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# Modern theory of Fermi acceleration: A new challenge to plasma physics\*

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One of the main features of astrophysical shocks is their ability to accelerate particles to extremely high energies. The leading acceleration mechanism, the diffusive shock acceleration, is reviewed. It is demonstrated that its efficiency critically depends on the injection of thermal plasma into acceleration which takes place at the subshock of the collisionless shock structure that, in turn, can be significantly smoothed by energetic particles. Furthermore, their inhomogeneous distribution provides free energy for magnetohydrodynamic (MHD) turbulence regulating the subshock strength and injection rate. Moreover, the MHD turbulence confines particles to the shock front controlling their maximum energy and bootstrapping acceleration. Therefore, the study of the MHD turbulence in a compressive plasma flow near a shock is a key to the understanding of the entire process. The calculation of the injection rate became part of the collisionless shock theory. It is argued that the further progress in diffusive shock acceleration theory is impossible without a significant advance in these two areas of plasma physics. © 2001 American Institute of Physics.

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## I. INTRODUCTION

Over the last few years, new observations and missions, e.g., the Energetic Gamma-ray Experiment Telescope (EGRET),<sup>1</sup> *Chandra*,<sup>2</sup> and TeV-astronomy,<sup>3</sup> have revolutionized the measurement of radiation from a variety of objects in the Universe. In most cases the *primary* source of the radiation is believed to be accelerated charged particles, often of remarkably high energies, such as  $10^{20}$  eV or even higher,<sup>4</sup> usually referred to as the ultra-high-energy cosmic rays (UHECR). The accelerated particles themselves (we will also use the term cosmic rays, CRs) are in many cases generated by shock waves (shocks). Note that the latter are the major events where the huge energy of stars, supernovas (SN) or black holes, is released in bulk gas motions and ultimately dissipated. The most successful particle acceleration mechanism is, perhaps, the diffusive shock acceleration (DSA),<sup>5,6</sup> which is a variant of the original Fermi idea (1949), also known as the first-order Fermi acceleration process. According to this mechanism particles gain energy by bouncing off hydromagnetic disturbances frozen into the converging upstream and downstream flow regions near a shock. Clearly, the understanding of this mechanism is critical for radiation models, since the primary particle spectrum is one of their most important “input” characteristics. The success of this mechanism has been due to the following appealing features:

- (1) reproduction of the power-law energy spectra with an index remarkably similar to that inferred from the observations of the galactic cosmic rays (CRs),<sup>5,6</sup>

- (2) convincing, direct observational evidence of its operation at interplanetary shocks and the earth bow shock,<sup>7-9</sup> and
- (3) the absence of any obvious intrinsic limitation to the maximum particle energy achievable by this process.

Often, however, this mechanism is applied in its simplified version, the so-called test particle (TP) or linear approximation (that neglects the backreaction of accelerated particles on the shock structure and produces a simple  $E^{-2}$  particle energy distribution). Although some more realistic, nonlinear theories do exist, they suffer from parametrization of particle transport coefficients, such as the pitch-angle scattering, spatial diffusivity, and plasma heating.

Also, it should be emphasized that the statement (2) above is only a *prima facie* evidence for the responsibility of this mechanism for galactic CRs, alluded to in (1). Despite similarity between physical parameters,<sup>6</sup> the lifetimes and extensions of shocks in these two environments are vastly different (up to a factor  $\sim 10^{10}$ ). This requires different approximations for these shocks. Namely, the treatment of large astrophysical shocks, such as Supernova remnant (SNR) shells, shocks in the lobes of radio galaxies, or even larger shocks in clusters of galaxies, *must* include the back reaction of accelerated particles on the shock structure, and thus be *intrinsically nonlinear*. This, in turn, necessitates the self-consistent treatment of the above-mentioned anomalous transport phenomena.

From the perspective of the widely accepted SNR shock hypothesis of the origin of galactic CRs, perhaps, the most critical impact of this treatment would be on the statement (3) above. It implies that the maximum energy is limited only by the time available for acceleration and by the size of accelerating object (roughly not to be exceeded by the diffu-

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sive escape length for particles, which grows with energy). This is becoming a “hot” issue in view of the lack of evidence for TeV-protons in SNR shocks (despite the signature of GeV-protons, EGRET), which might indicate that the particle spectrum either cuts off somewhere between GeV and TeV energies (i.e., in an energy range currently uncovered by any instrument) or probably significantly steepens there. We must await observations from the Gamma-ray Large Area Space Telescope (GLAST) and the next generation of the imaging atmospheric Cherenkov telescopes that are currently being built<sup>3</sup> to see where and how the spectrum disappears or changes. To understand how this *may* happen, one needs to consider all the crucial requirements for this mechanism, which are as follows:

- (i) the ability of some fraction of thermal particles downstream to make the first step to acceleration, i.e., to return upstream (injection problem);
- (ii) good confinement of accelerated particles near the shock front (to continue energy gain); and
- (iii) the ability of the shock to withstand the pressure of accelerated particles (the shock smoothing, or shock robustness problem).

These requirements are also associated with the main difficulties of the theory and are, in fact, strongly related. We consider them briefly, in order.

### A. Injection problem

Injection is the process of initial particle energization. Within the widely accepted “thermal leakage” scenario of injection, the way protons enter the shock acceleration process is physically the same as when they are accelerated afterwards (see, e.g., Ref. 10). After thermalization downstream, a certain fraction of particles will catch up with the shock. Their further leakage upstream generates Alfvén waves via the cyclotron resonance  $k p \mu \approx e B / c$ , where  $\mu$  is the cosine of the proton pitch angle, and  $p$  is the particle momentum. These waves do two things. First, they self-regulate the thermal particle leakage (injection) by trapping particles downstream when their leakage becomes too strong and, therefore, the wave amplitude too large as well.<sup>11</sup> Second, the large amplitude waves scatter already accelerating particles in pitch angle, thus ensuring their diffusive confinement near the shock.

According to the above resonance condition, waves excited by protons (with  $k p_p \sim 1$ ) are too long to scatter electrons, so that they interact with them adiabatically ( $k p_e \ll 1$ ). Therefore, electrons need a separate injection scenario. One suggestion is that they might be injected via scattering on self-generated whistler waves.<sup>12</sup> According to another mechanism suggested in Ref. 13, electrons are extracted directly from the thermal pool near the shock front via their interaction with proton-generated lower-hybrid waves and the electrostatic barrier formed there due to this interaction.

It should be noted that when the both species become relativistic and, if the synchrotron cooling time is longer than the acceleration time, the electron spectrum should be identical to that of the protons. However, it is electrons for which

we have convincing direct evidence of Fermi acceleration in SNR shells. There are well-documented measurements of nonthermal electron emission in radio, x rays and possibly also in  $\gamma$  rays, presented, e.g., in Refs. 14–16, respectively. Although electrons play perhaps a dynamically unimportant role in the shock structure, they should trace the dynamically important proton spectra.

### B. Particle confinement near the shock

As in the injection phase, further particle confinement near the shock is supported by self-generated Alfvén wave turbulence, since the energetic particle distribution ahead of the shock is ion-cyclotron unstable (see, e.g., Ref. 8). Due to rapid pitch-angle scattering on these waves, particles cross the shock repeatedly, thus gaining energy. Clearly, the scattering frequency (turbulence level) sets up the acceleration rate. Note that the existing background magnetohydrodynamic (MHD) turbulence in the interstellar medium (ISM) would support only very slow acceleration. A long-standing problem is that the wave generation process is likely to be so robust that wave amplitude may far exceed the level admissible by the quasi-linear theory. It is usually expected that the turbulence saturates at a level  $\delta B \sim B_0$ . Thus, particle scattering must occur via strongly nonlinear wave–wave and wave–particle interactions, so that the conventional quasi-linear description should be replaced by a nonperturbative approach.

MHD simulations have significantly advanced recently (see, e.g., Ref. 17 and references therein) but they would not suffice alone to self-consistently describe the wave generation by turbulently confined particles. The main difficulty is, as it will be seen in the sequel, the enormous extension of particle and wave spectra. It seems desirable to combine simulations in restricted but critical parts of the phase space (as the short-wave, low-energy part that controls the injection and the long-wave, high-energy part where particle losses occur) with an analytical approach in extended but tractable parts of the phase space where particle and wave spectra and even the flow profile exhibit relatively simple, scale invariant behavior.<sup>18</sup>

It should be evident that a purely numerical approach would encounter serious difficulties if applied to the shock acceleration of UHECRs. Indeed, even if we had a numerical solution of the problem of CR acceleration in SNR shocks, i.e., up to energies  $\sim 10^{15}$  eV, we would need to extend its dynamical range by a factor of  $\sim 10^5$ . Since the integration domain in configuration space is typically proportional to the maximum energy, this would mean  $\sim 10^{10}$  times larger phase space. It seems natural to use this large size of the phase space instead of letting a computer process decade after decade of it with basically the same physics. The analytic approach that we introduce in Sec. II B utilizes the corresponding small parameters and, therefore, appears particularly suitable to the problem of acceleration of UHECRs.

### C. Shock robustness

Due to the secular growth of the pressure of accelerated particles, the linear solution becomes invalid for shocks of

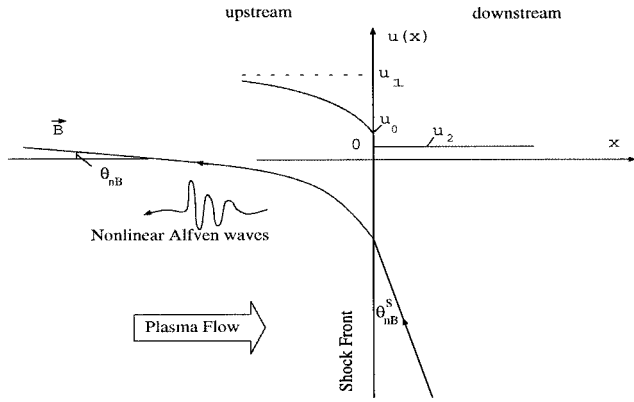


FIG. 1. Schematic representation of nonlinearly accelerating shock. Flow profile with a gradual deceleration upstream is also shown at the top.

sufficient life time, scale  $L$ , and magnetic field  $B$  [the most critical parameter is  $BL$  since the maximum particle energy scales as  $E_{\max} \sim (ulc)eBL$ , provided that enough time is available for acceleration, as discussed, e.g., in Ref. 6]. The main nonlinear effect is due to the back reaction of energetic particles on the flow, which reduces the velocity jump at the flow discontinuity (subshock) through the deceleration and heating of the plasma in front of it (i.e., in the so-called CR precursor) (Fig. 1). The total shock compression, however, increases due to the decrease in the adiabatic index  $\gamma$  caused by the presence of relativistic particles and, even more importantly, by their escape from the system (akin to radiative shocks). At the same time, the maintenance of a finite subshock within the global shock structure (subshock itself, plus CR precursor) is critical to the injection process and thus for acceleration in general. A mathematical limitation of the TP approximation can be obtained in terms of particle energy and injection rate  $\nu$ , namely,  $\nu\sqrt{E_{\max}} \lesssim 1$ ,<sup>19</sup> which states that the pressure of accelerated particles must remain smaller than the shock ram pressure. It is also clear from this condition that the above three issues are strongly coupled. Injection ( $\nu$ ) and particle confinement (which sets  $E_{\max}$ ) determine the shock structure and are, in turn, regulated by it. The acceleration time scale  $\tau_{\text{acc}}$  (or particle diffusivity  $\kappa$ ) and the precursor turbulent heating rate are also implicitly involved in this “feedback loop.” There are indeed too many shock variables to obtain reliable prediction by simply scanning the parameter space. Therefore, they need to be calculated self-consistently before the particle spectra can be calculated and compared with the observations.

## II. NONLINEAR THEORIES

A typical nonlinearly accelerating shock is illustrated in Fig. 1. The most “visible” nonlinear effects are (1) the deceleration of the flow upstream (see the flow profile on the top of Fig. 1) by the pressure of accelerated particles, (2) subshock reduction, as a consequence (its strength  $u_0/u_2$  may become small compared to the total compression  $u_1/u_2$ ), and (3) bending of the B-field due to the compression of its tangential component because of the frozen in condition,  $uB_{\parallel} = \text{const}$ , while the normal component is conserved due to  $\text{div } \mathbf{B} = 0$ .

## A. Two-fluid model

Initially, the back reaction of accelerated particles (CRs) on the shock structure was studied within the two-fluid model (TFM). This model treats CRs as a second fluid carrying the momentum and energy across the shock, but not the mass. Complete solutions given in Refs. 20 and 21 indeed revealed a very strong back reaction of the CRs onto the bulk plasma flow, leading to a bifurcation of the simple linear solution into a strongly nonlinear (efficient) solution with the acceleration efficiency approaching (in strong shocks) 100% due to the nonlinearly enhanced shock compression. This gives rise to a formally diverging (i.e., in reality, strongly dependent on the maximum energy) CR pressure, which makes the particle losses at the highest energies as important dynamically as injection, and strongly related to it through their feedback on the subshock strength. Moreover, these losses are controlled by the Alfvén turbulence which, in turn, depends on the CR distribution in the shock precursor and thus on the losses themselves. Unfortunately, the TFM, being a hydrodynamic theory, cannot be closed in such a way that these essentially kinetic effects are properly represented.

## B. Kinetic theory

The earlier kinetic theories also demonstrated that *upon accumulating enough CR energy, strong shocks develop into the nonlinear regime*. The minimal kinetic theory that captures bifurcations of the DSA can be formulated as follows.

We describe the distribution of high-energy particles (CRs) by the so-called diffusion-convection equation (see, e.g., Refs. 5 and 6). The gaseous discontinuity (the subshock) is assumed to be located at  $x=0$  and, for convenience, we flip the  $x$ -coordinate in Fig. 1, so that the upstream side is  $x>0$  half-space. Thus, the flow velocity in the shock frame can be represented as  $V(x) = -u(x)$ , where the (positive) flow speed  $u(x)$  jumps from  $u_2 \equiv u(0-)$  downstream to  $u_0 \equiv u(0+) > u_2$  across the subshock and then gradually grows upstream up to  $u_1 \equiv u(+\infty) \geq u_0$ . In a steady state the equation can be written as

$$u \frac{\partial f}{\partial x} + \kappa(p) \frac{\partial^2 f}{\partial x^2} = \frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p}, \quad (1)$$

where  $f(x, p)$  is the isotropic (in the local fluid frame) part of the particle distribution. This is assumed to vanish far upstream ( $f \rightarrow 0, x \rightarrow \infty$ ), while the only bounded solution downstream is obviously  $f(x, p) = f_0(p) \equiv f(0, p)$ . The most plausible assumption about the cosmic ray diffusivity  $\kappa(p)$  is that of the Bohm type, i.e.,  $\kappa(p) = Kp^2/\sqrt{1+p^2}$  (the particle momentum  $p$  is normalized to  $mc$ ). In other words,  $\kappa$  scales as the gyroradius,  $\kappa \sim r_g(p)$ . The reference diffusivity  $K$  depends on the  $\delta B/B$  level of the Alfvénic turbulence that scatters the particles in pitch angle. The minimum value for  $K$  would be  $K \sim mc^3/eB$  if  $\delta B \sim B$ . Note that the replacement of this plain parametrization by a self-consistent solution for the spectrum of turbulence driven by an inhomogeneous distribution of accelerated particles is a challenge to

plasma physics. The existing quasi-linear approaches clearly fail in the efficient acceleration regime due to unacceptable wave amplitudes.

The determination of  $u(x)$  in Eq. (1) requires three further equations. The first one is the conservation of the momentum flux in the smooth part of the shock transition ( $x > 0$ , i.e., in the CR-precursor),

$$P_c + \rho u^2 = \rho_1 u_1^2, \quad x > 0, \quad (2)$$

where  $P_c$  is the pressure of the high-energy particles

$$P_c(x) = \frac{4\pi}{3} mc^2 \int_{p_0}^{p_1} \frac{p^4 dp}{\sqrt{p^2 + 1}} f(p, x). \quad (3)$$

It is assumed here that there are no particles with momenta  $p > p_1$  (they leave the shock vicinity because there are no MHD waves with sufficiently long wave length  $\lambda$ , since the cyclotron resonance requires  $p \sim \lambda$ ). The momentum region  $0 < p < p_0$  cannot be described by Eq. (1) and the behavior of  $f(p)$  at  $p \sim p_0$  is described by the injection parameters  $p_0$  and  $f(p_0)$ .<sup>19</sup> The plasma density  $\rho(x)$  can be eliminated from Eq. (2) by using the continuity equation  $\rho u = \rho_1 u_1$ . Finally, the subshock strength  $r_s$  can be expressed through the Mach number  $M$  at  $x = \infty$

$$r_s \equiv \frac{u_0}{u_2} = \frac{\gamma + 1}{\gamma - 1 + 2R^{\gamma+1}M^{-2}}, \quad (4)$$

where the precursor compression  $R \equiv u_1/u_0$  and  $\gamma$  is the adiabatic index of the plasma.

The system of equations (1), (2), and (4) describes in a self-consistent manner the particle spectrum and the flow structure, although under parametrization of such critical quantities as  $\nu$  and  $p_1$ . Our poor knowledge of the maximum momentum  $p_1$  is related to the prescribed form of particle diffusivity  $\kappa(p)$ .

It is useful to reduce this system to one integral equation.<sup>19</sup> A key dependent variable is an integral transform of the flow profile  $u(x)$  with a kernel suggested by an asymptotic solution of the system (1) and (2), which has the form

$$f(x, p) = f_0(p) \exp\left[-\frac{q}{3\kappa} \Psi\right],$$

where

$$\Psi = \int_0^x u(x') dx'$$

is the flow potential and the spectral index downstream  $q(p) = -d \ln f_0 / d \ln p$ . The integral transform is as follows,

$$U(p) = \frac{1}{u_1} \int_{0^-}^{\infty} \exp\left[-\frac{q(p)}{3\kappa(p)} \Psi\right] du(\Psi), \quad (5)$$

and it is related to  $q(p)$  through the following formula

$$q(p) = \frac{d \ln U}{d \ln p} + \frac{3}{r_s R U(p)} + 3. \quad (6)$$

The physical meaning of the function  $U(p)$  is very simple. It reflects the degree of shock modification. Namely, a function

$U(p) + u_2$  is an effective flow velocity upstream as seen by a particle with momentum  $p$  that diffusively escapes ahead of the shock to a point  $x$  where the flow speed is  $u(x) = U(p) + u_2$ . Once  $U(p)$  is found, both the flow profile and the particle distribution can be determined by inverting transform (5) and integrating equation (6). Now, using the linearity of Eq. (2) ( $\rho u = \text{const}$ ), we derive the integral equation for  $U$  by applying the transformation (5) to the  $x$ -derivative of Eq. (2).<sup>19</sup> The result reads

$$U(t) = \frac{r_s - 1}{R r_s} + \frac{\nu}{K p_0} \int_{t_0}^{t_1} dt' \left[ \frac{1}{\kappa(t')} + \frac{q(t')}{\kappa(t) q(t)} \right]^{-1} \\ \times \frac{U(t_0)}{U(t')} \exp\left[ -\frac{3}{R r_s} \int_{t_0}^{t'} \frac{dt''}{U(t'')} \right], \quad (7)$$

where  $t = \ln p$  and  $t_{0,1} = \ln p_{0,1}$ . Here the injection parameter

$$\nu = \frac{4\pi}{3} \frac{mc^2}{\rho_1 u_1^2} p_0^4 f_0(p_0) \quad (8)$$

is related to  $R$  by means of the following equation:

$$\nu = K p_0 (1 - R^{-1}) \left\{ \int_{t_0}^{t_1} \kappa(t) dt \right. \\ \left. \times \frac{U(t_0)}{U(t)} \exp\left[ -\frac{3}{R r_s} \int_{t_0}^t \frac{dt'}{U(t')} \right] \right\}^{-1}. \quad (9)$$

The equations (4), (7), and (9) form a closed system that can be easily solved, e.g., numerically. Asymptotic analytic solutions are also available.<sup>19</sup> The physical quantities involved are the far upstream Mach number  $M$  (which is given, external parameter); internal parameters  $M_0$  (the Mach number at the subshock), the injection rate  $\nu$ , the particle maximum momentum  $p_{\text{max}} \equiv p_1$ , and particle diffusivity  $\kappa(p, x)$ . These internal parameters must be determined self-consistently, but currently are parametrized or calculated using some simplifying assumption. For example, assuming that there is no turbulent heating in the shock precursor (which is doubtful in such a turbulent environment), the parameter  $M_0 = M/R^{(\gamma+1)/2}$  [see Eq. (4)].

### III. CRITICAL NATURE OF ACCELERATION PROCESS

The presence of bifurcation in this acceleration process is best seen in variables  $R, \nu$ . The quantity  $R - 1$  is a measure of shock modification produced by CRs, in fact  $(R - 1)/R = P_c(0)/\rho_1 u_1^2$  [Eq. (2)] and may be regarded as an order parameter. The injection rate  $\nu$  characterizes the CR density at the shock front and can be tentatively treated as a control parameter. It is convenient to plot the function  $\nu(R)$  instead of  $R(\nu)$  [using Eqs. (9) and (4)] (Fig. 2).

In fact, the injection rate  $\nu$  at the subshock should be calculated given  $r_s(R)$  with the self-consistent determination of the flow compression  $R$  on the basis of the  $R(\nu)$  dependence obtained. However, in view of a very strong, even nonunique dependence  $R(\nu)$ , this solution can be physically meaningful only in regimes far from criticality, i.e., when  $R \approx 1$  (test particle regime) or  $R \gg 1$  (efficient acceleration). There are, however, self-regulating processes that should drive the system towards the critical region [where  $R(\nu)$

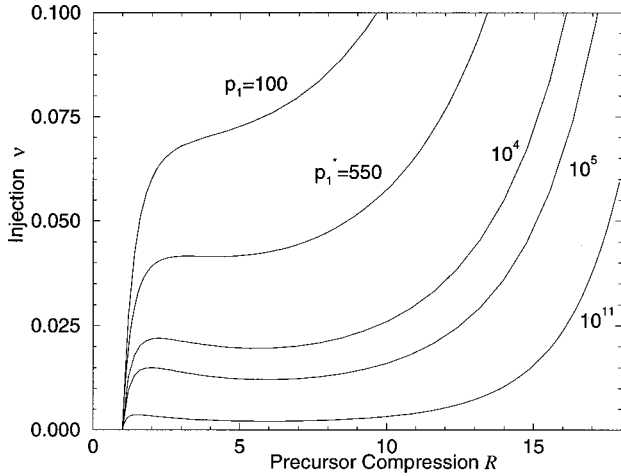


FIG. 2. Response of shock structure (bifurcation diagram) to the injection of thermal particles at the rate  $\nu$ . The strength of the response is characterized by the precompression of the flow in the CR shock precursor  $R = u_1/u_0$  (see Fig. 1). The flow Mach number  $M=150$ ; different curves correspond to different values of maximum momentum denoted here as  $p_1$  and normalized to  $mc$ . For each given  $\nu$  and  $p_{\max}$  one or three solutions exist.

dependence is very sharp].<sup>18,22</sup> First, if  $\nu$  is subcritical it will inevitably become supercritical when  $p_1$  grows in course of acceleration. Once this has happened, however, the strong subshock reduction [Eq. (4)] will reduce  $\nu$ , as well, and drive the system back to the critical regime (Fig. 3).

The maximum momentum  $p_1$  is subject to self-regulation, as well. Indeed, when  $R \gg 1$ , the generation and propagation of Alfvén waves is characterized by strong inclination of the characteristics of the wave transport equation towards larger wave numbers  $k$  on the  $k-x$  plane due to wave compression. Thus, considering particles with  $p \lesssim p_1$  inside the precursor, one sees that they are in resonance with waves that must have been excited by particles with  $p > p_1$  further upstream, but there are no particles with  $p > p_1$ . Therefore, the required waves can be excited only locally by the same particles with  $p \lesssim p_1$ , which substantially diminishes the amplitude of waves that are in resonance with par-

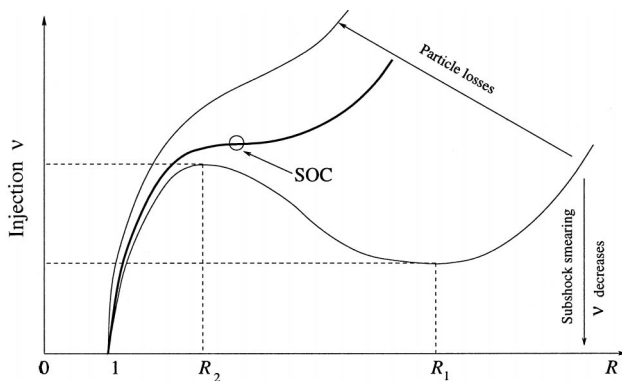


FIG. 3. Bifurcation diagram corresponding to the set of response curves shown in Fig. 2. Since  $\nu$  and  $p_{\max}$  are in reality dynamic rather than control parameters, the response curve moves towards the bifurcation curve drawn with the heavy line. The resulting state of the system corresponds to the critical (inflection) point on this curve, which can be described as a “self-organized critical” (SOC) state.

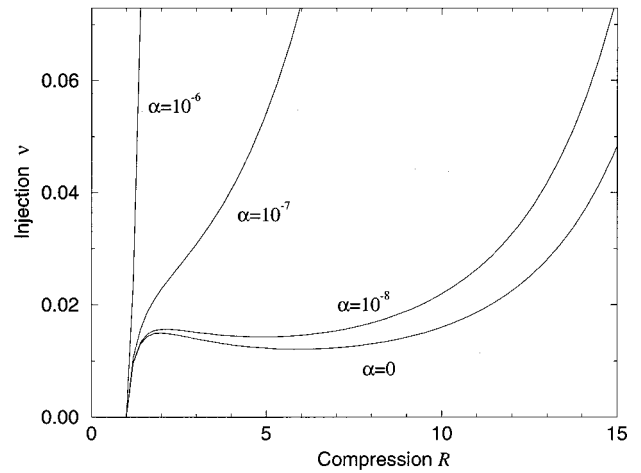


FIG. 4. Bifurcation diagrams as in Fig. 2 but for different heating rates  $\alpha$  (see text), for Mach number  $M=150$  and the cutoff momentum  $p_{\max}=10^5$ .

ticles from the interval  $p_1/R < p < p_1$ . [The left inequality arises from the resonance condition  $kcp \approx eB/mc$  and the frequency conservation along the characteristics  $ku(x) = \text{const}$ .] This will worsen the confinement of these particles to the shock front. The quantitative study of this process is another challenge to the theory of plasma turbulence. What can be inferred from Fig. 2 now is that the decrease of  $p_1$  straightens out and raises the curve  $\nu(R)$ , so that it returns to the monotonic behavior. However, once the actual injection becomes subcritical (and thus  $R \rightarrow 1$ ), then  $p_1$  will grow again restoring the two extrema on the curve  $\nu(R)$ .

Also the turbulent precursor heating straightens up the bifurcation diagram and returns it to the critical state. We illustrate this in Fig. 4 for different heating efficiencies  $\alpha$ , introduced phenomenologically through the decrease in the flow Mach number across the CR precursor:

$$M_0^{-2} = M^{-2} R^{8/3} + \frac{\alpha}{3} (R-1) p_{\max}. \quad (10)$$

Here the first term is due to the familiar adiabatic compression, while the second describes the turbulent heating, assumed to be proportional to the CR pressure contrast  $\propto R - 1$  (source of the free energy for turbulence), as well as to the precursor length  $\propto p_{\max}$ . The plots in Figs. 2 and 4 demonstrate that parametrization approaches in which the internal parameters (like  $\alpha$  and  $p_{\max}$  in many simulations) are treated as given may be useful only when (and if) these parameters are well outside the critical region, which is marked by very sharp or even nonunique parameter dependence. For example, as seen from Fig. 4, if  $\alpha$  (which is essentially unknown) lies within the interval  $10^{-8} - 10^{-7}$ , then there are no means to calculate the flow structure ( $R$ ), and thus the particle spectrum with a reasonable accuracy. To treat the injection rate in the critical region as a “control” parameter is equally useless, Figs. 2 and 4.

It appears to be more productive to assume a marginal state in which the maximum and the minimum of  $\nu(R)$  merge, at least in a sense of an averaged (in time and space) process. This means that  $\nu'(R) = \nu''(R) = 0$  at some  $R$

$=R_c$ . These two equations and the dependence  $\nu(R)$  itself not only determine  $R_c$  and  $\nu_c \equiv \nu(R_c)$ , but also provide an additional relation that involves other parameters of the problem which clearly enter the function  $\nu(R)$ . These are the Mach number  $M$ , the heating rate  $\alpha$ , and the maximum momentum  $p_{\max} \equiv p_1$ . For example, given  $M$  and  $\alpha$  we can easily calculate  $p_{\max}$ .<sup>22</sup> These results show that, at least in shocks with high Mach numbers and no significant turbulent heating, the nonlinear effects may significantly limit the maximum energy achievable by this acceleration mechanism. Before any relevance of this result to, e.g., the lack of observational evidence of TeV protons in SNR can be seriously debated, a number of difficult issues must be addressed, which we discuss briefly in the next section.

#### IV. DISCUSSION

Instead of relying on parametrization and guessing about the possible values of relevant parameters, it was suggested that, due to the self-regulation, the system should evolve precisely into the critical point within given parameter space.<sup>22</sup> A comprehensive evaluation of this suggestion and the study of self-regulation mechanisms poses a serious challenge to plasma physics. Note, however, that the critical self-organization approach was proven very useful in describing transport phenomena in laboratory plasmas, when the turbulence is generated by transport driving gradients, i.e., the pressure or density gradients, e.g., Ref. 23. The most important current issue is the construction of adequate links between the internal parameters (and so the Alfvénic turbulence), and their dependence upon the external parameters. This is an interdisciplinary problem in high-energy astrophysics and nonlinear plasma physics. It combines hydrodynamics, particle kinetics with a strong emphasis on the theory of dynamical chaos and ergodicity (injection), Alfvénic turbulence, collisionless nonlinear plasma phenomena, and, in particular, the theory of collisionless shocks.

Currently not all the aspects of this problem are well understood. We believe that we understand how the acceleration operates in the test particle (linear) regime, and have an analytic description of nonlinearly modified shocks including particle spectra, although under a prescribed Bohm-type particle diffusion. Injection can be calculated currently only for a relatively strong subshock, while we need it for varying shock parameters of dynamically evolving CR shocks. Turbulence dynamics, associated plasma heating, and the turbulent transport of CRs are understood to even lesser extent. A systematic determination of parameters involved should allow the self-consistent calculation of particle spectra.

Specifically, this requires detailed understanding of several physical phenomena in a shock environment, which include

- (1) generation of waves by energetic protons, wave transport, and spectral evolution;
- (2) turbulent transport of protons, in space and momentum, injection;
- (3) electron injection triggered by proton generated turbulence; and
- (4) heating by Alfvénic and (magneto-) acoustic turbulence, which controls the global shock structure

The resolution of these issues will lead to further progress in our understanding of particle acceleration and associated emission.

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