Title
STIFFNESS OF FNAL-TYPE STRUCTURAL RINGS

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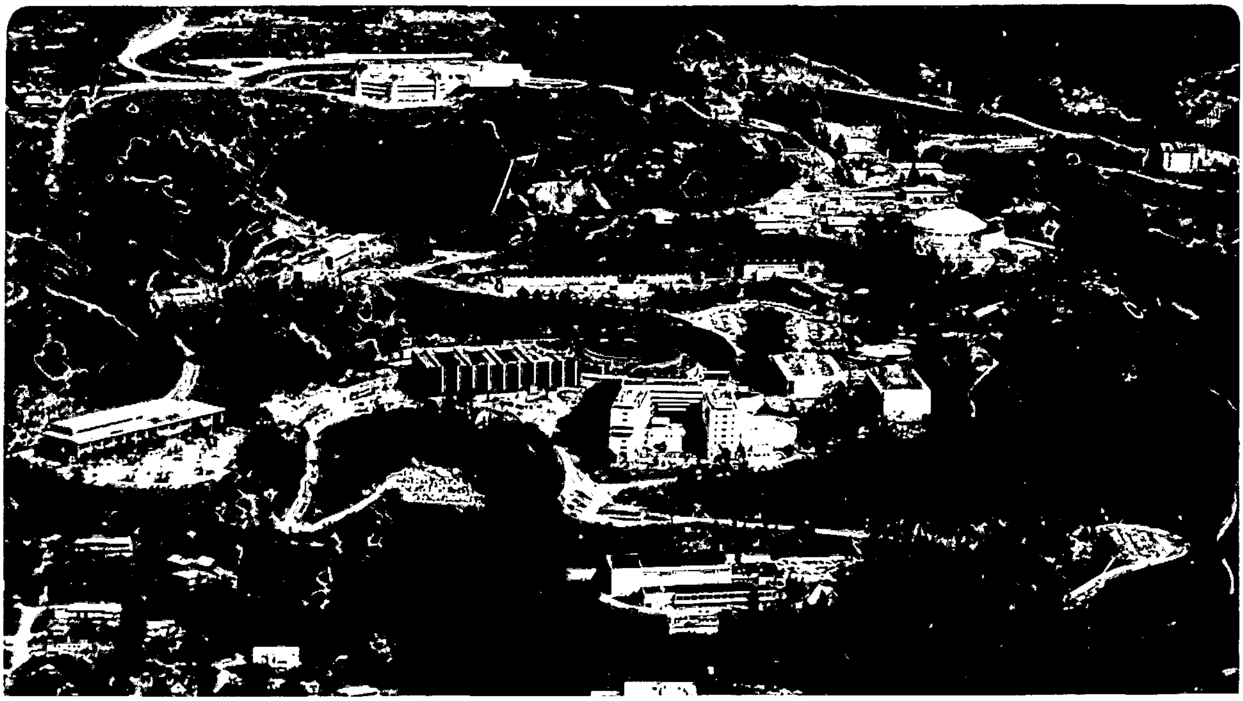

DISCLAIMER

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PROGRAM — PROJECT — JOB
HIGH-FIELD MAGNET DEVELOPMENT

ANALYSIS

TITLE
STIFFNESS OF FNAL-TYPE STRUCTURAL RINGS

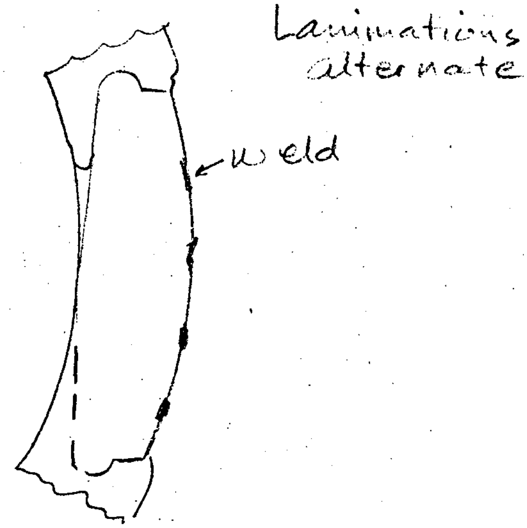
Inasmuch as we are planning to use FNAL type structural rings for our warm-iron alternative ISAbelle dipole magnet, it seems prudent to find out how stiff their weird joints are, as that effects both the stiffness and stresses in the rings.

We took a 1-inch stack of rings, cut away one side, applied a load as shown, and measured the deflections A and B.

From these measurements, we infer that the joint is 0.26 to 0.35 times as stiff as solid metal. I wouldn't have guessed it to be that low, so I am somewhat sceptical of the results.

Thanks to Al Borden for setting things up and running the test.

Joint



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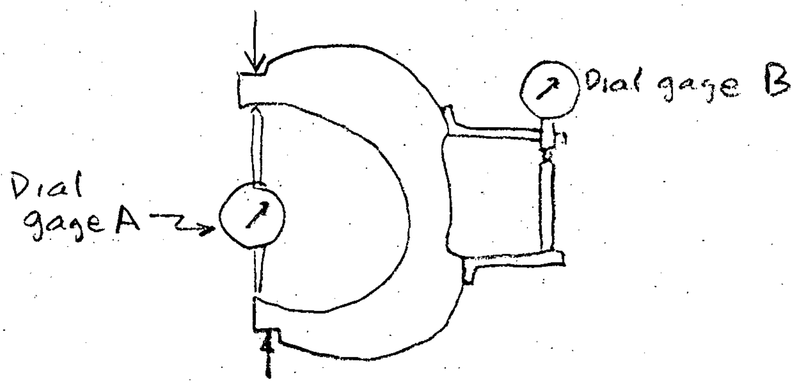
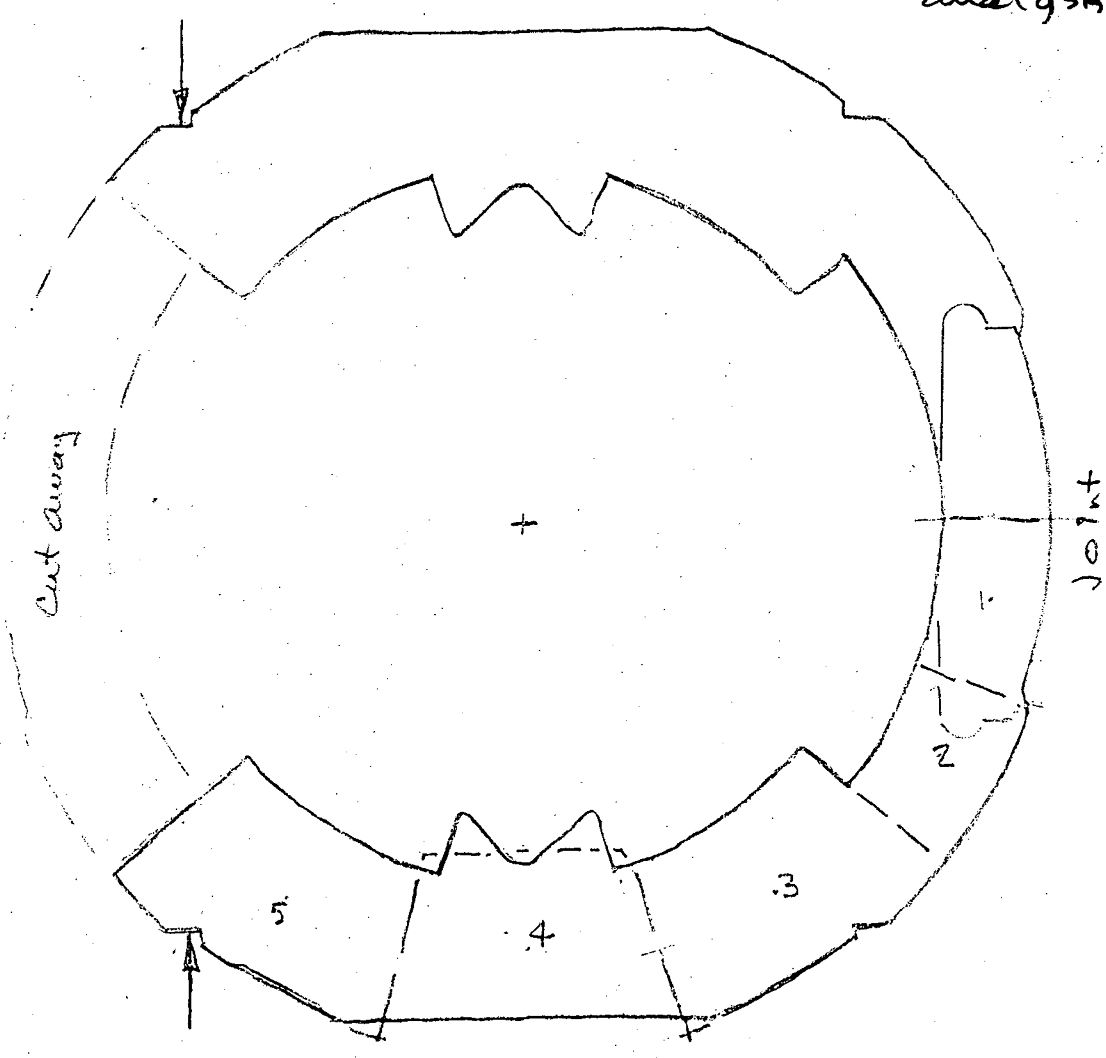
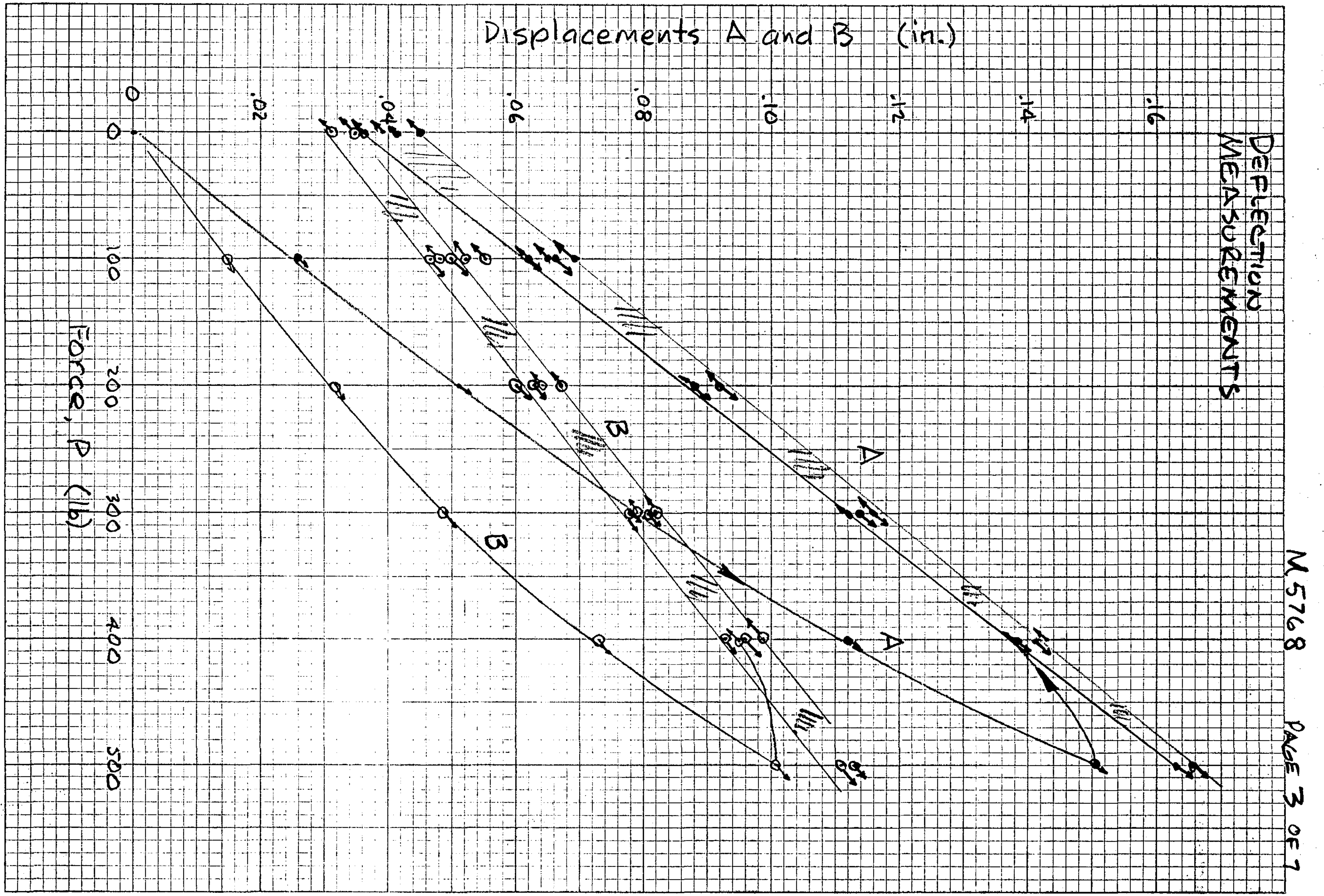
Set-Up



Ring Cross Section showing elements used for analysis




Displacements A and B (in.)
0
.02
.04
.06
.08
.10
.12
.14
.16
Force, P (lb)
0
100
200
300
400
500
A
A
B
B
DEFLECTION
MEASUREMENTS
M5768
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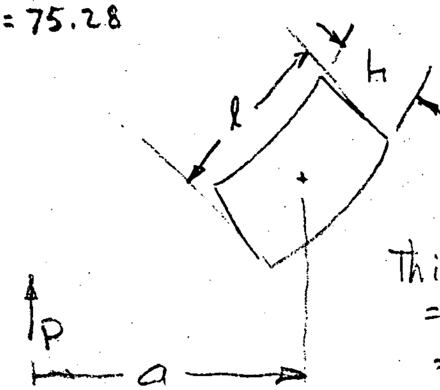
Dimensional Data

Element No	Dist from load, a	Length ℓ	Thickness h	$\frac{a^2\ell}{h^3}$	$\frac{\ell}{h}$
1	4.18	0.86	.572	80.29	1.50
2	3.91	0.83	.632	50.27	1.31
3	3.06	1.33	.885	17.97	1.50
4	1.76	1.30	.850	6.56	1.53
5	0.50	1.33	.885	.48	1.50

$$\sum_{i=1,2} \ell/h = 2.81$$

$$\sum_{i=3,5} \ell/h = 4.53$$

$$\sum_{i=2,5} a^2\ell/h^3 = 75.28$$



Thickness = b
= 16 × .061
= 0.976

Joint Efficiency Inferred From Deflection A

We use the deflection A as a measure of the angular rotation of the joint. But not all of the deflection is a result of the joint rotation; part of it results from shear and extension* of all parts of the ring, and from bending of parts other than the joint. So we correct for these other deflections

Displacement A resulting from extensions*

This is primarily due to the extension* of elements 1 and 2:

$$A = 2\sum_{i=1,2} \frac{P\ell}{bhE} = \frac{2P}{bE}\sum_{i=1,2}\frac{\ell}{h} = \frac{2P}{bE}\times 2.81 = 5.62\,\frac{P}{bE}$$

Displacement A resulting from shears

This is primarily due to the shear displacements of elements 3, 4, and 5:

$$A = 2\sum_{i=3,5}\frac{P\ell}{bhG} = 2\frac{P}{bG}\sum_{i=3,5}\frac{\ell}{h}$$

$$G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+.3)} = \frac{E}{2.6}\text{, so}$$

$$A = 2\times 2.6\,\frac{P}{bE}\sum_{i=3,5}\frac{\ell}{h} = 2\times 2.6\,\frac{P}{bE}\times 4.53 = 23.6\,\frac{P}{bE}$$

Displacement A resulting from rotations.

$$A = 2\sum\left(\frac{M\ell}{EI}\right)a_i \qquad I = \frac{bh^3}{12}, \quad M = Pa$$

$$A = 24\,\frac{P}{Eb}\sum_{i=2,5}\frac{\ell a^2}{h^3} = 24\,\frac{P}{Eb}\times 75.28 = 1806\,\frac{P}{bE}$$

The total of these gives

$$A/P = \frac{5.62 + 23.6 + 1806}{bE}$$

$$= \frac{1836}{.976 \times 30 \times 10^6} = 62.7 \times 10^{-6} \text{ in/lb}$$

The measured value is

$$A/P = 250 \times 10^{-6}$$

so the part resulting from joint rotation is

$$(A/P)_m = (250 - 62.7) \times 10^{-6} = 187 \times 10^{-6} \text{ in/lb}$$

measured

For a solid joint, the deflection A is

$$A = 2\left(\frac{Ml}{EI}a\right)_1 , \quad M = Pa_1 , \quad I_1 = \frac{bh_1^3}{12} , \text{ so}$$

$$(A/P)_t = 24 \frac{1}{Eb}\left(\frac{la^2}{h^3}\right)_1$$

"theoretical"

$$= \frac{24}{30 \times 10^6 \times .976} 80.29 = 65.81 \text{ in/lb}$$

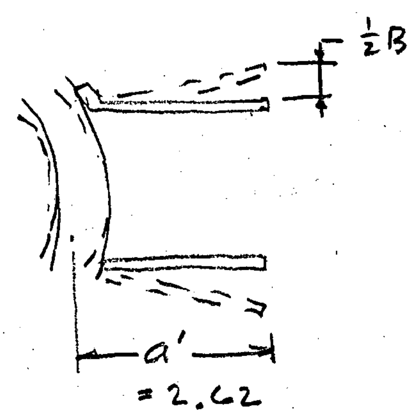
$$\text{Define } K = \frac{\text{stiffness, P/A, of actual joint}}{\text{" " " solid "}}$$

$$= \frac{(A/P)_t}{(A/P)_m} = \frac{65.81 \times 10^{-6}}{187 \times 10^{-6}} = \underline{\underline{0.35}}$$

Joint Efficiency Inferred From Deflection B

Measured displacement/load, $B/P = 160 \times 10^{-6}$ in/lb
Disp./load due to element extension (see previous calculation)

$$B/P = \frac{5.62}{bE} = \frac{5.62}{.976 \times 30 \times 10^{6}} = 0.2 \times 10^{-6},$$ ignore.



For a solid joint, the displacement B is

$$B/P = \frac{24}{Eb}\left(\frac{laa'}{h^3}\right)$$ (see previous calcs)

$$= \frac{24}{30 \times 10^{6} \times .976} \times \frac{.86 \times 4.18 \times 2.62}{.572^3} = 41.25 \times 10^{-6} \text{ in/lb}$$

The joint efficiency, then, is

$$K = \frac{41.25 \times 10^{-6}}{160 \times 10^{-6}} = \underline{\underline{0.26}}$$

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