Title
Impact of Multi-Packet Transmission and Reception on The Throughput Capacity of Wireless Ad Hoc Networks

Author
Garcia-Luna-Aceves, J.J.

Publication Date
2010-10-31

Peer reviewed

Impact of Multi-Packet Transmission and Reception on The Throughput Capacity of Wireless Ad Hoc Networks

Bita Azimdoost
Department of Electrical Engineering
University of California
Santa Cruz 95064
Email: bazimdoost@soe.ucsc.edu

Hamid R. Sadjadpour
Department of Electrical Engineering
University of California
Santa Cruz 95064
Email: hamid@soe.ucsc.edu

J.J. Garcia-Luna-Aceves
Department of Computer Engineering
University of California
Santa Cruz 95064
Email: jj@soe.ucsc.edu

***Abstract*—We study the capacity of random wireless ad hoc networks when nodes are capable of multi-packet transmission and reception (MPTR). This paper extends the unified framework of (n, m, k)-cast by Wang et al. [6] for single-packet reception (SPR) at each node to the case of MPTR. (n, m, k)-cast considers all types of information dissemination including unicast routing, multicast routing, broadcasting and anycasting. In this context, n, m, and k represent the total number of nodes in the network, the number of destinations for each communication group and the actual number of destinations that receive the packets, respectively. We show that the capacity of a wireless ad hoc network of n nodes in which nodes have a communication range of $r(n)$ and engage in an (n, m, k)-casting scales as $\Theta(n\sqrt{m}r^3(n)/k)$, $\Theta(nr^2(n)/k)$ and $\Theta(nr^4(n))$ bits per second when $m = O(1/r^2(n))$, $\Omega(k) = 1/r^2(n) = O(m)$ and $k = \Omega(1/r^2(n))$, respectively. We show that the use of MPTR leads to a gain of $\Theta(logn)$ compared to the capacity attained with multi-packet reception (MPR), and to a gain of $\Theta((logn)^2)$ compared to the capacity attained with SPR, when $\Omega(\sqrt{logn/n}) = r(n) = O(\sqrt{loglogn/3logn})$.**

I. Introduction

Gupta and Kumar [1] computed the throughput capacity of wireless networks with unicast traffic when nodes are endowed with single-packet reception (SPR) capability. Li et al. [4] studied the multicast capacity of wireless networks and proved that the per-node multicast capacity is $\Theta(1/\sqrt{nklogn})$ and $\Theta(1/n)$ for $k = O(n/logn)$ and $k = \Omega(n/logn)$ respectively, where k is the number of communication sessions in the network. Keshavarz et al. [8] focused on the broadcast capacity of wireless networks and observed that the capacity does not change with the transmission range.

Wang et al. in [6] presented a unified framework for the computation of throughput capacity of these networks and introduced the (n, m, k)-casting as a generalization of unicasting, multicasting, broadcasting and anycasting. In this framework, n is the total number of nodes in the network, m is the number of destinations for each communication group, and k is the actual number of destinations in each group that receive the packets. (n, m, k)-cast capacity was computed in [2] when nodes are only capable of SPR communications and the authors showed that the capacity is $\Theta(\sqrt{m}/nkr(n))$, $\Theta(1/nkr^2(n))$, $\Theta(1/n)$ for $m = O(1/r^2(n))$, $\Omega(k) = 1/r^2(n) = O(m)$ and $k = \Omega(1/r^2(n))$ respectively.

Multi-packet reception (MPR) has been introduced to increase capacity. It can be implemented by allowing a node to decode multiple concurrent packets using multiuser detection and directional antennas [9], [10], or distributed multiple input multiple output (MIMO) techniques. Wang et al. studied this technique and proved that a gain of $\Theta(logn)$ is achieved by using MPR instead of SPR [3].

The other way to increase the throughput order is to let all the nodes decode correctly multiple packets, and transmit concurrently multiple packets to different nodes. This multi-packet transmission and reception (MPTR) model was introduced in [5] and was shown to increase the unicast capacity by a factor of $\Theta(logn)$ in comparison with MPR model.

In this paper, we use the framework presented by Wang et al. and compute the (n, m, k)-cast throughput capacity when the nodes have the MPTR capability. Further, the relationship between capacity and delay as a function of transmission range and group size is derived.

The rest of the paper is organized as follows. In section II, we introduce the model which is used for the network, and the main results of our work on capacity and delay with MPTR are shown in section III. Section IV discusses the results.

II. Preliminaries

We first define the notations used throughout this paper. $|R|$ indicates the area of a continuous region R and $\#S$ shows the cardinality of a set S. The distance between two nodes x and y is denoted by $|x - y|$. The probability of event E is represented by $Pr(E)$ and if $Pr(E) > 1 - 1/n$ holds for sufficiently large n, the event E is assumed to occur with high probability (w.h.p.). Also the standard notations of Ω, Θ, and O are employed in this paper.

The protocol model defined in [7] is based on single-packet reception. The transmission range $r(n)$ is common for all nodes in the network. Node i at position X_i can successfully transmit to node j at position X_j if for any node k at position X_k, $k \neq i$, that transmits at the same time as i, then $|X_i - X_j| \leq r(n)$ and $|X_k - X_j| \geq (1+\Delta)r(n)$, where X_i, X_j and X_k are the cartesian positions in the unit square network for these nodes.

The protocol model for MPR is defined in [3]. In the MPR model, all nodes use a common transmission range $R(n)$ for all their communications. The network area is assumed to be a unit square area. In wireless networks with MPR capability, the protocol model assumption allows simultaneous decoding of packets for all nodes as long as they are within a radius of $R(n)$ from the receiver and all other transmitting nodes have a distance larger than $(1+\Delta)R(n)$.

In this paper the combination of MPR and multipacket transmission (MPT) is utilized for all nodes as defined in [5]. This model restricts the nodes to operate in a half-duplex mode (like all the other methods mentioned earlier), and similar to MPR model prohibits the transmission from a node k in the region $r(n) < |X_i - X_k| \leq (1+\Delta)r(n)$. The difference between MPTR and MPR protocol models is that, under the MPTR model, a node i transmitting a packet to node j can concurrently transmit packets to other nodes in the network.

This paper uses the concept of Total Active Area $(TAA(\Delta, r(n)))$ which is defined in [3] as the total area of the network multiplied by the average maximum number of simultaneous transmissions and receptions inside a communication region of $\Theta(r^2(n))$.

Minimum Area (n,m,k)-cast Tree $(MAMKT(r(n)))$ in a (n,m,k)-cast tree is the total area covered by the circles with radius $r(n)$ centered on sources and relays in the wireless ad hoc network, and $\overline{\#MEMKTC}$ is defined as the average total number of cells that contain all the nodes in an (n,m,k)-cast group.

III. Throughput Capacity of Network with MPTR Model

A. Upper Bound

A common technique to find the upper bound on the capacity of networks is to calculate the total number of simultaneous transmissions possible in the network area and we use this value to compute the upper bound throughput capacity for each (n,m,k)-cast group.

Lemma 1: The maximum number of simultaneous transmissions in a network with MPTR capability is $O(n^2r^2(n))$.

Proof: We divide the network area into equal-size cells each with a side-length of $r/\sqrt{5}$. Define the sub-graph $G_1 = (V_1, E_1)$ with V_1 including all the nodes as the network $(V_1 = V)$ and use the subset of edges E_1 such that each edge connects the nodes in adjacent cells; i.e., $E_1 = \{e \in E : e^+ = e^- \mp 1\}$. In this new graph the total number of cells is proportional to $\Theta(1/r^2(n))$, and the number of nodes in each cell is $\Theta(nr^2(n))$, so the total number of edges coming from or to a cell is $\Theta(nr^2(n)) \times \Theta(nr^2(n)) = \Theta(n^2r^4(n))$. Therefore, the maximum number of simultaneous transmissions in the network is $\Theta(1/r^2(n)) \times \Theta(n^2r^4(n)) = \Theta(n^2r^2(n))$. Note that this is the total number of simultaneous transmissions in sub-graph $G1$, and as the maximum number of simultaneous transmissions in graph G is at most a constant multiple of this value, which does not change the order, it would also be $\Theta(n^2r^2(n))$.

Lemma 2: The maximum rate that can be reached in a network with MPTR-capable nodes is $O(nr^2(n)/\overline{\#MEMKTC})$.

Proof: There are n multicast groups each sending data at rate λ, and the maximum average number of cells each bit has to travel to reach all destinations is $\overline{\#MEMKTC}$. Thus, the total number of simultaneous transmissions in such a network would be $n\lambda\overline{\#MEMKTC}$, which cannot be larger than $\Theta(n^2r^2(n))$.

$$n\lambda\overline{\#MEMKTC} \leq n^2r^2(n)$$

Then,

$$\lambda \leq \frac{nr^2(n)}{\overline{\#MEMKTC}}.$$

Lemma 3: The $\overline{\#MEMKTC}$ is tight bounded as

$$\overline{\#MEMKTC(r(n))} =$$
$$\begin{cases} \Theta(\frac{k}{r(n)\sqrt{m}}) & , for\ m = O(\frac{1}{r^2(n)}) \\ \Theta(k) & , for\ \Omega(\text{k}){=}\frac{1}{r^2(n)} = O(m) \\ \Theta(\frac{1}{r^2(n)}) & , for\ k = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

Proof: The proof is given in Lemma 4.7 and 5.5 in [6].

Theorem 4: In wireless ad hoc networks with MPTR, the upper bound on the per node throughput capacity of (n,m,k)-cast is:

$$C_{m,k}(n) = \begin{cases} O(\frac{n\sqrt{m}r^3(n)}{k}) & , for\ m = O(\frac{1}{r^2(n)}) \\ O(\frac{nr^2(n)}{k}) & , for\ \Omega(\text{k}){=}\frac{1}{r^2(n)} = O(m) \\ O(nr^4(n)) & , fork = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

Proof: Combining the results of Lemma 3 and 2 will lead to the result.

B. Lower Bound

To obtain a lower bound on capacity, we can use the TDMA scheme similar to the one used in [3] for MPR model. It has been shown [3] that there exists at least $\lceil 1/\ (Lr(n)/\sqrt{2})^2 \rceil$ simultaneous circular regions each one containing $\Theta\left(nr^2(n)\right)$ nodes w.h.p.. Note that the TDMA factor L is only a constant value and not a function of n.

Lemma 5: For any $r(n) = \Omega(\sqrt{\log n/n})$,

$$\lim_{n\to\infty} Prob(sup\{Number\ of\ trees\ intersecting\ cell\ S_{k,j}\})$$
$$= O(nr^2(n)\overline{\#MEMKTC(r(n))})$$

Proof: The proof of this lemma is provided in [3].

Lemma 6: The achievable lower bound for the (n, m, k)-cast capacity with MPTR is

$$C_{m,k}(n) = \Omega(nr^2(n) \times \overline{\#MEMKTC(r(n))}^{-1}).$$

Proof: There exists a transmitting schedule such that in every L^2 (L is constant) slots, each cell transmits or receives at rate W bits/second with maximum transmission distance $r(n)$. Therefore, the number of packets transmitted to and from a cell is $\Theta(n^2 r^4(n) W/L^2)$. From Lemma 5, each cell needs to transmit at rate $(C_{m,k}(n) nr^2(n)\overline{\#MEMKTC(r(n))})$ w.h.p.. In order to accommodate this requirement by all cells, we need $C_{m,k}(n) nr^2(n)\overline{\#MEMKTC(r(n))} \leq n^2 r^4(n)$ which proves the lemma.

Theorem 7: In wireless ad hoc networks with MPTR, the lower bound of the per node throughput capacity for (n, m, k)-casting is given by

$$C_{m,k}(n) = \begin{cases} \Omega(\frac{n\sqrt{m}r^3(n)}{k}) & ,for\ m = O(\frac{1}{r^2(n)}) \\ \Omega(\frac{nr^2(n)}{k}) & ,for\ \Omega(k) = \frac{1}{r^2(n)} = O(m) \\ \Omega(nr^4(n)) & .for\ k = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

Proof: Combining the results of lemmas 3 & 6 proves the theorem.

The obtained throughput capacity has been calculated without considering the maximum number of simultaneous receivers and transmitters that a node can accommodate. According to the bins and balls theorem, the maximum number of destinations which can be related to a single node is at most $\frac{3\log n}{\log\log n}$. The maximum rate at which a node can send or receive data cannot be less than the total traffic load that a node is required to accommodate. This constraint requires that,

$$nr^2(n) \geq C_{m,k}(n) \times \frac{3\log n}{\log\log n}.$$

It can be proved that if $r(n) = O(\sqrt{\frac{\log\log n}{3\log n}})$, then the above inequality holds in all regions of throughput capacity. We show the proof for region 1 as an example, i.e., $C_{m,k}(n) = O(\frac{n\sqrt{m}r^3(n)}{k})$.

$$\begin{aligned} nr^2(n) &\geq A_1 \frac{n\sqrt{m}r^3(n)}{k} \times \frac{3\log n}{\log\log n} \\ 1 &\geq A_1 \frac{\sqrt{m}r(n)}{k} \times \frac{3\log n}{\log\log n} \\ &\frac{\log\log n}{3A_1 \log n\sqrt{m}} k \geq r(n) \end{aligned} \tag{1}$$

In the first capacity region, m follows the following inequality.

$$A_2/r^2(n) \geq m \geq k$$

Now, we use this bound for m and k in eq. (1).

$$\begin{aligned} r(n) &\leq \frac{\log\log n}{3A_1 \log n\sqrt{m}} k \\ &\overset{a}{\leq} \frac{\log\log n}{3A_1 \log n}\sqrt{m} \\ &\overset{b}{\leq} \sqrt{A_2}\frac{\log\log n}{3A_1 \log n\ r(n)} \\ r(n) &\leq A_3\sqrt{\frac{\log\log n}{3\log n}} \end{aligned} \tag{2}$$

(a) is derived by replacing k with its upper bound m and (b) is derived by using the upper bound for m. The same results can be obtained for the other two capacity regions.

The above criterion gives us an upper bound on communication range, $r(n)$, such that the obtained capacity can be achieved without any congestion for each node. On the other hand, the connectivity criteria requires that $r(n) = \Omega(\sqrt{\frac{\log n}{n}})$. Thus, it is concluded that $r(n)$ should be in the region of $\Omega(\sqrt{\frac{\log n}{n}}) = r(n) = O(\sqrt{\frac{\log\log n}{3\log n}})$.

Combining Theorems 4 and 7 provides a tight bound on the throughput capacity of the network when each node is endowed with MPTR capability.

$$C_{m,k}(n) = \begin{cases} \Theta(\frac{n\sqrt{m}r^3(n)}{k}) & ,for\ m = O(\frac{1}{r^2(n)}) \\ \Theta(\frac{nr^2(n)}{k}) & ,for\ \Omega(k) = \frac{1}{r^2(n)} = O(m) \\ \Theta(nr^4(n)) & ,for\ k = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

C. Delay Analysis

In this section, we discuss the delay of (n, m, k)-casting and its relationship with the capacity.

Lemma 8: The delay of (n, m, k)-cast in a random dense wireless ad hoc network is

$$D_{m,k}(n) = \Theta(\overline{\#MEMKTC(r(n))})$$

Proof: The average total number of cells containing all the nodes in an (n, m, k)-cast group, $\overline{\#MEMKTC(r(n))}$), is proportional to the average number of hops traveled by the information from source to reach all its destinations. Thus it is the same order bound as the total delay. Complete proof is given in Lemma 4.7 in [2].

Lemma 9: The relationship between delay and capacity for (n, m, k)-casting is as follows:

$$D_{m,k}(n) C_{m,k}(n) = \Theta(nr^2(n))$$

Proof: The proof follows immediately by combining Lemmas 6 and 8.

IV. Discussion

In this paper we focused on the capacity of (n, m, k)-casting when the nodes are endowed with multi-packet transmission and reception capabilities. The capacity and delay have been obtained for different (n, m, k)-casting including unicast, multicast, broadcast and anycast communications.

A. Capacity as a function of transmission range and group size

The relationship between capacity and group size (m) as a function of communication range $r(n)$ is shown in figure 1. As can be seen, the throughput capacity does not change with the group size when $1 \leq m \leq \Theta(1)$. In this region the capacity is only a function of n and $r^3(n)$ and an increase in transmission range will increase the capacity.

Furthermore, when the number of receivers exceeds a threshold, the throughput capacity will be independent of m and it is just a function of n and $r^4(n)$.

In the first capacity region and for the multicast $(k = m < n)$ communications, the throughput capacity has its minimum value and decreases with the increase of m, $(\Theta(nr^3(n)/\sqrt{m}))$. In the same capacity region and for anycast communication, i.e., $k = 1$, the capacity reaches its maximum value and increases with the increase of m, $(\Theta(nr^3(n)\sqrt{m}))$.

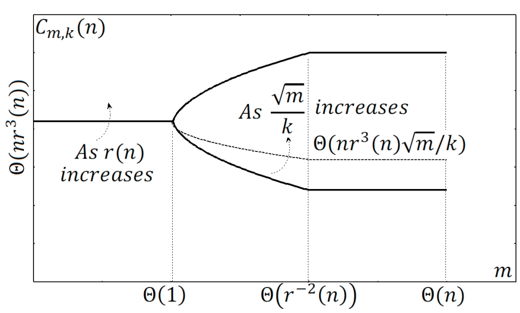

Fig. 1. The relationship between throughput capacity of (n, m, k)-cast and m as a function of $r(n)$.

B. Capacity and Delay Tradeoff

The tradeoff between delay and capacity using MPTR capability in the first (unicast), second (multicast), and third (broadcast) capacity regions are illustrated in figures 2, 3, and 4, respectively. In the unicast region, $(m = O(r^{-2}(n)))$, as $r(n)$ increases, delay decreases and capacity increases, so to have the minimum delay and maximum capacity, we just need to increase $r(n)$ to the maximum allowable value $(O(\sqrt{loglogn/3logn}))$. This condition clearly requires an increase in the computational complexity of the nodes in the network.

In the multicast region $(\Omega(k) = r^{-2}(n) = O(m))$, the delay does not change with $r(n)$ and to achieve the maximum capacity, the maximum $r(n)$ should be selected.

The broadcast region $(k = \Omega(r^{-2}(n)))$ has almost the same capacity-delay tradeoff similar to the unicast region, as we observe that by decreasing the delay, the capacity increases when the transmission range increases. Therefore, the maximum acceptable transmission range will result in minimum delay and maximum capacity.

Finally, our results demonstrate that in networks with MPTR capability, there is no need to sacrifice capacity to achieve lower delay. The main reason is the fact that MPTR takes care of interference and by increasing $r(n)$, more nodes can simultaneously communicate with each other.

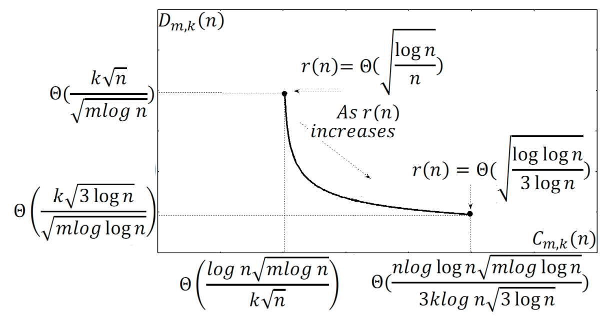

Fig. 2. Capacity and delay tradeoff in the first (unicast) capacity region.

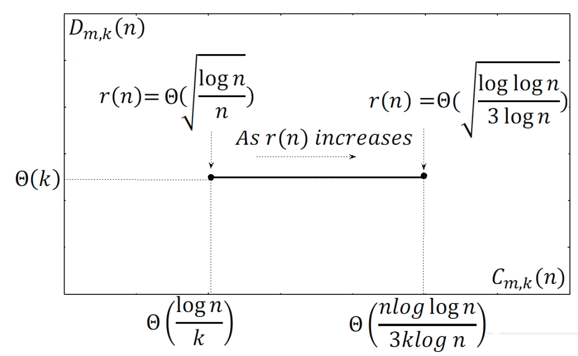

Fig. 3. Capacity and delay tradeoff in the second (multicast) capacity region.

V. Appendix

We compute the upper bound using another technique in this appendix. We introduce a circular cut of radius $r(n)$ as shown in figure 5 that divides the network into two regions of S and S^c. To compute the upper bound throughput capacity, we utilize the concepts of the average total active area and the

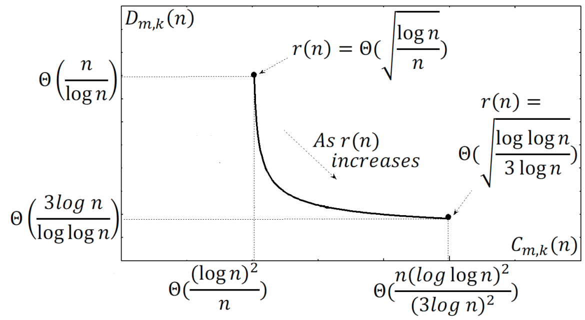

Fig. 4. Capacity and delay tradeoff in the third (broadcast) capacity region.

total area required to transmit information in an (n, m, k)-cast tree.

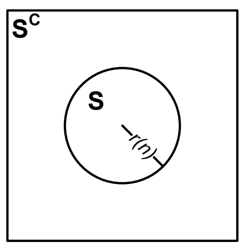

Fig. 5. A circular cut that divides the network into two regions of S and S^c.

Lemma A1: The maximum number of transmitters in S is $\Theta(nr^2(n))$.

Proof: In MPTR model, each node can receive from multiple nodes, so the existence of a transmitter in this circle does not prohibit the transmission from other nodes in this region. Thus, the maximum number of transmitters in this region equals to the maximum number of nodes that contains in this region which is equal to $\Theta(nr^2(n))$.

Lemma A2: The maximum number of transmissions per node is $\Theta(nr^2(n))$.

Proof: MPTR model allows each node to transmit to several nodes at a time. So the number of transmissions per node equals to the number of receivers in the circle with radius $r(n)$ centered on that node, which equals to the total number of nodes in this region $(\Theta(nr^2(n)))$.

Lemma A3: The Average Total Active Area, $\overline{TAA(\triangle, r(n))}$, in networks with MPTR is $\Theta(n^2r^4(n))$.

Proof: The radius of region S is $r(n)$. Let's consider all the nodes that are within a ring of greater than $r(n)/2$ and $r(n)$ with respect to the center of the circle. The number of nodes in this ring is proportional to $(\Theta(nr^2(n)))$. Because of the uniform distribution of nodes, there is on average $(\Theta(nr^2(n)))$ nodes in S^C that are within the communication range of the nodes inside this ring. Thus it can be assumed that each transmission from any transmitter inside this ring will pass through the cut, leading to a maximum flow equal to the multiplication of the number of transmitters in this ring and the number of transmissions per node in the ring which is equal to $\Theta(nr^2(n)) \times \Theta(nr^2(n)) = \Theta(n^2r^4(n))$.

Lemma A4: In random dense wireless ad hoc networks, the per-node throughput capacity of (n, m, k)-cast with MPTR is given by $O(\frac{1}{n} \times \frac{\overline{TAA(\triangle, r(n))}}{\overline{S(MAMKT(r(n)))}})$.

Proof: With MPTR, we observe that $\overline{S(MAMKT(r(n)))}$ represents the total area required to transmit information from a multicast source to all its m destinations. The ratio between average total active area, $\overline{TAA(\triangle, r(n))}$, and $\overline{S(MAMKT(r(n)))}$ represents the average number of simultaneous (n, m, k)-cast sessions that can occur in the network. Normalizing this ratio by n provides per-node throughput capacity which proves the Lemma.

Lemma A5: In (n, m, k)-cast applications, the average area of a (n, m, k)-cast tree with transmission range $r(n)$, $\overline{S(MAMKT(r(n)))}$ has the following lower bound as:

$$\overline{S(MAMKT(r(n)))} =$$

$$\begin{cases} \Omega(\frac{kr(n)}{\sqrt{m}}) & , for\ m = O(\frac{1}{r^2(n)}) \\ \Omega(kr^2(n)) & , for\ \Omega(k) = \frac{1}{r^2(n)} = O(m) \\ \Omega(1) & , for\ k = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

Proof: Note that $\overline{S(MAMKT(r(n)))}$ is the same value for MPTR, MPR and SPR, and they only depend on the communication range in the network. This value is derived in [2], [3].

Theorem A6: In wireless ad hoc networks with MPTR, the upper bound on the per node throughput capacity of (n, m, k)-cast is:

$$C_{m,k}(n) =$$

$$\begin{cases} O(n\sqrt{m}r^3(n)/k) & , for\ m = O(\frac{1}{r^2(n)}) \\ O(nr^2(n)/k) & , for\ \Omega(k) = \frac{1}{r^2(n)} = O(m) \\ O(nr^4(n)) & , for\ k = \Omega(\frac{1}{r^2(n)}) \end{cases}$$

Proof: The proof follows immediately by combining Lemmas A3, A4, and A5.

ACKNOWLEDGEMENT

This research was partially sponsored by the U.S. Army Research Laboratory under the Network Science Collaborative Technology Alliance, Agreement Number W911NF-09-0053, by the Army Research Office under agreement number W911NF-05-1-0246, by the National Science Foundation under grant CCF-0729230, and by the Baskin Chair of Computer Engineering. The views and conclusions contained in this document are those of the author(s) and should not be interpreted as representing the official policies, either expressed or implied, of the U.S. Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation hereon.

References

[1] P. Gupta, P. R. Kumar, *The capacity of wireless networks*, IEEE Transaction on Information Theory, vol. 46, no. 2, pp. 388-404, 2000.

[2] Z. Wang, H. Sadjadpour, J. J. Garcia-Luna-Aceves, and S. Karande, *Fundamental limits of information dissemination in wireless ad hoc networks- Part I: Single-packet reception*, IEEE Transactions on Wireless Communications, vol. 8, no. 12, pp. 5749-5754, 2009.

[3] Z. Wang, H. Sadjadpour, J. J. Garcia-Luna-Aceves, *Fundamental limits of information dissemination in wireless ad hoc networks- Part II: Multi-packet reception*, To appear in IEEE Transactions on Wireless Communications, 2010.

[4] X.-Y. Li, *Multicast capacity of wireless ad hoc networks*, IEEE/ACM Transaction on Networking, January, 2008.

[5] S. Karanade, Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, *Optimal unicast capacity of random geometric graph: impact of multipacket transmission and reception*, Journal of selected areas in communications, vol. 6, no. 1, January, 2009.

[6] Z. Wang, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, *A unifying perspective on the capacity of wireless ad hoc networks*, Proc. of IEEE INFOCOM 2008, Phoenix, Arizona, USA, April 13-18 2008.

[7] F. Xue, P. R. Kumar, *Scaling laws for ad hoc wireless networks: an information theoretic approach*, Now Publishers Inc., 2006.

[8] A. Keshavarz, V. Ribeiro, and R. Riedi, *Broadcast capacity in multihop wireless networks*, in Proc. of ACM MobiCom 2006, Los Angeles, California, USA, Septmber 23-29 2006.

[9] C. Peraki and S. Servetto, *On the maximum stable throughput problem in random networks with directional antennas*, in Proc. of ACM MobiHoc 2003, Annapolis, Maryland, USA., June 1-3 2003.

[10] S. Yi, Y. Pei, and S. Kalyanaraman, *On the capacity improvement of ad hoc wireless networks using directional antennas*, in Proc. of ACM MobiHoc 2003, Annapolis, Maryland, USA., June 1-3 2003.