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Effect of Mean Flow Shear on Cross Phase and Transport Reconsidered

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We reconsider the important question of the effect of a strong mean shear flow on the transport of a passive scalar field. By incorporating the effect of resonance, we show that the flux scales with the mean shear Ω as Ω^{-1}. The results also indicate that the scaling of the flux and cross phase with shear is rather weak and that the cross phase is not always more heavily suppressed than the amplitude of the turbulence. Furthermore, we show that the scalings of flux and cross phase with Ω depend on the statistics of the turbulent flow.

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One of the most promising mechanisms for regulating anomalous turbulent transport in magnetically confined plasmas is the enhanced decorrelation of turbulence by a mean shear flow [1]. Indeed, mean $\mathbf{E} \times \mathbf{B}$ shearing plays a central role in the $L \rightarrow H$ transition and in the development of internal transport barriers, which are crucial to advanced tokamak concepts. Any predictive model of transport barrier formation requires a quantitative understanding of the effects of flow shear on turbulent transport. More generally, the addition of a mean shear flow also defines an interesting and broadly relevant generalization of the classic problem of passive scalar transport [2].

The basic concept concerning the effect of shear on turbulence originated in the study of galactic dynamics [3]. A mean shear flow $U_0(x)\hat{\mathbf{y}}$ distorts turbulent eddies, generating smaller radial scales, until they are quenched by dissipation. Here, x and y represent local radial and poloidal directions. This process can reduce the radial transport $\Gamma = \langle \chi v_x \rangle = \sum_{\mathbf{k}} |\chi(\mathbf{k})||v_x(-\mathbf{k})| \cos\delta_{\mathbf{k}}$ of a scalar field χ, via the reduction of the amplitude of the turbulence $|\chi(\mathbf{k})|$ and/or via reduction of the phase shift $\cos\delta_{\mathbf{k}}$ between the scalar field and the radial velocity. Since the work by Biglari *et al.* [4] estimated the reduction in the amplitude of turbulence as $\langle \chi^2 \rangle \propto \Omega^{-2/3}$, several works [5–8] have been devoted to the determination of the dependence of cross phase $\cos\delta_{\mathbf{k}}$ on shear Ω. In particular, a recent work by Terry *et al.* [7] argued that a radial flux of a (passive) scalar field, which is advected by a random flow and a linear mean shear flow, is significantly reduced ($\sim \Omega^{-4}$ in the strong shear limit), with the cross-phase scaling with shear as Ω^{-3}, and claimed cross-phase suppression is the dominant transport reduction mechanism. Reference [7] also claimed agreements with several fluctuation measurements from experiments [1,9].

The purpose of this Letter is to revisit the important questions of cross phase and flux scaling with Ω. The basic physics that leads to our conclusion can be understood by recalling that a radial flux of a scalar field, representing the transport of the scalar field between different scales, requires dissipative process such as (effective) diffusion or decorrelation of the scalar field, *or* the presence of a resonant absorption point in the system where the Doppler shift frequency vanishes $\omega = \mathbf{k} \cdot \mathbf{U}_0 = x k_y (\partial_x U_0)$ (i.e., critical layer). Here $\mathbf{U}_0 = x(\partial_x U_0)\hat{\mathbf{y}}$ is a linear mean (poloidal) shear flow, and x is the distance from a rational surface where a spectral intensity is localized. The result in [7] was obtained by incorporating diffusion only for the limit $\omega \rightarrow 0$, overlooking the important and ubiquitous effect of resonance between the shear flow and the fluctuations. However, for a low frequency mode ($\omega \sim \omega_*$), a resonant point $x = \omega / k_y (\partial_x U_0)$ resides close to the rational surface, thereby contributing to the flux by introducing irreversibility (and thus transport) for small fluctuation amplitude (so long as phase space islands overlap). In this Letter, we show that the resonant contribution to the flux leads to a much weaker dependence on the shear, namely, $\Gamma \propto \Omega^{-1}$.

Before presenting our formal analysis, we provide a simple, but transparent, calculation of the flux, which sheds much light on the physics of the problem. We consider the advection of a passive scalar field χ by a turbulent flow $\mathbf{v}$ and a linear mean shear flow $U_0(x)\hat{\mathbf{y}} = x(\partial_x U_0)\hat{\mathbf{y}}$ in the local radial (x) and poloidal (y) planes, perpendicular to a magnetic field $\mathbf{B} = B_0 \hat{\mathbf{z}}$:

$$\partial_t \chi + \nabla \cdot (\mathbf{u}\chi) = 0, \tag{1}$$

where $\mathbf{u} = \mathbf{v} + U_0(x)\hat{\mathbf{y}}$ with $\mathbf{v} = -(c/B)\nabla\phi \times \hat{\mathbf{z}}$. The renormalized equation for the flutelike fluctuations $\tilde{\chi}$ (with $k_\parallel = 0$) takes the following form:

$$[-i\omega + i k_y x \Omega - \partial_x D(\mathbf{k}, \omega)\partial_x + k_y^2 d(\mathbf{k}, \omega)]\tilde{\chi}(\mathbf{k}, \omega) = \frac{c}{B} i k_y \phi(\mathbf{k}, \omega)\partial_x \chi_0. \tag{2}$$

Here, $\Omega = \partial_x U_0 = U_0'$, χ_0 is the mean component, and $D(\mathbf{k}, \omega)$ and $d(\mathbf{k}, \omega)$ are the renormalized radial and poloidal diffusivities, respectively [7]; x is the radial distance from the rational surface where the spectral intensity is localized, i.e., we assume that the fluctuation spectrum is localized at mode rational surfaces, where $\mathbf{k} \cdot \mathbf{B}_0 = 0$. Note that $\phi(\mathbf{k}, \omega)$ implicitly depends on x which varies on scales much larger than the characteristic scales of fluctuations. Note also that so long as the

spectral width (in radius) W_k satisfies $W_k > |\omega_k/k_y U_0'|$, a resonance between the flow and the fluctuation mode frequency ω_k is located *within* the spectral envelope (see Fig. 1).

It is appropriate to examine the basic time scales in the problem. Here, the dispersion rate for a fluid packet waveform (i.e., the autocorrelation rate, in the language of quasilinear theory) is $|k_y W_k \Omega|$ with the corresponding nonlinear decay rate $k^2 d(\mathbf{k}, \omega)$. *Thus, it follows that the applicability of the "strong shear" limit* [i.e., $k_y \Delta x \Omega > k^2 d(\mathbf{k}, \omega)$] *is precisely equivalent to the condition for the applicability of quasilinear theory* [i.e., $\tau_{\rm ac} < (k^2 d)^{-1}$], with $\Delta x \sim W_k$ and $\tau_{\rm ac} = (k_y \Delta x \Omega)^{-1}$. Note that this is eminently consistent with the expectation that shear should reduce turbulence intensity, thus rendering weak turbulence approaches appropriate.

In the limit of large shear (weak turbulence) such that $\omega, k_y W_k \Omega \gg \partial_x D(\mathbf{k}, \omega)\partial_x$, Eq. (2) is simply reduced to $\tilde{\chi}(\mathbf{k}, \omega) \sim (c/B) k_y \phi(\mathbf{k}, \omega) \partial_x \chi_0 / [\omega - k_y x \Omega + i k_y^2 d(\mathbf{k}, \omega)]$, as derived in [7], with the following flux:

$$\Gamma = \mathrm{Re} \sum_{\mathbf{k}} \frac{i\pi \partial_x \chi_0 |\overline{v}_x(\mathbf{k}, x)|^2}{\omega - k_y x \Omega + i k_y^2 d(\mathbf{k}, \omega)} \bigg|_{\omega = \omega_k + i\gamma_k}. \tag{3}$$

Here, the integration over frequency was performed by using Lorentzian frequency spectrum $|v_x(\mathbf{k}, x, \omega)|^2 = |\overline{v}_x(\mathbf{k}, x)|^2 \gamma_k / [(\omega - \omega_k)^2 + \gamma_k^2]$, and the dependence of $|\overline{v}_x(\mathbf{k}, x)|^2$ on a slowly varying x was explicitly indicated. The scaling of $\Gamma \propto \Omega^{-4}$ in [7] follows if $\omega, k_y^2 d, \gamma_k \ll |k_y W_k \Omega|$ (ignoring the resonant response) and also if $\gamma_k \propto d(\mathbf{k}, \omega) \propto \Omega^{-2}$, by expanding the denominator of Eq. (3). However, an exception to the latter scaling can be easily found in the limit of a delta-correlated flow $\mathbf{v}$, satisfying $\gamma_k \gg k_y^2 d, \omega_k, |k_y W_k \Omega|$, in which case the flux becomes independent of Ω.

We now examine the resonant contribution to the flux in a more general case. By using $\mathrm{Re}[i/(\omega - k_y x \Omega + i k_y^2 d(\mathbf{k}, \omega)] = \pi \delta(\omega - k_y x \Omega)$ in Eq. (3), we obtain

$$\Gamma = -\sum_{\mathbf{k}} \frac{\pi^2 |\overline{v}_x(\mathbf{k}, x)|^2}{|k_y \Omega|} \delta\left(x - \frac{\omega_k}{k_y \Omega}\right) \partial_x \chi_0. \tag{4}$$

Equation (4) shows that the flux due to resonance is proportional to Ω^{-1}, suggesting a rather modest dependence on Ω. The scaling of $D \sim 1/\Omega$ is precisely analogous to the well-known scaling of $D \sim 1/U_0$ in the classic 1D Vlasov quasilinear diffusion problem. In each case, the scaling is due to the proportionality of D to a fluid element dispersion time (phase space fluid, for the Vlasov case). Also, the flux Γ in the above equations is local in the radial direction. Thus, one can take a spatial average over scale λ in the radial direction (which lies between the scale of Ω and that of fluctuations) to remove the delta function.

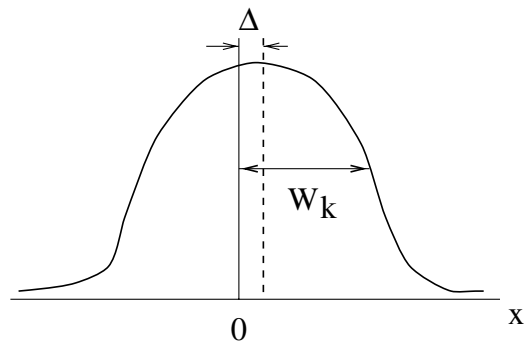

FIG. 1. Spectral intensity is localized within width W_k around rational surface $x = 0$. The maximum of the intensity is slightly shifted from $x = 0$ by Δ due to the shear. This shift Δ is exaggerated for clarity.

In order to obtain the correct scaling of the cross phase (in particular, that of the amplitude of turbulence) with the shear, the effect of the shear on scalar fluctuation levels should be carefully taken into account. This is because the amplitude of turbulence crucially depends on the radial diffusion, whose effect is very sensitive to the shear. Namely, a shear flow $U_0(x)\hat{\mathbf{y}}$ generates small scales in the x direction due to shearing, linearly increasing k_x in time. Thus, even if the radial diffusion at some time may be extremely small, its effect can be no longer neglected at later times on account of the reduction in the radial scale. This main effect of the shear, namely, the linear increase of the wave number, can be nonperturbatively incorporated by following a particle trajectory in the extended phase space $(\mathbf{x}, \mathbf{k}, t)$, with the help of the Gabor transform [10], as shown below. The Gabor transform is a localized Fourier transform (i.e., a type of wavelet transform), defined by $GT[\psi(\mathbf{x}, t)] = \hat{\psi}(\mathbf{k}, \mathbf{x}, t) = \int_{-\infty}^{\infty} d^2x' f(|\mathbf{x} - \mathbf{x}'|) e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \psi(\mathbf{x}', t)$. Here, $f(x) = f(0) \times \exp(-x^2/\lambda^2)$ is a filter function at scale λ. λ can be chosen to be the scale of the localization of a mode near resonant surface (i.e., $\lambda \sim W_k$) to ensure a roughly homogeneous turbulence within the localized regime. By using this method, we now determine the flux, amplitude, and cross phase.

In terms of the Gabor transform, Eq. (1) is written as

$$D_t \hat{\chi} = -\hat{v}_x \partial_x \chi_0 - D(k_x^2 + k_y^2)\hat{\chi}, \tag{5}$$

where $D_t = \partial_t + U_0 \partial_y + k_y \Omega \partial_{k_x}$ is the total time derivative in the extended phase space $(\mathbf{x}, \mathbf{k}, t)$, $\Omega = -\partial_x U_0$, which is assumed to be positive without a loss of generality, and D is a normalized diffusivity. Note that $D_t k_x = k_y \Omega$, by the eikonal equations. The main effect of shearing, namely, the linear increase or decrease of the wave number in time, can be explicitly incorporated by integrating Eq. (5) along a particle trajectory, for a given v_x, as

$$\hat{\chi}(\mathbf{x}, \mathbf{k}, t) = -\int d^2k_1 d^2x_1 dt_1 g(\mathbf{x}, \mathbf{k}, t{:}\mathbf{x}_1, \mathbf{k}_1, t_1) \hat{v}_x(\mathbf{x}_1, \mathbf{k}_1, t_1) \partial_x \chi_0, \tag{6}$$

where g is the Green's function

$$g(\mathbf{x}, \mathbf{k}, t{:}\mathbf{x}_1, \mathbf{k}_1, t_1) = \delta(x - x_1)\delta(y - y_1 - U_0(t - t_1))\delta(k_x - k_{1x} - k_y\Omega(t - t_1))\delta(k_y - k_{1y}) \times \exp\left\{-D\left(k_y^2 t + \frac{k_x^3}{3\Omega k_y}\right)\right\}\exp\left\{D\left(k_{1y}^2 t_1 + \frac{k_{1x}^3}{3\Omega k_{1y}}\right)\right\}. \tag{7}$$

To compute the flux in the long time limit, we first use the inverse Gabor transform $a(\mathbf{x}, t) = \int d^2k\hat{a}(\mathbf{x}, \mathbf{k}, t)/f(0)(2\pi)^2$ and then approximate the correlation function of the random velocity $\hat{v}_x$ in Gabor space in terms of that in Fourier space by assuming homogeneous v_x as $\langle\hat{v}_x(\mathbf{x}_1, \mathbf{k}_1, t_1)\hat{v}_x(\mathbf{x}_2, \mathbf{k}_2, t_2)\rangle \sim (2\pi)^2\delta(\mathbf{k}_1 + \mathbf{k}_2)f^2(|(\mathbf{x}_1 - \mathbf{x}_2)/2|)e^{i(\mathbf{x}_1-\mathbf{x}_2)\cdot\mathbf{k}_2} \times \tilde{\psi}(\mathbf{k}_2, t_2 - t_1)$, where $\tilde{\psi}$ is the Fourier transform of $\psi(\mathbf{r}, t) \equiv \langle v_x(\mathbf{x}, t_1)v_x(\mathbf{x} + \mathbf{r}, t_2)\rangle$ (e.g., see [10]). Straightforward algebra then gives us

$$\langle\tilde{\chi}(\mathbf{x}, t)v_x(\mathbf{x}, t)\rangle \simeq \frac{-\partial_x\chi_0}{(2\pi)^3 f^2(0)}\int d^2k_1 d\omega \int_0^\infty d\tau e^{i(\omega - k_{1y}x\Omega)\tau - DQ(\mathbf{k}_1,\tau)} f^2\left(\left|\frac{x\Omega\tau}{2}\right|\right)\psi(\mathbf{k}_1, \omega). \tag{8}$$

Here, $Q(\mathbf{k}, \tau) = \tau(k^2 + k_x k_y\Omega\tau + k_y^2\Omega^2\tau^2/3)$, and $\psi(\mathbf{k}_1, \omega)$ is the power spectrum of v_x in Fourier space, i.e., $\langle v_x^2(\mathbf{x}, t)\rangle = \int d^2k d\omega\psi(\mathbf{k}, \omega)/(2\pi)^3$. In the following, we focus on the strong shear limit such that $Dk_1^2/\Omega \ll 1$, which corresponds to the weak turbulence case as previously discussed. Note that the condition $Dk_1^2/\Omega \ll 1$ applies to the spectrum of the (prescribed) turbulent flow and is thus satisfied in the long time limit despite the linear increase of k_x of the advected scalar field $\tilde{\chi}$. For a sufficiently strong shear, satisfying $Dk_1^2/\Omega \ll (x/\lambda)^2 \lesssim 1$, Eq. (8) simplifies to

$$\langle\tilde{\chi}(\mathbf{x}, t)v_x(\mathbf{x}, t)\rangle = \frac{-\partial_x\chi_0}{8\pi^{5/2}}\left[1 + \left(\frac{\gamma_k\lambda}{x\Omega}\right)^2\right]^{-1/2}\frac{\lambda}{x\Omega}\int d^2k_1\overline{\psi}(\mathbf{k}_1)\exp\left[-\frac{(\omega_k - k_{1y}x\Omega)^2}{\gamma_k^2 + (x\Omega/\lambda)^2}\right]. \tag{9}$$

Here, the integration over frequency was performed, for simplicity, by assuming a Gaussian frequency spectrum $\psi(\mathbf{k}, \omega) = \overline{\psi}(\mathbf{k})\exp\{-(\omega - \omega_k)^2/\gamma_k^2\}/\sqrt{\pi}\gamma_k$. One can immediately check that the flux becomes independent of the shear in the case of a delta-correlated flow with $\lambda\gamma_k/x\Omega \gg 1$, as $\langle\tilde{\chi}(\mathbf{x}, t)v_x(\mathbf{x}, t)\rangle = [(-\partial_x\chi_0)/(8\gamma_k\pi^{5/2})] \times \int d^2k_1\overline{\psi}(\mathbf{k}_1)$. This is consistent with the result obtained previously. In a physically more relevant case of a localized frequency spectrum with $\lambda\gamma_k/x\Omega \ll 1$, Eq. (9) may be approximated as

$$\langle\tilde{\chi}(\mathbf{x}, t)v_x(\mathbf{x}, t)\rangle \sim -\frac{\partial_x\chi_0}{8\pi^{5/2}}\frac{\lambda}{x\Omega}\int d^2k_1\overline{\psi}(\mathbf{k}_1) \times \exp\{-\lambda^2[k_{1y} - (\omega_k/x\Omega)]^2\}. \tag{10}$$

Here, the power spectrum $\overline{\psi}(\mathbf{k}_1)$ was assumed to be localized in $\mathbf{k}_1$ to insure the convergence of the integral over $\mathbf{k}_1$ (especially k_{1x}). The argument of the exponential function takes on its maximum value at the resonant point where $\omega_k = k_{1y}x\Omega$. At that point the flux becomes proportional to Ω^{-1}. As one moves away from the resonance point towards larger x ($\omega_k \ll k_{1y}x\Omega$), the exponential function becomes $\propto \exp\{-\lambda^2 k_{1y}^2\}$, independent of Ω, again leaving the flux $\propto \Omega^{-1}$. This can also be viewed as the low frequency limit $\omega_k \to 0$ for a fixed x. Note that the flux in Eq. (10) decreases with x because the amplitude of the mode decreases far from the rational surface. On the other hand, the flux in the small x limit such that $k_{1y}x < \omega/\Omega, Dk_1^2/\Omega, (2k_{1y}\lambda)\sqrt{Dk_1^2/\Omega}$ can be computed from Eq. (8) as $\langle\tilde{\chi}(\mathbf{x}, t)v_x(\mathbf{x}, t)\rangle = [(-\partial_x\chi_0)/(8\gamma_k\pi^{5/2})] \times \int d^2k_1\overline{\psi}(\mathbf{k}_1)\exp(-\omega_k^2/\gamma_k^2) \propto \Omega^0$. Interestingly, this result is similar to that in the case of delta-correlated flow. Note, however, that despite the different behavior of the flux for small and large x limits, the integral over wave number $\mathbf{k}_1$ [in Eq. (8)] will drastically smooth over the spatial variation of the diffusion coefficient, making the flux $\sim\Omega^{-1}$.

To determine the cross phase which convolves both the effective autocorrelation time and the scalar response function, we now compute the amplitude of the turbulence $\langle\tilde{\chi}^2\rangle$ by using Eq. (6) in the long time limit:

$$\langle\tilde{\chi}^2(\mathbf{x}, t)\rangle = \frac{(\partial_x\chi_0)^2}{(2\pi)^3 f^2(0)}\int d^2k_1 d\omega \int_0^\infty d\tau_1 \int_0^\infty d\tau_2 e^h f^2\left(\left|\frac{x\Omega(\tau_2 - \tau_1)}{2}\right|\right)\psi(\mathbf{k}_1, \omega), \tag{11}$$

where $h = i(\omega - k_{1y}x\Omega)(\tau_1 - \tau_2) - D[Q(\mathbf{k}_1, \tau_1) + Q(-\mathbf{k}_1, \tau_2)]$, and $Q(\mathbf{k}, \tau) = \tau(k^2 + k_x k_y\Omega\tau + k_y^2\Omega^2\tau^2/3)$. For a sufficiently strong shear, satisfying $Dk_1^2 \ll \Omega(x/\lambda)^2 \lesssim \Omega$, Eq. (11) becomes

$$\langle\tilde{\chi}^2(\mathbf{x}, t)\rangle \simeq \frac{(\partial_x\chi_0)^2}{12\pi^{5/2}}\frac{\lambda}{x\Omega^2}\Gamma\left(\frac{1}{3}\right)\left[1 + \left(\frac{\gamma_k\lambda}{x\Omega}\right)^2\right]^{-1/2}\int d^2k_1\overline{\psi}(\mathbf{k}_1)\left(\frac{3\Omega}{2Dk_1^2}\right)^{1/3}\exp\left[-\frac{(\omega_k - k_{1y}x\Omega)^2}{\gamma_k^2 + (x\Omega/\lambda)^2}\right]. \tag{12}$$

Here, again the integration over frequency was performed by using Gaussian frequency spectrum. For a delta-correlated flow with $\lambda\gamma_k/x\Omega \gg 1$, Eq. (12) can be simplified to $\langle\tilde{\chi}^2(\mathbf{x}, t)\rangle \sim [(\partial_x\chi_0)^2/(12\gamma_k\pi^{5/2}\Omega)]\Gamma(1/3) \times \int d^2k_1\overline{\psi}(\mathbf{k}_1)(3\Omega/2Dk_1^2)^{1/3}$. Since the flux is independent of the shear for a delta-correlated flow (as previously shown), the cross phase $\propto [\langle v_x\tilde{\chi}\rangle/(\langle\tilde{\chi}^2\rangle^{1/2}\langle v^2\rangle^{1/2})] \propto \Omega^{1/3}$, which increases with shear. It can be easily shown that without shear,

$\langle\tilde{\chi}^2\rangle \propto (\partial_x\chi_0)^2 \int d^2k_1\overline{\psi}(\mathbf{k}_1)(\Omega/Dk_1^2)\Omega^{-1}$. Thus, a shear reduces $\langle\tilde{\chi}^2\rangle$ by a factor of $(Dk_0^2/\Omega)^{2/3}$, in agreement with the estimate given in [4]. Here, k_0 is the characteristic scale of fluctuations.

In a physically more relevant case of a localized frequency spectrum with $\lambda\gamma_k/x\Omega \ll 1$, Eq. (12) is approximated to

$$\langle\tilde{\chi}^2(\mathbf{x},t)\rangle \sim \frac{(\partial_x\chi_0)^2}{12\gamma_k\pi^{5/2}\Omega^2}\Gamma\left(\frac{1}{3}\right)\frac{\lambda}{x}\int d^2k_1\overline{\psi}(\mathbf{k}_1)\left(\frac{3\Omega}{2Dk_1^2}\right)^{1/3} \times \exp\{-\lambda^2[k_{1y}-(\omega_k/x\Omega)]^2\}. \quad (13)$$

That is, in this limit, $\langle\tilde{\chi}^2\rangle$ is proportional to $(\Omega/Dk_1^2)^{1/3}\Omega^{-2} \propto \Omega^{-5/3}$ for $\omega \leq k_{1y}x\Omega$. Therefore, Eqs. (10) and (13) give us the cross phase $\cos\delta_{\mathbf{k}} \sim (\lambda/x)^{1/2}(Dk^2/\Omega)^{1/6}$. Interestingly, the suppression factor $\Omega^{-1/6}$ for the cross phase is much weaker than that $\Omega^{-5/3}$ for the mean square amplitude. *This result clearly indicates that it is not universally true that the cross phase is more heavily suppressed than the amplitude of the turbulence.* Note that $\langle\tilde{\chi}^2\rangle \propto \Omega^{-5/3}$ in Eq. (13) for a localized frequency spectrum is different from the estimate $\langle\tilde{\chi}^2\rangle \propto \Omega^{-2/3}$ given in [4]. It is because [4] considered the evolution of two particles with a small initial separation while the result (13) is valid in the long time limit, for which the spatial separation is much larger than the correlation length. In the long time limit, the evolution of two particles with a small separation can, however, be isolated by considering a delta-correlated flow which quickly randomizes particle trajectories to compensate for the linear increase of the separation between two particles (due to the shear). This is why the estimate in [4] is recovered for a delta-correlated flow, but not in general. Finally, the square amplitude of fluctuations in the limit as $x \to 0$ can be obtained from Eq. (11) as $\langle\tilde{\chi}^2(\mathbf{x},t)\rangle \sim [(\partial_x\chi_0)^2/12\gamma_k\pi^{5/2}\Omega]\Gamma(1/3)\int d^2k_1\overline{\psi}(\mathbf{k}_1) \times (3\Omega/2Dk_1^2)^{1/3}\exp(-\omega_k^2/\gamma_k^2) \propto \Omega^{-2/3}$, which is similar to the result in the case of delta-correlated flow and also agrees with the estimate in [4]. Here, we again note that the mean square amplitude becomes proportional to $\Omega^{-2/3}$ once the integration over $\mathbf{k}_1$ in Eq. (11) is carried out.

In conclusion, we have reexamined the important issue of how a mean shear flow affects the transport of a passive scalar field, which is advected by a random flow together with a mean linear shear flow. By incorporating the resonant response, we demonstrate that in the strong shear limit, (i) for a random flow with a localized frequency spectrum, the flux decreases as Ω^{-1} while the square amplitude of the turbulence scales with $\Omega^{-5/3}$; (ii) for a random flow with a white noise frequency spectrum, the flux becomes independent of Ω while the mean square amplitude of turbulence is $\propto \Omega^{-2/3}$, with the cross phase $\propto \Omega^{1/3}$. These results suggest that the suppression factors for the flux and cross phase due to shear are generally rather weak. Furthermore, it also illustrates that the cross phase is not always more vulnerable to shear suppression than the intensity of the turbulence is [11].

These results imply that the scaling of transport of a scalar field with the shear is unlikely to be universal. For instance, in a self-consistent model where the turbulence arises due to the instability of the system (e.g., resistive ballooning mode, etc.), the scalings of flux and cross phase with a shear are likely to be different from those in the case of a passive scalar field considered in this Letter. Furthermore, the flux of particle and that of heat may respond differently to the effect of the shear flow. This is a particularly interesting issue in view of the fact that barriers for heat and particle do not always form simultaneously [6,11]. Another interesting problem is the effect of zonal flows on the transport and cross phase. Unlike a mean shear flow, zonal flows exhibit detailed, spectral structure in space. Moreover, in contrast to mean flows, zonal flows are not slowly evolving in time, but rather have a finite lifetime, roughly comparable to characteristic turbulence intensity time scales. These issues are currently under investigation.

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