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Author

Rook, J.R.

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University of California

Ernest O. Lawrence

Radiation Laboratory

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J. R. Rook

September 1966

ABSOLUTE CROSS SECTIONS FOR TRITON AND ^3He INDUCED REACTIONS*

J. R. Rook

Lawrence Radiation Laboratory
University of California
Berkeley, California

and

Nuclear Physics Laboratory
Oxford University
Oxford, England

Abstract: The Ohmura wavefunction for the three body system is used to obtain estimates of the absolute cross sections for (t,d) and (^3He ,d) reactions. Qualitative agreement with the experimental data is obtained but it is found that the results are too sensitive to the parameters of the calculation for any quantitative conclusions to be drawn. The same three body wavefunction is used to investigate the relative amplitudes for singlet and triplet spin transitions in (^3He ,p) reactions. These transitions are calculated to be about equally probable but a precise comparison with experiment is difficult as it is necessary to make detailed assumptions concerning the wavefunctions of the nuclei concerned. A rough estimate gives satisfactory agreement with experiment.

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1. Introduction

A considerable amount of attention has recently been given to the problem of calculating the absolute cross sections for stripping and pick up reactions in the distorted wave theory^{1,2,3}). The interest in this calculation arises partly from its importance as a verification of the correctness of the theory and partly because it may be possible in this way to extract the absolute values of spectroscopic factors. We think that the second of these points is not of practical interest at present, except possibly for the case of (d,p) reactions, since in other cases the accuracy obtained by calculation is less than that obtained by normalising the process to a known single particle transition. Thus our major interest is that these calculations provide further information concerning the accuracy of the theory.

We shall consider two major types of process, the single nucleon transfer reactions (t,d) and (³He,d) and the two nucleon transfer reactions (t,p) and (³He,p). The first type has recently been considered by Bassel¹), who has shown that by using the Irving wavefunction for ³He it is possible to obtain very good agreement with the experimental (³He,p) cross sections. Other calculations by Glover et al²) have shown that agreement can be obtained for (t,d) reactions although these calculations are rather artificial and agreement is forced by adjusting the parameters.

For the two nucleon stripping reactions Mathur³) has presented calculations using the Ohmura⁴) wavefunction to describe the internal motion of the nucleons in the triton. A fit is obtained in this case with a suitable but very reasonable choice of parameters although further calculations by Glover⁵) have shown that the fit obtained by Mathur is rather fortuitous in view of the uncertainties of the distorted wave theory.

Our first object in this paper is to repeat the calculations of Bassel¹⁾ using the Ohmura wavefunction. In this way we show that the wavefunction used in ref.³⁾ to reproduce the (t,p) data also accounts moderately well for the (t,d) data. Our results further suggest that the good agreement obtained in ref.¹⁾ is a consequence of a fortunate choice of wavefunction for ^3He .

Our second object is to consider the ratio of singlet to triplet transition probabilities for ($^3\text{He},p$) reactions. This is a quantity which is difficult to measure experimentally but is required before information concerning nuclear wavefunctions can be obtained. Also a calculation should be fairly reliable as only the ratio of the singlet to triplet amplitudes is required and not the absolute value of either.

2. The Reactions (t,d) and ($^3\text{He},d$)

A number of authors^{1,2)} have shown that the zero range distorted wave theory for (t,d) reactions should be multiplied by

$$C^2 g^2 = 2 |\langle \psi_D X_n | V | \psi_t \rangle|^2 \quad (1)$$

where ψ_D and ψ_t are the deuteron and triton wavefunctions, X_n is the spin wavefunction of the captured neutron, V is the deuteron-neutron interaction and the factor 2 arises since these are two neutrons. The symbol C in eq. (1) denotes the Clebsch-Gordon coefficient representing the coupling of the angular momenta on the right hand side. It is normally included in the distorted wave theory and henceforth we shall ignore it. If we let 1 and 2 refer to neutrons and 3 to the proton, then V is given by the sum of the internucleon

potentials V_{13} and V_{12} . We now assume the spatial part of the triton wavefunction to be completely symmetric. Consideration of the spin part of eq. (1) gives

$$g^2 = \frac{3}{2} \left| \frac{3}{2} U_0 + \frac{1}{2} U_1 \right|^2 \quad (2)$$

where

$$U_S = \int \phi_t(r_1, r_2, r_3) V_S(|r_1 - r_3|) \phi_D(r_2, r_3) d\tau \quad (3)$$

In this formula $V_S(|r_1 - r_3|)$ is the internucleon interaction in the spin state S, ϕ is the spatial parts of the wavefunctions ψ and the integration $d\tau$ is for fixed center of mass of the three nucleons.

For the case of $(^3\text{He}, d)$ reactions we obtain in place of eq. (2)

$$g^2 = \frac{3}{2} \left| \frac{3}{2} U'_0 + \frac{1}{2} U'_1 - U_C \right|^2 \quad (4)$$

when U'_0 and U'_1 are given by eq. (3) with the triton wavefunction replaced by that for ^3He while U_C is given by a similar expression with V_S replaced by the Coulomb potential e^2/r . We shall assume throughout that the spatial parts of the triton and ^3He wavefunctions are identical so that $U'_S = U_S$.

Comparing our results with those of Bassel¹⁾ we see that the quantity inside the modulus signs of eqs. (2) and (4) is just labeled D_0 in his paper. We shall compare our respective values of D_0 and it should be noted that the cross sections are proportional to the square of this quantity.

In refs.^{1,2)} D_0 is evaluated by a transformation using the Schrödinger equations for ϕ_t and ϕ_D , but using the Ohmura method it is easier to evaluate eq. (3) directly. For ϕ_t we have⁴⁾

$$\phi_t = N_t F(|r_1 - r_2|) F(|r_1 - r_3|) F(|r_2 - r_3|) \quad (5)$$

where

$$F(x) = e^{-\mu(x-D)} - e^{-\nu(x-D)} \quad x > D \quad (6a)$$

$$= 0 \quad x < D \quad (6b)$$

and N_t is a normalising constant. We further take

$$\phi_D(x) = N_D (e^{-\gamma(x-D)} - e^{-\alpha(x-D)})/x \quad x > D \quad (7a)$$

$$= 0 \quad x < D$$

with N_D a normalising constant. Finally the internucleon potential is taken to have the form

$$V_S(x) = V_S^0 e^{-\beta_S(x-D)} \quad x > D \quad (8a)$$

$$= \infty \quad x < D \quad (8b)$$

We take $\gamma = 0.14 \text{ fm}^{-1}$, $\alpha = 1.6 \text{ fm}^{-1}$ and the values of μ , ν , β_S and V_S^0 tabulated in ref.⁴⁾ for various values of D . The parameter V_0^0 is known only if the singlet effective range r_{0S} is known and in ref.⁴⁾ values of the

parameters are given for $r_{OS} = 2.4$ fm and 2.7 fm, the latter being a better value. We present results corresponding to both values. The analytical evaluation of eq. (3) is now a straightforward calculation and we shall not give the detailed formulae.

The results of our calculations for (t,d) reactions together with that of Bassel¹⁾ using the Irving wavefunction are given in table 1. We see first of all that our calculated cross section is very sensitive to the assumed value of the hard core radius D so that any precise comparison with experiment must be meaningless since this quantity is not well determined. Further we note that our values of D_0 are larger than Bassel's which is itself a little larger than is required to agree with experiment. A result larger than the experimental one was also obtained by Mathur³⁾ for (t,p) reactions but it is not clear whether these discrepancies are caused by an inadequate three body wavefunction or a deficiency in the distorted wave theory.

We consider next the comparison of ($^3\text{He},d$) reactions with (t,d) reactions. In table 2 we give the ratio of the respective values of D_0^2 , that is the ratio of the values of the constants which multiply the results obtained from the distorted wave theory. The Bassel calculations labeled a and b correspond respectively to excluding and including the Coulomb term in V of eq. (1). In each case his ^3He wavefunction differs from that for tritium through the binding energy difference. We see that our calculations suggest that the ($^3\text{He},d$) and (t,d) cross sections will be within 10% of each other while Bassel's calculations, especially the one labeled a, suggest rather more different results. A detailed experimental test of this point would perhaps be worthwhile.

We should mention that while we have assumed that tritium and ^3He have the same wavefunctions the respective root mean square charge radii are rather different. This point has been considered in ref.³⁾ for the two nucleon stripping process and similar results may be obtained in the present case. It turns out that the difference in radii does not imply that the absolute cross sections for (t,d) and $(^3\text{He},d)$ are necessarily very different.

3. Two Nucleon Transfer Reactions

In ref.³⁾ a comparison was made of the cross sections for $(^3\text{He},p)$ and (t,p) reactions between analogue states. From the point of view of the reaction process the (t,p) reaction is simpler than the $(^3\text{He},p)$ reaction since for the former we need consider only transfer of the two nucleons in singlet spin states. This result follows from the identity of the transferred nucleons providing we assume a spatially symmetric triton wavefunction. For $(^3\text{He},p)$ reactions the transferred nucleon pair is not composed of identical particles and so triplet spin transfer is possible. In ref.³⁾ this point was not considered since the nuclear states involved there yielded only singlet transfer, but in general both singlet and triplet transfers are allowed so that a knowledge of their relative amplitudes is required before information can be obtained concerning the nuclear wavefunctions.

For $(^3\text{He},p)$ reactions the scaling factor g is defined similarly to eq. (1) by

$$C_g = \langle \chi(1)\phi(2,3) | V_{12} + V_{13} | \psi(1,2,3) \rangle \quad (9)$$

where 1 and 2 represent protons and 3 the neutron, V_{12} is the interaction between nucleons 1 and 2, χ is the spin wavefunction of the emitted proton, ϕ is the spin wavefunction of the captured pair and ψ is the internal wavefunction of ^3He . We are interested in the relative values of g when ϕ is a singlet, $S = 0$, state or a triplet, $S = 1$, state. Assuming that the spatial part of ψ is symmetric the spin part of eq. (9) is easily evaluated. We obtain

$$\frac{g(S=0)}{g(S=1)} = -\frac{1}{\sqrt{3}} \frac{\frac{1}{4} U_0 + \frac{3}{4} U_1 - \frac{1}{2} U_C}{\frac{3}{4} U_0 + \frac{1}{4} U_1 - \frac{1}{2} U_C} \quad (10)$$

where

$$U_i = \int \phi_{\text{He}}(r_1 r_2 r_3) V_i(|r_1 - r_2|) d\tau \quad (11)$$

and $i = 0, 1$ refers to the singlet and triplet internucleon potentials of eqs. (8) while $i = C$ refers to the Coulomb potential $\frac{e^2}{|r_1 - r_2|}$.

We shall write eq. (10) as

$$\frac{g(S=0)}{g(S=1)} = -\frac{1}{\sqrt{3}} R \quad (12)$$

and present the values of R in table 3. We see that a priori the values of R corresponding to singlet and triplet transfer are roughly comparable and that the best estimate from our calculation is $R = 1.2$.

It is interesting to see how this result works out in practice. We consider the data of Hardy and Skyrme⁶⁾ on the $^{31}\text{P}(p, ^3\text{He})^{29}\text{Si}$ and $^{31}\text{P}(p, t)^{29}\text{P}$

reactions and in particular we shall consider the transition to the ground states of ^{29}Si and ^{29}P . The purpose of considering both the (p,t) and $(p,^3\text{He})$ reactions is that in this way we obtain an estimate of R without the necessity of considering absolute cross sections. We assume that the ground state of ^{31}P consists of $(2s_{\frac{1}{2}})^3$ while the ground states of ^{29}Si and ^{29}P consist of $(2s_{\frac{1}{2}})$. Given these configurations it is a straightforward matter to calculate the ratio of $S = 0$ to $S = 1$ fractional parentage coefficients using the formulae of Rook and Mitra⁷⁾. The ratio turns out to be -1 multiplied by the isospin Clebsch Gordon coefficient. Collecting all the terms together we obtain

$$\frac{\sigma(p, ^3\text{He})}{\sigma(p, t)} = \frac{|1 - \frac{1}{3} R|^2}{|\frac{2}{3} R|^2} \quad (13)$$

Experimentally, this ratio is .25 giving from eq. (13) $R = 1.5$. Comparing with table 3 we see surprisingly good agreement with our calculated value of 1.2.

4. Conclusion

We have presented here and in refs.^{2,3)} a number of calculations concerning the absolute cross sections for stripping reactions none of which are in gross disagreement with the experimental data and we can thus conclude that distorted wave theory gives qualitatively the correct answers for the absolute cross sections. We believe, however, that our calculations taken together with those of Mathur³⁾ indicate that no statement concerning quantitative agreement can be made in view of the uncertainties apparent from our results. The only hope of real improvement appears to lie in consideration of a far better three body wavefunction including D states.

I would like to thank Dr. V. S. Mathur for introducing me to the techniques of the Ohmura wavefunction, the Director of the Oxford University Computing Laboratory for computing time and the Science Research Council for financial support.

References

- 1) R. H. Bassel, Phys. Rev., to be published.
- 2) R. H. Glover, A. D. W. Jones and J. R. Rook, Nuclear Physics 81 (1966), 289.
- 3) V. S. Mathur and J. R. Rook, Nuclear Physics, to be published.
- 4) T. Kikuta (Ohmura), M. Morita and M. Yamada, Prog. Theor. Phys. 15 (1956), 222.
- 5) R. N. Glover, private communication.
- 6) J. C. Hardy and D. J. Skyrme, to be published.
- 7) J. R. Rook and D. Mitra, Nuclear Physics 51 (1966), 96.

Table 1
Value of D_0 for (t,d) reactions

D fm	D_0 MeV - fm ^{3/2}	
	$r_{0s} = 2.7$ fm	$r_{0s} = 2.4$ fm
0.0	213	216
0.2	228	236
0.4	282	300
0.6	368	400
Bassel (Irving)	184	---

Table 2
Ratio of D_0^2 for (³He,d) and (t,d) reactions

D fm	$D_0^2(^3\text{He},d)/D_0^2(t,d)$	
	$r_{0s} = 2.7$ fm	$r_{0s} = 2.4$ fm
0.0	0.89	0.91
0.2	0.93	0.93
0.4	0.93	0.94
0.6	0.94	0.95
Bassel (Irving)	0.77 (a)	0.88 (b)

Table 3

Comparison of $S = 0$ and $S = 1$ two nucleon transfer reactions

D fm	R	D fm	R
0.0	1.23	0.4	1.12
0.2	1.18	0.6	1.11

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