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NON-PERTURBATIVE BREAKDOWN OF SCALE INVARIANCE IN 4n+22+n-1

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NON-PERTURBATIVE BREAKDOWN OF SCALE INVARIANCE IN $\phi^{4n+2}_{2+n^{-1}}$

M.B. Halpern

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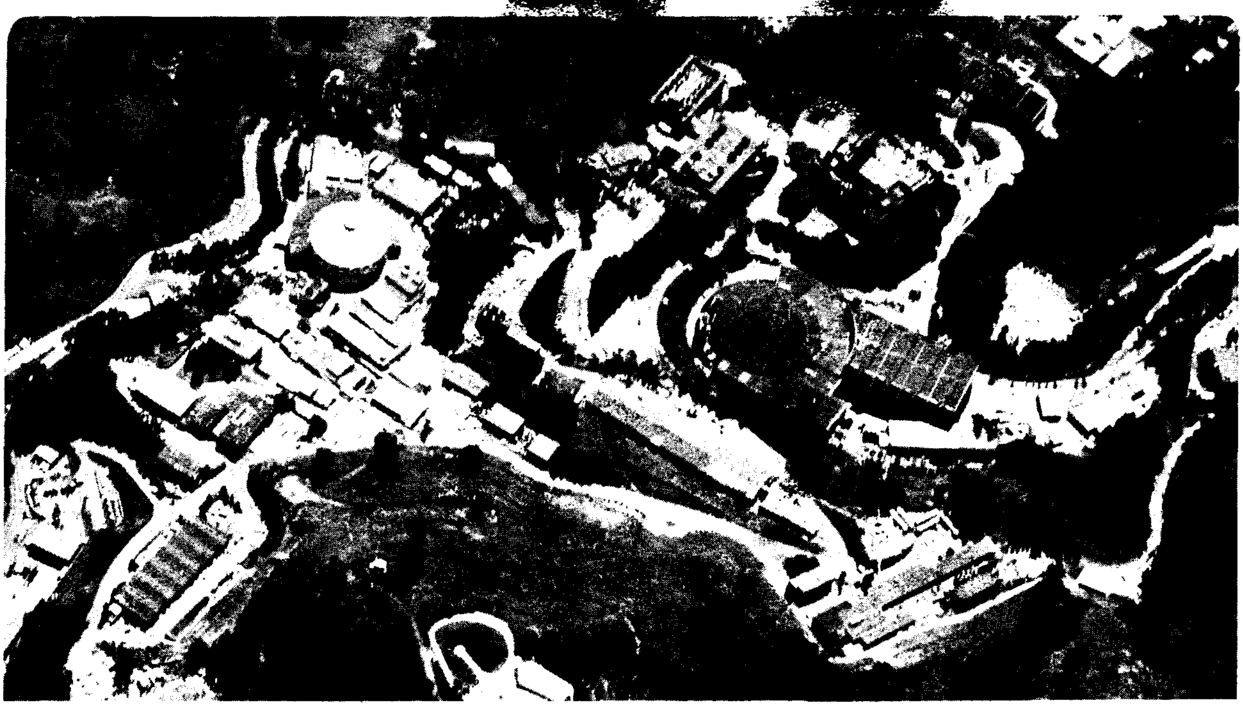

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NON-PERTURBATIVE BREAKDOWN OF SCALE INVARIANCE IN $\phi^{4n+2}{}_{2+n^{-1}}$

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Abstract

A large N analysis of the renormalizable interactions $(\phi^2)^M{}_{2M|M-1}$, $M \geq 3$ is reported, generalizing a recent work of Bardeen, Moshe and Bander. At "multi-critical" points, each theory is perturbatively scale invariant. Each member of the M odd sequence exhibits a non-perturbative ultra-violet fixed point, spontaneous breaking of scale invariance and a dilaton.

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Some of the most beautiful theories of physics are classically scale invariant, and nature is not. The question of quantum breakdown of scale invariance (running coupling, $\beta \neq 0$) is therefore physically relevant. In contrast to QCD, in which $\beta \neq 0$ by perturbative renormalization, certain finite theories, like N = 4 Yang-Mills, exhibit $\beta = 0$ to all orders [1]. It is important to know whether such theories will experience non-perturbative breaking of scale invariance. As a prelude to this, it may be important to have models of the phenomenon. Recently, Bardeen, Moshe and Bander [2] (BMB) have found precisely this phenomenon in large N $\phi^6{}_3$ at the tricritical point: there is a non-perturbative ultra-violet fixed point, associated with a spontaneous breakdown of scale invariance, and a dilaton. In this note, I report that the infinite sequence of theories $(\phi^2)^{2n+1}{}_{2+n^{-1}}$, $n \geq 1$, enjoys the same phenomenon.

Scalar interactions of the form $(\phi^2)^\nu$ in $D = 2\nu|\nu-1$ dimensions are characterized by dimensionless couplings, even for non-integer ν and D, and are presumably renormalizable theories. I restrict myself here to the conventionally renormalizable polynomial subset, $\nu = M \geq 3$, so

$$2 < D = \frac{2M}{M-1} \leq 3 \,. \qquad (1)$$

As seen below, for this range of M, the dimensionless couplings experience no perturbative renormalization at large N. Any running of these couplings will be purely non-perturbative.

The large N computations ultimately involve solution of the gap equation. Since these are quadratic building-block theories, many equivalent techniques are available [3]. I follow here the Hartree-Fock method of Ref. [2] The Hamiltonian is (a = 1...N)

$$H = \int (d^{D-1}x) \left[\frac{1}{2}\pi_a\pi_a + \frac{1}{2}\partial\varphi_a\partial\varphi_a + N\sum_{n=1}^{M} \frac{g^0_{2n}}{2n}\left(\frac{\varphi^2}{N}\right)^n \right]. \qquad (2)$$

The trial wave functional,[F1]

$$\Psi_{trial} \sim \exp\left\{-\frac{1}{4}\int (d^{D-1}x)\, \varphi_a \left[-\partial^2+m^2\right]^{\frac{1}{2}} \varphi_a\right\} \tag{3}$$

results at large N in the trial energy/volume W,

$$\frac{W}{N} = K+V$$
$$K(m^2) \equiv -\frac{1}{2}\int_0^{m^2} d\mu^2\, \mu^2 \frac{\partial}{\partial\mu^2}\left\langle \frac{\varphi^2}{N}\right\rangle_\mu \tag{4}$$
$$V \equiv \sum_{n=1}^{M} \frac{g_{2n}^0}{2n}\left\langle\frac{\varphi^2}{N}\right\rangle^n, \quad \left\langle\frac{\varphi^2}{N}\right\rangle = \int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2+m^2}$$

where m is the variational mass. For $2 < D < 4$, a Euclidean momentum cutoff Λ gives

$$\left\langle\frac{\varphi^2}{N}\right\rangle = \Omega\left[\frac{\Lambda^{D-2}}{D-2} - m^{D-2}F + m^2\frac{\Lambda^{D-4}}{4-D} + \dots\right]$$
$$\Omega^{-1}(D) \equiv \frac{1}{2}(4\pi)^{\frac{D}{2}}\Gamma\left(\frac{D}{2}\right), \quad F(D) \equiv \frac{\pi}{2}\csc\left[\frac{\pi}{2}(D-2)\right]$$
$$\Omega F = -(4\pi)^{-\frac{D}{2}}\Gamma\left(1-\frac{D}{2}\right) \tag{5}$$

Both Ω and F are positive for $2 < D < 4$. Renormalized couplings g_{2n} are defined as in [2],

$$V \equiv V_0 + \sum_{n=1}^{M}\frac{g_{2n}}{2n}\Omega^n\left[-m^{D-2}F + m^2\frac{\Lambda^{D-4}}{4-D}+\dots\right]^n. \tag{6}$$

V_0 is independent of m, and irrelevant, while from (4), (5) and (6),

$$\frac{g_{2n}}{2n} = \sum_{\ell=n}^{M}\frac{g_{2\ell}^0}{2\ell}\binom{\ell}{n}\left(\frac{\Omega\Lambda^{D-2}}{D-2}\right)^{\ell-n}. \tag{7}$$

Note that $g_{2M} = g_{2M}^0$; the dimensionless coupling is not (perturbatively) renormalized. A perturbatively scale-invariant "multi-critical point" is defined by setting all dimensionful renormalized couplings to zero, $g_{2n} = 0$ $(n < M)$. After a little algebra, the resulting trial energy is

$$\frac{W}{N} = \frac{m^D\Omega F}{2M}\left\{1+(-1)^M\frac{g_{2M}}{g_{2M}^*}\right\} - m^4\frac{\Lambda^{D-4}\Omega}{2(D-4)}\left\{\frac{1}{2}+(-1)^M\frac{g_{2M}}{g_{2M}^*}\right\}$$
$$g_{2M}^* \equiv (\Omega F)^{1-M}. \tag{8}$$

For <u>even</u> M, the only cutoff-independent minimum is at $m = 0$. For all $g_{2M} > 0$ (M even), these theories are therefore in an expected massless (perturbative) phase, with $\beta_{2M} = 0$ (M even). In what follows, I assume M is odd.

For <u>odd</u> M, g_{2M}^* becomes a critical coupling,

$$g_{2M}^* = (4\pi)^M\left[\Gamma\left(\frac{1}{1-M}\right)\right]^{1-M}. \tag{9}$$

For $g_{2M} < g_{2M}^*$, the correct minimum is at $m = 0$. This weak-coupling phase is again massless, perturbative and $\beta_{2M} = 0$. For $g_{2M} > g_{2M}^*$, however, a new minimum is seen, proportional to the cutoff (BMB instability [2])

$$m = \Lambda\left[\frac{(g_{2M}-g_{2M}^*)}{(g_{2M}-\frac{1}{2}g_{2M}^*)}\,\frac{F(D-2)(4-D)}{4}\right]^{\frac{1}{D-4}}. \tag{10}$$

Therefore a new massive strong-coupling phase is obtained if $g_{2M} \to g_{2M}^*$ from above as the cutoff is removed. The correct rate of approach, for arbitrary fixed m is, from (10),

$$g_{2M} \underset{\Lambda}{=} g_{2M}^* \left[1 + \left(\frac{m}{\Lambda}\right)^{4-D} \frac{2}{F(D-2)(4-D)} + \cdots\right]. \qquad (11)$$

In the continuum limit, $g_{2M} = g_{2M}^*$ and the mass m is arbitrary, the entire odd-M sequence of theories exhibiting dimensional transmutation.

The β-functions are immediately computed from (11),

$$\beta_{2M} \equiv \Lambda \frac{\partial g_{2M}}{\partial \Lambda} = (4-D)(g_{2M}^* - g_{2M})\,\theta(g_{2M} - g_{2M}^*) \qquad (12)$$

valid for g_{2M} from zero through a neighborhood of g_{2M}^*. The couplings run non-perturbatively in the massive strong coupling phases ($g_{2M} > g_{2M}^*$) toward the ultra-violet fixed points g_{2M}^*.

In order to compute physical amplitudes in the continuum limit at the multi-critical points, it is convenient to work in terms of "effective" couplings $\bar{g}_{2n}$, induced by normal ordering,

$$\sum_{n=1}^{M} \frac{g^0_{2n}}{2n}\left(\frac{\varphi^2}{N}\right)^n \equiv \bar{V}_0 + \sum_{n=1}^{M} \frac{\bar{g}_{2n}}{2n} :\left(\frac{\varphi^2}{N}\right)^n: . \qquad (13)$$

The effective couplings are determined as follows. Wick's theorem at large N is

$$(\varphi^2)^n \underset{N}{=} \sum_{m=0}^{n} :(\varphi^2)^m: \binom{n}{m} \langle \varphi^2 \rangle^{n-m}. \qquad (14)$$

So, using Eqs. (4) and (14),

$$\frac{\bar{g}_{2n}}{2n} = \sum_{\ell=n}^{M} \frac{g^0_{2\ell}}{2\ell}\binom{\ell}{n}\left\langle \frac{\varphi^2}{N}\right\rangle^{\ell}. \qquad (15)$$

At the multi-critical points, Eq. (7) can be solved for the bare couplings in terms of g_{2M}^*,

$$g^0_{2n} = g^*_{2M}\binom{M-1}{n-1}\left(\frac{-\Omega\Lambda^{D-2}}{D-2}\right)^{M-n}. \qquad (16)$$

Together with Eq. (15), the effective couplings are finally determined. After some algebra,

$$\bar{g}_{2n} = g^*_{2M}\binom{M-1}{n-1}\left[\frac{m^{D-2}\,\Gamma(1-\frac{D}{2})}{(4\pi)^{D/2}}\right]^{M-n}. \qquad (17)$$

As an example, I construct the four-point functions for the odd-M sequence,

$$\Gamma_4 = \frac{2\bar{g}_4}{N}\left[1 + \bar{g}_4 B(p)\right]^{-1} \qquad (18)$$

$$B(p) = \int \frac{d^D k}{(2\pi)^D}\frac{1}{k^2+m^2}\frac{1}{(p-k)^2+m^2}.$$

Rotating to Minkowski space, I obtain, with (9) and (17),

$$\Gamma_4 = \frac{-2(4\pi)^{D/2}(m^2)^{2-D/2}}{N\,\Gamma(2-\frac{D}{2})}\left\{1 - \int_0^1 d\alpha\left[1-\alpha(1-\alpha)\frac{p^2}{m^2}\right]^{\frac{D}{2}-2}\right\}^{-1}. \qquad (19)$$

For $p^2 << m^2$,

$$\Gamma_4 \sim \frac{12(4\pi)^{D/2}(m^2)^{3-\frac{D}{2}}}{N\,\Gamma(3-\frac{D}{2})\,p^2} \qquad (20)$$

so every theory in the sequence shows a dilaton as a pole with positive residue; scale invariance is realized in the Goldstone mode.

I have also examined the physical 2M-point function. At large N, the surviving graphs consist of M strings of effective bubbles, as above, joined either by a single 2M-point coupling, or by an M-sided polygon of 4-point couplings. The bubbles (B), and the polygon vanish at large momentum, so $\Gamma_{2M} \to g_{2M}{}^*$ in the ultraviolet, as expected. Finally, I have checked the existence of the new fixed points in dimensional regularization $[D = (2M|M-1)-\varepsilon,\ \varepsilon > 0]$. With $g_{2M} \equiv g_{2M}\ \mu^{\varepsilon(M-1)}$ (g_{2M} dimensionless), the dominant effect at the multi-critical point is the stabilization of the potential term by a factor $(\mu/m)^{\varepsilon(M-1)}$. It is clear that $g_{2M} \to g_{2M}{}^*$ as $\varepsilon \to 0^+$, but it seems difficult to compute the physical β directly in this approach. Curiously, the sequence is also stabilized, with the same results, by a small amount of $g_{2(M+1)}$.[F2]

In summary, approximately one-half (M odd) the infinite class of perturbatively scale-invariant theories considered undergoes a non-perturbative breakdown of scale-invariance. The other half (M even) does not. The issue in other finite theories, such as N = 4 Yang Mills, remains open.

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[2]. W. Bardeen, M. Moshe and M. Bander. Fermilab-Pub-83/53-THY. To be published in *Physics Letters*.

[3]. M. B. Halpern, *Nucl. Phys.* B173, *504 (1980).*

Footnotes

F.1. It is well known that Hartree-Fock gives the correct result in the large N limit for quadratic building-block theories. Note that (3) is not the true ground state wave functional of the system, even at large N, and at the minimum. Evidently, many trial functions can give correct results at large N, perhaps even for matrix models. In the quadratic building-block case, the phenomenon is easily traced to master fields.

F.2. This was worked out in a conversation with M. Moshe.

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