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Time Domain Double Diffraction at a Pair of Coplanar Skew Edges

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I. Introduction

A time domain version of the uniform description of double diffraction at a pair of coplanar skew edges is here presented, with source and observation at a finite distance. The time domain (TD) field description is obtained by direct Fourier inversion of the frequency domain (FD) doubly diffracted (DD) field recently developed in [1], [2] for point source excitation and in [3] for line source excitation. There, a high-frequency uniform approximation of the DD field was given using a special transition function that is conveniently expressed in terms of generalized Fresnel integrals (GFIs) [4]. Thus, the TD-DD field response to an impulsive delta excitation, is obtained by Fourier inversion of the high-frequency DD field. This is valid only for *early times*, on and close to (behind) the wavefronts. The TD-DD field response to a more general pulsed excitation is obtained via convolution. If the exciting signal has no low-frequency components and is thus dominated by high frequencies, the range of validity of the resulting pulsed response is enlarged to later observation times behind the wavefront. The present TD-DD field is limited to real time, and matches and compensates the spatial discontinuity of the TD singly diffracted field developed in [5], [6]. Analytic extension of the DD mechanisms to complex time, as in [7], [8], [6] for singly diffracted field, is currently under investigation.

II. Doubly Diffracted Field

Let us consider a pair of wedges with soft/hard boundary conditions (BCs) and coplanar edges, illuminated by a spherical source. It is useful to define a cylindrical (ρ_i, ϕ_i, z_i) and a spherical (r_i, β_i, ϕ_i) ray fixed coordinate system at each edge with origin at the diffraction point Q_i ($i = 1, 2$). Our description of the double diffraction mechanism is constructed first in the FD as the superposition of two analogous mechanisms: a field diffracted from edge 2 when it is illuminated by the field diffracted from edge 1 (12), and that from 1 when it is illuminated by 2 (21), as in [1], and here reported for clarity. In the following, only the contribution 12 will be considered. The ray geometry from the field DD at Q_1 and Q_2 is depicted in Fig.1 with ℓ the distance between the two diffraction points Q_1 and Q_2 , and ϕ_{12} (ϕ'_{12}) the azimuthal coordinate of Q_2 (Q_1) measured in the system at edge 1 (2). FD and TD quantities are related by the Fourier transform pair $\psi(\omega) = \int_{-\infty}^{\infty} \hat{\psi}(t)e^{-j\omega t} dt$, $\hat{\psi}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(\omega)e^{j\omega t} d\omega$ (a caret $\hat{}$ tags time-dependent quantities). We present first the FD high-frequency doubly diffracted field obtained in [1], that is successively transformed into its TD counterpart.

Frequency Domain The singly diffracted field from the first wedge illuminated by a spherical source at $P'(\phi'_1) \equiv (r'_1, \pi - \beta'_1, \phi')$ evaluated at edge 2, is expressed as superposition of spectral spherical sources at $P'(\alpha_1 + \phi_{12} + \pi)$ weighted by the spectral $G^{s,h}(\phi'_1, \alpha_1 + \phi_{12})$ [1], where $G^{s,h}(\phi, \phi') = -\frac{1}{2}[\sec((\phi' - \phi)/2) \mp \sec((\phi' + \phi)/2)]$, with the $- (+)$ sign referring to the soft (hard) BC. Each spherical source provides a diffracted field contribution from edge 2 at the observation point $P(\phi_2) \equiv (r_2, \beta_2, \phi_2)$, that is conveniently calculated using reciprocity, i.e., the diffracted field from edge 2 at $P'(\alpha_1 + \phi_{12} + \pi)$ due to a point source at $P(\phi_2)$ is represented as a summation of spectral spherical sources at $P(\alpha_2 + \phi'_{12} + \pi)$ weighted by the spectral $G^{s,h}(\phi_2, \alpha_2 + \phi'_{12})$. Thus, using the spectral superposition of the field radiated by wedge 2 for all the spectral spher-

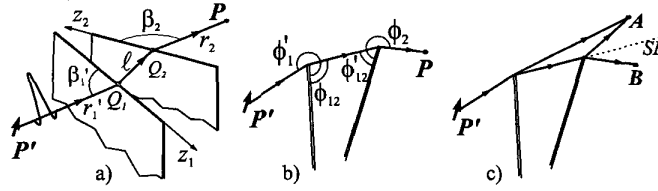


Fig. 1. Geometry of the two half planes. (a) Angles with respect to the edges. (b) Transverse angles. (c) Observer A is reached by both singly and DD fields, while B is reached only by the DD field. The shadow boundary (SB) plane truncates the domain of existence of the singly diffracted field.

ical sources representing the radiation by edge 1, leads to the the double integral representation

$$\psi_{12}^{dd} = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-jkR(\alpha_1, \alpha_2)}}{4\pi R(\alpha_1, \alpha_2)} G(\phi_1', \alpha_1 + \phi_{12}) G(\phi_2, \alpha_2 + \phi_{12}') d\alpha_1 d\alpha_2, \quad (1)$$

in which $R(\alpha_1, \alpha_2) = |P'(\alpha_1 + \phi_{12} + \pi) - P(\alpha_2 + \phi_{12}' + \pi)|$ is the distance between the spectral source and observation whose explicit expression is given in [1], and $k = \omega/c$ with c the ambient wavespeed. The integrand in (1) exhibits a two-dimensional stationary phase point at $(\alpha_1, \alpha_2) = (0, 0)$ that provides the DD ray field contribution, and poles in each variable given by the G functions at $\alpha_1 = \Phi_1^p = \phi_1' + (-1)^p \phi_{12} + \pi$ ($p = 1, 2$), and $\alpha_2 = \Phi_2^q = \phi_2 + (-1)^q \phi_{12}' + \pi$ ($q = 1, 2$). As shown in [1], first the integrand is decomposed in its even and odd part with respect to $(\alpha_1, \alpha_2) = (0, 0)$, then it is evaluated asymptotically through saddle point in a uniform way with respect to the poles of the G functions, leading to $\psi_{12}^{dd} \sim \psi^{inc} A(r_1', \ell, r_2) \exp(-jk(r_1' + \ell + r_2)) D_{12}^{s,h}$, in which $\psi^{inc} = \exp(-jkr_1')/(4\pi r_1')$ is the incident field at edge 1 (at Q_1), and $A(r_1', \ell, r_2) = \sqrt{r_1'}/\sqrt{\ell r_2(r_1' + \ell + r_2)}$ is the spreading factor. The diffraction coefficient for the soft (s) and hard (h) cases are represented as $D_{12}^{s,h} = D_{12}^{I,s,h} + D_{12}^{II,s,h}$ where, for space limitations, only the first order

$$D_{12}^{I,s,h} = \frac{1}{8\pi jk \sin \beta_1' \sin \beta_2} \sum_{p,q=1}^2 (\mp 1)^{p+q} \frac{1}{\sin \frac{\phi_1^p}{2} \sin \frac{\phi_2^q}{2}} T^I(\sqrt{\omega} \hat{a}_p, \sqrt{\omega} \hat{b}_q, w) \quad (2)$$

is treated and TD-inverted in the following. According to [1], the transition function

$$T^I(a, b, w) = \frac{jab}{\sqrt{1-w^2}} \sum_{i=1,2} \left[G\left(a, \frac{b + (-1)^i wa}{\sqrt{1-w^2}}\right) + G\left(b, \frac{a + (-1)^i wb}{\sqrt{1-w^2}}\right) \right] \quad (3)$$

is here conveniently represented as combination of generalized Fresnel integrals (GFI) [4]

$$G(x, y) = ye^{jx^2} \int_x^\infty \frac{e^{-jv^2}}{v^2 + y^2} dv, \quad G(-x, y) = -G(x, y). \quad (4)$$

The parameters of the T^I function in (2), $\hat{a}_p = \sqrt{2/c} \sin \beta_1' \sqrt{r_1' \ell / (r_1' + \ell)} \sin(\Phi_1^p/2)$, $\hat{b}_q = \sqrt{2/c} \sin \beta_2 \sqrt{r_2 \ell / (r_2 + \ell)} \sin(\Phi_2^q/2)$, and $w = (r_1' r_2)^{1/2} [(r_1' + \ell)(r_2 + \ell)]^{-1/2}$, are all independent of ω . The normalization with respect to $\sqrt{\omega}$ has been made

to prepare T^I for its Fourier inversion. Similar expressions apply for the higher order contribution $D_{12}^{I,s,h}$ (see [1]).

Time Domain The TD version of the DD field $\hat{\psi}_{12}^{dd}$ is obtained through direct Fourier inversion of the FD high-frequency field $\hat{\psi}_{12}^{dd}$. Except for the exponential term $e^{-jk(r'_1 + \ell + r_2)}$ ($k = \omega/c$), the ω variable compares at the denominator of the coefficient $D_{12}^{I,s,h}$ and in the parameters of the transition function T^I in (2). Therefore, the TD-DD field is given by

$$\hat{\psi}_{12}^{dd} \sim A^{inc} A(r'_1, \ell, r_2) \hat{D}_{12}^{s,h}(\tau_r), \quad \tau_r = t - (r'_1 + \ell + r_2)/c \quad (5)$$

where $A^{inc} = 1/(4\pi r'_1)$ is the incident spreading factor at Q_1 , and $\hat{D}_{12}^{s,h}(t) = \hat{D}_{12}^{I,s,h}(t) + \hat{D}_{12}^{II,s,h}(t)$ is the Fourier inverted TD double diffraction coefficient evaluated at the retarded time τ_r . As before, only the first order

$$\hat{D}_{12}^{I,s,h}(t) = \frac{1}{8\pi \sin \beta'_1 \sin \beta_2} \sum_{p,q=1}^2 (\mp 1)^{p+q} \frac{1}{\sin \frac{\phi_p}{2} \sin \frac{\phi_q}{2}} \hat{T}^I(t, \hat{a}_p, \hat{b}_q, w) \quad (6)$$

is here discussed. Note that the factor jk at the denominator of (2) is included in the Fourier inversion of $(jk)^{-1} T^I(\sqrt{\omega} \hat{a}_p, \sqrt{\omega} \hat{b}_q, w)$ leading by definition to the TD transition function $\hat{T}^I(t, \hat{a}_p, \hat{b}_q, w)$ represented as combination of TD-GFI

$$\hat{T}^I(t, \hat{a}, \hat{b}, w) = \frac{c\hat{a}\hat{b}}{\sqrt{1-w^2}} \sum_{i=1}^2 \left[\hat{G} \left(t, \hat{a}, \frac{\hat{b} + (-1)^i w \hat{a}}{\sqrt{1-w^2}} \right) + \hat{G} \left(t, \hat{b}, \frac{\hat{a} + (-1)^i w \hat{b}}{\sqrt{1-w^2}} \right) \right] \quad (7)$$

It is convenient to limit our analysis to the real part of the positive ω spectrum, therefore using $\hat{G}(t, x, y) = \Re e \frac{1}{\pi} \int_0^\infty G(\sqrt{\omega} x, \sqrt{\omega} y) e^{j\omega t} d\omega$, to avoid definitions of the square root $\sqrt{\omega}$ for $\omega < 0$ and inherent definition of the FD-GFI in (4) for complex parameters. Using the integral representation (4), the ω and v orders of integration are interchanged (allowed specifying $\Im m t > 0$, that is eventually removed)

$$\hat{G}(t, x, y) = \Re e \frac{y}{\pi} \int_x^\infty dv \frac{1}{v^2 + y^2} \int_0^\infty d\omega e^{j\omega(t+x^2-v^2)} \quad (8)$$

We limit our present preliminary formulation to real time signals, leaving the more general analytic signal formulation to future studies. To this end, we use the identity $\Re e \frac{1}{\pi} \int_0^\infty d\omega e^{j\omega(t+x^2-v^2)} = \delta(t+x^2-v^2)$ for t real. Next, recalling that $\delta[f(v)] = \delta(v-v_s) |df/dv|_{v_s}^{-1}$ when $f(v_s) = 0$, in the evaluation of the v integral in (8) we consider only positive solutions $v_s = \sqrt{t+x^2}$ on the integration domain (x, ∞) , i.e., $v_s > x$ for $t > 0$. Eventually, using $|df/dv|_{v_s} = 2v_s = 2\sqrt{t+x^2}$, the evaluation of the v integral leads to the TD-GFI

$$\hat{G}(t, x, y) = \frac{y}{2} \frac{U(t)}{(t+x^2+y^2)\sqrt{t+x^2}}, \quad \hat{G}(t, -x, y) = -\hat{G}(t, x, y). \quad (9)$$

Far from the shadow boundaries (SBs) (see Fig.1c), where $|\hat{a}|$ and $|\hat{b}| \rightarrow \infty$, it can be easily verified that $T^I(t, \hat{a}, \hat{b}, w) \rightarrow cU(t)$, whence the DD wave phenomenology acts like a time integrator. As shown in Fig.2a, the time dependence of the TD-DD field is shaped as the primitive of the exciting Rayleigh pulse. This is consistent with the $1/\omega$ -frequency dependence of the FD-DD field.

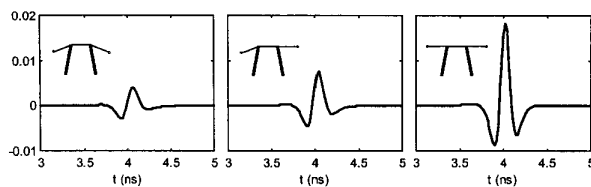


Fig. 2. TD-DD field ψ_{12}^{dd} in three distinct wave regimes. Geometry: $r_1 = 42\text{cm}$; $\ell = 45\text{cm}$; $r_2 = 33\text{cm}$; $\beta_1 = 100^\circ$; $\beta_2 = 50^\circ$; $\phi_{12} = 100^\circ$; $\phi_{12} = 100^\circ$, hard BC. Band limited excitation: normalized Rayleigh pulse $\hat{G}(t) = \Re\{e^{j(j + 2\pi f_M t/4)^5}\}$; central frequency $f_M = 3\text{GHz}$ ($\lambda_M = c/f_M = 10\text{cm}$). a) Far from transition regions ($\phi_1 = 310^\circ$, $\phi_2 = 310^\circ$). b) Observer in transition region ($\phi_1 = 310^\circ$, $\phi_2 = 281^\circ \approx \phi_1 + 180^\circ$). c) Source and observer both in transition region ($\phi_1 = 281^\circ$, $\phi_2 = 281^\circ$).

Shadow Boundary Limits. Let us consider first the case when the observation point crosses the plain containing edges 1 and 2, depicted in Fig.1c. There, the singly diffracted field from edge 1 is spatially discontinuous at any time, due to the shadowing by edge 2. At this aspect, $\phi_2 \rightarrow \phi_{12} + \pi$, so that $\Phi_{12}^{\frac{1}{2}} = 2\pi$ and consequently $\hat{b}_1 = 0$ (see [1]). Using the limit $\hat{G}(t, x \rightarrow 0, y) = \frac{\sqrt{\pi/c}}{2y} f(y^2, t)$, being $f(y^2, t) = \sqrt{c/\pi y^2} U(t)/[\sqrt{i(t+y^2)}]$ the TD transition function of the TD-UTD [6], it can be shown after substitution in (7), (6), and (5) that $\psi_{12}^{dd} = -\frac{1}{2}\text{sgn}(\hat{b}_1)\psi_1^d$, with ψ_1^d the TD-UTD singly diffracted field [6], so that compensating for the discontinuity of ψ_1^d at the SB $\phi_2 = \phi_{12} + \pi$, at and after the wavefront. At this SB limit, the ψ_{12}^{dd} time dependence recovers the well known $1/\sqrt{t}$ behaviour of the singly diffracted field, as shown in Fig.2b. Analogously, when only the source crosses the plane containing the edges, is the singly diffracted field at edge 2 (ψ_2^d) that is now shadowed. The DD field ψ_{12}^{dd} compensates for the spatial discontinuity of ψ_2^d given by its abrupt appearance or disappearance. When the source and the observer are both close to the plane containing the edges, both $\hat{a}, \hat{b} \rightarrow 0$ and $\psi_{12}^{dd} \sim 1/4\text{sgn}(\hat{a}\hat{b}) \hat{\psi}^{inc}(P)$, i.e. the DD field reduces to one fourth of the free-space direct contribution of the source, allowing the simultaneous compensation for the appearing/disappearing of the Geometrical Optics and of the two close-to-transition singly diffracted fields, restoring the continuity of the total field at and after the wavefront. At this double transition regime, the DD field time dependence is the same of the exciting pulse (Fig.2c).

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