose that the consumer has selected good j. Conditional on this decision his expenditure, $\tilde{e}_j$, is now a random variable from the point of view of the econometric investigator

(2.11') $\tilde{e}_j = g_j(q_j, u - \tilde{\varepsilon}_j) + p_j\bar{x}_j \qquad j = 1, \ldots, N,$

where $g_j(\cdot, \cdot)$ is the same function as in (2.11). Note that, whereas the random element $\tilde{\varepsilon}_j$ enters linearly into the conditional indirect utility function (2.6') in general it enters non-linearly into the conditional expenditure function (2.11').

Associated with the expenditure minimization is a set of unconditional compensated demand functions $\tilde{x}_j = x_j(p, q, u, \tilde{\varepsilon}) = \delta_j(p, q, u, \tilde{\varepsilon})\bar{x}_j$, where $\delta_j(p, q, u, \tilde{\varepsilon})$ is defined as in (2.12) but using $\tilde{e}_j$ in place of e_j. Substituting these demand functions into the minimand yields the unconditional expenditure function

(2.13') $\tilde{e} = e(p, q, u, \tilde{\varepsilon}) = \min[g_1(q_1, u - \tilde{\varepsilon}_1) + p_1\bar{x}_1, \ldots, g_N(q_N, u - \tilde{\varepsilon}_N) + p_N\bar{x}_N]$.

For the investigator $\tilde{e}$ is a random variable with a c.d.f. $F_e(w) = 1 - \Pr\{g_1(q_1, u - \tilde{\varepsilon}_1) > w - p_1\bar{x}_1, \ldots, g_N(q_N, u - \tilde{\varepsilon}_N) > w - p_N\bar{x}_N\}$. The mean of this distribution will be denoted $\mathcal{E}\{e(p, q, u, \tilde{\varepsilon})\} \equiv E(p, q, u)$. It can be shown that, whereas $e(p, q, v(p, q, y, \tilde{\varepsilon}), \tilde{\varepsilon})) \equiv y$, this duality relationship does not generally apply to the means of these random variables

(2.19) $E(p, q, V(p, q, y)) \neq y.$

This is because of the nonlinearity mentioned above combined with the linearity of the expectation operator.[9]

Accordingly, if one were to employ the expected value of the expenditure function to formulate welfare measures analogous to (2.15) and (2.16),

(2.14') $CV' = E(p^1, q^1, V^0) - E(p^0, q^0, V^0)$

(2.15') $EV' = E(p^1, q^1, V^1) - E(p^0, q^0, V^1)$

where $V^t = V(p^t, q^t, y)$, $t = 0, 1$, these quantities would <u>not</u> generally coincide with those based on the expected value of the indirect utility function, i.e., $CV \neq CV'$ and $EV \neq EV'$. Since in most practical situations the data on an individual's qualitative responses pertain to the primal problem (2.1')-(2.4) rather than the dual expenditure minimization problem, it would be natural to concentrate on the CV and EV welfare measures. I will follow this course in the examples presented below.

The moral of this analysis is that, when one switches from a deterministic utility framework to the random utility setting and works with the means of the resulting random demand and utility functions, several of the standard relationships fail to carry over. This point is not adequately stressed by SR, partly because they tend to focus on the special case of no income effects in which these difficulties disappear.

2.3 The Case of No Income Effects

In the random utility context Assumption B, that the ordinary demand functions $x_j(p, q, y, \tilde{\varepsilon})$ coincide with the compensated demand functions $x_j(p, q, u, \tilde{\varepsilon})$, implies both that the non-stochastic component of the utility function (2.1') has the form given in (2.17) and that the moments of the joint distribution F_ε are independent of (p, q, y). Thus (2.6') and (2.11') become

(2.20) $\tilde{v}_j(q_j, y - p_j\bar{x}_j) + \tilde{\varepsilon}_j = h_j(q_j) + \gamma y - \gamma p_j\bar{x}_j + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N$

(2.21) $g_j(q_j, u - \tilde{\varepsilon}_j) + p_j\bar{x}_j = p_j\bar{x}_j + [(u - \tilde{\varepsilon}_j - h_j(q_j))/\gamma] \qquad j = 1, \ldots, N.$

Substituting (2.20) into (2.8') and taking the expectation yields

$$(2.22)\qquad V(p, q, y) = \gamma y + E\{\max[h_1(q_1) - \gamma p_1\bar{x}_1 + \tilde{\varepsilon}_1, \ldots, h_N(q_N) - \gamma p_N\bar{x}_N + \tilde{\varepsilon}_N]\}$$

$$\equiv \gamma y + T(p, q)$$

where $T(\cdot)$ can be calculated using the formulas in Table I. Applying the welfare formulas (2.9') and (2.10'), one finds that

$$(2.23)\qquad CV = EV = [T(p^0, q^0) - T(p^1, q^1)]/\gamma .$$

Moreover, if one substitutes (2.21) into (2.13') and evaluates $E(p, q, u)$, it turns out that (2.19) becomes an identity and CV' and EV' calculated from (2.14') and (2.15') satisfy CV' = EV' = CV = EV. Thus, under the assumption of no income effects there is but a single welfare measure.[10]

SR [15, equation (5.5)] present a welfare formula which can be shown to be equivalent to (2.23). They derive it in a somewhat different manner, and they claim to require Assumptions A, B, and C. However, as demonstrated above, only Assumption B is required: it already implies Assumption A, and it precludes Assumption C, which plays no role in the derivation of (2.23).[11] One consequence of this assumption should be noted. The income variable drops out of the utility differences which, from (2.20), take the form $v_j - v_i = h_j(q_j) - h_i(q_i) - \gamma(p_j\bar{x}_j - p_i\bar{x}_i)$. Since it is the utility differences which enter into the formula for the discrete choice probabilities (2.18), it follows that these choice probabilities are independent of income. The marginal utility of income, γ, can still be estimated because it is also the coefficient of the price variables, but income itself cannot appear as an explicit variable in a qualitative response model under the no income effects assumption.

2.4 Econometric Applications

In the light of the above discussion, the following procedure is suggested for performing welfare evaluations with qualitative response models of discrete consumer choice. Formulate a budget-constrained RUM model (2.1')-(2.4) in which the conditional indirect utility functions (2.6') can be cast in the regression form $\tilde{u}_j = W_j\beta_j + \tilde{\varepsilon}_j$ and specify either an extreme value or normal distribution for $\varepsilon_1, \ldots, \varepsilon_N$. Fit the resulting logit or probit model to obtain estimates of the coefficients in $v_j(\cdot)$ and the parameters of the distribution F_ε. For any given change from (p^0, q^0) to (p^1, q^1) calculate CV or EV by evaluating $V(p, q, y)$ from the formulas in Table I and applying (2.9') or (2.10'). Alternatively, if the utility model satisfies the no income effects condition, evaluate $T(p, q)$ and apply (2.23).

In the no income effects case (2.23) provides a closed-form expression for CV. For example, with the independent logit model one obtains

$$(2.23a)\quad CV = EV = \frac{1}{\gamma}[\ell n(\Sigma e^{v_j^0}) - \ell n(\Sigma e^{v_j^1})]$$

while with the binary independent probit model where Σ is diagonal, independent of (p, q, y), and normalized so that $\kappa_2 = 1$, one obtains

$$(2.23b)\quad CV = EV = \frac{1}{\gamma}[\Delta^0\Phi(\Delta^0) + v_2^0 + \phi(\Delta^0) - \Delta^1\Phi(\Delta^1) - v_2^1 - \phi(\Delta^1)]$$

where $v_j^t = v_j(q_j^t, y - p_j^t)$, $t = 0, 1$, and $\Delta^t = v_1^t - v_2^t$. In the presence of income effects, however, $V(p, q, y)$ is generally nonlinear in y and therefore one must employ numerical techniques such as Newton's method in order to solve (2.9') or (2.10') for CV or EV. Nevertheless, in some cases approximations are available which yield closed-form solutions. Consider, for example, the RUM models associated with the conditional indirect utility functions[12]

$$(2.24)\quad \tilde{u}_j = h_j(q_j) + \gamma_j y - \gamma_j p_j + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N$$

$$(2.25)\quad \tilde{u}_j = h_j(q_j) + \gamma_j\ \ell n(y - p_j) + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N.$$

The model (2.24) is the RUM analog of (2.16a) and satisfies Assumption A but not B, while (2.25) satisfies neither A nor B.[13]

I first assume that F_ε is the standard extreme value distribution, so that (2.24) and (2.25) generate independent logit models. In both cases $V(p^0, q^0, y) = \ell n(\Sigma e^{v_j^0}) + 0.57722$, where v_j^0 is the non-stochastic component on the RHS of (2.24) and (2.25) evaluated at (p_j^0, q_j^0, y). In the case of (2.24)

$$V(p^1, q^1, y + CV) - 0.57722 = \ell n(\Sigma e^{h_j(q_j^1) + \gamma_j(y + CV) - \gamma_j p_j^1}) = \ell n(\Sigma e^{v_j^1} e^{\gamma_j CV}),$$

where v_j^1 is evaluated at (p_j^1, q_j^1, y). Applying (2.9'), CV satisfies

$$(2.26) \quad \Sigma e^{v_j^1} e^{\gamma_j CV} = \Sigma e^{v_j^0} .$$

Rather than employing numerical methods to solve (2.26) one can use the approximation $e^z \approx 1 + z$ to obtain

$$\Sigma(1 + \gamma_j CV) e^{v_j^1} \approx \Sigma e^{v_j^0}$$

or

$$(2.27) \quad CV \approx (\Sigma e^{v_j^0} - \Sigma e^{v_j^1}) / \Sigma \gamma_i e^{v_j^1} .$$

The quantity EV can be approximated similarly.[14] For the model (2.25)

$$V(p^1, q^1, y + CV) - 0.57722 = \ell n[\Sigma e^{h_j(q_j^1) + \gamma_j \ell n(y + CV - p_j^1)}] = \ell n[\Sigma e^{h_j(q_j^1)} (y + CV - p_j^1)^{\gamma_j}] .$$

Using the approximation $(a + z)^\gamma \approx a^\gamma + \gamma z a^{\gamma - 1}$ with $a = y - p_j^1$ and $z = CV$ one obtains

$$\Sigma e^{h_j(q_j^1)} (y + CV - p_j^1)^{\gamma_j} \approx \Sigma e^{h_j(q_j^1)} [(y - p_j^1)^{\gamma_j} + \gamma_j CV (y - p_j^1)^{\gamma_j - 1}]$$

$$= \Sigma e^{v_j^1} [1 + \gamma_j CV (y - p_j^1)^{-1}] .$$

Thus,[15]

$$(2.28) \quad CV \approx (\Sigma e^{v_j^0} - \Sigma e^{v_j^1}) / \Sigma \gamma_j (y - p_j^1)^{-1} e^{v_j^1} .$$

Similar approximations can be applied to the GEV model based on the c.d.f. given in Table I. As an example I will consider what Amemiya [2] calls the standard GEV model. Suppose that the N indices can be naturally partitioned into $S < N$ groups so that each group consists of similar alternatives. Then, the function $G(\cdot)$ in Table I takes the form

$$(2.29) \quad G(t_1, \ldots, t_N) = \sum_{s=1}^{S} \alpha_s \left(\sum_{j \varepsilon I_s} t_j^{1/1 - \sigma_s} \right)^{1 - \sigma_s}$$

where $\alpha_s \geq 0$ and $0 \leq \sigma_s < 1$. Thus

$$V(p^0, q^0, y) = \ell n[\Sigma_s \alpha_s (\sum_{j \varepsilon I_s} e^{v_j^0 / 1 - \sigma_s})^{1 - \sigma_s}] + 0.57722.$$

In the case of (2.24), note that

$$\{ \sum_{j \varepsilon I_s} \exp[v_j(q_j^1, y + CV - p_j^1) / 1 - \sigma_s] \}^{1 - \sigma_s} = \{ \sum_{j \varepsilon I_s} e^{v_j^1 / 1 - \sigma_s} e^{\gamma_j CV / 1 - \sigma_s} \}^{1 - \sigma_s}$$

$$\approx \{\Sigma\, e^{v_j^1/1-\sigma_s} + \frac{CV}{1-\sigma_s}\ \Sigma\gamma_j e^{v_j^1/1-\sigma_s}\}^{1-\sigma_s}$$

$$\approx (\Sigma e^{v_j^1/1-\sigma_s})^{1-\sigma_s} + CV(\Sigma\gamma_j e^{v_j^1/1-\sigma_s})(\Sigma e^{v_j^1/1-\sigma_s})^{-\sigma_s} .$$

Substituting this into the formula for $V(p^1, q^1, y+CV)$ and applying (2.9'), one obtains

$$CV \approx [\sum_s \alpha_s(\sum_{j\in I_s} e^{v_j^0/1-\sigma_s})^{1-\sigma_s} - \sum_s \alpha_s(\sum_{j\in I_s} e^{v_j^1/1-\sigma_s})^{1-\sigma_s}]/\sum_s \alpha_s(\sum_{j\in I_s} \gamma_j e^{v_j^1/1-\sigma_s})(\sum_{j\in I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s} . \tag{2.30}$$

If one sets $\alpha_s = 1$ and $\sigma_s = 0$, this collapses to the formula in (2.27). Similar manipulations applied to (2.25) yield the approximation

$$CV \approx [\sum_s \alpha_s(\sum_{j\in I_s} e^{v_j^0/1-\sigma_s})^{1-\sigma_s} - \sum_s \alpha_s(\sum_{j\in I_s} e^{v_j^1/1-\sigma_s})^{1-\sigma_s}]/\sum_s \alpha_s(\sum_{j\in I_s} \gamma_j (y-p_j^1)^{-1} e^{v_j^1/1-\sigma_s})(\sum_{j\in I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s} . \tag{2.31}$$

In the binary independent probit case with fixed Σ, $V(p^t, q^t, y) = \Delta^t\Phi(\Delta^t) + v_2^t + \phi(\Delta^t)$, $t = 0, 1$. With the utility model (2.24)

$$V(p^1, q^1, y+CV) = [\Delta^1 + (\gamma_1 - \gamma_2)CV]\Phi(\Delta^1 + (\gamma_1 - \gamma_2)CV) + v_2^1 + \gamma_2 CV + \phi(\Delta^1 + (\gamma_1 - \gamma_2)CV) . \tag{2.32}$$

Using the first-order approximations $\Phi(a+z) \approx \Phi(a) + z\phi(a)$ and $\phi(a+z) \approx \phi(a)[1 - az]$ with $a = \Delta^1$ and $z = (\gamma_1 - \gamma_2)CV$, (2.32) becomes

$$V(p^1, q^1, y + CV) \approx A_1 CV^2 + A_2 CV + V(p^1, q^1, y)$$

where $A_1 \equiv (\gamma_1 - \gamma_2)^2\phi(\Delta^1)$, and $A_2 \equiv (\gamma_1 - \gamma_2)\Phi(\Delta^1) + \gamma_2$. Thus CV satisfies (approximately) the quadratic equation $A_1 CV^2 + A_2 CV + A_3 = 0$, where $A_3 \equiv V(p^1, q^1, y) - V(p^0, q^0, y)$. Hence,

$$CV \approx (-A_2 \pm \sqrt{A_2^2 - 4A_1A_3})/2A_1 \tag{2.33}$$

where one should remember that $\text{sign}(CV) = \text{sign}(-A_3)$ in selecting the positive or negative root. For the model (2.25) one needs to employ the approximation

$$\ell n(y + CV - p_j^1) \approx \ell n(y - p_j^1) + \frac{CV}{y - p_j^1} \qquad j = 1, 2 .$$

Hence,

$$V(p^1, q^1, y+CV) \approx [\Delta^1 + (\lambda_1 - \lambda_2)CV]\Phi(\Delta^1 + (\lambda_1 - \lambda_2)CV) + v_2^1 + \lambda_2 CV + \phi(\Delta^1 + (\lambda_1 - \lambda_2)CV) \tag{2.34}$$

where $\lambda_j \equiv \gamma_j/(y - p_j^1)$, $j = 1, 2$. Using the same approximations as those applied to (2.32) one obtains the formula (2.33) where A_3 is defined as before but $A_1 \equiv (\lambda_1 - \lambda_2)^2\phi(\Delta^1)$ and $A_2 \equiv (\lambda_1 - \lambda_2)\Phi(\Delta^1) + \lambda_2$.

It would however be difficult to apply such approximations to polychotomous probit models based on (2.24) or (2.25) because of the complexity of the formula for V. An additional complication arises in random coefficient versions of the probit model where γ_j itself is normally distributed — see [10] for example — because then the covariance matrix Σ depends on y. One would have to allow for this dependence in evaluating $V(p^1, q^1, y + CV)$ and solving (2.9'). In these cases numerical solution techniques would certainly be required.

Finally, it is instructive to consider the special case where only one price changes — i.e., $q^0 = q^1$ and $p_j^0 = p_j^1$, $j \geq 2$. In the discussion of the deterministic utility model (2.1)-(2.4) the following result was mentioned: if the consumer selects good 1 and its price falls, the compensating variation is measured by the change in price, $cv = (p_1^1 - p_1^0)$. One might wonder whether a similar result carries over to the RUM model (2.1')-(2.4).

For the independent logit model with no income effects, the exact formula for CV (2.23a) may be approximated by $CV \approx (\Sigma e^{v_j^0} - \Sigma e^{v_j^1})/\gamma\Sigma e^{v_j^1}$. With a single price change

$$\Sigma e^{v_j^0} - \Sigma e^{v_j^1} = e^{v_1^0} - e^{v_1^1} = e^{v_1^1}(e^{\gamma(p_1^1 - p_1^0)} - 1) \approx e^{v_1^1}\gamma(p_1^1 - p_1^0).$$

Thus,

$$CV \approx e^{v_1^1}\gamma(p_1^1 - p_1^0)/\gamma\Sigma e^{v_j^1} = \pi_1^1(p_1^1 - p_1^0)$$

where π_1^1 is the probability of selecting good 1 evaluated at the new price. This result, that the compensating variation for a price change is approximately equal to the probability of buying the good multiplied by the change in price, may be regarded as an analog of the deterministic utility result. For the logit model based on (2.24) which yields the CV formula (2.27), applying the approximation $e^z \approx 1 + z$ for a second time one obtains

(2.35) $CV \approx e^{v_1^1}\gamma_1(p_1^1 - p_1^0)/\Sigma\gamma_j e^{v_j^1}$.

Similarly, for the logit model based on (2.25), (2.28) becomes

(2.36) $CV \approx e^{v_1^1}(\gamma_1/y)(p_1^1 - p_1^0)/\Sigma\gamma_j(y - p_j^1)^{-1}e^{v_j^1}$.

In both cases one can argue that the RHS is likely to be of the same order of magnitude as $e^{v_1^1}(p_1^1 - p_1^0)/\Sigma e^{v_j^1} = \pi_1^1(p_1^1 - p_1^0)$.[16] It is more difficult to perform similar manipulations with the probit models, except for the binary independent probit model with no income effects which yields the exact formula (2.23b). When only p_1 changes $\Delta^0 = \Delta^1 + \gamma(p_1 - p_1)$ and to a first-order approximation (2.23b) becomes

(2.37) $CV \simeq \pi_1^1(p_1^1 - p_1^0) + \gamma(p_1^1 - p_1^0)^2\phi(\Delta^1)$

where the first term is likely to dominate the RHS.

In all of these cases it must be stressed that the formula $CV \simeq \pi_1^1(p_1^1 - p_1^0)$ is only a very rough approximation and <u>not</u> an exact result. It may be useful as a rule of thumb for quick calculations, but its accuracy remains to be determined empirically. The ultimate solution for any welfare evaluation is to apply numerical techniques to the fundamental equations (2.9') and (2.10'), or to employ the approximations (2.27), (2.28), (2.30), (2.31) or (2.33).

3. Other RUM Models

The budget-constrained RUM model (2.1')-(2.4) implies that the conditional indirect utility functions have the form $\tilde{u}_j = v_j(q, y - p_j) + \tilde{\varepsilon}_j$, which imposes a substantive restriction on the manner in which the price and income variables enter the formulas for the discrete choice probabilities. The literature contains many examples of logit or probit models of discrete consumer choices which violate these restrictions. For example, one finds RUM models based on utility functions of the form

(3.1) $\tilde{u}_j = h_j(q_j) - \beta_j p_j + \gamma_j y + \tilde{\varepsilon}_j, \quad \beta_j \neq \gamma_j \qquad j = 1, \ldots, N$

(3.2) $\tilde{u}_j = h_j(q_j) - \beta p_j + \gamma y p_j + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N$

which are incompatible with the budget constraint (2.2). In these and other

RUM models which violate (2.2) one still might wish to evaluate the welfare effects of a change in the choice set. The appropriate procedure would be to calculate $V \equiv E\{\max[\tilde{u}_1, \ldots, \tilde{u}_N]\}$ using the formulas in Table I and solve the equations analogous to (2.9') or (2.10') for CV or EV.

As before this generally requires numerical solution techniques, but in some cases a closed-form approximation to the solution can be obtained. The models (3.1) and (3.2) are particular cases of the more general model

$$(3.3) \qquad \tilde{u}_j = \psi_j(q_j, p_j) + \lambda_j(p_j, q_j)y + \tilde{\varepsilon}_j \qquad j = 1, \ldots, N.$$

For any standard GEV model based on (3.3), by applying the approximations used in connection with (2.30) one obtains

$$(3.4) \qquad CV \simeq [\Sigma_s \alpha_s (\sum_{j\varepsilon I_s} e^{v_j^0/1-\sigma_s})^{1-\sigma_s} - \Sigma_s \alpha_s (\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{1-\sigma_s}]/\Sigma_s \alpha_s (\sum_{j\varepsilon I_s} \lambda_j^1 e^{v_j^1/1-\sigma_s})(\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s}.$$

where v_j^t is the non-stochastic component on the RHS of (3.3) evaluated at (p_j^t, q_j^t, y), $t = 0, 1$, and $\lambda_j^1 = \lambda_j(p_j^1, q_j^1)$. Setting $\alpha_s = 1$ and $\sigma_s = 0$ yields the formula for the independent logit model. Similarly, for any binary independent probit model with fixed covariance matrix based on (3.3), by applying the approximations used in connection with (2.31) one obtains the formula (2.33) where $A_1 \equiv (\lambda_1^1 - \lambda_2^1)^2 \phi(\Delta^1)$, $A_2 \equiv (\lambda_1^1 - \lambda_2^1)\Phi(\Delta^1) + \lambda_2^1$, $A_3 \equiv \Delta^1\Phi(\Delta^1) + v_2^1 + \phi(\Delta^1) - \Delta^0\Phi(\Delta^0) - v_2^0 - \phi(\Delta^0)$, and $\Delta^t \equiv v_1^t - v_2^t$, $t = 0, 1$.

Another example of a RUM model not consistent with (2.1')-(2.4) occurs when the qualitative responses represent choices among actions with uncertain consequences by a von Neumann-Morgenstern expected utility maximizing individual. Suppose an individual has wealth y and a utility of wealth function whose non-stochastic component is denoted by $\psi(y)$. The individual must choose among N actions whose consequences depend on the state of the world $s = 1, \ldots, S$. Associated with act j is a vector of state probabilities $\rho_j = (\rho_{j1}, \ldots, \rho_{jS})$ and a vector of monetary consequences $z_j = (z_{j1}, \ldots, z_{jS})$. Conditional on the choice of act j the individual's utility is

$$(3.5) \qquad \tilde{u}_j = \sum_s \rho_{js}\psi(y + z_{js}) + \tilde{\varepsilon}_j \equiv v_j + \tilde{\varepsilon}_j, \qquad j = 1, \ldots, N.$$

This generates an MRQR model which may or may not be linear in the unknown parameters depending on the form of the utility function $\psi(y)$. Given that the individual has chosen optimally, his utility is $\tilde{v} = \max[\tilde{u}_1, \ldots, \tilde{u}_N]$. This is a random variable for the econometric investigator with a mean $E\{\tilde{v}\} \equiv V(\rho, z, y)$. Suppose the state probabilities and/or payoffs change from (ρ^0, z^0) to (ρ^1, z^1). By analogy with (2.9'), the compensating variation measure of the effect of this change on the individual's welfare is defined by

(3.6) $$V(\rho^1, z^1, y + CV) = V(\rho^0, z^0, y).$$

Again, numerical solution techniques will generally be required, but sometimes one can obtain a closed-form approximation to the solution. For example, consider the model based on (3.5) with $\psi(y) = \gamma \ln y$ which arises in a study of seat-belt usage by motorists [9]. In the independent logit case, with $S = 2$, by an argument similar to that leading to (2.28), one obtains a quadratic in CV with the solution (2.33) where $A_1 \equiv \gamma^2 \Sigma \lambda^1_{j1} \lambda^1_{j2} e^{v^1_j}$, $A_2 \equiv \gamma \Sigma \lambda^1_j e^{v^1_j}$, $A_3 \equiv (\Sigma e^{v^1_j} - \Sigma e^{v^0_j})$, $\lambda^1_{js} = \rho^1_{js}/(y + z^1_{js})$, $s = 1, 2$, and $\lambda^1_j \equiv \lambda^1_{j1} + \lambda^1_{j2}$. Likewise, in the independent probit case one obtains the solution (2.33) with $A_1 \equiv \gamma^2 (\lambda^1_1 - \lambda^1_2)^2 \phi(\Delta^1)$, $A_2 \equiv \gamma(\lambda^1_1 - \lambda^1_2)\Phi(\Delta^1) + \gamma\lambda^1_2$, and $A_3 = \Delta^1\Phi(\Delta^1) + v^1_2 + \phi(\Delta^1) - \Delta^0\Phi(\Delta^0) - v^0_2 - \phi(\Delta^0)$, where $\Delta^t = v^t_1 - v^t_2$, $t = 0, 1$, and $\lambda^1_j = \Sigma\rho^1_{js}/(y + z^1_{js})$, $j = 1, 2$. Similar results can be obtained for some other common specifications of $\psi(y)$.

4. Conclusions

The purpose of this paper has been to describe a general procedure for performing welfare evaluations with MRQR models which are derivable from a RUM model of consumer choice. The procedure is based on the equations (2.9'), (2.10') and (3.6), which in general require numerical solution techniques. With current computer software, however, this should not be a serious obstacle. Moreover, I have shown that for many common logit and probit models closed-form approximations to the solution are readily available. Much attention has been devoted to budget-constrained RUM models because these have an exact analog in deterministic utility theory. This makes it possible to highlight the similarities as well as the differences between the deterministic and random utility frameworks. However, the same general approach to welfare evaluations carries over to the other RUM models which are found in the literature.

The emphasis throughout the paper has been on the welfare theory of an individual consumer. An implication is that the welfare measures described above must be calculated separately for each consumer. The problems of estimating MRQR models from aggregate data and developing welfare inferences on the basis of aggregate utility functions have not been addressed here although, as McFadden [13] shows, the no income effects utility function (2.20) can be employed for this purpose. My approach presupposes that the MRQR model is estimated from disaggregated micro data and that all individuals in the sample have utility functions with the same non-stochastic component and the same probability law governing the random elements. These conditions are met by virtually all of the empirical MRQR models which have appeared in the literature. Hence, the techniques of welfare analysis described here should be widely applicable.

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FOOTNOTES

1. I am grateful to Angus Deaton and two referees for their helpful comments on an earlier version of this paper.
2. A tilde will be used to denote random variables.
3. The question of mixed qualitative and quantitative choices will not be considered here. Welfare measures for a class of these models are presented in [8].
4. For simplicity I will treat q_j as a scalar variable, but in empirical applications it is usually a vector.
5. I assume that $y \geq \max_i \{p_i \bar{x}_i\}$, so that the non-negativity condition on z is not violated.
6. SR [15, fn. 16] note an error in Mäler's analysis of this point.
7. The proof is presented in the earlier version of this paper, which is available on request.
8. In the case of the binary probit model the formula for V is exact. In the case of the trichotomous probit model the formula is approximate and relies on Clark's [4] approximation, which could also be employed for higher dimensional probit models.
9. Further details are provided in the earlier version of this paper, where it is also shown that Roy's Identity does not apply to $X_j(p, q, y)$ and $V(p, q, y)$, i.e., $X_j \neq -(\partial V/\partial p_j)/(\partial V/\partial y)$. Similarly, $E(p, q, u)$ does not satisfy Shepherd's Lemma.
10. It can also be shown that in this case V and E satisfy Roy's Identity and Shepherd's Lemma.
11. An analysis of SR's derivation of their welfare measure and a demonstration of the irrelevance of Assumption C is contained in the earlier version of this paper.
12. I now assume the units of measurement are chosen so that $\bar{x}_j = 1$.
13. In (2.25) one can set $\gamma_j = \gamma$ all j while still preserving an income effect.

14. EV can be calculated from the formulas presented here by observing from (2.9') and (2.10') that the EV for a change from (p^a, q^a) to (p^b, q^b) is the negative of the CV for the change from (p^b, q^b) to (p^a, q^a).

15. Using the same approximations one can obtain similar formulas for CV for the utility models $\tilde{u}_j = h_j(q_j) + e^{\gamma_j(y - p_j)} + \tilde{\varepsilon}_j$ and $\tilde{u}_j = h_j(q_j) + (y - p_j)^{\gamma_j} + \tilde{\varepsilon}_j$ in the logit case. The latter model is a CES version of the Cobb-Douglas model (2.25). However, these utility functions generate MRQR models which are nonlinear in the parameters.

16. By a similar argument it can be shown that EV is roughly of the same order of magnitude as $\pi_1^0(p_1^1 - p_1^0)$.

TABLE I

FORMULAS FOR $V \equiv E\{\max[v_1 + \tilde{\varepsilon}_1, \ldots, v_N + \tilde{\varepsilon}_N]\}$

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1. Generalized Extreme Value

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = \exp[-G(e^{-\varepsilon_1}, \ldots, e^{-\varepsilon_N})]$$

$$V = \ln G(e^{v_1}, \ldots, e^{v_N}) + 0.57722$$

2. Independent Logit

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = \exp[-\Sigma e^{-\varepsilon_j}]$$

$$V = \ln \Sigma e^{v_j} + 0.57722$$

3. Probit

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = N(0, \Sigma), \qquad \Sigma = \{\sigma^2_{ij}\}$$

a. Binary Probit, N = 2

$$V_2 = (v_1 - v_2)\Phi(\frac{v_1 - v_2}{\kappa_2}) + v_2 + \kappa_2\,\phi(\frac{v_1 - v_2}{\kappa_2}), \quad \kappa_2 \equiv (\sigma^2_{11} + \sigma^2_{22} - 2\sigma^2_{12})^{\frac{1}{2}}$$

b. Trichotomous Probit, N = 3

$$V_3 \approx (V_2 - v_3)\Phi(\frac{V_2 - v_3}{\kappa_3}) + v_3 + \kappa_3\,\phi(\frac{V_2 - v_3}{\kappa_3})$$

$$\kappa_3 \equiv (\sigma^2_{33} + S^2_2 - 2S^2_{2,3})^{\frac{1}{2}}$$

$$S^2_{2,3} \equiv \sigma^2_{23} + (\sigma^2_{13} - \sigma^2_{23})\Phi(\frac{v_1 - v_2}{\kappa_2})$$

$$S^2_2 \equiv v^2_2 + \sigma^2_{22} + (v^2_1 + \sigma^2_{11} - v_2 - \sigma^2_{22})\Phi(\frac{v_1 - v_2}{\kappa_2}) + (v_1 + v_2)\kappa_2\phi(\frac{v_1 - v_2}{\kappa_2}) - V^2_2$$

ϕ and Φ are, respectively, the standard univariate normal p.d.f. and c.d.f.

$$(2.30)\quad CV \approx [\sum_s \alpha_s (\sum_{j\varepsilon I_s} e^{v_j^0/1-\sigma_s})^{1-\sigma_s} - \sum_s \alpha_s (\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{1-\sigma_s}] / \sum_s \alpha_s (\sum_{j\varepsilon I_s} \gamma_j e^{v_j^1/1-\sigma_s}) (\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s} .$$

$$(2.31)\quad CV \quad {}_s\,{}_s(\qquad / \sum_s \alpha_s (\sum_{j\varepsilon I_s} \gamma_j (y - p_j^1)^{-1} e^{v_j^1/1-\sigma_s}) (\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s} .$$

$$(3.4)\quad CV \qquad / \sum_s \alpha_s (\sum_{j\varepsilon I_s} \lambda_j^1 e^{v_j^1/1-\sigma_s}) (\sum_{j\varepsilon I_s} e^{v_j^1/1-\sigma_s})^{-\sigma_s}$$