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## ABSTRACT

Two methods of placing high-energy electrons in a synchrotron on an internal target within one microsecond are proposed. One method uses the forced radial oscillations that occur at n = 0.75 in the presence of a first harmonic azimuthal variation in the magnetic field which gives rise to a Mathieu Equation. The effect of the first harmonic in general and at n = 0.75 is examined. The requirements for changing the radial field gradient to n = 0.75 are determined. An operating cycle is suggested. The second method proposes the application of a radiofrequency electric field resonant with the radial oscillation thus producing blowup in an unmodified magnetic field. The axial and radial oscillation frequencies are not commensurable, thus resonance blowup is avoided in the axial direction. The conditions to accomplish these are determined in general and specifically for the Berkeley synchrotron.

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## INTRODUCTION

Two methods of placing the circulating electron beam in a synchrotron on an internal target in one microsecond are proposed. The first method requires the modification of the magnetic field of the synchrotron at the end of the acceleration cycle to permit the use of a resonance between the electron oscillations and an azimuthal field variation. The second method uses the resonant blow-up caused by an rf electric field perturbing the electrons at the frequency of their oscillations at the end of the acceleration cycle. The conditions to accomplish this are determined in general and specifically for the Berkeley synchrotron. These methods may be applieu to betatrons as well.

At the end of the acceleration cycle in synchrotrons, the radiofrequency voltage is removed and the magnetic field is static or changing very slowly. Under these conditions, the electrons spiral slowly inward because of their loss of energy by radiation.[1,2] A target placed at an inner radius intercepts the beam as it moves inward. In the Berkeley synchrotron the beam consumes about 15-25 microseconds in being decelerated by the target. The bremsstrahlung spectrum is present throughout.

In some experiments, an example of which is the measurement of the decay of a meson in flight, it is necessary to obtain a very short burst of radiation from the machine with no subsequent radiation. The first part of this paper tells how one-microsecond duration may be achieved by making use of a forced oscillation occurring at an n value of 0.75 with a first harmonic variation in the magnetic field asymmetry. In the Berkeley synchrotron the variation of the field with radius must be modified to change the n value from 0.67 to 0.75 at the time the beam pulse is desired. The requirements for doing this are determined below.

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1. L. I. Schiff, Rev. Sci. Instr. 17, 8 (1946).
2. J. Schwinger, Phys. Rev. 75, 1912 (1949).

The second part of this paper considers the application of a radio-frequency voltage resonant with the radial oscillation frequency will induce a rapid increase of oscillation amplitude and the precession will carry the particle onto the target. The requirements to accomplish this are also determined below.

A pulsed electrostatic deflection system has been tried and is unsuccessful for the following reasons: First, the voltage gradient required could not be maintained in the accelerating chamber, and second, the strong focusing action of the field reduces the initial deflection into an oscillatory path and further increases the voltage requirements.

Attempts to suddenly pulse the magnetic field to a higher value and stepwise reduce the radius of the orbit of the particles all involve a longer time in making the required change of field than is used by the beam in spiraling inward because of radiation loss.

# PART I USE OF MODIFIED MAGNETIC FIELD

## PARTICLE MOTION IN THE PRESENCE OF A FIRST HARMONIC OF AZIMUTHAL FIELD VARIATION

The derivation for the particle motion is made to show the effect of the first harmonic in general, and at the condition $n = 0.75$ in particular.

For a circular path in a magnetic field the equilibrium orbit is obtained when the centrifugal forces equal the magnetic forces.

$$\frac{mv^2}{r_o} = \frac{eB_{r_o} v}{c} \tag{1}$$

where $r_o$ is the radius of the equilibrium orbit and $B_{r_o}$ is the magnetic field at the radius. For a particle not on the equilibrium orbit a force arises given by

$$F = \frac{mv^2}{r} - \frac{eBv}{c} . \tag{2}$$

Letting $r = r_o + \rho$ and $B = B_{r_o} + \frac{\partial B_{r_o}}{\partial r} \rho$, and letting the azimuthal variation be such that $B_{r_o} = B_{r_o} (1 + h \cos \theta)$, we have a force

$$F = \frac{mv^2}{r_o + \rho} - \frac{ev}{c} \left[ B_{r_o} (1 + h \cos \theta) + \frac{\partial B_{r_o}}{\partial r} \rho (1 + h \cos \theta) \right] ,$$

noting that $\frac{1}{r_o + \rho} = \frac{1}{r_o} - \frac{\rho}{r_o^2} + \frac{\rho^2}{r_o^3} \ldots$ .

Since $\rho << r_o$, this series may be cut off after the second term. Then the force may be written

$$F = \left[ \frac{mv^2}{r_o} - \frac{ev}{c} B_{r_o} \right] - \frac{mv^2}{r_o^2} \rho - \frac{ev}{c} \frac{\partial B_{r_o}}{\partial r} \rho -$$

$$\frac{evh}{c} \left[ B_{r_o} + \frac{\partial B_{r_o}}{\partial r} \rho \right] \cos \theta;$$

but--from Eq. 1--the first two terms on the right-hand side are equal, and thus the bracket is zero. The force may be replaced by the product of the mass of the particle and its acceleration (since the particle energy is assumed constant):

$$F = m \frac{d^2\rho}{dt^2} = \frac{mv^2}{r_o^2}\rho - \frac{e}{c}\frac{v}{r_o} r_o \frac{\partial B_{r_o}}{\partial r}\rho -$$

$$\frac{eh}{c}\frac{v}{r_o} r_o \left[ B_{r_o} + \frac{\partial B_{r_o}}{\partial r}\rho \right] \cos\theta. \qquad (3)$$

Letting $\frac{eB}{mc} = \omega$ and $\frac{v}{r_o} = \omega$, and defining $n = -\frac{r}{B_{r_o}}\frac{\partial B_{r_o}}{\partial r}$, we may write the equation

$$\frac{d^2\rho}{dt^2} + (1 - n)\,\omega^2\rho = \omega^2 h \cos\theta \left[ n\rho - r_o \right]. \qquad (4)$$

Since $n < 1$ and $\rho \ll r_o$, this equation may be written as

$$\frac{d^2\rho}{dt^2} + (1 - n)\,\omega^2\rho = -\,\omega^2 h\, r_o \cos\theta. \qquad (5)$$

Letting $\frac{d^2}{dt^2} = \omega^2 \frac{d^2}{d\theta^2}$, one obtains

$$\frac{d^2\rho}{d\theta^2} + (1 - n)\,\rho = -\,h r_o \cos\theta. \qquad (6)$$

The solution of equation (6) is of the form

$$\rho = A \sin\sqrt{1-n}\,\theta + B\cos\sqrt{1-n}\,\theta + \frac{hr_o}{n}\cos\theta. \qquad (7)$$

Since for the synchrotron $\sqrt{1-n}$ is not close to unity and since h is small, the third term is a forcing term far from resonance and it does not contribute a unidirectional increase in $\rho$. Thus, except when n = 0.75, the presence of a first harmonic of azimuthal field variation has no serious effect on the particle motion, as revealed by this linearized equation.

If the term $n\rho$ is not dropped, the equation becomes

$$\frac{d^2\rho}{d\theta^2} + \left[(1 - n) - nh \cos \theta\right] \rho = - hr_o \cos \theta. \tag{8}$$

We may consider the auxiliary equation for Eq. 8 and substitute for the independent variable $\theta = 2\phi$ to obtain

$$\frac{d^2\rho}{d\phi^2} + 4 \left[(1 - n) - nh \cos 2\phi\right] \rho = 0. \tag{9}$$

Eq. 9 is a Mathieu type equation whose canonical form[3] is

$$\frac{d^2y}{dz^2} + (a - 2q \cos 2z)\, y = 0, \tag{10}$$

in which 4 (1 - n) = a and 4 nh = 2q. For small values of h, i.e., 5% - 10%, and values of n $\neq$ 0.75, this equation is stable and its solution is oscillatory. However, at n = 0.75, Eq. 8 becomes

$$\frac{d^2\rho}{d\phi^2} + (1 - 3h \cos 2\phi)\, \rho = - hr_o \cos 2\phi, \tag{11}$$

whose solution is

$$\rho = Ae^{\mu\phi}\, \emptyset(\phi) + Be^{-\mu\phi}\, \emptyset(-\phi) - \frac{hr_o}{3(1 - h \cos 2\phi)} \cos 2\phi, \tag{12}$$

where $\emptyset(\phi)$ is a periodic function of period $\phi$ and $\mu$ is an exponent which is

3. H. McLachlan, Theory and Applications of Mathieu Functions, Oxford, 1947.

real. The first term of the solution represents an exponential growth of the oscillatory motion.

For the conditions $a = 1$, $2q = 3h$, the solution may be obtained from McLachlan[3] (Section 4.30) as $\mu = \pm \frac{3}{4} h$. The amplitude of oscillation is increased by a factor of e each time $\phi$ becomes $\frac{1}{\mu}$. Thus the number of turns required to give the growth of e is

$$N = \frac{4}{3\pi} \frac{1}{h}. \quad (13)$$

Thus as an example, a 1-percent first harmonic will require 43 turns to grow a factor of e. A 5-percent first harmonic will give an increase of e in about 9 turns. In 1 microsecond the electrons make 50 turns, so the increase in the latter case will be given by $e^{50/9} \sim 260$. It is expected this increase will destroy the beam by bringing all particles onto the target.

## MODIFICATION OF THE SYNCHROTRON FIELD

The modification of the field pattern of the synchrotron may be made by adding current loops in the region of interest. The intention is not to change the mean field value, but merely to change the distribution within the gap.

Figure 1 shows field distribution under normal operating conditions. Figure 2 shows the physical arrangement of the gap, including compensating coils. The compensating coils are closed through resistors whose value for each set was arrived at experimentally. Figure 3 shows the magnetic cycle and the cutoff for the radiofrequency signal.

From the definition of $n = -\frac{r}{B} \frac{\partial B}{\partial r}$ at the peak field $B_o = 11,400$ gauss at $r_o = 39$ in., one may obtain the change in radial gradient required.

Table I

| r | $\partial H/\partial r$ at n=0.67 | $\partial H/\partial r$ at n=0.75 | $\Delta \frac{\partial H}{\partial r}$ | $\Delta \frac{\partial H}{\partial r}$ |
|---|---|---|---|---|
| 36" | -1.96%/inch | -2.19%/inch | -0.23%/inch | 26.2 gauss/in |
| 39" | -1.72 | -1.93 | -0.21 | 23.9 |
| 42" | -1.52 | -1.70 | -0.18 | 20.5 |

The change in the gradient due to one pair of coils is estimated as follows. The field from a current-carrying conductor is given by $B = \frac{0.2\,I}{r}$ where B is in gauss, I in amperes, and r in cm. The presence of nonsaturated iron increases this by a factor of two, and the presence of the other member of the pair gives an additional factor of two. The vertical component at the median plane may be computed then from $B_y = \frac{0.8\,Ix}{x^2 + y^2}$. Figure 4 shows the field from the No. 1 pair. The additional contribution from the first set of images is given for the first pair of coils. The sum of the field from the coil and its first reflection is shown. Finally, an approximation shown was taken as the contribution of the coil and all its reflections.

From the approximation, currents in the eight sets of coils were assumed to have various values to give the distribution desired. The best fit is obtained when coils 4 and 5 have about 192 amperes in each conductor and all the other coils have no current. The spacing between coils 4 and 5 is the optimum for this gap. Figure 5 shows the field from these two pairs.

The effect of changing gap width has not been considered in this approximation. The effect is to increase the gradient in the smaller gap region and decrease it in the larger gap region. This is just the proper compensation to give the form implied in Table I. Adjustments in the currents in the pairs of coils permit additional trimming.

The force on the conductors is radial and is about one and one-quarter pounds per inch.

A sizable first harmonic in the full field of the synchrotron is believed to exist. A more pronounced harmonic may be induced by adjusting the quadrant-coil currents during the final portion of the cycle.

## OPERATION SEQUENCE

The change of the radial field gradient may take place slowly, with the rapid increase in radial oscillation amplitude occurring only when the $n = 0.75$ condition is reached. Thus the beam may be destroyed in a very short time without the pulse difficulties of the other methods.

The rate of rise of current in the pairs of coils must meet two requirements. First, the rate must be small enough to avoid substantial cancellation of the field by the induced currents. Second, the duty cycle of the current must be short enough to avoid overheating the conductors.

A suitable cycling operation may be as follows. Near the end of the acceleration cycle current may be supplied to the pairs of coils, perhaps each turn being in series with the others. The rate of rise of this current may be not more than perhaps five times the maximum rate of rise of the main field. The calculated condition for radial blowup may be achieved after a sufficient period of inward spiraling to carry the beam near the target. The current may be stopped in any manner.

The slow rate at which the beam loses energy and decreases radius appears to preclude any extreme exactness in the cycling operation.

## PART II USE OF RADIOFREQUENCY ELECTRIC PERTURBATION

### PARTICLE MOTION IN THE PRESENCE OF A PERIODIC DISTURBANCE

The equations of motion of the particles undergoing free oscillations are:

$$d^2\rho/dt^2 + (1 - n)\,\omega_o^2\rho = 0, \qquad (14)$$

$$d^2z/dt^2 + n\,\omega_o^2\,z = 0, \qquad (15)$$

where $\rho$ and $z$ are the displacements from the equilibrium orbit, $n$ is $-\frac{r}{B}\frac{\partial B}{\partial r}$ and defines the radial field gradient, $\omega_o$ is the angular frequency of the particle in the synchrotron, and $t$ is time.

The relation between the radial and axial oscillation angular frequencies and $\omega_o$ are:

$$\omega_r = \sqrt{1 - n}\,\omega_o,$$

$$\omega_z = \sqrt{n}\,\omega_o.$$

In the Berkeley synchrotron the value of $n$ is 0.67 and remains unchanged. Likewise $\omega_o$ remains practically unchanged. As the particles travel at almost the velocity of light, $\omega_o$ is given by

$$\omega_o = v/r = c/r.$$

The equilibrium orbit is very close to one meter, giving $\omega_o$ = 300 megaradians per second. Thus $f_o$ = 47.8 megacycles per second.

The change in $\omega_o$ occurring as the particles spiral inward is very small because of the small change in the radius. The maximum change in $\omega_o$ is about 0.5 percent.

The radial and axial frequencies are related by

$$\frac{f_r}{f_z} = \frac{\omega_r}{\omega_z} = \sqrt{\frac{1 - n}{n}} . \tag{16}$$

For $n = 2/3$, $f_r/f_z = 1/2 = 0.707$, which is not commensurable.

The radial oscillation frequency is

$$f_r = \sqrt{1 - n}\, f_o$$

$$= 27.6 \text{ megacycles per second.}$$

Now we shall consider the differential equations when there is a forcing term present.

$$d^2\rho/dt^2 + (1 - n)\,\omega_o^2\,\rho = K \sin \sqrt{(1 - n)}\,\omega_o t, \tag{17}$$

$$d^2 z/dt^2 + n\,\omega_o^2\, z = K \sin \sqrt{1 - n}\,\omega_o t. \tag{18}$$

The solutions of these equations are:

$$\rho = A_r \sin \omega_r t + B_r \cos \omega_r t - \frac{Kt}{2\omega_r} \cos \omega_r t, \tag{19}$$

$$z = A_z \sin \omega_z t + B_z \cos \omega_z t + \frac{K}{\omega_z^2 - \omega_r^2} \sin \omega_r t. \tag{20}$$

In the z solution, the third term on the right has a constant amplitude but a different frequency than the free oscillations. This may be considered as a modulation of the free-oscillation amplitude.

In the $\rho$ solution, the linear increase of the amplitude of the coefficient of the third term on the right in time is observed. This amplitude increase will predominate the situation and large oscillations will cause the particles to strike the target.

## RADIOFREQUENCY FIELD REQUIRED FOR RAPID BLOW-UP

The periodic force on the particle in the synchrotron occurs only in the region of the deflecting plates. Let their length along the path be $\ell$. The force is $e\epsilon$ where $e$ is the electronic charge and $\epsilon$ is the electric field gradient from the deflector.

From the solution of the radial equation, setting $A_r = 0$, we obtain

$$\rho = \left[B_o - \frac{Kt}{2\omega_o\sqrt{1-n}}\right] \cos\sqrt{1-n}\,\omega_o t.$$

At $t = 0$, let $\rho = B_o$, while at $t = t$, let $B_o - \frac{Kt}{2\omega_o\sqrt{1-n}} = B$ where $B \gg B_o$, and let $B = xB_o$, thus considering the magnitudes

$$\left|\frac{Kt}{2\omega_o\sqrt{1-n}}\right| = \left|(x-1)\,B_o\right|,$$

thus

$$K = \frac{2(x-1)\,B_o\,\omega_o\sqrt{1-n}}{t}.$$

Since $K = \frac{e\epsilon}{m} = \frac{e\epsilon\,c^2}{mc^2} = \frac{e\epsilon\,c^2}{E}$, we obtain

$$e\epsilon = \frac{2\omega_o\sqrt{1-n}\,(x-1)\,B_o\,E}{c^2 t}. \qquad (21)$$

This is the total force normal to the synchronous orbit required to give an increase in amplitude of $(x - 1)$ in time $t$.

Since the length of the deflector is $\ell$, and the electrons are perturbed only in this region, the field gradient must be increased to make the net force equal the value computed above. This ratio is given by

$$\frac{2\pi R}{\ell}$$

where R is the synchronous radius of the electrons.

Thus the required electric field gradient in the deflector is

$$e\epsilon = \frac{2\omega_o\sqrt{1 - n}\,(x - 1)\,B_o\,E}{c^2 t} \cdot \frac{2\pi R}{\ell}. \qquad (22)$$

As an example, for 350-Mev electrons, $E = 3.5 \times 10^8$ ev, $x = 500$, $\omega_o = 3 \times 10^8$ rad/sec, $R = 1$ meter, $\ell = 25$ cm, $t = 10^{-6}$ sec, $n = 2/3$

$$e\epsilon = 1.7 \times 10^6\, B_o \text{ ev/cm}$$

In the synchrotron the value of $B_o$ at the time a perturbation would be applied is small, about 0.1 cm, owing to the damping observed. The amplification factor x must be smaller for larger $B_o$; thus the value computed in the example is certainly higher than required. The product of gradient and deflector length below is believed to be sufficient to cause the beam destruction in one microsecond in the Berkeley synchrotron.

$$e\epsilon\ell = 1.7 \times 10^5 \qquad \text{electron volts cm/cm}$$

One notes the compensation between $B_o$ and x required to produce a given amplitude; thus the final amplitude is independent of the initial amplitude and is a measure of the energy supplied to the electrons in the radial direction.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

## FIGURE CAPTIONS

Fig. 1 Synchrotron magentic field.

Fig. 2 Synchrotron cross section.

Fig. 3 Synchrotron magnetic field variation (not to scale).

Fig. 4 The verticle component of the magnetic field at the median plane from No. 1 pair of wires.

Fig. 5 Magnetic field from equal currents in pairs 4 and 5 using the approximation of Fig. 4.

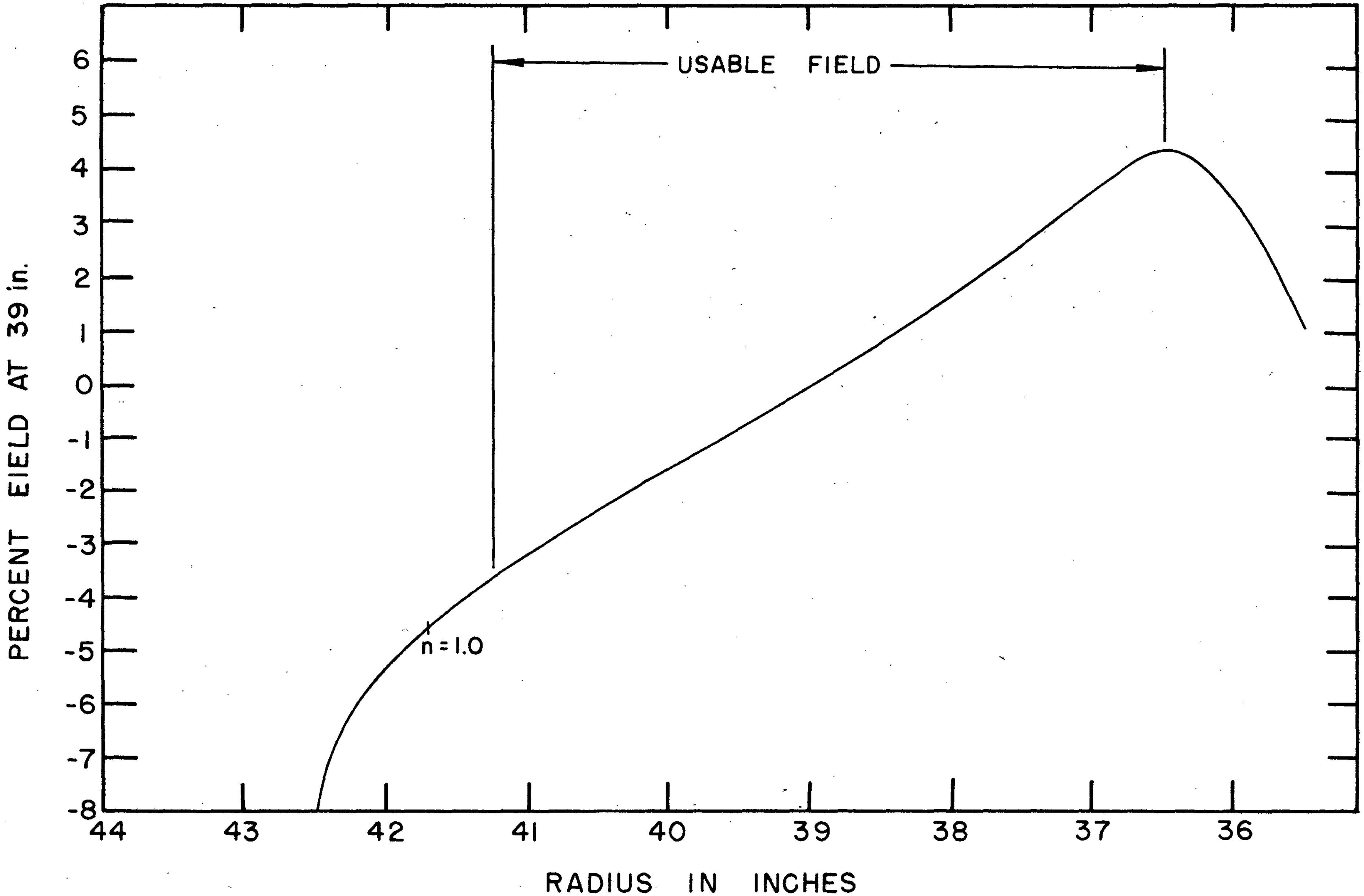


Fig. 1 --- Synchrotron Magnetic Field

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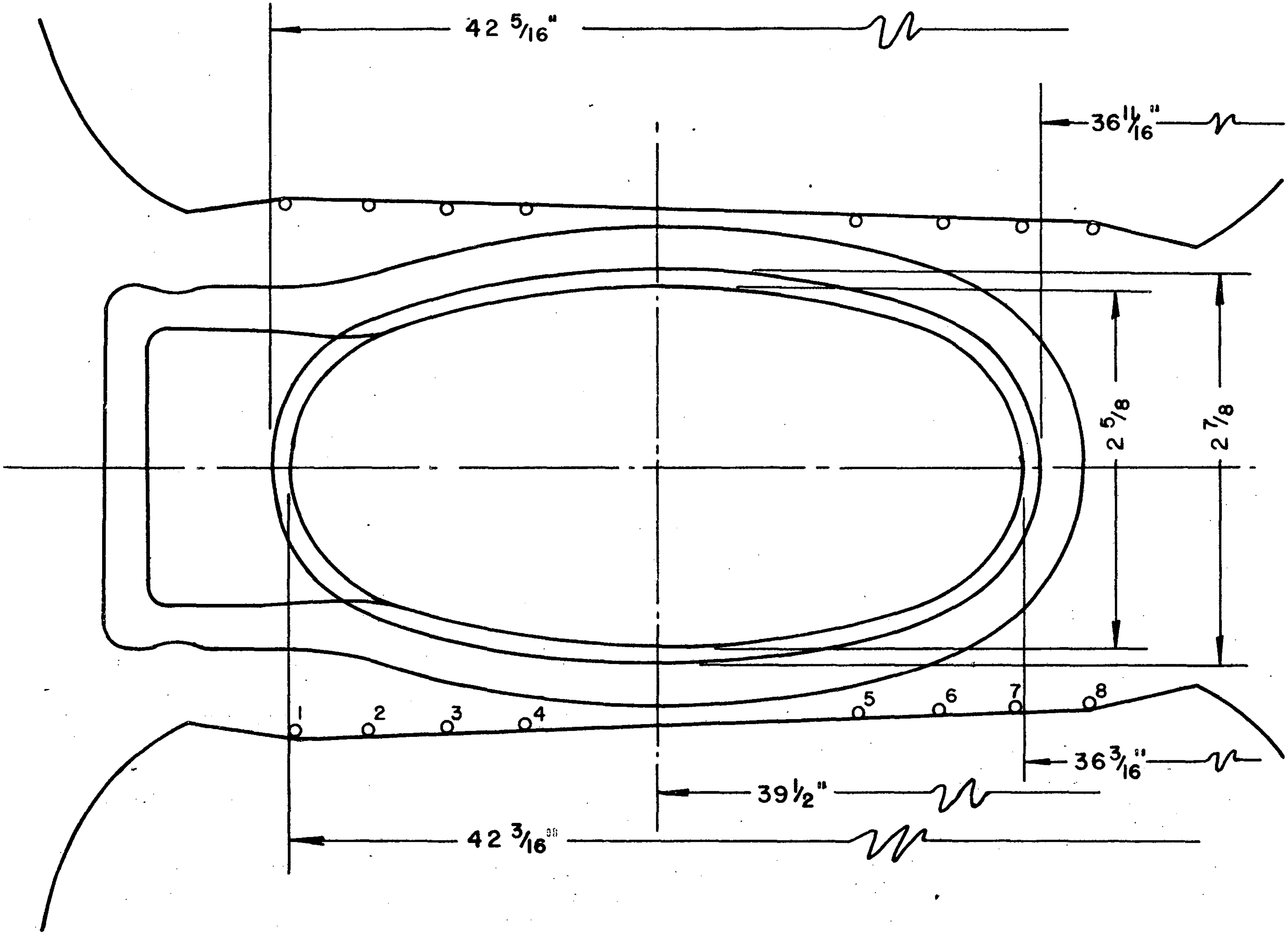


Fig. 2 --- Synchrotron Cross Section

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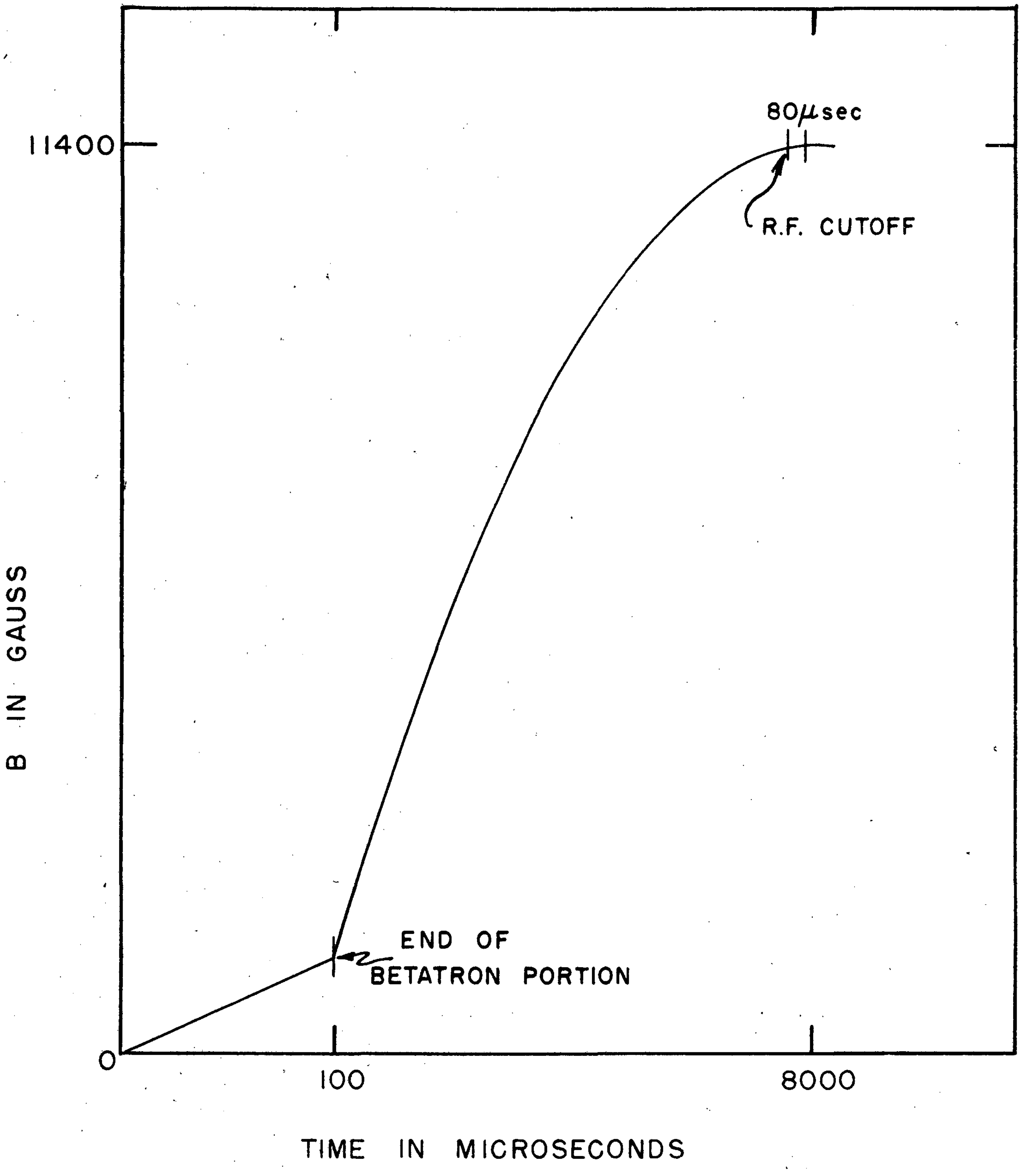


Fig. 3---Synchrotron Magnetic Field Variation
(NOT TO SCALE)

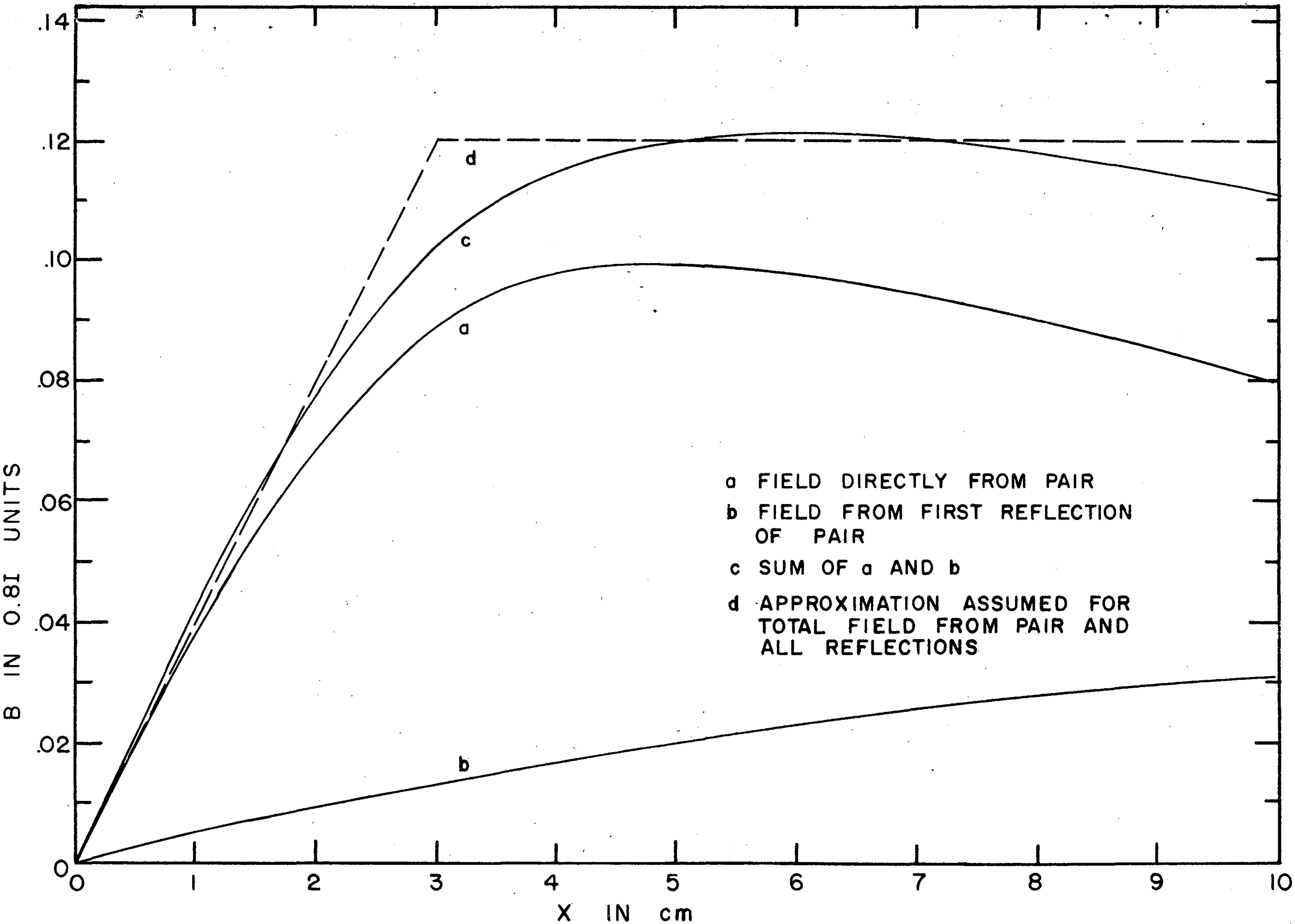


Fig. 4---The Verticle Component of the Magnetic Field at the Median Plane from No. 1 Pair of Wires.

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