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CALCULATION OF GAS VISCOSITY AS A FUNCTION OF TEMPERATURE

LeRoy A. Bromley

November 1949

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Calculation of Gas Viscosity as a Function of Temperature.

by

LeRoy A. Bromley

Abstract

Nomographs and a tabular function based on Hirschfelder, Bird and Spotz⁽²⁾ equations and tables for non-polar, smooth spherical molecules are presented, which allow easy, quick and accurate extrapolation of gas viscosity. The method is restricted to gases at moderate pressures where viscosity is essentially independent of pressure.

In all calculations involving gas flow and heat transfer, a knowledge of the viscosity of the gases as a function of temperature is essential. A completely rigorous calculation of this quantity for all gases is not possible at the present time. Recently, Hirschfelder, Bird and Spotz⁽²⁾ have published tables of collision integrals based on the refined kinetic theory of Chapman and Cowling⁽¹⁾ for the energy of interaction potential between two molecules,

$$E(r) = 4\epsilon \left[- (r_0/r)^6 + (r_0/r)^{12} \right]. \quad (1)$$

These tables allow quite accurate calculation of the viscosity for non-polar smooth spherical molecules. Although most real gases do not meet these conditions, nevertheless, Hirschfelder et al⁽³⁾ have shown that these tables may be used to calculate adequately the viscosity of some forty-five common gases.

The viscosity, μ , of a pure gas may be written:

$$\mu = \left(\frac{5}{16r_0^2} \right) \left(\frac{mkT}{\pi} \right)^{1/2} \left(\frac{V}{W^2(2)} \right). \quad (2)$$

The symbols, r_0 , m , k , and T represent respectively: the collision diameter for low velocity head-on collisions, the mass of an individual molecule, the Boltzmann constant, and the absolute temperature. V and $W^2(2)$ are functions of kT/ϵ and have been tabulated by Hirschfelder, Bird and Spotz⁽²⁾. The quantity, ϵ/k , may be regarded as a temperature and has

a characteristic value for each gas.

Equation (2) may be rewritten for the viscosity, μ , expressed in centipoises as follows:

$$\mu_{\text{cps}} = 0.0026693 \left(\frac{1}{r_0^2} \sqrt{\frac{ME}{k}} \right) f\left(\frac{kT}{\epsilon}\right) \quad (3)$$

Here, $\left(\frac{1}{r_0^2} \sqrt{\frac{ME}{k}} \right)$ is a constant, characteristic of the gas with r_0 in Å and ϵ/k in °K. and

$$f\left(\frac{kT}{\epsilon}\right) = \left(\frac{kT}{\epsilon}\right)^{1/2} \left(\frac{V}{W^2(2)}\right) \quad (4)$$

is a unique function of $\frac{kT}{\epsilon}$.

Table 1 lists the values of the two characteristic constants for forty-five common gases as determined from Hirschfelder et al⁽³⁾.

For other gases the following equations may be used:

$$\epsilon/k \approx 0.75 T_c \quad (5)$$

$$\epsilon/k \approx 1.39 T_b \quad (6)$$

$$r_0 \approx 0.833 (V_c)^{1/3} \quad (7)$$

T_c , T_b and V_c are respectively the critical temperature, the normal boiling point and the critical volume expressed in cubic centimeters. r_0 is expressed in Ångstroms.

Table 2 gives the value of $f\left(\frac{kT}{\epsilon}\right)$ as a function of $\frac{kT}{\epsilon}$. The absolute accuracy of these values is only one part in three hundred. However, the values are given to more significant figures to maintain internal consistency. If a value of the viscosity of the gas at one temperature is given, then we may calculate the value at a second temperature from the following equation:

$$\mu_2 = \mu_1 \frac{f\left(\frac{kT_2}{\epsilon}\right)}{f\left(\frac{kT_1}{\epsilon}\right)} \quad (8)$$

Hence, in this case it is only necessary to estimate ϵ/k from equations (5) or (6). An error of ten percent in ϵ/k will produce a maximum error of about 1 1/2 percent in μ_2 when μ_2 is twice μ_1 . Even in the case of steam for which the above equations should be at their worst, the calculated value of the viscosity is 0.01781 compared to

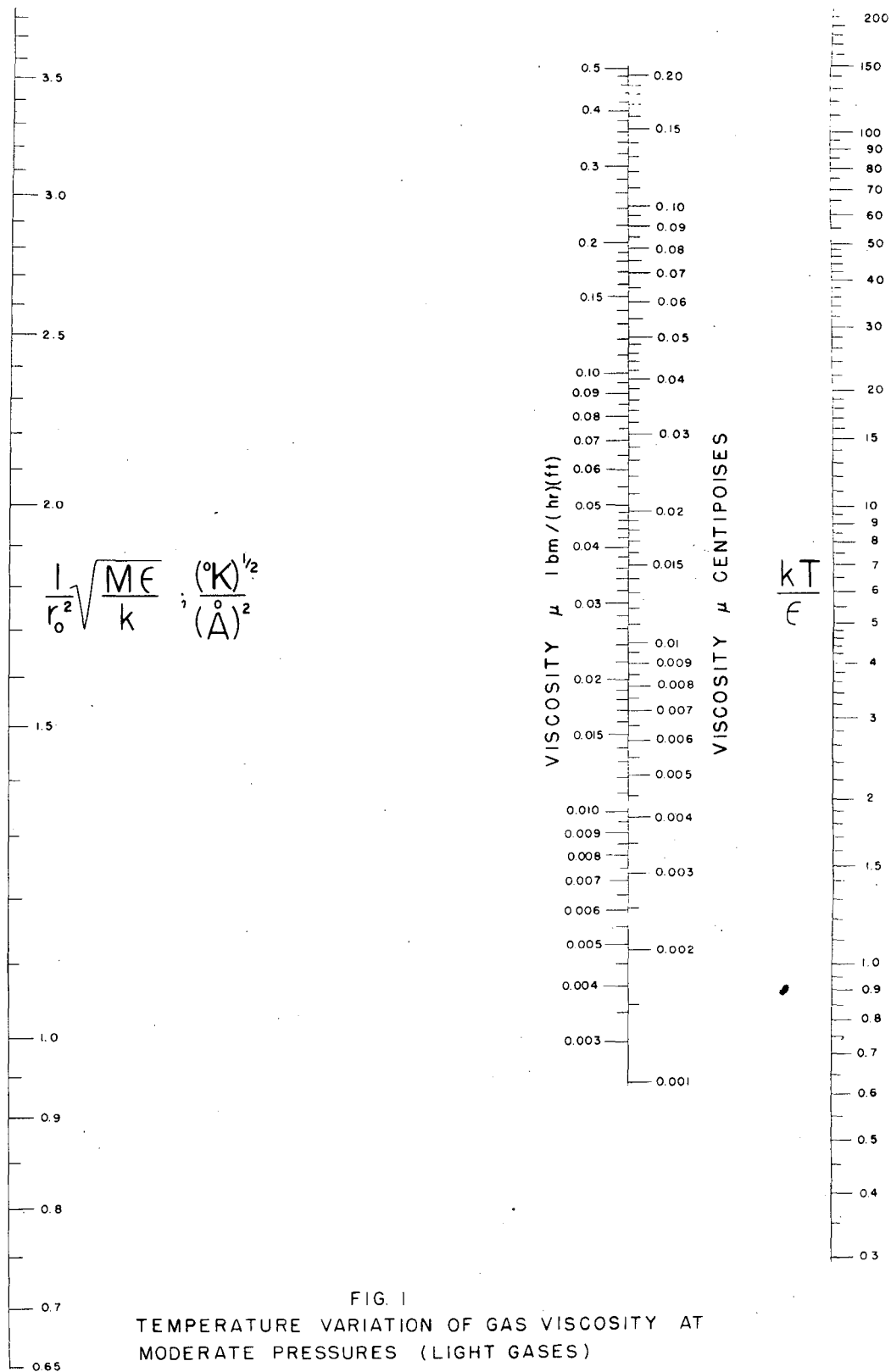


FIG. 1
 TEMPERATURE VARIATION OF GAS VISCOSITY AT
 MODERATE PRESSURES (LIGHT GASES)

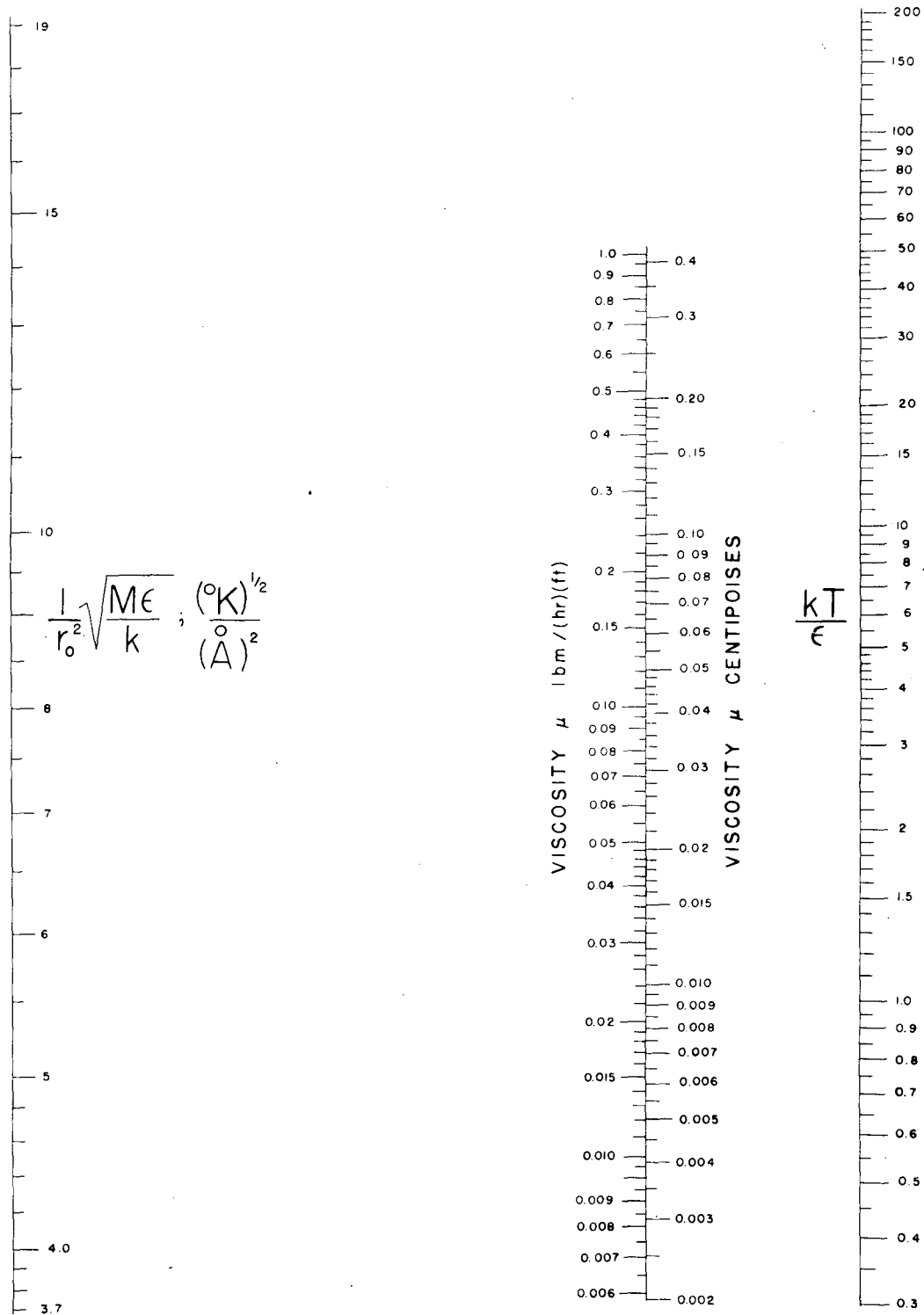


FIG. 2
 TEMPERATURE VARIATION OF GAS VISCOSITY AT MODERATE
 PRESSURES (HEAVY GASES)

the experimental value 0.0190 centipoises at 261.3°C. The experimental value of 0.00904 centipoises at 0°C and an ϵ/k of 519°K calculated from equation (6) were used. This represents an error of only 6.3 percent for a viscosity change of a factor of 2.1. The experimental and calculated quantities should be in much better agreement for nearly all other gases; and for most gases the agreement with this equation should be much better than with the simple exponential equation or the Sutherland type equation.

For rapid calculations nomographs, Figures 1 and 2, based on equation (3) are presented. The general procedure for the use of these nomographs is:

1. Read ϵ/k from Table 1 or estimate this quantity by means of equation (5) or (6).
2. Read $\frac{1}{r_0^2} \sqrt{\frac{ME}{k}}$ from Table 1 or look up a value of the viscosity at some known temperature.
3. Calculate the values of $\frac{kT}{\epsilon}$ and after having located the pivot point on the $\frac{1}{r_0^2} \sqrt{\frac{ME}{k}}$ scale read off the values of the viscosity by laying a straight edge between the pivot point and the respective values of $\frac{kT}{\epsilon}$.

In engineering calculations particularly heat transfer, the unit of viscosity, the pounds mass per hour foot is frequently used.

$$\mu \left(\frac{\text{lbm}}{\text{hr ft}} \right) = 0.0064573 \left(\frac{1}{r_0^2} \sqrt{\frac{ME}{k}} \right) f \left(\frac{kT}{\epsilon} \right) \quad (9)$$

where again r_0 is in Å and $\frac{\epsilon}{k}$ in °K.

In figures 1 and 2 the viscosity is given both in centipoises and pounds mass per hour foot.

At high pressures the method of Comings and Egly⁽⁴⁾ may be used to correct for the effect of pressure.

This work was performed under the auspices of the Atomic Energy Commission.

Table I

Constants for the Calculation of Gas Viscosity.

Name	Formula	ϵ/k °K	$\frac{1}{r_0^2} \sqrt{\frac{M\epsilon}{k}}$ °K ^{1/2} /Å
Acetylene	C ₂ H ₂	185	3.895
Air		97.0	4.054
Argon	A	124.0	6.024
Arsine	AsH ₃	281	8.979
Benzene	C ₆ H ₆	440	6.676
Bromine	Br ₂	400	16.98
i-butane	C ₄ H ₁₀	313	4.887
n-butane	C ₄ H ₁₀	410	6.183
Carbon Dioxide	CO ₂	190	5.726
Carbon Disulfide	CS ₂	488	9.784
Carbon Monoxide	CO	110.3	4.313
Carbon Tetrachloride	CCl ₄	327	6.485
Carbonyl Sulfide	COS	335	8.319
Chlorine	Cl ₂	357	9.3484
Chloroform	CHCl ₃	327	6.702
Cyanogen	C ₂ N ₂	339	6.922
Cyclohexane	C ₆ H ₁₂	324	4.447
Ethane	C ₂ H ₆	230	4.261
Ethyl Alcohol	C ₂ H ₅ OH	391	6.762
Ethylene	CH ₂ = CH ₂	205	4.234
Helium	He	6.03	0.6739
n-Hexane	C ₆ H ₁₄	413	5.401
Hydrochloric Acid	HCl	360	10.49
Hydrogen	H ₂	33.3	0.9301

Table I (continued)

Name	Formula	ϵ/k °K	$\frac{1}{r_0^2} \sqrt{\frac{M\epsilon}{k}}$ °K ^{1/2} /Å
Iodine	I ₂	550	15.05
Krypton	K _r	190	9.676
Mercuric Bromide	HgBr ₂	530	14.91
Mercuric Iodide	HgI ₂	698	17.80
Methane	CH ₄	136.5	3.105
Methyl Alcohol	CH ₃ OH	507	9.920
Methyl Chloride	CH ₃ Cl	855	18.24
Methylene Chloride	CH ₂ Cl ₂	406	8.199
Neon	Ne	35.7	3.423
Nitric Oxide	NO	119	4.963
Nitrogen	N ₂	91.46	3.736
Nitrous Oxide	N ₂ O	220	6.540
n-Nonane	C ₉ H ₂₀	240	2.458
n-Octane	C ₈ H ₁₈	320	3.444
Oxygen	O ₂	113.2	5.107
n-Pentane	C ₅ H ₁₂	345	4.741
Propane	C ₃ H ₈	254	4.132
Stannic Bromide	SnBr ₄	465	10.16
Stannic Chloride	SnCl ₄	1550	30.83
Sulfur Dioxide	SO ₂	252	6.906
Xenon	Xe	230	10.59

Table II

Temperature Function $f\left(\frac{KT}{\epsilon}\right)$ for Gas Viscosity

$\frac{KT}{\epsilon}$.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Average Difference
0.3	0.1969	0.2025	0.2081	0.2138	0.2195	0.2252	0.2309	0.2366	0.2424	0.2482	0.0057
0.4	0.2540	0.2598	0.2657	0.2716	0.2775	0.2834	0.2893	0.2953	0.3013	0.3073	0.0059
0.5	0.3134	0.3195	0.3256	0.3317	0.3378	0.3440	0.3502	0.3564	0.3626	0.3688	0.0061
0.6	0.3751	0.3814	0.3877	0.3940	0.4003	0.4066	0.4129	0.4192	0.4256	0.4320	0.0063
0.7	0.4384	0.4448	0.4512	0.4576	0.4640	0.4704	0.4768	0.4832	0.4897	0.4961	0.0064
0.8	0.5025	0.5089	0.5153	0.5218	0.5282	0.5346	0.5410	0.5474	0.5538	0.5602	0.0064
0.9	0.5666	0.5730	0.5794	0.5858	0.5922	0.5985	0.6049	0.6112	0.6176	0.6239	0.0064
1.0	0.6302	0.6365	0.6428	0.6491	0.6554	0.6616	0.6679	0.6741	0.6804	0.6866	0.0063
1.1	0.6928	0.6990	0.7052	0.7114	0.7176	0.7237	0.7299	0.7360	0.7422	0.7483	0.0062
1.2	0.7544	0.7605	0.7666	0.7727	0.7788	0.7849	0.7910	0.7970	0.8031	0.8091	0.0061
1.3	0.8151	0.8211	0.8270	0.8330	0.8390	0.8449	0.8508	0.8567	0.8626	0.8685	0.0059
1.4	0.8744	0.8803	0.8861	0.8920	0.8978	0.9036	0.9094	0.9152	0.9210	0.9268	0.0058
1.5	0.9325	0.9383	0.9440	0.9497	0.9554	0.9611	0.9668	0.9724	0.9781	0.9837	0.0057
1.6	0.9894	0.9950	1.0006	1.0062	1.0118	1.0174	1.0230	1.0286	1.0342	1.0397	0.0056
1.7	1.0453	1.0509	1.0564	1.0619	1.0674	1.0729	1.0783	1.0837	1.0892	1.0946	0.0055
1.8	1.0999	1.1052	1.1105	1.1158	1.1211	1.1264	1.1317	1.1370	1.1423	1.1476	0.0054
1.9	1.1529	1.1582	1.1634	1.1686	1.1738	1.1790	1.1842	1.1894	1.1945	1.1997	0.0052
2.0	1.2048	1.2099	1.2150	1.2201	1.2252	1.2303	1.2354	1.2405	1.2456	1.2507	0.0051
2.1	1.2558	1.2608	1.2658	1.2708	1.2758	1.2808	1.2858	1.2908	1.2958	1.3008	0.0050
2.2	1.3057	1.3106	1.3155	1.3204	1.3253	1.3302	1.3351	1.3400	1.3449	1.3498	0.0049
2.3	1.3547	1.3596	1.3644	1.3692	1.3740	1.3788	1.3836	1.3884	1.3932	1.3980	0.0048
2.4	1.4028	1.4076	1.4124	1.4172	1.4219	1.4266	1.4313	1.4360	1.4407	1.4454	0.0047
2.5	1.4501	1.4548	1.4594	1.4640	1.4686	1.4732	1.4778	1.4824	1.4870	1.4916	0.0046
2.6	1.4962	1.5008	1.5054	1.5100	1.5146	1.5192	1.5237	1.5282	1.5327	1.5372	0.0045
2.7	1.5417	1.5462	1.5507	1.5552	1.5597	1.5641	1.5685	1.5729	1.5773	1.5817	0.0044
2.8	1.5861	1.5905	1.5949	1.5993	1.6037	1.6081	1.6125	1.6169	1.6212	1.6255	0.0044
2.9	1.6298	1.6341	1.6384	1.6427	1.6470	1.6513	1.6556	1.6599	1.6642	1.6685	0.0043
3.0	1.6728	1.6771	1.6814	1.6857	1.6900	1.6943	1.6986	1.7028	1.7070	1.7112	0.0043
3.1	1.7154	1.7196	1.7238	1.7280	1.7322	1.7364	1.7406	1.7448	1.7490	1.7532	0.0042
3.2	1.7573	1.7614	1.7655	1.7696	1.7737	1.7778	1.7819	1.7860	1.7901	1.7942	0.0041
3.3	1.7983	1.8024	1.8065	1.8106	1.8147	1.8188	1.8228	1.8268	1.8308	1.8348	0.0040
3.4	1.8388	1.8429	1.8469	1.8509	1.8549	1.8589	1.8629	1.8669	1.8709	1.8749	0.0040
3.5	1.8789	1.8829	1.8869	1.8909	1.8949	1.8989	1.9029	1.9068	1.9108	1.9147	0.0040
3.6	1.9186	1.9225	1.9264	1.9303	1.9342	1.9381	1.9420	1.9459	1.9498	1.9537	0.0039
3.7	1.9576	1.9615	1.9654	1.9693	1.9732	1.9771	1.9810	1.9848	1.9886	1.9924	0.0039
3.8	1.9962	2.0001	2.0039	2.0077	2.0115	2.0153	2.0191	2.0229	2.0267	2.0305	0.0038

Table II (Continued)

$\frac{kT}{\epsilon}$.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	Average Difference
3.9	2.0343	2.0381	2.0419	2.0457	2.0495	2.0533	2.0571	2.0608	2.0645	2.0682	0.0038
4.0	2.0719	2.0757	2.0794	2.0831	2.0868	2.0905	2.0942	2.0979	2.1016	2.1053	0.0037
4.1	2.1090	2.1127	2.1164	2.1201	2.1238	2.1275	2.1312	2.1349	2.1385	2.1421	0.0037
4.2	2.1457	2.1494	2.1531	2.1567	2.1604	2.1640	2.1676	2.1712	2.1748	2.1784	0.0036
4.3	2.1820	2.1856	2.1892	2.1928	2.1964	2.2000	2.2036	2.2072	2.2108	2.2144	0.0036
4.4	2.2180	2.2216	2.2252	2.2288	2.2324	2.2360	2.2396	2.2431	2.2466	2.2501	0.0036
4.5	2.2536	2.2572	2.2607	2.2643	2.2678	2.2713	2.2748	2.2783	2.2818	2.2853	0.0035
4.6	2.2888	2.2923	2.2958	2.2993	2.3028	2.3063	2.3098	2.3133	2.3168	2.3203	0.0035
4.7	2.3237	2.3272	2.3307	2.3342	2.3376	2.3411	2.3446	2.3481	2.3515	2.3549	0.0035
4.8	2.3583	2.3618	2.3653	2.3687	2.3722	2.3756	2.3790	2.3824	2.3858	2.3892	0.0034
4.9	2.3926	2.3960	2.3994	2.4028	2.4062	2.4096	2.4130	2.4163	2.4197	2.4231	0.0034
5.0	2.4264	2.4298	2.4332	2.4365	2.4399	2.4432	2.4466	2.4499	2.4532	2.4565	0.0033

	.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
5.0	2.426	2.460	2.493	2.526	2.559	2.591	2.623	2.655	2.687	2.719	0.032
6.0	2.751	2.782	2.813	2.844	2.874	2.904	2.934	2.964	2.994	3.024	0.030
7.0	3.053	3.082	3.111	3.140	3.169	3.197	3.225	3.253	3.281	3.309	0.028
8.0	3.337	3.365	3.392	3.419	3.446	3.473	3.500	3.527	3.554	3.581	0.027
9.0	3.607	3.634	3.660	3.686	3.712	3.738	3.764	3.790	3.816	3.841	0.026
10.0	3.866	3.892	3.917	3.943	3.968	3.993	4.018	4.043	4.068	4.093	0.025

$$10 \frac{kT}{\epsilon} \quad 400 \quad f\left(\frac{kT}{\epsilon}\right) = 0.878 \left(\frac{kT}{\epsilon}\right)^{0.645}$$

For $\frac{kT}{\epsilon}$ between 40 and 400, replacing 0.878 by 0.8788 still using 0.6450 as the exponent, gives slightly better agreement with the published values. Tabular values of $f\left(\frac{kT}{\epsilon}\right)$ for $\frac{kT}{\epsilon}$ between 10 and 400 are as follows:

$\frac{kT}{\epsilon}$	10	20	30	40	50	60	70	80	90	100
$f \frac{kT}{\epsilon}$	3.866	6.063	7.880	9.488	10.958	12.324	13.615	14.839	16.010	17.137
$\frac{kT}{\epsilon}$	100	200	300	400						
$f \frac{kT}{\epsilon}$	17.137	26.80	34.81	41.90						

Nomenclature

\AA	Angstroms
$E(r)$	Potential energy between molecules as a function of separation r (see equation (1)).
$f(\frac{kT}{\epsilon})$	Temperature function of viscosity (see equation (4)).
k	Boltzmann Constant.
M	Molecular weight of gas.
m	Mass of an individual molecule.
r	separation between molecules.
r_0	Collision diameter for low velocity head-on collisions.
T	Temperature $^{\circ}\text{K}$.
V	Function of $\frac{kT}{\epsilon}$ from Hirschfelder et al.
V_c	Critical volume, cubic cm.
$W^2(2)$	Function of $\frac{kT}{\epsilon}$ from Hirschfelder et al.
ϵ	Energy difference between the separated molecules and the molecules in the configuration for which they have the maximum energy of attraction.
$\frac{\epsilon}{k}$	An effective temperature (see Table 1 and equations (5) and (6)).
μ	Viscosity.

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