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ABSTRACT

Different types of spectra in even-even nuclei are discussed. Properties of those nuclei for $40 < A < 154$ and $180 < A < 222$ are summarized.

Several theoretical predictions of the existence of a complex vibrational second excited state are described and compared with experimental data.

REMARKS ON EXCITED STATES OF EVEN-EVEN NUCLEI AND ON THE EXISTENCE OF A COMPLEX VIBRATIONAL SECOND EXCITED STATE*

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As is well known, the excited states of even-even nuclei show certain regularities, and one is able to distinguish different "types" of spectra characterized by the energy ratios between the excited levels and the character of these levels. The following types of spectra can be mentioned:[1, 2]

(a) Spectra characterized by ratios of energies of excited states and character of excited states corresponding to closed shells (protons and (or) neutrons).

(b) Spectra characterized by collective vibrations of spherical nuclei. In this case the character of the ground state is always 0+; that of the first excited state is 2+; the second excited state is either 0+, 2+, or 4+; and the third excited state either 0+, 2+, 3+, 4+, or 6+. This type of nuclei lies in the regions $40 < A < 154$ and $180 < A < 226$.

(c) Spectra characterized by collective vibrations and rotations of non-spherical nuclei. This type of spectra is:

0+, 2+, 4+, ... corresponding to the ground-state rotational band (K=0)
0+, 2+, 4+, ... corresponding to the beta vibrational band (K=0)
2+, 3+, 4+, ... corresponding to the gamma vibration band (K=2) (reinterpreted by Davydov and Filippov as rotations).
These nuclei lie between the regions $19 < A < 25$; $154 < A < 180$; and $A > 226$.

(d) Spectra that exhibit one level of character 1- with an energy smaller than that of group (d). These are due to collective excitations of a pear form which correspond to the octupole vibrational band (K=0).

We are interested in showing the characteristics of the spectra of type (b)

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and particularly that of the complex structure of the second excited state. As was pointed out by Scharff-Goldhaber,[3] a further search for triplet states is clearly needed, because if one can determine their relative positions and their separation energies, one is able to find the actual coupling that takes place and the strength of the corresponding parameter. Up to the present there is only one example (Cd^{114}) in which the three levels of the triplet appear, at an energy about twice that of the first excited state.[4] There are several nuclei (Ge^{72}, Kr^{82}, Ba^{134}, Pt^{192}, and Hg^{198}) that exhibit two levels of the triplet with character 2+ and 4+ which have an energy between 1.74 to 2.63 times the energy of the first excited state.[2] There are nuclei (Ru^{100}, Pd^{106}, Pd^{108}, Po^{214}) that exhibit two levels of the triplet with character 2+ and 0+ which have an energy between 2.10 and 2.50 times the energy of the first excited state.[2] In Fig. 1 the ratio ϵ_2/ϵ_1 is represented as a function of the neutron number, according to Way et al.[5] (ϵ_2 and ϵ_1 are the energy of the second and first excited states, respectively). One can see the characteristic ratio ϵ_2/ϵ_1 for two different types of excited levels, vibrational and rotational.

In the rest of the text, the following shortened nomenclature will be used. The parity of the levels, if it is not indicated, is always +. In our notation, 0 is the character of the ground state; 2 is the character of the first 2+ excited state; 0' is the character of the 0+ member of the second excited state; 2' is the character of the 2+ member of the second excited state, and 4 is the character of the 4+ member of the second excited state. Here M1 and E2 represent the magnetic-dipole and electric-quadrupole transition probabilities, respectively, and B(E2) is the reduced electric-quadrupole transition probability.

The following well-known regularities of the vibrational pattern can be mentioned:

(a) The ratio ϵ_2/ϵ_1 is approximately 2.

(b) For $B(E2;\ 2 \longrightarrow 0)_{sp} = 0.45\,R^4S$ (where sp means single-particle model, R is the radius of the nucleus, and S is the statistical factor), the ratio

$$F = \left[B(E2;\ 2 \longrightarrow 0)\ \mathrm{exp}\right] / \left[B(E2;\ 2 \longrightarrow 0)\ \mathrm{sp}\right]$$

is approximately 35 for an average nucleus, although it is a function of ϵ_1.[2, 6]

(c) When the second excited state is 2+, the M1/E2 ratio in the $2' \longrightarrow 2$ transition is less than 1, and $B(E2;\ 2' \longrightarrow 0)$ is much smaller than $B(E2;\ 2' \longrightarrow 2)$.

(d) For nuclei with $30 < N < 40$, ϵ_2/ϵ_1 is less than 2.[7]

(e) For nuclei with $22 < N < 30$, the measured spin of the second excited state is always 4+, and for $N > 32$ is always 2+.[8] There is some indication that the 4+ and 2+ levels may cross over at N = 30 to 32. There have been several theoretical attempts to describe this type of spectrum due to collective vibrations of the nucleus, and different predictions have been made for the relative positions of the levels 0', 2', and 4 of the triplet second excited state. We shall review briefly the theoretical predictions that arise from different coupling schemes.

1. The first investigation was made by Scharff-Goldhaber and Weneser using the Bohr-Mottelson model in the region of weak to moderate coupling.[9] In a more recent review, Scharff-Goldhaber[3] made a survey of the theoretical and experimental work done in the region $66 < A < 150$. The following predictions arise:

(a) The ratio of the energy of the second to the first excited state is ~ 2.

(b) The ratio of the transition probability of the first excited state to that predicted by the shell model is ~ 25.

(c) When the second excited state is 2', the ratio of the reduced E2 transition probability between the second to first excited states and the first to the ground states is ~ 2.

(d) For the value of about 1.5 for the coupling strength parameter, there appears a triplet excited state with character 0', 2', and 4, although this order can be changed by adding anharmonic terms to the coupling.

2. Later Wilets and Jean made a new attempt to explain the behavior of the nuclei in the above-mentioned region, using strong coupling in the Bohr-Mottelson model with a γ-unstable potential.[10] The conclusions are to some extent similar to those of Scharff-Goldhaber and Weneser. When the second excited state is 2', the Ml radiation and the cross-over transition are forbidden, but for a small change in the potential they can be allowed. The ϵ_2/ϵ_1 ratio ranges from 2 to 2.5; $[B(E2;\ 2' \longrightarrow 2)]/[B(E2;\ 2 \longrightarrow 0)]$ is 2. The 0' and 2' level lie very close and the 4 level much higher for large deformations.

3. New theoretical results are disclosed when a weak to moderate surface coupling is added to the two-body particle interaction, using an appropriate choice of both the two-body interaction (D) and the deformation (x) parameters. Raz has applied this combination of interactions and has obtained the following predictions:[11]

(a) For $x > 0.25$ and all values of D, the 2' and 4 levels are about twice the energy of the first excited state and the 0' level is higher.

(b) For $D = 0.4$, the energy of the first excited state increases with x and the second excited state has a character of 4 for all values of x.

(c) For $D > 0.4$, the energy of the first excited state decreases as x increases, and for $D = 1$, the 2' state is lower than the 4 state for $x < 0.7$ (see Fig. 2).

(d) For $x = 1.0$, the spectrum becomes almost independent of D.

(e) $B(E2;\ 0 \longrightarrow 2)$ is a rapidly increasing function of x.

(f) For $D = 1.0$ and $x = 0.35$ the direct transitions are favored over the cross over transition.

(g) The ratio Ml/E2 in $2' \longrightarrow 2$ transition is always < 1.0 for all values of x.

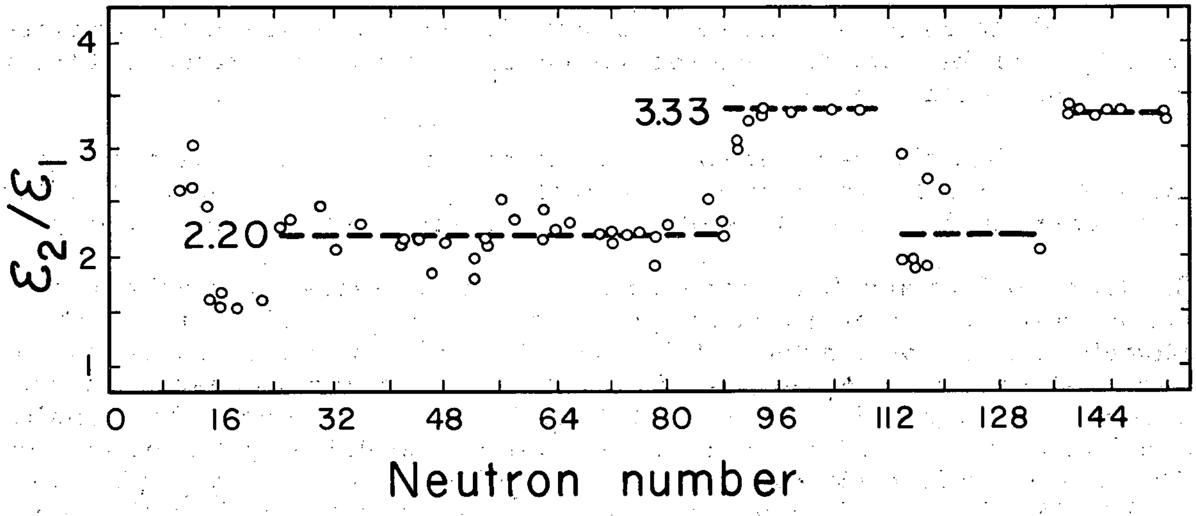

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Fig. 1. Ratio of the energy of the second excited state to the first excited state as a function of the number of neutrons, in even-even nuclei. (Simplified version of the corresponding figure from reference 5.).

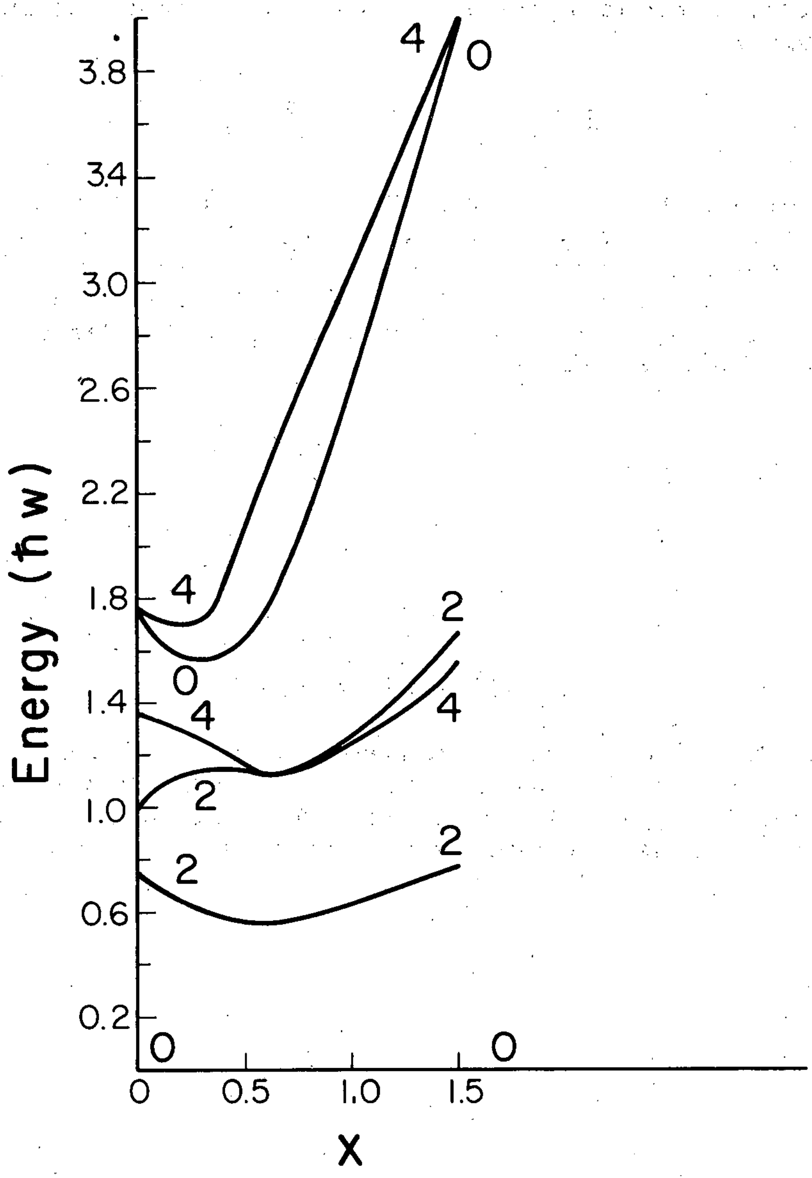

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Fig. 2. Computed variation of the energy levels as a function of the deformation parameter x. The calculation is for the configuration $(7/2)^2$ and for D = 1.0. (From Fig. 3 of reference 11.)

4. A new attempt to explain a triplet structure in the vibrational nuclei was made recently by Tamura and Komai.[12] They modified slightly the Wilets and Jean model, introducing to the $1/2\ C(\beta - \beta_0)^2$ term a potential-energy term of the form

$$\sum_n k_n (\beta - \beta_n)^n \cos^n(3\gamma),$$

which produces a right-left asymmetry in the potential-energy curve. Assuming this kind of potential, the well-known[4] triplet in Cd^{114} can be explained theoretically, as is shown in Fig. 3,[12] for certain values of the parameters C, k_i, β_0, and β_i. All predictions from different coupling schemes together with experimental data are given in Table I. This table has to be considered only as tentative.

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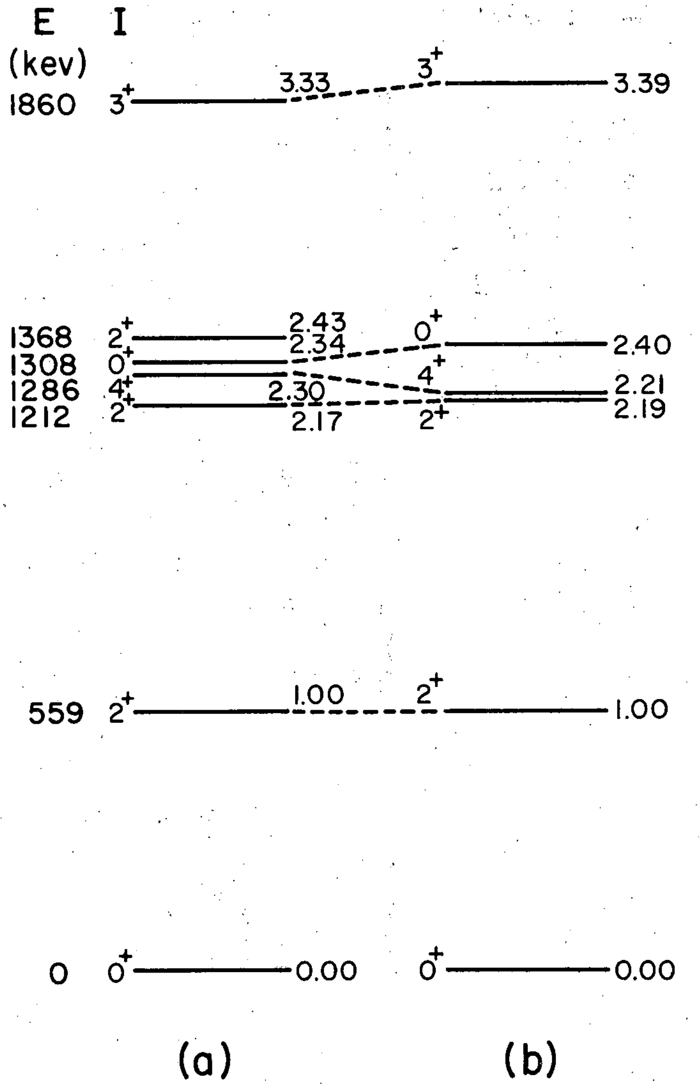

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Fig. 3. Spectrum of the lower states of Cd^{114}: (a) experimental; (b) theoretical. On the left-hand side of (a) are given the excitation energy in kev and the spin of each state, while on the right-hand side the ratio $E_n(I)/E_1(2)$ is given. The 2+ state at 1368 kev seems to be due to a single-particle excitation and the absence of the corresponding state in (b) is without significance since we here consider only collective states. (From Fig. 1 of reference 12.)

Table I

Theoretical predictions for different properties of even-even nuclei in the region $40 < A < 154$ and $180 < A < 226$ compared with experimental results

Properties	Authors				Experimental data[2]
	Scharff-Goldhaber and Weneser[9]	Wilets and Jean[10]	Raz[11]	Tamura and Komai[12]	
Energy ratio ϵ_2/ϵ_1	2.2	$2 \longrightarrow 2.5$	$\gtrless 2$	either 2	$1.8 \longrightarrow 2.5$
Sequence of the levels of the triplet	0 2 4	4 0 2	0 4 2 for $\chi < 0.7$ and $D = 1.0$	0 4 2[a]	0 4 2
$2' \longrightarrow 0$ transition	Allowed	forbidden, but for a change in the potential, can be allowed	Allowed	Allowed	Allowed
$F = \dfrac{B(E2;\, 0 \longrightarrow 2)\text{coll.}}{B(E2;\, 0 \longrightarrow 2)\text{ sp}}$	$F \sim 25$ for $\xi = 0.75$ and $Z = 40$				$F = 35 \pm 10$ (average)
$R_I = \dfrac{B(E2;\, 2' \longrightarrow 2)}{B(E2;\, 2' \longrightarrow 0)}$			$1 \longrightarrow 100$ for $0 \leq \chi \leq 1.5$ $D = 1.0$		$9 \longrightarrow 500$
$R_{II} = \dfrac{B(E2;\, 2' \longrightarrow 0)}{B(E2;\, 2 \longrightarrow 0)}$			$1 \longrightarrow 0.01$ for $0 \leq \chi \leq 1.5$ and $D = 1.0$	0.014	0.004 to ~ 0.04
$R_{III} = \dfrac{B(E2;\, 2' \longrightarrow 0)}{B(E2;\, 2 \longrightarrow 0)}$	2	2	$0 \longrightarrow 1$ for all values of $\underline{\chi}$ and $D = 1.0$	1.7	1.5 ± 1 (average)

[a] This sequence was calculated for only one example.

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