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THE WELFARE EFFECTS OF PUBLIC INFORMATION IN BOTH COMPLETE AND ASYMMETRIC INFORMATION MARKETS

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1 Introduction

An important theoretical question in financial reporting is under what conditions does (additional) public information increase the welfare of market participants. Research on the question has primarily used the Pareto criterion to measure an increase in welfare and an Arrow-Debreu exchange model of financial markets. When the comparison is a direct one between public information and no public information while assuming the set of securities is the same in both cases, welfare has not been found to increase with public information. Indeed, in some situations no information is Pareto superior to public information. The accepted explantion for these negative results is that information can reduce risk-sharing. For example, in a Rod Serling world where everyone's date of death is public information, the life insurance market disappears. However, if we start with a scenario of trading with no public information the follow it with a second round of trading where trading with public information is contrasted with trading with no public information, then it is immediate that the second round of trading with public information cannot hurt, and it may result in an increase in welfare under various interesting and plausible conditions Hakansson, Kunkel, and Ohlson (1982, Theorem 1) and Ng (1977,)). Proposition 1)). We will consider the welfare effects of public information in the direct-comparison single-period case only. The first part of this paper uses the complete market assumption, and we distinguish between (a) conditions where there is no Pareto improvement with information, and (b) conditions where there is a Pareto impairment with information. In both situations we extend previous negative results.

This lack of positive results suggests that the Arrow-Debreu exchange model may fail to capture some major aspects of information. One possible modification is to allow production (see Ohlson (1986)). The approach taken in the second part of this paper is to consider an asymmetric information financial market of informed sellers and less- informed buyers. In this market, money, low quality stocks, and high quality stocks are traded by individuals with exponential utility functions and different risk tolerances. Public information is assumed to eliminate information asymmetries, and in general some individuals will gain and some will lose with public information. However, our main result shows that public information satisfies the Kaldor-Hicks-Scitovsky criterion that there is a zero sum set of side payments such that the equilibrium with public information adjusted by these side payments is pareto superior to the no public information equilibrium. The Kaldor Hicks-Scitovsky criterion plays a prominent role in welfare economics and, for example, it was used by Samuelson (1939) to compare free trade with no free trade.

Thus we show how public information can result in increased welfare. Public information does this in an intuitively plausible way by countering the effect described by Akulof (1970) of indistinguishable "lemmas" diluting a market.

2 Complete Markets

2.1 The model

Our notation and formulation will follow closely but not completely that of Hakansson, Kunkel, and Ohlson (1982). We consider a single-period exchange economy with a single commodity, "wealth". There are no taxes or transaction costs and at the end of the period the economy will be in some state $s \in S$, where $s = 1, 2, ..., m$. There are I individuals (market participants) whose probability beliefs are given by the vectors $\pi_i = (\pi_{i1}, ..., \pi_{im})$ where $\pi_{i2} > 0$ for all s. We will assume that there is a complete market with respect to consumption claims, and that all trading takes place in these claims directly. The endowment of individual i of claims to state s is denoted by $\bar{z}_{is}, S = 1, 2, ...m$. The economy's endowment of claims to state s is denoted by $Z_s = \sum_i \bar{z}_{is}$.

Let U_i be the strictly increasing, strictly concave utility function of individual i and z_{is} be his wealth (consumption) after trading when the economy is in state s. Individual i seeks to maximize expected utility,

$$V_i = \sum_s \pi_{is} U_i(z_{is}) \tag{1}$$

subject to his budget constraint

$$\sum_i P_s(\bar{z}_{is} - z_{is}) = 0 \tag{2}$$

as a price-taker, where P_s is the price to a claim when state s occurs. The basic optimality condition for the allocation vector z is that

$$\pi_{is} \frac{dU_i(z_{is})}{dz_{is}} / P_s = \gamma_i \text{ for } s = 1, ..., m \cdot \tag{3}$$

Here γ_i is a strictly positive Lagrange multiplier.

Prior to trading, investors may obtain information on the state that will occur via a public information system Y. The information is conveyed by $m \times n$ information matrix Q whose (s, y) element is $q(y|s)$, the probability of signal $y, y = 1, 2, ..n$, when the true state is s. In order to avoid uninteresting special cases, we assume that each column of Q has at least one non-zero element. This assumption combined with the assumption $\pi_{is} > 0$ will imply that the probability of observing any signal y is strictly positive. When the public signal y is observed, all individuals update their prior probabilities according to Bayes law:

$$\pi_{is}^y = \frac{\pi_{is} q(y|s)}{\sum_s \pi_{is} q(y|s)} \,. \tag{4}$$

With public information, each individual maximizes (1) subject to (2) with π_{is}^y replacing π_{is} and P_s^y replacing P_s. The maximized value is denoted by V_i^y, and the after-trading vector of wealth is denoted by Z_i^y.

As in Hakansson, Kunkel, Ohlson, p.1173, the relevent expected utility in the information case is

$$V_i(Y) = \sum_y q_i(y) V_i^y \tag{5}$$

where $q_i(y) = \sum_s \pi_{is} q(y|s)$, is individual $i's$ subjective probability of observing a signal y. Following standard terminology, information will be said to be Pareto superior to no information if

$$\begin{aligned} V_i(Y) &\geq V_i \quad \text{all } i \\ V_i(Y) &> V_i \quad \text{some } i \,. \end{aligned}$$

2.2 The Welfare Effects of Public Information

Our first result is Theorem 1 which says that public information does not lead to a Pareto improvement. As mentioned in the introduction the accepted explanation is that information can preclude risk sharing, and the reader is directed to Verrecchia (1982) for a leisurely discussion.

Theorem 1. In a single period exchange economy, public information is not Pareto superior to no information if individuals have heterogeneous prior beliefs and strictly increasing, strictly concave utility functions. Theorem 1 is proved in the

Appendix. This result has been previouisly stated as part of Lemma 3 in Hakansson, Kunkel, and Ohlson (1982). However their proof of Lemma 3, which is almost exclusively concerned with the two period homogeneous signal belief case, does not work for heterogeneous beliefs because their key inequality (20) holds only for homogeneous signals beliefs. Examples can be given which show that the "average allocations" used in their proof may be infeasible under heterogenous beliefs. Our proof is the same as theirs except for using a different method to determine average allocations.

Perhaps just as important as a general result on no Pareto improvement with public information, are conditions where there is a Pareto impairment with public improvement. The Pareto criterion is a strigent one and there can be improvements with respect to a weaker criterion (such as the Kaldor-Hicks-Scitovsky criterion) which are not Pareto improvements. However when no public information is Pareto superior to no public information there seems to be little one can say in defense of public information. Our result on Pareto impairment with public information is:

Theorem 2. In a single-period exchange economy, non-trivial public information is Pareto inferior to no public information if all individuals have homogeneous beliefs and either identical isoelastic utility functions or exponential utility functions.

Specifically in Theorem 2 individuals are assumed to have identical utility functions of either the form

$$U(z) = \frac{z^{\lambda}}{\lambda} \text{ where } r_i > 0, \tag{6}$$

and $\lambda = 0$ is interpreted as the log utility, or of the form

$$U_i(z) = e^{\frac{-z}{r_i}} \text{ where } r_i >) \cdot \tag{7}$$

The parameter r_i is the risk tolerance of individual i. The definition of non-trivial public information is that there is ome individual i and signals y and y^i such that $z_i^y \neq z_i^{y'}$. If a public information system is trivial, then the z_i^y, $y = 1, ..., n$, ha have a common value, and it follows from equations (25) and (26) in the Appendix that this common value is z_i. Therefore all individuals are indifferent between trivial public information and no information.[1]

[1] The hypothesis of non-trivial information is a hypothesis about endowments as well as the information matrix Q. For example if endowments represent equilibrium allocations with respect to no information, then all public information is trivial.

The proof of Theorem 2 is given in the Appendix. It generalizes a previous result of Wilson (1975,p.185) who proves this result for the log utility function, and Ng who proves this result when the information matrix Q is a partitioning matrix, that is each row has all zeros except for a single one.[2] Such an information matrix is called a partitioning information matrix since it partitions S into sets $S_y, y = 1, ..., n, \text{where} S_y = s : q(y|s) = 1$. The proofs of Wilson, Ng, and Theorem 2 are quite different from each other.

Theorem 1 and 2 and other negative results in the literature suggest that the Arrow-Debreu exchange model may fail to capture important aspects of information. One of these aspects is asymmetric information. As we have seen the Arrow-Debreu formulation allows for heterogeneous beliefs. However, anyone's beliefs are considered as valid as those of anyone else, and there is no motivation for an individual to change his beliefs upon leaving the beliefs of others. Beliefs are only modified by outside signals. The above is contained in Grossman's criticism (1981) of the standard Walras-Arrow-Debreu model for its failure to aggregate information through prices in markets with uncertainty and asymmetric information. In the second half of this paper we will consider an asymmetric information financial market and conclude that public information can improve welfare.

3 Asymmetric Information

3.1 The Model

As before we consider a single-period exchange economy with a single commodity "wealth", and no taxes or transaction costs. There are three types of marketed securities. First there is a riskless asset which can be interpreted as either money or a zero-coupon bond. Second there are low quality firms. At the end of the period, each share of a low quality firm pays some multiple of $\mu_2 + \theta$ where μ_2 is a scalar and θ a unit normal variable with a mean of 0 and a variance of 1. Third there are high quality firms, and at the end of the period each share of a high quality firm pays some multiple of $\mu_1 + \theta$ where $\mu_1 > \mu_2$. The θ random variable is perfectly correlated across the same and represents non-diversifiable market risk. Without

[2]If one redefines the state space to be states ss s cross signals, then a general information matrix for the original state space can be represented by a partitioning matrix on the more general state space. However a complete set of securities for the original state space will not, without additional assumptions, be complete with respect to the more general state space.

loss of generality we can rescale shares so that any share of a low quality firm pays $\mu_2 + \theta$ and any share of a high quality firm pays $\mu_1 + \theta$.

There are I individuals (market participants). Each individual i has an exponential utility function and seeks to maximize expected utility, $E(-e^{(w_i/r_i)})$, where w_i and $r_i > 0$ are the end of period wealth and risk tolerance respectively of individual i. The total risk tolerance in the economy is denoted by $R = \sum_i r_i$. We let $\bar{z}_{i1}, \bar{z}_{i2}, \text{and} \bar{z}_{i3}$ be individual $i's$ endowment of high quality shares, low quality shares, and money respectively. The total holdings in the economy of high quality shares is $Z_1 = \sum_i \bar{z}_{i1}$, and the total holdings of low quality shares is $Z_2 = \sum_i \bar{z}_{i2}$.

With no public information there is asymmetric information and individuals know the quality of the firms whose shares they are endowed with but not the quality of other firms. With public information everyone can distinguish between low and high quality firms so that there are no information asymmetries. Before turning to the easier public information equilibrium, we want to interpret the asymmetric information assumptions and to place some restrictions on economy parameters.

Without a restriction on the ratio of economy risk tolerance to risky assets, shares could have a negative price. Therefore, we will require that

$$R/(Z_1 + Z_2) \geq \mu_2 \tag{8}$$

In the no public information and hence asymmetric information situation, shareholders of high quality firms are motivated to credibly communicate their high quality to potential buyers.[3] In our model the number of shares retained by the owners if known, is a strong signal as to firm quality. We will assume that the number of shares retained by the owners is not public information, and our model will not allow for any other methods to communicate firm quality. This asymmetric information assumption is easier to go along with if there is less of a difference between high and low quality firms and hence less motivation to communicate high quality. Therefore our model should be interpreted as having the restriction that μ_1 is not "too" much larger than μ_2. It turns out that in order to obtain an equilibrium in the asymmetric information case we must restrict $\mu_1 - \mu_2$, and we

[3] This phenonomen has been analyzed by a number of authors including Leland and Pyle (1977), Titman and Trueman (1986) and Hughes (1986) where the signals of higher quality are respectively the amount of shares retained by the owner, the choice of auditor, and a choice of double signal of shares retained by the owners and a manager forecast.

will require that

$$\frac{(Z_1 + Z_2)^2}{4RZ_2} \frac{R_2}{R_1} \geq \mu_1 - \mu_2 \cdot \tag{9}$$

Here R_1 is the sum of risk tolerances over individuals such that $\overline{z}_{i1} > 0, \text{and} R_2$ is the sum of risk tolerances over the complementary set of individuals with $\overline{z}_{i1} = 0$.

The assumption that individuals know the quality of firms whose shares they are endowed with is more credible if they have endowments in at most a few firms, and our model should be interpreted as having this restriction. A somewhat related technical assumption we will make is that if $\overline{z}_{i1} > 0$, then

$$\overline{z}_{i1} \geq r_i \mu_1 \cdot \tag{10}$$

The restriction (10) will turn out to be a sufficient condition for individuals with endowments of high quality shares (and recall that they are aware that they are high quality) not to desire to purchase any more shares of these firms.

4 The Public Information Equilibrium

Recall that in the public information equilibrium all individuals can distinguish between low and high quality firms. Although there are three marketed securities, there are only two elemental securities, money and "risk". As represented by 1 share with a payoff of 0, the standard normal distribution. We let $P > 0$ be the price paid to a purchaser for absorbing a share of risk. Low quality stock sells for $\mu_2 - P$ and high quality stock for $\mu_1 - P$. Individual $i's$ endowment has a value of $\overline{W}_i = \overline{z}_{i1}(\mu_1 - P) + \overline{z}_{2i}(\mu_2 - P) + \overline{z}_{3i}$. His portfolio problem is to determine the amount of risk to purchase so as to maximize expected utility. If x_i is the amount of risk purchased, his expected utility is

$$V_i = -e^{-[(\overline{W}_i + Px_i)/r_i]} + \frac{1}{2}\frac{x_i^2}{r_i^2}. \tag{11}$$

The maximizing value of $x, x*_i$; is obtained by standard calculus procedures and $x_i^* = r_i P$ which is positive since both r_i and P are positive. The equilibrium value of price must equate supply and demand so that

$$P = (Z_1 + Z_2)/R \cdot \tag{12}$$

By (8), P will not exceed μ_2. Substituting $\overline{W}_i$ and X_i^* into (11) says that the expected utility of individual i in the public information equilibrium is

$$V_i = -e^{-[(\overline{z}_{i1}+\overline{z}_{i2})(\mu_2-P)+\overline{z}_{i1}(\mu_1-\mu_2)+\overline{z}_{i3}+\frac{1}{2}P^2]/r_1} \quad . \tag{13}$$

5 The No Public Information Equilibrium

In the no public information equilibrium there is asymmetric information and individuals know the quality of firms whose shares they are endowed with but not the quality of other firms. In this situation there is a motivation to form a market protfolio whose shares have a payoff of $\mu, +\theta$, where $\mu_1 > \mu > \mu_2$. Although buyers cannot distinguish between low and high quality shares, they are assumed to know μ. This a common type of rational expectations assumption. The value of μ will depend on the proportion of high and low quality shares which make up the portfolio. The price of a share of the market portfolio is $\mu - P_n$ where P_n is the price paid for absorbing a share of risk. Since there are no transaction costs, sellers of shares to market portfolio receive $\mu - P_n$ per share, regardless of whether they are high or low qualify shares since both are indistinguishable. Individuals, aware of the quality of their endowments are, of course, more prone to sell their low quality shares than their high quality shares to the market portfolio. The market portfolio will dominate other forms of trading because it eliminates uncertainty about the mean of the payoff distribution.

Let I_1 be the set of individuals with endowments of high quality stock ($\overline{z}_{i1} > 0$), and I_2 be the complement of $I_1(\overline{z}_{i1} = 0)$. All individuals $i \in I_2$, have an endowment with a value of $\overline{W}_i = \overline{z}_{i2}(\mu - P_n) + \overline{z}_{3i}$, and will want to sell all their stock to the market portfolio. The problem is to determine the number of shares of the market portfolio to purchase. If x_i shares of the market portfolio are purchased, their expected utility is

$$V_i = -e^{(\frac{\overline{W}_i+P_n x_i)}{r_i}+\frac{1}{2}\frac{X_i^2}{r_i^2}} . \tag{14}$$

The maximizing value of x_i is $r_i P_n$. Substituting $\overline{W}_i$ and x_i^* into (12) yields

$$V_i = -e^{-[\frac{(\overline{z}_{i2}(\mu-P_n)+\overline{z}_{i3})}{r_i}+\frac{1}{2}P_n^2]}. \tag{15}$$

Now let us turn to individuals $i \in I_1$. Like the individuals in I_2 they will sell all their low quality stock to the market portfolio. They could conceivably want to purchase more of high quality shares of stock they already own from knowledgeable sellers, or perhaps shares from the market portfolio. The assumption (10) concerning the risk tolerance and endowments of individuals in I_1 significantly simplifies our analysis since it implies that the above will not be the case. It implies that individuals in I_1, will purchase no risky assets and sell some of their high quality stock to the market portfolio for $\mu - P_n$ a share. Individual $i's$ problem is to determine the amount of high quality shares to retain. If x_i is the number of shares of high quality stock retained, his expected utility is

$$V_i = -e^{\frac{-\mu_1 x_i + (\overline{z}_{i1} + \overline{z}_{2i} - X_i)(\mu - P_n) + \overline{z}_{i3})}{r_i} + \frac{1}{2}\frac{x_i^2}{r_i^2}} \quad . \tag{16}$$

The maximizing value of $x_i, x_i^*, \text{is} r_i(\mu_1 - (\mu - P_n))$. Since $\mu - P_n$ will be shown to be positive, assumption (16) implies $x_i* \leq \overline{z}_{i1}$. Substituting x_i into (16) yields.

$$V_i = -e^{-[\frac{(\overline{Z}_{i1} + \overline{Z}_{2i})(\mu - P_n) + \overline{Z}_{3i})}{r_i} + \frac{1}{2}(\mu - (\mu - P_n))^2]} \tag{17}$$

Let us now consider the equilibrium values of μ and P_n. The number of shares of high quality stock in the market portfolio is $\sum_{i \in I_1}(\overline{z}_{i1} - x_i^*) = Z_1 R_1(\mu_1 - (\mu - P_n))$. Therefore,

$$\mu = \frac{Z_2\mu_2 + [Z_1 - R_1(\mu_1 - (\mu - P_n))]\mu_1}{Z_2 + Z_1 - R_1(\mu_1(\mu - P_n))} \tag{18}$$

The demand for shares for the market portfolio comes from individuals belonging to I_2 and equals $R_2 P_n$. Equating supply and demand leads to

$$Z_2 + Z_1 - R_1(\mu_1 - (\mu_{P_n})) = R_2 P_n. \tag{19}$$

Equation (19) can be written as

$$P_n = \frac{Z_2 + Z_1 - R_1\mu_1}{R_1 + R_2} + \frac{R_1\mu}{R_1 + R_2}. \tag{20}$$

Equation (18) can be rewritten as

$$\mu = \mu_1 - \frac{Z_2(\mu_1 - \mu_2)}{R_2 P_n} \, . \tag{21}$$

The system (20) and (21) leads to the quadratic equation in either μ or P_n. By straight-forward calculations the quadratic equation has a real solution if and only if equation (9) holds, and we have assumed that it does. From (20) it follows that $\mu - P_n = (R_2\mu_1 - Z_2 - Z_1)/(R_1 + R_2)$ which is positive by (8).

6 The Welfare Effect of Public Information

Having determined both the public information and the no public information equilibrium, we are now in a position to compare them. A major topic in welfare economics is developing criteria for preferring equilibirum A to equilibrium B. If everyone is at least as well off in equilibrium A and some are better off (the Pareto criterion) then there is general agreement to A is preferred to B. But what if some prefer A and other prefer B. In order to expand the scope of welfare comparisons, Kaldor and Hicks proposed that equilibrium A be considered preferable to equilibrium B if there is a zero-sum set of side payments which transforms A to A', and A' is Pareto preferable to B. Unfortunately, there are examples of equilibria A and B where the Kaldor-Hicks criteria says that A is prefered to B and B is preferred to A. Therefore we will add the additional requirement associated with Scitovosky which rules out this possibility.

Following the terminology in Rothenberg (1961, Chapter 4) we say that equilibrium A is potentially preferred to equilibrium B if it meets the Kaldor-Hicks requirement to be preferred to B, and B does not meet the Kaldor-Hicks requirement to be preferred to A. The cautious word "potentially" is appropriate in this definition, because it warns the reader that there is no reason to believe that the zero-sum set of side payments will actually be made.

It will streamline the proof of our main result if we establish two short lemmas.

Lemma 1. The prices for absorbing risk in the two equilibria satisfy the relation $\mu_1 - (\mu - P_n) > P > P_n$.

Proof. By (12) and (19) $P = (R_1(\mu_1 - (\mu_{Pn})$. By (21) $\mu_1 > \mu$ so that $\mu_1 - (\mu_{Pn}) > P_n$. Q.E.D.

The next lemma is concerned with the no public information equilibrium, and it makes the fairly obvious statement that the total "mean" of stocks in the economy, $Z_1\mu_1 + Z_2\mu_2$ equals the "mean" of stocks in the market portfolios plus those of high quality stocks retained by individuals in I_1.

Lemma 2. The mean return of the market portfolio satisfies

$$R_1(\mu_1 - (\mu - P_n)\mu_1 + (Z_1 + Z_2 - R_1|\mu_1 - (\mu - P_n)) = Z_1\mu_1 + Z_2\mu_2 \cdot \qquad (22)$$

Proof: By (18), the left-hand side of (20) equals $R_1(\mu_1 - (\mu - P_n))\mu_1 + Z_2\mu_2 + [\overline{Z}_1 - R_1|\mu_1 - (\mu - P_n)]\mu_1$ which equals $Z_1\mu_1 + Z_2\mu_2$. Q.E.D.

A more convenient form of (22) for our purpose is

$$R_1(\mu_1 - (\mu - P_n))(\mu_1 - \mu) + (Z_1 + Z_2)(\mu - \mu_2) - Z_1(\mu_1 - \mu_2) = 0 \cdot \qquad (23)$$

We are now in a position to prove our result in the asymmetric information case.

Theorem 3: Consider the single period exchange economy described above satisfying the restrictions (8), (9), and (10). The public information equilibrium with expected utility given by (13) is potentially preferred to the no public information equilibrium with expected utilities given by (15) and (17).

Proof. To show potential preference it is necessary to demonstrate a zero-sum set of side payments such that the public information equilibrium with side payments is Pareto superior to the no information equilibrium. We begin with individuals in I_2, that is those with no endowments of high quality stocks. By Lemma 1 $P > P_n$ so that by $\frac{1}{2}[P^2 - P_n^2] > P_n(P - P_n)$. Therefore, comparing (13) and (15), any individual in I_2 is better off with public information and a side payment of $-r_i P_n(P - P_n) + \overline{z}_{i2}((\mu - \mu_2) + (P - P_n))$ than with no public information. Summing over $i \in I_2$, we have side payments of $-R_2 P_n(P - P_n) + Z_{2,2},((\mu - \mu_2) + (P - P_n))$ where $z_{2,2} = \sum_{i \in I_2} \overline{Z}_{i2}$.

Now consider individuals in I_1, that is those with endowments of high quality stock. By Lemma 1 $(\mu_1 - (\mu - P_n)) > P$ so that $\frac{1}{2}[(\mu_1(\mu - P_n))^2 - P^2] < (\mu_1 - (\mu - P_n))(\mu_1 - (\mu - P_n) - P)$. Therefore, comparing (13) and (17), any individual in I_1 is better off with public information and a side payment of $r_i(\mu_1 - (\mu - P_n))(\mu_1 - (\mu - P_n) - P) + (\overline{z}_{i1} + \overline{z}_{i2})((\mu - \mu_2) + (P - P_n)) - \overline{z}_{i1}(\mu_1 - \mu_2)$. Summing up over individuals in I_1 we have side payments of $R_1(\mu_1 - (\mu - P_n))(\mu_1 - (\mu - P_n) - P) + [Z_1 + z_{2,1}]((\mu - \mu_2) + (P - P_n)) - Z_1(\mu_1 - \mu_2)$ where $z_{2,1} = \sum_{i \in I_1} \overline{z}_{i2}$.

The total value of all side payments, using (19), is $R_1(\mu_1 - (\mu - P_n))(\mu - (\mu - P_n) - P) + (Z_1 + Z_2)[(\mu - \mu_2) + (P - P_n)] - Z_1(\mu_1 - \mu_2) - (Z_1 + Z_2 - R_1(\mu_1 - (\mu - P_n))(P - P_n)$. Now apply (23) to see that the total value of side payments is 0.

It remains to show that there is no set of side payments such that no public information is preferred to public information. The above argument can be repeated to show that only a set payments greater than zero would make individuals prefer no public information to public information. For example any individual in I_2 would require a side payment strictly greater than $r_i P_n(P - P_n) - \overline{z}_{i2}((\mu - \mu_2) + (P - P_n))$. Q.E.D.

The gains that make the public information equilibrium potentially better than the no public information equilibrium come from a more efficient allocation of risk.

Lemma 1 says that the price for absorbing risk in the public information case is a weighted average between the price of risk to individuals belonging to I_2 and the implicit price of risk to individuals belonging to I_1 in the no public information euqilibrium. Therefore in the no public information equilibrium, individuals in I_2 will hold a less than optimal amount of risks and individual in I_1 will hold a greater than optimal amount of risk. Public information eliminates this y. inefficiency.

7 Conclusion

We have examined the welfare effect of public information in a single period exchange economy. In a complete market, we have given a corrected proof that even with heterogenous prior beliefs, public information is not Pareto superior to no information. With homogeneous prior beliefs, we have extended previous results to show that if individuals have either identical inelastic utility functions or the exponential utility function then no information is Pareto superior to information. Thus, for complete markets, this paper adds to literature on the lack of social value of public information.

In the second part of the paper we consider a single period exchange model with asymmetric information. There are three marketed securities, money, low quality stock, and high quality stock. Individuals have exponential utility functions. They know the quality of the shares they hold, but not those of other firms. Public information is assumed to eliminate this asymmetry by making the quality of stock known to all. It is shown that there is a zero-sum set of side payments such that public information with side payments if Pareto superior to no public information. Public information achieves this improvement by allocating risk efficiently throughout the economy.

These two contrasting results are obtained because (1) they treat asymmetric information differently and (2) the signal in the asymmetric information model is not random so the "distribution risk" that drives the complete market results is not present. In the complete market model, heterogeneous beliefs are considered equally valid and no one has anything to learn from any other individual's beliefs. In the asymmetric information model there are informed and uninformed individauls, and each is aware of her situation.

Appendix

In this appendix we will prove the two results for the complete markets.

Proof of Theorem 1. For each state $s, s = 1, ..., m$, and individual i define the average allocation vectors $\hat{Z}_i$ by $\hat{z}_{is} = \sum_y q(y|s) z^y_{is}$. These allocation vectors are feasible since $\sum_i \hat{z}_{is} = \sum_i \sum_y q(y|s) z^y_{is} = \sum_y q(y|s)(\sum_i z^y_{is})$, $\sum_y q(y|\theta) = 1$, and the allocations z^y_{is}, satisfy $\sum_i z^y_{is} \leq Z_s$. The rest of the proof follows Lemma 3 of Hakansson, Ohlson, and Kunkel (1982).

$$\begin{aligned}
\mathrm{V}_i^Y &= \sum_y q_i(y) V_i^y \\
&= \sum_y (\sum_{s'} \pi_{is'} q(y|s')) \sum_s \pi^y_{is} U(z^y_{is}) \\
&= \sum_y (\sum_{s'} \pi_{is'} q(y|s'))(\sum_s) \frac{\pi_{is} q(y|s')}{(\sum_{s'} \pi_{is'} q(y|s'))} U_i(Z^y_{is}) \\
&= \sum_y \sum_s \pi_{is} q(y|s') U_i(Z^y_{is}) \leq \sum_s \pi_{is} U_i(\hat{Z}_{is}) = V_i(\hat{Z}_i) \cdot
\end{aligned}$$

The inequality follows from Jensen's inequality. Of course $\hat{Z}_i \neq Z_i$, individual $i's$ vector of a after-trading holdings without information. However, since the $\hat{Z}_i$ are feasible allocations, the $V_i(\hat{Z}_i)$ cannot be Pareto superior to the $V_i(z_i)$. The above inequality shows that the V_i^y cannot be Pareto superior to the $V_i(Z_i)$. Q.E.D.

The proof of Theorem 2 will be carried out in detail for utility functions of the form (6) only. This proof works for the log utility function with trivial modifications. After the proof we will outline the modification needed for the exponential utility function.

A review of some known facts about the market solution our model with homogeneous beliefs when each individual has an identical utility function of the form (6) is helpful. From the optimality condition (3) it follows that for any two individuals i and j and any two state s and t that $(z_{is}/z_{it})^{\lambda-1} = (z_{js}/z_{jt})^{\lambda-1}$ so that $z_{is}/z_{it} = z_{js}/z_{jt}$. Consequently each individual i will consume some fraction α_i of the available quantity of contingent claims s, Z_s, where $Z_s = \sum_i \bar{z}_{is}$. A second major fact is that an equilibrium price vector is $P_s = \pi_s Z_s^{\lambda-1}$. The value of α_i is given by

$$\alpha_i = \sum_s P_s \bar{z}_{is} / \sum_s P_s Z_s \cdot \tag{24}$$

When a signal y is received, the value of individual $i's$ wealth is $\sum_s P^y_s \bar{z}_{is}$, where $P^y_s = \pi^y_s Z_s^{\lambda-1}$. Individual $i's$ wealth can be rewritten as $\sum_s P^y_s(\alpha_i Z_s) + \sum_s P^y_s(\bar{z}_{is} - \alpha_i Z_s)$. If we add equation (2), $\sum_s P_s(\bar{z}_{is} - \alpha_i Z_s) = 0$, to individual $i's$ wealth we have

$$\sum_s P_s^y \bar{z}_{is} = \sum P_s^y(\alpha_i Z_s) + \sum_s P_s^y(\bar{z}_{is} - \alpha_i Z_s) \tag{25}$$

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$$\sum_s P_s^y \bar{z}_{is} = \sum P_s^y(\alpha_i Z_s) + \sum_s W_i^y \text{ for each } i \tag{26}$$

where $W_i^y = \sum_s(P_s^y - P_s)(\bar{z}_{is} - \alpha_i Z_s)$. An interpretation of W_i^y is the change in individual $i's$ wealth when signal y is received. The quantity W_i^y may be either positive or negative. These wealth changes average to zero in the following sense. For every individual i,

$$\sum_y q(y) W_i^y = 0 \cdot \tag{27}$$

Equation (26) follows from the fact (Wilson 13, p.185)) that $\sum_y q(y)(\pi_s^y - \pi_s) = 0$. (Then multiply both sides of the equation by $Z_s^{\lambda-1}(\bar{z}_{is} - \alpha_i Z_s)$ and sum over s to get (26)). The following lemma, whose easy proof will not be given, relates to (26) and will be used in the formal proof below.

Lemma 1. Let $d_y, y = 1, ..., n$, be strictly positive numbers and suppose that $\sum_{y=1}^n d_y x_y = 0$ where the x_y are numbers, not all of which are zero. Now consider non-negative numbers $e_y, y = 1, ..., n$, with the property that max $\{e_y : y \in Y^+\} <$ min $\{e_y : y \in Y^-\}$ where $Y^+ = \{y : x_y > 0\}$ and $Y^- = : x_y < 0\}$. Then $\sum_{y=1}^n d_y x_y e_y < 0$.

Proof of Theorem 2 for Identical Isoelastic Utility Functions. With no information each individual $i's$ expected utility, V_i, equals $\frac{1}{\lambda}\sum_s \pi_s Z_s^\lambda \alpha_i^\lambda$ where α_i is given by (24). Now consider the function

$$U_i(\theta) = \frac{1}{\lambda}\sum_y q(y) \sum_s \pi_s^y Z_s^\lambda \alpha_i +$$

Now apply Lemma 1 letting $q(y) = d_y, W_i^y = x_y$ and $[\alpha_i + \theta W_i^y / \sum_s P_s^y Z_s]^{\lambda-1} = e_y$ to show that $U_i^{'(\theta)<0}$, when $\theta > 0$. As pointed out in the model description, the $d_y = q(y)$ are strictly positive. By (26), $\sum_{y=1}^n d_y x_y = 0$. The e_y have the desired proprety since $\lambda < 1$. Either the x_y have the desired property or all the x_y equal 0. In the latter case $U_i(1) = U_i(0)$. In the former case $U_i(1) < U_i(0)$. By the non-trivial public information hypothesis, the x_y do not equal zero for at least one individual, which concludes the proof.

The same method of proof works for the exponential utility function where the risk tolerance parameter can vary with each individual, and we now outline that proof. Letting $R = \sum_i r_i$, it is known that z_{is}, in the exponential utility case, will be of the form $q_i + r_i Z_s$. The price for a security s, P_s, equals $\pi_s e^{-\frac{Z_s}{R}}\cdot$ The value of q_i is $\frac{\sum_s (P_s(\sum_{is} - \frac{r_i Z_s}{R}))}{\sum_s P_s}$.

When a signal y is received, $P_s^y = \pi_s^y e^{-\frac{Z_s}{R}}$. Equation (25) changes to $\sum_s P_s^y \overline{z}_{is} = \sum_s P_s^y (q_i + \frac{r_i Z_s}{R}) + W_i^y$ where $W_i^y = \sum_s (P_s^y - P_s)(\overline{Z}_{is} - (q_i + \frac{r_i Z_s}{R}))$. Equation (26) and the rest of the proof work as before with $U_i(\theta) = -\sum_y q(y) \sum_s \pi_s^y e^{-[\frac{q_i}{r_i} + \frac{\theta W_i^y}{r_i \sum_s P_s} + \frac{Z_s}{R}]}$, and $U_i'(\theta) = W\frac{i}{r_i} \sum_y q(y) W_i^y e^{-[\frac{q_i}{r_i} + \frac{\theta W_i^y}{r_i \sum_s P_s}]}$.

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