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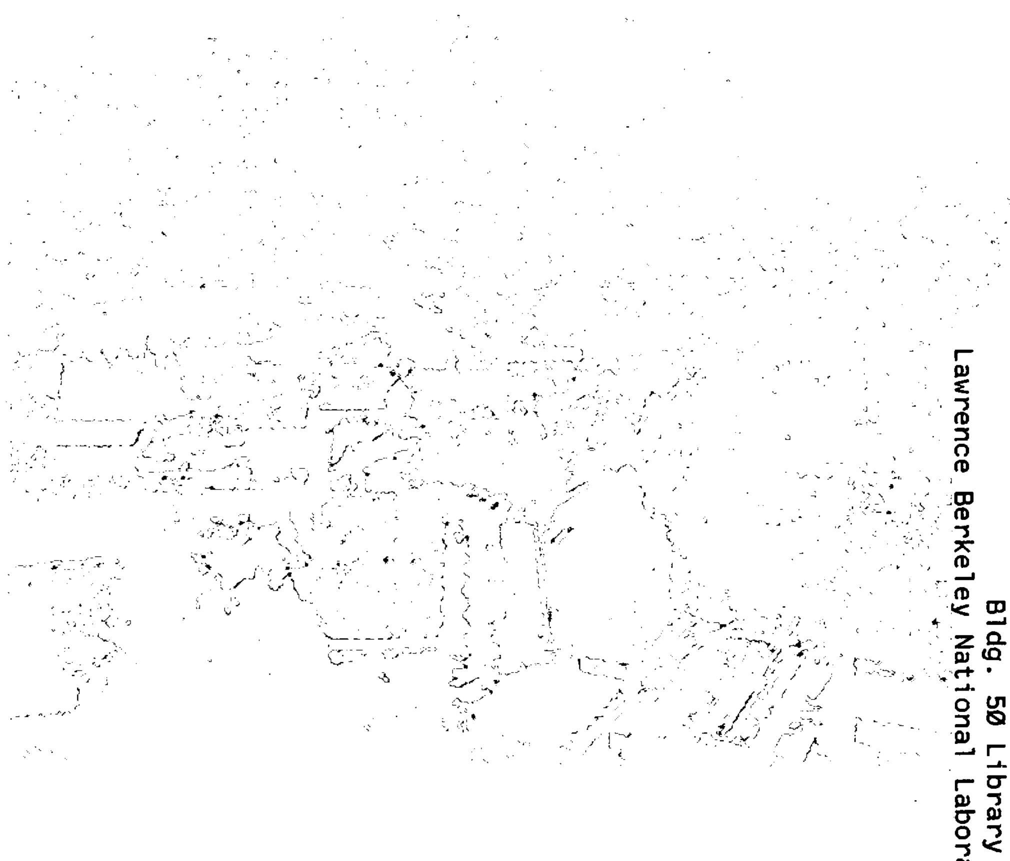

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We propose descriptions of interacting $(1,0)$ supersymmetric theories without gravity in six dimensions in the infinite momentum frame. They are based on the large N limit of quantum mechanics or $1+1$ dimensional field theories with $SO(N)$ gauge group and four supercharges. We argue that this formulation allows for a concrete description of the chirality-changing phase transitions which connect $(1,0)$ theories with different numbers of tensor multiplets.

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1. Introduction

In the past few years, large classes of interacting superconformal field theories with between 4 and 16 supercharges have been discovered in three, four, five and six space-time dimensions. Most of these theories are not described by weakly-coupled Lagrangians, and there is not even a known Lagrangian which flows to them in many cases. Therefore, we require a different approach to analyze them. This is an interesting abstract problem in itself, and it is rendered more urgent by the many applications these theories have in M theory. In the matrix formulation of M theory [1] these theories are relevant for compactifications on four dimensional spaces [2-4]. These theories also arise in the study of certain black holes in string theory [5], and it has been suggested that an improved understanding of some of these theories may lead to progress in solving large N nonsupersymmetric QCD [6]. The fixed points with 8 or fewer supercharges are important in the problem of unifying M-theory vacua, since they are crucial in connecting vacua with different spectra of chiral fields [7-10].

Fixed point theories with $(2,0)$ supersymmetry in six dimensions [11,12] were recently studied in a matrix model formulation in [13]. The purpose of this paper is to move on to theories with $(1,0)$ supersymmetry in six dimensions. We will formulate a matrix description of these theories and follow the chirality-changing phase transitions of [7,8] in this language. We begin in section 2 with the definition of the theory. In section 3 we analyze deformations away from the fixed point, where we can see the low-energy spectrum in the spacetime theory, and observe the chirality-changing phase transition. We discuss various interesting issues, which we are not able to fully resolve, concerning the matrix description of these deformations. Section 4 contains the $1+1$ dimensional generalization of the quantum mechanical theory, which corresponds to a six dimensional "little string" theory in spacetime.

As this paper was being completed, similar results were independently obtained in [14].

2. The Quantum Mechanical Definition of the Fixed Point Theory

We will study here the simplest example of a fixed point with $(1,0)$ supersymmetry, which is the low energy theory of a small instanton in the $E_8 \times E_8$ heterotic string. In M theory this is described by a fivebrane at the end of the world ninebrane [7,8]. This theory has a Coulomb branch of the form $\mathbb{R}/\mathbb{Z}_2$ (times a decoupled $\mathbb{R}^4$ factor), on which the low energy spectrum consists of a tensor multiplet and a hypermultiplet. The scalars in these multiplets label the transverse position of the fivebrane in M theory. The scalar in the tensor multiplet parametrizes the distance between the fivebrane and the ninebrane, and when its expectation value vanishes the low-energy theory is superconformal. Another branch coming out of the superconformal point is the Higgs branch, corresponding to enlarging the size of the instanton. On this branch, the low-energy theory has 30 hypermultiplets, which are in the $\frac{1}{2}(\mathbf{56})+\mathbf{1}+\mathbf{1}$ representation of the E_7 symmetry left unbroken by the instanton. We would like to propose an infinite momentum frame quantum mechanical description of this theory, which reproduces this moduli space and low-energy spectrum. In particular, we will consider in this framework the chirality-changing phase transitions of [7,8]. There is an obvious generalization of this theory to k coincident fivebranes (or small instantons), which will also be discussed.

The arguments used in [13] for the construction of $(2,0)$ theories in six dimensions can also be used for the construction of theories with $(1,0)$ supersymmetry. To get a light-cone description of this system, we start with M theory on $S^1/\mathbb{Z}_2$ [15] with k fivebranes, and compactify a longitudinal direction (of the fivebranes) on a circle of radius R. The theory then becomes the type IIA string theory on $S^1/\mathbb{Z}_2$ (a.k.a. type I'), with 8 D8-branes at each orientifold fixed point [16] and k D4-branes.

In the next subsection we will discuss the full matrix description of this theory. We will introduce the degrees of freedom of the matrix description of this system, their interactions, and their representations under the various symmetries. In §2.2 we will consider the limit $M_p \to \infty$ in spacetime, and determine what remains of the degrees of freedom in the matrix description in this limit. This surviving quantum mechanics is our formulation of the $(1,0)$ SCFT.

2.1. Heterotic Fivebranes in Matrix Theory

The above type I' system is equivalent to the $E_8 \times E_8$ heterotic theory on a circle, with a Wilson line A_E breaking the gauge symmetry to $SO(16)\times SO(16)$ (and k fivebranes wrapped around the circle). Let the radius of this circle in the $E_8 \times E_8$ theory be denoted r_E. This vacuum is related by T-duality [17] to the $SO(32)$ heterotic string on a circle of radius $r_S = 1/4r_E$, with a Wilson line A_S breaking the gauge group to $SO(16) \times SO(16)$. The winding number n_S of the $SO(32)$ theory maps to the D0-brane number N in the type I$'$ description. The $SO(17,1)$ T-duality transformation maps this to a linear combination of momentum, winding, and $E_8 \times E_8$ lattice quantum numbers in the $E_8 \times E_8$ theory:

$$N = n_S \leftrightarrow 2m_E - A_E^2 n_E - 2A_E \cdot P_E, \tag{2.1}$$

where m_E, n_E and P_E are the momentum, winding, and $E_8 \times E_8$ lattice quantum numbers in the $E_8 \times E_8$ theory.

For the infinite momentum frame description we are interested in states with large momentum m_E around the circle in the $E_8 \times E_8$ theory. From (2.1) we see that this corresponds to large D0-brane number $N = n_S$, though the two quantum numbers are not exactly the same.

Let us now describe the quantum mechanics of the D0-branes in this theory, near one of the orientifolds. This quantum mechanics without the D4-branes was studied in [18-20]. It is an $SO(N)$ gauge theory with 8 supersymmetries, containing 16 fermions in the fundamental representation which arise from the 0-8 strings. Adding the D4-branes (longitudinal fivebranes [21]) is done simply by adding the 0-4 strings. These are k "hypermultiplets" in the fundamental representation, and there is an $Sp(k)(\equiv USp(2k))$ global symmetry corresponding to these. For $N = 1$ this theory was described in [22] (see also [23]). Altogether we are left with four linearly realized supersymmetries, which is the correct number for a lightcone description of a spacetime theory with 8 supersymmetries.

The global symmetry of the quantum mechanics is

$$SO(4)_{\parallel} \times SO(4)_{\perp} \times SO(16) \times Sp(k), \tag{2.2}$$

where $SO(4)_{\perp}$ corresponds to the rotation symmetry transverse to the 4-branes (but inside the 8-branes), $SO(4)_{\parallel}$ corresponds to the rotations inside the 4-branes, $SO(16)$ is the gauge symmetry on the 8-branes and $Sp(k)$ is the gauge symmetry of the 4-branes. The 4 supersymmetry generators transform in the $\{(\mathbf{2},\mathbf{1})\ (\mathbf{2},\mathbf{1})\ \mathbf{1}\ \mathbf{1}\}$ representation of this group, so that two of its $SU(2)$ factors are in fact R-symmetries. The representations of the fields under the $SO(N)$ gauge symmetry and the global symmetries are given in the following table :

		$SO(N)$	$SO(4)_{\parallel}$	$SO(4)_{\perp}$	$SO(16)$	$Sp(k)$	
$0-0$ states :	A_0, X_9	$\mathbf{N(N-1)/2}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}$	$\mathbf{1}$	
	α_L	$\mathbf{N(N-1)/2}$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{2},\mathbf{1})$	$\mathbf{1}$	$\mathbf{1}$	
	β_L	$\mathbf{N(N-1)/2}$	$(\mathbf{1},\mathbf{2})$	$(\mathbf{1},\mathbf{2})$	$\mathbf{1}$	$\mathbf{1}$	
	$X_{\parallel}$	$\mathbf{N(N+1)/2}$	$(\mathbf{2},\mathbf{2})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}$	$\mathbf{1}$	
	ρ_R	$\mathbf{N(N+1)/2}$	$(\mathbf{1},\mathbf{2})$	$(\mathbf{2},\mathbf{1})$	$\mathbf{1}$	$\mathbf{1}$	(2.3)
	$X_{\perp}$	$\mathbf{N(N+1)/2}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{2})$	$\mathbf{1}$	$\mathbf{1}$	
	σ_R	$\mathbf{N(N+1)/2}$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$\mathbf{1}$	$\mathbf{1}$	
$0-4$ states :	v	$\mathbf{N}$	$(\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}$	$\mathbf{2k}$	
	ψ_R	$\mathbf{N}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{1})$	$\mathbf{1}$	$\mathbf{2k}$	
	ψ_L	$\mathbf{N}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$\mathbf{1}$	$\mathbf{2k}$	
$0-8$ states :	χ_L	$\mathbf{N}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{16}$	$\mathbf{1}$.	

Here $X_{\|}$ gives the positions of the zero branes along the fourbranes, $X_\perp$ gives the positions perpendicular to the fourbranes, and X_9 gives the positions in the $S^1/\mathbb{Z}_2$ direction. In addition we have scalars v in the fundamental representation. The fermions (which are all real) are denoted with subscripts R or L, according to their chirality in the corresponding $1+1$ dimensional theory of 1-branes, fivebranes and ninebranes, which is related to the quantum mechanics we describe by a T duality in the x_9 direction. That theory has $(0,4)$ supersymmetry, and we will discuss it further in section 4. Supersymmetry pairs the right moving fermions with the bosons appearing directly above them in the table, and α_L with the gauge field.

The moduli of the spacetime theory are parameters in the quantum mechanics. These moduli are the scalars in the theory of the 4-branes and the 8-branes, which are in the following representations :

		$SO(N)$	$SO(4)_{\|}$	$SO(4)_\perp$	$SO(16)$	$Sp(k)$	
$4-4$ states :	$X_\perp^{(4)}$	$\mathbf{1}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{2})$	$\mathbf{1}$	$\mathbf{2k(2k-1)/2}$	
	$X_9^{(4)}$	$\mathbf{1}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{1}$	$\mathbf{2k(2k+1)/2}$	(2.4)
$4-8$ states :	H	$\mathbf{1}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{2},\mathbf{1})$	$\mathbf{16}$	$\mathbf{2k}$	
$8-8$ states :	$X_9^{(8)}$	$\mathbf{1}$	$(\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1})$	$\mathbf{120}$	$\mathbf{1}$.	

Most of the interactions of this system may be easily derived from those of the 0-brane/4-brane system, which is the dimensional reduction of a 6D $(1,0)$ theory, and from those of the 0-brane/8-brane system [18-20]. Among the terms appearing in the Lagrangian are terms of the following (schematic) form[1] :

$$\begin{aligned}&\chi_L(X_9-X_9^{(8)})\chi_L+\psi_L(X_9-X_9^{(4)})\psi_L+\psi_R(X_9-X_9^{(4)})\psi_R+v^2(X_9-X_9^{(4)})^2+\\&\psi_L(X_\perp-X_\perp^{(4)})\psi_R+v^2(X_\perp-X_\perp^{(4)})^2+([X_{\|},X_{\|}]+v^2)^2+[X_\perp,X_\perp]^2+v\sigma_R\psi_L+v\alpha_L\psi_R+\\&\alpha_L[X_{\|},\rho_R]+\beta_L[X_{\|},\sigma_R]+\alpha_L[X_\perp,\sigma_R]+\beta_L[X_\perp,\rho_R]+[X_\perp,X_{\|}]^2+(Hv)^2+H\psi_R\chi_L.\end{aligned} \tag{2.5}$$

The singlet components of the fermions ρ_R are completely decoupled, and their shifts generate four non-linearly realized supersymmetries, completing the 8 spacetime supersymmetries. Quantization of the zero modes of these fields will multiply the representation of each state we get by $\{(\mathbf{1},\mathbf{2})\ (\mathbf{1},\mathbf{1})\}+\{(\mathbf{1},\mathbf{1})\ (\mathbf{2},\mathbf{1})\}$ (which is the content of a half-hypermultiplet in spacetime).

In the quantum mechanics describing the superconformal point in space-time, all the parameters (2.4) vanish. Then, the quantum mechanical theory has a Coulomb branch

[1] This formula does not include the powers of the gauge coupling g_{QM}, which may be put in on dimensional grounds.

(in the usual sense of a Born-Oppenheimer approximation) in which $X_\perp \neq 0$ and $v = 0$. In the matrix model interpretation, graviton states live here, as well as E_8 gauge bosons localized near $X_9 = 0$ [19,20]. In addition, there is a Higgs branch in which $X_\perp = 0$. It is parametrized by expectation values of $X_{||}$ and v, and has (real) dimension

$$\dim \mathcal{M}_H = 4Nk + 4\frac{N(N+1)}{2} - 4\frac{N(N-1)}{2} = 4N(k+1). \tag{2.6}$$

2.2. Decoupling Gravity: Formulation of the $(1,0)$ *SCFT*

As in the case of the $(2,0)$ theories discussed in [13], the gauge coupling in the quantum mechanics is related to the eleven dimensional Planck mass M_p and the compactification radius R by $g_{QM}^2 \sim M_p^6 R^3$. Thus, taking $M_p \to \infty$ corresponds to the $g_{QM} \to \infty$ (or IR) limit of the quantum mechanics, where we expect the conformal theory in spacetime to decouple from gravity, as well as from the E_8 gauge bosons whose gauge coupling goes to zero in this limit.

As in [13], the presence of the Higgs branch (with no apparent spacetime interpretation) is what signals the presence of the nontrivial conformal theory in spacetime. In the limit $g_{QM} \to \infty$, some degrees of freedom become infinitely massive on the interior of the Higgs branch. In other words, the Coulomb and Higgs branches of the quantum mechanics decouple. Integrating out the 0-4 states leads to an infinite tube on the Coulomb branch, so the origin is infinitely far away on that branch (where the gravitons and the E_8 gauge bosons live). The degrees of freedom from (2.3) that are lifted in the interior of the Higgs branch ($v \neq 0$, $X_{||} \neq 0$) are :

$$\begin{gathered} A_0, X_9, \alpha_L, 2N(N-1) \text{ of the fields } v \text{ and } X_{||} \text{ (and their superpartners } \psi_R, \rho_R), \\ X_\perp, \sigma_R, 2N(N+1) \text{ of the fields } \beta_L \text{ and } \psi_L. \end{gathered} \tag{2.7}$$

We are left with :

$$\begin{gathered} 4N(k+1) \text{ of the fields } v \text{ and } X_{||} \text{(and their superpartners } \psi_R, \rho_R), \\ \chi_L, 4N(k-1) \text{ of the fields } \beta_L \text{ and } \psi_L. \end{gathered} \tag{2.8}$$

It is the $g_{QM} \to \infty$ theory of these degrees of freedom that constitutes the matrix formulation of the spacetime SCFT. Note that, unlike in [13], even for $k = 1$ there is a non-trivial Higgs branch here. This corresponds to the non-trivial SCFT in spacetime which exists even in this case.

The classical Higgs branch of the quantum mechanics is the moduli space of $Sp(k)$ instantons [24]. There is no non-renormalization theorem for the moduli space in this case.

In the quantum mechanics there could be loop corrections (say, involving the χ_Ls) to the metric of this space. However, there is some fixed point governing the Higgs branch in the infrared ($g_{QM} \to \infty$) limit. We conjecture that, for $N \to \infty$, it correctly describes the $(1,0)$ superconformal theories in the infinite momentum frame. In fact, it follows from the results of [25] that the corresponding $1+1$ dimensional $(0,4)$ sigma model, with target space $\mathcal{M}_H$ and with the additional left-moving fermion multiplets, is finite. This is not to say that the infrared physics will necessarily be transparent in terms of the degrees of freedom (2.8). The IR theory may have complicated interactions, arising for instance, from the gauge constraint (the A_0 equation of motion) in the original gauge theory we start from [20].

Note that since N here is not the same as the spacetime momentum m_E, the finite N theory does not directly give us a discrete light-cone description of the $(1,0)$ superconformal theories, as suggested in [26]. Presumably, as in [27], the finite N theory is a discrete light-cone quantization of these theories compactified on a light-like circle with a Wilson loop breaking the E_8 symmetry to $SO(16)$. In the quantum mechanics only an $SO(16)$ subgroup of the E_8 global symmetry of the $(1,0)$ superconformal theory in spacetime is visible. As in [19], the full E_8 representations get filled out as the type I$'$ coupling goes to infinity and states of energy $1/\lambda_{I'}$ come down.

3. Low Energy States Away from the Fixed Point

As discussed in the introduction, the six dimensional $(1,0)$ theories play a very interesting role in giving chirality-changing phase transitions. Within Lagrangian field theory there is no way to lift chiral matter, so it is interesting to consider how this occurs in our formulation. Let us perturb the spacetime theory away from the conformal point, going into its Higgs or Coulomb branches. Along these branches the low energy theory in spacetime is free, and we should be able to find the correct low energy spectrum in our quantum mechanical description. We will see how the quantum numbers for these states arise in this section.

It is not clear to us that the deformed theory can be described using only the degrees of freedom (2.8) that were involved in formulating the critical theory. In principle, there are two ways to analyze the theory away from the conformal point. We could either perform the perturbation in the full gauge theory and then take the $g_{QM} \to \infty$ limit (while keeping the perturbation parameters finite), or work directly in the theory which describes the Higgs branch of the quantum mechanics in the $g_{QM} \to \infty$ limit, and analyze the perturbations in that model. As we will discuss in some detail below, we have difficulty finding the correct spacetime spectrum using the second approach.

We interpret this difficulty as resulting from the fact that this approach does not include information about states localized at the singularities at the boundaries of the Higgs branch. These quantum-mechanical variables, though decoupled from the interior of the Higgs branch at the conformal point, may still be important after we deform the theory away from the conformal point. Because of the tube metric, the singularities of the Higgs branch still decouple from the graviton/gauge boson states which live on the Coulomb branch. After turning on the deformations (2.4), the quantum mechanical Higgs branch is (generically) lifted, and the wave functions of all states are concentrated near the origin of the Higgs branch. Thus, it is not a surprise that the degrees of freedom related to the singularities in the Higgs branch are required to describe the states after the deformation. It would be interesting to understand better the role of the singularities at the boundaries of the Higgs branch, both in this theory and in the $(2,0)$ theories described in [13].

3.1. The Coulomb Branch

First, let us discuss the Coulomb branch of the spacetime theory (this is not to be confused with the Coulomb branch of the quantum mechanics). On this branch the fivebranes are (generically) all separated from each other and from the ninebrane. There is a tensor multiplet and a hypermultiplet (forming a tensor multiplet of $(2,0)$ supersymmetry) living on each fivebrane. For simplicity, let us focus on the case $k=1$ (the other cases generically give k copies of this). Moving into the Coulomb branch away from the critical point is done by turning on $X_9^{(4)}$, and we expect to find the fivebrane states localized in the moduli space near $X_9 = X_9^{(4)}$ (specifically, when half of the eigenvalues of X_9 are equal to one of the eigenvalues of $X_9^{(4)}$). In this region the 0-8 strings are all massive and the $SO(N)$ gauge theory is broken by the VEV of X_9 to $U(N/2)$ (here we take N to be even). In the IR, the theory reduces exactly to the quantum mechanics of D0-branes and D4-branes (with 8 supersymmetries) discussed in [13], which is a supersymmetric quantum mechanics on the moduli space of $U(k)$ instantons. In both cases the spacetime spectrum should include a tensor multiplet and a hypermultiplet for $k=1$ [2]. Thus, we should find 16 ground states of this theory, which should be in the $\{(\mathbf{1},\mathbf{3})\ (\mathbf{1},\mathbf{1})\} + \{(\mathbf{1},\mathbf{1})\ (\mathbf{1},\mathbf{1})\} + \{(\mathbf{1},\mathbf{1})\ (\mathbf{2},\mathbf{2})\} + \{(\mathbf{1},\mathbf{2})\ (\mathbf{1},\mathbf{2})\} + \{(\mathbf{1},\mathbf{2})\ (\mathbf{2},\mathbf{1})\}$ representation of the $SO(4)_{\|} \times SO(4)_{\perp}$ global symmetry. This representation arises by quantizing the fermion zero modes of the $U(N/2)$-singlet components of the fermions β_L and ρ_R appearing in table (2.3).

[2] In the $(2,0)$ case it was not clear if a $k=1$ theory which was decoupled from the Coulomb branch existed or not [28], but here we are reaching this theory by a perturbation from a theory that was already decoupled from gravity, so there is no problem.

Note that in formulating the critical theory for $k = 1$, we discarded β_L because it became infinitely massive on the interior of the Higgs branch (2.7). But, as just noted, quantizing its zero modes gives the correct degeneracy and quantum numbers to describe the tensor multiplet on the spacetime Coulomb branch. This is the first difficulty we find in attempting to describe the deformations away from the critical theory using only the degrees of freedom involved in formulating the fixed point itself.

In general, there is a correspondence between ground states of the supersymmetric quantum mechanics on a space X and cohomology classes of X. Thus, we expect the modes of the tensor multiplet, which should exist for any integer value of momentum around the circle in the $E_8 \times E_8$ theory (i.e. for all even values of N in the original $SO(N)$ theory), to correspond to cohomology classes of the moduli space of our theory. In the case $k = 1$ and for non-zero X_9, this space is simply the moduli space $\mathcal{M}_{\widetilde{N}}(U(1))$ of $\widetilde{N} = N/2$ instantons in a $U(1)$ gauge group. Since these instantons are necessarily all of zero size, this space is just

$$\mathcal{M}_{\widetilde{N}}(U(1)) = \mathbb{R}^{4\widetilde{N}}/S_{\widetilde{N}}. \tag{3.1}$$

For $\widetilde{N} = 1$, we have simply a 0-brane/4-brane system, and the required state is just the bound state of [29]. Indeed, this state becomes completely localized on the 4-brane in the $M_p \to \infty$ limit.

For higher values of $\widetilde{N}$, it is not apriori clear which cohomology should be used, since the states are all associated with the (orbifold) singularities of the moduli space. It is natural to conjecture that the quantum mechanical ground states are given by the orbifold cohomology of this space [30] (this is more justified in the 1+1 dimensional theory described in section 4, but our theory is just a dimensional reduction of that theory). This gives a state for every partition of $\widetilde{N}$ [31], in agreement with our expectation of finding a single state for any integer value of momentum of the tensor multiplet. Quantizing the zero modes of β_L and ρ_R then gives this state the Lorentz quantum numbers of a tensor multiplet and a hypermultiplet in the $(1,0)$ spacetime theory. These states are examples of states living at the singularities of the Higgs branch, as discussed above.

3.2. The Higgs Branch

The other branch of the spacetime theory is the Higgs branch, in which the fivebranes in spacetime turn into large E_8 instantons. We are only interested in the regime in which a quantum field theory description, decoupled from gravity, remains valid. Let us denote by $\widetilde{H}$ the canonically normalized (dimension 2) scalar field in spacetime whose VEV parameterizes the Higgs branch. The field theory regime is

$$\widetilde{H} << M_p^2. \tag{3.2}$$

On dimensional grounds, $\widetilde{H}$ is related to the scale size ρ of the instanton/fivebrane by

$$\widetilde{H} = M_p^3 \rho. \tag{3.3}$$

Thus, the field theory regime is

$$\rho << l_p, \tag{3.4}$$

where the fivebrane is still thin in Planck units. In the regime $\rho > l_p$, the fivebrane becomes thick, gravity fails to decouple, and the matrix description necessarily involves the degrees of freedom (2.7) as well as (2.8). In the field theory regime (3.4), as discussed above, one might hope to describe the theory using only the degrees of freedom (2.8). However, as with the spacetime Coulomb branch, we will encounter difficulties in realizing this.

We will analyze here only the case where the instantons are all embedded in a single $SU(2)$. In this case, the E_8 gauge symmetry in spacetime is broken to E_7, and its $SO(16)$ subgroup (which appears explicitly in the quantum mechanics) is broken to $SO(12) \times SU(2)$. In the quantum mechanics, we go into this branch by turning on the parameters corresponding to the 4-8 strings H and to the 4-4 strings $X_\perp^{(4)}$. Note that turning on only the 4-8 strings when the instantons are all in the same $SU(2)$ still leaves all but one of the fivebranes/instantons at zero-size, so we still have a non-trivial conformal theory for $k > 1$. In the quantum mechanics we see that not all of the Higgs branch is lifted in that case. In contrast, from (2.5) we can easily see that turning on both H and $X_\perp^{(4)}$ gives a mass to all the fields v and ψ_R, and to $4k$ of the fields ψ_L and χ_L. The first 12 components of χ_L (in the fundamental representation of the unbroken $SO(12)$ and of $SO(N)$) remain massless, as do 4 combinations of χ_L and ψ_L (again, in the fundamental of $SO(N)$).

Naively, when we turn on H the fields v and ψ_R become massive, and there is no longer an infinite tube in the Coulomb branch of the gauge theory, so gravity does not seem to decouple from our theory. However, as discussed above, we should be careful in how we normalize H. In spacetime, we want $\widetilde{H}$ to remain finite as M_p goes to infinity. This corresponds to having a finite H in the theory describing the Higgs branch in the $g_{QM} \to \infty$ limit. In this limit, even for finite H there is still an infinite tube in the Coulomb branch, and gravity still decouples from the Higgs branch of the 6D SCFT.

For simplicity, we will analyze here only the case $k = 1$, where the combinations that remain massless are exactly the 4 fields ψ_L[3]. The hypermultiplet H which obtains a VEV on the Higgs branch is (using (2.4)) charged under $SU(2)_R \times SO(16) \times Sp(1)$, where $SU(2)_R$

[3] Since to get a free low-energy theory in spacetime for $k > 1$ we are forced to turn on 4-4 strings, the general case decomposes in the IR into k copies of this case (living at different values of $X_\perp$, corresponding to the eigenvalues of $X_\perp^{(4)}$).

is the first $SU(2)$ factor in $SO(4)_\perp$ (which is identified with the $SU(2)_R$ symmetry of the spacetime theory). Giving it a VEV breaks this symmetry to $SU(2)_{R'} \times SO(12) \times SU(2)$, where $SU(2)_{R'}$ is a subgroup of $SO(16)$ and $SU(2)_R$, and the last $SU(2)$ is a subgroup of $SO(16)$ and $Sp(1)$ (but note that away from the small instanton point this is a perturbative gauge symmetry from the heterotic point of view). The fermions in the fundamental representation of $SO(N)$ which remain massless are the χ_L, in the $(\mathbf{1},\mathbf{12},\mathbf{1})$ representation, and ψ_L, in the $(\mathbf{1},\mathbf{1},\mathbf{2})$ representation (and in the $\mathbf{2}$ of the other $SU(2)$ factor in $SO(4)_\perp$).

Since the v fields are all massive, the Higgs branch of the theory after the perturbation is given simply by the space of $X_\parallel$s, which is $\mathbb{R}^{4N}/S_N$. What states do we expect to find in this case? The massless states of the spacetime theory on the Higgs branch are 30 hypermultiplets. One of these hypermultiplets, which corresponds to the transverse position of the instanton / fivebrane (and is free everywhere in the moduli space) is in the $\{(\mathbf{1},\mathbf{1})\ (\mathbf{2},\mathbf{2})\} + \{(\mathbf{1},\mathbf{2})\ (\mathbf{1},\mathbf{2})\}$ representation of the $SO(4)_\parallel \times SO(4)_\perp$ global symmetry (where $SO(4)_\perp$ now includes the new $SU(2)_{R'}$ group instead of the old $SU(2)_R$). The other hypermultiplets are all in the $2(\{(\mathbf{1},\mathbf{1})\ (\mathbf{2},\mathbf{1})\} + \{(\mathbf{1},\mathbf{2})\ (\mathbf{1},\mathbf{1})\})$ representation, and in the $\frac{1}{2}\mathbf{56}+\mathbf{1}$ representation of the unbroken E_7 gauge group in spacetime. This representation decomposes into a $\frac{1}{2}(\mathbf{32},\mathbf{1}) + \frac{1}{2}(\mathbf{12},\mathbf{2}) + (\mathbf{1},\mathbf{1})$ of the $SO(12) \times SU(2)$ that we see in the quantum mechanics. According to (2.1), the momentum modes of the first representation should appear for odd values of N, while all the others should appear for even values of N. Of course, this does not mean that the momentum quantum number in the $E_8 \times E_8$ string theory depends on the representation: from (2.1) one sees that for $N = n_S$ odd, P_E must be a spinorial representation of $SO(16) \times SO(16)$ but m_E can be odd or even.

As in the 0-8 system [18-20], we expect the structure of the ground states for odd (even) values of N to be the same as for $N = 1$ ($N = 2$), with the only change being in the structure of the wave functions for the 0-8 bound states. T duality and S duality relate our system to a heterotic $SO(32)$ string theory with some non-trivial instanton bundle, and there we can show that the appropriate states exist (the calculation is essentially as in [32], and the presence of torsion does not change the results in this case [33]).

Let us analyze first the case $N = 1$. In this case, the moduli space is just $\mathbb{R}^4$, so we have only the ground state. The 12 remaining fermions χ_L are completely free in this case (since the gauge symmetry is just $O(1) \equiv \mathbb{Z}_2$), so they have zero modes. On the other hand, as explained in [22], the ψ_Ls are sections of the $SU(2)$ instanton bundle that lives in the $X_\perp$ directions. In this background they do not contribute any additional degeneracy. Quantization of the χ_L zero modes gives states in the $\mathbf{32}+\mathbf{32}'$ representation of the $SO(12)$ group. As in [19,20], imposing a $\mathbb{Z}_2 \equiv O(1)$ gauge constraint removes half of these states and leaves us just with a $\mathbf{32}$. Adding the ρ_R zero modes turns each of these states into a half-hypermultiplet in spacetime, so we get exactly the expected spectrum of states for this value of N.

In fact, for $N = 1$ we can find the right states also if we work only with the degrees of freedom (2.8) involved in the critical theory. Then v and its superpartners, as well as four of the fields χ_L, are lifted by H, and quantizing the zero modes of the remaining fermions χ_L and ρ_R provides us with the required $\frac{1}{2}\mathbf{32}$ hypermultiplets (after taking into account the $\mathbb{Z}_2$ constraint).

For $N = 2$, the situation is more complicated (as it was also in the 0-8 case), since the interactions between the fields play an important role in constructing the states. To realize these states in our formulation, we turn the operators (including χ_L and ψ_L) into creation and annihilation operators (as in [20]). We expect the ground states in the quantum mechanics to be the same as those in the corresponding $1+1$ dimensional sigma model, where a level-matching constraint will force us to have two χ_L or ψ_L oscillators in the sector where they are anti-periodic (and no states will arise from other sectors). In the quantum mechanics, there will be a gauge constraint (analogous to the level-matching constraint of the heterotic string) which will force the total charge of a state under the $SO(2)$ gauge symmetry to be equal to one [20]. We expect to find ground states of the form $\chi_L \psi_L |0\rangle$ (where χ and ψ are now creation operators), multiplied by an appropriate wave function which turns this state into an $SO(4)_{||} \times SO(4)_{\perp}$-singlet. These states will be in the $(\mathbf{12}, \mathbf{2})$ representation of the $SO(12) \times SU(2)$ global symmetry corresponding to the remaining spacetime gauge symmetry, and again the ρ_R zero modes will turn them into half-hypermultiplets. The 29th and 30th hypermultiplets will arise from states involving two ψ_Ls (contracted to form a singlet of the $SU(2)$ gauge symmetry), again with an appropriate wave function for the rest of the fields. It would be interesting to perform the Born-Oppenheimer calculations explicitly, and see that exactly states of this form arise.

The IR theory of the degrees of freedom (2.8) is complicated in this case, and we have not been able to find these states directly by deforming that theory. Presumably, this is again a result of the theory at the singularities of the Higgs branch mixing with the theory describing the interior of the Higgs branch as we deform away from the conformal point.

4. String Theories for string Theories

The Higgs branch of the quantum mechanics formulated above is expected to describe the $(1,0)$ superconformal theory in spacetime. In [13], a similar quantum mechanics described the $(2,0)$ superconformal theories in spacetime. The corresponding $1+1$ dimensional theory (which gives the quantum mechanics upon dimensional reduction) was conjectured [13,28] to correspond to the "little string" theory of the type IIA NS fivebrane [34,35], which reduces at low energies to the superconformal theory. Similarly, we expect the $1+1$ dimensional theory with $(0,4)$ supersymmetry to describe the "little string" theory of the heterotic $E_8 \times E_8$ fivebrane, defined by the limit $g_s \to 0$ in that theory [34].

The field content and interactions of this theory are the same as those described above, with X_9 now becoming part of the $1+1$ dimensional gauge field. The only difference is that there are now 32 chiral fermions χ_L, since we can no longer ignore the states of the "other wall" (these states are also required for anomaly cancellation). As in the Matrix theory descriptions of the heterotic string [36-38], half of these fermions have periodic boundary conditions and the other half have anti-periodic boundary conditions. The X_9 positions of the D0-branes turn into the Wilson loop around the circle, and half of the χ_L fermions are massless when the value of this Wilson loop corresponds to the D0-branes being at each of the two walls. However, this theory should still have a parameter $X_9^{(4)}$, corresponding to the X_9 position of the fivebranes[4], and the ψ fermions (as well as their bosonic partners) should only be massless when the Wilson loop is equal to the eigenvalues of $X_9^{(4)}$. This is realized in the $1+1$ dimensional field theory by having the boundary conditions for the ψ fields around the circle twisted by an arbitrary $X_9^{(4)}$ matrix (in the adjoint representation of $Sp(k)$), namely

$$\psi(x+2\pi) = \exp(X_9^{(4)})\psi(x) . \tag{4.1}$$

The vs have similar boundary conditions. Note that such boundary conditions are not possible for the χ_L fields since a potential would be generated if their boundary condition were different [40].

We conjecture that the Higgs branch of this theory, in the $g_{YM} \to \infty$ and large N limits, gives an infinite momentum frame description of the "little string" theory of the heterotic $E_8 \times E_8$ fivebrane at zero coupling. At low energies this theory goes over to the quantum mechanics of the previous sections, as expected. Note that the spacetime theory in this case includes two strings even for a single fivebrane, coming from the membranes stretched between the fivebrane and the two end of the world ninebranes. The sum of the tensions of these two strings is the heterotic string tension M_s^2, but their ratio depends on the parameter $X_9^{(4)}$ described above.

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[4] As noted in [39], this parameter actually lives in the "Coxeter block" of $Sp(k)$.

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