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Author
Genz, H.

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ON BARYON SPECTRAL FUNCTION SUM RULES*

H. Genz

Lawrence Radiation Laboratory
University of California
Berkeley, California

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ABSTRACT

Necessary and sufficient conditions for baryon spectral-function sum rules are obtained under the assumptions that (1) the equal-time commutator of the axial charges $Q_5^{\ a}(x_o)$ $(a = 1,2,3)$ and the nucleon field $\bar{\psi}(y)$ is given by $\left[Q_5^{\ a}(y_o), \bar{\psi}(y)\right] = - r_A \bar{\psi}(y)\gamma_5 \tau^a + (\Delta I = \frac{3}{2} - \text{terms})$ and that (2) the axial current $A_\mu^{\ a}(x)$ is conserved. For each of these sum rules (enumerated by $n = 1,2,3\ldots$) the equivalence to

$$\int d^3z \left\langle \left[Q_5^{\ a}(y_o), \left[\left(\frac{\partial}{\partial y_o}\right)^{2n-1} \psi(y), \bar{\psi}(z)\right]_+\right]\right|_{y_o=z_o} \Big\rangle_o = 0$$

is actually shown under the weaker conditions, assumption (1) and, instead of (2),

$$\sum_{j=0}^{2n-2} \left\langle \left[\left\{\left(\frac{\partial}{\partial y_o}\right)^j \left[\int d^3x\, \partial^\mu A_\mu^{\ a}(y_o, \underset{\sim}{x}) \left(\frac{\partial}{\partial y_o}\right)^{2n-2-j} \psi(y)\right]\right\}, \psi(z)\right]_+ \right|_{y_o=z_o} \Big\rangle_o = 0 \; .$$

Further equivalences are given. The sum rules connect the $(I = \frac{1}{2}, J = \frac{1}{2}^{+})$ and $(I = \frac{1}{2}, J = \frac{1}{2}^{-})$ baryon spectrum and include (for $n = 1$) a sum rule, obtained independently by J. Rothleitner and (in the one-particle approximation) by M. Sugawara. In our derivation we make no assumptions on high-energy behavior and we use an identity of the Jacobi type.

Assuming the first two sum rules to be valid, the model then predicts a $P_{11}(m \geq 1470$ MeV) resonance [which may be identified as the observed $P_{11}(1750)$] from the existence of the four nucleon resonances $P_{11}(940)$, $P_{11}(1470)$, $S_{11}(1550)$, and $S_{11}(1710)$.

The spectral-function sum rules, derived by Weinberg[1] for the chiral $SU(2) \otimes SU(2)$ currents have been extended by several authors[2-4] and various proofs have been given.[1-6] Among these, Glashow, Schnitzer, and Weinberg[3] have described a derivation of the first Weinberg sum rule using the Jacobi identity, and Jackiw[5] has used the Jacobi identity in order to derive a condition for the second Weinberg sum rule. The main difference between Weinberg's[1] original proof of the second sum rule and the one given by Jackiw lies in the replacement of the assumption on high-energy behavior, made in Ref. 1, by the assumption that a certain vacuum expectation value of a triple commutator vanishes.

Among the extensions of the Weinberg sum rules, J. Rothleitner[4] has derived a sum rule for baryon spectral functions, assuming that[7]

$$\lim_{p^2 \to \infty} \lim_{q_\mu \to 0} \int d^4x\, d^4y\, e^{-iqx + ipy} \langle T\left\{(q_\mu - \partial_\mu) A_\mu{}^a(x), \psi(y), \bar{\psi}(o)\right\}\rangle_o = 0 \tag{1}$$

and that

$$\left[A_o{}^a(x), \bar{\Psi}(y)\right]\Big|_{x_o = y_o} = - r_A \bar{\Psi}(x)\gamma_5 \tau^a \delta(\underset{\sim}{x} - \underset{\sim}{y}) + (\Delta I = \tfrac{3}{2} - \text{terms}). \tag{2}$$

In the above, we have denoted (for $a = 1,2,3$) the axial vector current by $A_\mu{}^a(x)$ and the nucleon field by $\bar{\psi}(y)$. The sum rule derived in Ref. 4 from Eqs. (1) and (2) reads

$$S_1 = \int_0^\infty dm^2\, m \left(F_+^{\,2}(m^2) - F_-^{\,2}(m^2) \right) = 0 \,, \qquad (3)$$

where we have defined

$$(2\pi)^{\frac{3}{2}} \langle o \left| \psi(o) \right| \begin{matrix} m^2 & \epsilon \\ \underset{\sim}{p} & r \end{matrix}\, \alpha \rangle = \begin{cases} w_r(p)\; F_+^{\,\alpha}(m^2) & \text{for } \epsilon = 1 \\ i\, \gamma_5 w_r(p)\; F_-^{\,\alpha}(m^2) & \text{for } \epsilon = -1 \end{cases} \qquad (4)$$

Here, $\left| \begin{matrix} m^2 & \Sigma \\ p & r \end{matrix}\, \alpha \right\rangle$ denotes a state with the same baryon number, spin, isospin, and strangeness as the nucleon; α stands for additional quantum numbers. We have also

$$F_\pm^{\,2}(m^2) = \sum_\alpha F_\pm^{\,\alpha}(m^2)^2 \,. \qquad (5)$$

If we saturate the sum rule (3) by one-particle intermediate states, it reads

$$\sum_i \epsilon_i\, F_{\epsilon_i}^{\,2}(m_i^{\,2})\, m_i = 0 \,. \qquad (6)$$

This is the sum rule derived by M. Sugawara[8] as a consequence of his self-consistency conditions. The proof of these conditions[8] uses, in addition to Eq. (7) below, assumptions on analyticity and high-energy behavior.

The purpose of the present note is two-fold. First, in analogy to the derivations of the Weinberg sum rules using the Jacobi identity,[3,5] we will derive the following statement by means of an algebraic identity.

<u>Statement 1</u>. Let[9,11]

$$\left[Q_5{}^a(y_o),\ \overline{\Psi}(y)\right] = - r_A\overline{\Psi}(y)\ \ \gamma_5\tau^a + \overline{\Delta}^{\frac{3}{2}}(y)\ \gamma_5 \tag{7}$$

and

$$\langle \left[\left[\int d^3x\ \partial^\mu A_\mu{}^a(x),\ \psi(y)\right],\ \overline{\Psi}(z)\right]_+ \rangle_o = 0\ , \tag{8}$$

where $\Delta^{\frac{3}{2}}(y)$ denotes possible $\Delta I = \frac{3}{2}$ terms.

Then we have

$$2r_A i\tau^a\gamma_5\int dm^2\ m\left(F_+{}^2(m^2) - F_-{}^2(m^2)\right)\delta(\underset{\sim}{y} - \underset{\sim}{z}) = \langle\left[Q_5{}^a(y_o),\left[\dot{\psi}(y),\overline{\psi}(z)\right]_+\right]\rangle_o\ . \tag{9}$$

In the above statement, $Q_5{}^a(x_o)$ is defined by

$$Q_5{}^a(x_o) = \int d^3x\ \ A_o{}^a(x)\ . \tag{10}$$

Note that Eqs. (8) and (9) have <u>anti</u>commutators for fermion operators. The statement shows that given Eqs. (7) and (8), which we discuss below, at most the non-Schwinger part of the anticommutator $\left[\dot{\psi}(y), \bar{\psi}(z)\right]_+$ survives in Eq. (9). The vanishing of this expression itself is then equivalent to the sum rule Eq. (3).

As to the validity of the assumptions made, Eq. (7) is a consequence of the more restrictive assumption Eq. (2), allowing for additional arbitrary Schwinger terms. Models in which Eq. (7) holds have been investigated by several authors,[4,8,10,12 - 16] and in neither case a contradiction with Eq. (7) was found. On the contrary, assuming Eq. (7) without $\Delta_{\frac{3}{2}}(y)$ terms, M. Sugawara[16] has reached reasonable agreement with experiment in a number of cases. Rothleitner[4] obtained agreement with experiment, too.[17]

The main advantage of Eq. (7) as compared to Eq. (2) is that Eq. (7) is more likely to hold for fermion operators introduced into a field theory of currents.[18] As was shown in Ref. 15, for $\Delta_{\frac{3}{2}}(y) = 0$, Schwinger terms are then present in the equal-time commutator of the time components of the currents with $\psi(y)$.[19] As to the second assumption, Eq. (8) is [and so are the later Eqs. (13)] an obvious consequence of $\partial^{\mu}A_{\mu}{}^{a}(x) = 0$. If PCAC holds for massive pions and the so-defined pion field and $\psi(y)$ are canonical fields, Eq. (8) follows from the canonical rule[12,20]

$$\left[\partial^{\mu}A_{\mu}{}^{a}(x), \psi(y)\right]\Big|_{x_o=y_o} = 0 \ . \qquad (11)$$

However, Eq. (11) does not prove the assumption in Eq. (13) of statement 2 below [as does the assumption $\partial^{\mu}A_{\mu}{}^{a}(x) = 0$].

Assuming the local commutator Eq. (2), it was shown in Ref. 4 that Eqs. (1) and (3) are equivalent, and thus

$$\langle \left[Q_5^{\,a}(x_o), \left[\dot{\psi}(y), \bar{\psi}(z)\right]_+ \right] \rangle_o \Big|_{x_o = y_o = z_o} = 0, \qquad (12)$$

if and only if Eq. (1) holds, under the above assumptions.

The other purpose of the present note is to give conditions for additional sum rules. We will prove:

<u>Statement 2.</u> Let Eq. (7) be valid and let[11] for $n \geq 1$

$$0 = \sum_{j=0}^{2n-2} \langle \left[\left\{ \left(\frac{\partial}{\partial y_o}\right)^j \left[\int d^3x\, \partial^\mu A_\mu^{\,a}(x), \left(\frac{\partial}{\partial y_o}\right)^{2n-2-j} \psi(y) \right] \right\}, \bar{\psi}(z) \right]_+ \rangle_o \ .$$

(13)

Then we have

$$\frac{1}{2} \langle \left[Q_5^{\,a}(x_o), \left[\left(\frac{\partial}{\partial y_o}\right)^{2n-1} \psi(y), \bar{\psi}(z) \right]_+ \right] \rangle_o$$

$$= i\tau^a \gamma_5 r_A \int dm^2\, m \left(F_+^{\,2}(m^2) - F_-^{\,2}(m^2) \right) \left(\frac{\partial}{\partial \underset{\sim}{y}} \frac{\partial}{\partial \underset{\sim}{y}} - m^2 \right)^{n-1} \delta(\underset{\sim}{y} - \underset{\sim}{z})$$

$$= i\tau^a \gamma_5\, r_A \sum_{\nu=0}^{n-1} S_{1+2\nu} \binom{n-1}{\nu} \left(\frac{\partial}{\partial \underset{\sim}{y}} \frac{\partial}{\partial \underset{\sim}{y}} \right)^{n-1-\nu} \delta(\underset{\sim}{y} - \underset{\sim}{z}) \ .$$

(14)

In the above statement we have defined S_ν by

$$S_\nu = \int dm^2\, m^\nu \left(F_+{}^2(m^2) - F_-{}^2(m^2)\right). \tag{15}$$

Note again the <u>anti</u>-commutators in Eqs. (13) and (14). Conditions under which Eq. (13) is valid have been investigated above. In Eq. (14), the highest-order Schwinger term is of order $2(n - 1)$. That this term vanishes is equivalent to the sum rule Eq. (3).

The rule (for $0 \leq \nu \leq n-1$),

$$S_{2\nu + 1} = 0 \,, \tag{16}$$

is valid if and only if the Schwinger term of order $2(n - 1 - \nu)$ is absent in Eq. (14). Note that each $S_{2\nu + 1}$ is present in all the expressions (14), for which $n \geq \nu+1$. In Eq. (14) S_{2n+1} multiplies the non-Schwinger term. These remarks establish a set of conditions for each sum rule, as well as identities between Schwinger terms in Eq. (14). These can be read off easily.

For all integers $\nu \geq 0$, Eq. (16), would imply

$$m \left(F_+{}^2(m^2) - F_-{}^2(m^2)\right) = 0 \,. \tag{17}$$

That is, up to massless fermions, the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ spectral functions are identical. As there are no $J=I=\frac{1}{2}$ parity doublets, $\psi(y)$ would not allow any particle interpretation. Unless this is the case, the anticommutators $\left[\left(\frac{\partial}{\partial y_0}\right)^{2k-1} \psi(y),\, \bar{\psi}(z)\right]_+ \Big|_{x_0=y_0}$ are not c numbers for all integers $k \geq 1$ [and Schwinger terms are present in some of the Eqs. (14)].

Finally, from Ref. 4 and the high-energy expansion[6,21] of the spectral representation for $\langle T(\bar{\psi}\psi)\rangle_o$, one derives that, if Eq. (2) holds in addition, Eq. (16) is equivalent to a vanishing of the expression in Eq. (1), like $(p^2)^{-\nu-1}$ in the limit $p^2 \to \infty$.

In order to prove the above statements, it would be sufficient to prove the second one (Statement 1 is Statement 2 for $n = 1$). However, we would rather prove statement 1 and generalize the proof. We start with the following algebraic identity of the Jacobi type:

$$\Big[[a,b], c\Big]_+ + \Big[[b,c]_+, a\Big] - \Big[[c,a], b\Big]_+ = 0 \quad . \qquad (18)$$

Then Eq. (7) allows us to write[12,20]

$$\left[Q_5{}^a(x_o), \dot{\psi}(y)\right] = -\gamma_5\tau^a\, r_A\dot{\psi}(y) + \gamma_5\dot{\Delta}^{\frac{3}{2}}(y) - \left[\int d^3x \frac{\partial}{\partial x_o} A_o{}^a(x), \psi(y)\right]$$

$$= -\gamma_5\tau^a\, r_A\dot{\psi}(y) + \gamma_5\dot{\Delta}^{\frac{3}{2}}(y) - \left[\int d^3x\, \partial^\mu A_\mu{}^a(x), \psi(y)\right] \quad . \qquad (19)$$

We have used the Jacobi identity for $\left[Q_5{}^a(x_o), [H,\psi(y)]\right]$ and have added $-\left[\int d^3x\ \partial^k A_k{}^a(x), \psi(y)\right] = 0$ to the first line in Eq. (19). Then one derives

$$\left[Q_5^{\;a}(x_o),\ (\frac{\partial}{\partial y_o})^{2n-1}\psi(y)\right] = \frac{\partial}{\partial y_o}\left[Q_5^{\;a}(x_o),(\frac{\partial}{\partial y_o})^{2n-2}\psi(y)\right]$$

$$-\left[\int d^3x\,\frac{\partial}{\partial x_o}A_o^{\;a}(x),\ (\frac{\partial}{\partial y_o})^{2n-2}\psi(y)\right]$$

$$= \ldots = -\,r_A\gamma_5\tau^a\,(\frac{\partial}{\partial y_o})^{2n-1}\psi(y) + (\frac{\partial}{\partial y_o})^{2n-1}\gamma_5\,\Delta^{\frac{3}{2}}(y)$$

$$-\sum_{j=o}^{2n-2}(\frac{\partial}{\partial y_o})^{j}\left[\int d^3x\,\partial^\mu A_\mu^{\;a}(x_o),\ (\frac{\partial}{\partial y_o})^{2n-2-j}\psi(y)\right]. \qquad (20)$$

First we prove statement 1. We write the identity Eq. (18) with $a = Q_5^{\;a}(x_o)$, $b = \dot{\psi}(y)$, and $c = \bar{\psi}(z)$. Thus, from Eqs. (7) and (19) we get

$$\left[r_A\,\gamma_5\tau^a\dot{\psi}(y),\bar{\psi}(z)\right]_+ + r_A\left[\psi(z)\gamma_5\tau^a,\ \dot{\psi}(y)\right]_+$$

$$+\left[Q_5^{\;a}(x_o),\ \left[\dot{\psi}(y),\ \bar{\psi}(z)\right]_+\right] = \left[\bar{\Delta}^{\frac{3}{2}}(z),\ \dot{\psi}(y)\right]_+ \qquad (21)$$

$$+\left[\dot{\Delta}^{\frac{3}{2}}(y),\ \bar{\psi}(z)\right]_+ - \left[\left[\int d^3x\ \ \partial^\mu A_\mu^{\;a}(x),\ \psi(y)\right],\ \bar{\psi}(z)\right]_+ .$$

If we take vacuum expectation value, the right-hand side vanishes due to our assumptions, and we are left with

$$-r_A\left\{\gamma_5\tau^a \left\langle \left[\dot{\psi}(y), \bar{\psi}(z)\right]_+\Big|_{y_o = z_o} \right\rangle_o + \left\langle \left[\dot{\psi}(y), \bar{\psi}(z)\right]_+\Big|_{y_o = z_o} \right\rangle_o \gamma_5\tau^a \right\}$$

$$= \left\langle \left[Q_5^a(x_o), \left[\dot{\psi}(y), \bar{\psi}(z)\right]_+\right]\Big|_{x_o=y_o=z_o} \right\rangle_o . \qquad (22)$$

Using next the spectral representation ,

$$\left\langle \left[\psi(y), \bar{\psi}(z)\right]_+ \right\rangle_o \qquad (23)$$

$$= i\int dm^2 \left[F_+^2(m^2)\left(i\frac{\partial}{\partial y_\mu}\gamma_\mu + m\right) + F_-^2(m^2)\left(i\frac{\partial}{\partial y_m}\gamma_\mu - m\right)\right]\Delta(y-z;m^2) ,$$

we see that, due to the presence of γ_5 in Eq. (22), no term proportional to γ_μ contributes upon substituting Eq. (23) into Eq. (22). Finally, performing the time differentation under the integral we get Eq. (9) in the equal time limit.

To prove statement 2, we write Eq. (18) for $a = Q_5^a(x_o)$, $b = \left(\frac{\partial}{\partial y_o}\right)^{2n-1}\psi(y)$, $c = \bar{\psi}(z)$. Performing precisely the same manipulations as above but this time using Eq. (20) instead of (19) we have

$$r_A\left\{\gamma_5\tau^a\left\langle \left[\left(\frac{\partial}{\partial y_o}\right)^{2n-1}\psi(y), \bar{\psi}(z)\right]_+\Big|_{y_o=z_o} \right\rangle_o + \left\langle \left[\left(\frac{\partial}{\partial y_o}\right)^{2n-1}\psi(y),\bar{\psi}(z)\right]_+\Big|_{y_o=z_o} \gamma_5\tau^a\right\}$$

$$= -\left\langle \left[Q_5^a(x_o), \left[\left(\frac{\partial}{\partial y_o}\right)^{2n-1}\psi(y), \bar{\psi}(z)\right]_+\right]\right\rangle_o . \qquad (24)$$

Note that due to Eq. (13) there is no contribution from the sum in Eq. (20). We again insert the spectral representation and observe that terms porportional to γ_μ drop. Then, using

$$\frac{\partial}{\partial y_0}\left(\frac{\partial}{\partial y_0}\right)^{2n-2}\Delta(y-z;\, m^2)\Big|_{y_0=z_0} = -\left(\frac{\partial}{\partial \underset{\sim}{y}}\,\frac{\partial}{\partial \underset{\sim}{y}} - m^2\right)^{n-1}\delta(\underset{\sim}{z}-\underset{\sim}{y}) \,, \tag{25}$$

we reach Eq. (14), the desired result.

As to the consequences of Eq. (16), restrictions follow from the positivity

$$F_{\pm}^{2}(m^2) \geq 0 \,. \tag{26}$$

Evidently, any of the Eqs. (16) -- if saturated by one-particle intermediate states -- can hold only if baryons of opposite parities exist. For $S_1 = 0$, this has been noted in Refs. 4 and 8.

To derive a further consequence, let us enumerate by $N_1, \ldots, N_4$ the four nucleon resonances $P_{11}(940)$, $P_{11}(1466)$, $S_{11}(1548)$, and $S_{11}(1709)$, and let us denote $F_{\epsilon i}^{2}(m_i^2)$ by F_i^2. We assume $F_1^2 \neq 0$, and we normalize to $F_1^2 = 1$. The assumptions of statement 2 for $n = 2$, together with assuming

$$\langle \left[Q_5^a(x_0), \left[\dddot{\psi}(y),\, \bar{\psi}(z) \right]_+ \right] \Big|_{x_0=y_0=z_0} \rangle = 0 \,, \tag{27}$$

give us the sum rules

$$S_1 = S_3 = 0 \,. \tag{28}$$

If saturated by one-particle intermediate states, Eqs. (28) allow us to predict the existence of at least one further nucleon resonance, N_5, from $N_1, \ldots, N_4$. Concerning its mass and parity, there are two possibilities. Either we have $m_5 < m_2$ and $\epsilon_5 = -1$ or $m_2 < m_5$ and $\epsilon_5 = +1$. As the existence of an undiscovered resonance with a mass smaller than m_2 is very unlikely, the actual prediction is

$$m_5 > m_2 \ , \ \epsilon_5 = +1 \ . \tag{29}$$

This agrees with the existence of the $P_{11}(1750)$.

In order to derive the conclusion, we write Eq. (28) as

$$\begin{aligned} m_1 + m_2 F_2^{\,2} &= m_3 F_3^{\,2} + m_4 F_4^{\,2} - \sum_{i=1}^{N} \epsilon_i \, m_i \, F_i^{\,2} \\ m_1^{\,3} + m_2^{\,3} F_2^{\,2} &= m_3^{\,3} F_3^{\,2} + m_4^{\,3} F_4^{\,2} - \sum_{i=1}^{N} \epsilon_i \, m_i^{\,3} \, F_i^{\,2} \ . \end{aligned} \tag{30}$$

Thus we have:

$$\begin{aligned} m_1 (m_2^{\,2} - m_1^{\,2}) &= m_3 (m_2^{\,2} - m_3^{\,2}) \, F_3^{\,2} \\ &+ m_4 (m_2^{\,2} - m_4^{\,2}) \, F_4^{\,2} - \sum_{i=1}^{R} \epsilon_i \, m_i (m_2^{\,2} - m_i^{\,2}) \, F_i^{\,2} \ , \end{aligned} \tag{31}$$

with R being the total number of nucleon resonances. The left-hand side is positive and the first two terms on the right-hand side are not positive. Therefore, at least one term in the sum is negative. Giving the number 5 to it, we have

$$\epsilon_5(m_2^2 - m_5^2) < 0 \, . \tag{32}$$

This is the desired result.

The content of the paper is summarized in statements one and two and in the prediction, Eq. (29).

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TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720