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There are at present several models describing the nucleon-nucleon interaction below one BeV. The low energy S-wave proton-proton scattering should provide a test for the singlet even interaction predicted by the models. Between 0 and 10 MeV the S-wave scattering can be represented by a convergent power series,[1] and therefore it can be approximated by a polynomial

$$K = \sum_{0}^{n} A_n E^n \tag{1}$$

where E is the laboratory energy, usually expressed in MeV. The explicit relation of Eq. (1) with the $^1S_0$ p-p phase shifts and currently used scattering parameters is obtained through the equation $K = RF$

$$F = C^2 k \cot \delta_0 + \frac{1}{R} h(\eta) = -\frac{1}{a_p} + \frac{1}{2} r_e k^2 - P r_e^3 k^4 + Q r_e^5 k^6 - \ldots \tag{2}$$

where $C^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$, $k^2 = \frac{M_p E}{2\hbar^2}$, $R = \frac{\hbar^2}{M_p \epsilon^2}$, $h(\eta) = \mathrm{Re}\frac{\Gamma'(-i\eta)}{\Gamma(-i\eta)} - \ln \eta$

E is again the laboratory energy, M_p is the proton mass, $\eta = \frac{\epsilon^2}{\hbar v_{LAB}}$, usually called Coulomb parameter (ϵ is the proton charge, $\hbar$ is Planck's constant divided by 2π, v_{LAB} is the relative velocity), a_p is the proton-proton scattering length, r_e is the effective range, P and Q are shape-dependent parameters. Existing calculations of Q for different well shapes[2] do indicate that the term in k^6 of Eq. (2) is small compared with the term in k^4, for energies between 0 and 4 MeV lab. Correspondingly Eq. (1) can be approximated by

$$K = A + BE + CE^2. \tag{3}$$

H. Pierre Noyes[3] has calculated the scattering parameters from recent very accurate experimental phase shifts[4,5] at five low energies between 0.3825 and 3.037 MeV, with the aim of comparing theoretical predictions of the shape parameter P with the experimental value. Such calculations have been repeated by different authors[6,7] and the results are substantially in agreement, except for fluctuations well within the experimental errors. Nevertheless, some of the problems pointed out long time ago by Foldy and Eriksen[8] in relation to the empirical evidence for the accuracy of the vacuum polarization correction (from now on called VPC) still remain unresolved if the analysis is restricted to the five above mentioned experimental points. It can be shown that the determination of the parameter P depends critically on the experimental point at 0.3825 MeV and also on the correctness of the VPC.[7] On the other hand Gursky and Heller[6] have reported that the Yale[9] and Hamada-Johnston[10] potentials predict a k^6 term in the expansion (2), or correspondingly a cubic term DE^3 in (1) of "comparable importance" with the quadratic term at 3-MeV laboratory energy.[11] Gursky and Heller[6] attempted a cubic fit for the five data points between 0.3825 and 3.037 MeV. Figure 1 shows the cubic fit as calculated by the present author, in agreement with their calculation. The values obtained for P and Q are completely unreasonable. Clearly, when four parameters are to be determined with only five experimental points the requisites on the size of the errors and experimental fluctuations are very stringent. An inversion of curvature is produced by the point at 3.037 MeV. The function K thus determined is unacceptable because it diverges strongly from experimental values at higher energies, calculated from the data of Worthington, Mc Gruer, and Findley[12] (from now on called WMF data). The WMF data have been subject to a phase shift analysis originally by Hall and Powell[13] and subsequently by de Wit and Durand,[14] who also discussed the VPC with regard to the calculation of proton-proton scattering parameters, as well as relativistic effects, in S and P waves. The singlet S phase shifts themselves were substantially in agreement in both analyses, within statistics. The WMF data overlap with the more recent measurements at 1.855 MeV, 2.425 MeV and 3.037 MeV. The values of the K function at those energies are in good mutual agreement in both sets of data, as is born out by Table I, and hence one should assume that the higher energy values are as reliable as the lower energy values. The function K is corrected for vacuum polarization effects.[8,14] The error bars are larger in the WMF data, but, as they extend about 1.2 MeV beyond the highest energy point of the more recent measurements, they should prove useful in reducing the uncertainties presently existing in the analyses of the five recent measurements. Table I contains altogether twelve experimental values of K between 0.3825 and 4.203 MeV, and also the values K' obtained after correction for electromagnetic structure effects using models without a static core, with a hard core of radius $r_c = 0.4$ f and with a soft core.[15]

The purpose of this note is to report results of least squares fits to the values of the functions K and K' of Table I, and compare the scattering parameters thus obtained with predictions for the shape dependent parameters P[2,15] and Q.[2] It will be shown that the situation is less uncertain than reported by Gursky and Heller[6] in their analysis of the five recent experimental values of the $^1S_0$ phase shift. For convenient reference to the theoretical predictions of the scattering parameters Table II contains a list of them.

The first logical step should be an attempt to obtain a four parameter fit to the twelve values of K. Table III contains such fits together with several others. The fit after performing the electromagnetic corrections (EMC) in a model without a core (NC) is slightly better than the fit to the uncorrected function. In both cases the pair P and Q acquires values not predicted by any existing model. The exclusion of the experimental point at 0.3825 MeV does not alter this conclusion, because P remains small, and Q varies within the probable errors. This result is thus fairly independent of the accuracy of the VPC. In order to compare with current fits[3,6,7] to the five recently measured experimental points[4,5] Table III contains fits to the twelve values of K given in Table I assuming that Q=0. There is a drift of all the scattering parameters towards higher values than in the five point fit, and, in particular, P is almost doubled. Therefore, the agreement of the experimental values obtained for a_p, r_e, and P from five values of K, with the predictions of the Coulomb-corrected-partial-wave-dispersion relation (PWDR) claimed in Reference (3) was most likely fortuitous. For completeness a shape independent fit (SI) is also transcribed in Table III, and it is clearly inadequate.

Table III contains also the function $\chi^2-\chi^2_{3.037}$ resulting from the subtraction of the χ^2 due to both values at 3.037 MeV. The WMF[12] measurement, as well as the recent remeasurement[4] are in good mutual agreement, but both values tend to induce an inversion of curvature, apparently unwarranted by the higher energy values. It is clear from the table that both measurements dominate the behavior of the χ^2 function. The minimum of the function $\chi^2-\chi^2_{3.037}$ yields values for a_p, r_e, and P in remarkable agreement with values that can be easily interpolated between calculations of V. V. Babikov,[16] in the context of a soft core model due to Babikov et al.[17] It is even more remarkable that the fit to the function K' with EMC in a SC model automatically produces values in agreement with the prediction, through a straightforward least squares routine with a reasonable χ^2, and also the absolute minimum for $\chi^2-\chi^2_{3.037}$, as it is shown in Table III.

If the values of P predicted by the hard core potential models of Hamada-Johnston[10] and Yale[9] are used to calculate the remaining parameters, the coefficient D of the cubic term of the polynomial (1) is 400 times and 20 times smaller respectively than the coefficient C. Thus the experimental data in conjunction with such models do not favor a cubic term of comparable importance to the quadratic term, over the investigated energy range.

The results of the present reanalysis are to a certain extent ambiguous, particularly due to the strong contribution to the χ^2 function of the experimental values at 3.037 MeV. Nevertheless it is clear that the twelve values of K restrict the possible final solutions to two, such that the quadratic term CE^2 is small and the cubic term DE^3 is large, or vice versa. The latter alternative is in excellent agreement with a soft core model, like the one proposed by Babikov et al.[17] On the theoretical side it would be desirable to have available the prediction of Q for the models currently in vogue, as well as an exploration of core effects on it. It is already known[18] that P varics rapidly as a function of the core parameters, and can be zero. Therefore, a solution with P=0, or very small, cannot be discarded, although presumably in such case Q would also be small due to core effects. From an experimental viewpoint it would be desirable that additional measurements be carried out in the neighborhood of 3.037 MeV, due to the anomalously large contribution to the χ^2 by the values of K at that energy.

Table I. Experimental values of the function K and of K', corrected for electromagnetic structure effects in a model without a static core (NC), with a hard core (HC) of radius r_c=0.4 f and with a soft core (SC).[15]

E_{LAB} MeV	K	K' NC	K' HC	K' SC
0.3825[a]	3.86501±0.00274	3.88899	3.84631	3.84042
1.397[a]	4.35428±0.00156	4.37810	4.33517	4.32914
1.855[a]	4.57406±0.00219	4.59780	4.55479	4.54864
1.855[b]	4.57398±0.00243	4.59772	4.55471	4.54856
1.858[b]	4.57232±0.00294	4.59606	4.55305	4.54690
2.425[a]	4.84212±0.00155	4.86575	4.82263	4.81637
2.425[b]	4.84104±0.00320	4.86467	4.82155	4.81529
3.037[a]	5.13318±0.00237	5.15670	5.11347	5.10708
3.037[b]	5.13418±0.00473	5.15770	5.11448	5.10808
3.527[b]	5.35126±0.00575	5.37469	5.33139	5.32488
3.899[b]	5.52449±0.00744	5.54785	5.50449	5.49790
4.203[b]	5.66355±0.01093	5.68687	5.64343	5.63680

[a]Data of several authors, Refs. 4 and 5.

[b]Data of Worthington, McGruer, and Findley, Refs. 12, 13, and 14.

Table II. The ^1S_o scattering parameters as calculated by several authors for various models describing the proton-proton interaction.

No.	Model	Parameter $-a_p$	r_e	P	Q
1	Yukawa[a]	7.6512	2.6756	0.05540	0.019
2	Hamada-Johnston[b] (x_o=0.343)	7.729	2.749	0.0478	-
3	Hamada-Johnston[b] (x_o=0.341)	8.542	2.664	0.0499	-
4	Breit et al[b] (x_o=0.350)	7.078	2.829	0.0372	-
5	Babikov et al[b] ($g_\omega^2-2f_\omega^2$=29.2)	8.710	2.721	0.0371	-
6	Babikov et al[b] ($g_\omega^2-2f_\omega^2$=29.6)	7.572	2.840	0.0357	-
7	CSF[c]	7.8426	2.853	0.0612	-
8	PWDR[c]	7.8259	2.786	0.024	-
9	BC[c]	7.8009	2.687	-0.036	-
10	Gaussian well[a]	7.7797	2.6055	-0.01936	-0.00073

[a]Values taken from Table VIII of Ref. 2.

[b]Values taken from Ref. 15.

[c]Values taken from Table I of Ref. 3.

Table III. Experimental values of the $^1S_0$ proton-proton scattering parameters, obtained from fits to the function K of Table I. The probable errors of the fits that were considered more relevant are given.

EMC	$-a_p^F$	r_e^F	P	Q	X^2	$P(X^2>X^2_{fit})$	$X^2-X^2_{3.037}$	Remarks
	7.8320±0.0035	2.7869±0.0042	-0.0015±0.0035	-0.084±0.024	5.61	0.69	2.98	
	7.8353	2.7970	0.0054	-0.074	-	-	-	11 points fit[a]
	7.8404±0.0035	2.8391±0.0058	0.049 ±0.003	0	7.08	0.63	2.45	
	7.8603	2.8789	0.059	0	-	-	-	11 points fit[a]
no	7.7939	2.7087	0	0	53.7	~0	39.5	SI fit
	7.8424±0.0034	2.8515±0.0037	0.061 ±0.002	0.019±0.008	7.86	0.44	2.75	Q of Yukawa well
	7.8402	2.8379	0.0478	-0.0014	7.24	0.52	2.64	P of Hamada-Johnston[b]
	7.8384	2.8267	0.0372	-0.018	6.63	0.57	2.51	P of Breit-Babikov[c]
	7.8381±0.0038	2.8247±0.0019	0.0357±0.0025	-0.021±0.0011	6.53	0.59	2.50	P of Babikov[d]
	7.8365	2.8148	0.026	-0.037	6.22	0.62	2.58	P of Ref. 3[e]
yes NC	7.7790±0.0039	2.7746±0.0051	-0.0124±0.0036	-0.102±0.013	5.47	0.71	3.08	
" HC	7.8808	2.8399	0.0518	0.0042	7.47	0.48	2.96	
" SC	7.8930±0.0030	2.8254±0.0034	0.0365±0.0047	-0.022±0.009	5.98	0.65	2.06	
" NC	7.7888±0.0035	2.8364±0.0058	0.0487±0.0025	0	7.25	0.61	2.54	
" HC	7.8802	2.8368	0.0491	0	7.35	0.60	2.70	errors as in NC
" SC	7.8957	2.8420	0.0510	0	7.42	0.59	3.11	errors as in NC
" NC	7.7908	2.849	0.0602	0.019	7.47	0.48	2.54	Q of Yukawa well
" NC	7.7859	2.8181	0.0315	-0.028	6.30	0.62	2.36	min of $X^2-X^2_{3.037}$

[a] The point at 0.3825 MeV was excluded. Such fits are less dependent on the correctness of the VPC, with regard to P and Q.

[b] See Table II, No. 2.

[c] See Table II, No's. 4 and 5.

[d] See Table II, No. 6. This value also furnishes the minimum of $X^2-X^2_{3.037}$.

[e] This value of P is consistent with fits to the more recent experimental values by several authors (Refs. 3, 6, and 7).

FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U. S. Atomic Energy Commission.

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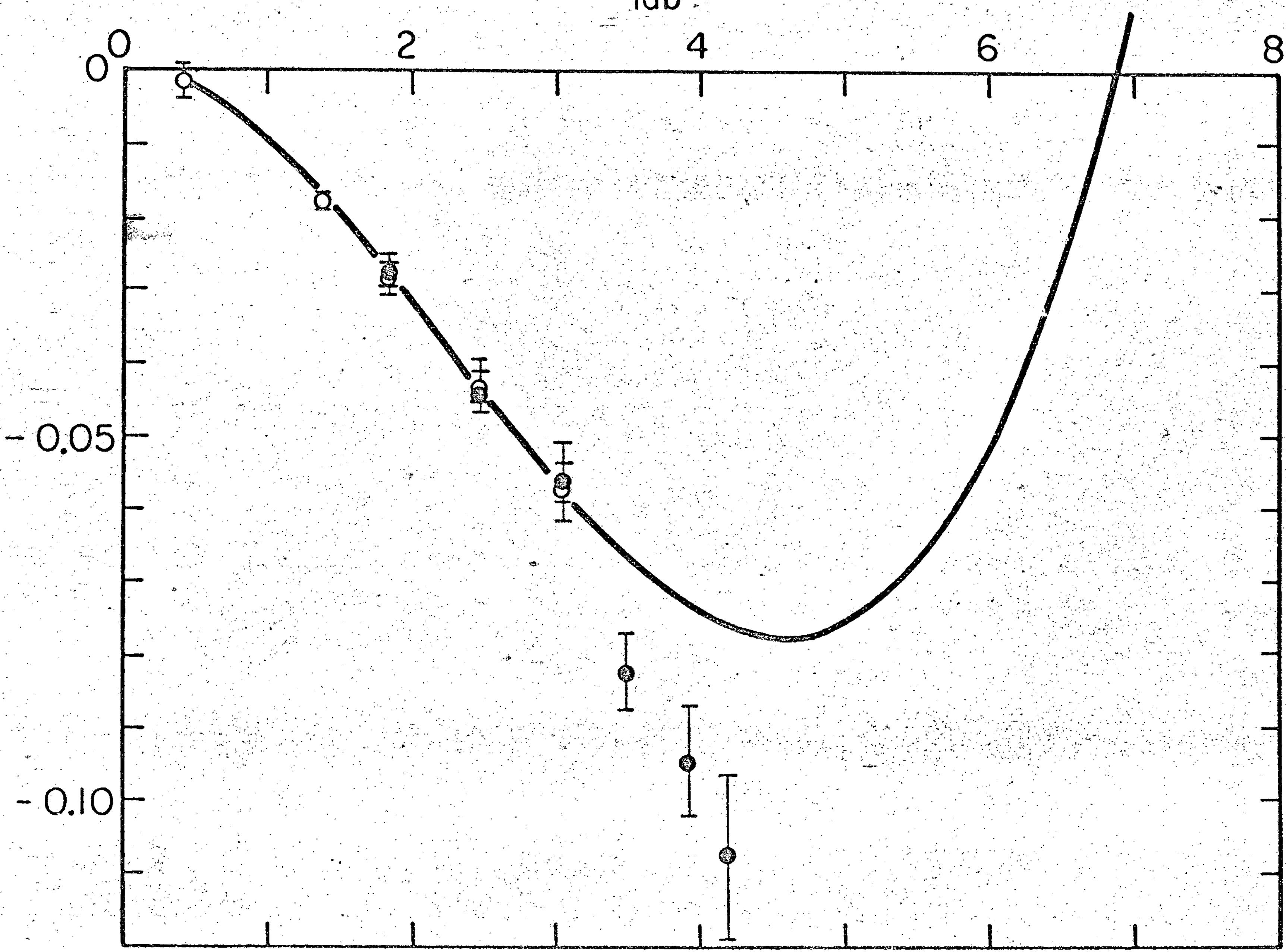

Fig. 1. Values of the function $\Delta K = K - (A + BE)$ obtained from the experimental phase shifts. The solid line results from the four parameter fit $K = A + BE + CE^2 + DE^3$, to the five recent measurements between 0.3825 MeV and 3.037 MeV. The open circles correspond to the data of Refs. 4 and 5. The solid dots are obtained from the WMF data.