Single-Cell Forward Link Power Allocation Using Pricing in Wireless Networks

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***Abstract*—We consider forward link power allocation for voice users in a code-division multiple-access wireless network. Admission control policies are investigated, which base a new call admission decision not only upon available capacity, but also upon the required forward link transmit power and upon the user's willingness to pay. We assume that each voice user has a utility function that describes the user's willingness to pay as a function of forward link signal-to-interference plus noise ratio. The network objective is to maximize either total utility summed over all users or total revenue generated from all users. Properties of the optimal power and code allocations are presented. Our key results show how these optimal allocations can be achieved using pricing. The analysis is complemented with a numerical study, which shows how the optimal prices and corresponding utility or revenue vary with load.**

***Index Terms*—Cellular resource allocation, power control, pricing, utility maximization.**

I. Introduction

THE FORWARD link capacity of a single cell in a code-division multiple-access (CDMA) wireless network is limited by available resources, namely, power and codes. The number of available codes is limited by a bandwidth constraint, whereas, the transmitted power may be limited by a physical constraint, or by the associated interference received in neighboring cells. As the demand for service increases, so does the need for an efficient allocation of available resources to user requests.

In this paper, we study forward link power allocation for voice users in a CDMA system. The system capacity, i.e., the number of voice users that can obtain service, is assumed to be limited by forward link resources. Current CDMA networks generally admit calls on a first-come first-served basis. As an alternative, our approach to admission control bases call admission decisions not only on the availability of resources (i.e., codes and transmit power), but also upon the users' willingness to pay. Here, we consider a single cell, and account for interference to adjacent cells by imposing a transfer payment, or cost, which is proportional to the transmitted power. The network objective is to maximize either the total utility summed over all users, or total revenue generated by the users.

We assume that each voice user has a utility function that specifies the perceived utility derived from the service, or willingness to pay, as a function of the received signal-to-interference plus noise ratio (SINR). This utility function might be assigned to the user by the service provider, according to a choice of service plan. In that case, the utility function can be announced to the network at the onset of a call request.

The resource allocation proceeds with an exchange of price and demand information. The base station announces a price per unit transmitted power and a price per code. Each user responds by requesting the amount of each resource that maximizes his/her individual surplus, defined as utility minus cost. The goal is to set prices to maximize total utility or revenue. We remark that the users may not actually pay the prices set by the network. In that case, the prices serve only as internal network parameters that guide the resource allocation.

Unlike other approaches to resource allocation, pricing can allocate resources according to perceived user utility, thereby increasing the overall utility of the network. Other attractive properties include the accommodation of a wide range of traffic flows, and potential simplification or elimination of explicit admission control policies [1].

After problem formulation, we express total utility and revenue in terms of the resource prices and distribution functions of user utility and transmitted power. We also characterize the optimal power and code allocations. Our key results show how these optimal allocations can be achieved using pricing. We complement this analysis with a numerical study. We display the optimal power allocation to each user, as a function of the geographical distribution of users, for a selection of different utility function distributions. We demonstrate how prices per code and per unit transmitted power can be used to achieve the optimal power allocation in a distributed fashion, and the variation of these prices with system load.

A sampling of papers that propose pricing of resources in wireline networks to control congestion and/or implement connection admission control include [1]–[4]. These papers demonstrate that the overall utility of the network can be greatly increased by allocating resources to flows based on utility and on congestion. Utility-based power control for wireless networks has been proposed in [5]–[9] and [18]. Power control for the reverse link of a data cellular network is considered in [5]–[8] and [18] and the forward link is considered in [9].

Our model differs from prior models in several key respects. First, we focus on voice users rather than data users. Our utility

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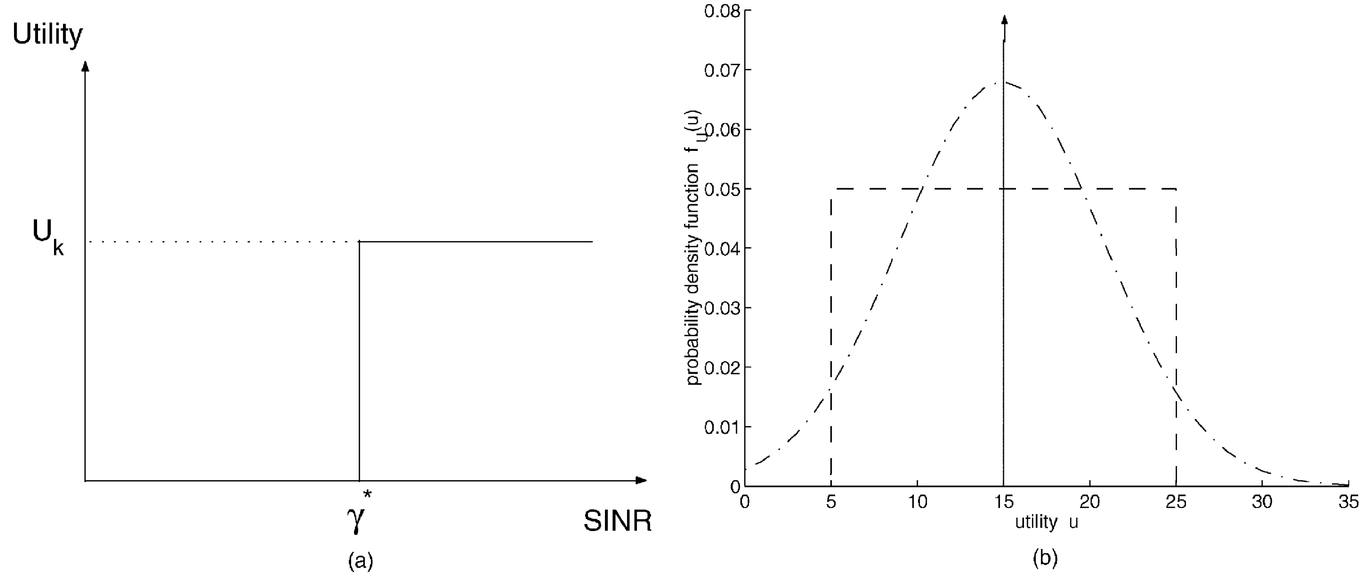

Fig. 1. (a) Utility function for voice traffic. (b) Uniform, Gaussian, and delta distribution for utility step U.

functions correspondingly have a different form—a step function in SINR, in contrast to a function of bits/joule, as in [5]–[8], a function of throughput, as in [9], or a concave function of SINR, as in [18]. The focus on voice leads to significantly different results than have been obtained for data. Second, we focus on the forward link, whereas, [5]–[8] and [18] focus on the reverse link. On the reverse link, the key impairment is typically multiuser interference. In contrast, on the forward link orthogonal codes are often assumed within a single cell (although often not between multiple cells), and the complexity comes from constraints on total power and on codes. Both the power and code constraints have a significant impact upon the results. Our main focus in this paper is on characterizing optimal resource allocation policies and implementing them in a distributed manner. In contrast, [5]–[8] do not address optimality; indeed, although their use of pricing produces a Pareto improvement, no notion of optimality is defined or achieved.

The rest of our paper is organized as follows. In Section II, we present our system model for a finite number of users, and introduce the notion of utility functions. In Section III, we formulate utility and revenue optimization problems, present characterizations of the optimal power allocation policies, and show how these policies can be distributed using pricing. In Section IV, we introduce a large system model in which the number of users and codes increase at a fixed ratio. In Section V, we reformulate utility maximization problems under this large system limit, and propose pricing methods for power and code allocation. In Section VI, numerical results are presented, which illustrate system behavior. Finally, in Sections VII and VIII, we consider maximizing revenue instead of utility, and illustrate the differences in resource allocation.

II. Finite System Model

In this section, we introduce the user and network models for a two-dimensional single-cell forward link system with a finite number of users. In what follows, user locations are specified by the distance from the base station r, where $r \leq 1$.

Each user k has a *utility function* $u_k(\gamma)$, where γ denotes the received SINR. For the voice service considered, we model $u_k(\gamma)$ as a step, as shown in Fig. 1(a), rising from zero utility when $\gamma < \gamma^*$ to a positive utility $U_k > 0$ when $\gamma > \gamma^*$. The SINR threshold γ^* is assumed to be the same for all users. Typical values may range from 3 to 7 dB depending on the specific CDMA system. The height of the step indicates the call's priority level. More important calls are associated with higher values of U_k, i.e., the user is willing to pay more to gain admission to the system.

The codes within the cell are assumed to be orthogonal, but they are not orthogonal to codes in adjacent cells. We lump together the background noise and the interference from other cells, and represent the sum as σ^2. The interference from other cells is a function of the total power transmitted from other cells, and the associated path loss. We assume that the multipath is negligible or is equalized [10], so σ^2 is fixed. The received SINR for user k is given by $\gamma_k = h_k P_k / \sigma^2$, where h_k is the channel attenuation to user k and P_k is the forward link transmitted power to user k. The transmitted power needed to achieve the target SINR γ^* is then given by

$$P_k = \frac{\gamma^* \sigma^2}{h_k}. \tag{1}$$

The attenuation h_k depends on the location of the user and on random shadowing.

III. Optimization With a Finite Set of Users

We start with a single cell containing a finite number of users. We consider two optimization metrics: total utility and revenue.

A. Utility Maximization

A resource allocation policy maps a user's channel h_k and utility U_k to a power assignment P_k and a code assignment C_k. A user with positive assigned power ($P_k > 0$) uses a single code ($C_k = 1$); a user with zero assigned power ($P_k = 0$) uses no codes ($C_k = 0$).

In a multiple cell network, we should explicitly account for the increase in the required transmission power in one cell caused by the interference from other cells. Under a power constraint, this increase in required transmission power can cause a reduction in the achieved utility. In order to achieve the maximum total utility over all cells in the network, we must account for this intercell effect. We do so here by penalizing each cell linearly as a function of its total transmitted power. Specifically, we assume that each base station must pay a transfer payment, equal to β per unit transmitted power, to neighboring cells. It can be shown [16], [17] that the maximum utility summed over all cells is achieved by setting β equal to -1 times the sum of the marginal utilities in other cells due to an increase in transmission power in this cell. As a consequence, β will fluctuate with the loads in each cell. However, in the single-cell model in this paper, we hold the interference from neighboring cells and β constant.

We wish to find the policy that maximizes the net utility of the cell. The net utility is defined as the total utility of all active users minus the transfer payment, and is given by

$$U_{\text{tot}} = \sum_k (U_k - \beta P_k). \tag{2}$$

The resulting optimization problem is:

Problem FU1:

$$\max_{\{P_k\}} U_{\text{tot}}, \qquad \text{such that } \sum_k P_k \le \mathcal{P} \tag{3}$$

$$\sum_k C_k \le M. \tag{4}$$

where $\mathcal{P}$ represents the maximum total transmitted power in the cell and M is the number of available codes in the cell.

This optimization problem is an integer programming problem, and is related to traditional bin-packing problems. The solutions to integer programming problems are typically very sensitive to the parameters of the problem. (In problem FU1, these parameters are the sets of user attenuations and user utilities, the power limit $\mathcal{P}$, and the number of codes M.) An infinitesimal change in a continuous parameter, e.g., attenuation, can produce a noninfinitesimal change in the optimal policy. In particular, when there is more than one constraint, the optimal policy can change in a very complicated manner with infinitesimal changes in parameters. As a consequence, characterizations of optimal policies for integer programming problems are notoriously difficult to derive.

The integer programming problem can be simplified if we remove the power constraint:

Problem FU2:

$$\max_{\{P_k\}} U_{\text{tot}}, \qquad \text{such that } \sum_k C_k \le M.$$

Our first step toward deriving an optimal policy is to reduce the set of policies that need be considered.

Theorem 3.1: Consider the class of policies in which each active user is assigned one code and a transmit power required to obtain the target SINR, and each inactive user is assigned no codes and no power. The solution to problem FU2 belongs to this class of policies.

This can be easily proven by contradiction. Namely, given a feasible policy not in this class, lowering the power for a particular user to either that required to achieve the target SINR, or zero always results in a higher net utility without violating the code constraint. The optimal policy can now be stated in terms of the optimal set of active users.

Theorem 3.2: Order the users according to decreasing net utilities $U_k - \beta P_k$. Let N represent the number of users with positive net utility. Consider the policy given by setting the first $M' = \min(N, M)$ users active and the remaining users inactive. Each active user is assigned one code and a transmit power that allows the user to obtain the target SINR and each inactive user is assigned no codes and no power. This policy solves problem FU2.

This follows from a simple interchange argument. Namely, given any other active user set, a higher net utility can always be achieved by activating a user that belongs to the optimal set as described in Theorem 3.2 and/or deactivating a user that does not belong to the optimal set. The optimal policy, therefore, chooses those users who have the best combination of low path loss and high utility.

B. Utility Maximization Using Pricing

The optimal policy described in Theorem 3.2 has a simple structure, but requires that the base station know each user's utility function. In this section, we consider methods to distribute the optimization process.

We propose a distributed resource allocation method using the following *pricing process*. The base station announces a price per unit transmitted power α_p and a price per code α_c. Each user responds by requesting the amount of each resource that maximizes his/her individual surplus, defined as utility minus cost. The total charge for service to an active user with transmit power P_k is, therefore, $\alpha_c + \alpha_p P_k$. For a step utility function, the user's surplus is maximized either by buying one code and exactly enough power to achieve an SINR equal to the threshold γ^* (if $U_k > \alpha_c + \alpha_p P_k$), or by remaining inactive.

The principal result of this section is that such a pricing policy can always achieve the optimal power allocation.

Theorem 3.3: Consider the preceding pricing process with prices set as follows. Set $\alpha_p = \beta$ and α_c to be the minimum value ≥ 0 such that (4) is satisfied. The corresponding set of transmit powers solves problem FU2.

Proof: Order the users according to decreasing net utilities $U_k - \beta P_k$. Let N represent the number of users with positive net utility, and let $M' = \min(N, M)$. Using the pricing process, a user is active if $U_k > \alpha_c + \alpha_p P_k$. The proposed prices will result in $\alpha_c = U_{M'+1} - \beta P_{M'+1}$, if $M' < N$ and $\alpha_c = 0$, otherwise. With the proposed prices, therefore, the first M' users will choose to be active and the remaining users will choose to be inactive. By Theorem 3.2, the result follows. Q.E.D.

The optimal set of prices (α_c, α_p) is a function of the set of user utilities U_k and the set of required power levels P_k. Theorem 3.3, however, states that the optimal α_c is the price at which demand for codes equals supply for codes (or zero if the number of users in the cell with positive net utility is less than the number of codes). The optimal α_c can, thus, be found using a simple line search algorithm; such an algorithm would increase α_c while demand exceeds supply, and reduce α_c while supply exceeds demand (but not below $\alpha_c = 0$). Since demand for codes decreases monotonically as α_c increases, it is easy to construct a line search algorithm for the optimal α_c that is guaranteed to converge. As a result, the optimal α_c can be found without explicitly asking the users for their utility functions. This is the general idea behind using pricing; it can reduce the amount of information that must be exchanged between users and the network.[1]

In the finite user model, the optimal prices may not be unique. Any (α_c, α_p) that satisfies $\max(0, U_{M'+1} - \beta P_{M'+1}) \leq \alpha_c < U_{M'} - \beta P_{M'}, \alpha_p = \beta$ activates the same set of users, and therefore, achieves the same total net utility. Of the optimal set of prices, the pair achieved by Theorem 3.3 has a particularly nice interpretation: $\alpha_p = \beta$ is negative one times the marginal utility in other cells for an increase in power in this cell, and the corresponding α_c is the shadow cost for the code constraint, namely $\delta U_{\mathrm{tot}}/\delta M^+ = \alpha_c$.

C. Revenue Maximization

We now pose a revenue maximization problem. A pricing process is assumed, and the network desires to maximize total net revenue rather than total net utility. For a fixed set of prices, denote the set of active users as $S = \{k : U_k \geq \alpha_c + \alpha_p P_k\}$. We will call the set of prices that activates no more than M users the "*feasible region*:" $F = \{(\alpha_c, \alpha_p) :\mid S \mid /M \leq 1\}$.

The resulting optimization problem is:

Problem FR1:

$$\max_{(\alpha_c, \alpha_p)\in F} \sum_{k\in S} (\alpha_c + \alpha_p P_k - \beta P_k). \tag{5}$$

This optimization problem is generally not easy to solve. Unlike the utility maximization problem, the network may activate fewer than M users even if M users can contribute positive net revenue. The total revenue is an irregular function of the price set, with jumps at the specific prices at which users become activated or deactivated. Related work on pricing to maximize revenue, where users can be charged different prices is presented in [11].

IV. Large System Model

In this section, we introduce a large system model in order to avoid the analytical problems associated with a finite number of users. This approach will allow us both to introduce a power limit and to analyze revenue maximization. As mentioned above, the finite user model leads to an integer programming problem, and the optimal policy is not a continuous function of the problem parameters. We can eliminate this discontinuity by modeling a system with an infinite number of users, each using an infinitesimal percentage of the total power and codes. This approach is motivated by the large system analysis of CDMA receivers in [12]. Our "large system model" can be obtained by starting with a finite user model and taking the limit as the number of users, number of codes, and power limit all approach infinity with fixed ratios. In the limit, the distribution of each resource over users is specified by a function of a continuous variable, and the optimal policy typically varies in a continuous manner with the resource constraints. In addition, the difference between the values of the optimization metric under the optimal policies for the continuous and finite-size discrete problems of interest is often small.

Formally, the large system limit is obtained by starting with problem FU1, scaling the power limit $\mathcal{P}$ with the number of codes $\mathcal{P} = \mathcal{P}'M$, scaling the number of users with the number of codes $K = \rho M$, and letting the number of codes M approach infinity. The scaling parameter $\mathcal{P}'$ is interpreted as the maximum average power per code, and the scaling parameter ρ is interpreted as the offered load (measured in users per code).

Since the large system model has an infinite number of users, we now assume there exists a distribution of users' utilities and a distribution of the channel attenuations. We denote f_U as the density function of an user's utility and denote f_h as the density function of channel attenuations. Using (1), we can determine the corresponding distribution for transmitted power, denoted by f_P. In what follows, we assume that the cumulative distribution function for power is continuous, i.e., f_h and f_P do not contain impulses.

Let Q denote the set of active users. The constraint on total transmitted power, which for the finite user model was expressed in (3), now becomes $K\mathbf{E}P \leq \mathcal{P}$, where $\mathbf{E}P = \int\int_Q p f_U f_P du dp$ is the average transmitted power per user. Similarly, the constraint on codes, which for the finite user model was expressed in (4), now becomes $K\mathbf{E}C \leq M$, where $\mathbf{E}C = \int\int_Q f_U f_P du dp$ is the fraction of active users. Normalizing both equations by the number of codes M results in

$$\rho \int\int_Q p f_U f_P du dp \leq \mathcal{P}' \tag{6}$$

as the new power constraint and

$$\rho \int\int_Q f_U f_P du dp \leq 1 \tag{7}$$

as the new code constraint.

In the analytical work, the density functions f_U and f_h are general. In numerical work, we will consider three distributions for f_U, as shown in Fig. 1(b). The first density function is a delta function $f_U = \delta(u - u_0)$, meaning that all users have identical utility functions. The second density function is uniform from u_1 to u_2. The third density function is a truncated Gaussian with mean m_G and standard deviation σ_G. The latter two functions allow for a range of users with different priority levels.

In numerical work, we use a distance-based attenuation model with an exponent of four, i.e., $h(r) = (d_0/r)^4$. We assume that the users are spatially uniformly distributed throughout the cell, i.e., with a probability density function given by $f_r(r) = 2r$. The resulting received power at distance r from the base is given by $P_R(r) = P_T(r)(d_0/r)^4$, where $P_T(r)$ is the transmitted power and d_0 is a reference point in the far-field region of the transmitter antenna. If all users in the cell are active, then the resulting power distribution is given by

$$f_P(p) = \frac{1}{2}\left(\frac{d_0^4}{\gamma^*\sigma^2 p}\right)^{1/2}, \quad p \in \left(0, \frac{\gamma^*\sigma^2}{d_0^4}\right). \tag{8}$$

[1]In a multicell network, a similar approach can be taken to reduce the amount of information that must be exchanged between cells using the transfer charges β.

Of course, other distributions can be derived which account for additional propagation and system effects such as random shadowing and soft handoff.

V. Large System Utility Maximization

We return to the utility maximization problem initially posed as Problem FU1. Recall that the power assigned to each user depends only on the channel attenuation h and the user's utility u. Our objective is, therefore, to find a mapping from each point in the (h, u) plane to a power p, which maximizes the average net utility per code, given by

$$U_{\text{ave}} = \rho \int \int_{Q} (u - \beta p) f_U f_P du dp. \tag{9}$$

Let $\Pi : (\Re^+, \Re^+) \to \Re^+$ denote this map, where $\Re^+$ is the set of nonnegative real numbers.

The large system optimization problem is:

Problem LU1:

$$\max_{\Pi} U_{\text{ave}}$$

such that inequalities (6) and (7) are satisfied.

Note that this is a nonlinear maximization problem with nonlinear inequality constraints. The Kuhn–Tucker theorem [13] states that at the optimal allocation, there is a shadow cost associated with each constraint. The shadow cost is equal to the derivative of the optimization metric (utility or revenue) with respect to the right-hand side of the associated inequality constraint (power or codes).

At the optimal allocation, we say the cell is power-limited (PL) if the power constraint is binding and code-limited (CL) if the code constraint is binding. In addition, we say the cell is interference-limited (IL) if the transfer price β is strictly positive, indicating that power usage in this cell has a negative effect upon utility or revenue in neighboring cells. Finally, we say the cell is demand-limited (DL) if it is neither PL, CL, nor IL.

There are three types of shadow costs in the system. First, if the cell is PL, then there is a positive shadow cost associated with the cell's power limit. Second, if the cell is CL, then there is a positive shadow cost associated with the cell's code limit. Finally, if the cell is IL, the transfer payment can be interpreted as a shadow cost associated with the loss of utility or revenue in other cells due to the interference generated by this cell.

We start with a version without the power constraint.

Problem LU2:

$$\max_{\Pi} U_{\text{ave}}$$

such that the inequality (7) is satisfied.

As before, our first step toward deriving an optimal policy is to reduce the set of policies that need be considered.

Theorem 5.1: Consider the class of policies in which each active user is assigned one code and a transmit power required to obtain the target SINR and each inactive user is assigned no codes and no power. The solution to problem LU2 belongs to this class of policies.

This can be proven by contradiction in analogy to the proof of Theorem 3.1. The optimal policy can now be stated in terms of the optimal set of active users.

Theorem 5.2: Order the users according to decreasing net utilities $u - \beta p$. Consider the policy given by setting users active in this order until either the code constraint becomes binding or until all users with positive net utility are active. Set the remaining users to be inactive. Each active user is assigned one code and a transmit power that allows the user to obtain the target SINR and each inactive user is assigned no codes and no power. This policy solves problem LU2.

The proof again uses an interchange argument similar to that for Theorem 3.2. The optimal policy can be distributed using the same *pricing process* as in the finite user case: The base station announces a price per unit transmitted power α_p and a price per code α_c. Each user responds by requesting the amount of each resource that maximizes his/her individual surplus, defined as utility minus cost. The total charge for service to an active user with transmit power p is $\alpha_c + \alpha_p p$. This pricing process can always achieve the optimal power allocation.

Theorem 5.3: Consider the preceding pricing process with prices set as follows. Set $\alpha_p = \beta$, and α_c to be the minimum value ≥ 0 such that the code constraint (7) is satisfied. The corresponding set of transmit powers solves problem LU2.

The proof is similar to that for Theorem 3.3. This choice for α_c is again the price that equates demand with supply. This price is positive if there are more users with positive net utility than codes, and zero otherwise. However, unlike the finite user case, the optimal prices are now unique. Note that the shadow cost corresponding to the code constraint is $\alpha_c = \partial(U_{\text{ave}}/\rho)/\partial(1/\rho)$.[2]

We now return to the version with the power constraint, problem LU1. As before, our first step toward deriving an optimal policy is to reduce the set of policies that need be considered.

Theorem 5.4: Consider the class of policies in which each active user is assigned one code and a transmit power required to obtain the target SINR and each inactive user is assigned no codes and no power. The solution to problem LU1 belongs to this class of policies.

The proof is similar to that for Theorem 5.1. The optimal policy, however, no longer necessarily corresponds to activating users in decreasing order by their net utility, as in Theorem 5.2. Such an approach could result in a tight power constraint and a loose code constraint, while an alternate approach that trades off a few high power users for more low power users might be superior.

However, we can still describe and implement the optimal policy using pricing. We use the same process as above, but potentially with different prices.

Theorem 5.5: Consider the above pricing process with prices set as follows. Jointly set α_p to be the minimum value $\geq \beta$ such

[2]Alternatively, defining the average utility per user $\overline{U_{\text{ave}}} = U_{\text{ave}}/\rho$, and the number of codes per user $\overline{M} = 1/\rho$, we have $\alpha_c = \partial\overline{U_{\text{ave}}}/\partial\overline{M}$.

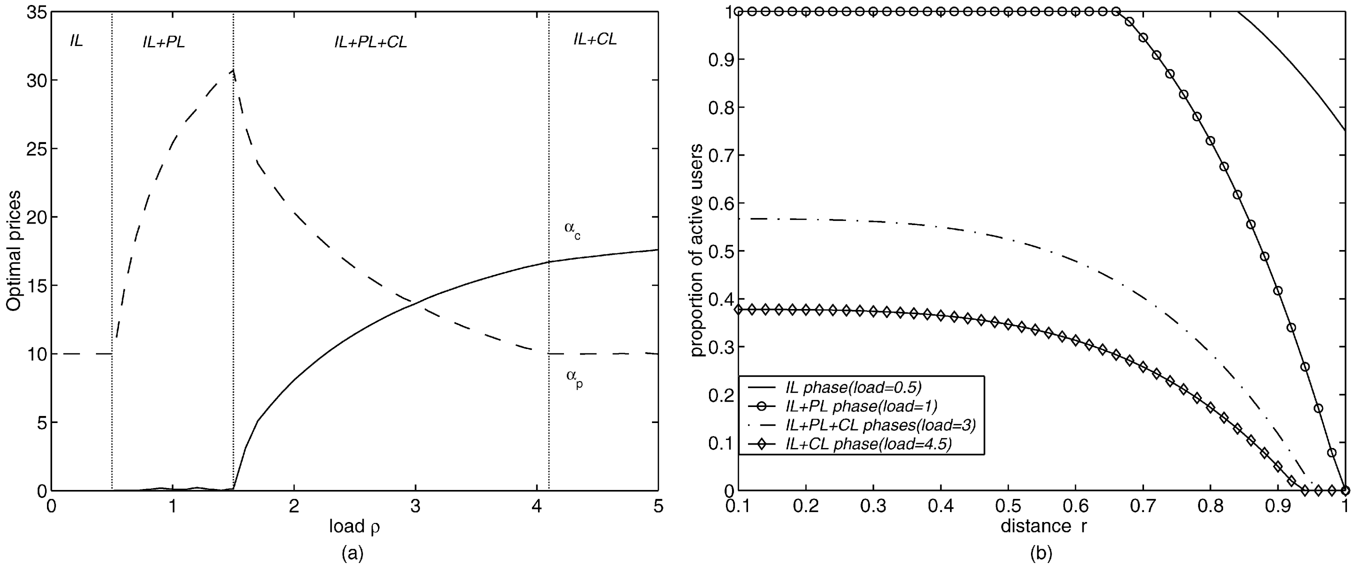

Fig. 2. Utility maximization with uniform distribution at $\mathcal{P}/\sigma^2 = 37$ dB, $\beta = 10$. (a) Optimal prices versus load. (b) Proportion of users at distance r who are active.

that the power constraint (6) is satisfied and set α_c to be the minimum value ≥ 0 such that the code constraint (7) is satisfied. The corresponding set of transmit powers solves problem LU1.

The proof is given in the Appendix. The optimal prices can again be interpreted as shadow costs for their corresponding constraints, i.e., $\partial(U_{\text{ave}}/\rho)/\partial(1/\rho) = \alpha_c$ and $\partial U_{\text{ave}}/\partial\mathcal{P} = \alpha_p - \beta$. We remark that this pricing process may not produce a unique optimal (α_c, α_p). An example with an infinite number of optimal price combinations is given in Section VI-C, when all the users have the same utility function.

The primary advantage of pricing over direct centralized control of power and codes is the lower complexity required to find the optimal policy. As we discussed above, in utility maximization problems with only a code constraint (FU2 and LU2), pricing can be used to reduce the optimization problem to a line search for the optimal α_c. Similarly, in the large system model with both code and power constraint (LU1), pricing can be used to reduce the optimization problem to a search for the optimal set of prices (per code and per unit power). This search can be implemented using standard fixed point or gradient algorithms. A centralized algorithm would require full knowledge of each user's utility function. While this may not be too much of a burden in a single cell with step utility functions, the complexity of the centralized scheme grows quickly both with more general utility functions and with more cells. In contrast, the complexity of the distributed pricing algorithm remains low. In particular, in a multicell network, a centralized algorithm may require knowledge of the utility functions of users in neighboring cells, while the distributed pricing algorithm only requires communication of externality prices between cells.

VI. Numerical Study of Utility Maximization

In this section, we present numerical results to show how the system phases (DL, IL, PL, and CL) and optimal prices vary with the offered load and the transfer price. In the following sections, we consider utility distributions given by uniform, Gaussian, and delta densities. Throughout our numerical study, the target SINR $\gamma^* = 5$ dB, the far-field reference point $d_0 = 0.1$, the background noise plus interference level σ^2 is set so that $\gamma^*\sigma^2/d_0^4 = 1$.

A. Uniform Utility Distribution

We assume the height of the step in the user's utility function is uniformly distributed between $u_1 = 5$ and $u_2 = 25$, assume the power limit is $\mathcal{P}/\sigma^2 = 37$ dB, and set the transfer price to $\beta = 10$, which indicates the level of interference that users in this cell are generating to users in other cells. Because $\beta > 0$, the cell is always IL. The optimal price per code α_c and price per unit transmitted power α_p are shown in Fig. 2(a). At low loads, the system is only IL, meaning that there are excess power and codes in the cell, but that the transmitted power in this cell decreases the utility in neighboring cells. Optimal prices are equal to the associated shadow costs. The price per code α_c is set to the shadow cost associated with the code constraint. The price per unit transmitted power α_p is set to the *sum* of the shadow cost associated with the power constraint and the transfer price. In the interference-only-limited phase, therefore, the cell sets the price per code equal to zero, since there are excess codes, and sets the price per unit power equal to β, since there is excess power.

At a load of 0.5, the cell exhausts its power supply, and it is then both IL and PL for loads between 0.5 and 1.5. Correspondingly, the cell must raise its price per unit transmitted power above β to restore equality between demand and supply for power. At a load of 1.5, the cell runs out of codes as well. Between loads of 1.5 and 4, the cell is IL, PL, and CL. Correspondingly, the optimal price per code is strictly positive and increases with load in this phase, since a higher price for code is required to reduce the demand for codes (to equal the supply) at higher loads. This price is also equal to the shadow cost associated with the code constraint; at higher loads, the marginal

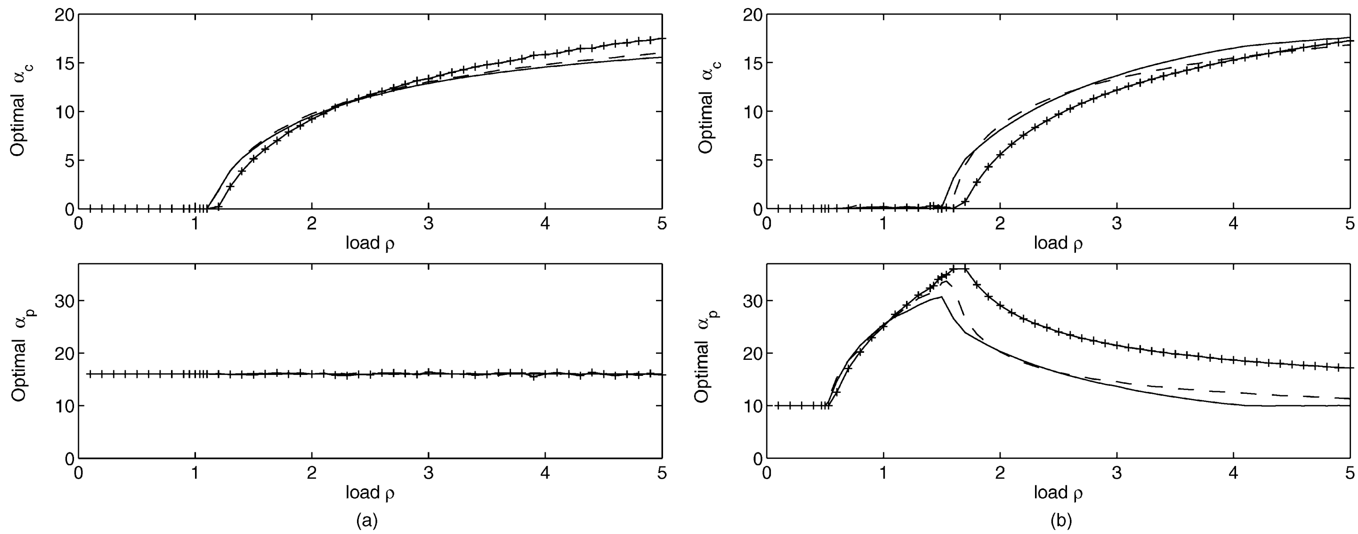

Fig. 3. Comparison of optimal prices for utility maximization under different utility distributions at (a) $\mathcal{P}/\sigma^2 = 40$ dB, $\beta = 16$ and (b) $\mathcal{P}/\sigma^2 = 37$ dB, $\beta = 10$. Solid line is uniform distribution, dashed line is Gaussian with standard deviation of 5.77, and cross–hatched line is Gaussian with standard deviation of 10.

utility that would be generated by an additional code in the cell is higher since that code would be allocated to users with higher utilities. However, this increase in price per code causes some users to become inactive, which lowers the demand for power. Consequently, the price per unit power can be lowered. At loads above four, the cell is IL and CL but no longer PL; the price per unit power has returned to its floor of β.

This behavior can be better understood by examining the proportion of users at distance r who are active, shown in Fig. 2(b). Recall that at a load of 0.5, the optimal prices are $\alpha_c = 0$ and $\alpha_p = 10$. Within the region close to the base station ($r < 0.84$), the resulting power charge is less than $u_1 = 5$, and consequently all users are active. Beyond $r = 0.84$, the power charge rises above five, and therefore, some users choose not to transmit. The proportion of active users, therefore, falls with increasing distance, in proportion to the rise in required power to obtain the target SINR.

As the load rises from 0.5 to 1, the cell also becomes PL, and the optimal power price rises. This causes a decrease in active users, since it results in an increased charge for all active users that is also increasing with distance. Consequently, the proportion of active users falls more steeply. As the load rises from one to three, the cell also becomes CL, the optimal code price rises and the optimal power price falls. A rise in code price lowers the proportion of active users by an equal amount at all distances, while a fall in power price makes the proportion of active users less sensitive to distance. These trends continue as the load rises from 3 to 4.5.

Empirical evidence suggests that the transition among phases, from IL, possibly through IL + PL and IL + PL + CL, to IL + CL, depends on the relative values of the cell parameters. Consider fixing the utility distribution (at $u_1 = 5$ and $u_2 = 25$) and the transfer price (at $\beta = 10$), but varying the power limit $\mathcal{P}$. We find that when $\mathcal{P}/\sigma^2 < 39.8$ dB, the cell progresses through the same series of phases as above—from IL through IL + PL and IL + PL + CL to IL + CL. The transition from IL to IL + PL always occurs at a load less than one, but this load threshold is increasing with $\mathcal{P}$. At high power limits, $\mathcal{P}/\sigma^2 > 39.8$ dB, the cell progresses through the series of phases—from IL directly to IL + CL. Similar trends occur with different choices for u_1, u_2, and β. The $\mathcal{P}$ threshold for the presence of a PL phase (39.8 dB relative to σ^2 in the above example) increases with u_1, u_2, since this increases average power usage, and decreases with β, since this decreases average power usage.

B. Relationship of Optimal Prices to Utility Distribution

We now compare the optimal prices associated with different utility distributions. Here, we include a Gaussian distribution with mean 15 and standard deviation 5.77, truncated so that negative values do not occur, as shown in Fig. 1(b). The optimal prices for this distribution are compared with those for the uniform distribution used in Section V, and another truncated Gaussian with mean 15 and standard deviation 10.

Optimal prices for the three distributions are shown in Fig. 3 for two cases of parameter setting. In each case, the optimal prices show similar trends. For the uniform and Gaussian distributions with the same first and second moments, the prices are virtually indistinguishable. The code price associated with the larger standard deviation starts below the other code prices and the curves cross as the load increases. Similarly, for most loads the power price for the larger standard deviation is above the power prices for the smaller standard deviation. This is due to the higher proportion of active users with utilities in both low and high ranges.

C. Delta Utility Distribution

We now assume that all users have the same utility function, a step with a height of $u_0 = 15$. There are two significant differences from the uniform and Gaussian cases. First, the proportion of users at distance r who are active now falls from one to zero at some ”cutoff” distance, which we denote by r_{co}. Second, there are an infinite number of price combinations that can induce the same set of active users, as shown in Fig. 4.

One set of optimal prices is shown in Fig. 5(a), and the resulting cutoff distances are shown in Fig. 5(b). In the first scenario ($\mathcal{P}/\sigma^2 = 40$ dB, $\beta = 16$), at loads less than one the

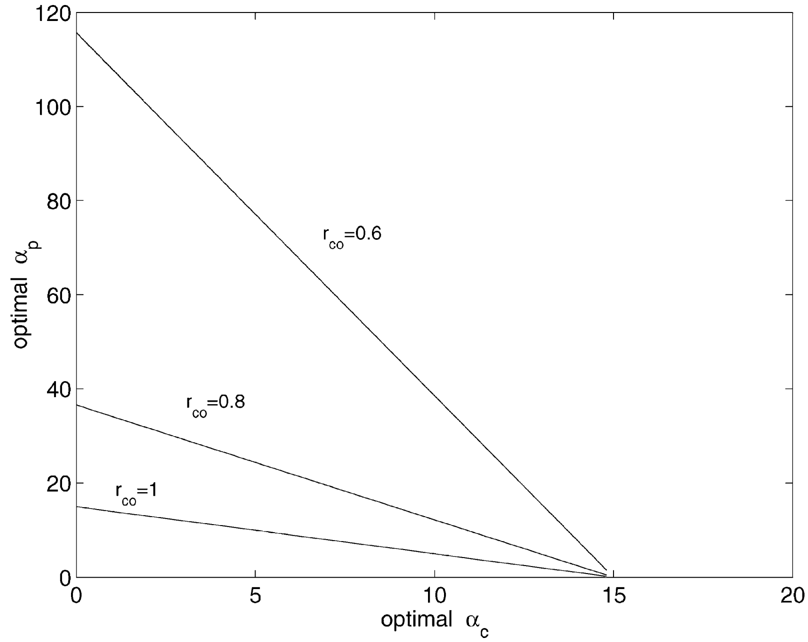

Fig. 4. Equivalent price combinations for utility maximization with delta distribution.

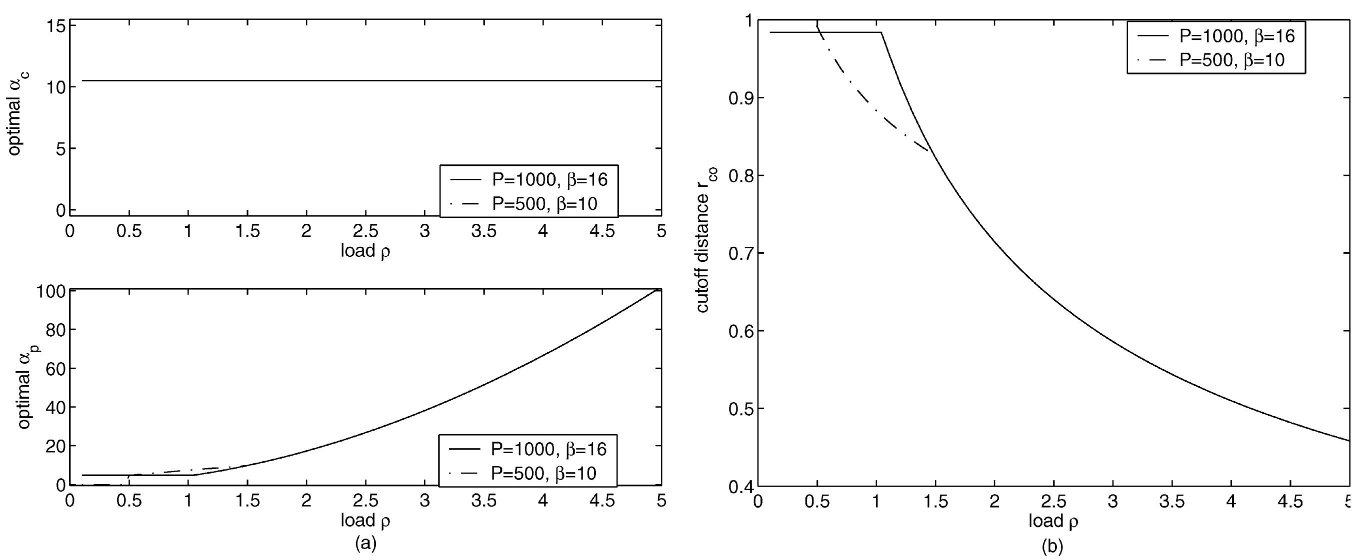

Fig. 5. Utility and revenue maximization with delta distribution. (a) Optimal prices versus load. (b) Cutoff distance versus load.

system is IL, i.e., $r_{\rm co} < 1$, as it was in the uniform and Gaussian utility cases [Fig. 3(a)]. Note, however, that it is no longer necessary that optimal prices be equal to the associated shadow costs. As the offered load rises above one, the supply of codes are exhausted. The optimal prices rise, and the cutoff distance $r_{\rm co}$ shrinks.

In the second scenario ($\mathcal{P}/\sigma^2 = 37$ dB, $\beta = 10$), the system progresses from IL, through IL + PL and IL + PL + CL, to IL + CL. This pattern is the same as in the uniform and Gaussian cases [Fig. 3(b)]. At loads less than 0.5, all users are active, since the transfer price β is less than the utility u_0. At loads between 0.5 and 1.5, the power price is higher than in the first scenario, resulting in a lower cutoff distance. Again note that optimal prices need not follow the shadow costs.

VII. Large System Revenue Maximization

We now return to the revenue maximization problem initially posed as Problem FR1. The set of active users is given by $Q = \{(p,u) : u \geq \alpha_c + \alpha_p p\}$, and the large system net revenue per code is

$$R_{\rm ave} = \rho \int\int_Q (\alpha_c + (\alpha_p - \beta)p) f_U f_P du dp. \qquad (10)$$

The corresponding optimization problem is:
Problem LR1:

$$\max_{(\alpha_c, \alpha_p)} R_{\rm ave}$$

such that inequalities (6) and (7) are satisfied.

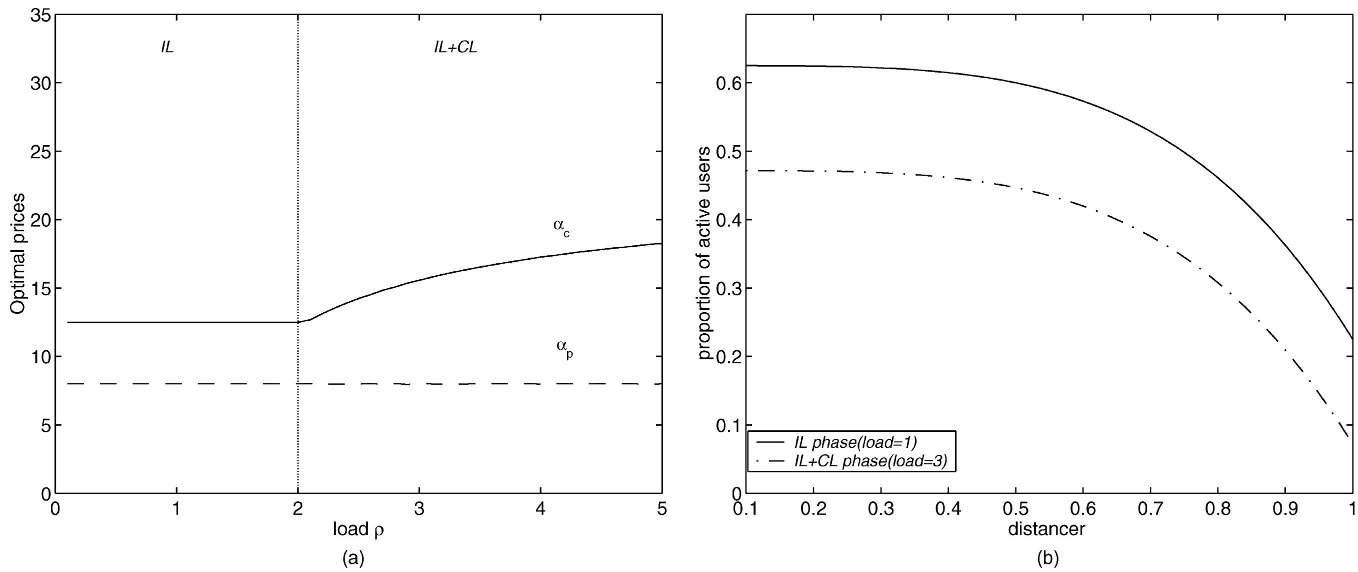

Fig. 6. Revenue maximization with uniform distribution at $\mathcal{P}/\sigma^2 = 40$ dB, $\beta = 16$. (a) Optimal prices versus load. (b) Proportion of users at distance r who are active.

To find the optimal α_c, we set $\partial R_{\rm ave}/\partial \alpha_c = 0$, which gives

$$\int_{\alpha_c}^{\infty} f_U(u)\left(\int_0^{(u-\alpha_c)/\alpha_p} f_P(p)\,dp\right)du = \int_0^{\infty}[\alpha_c + (\alpha_p - \beta)p] f_P(p) f_U(\alpha_c + \alpha_p p)\,dp. \quad (11)$$

The right side of (11) represents the gain in revenue due to a marginal increase in α_c, and the left side of (11) represents the corresponding loss in revenue due to users becoming inactive after the price increase.

Similarly, setting $\partial R_{\rm ave}/\partial \alpha_p = 0$ gives

$$\int_{\alpha_c}^{\infty} f_U(u)\left(\int_0^{(u-\alpha_c)/\alpha_p} p f_P(p)\,dp\right)du = \int_0^{\infty}[\alpha_c + (\alpha_p - \beta)p] p f_P(p) f_U(\alpha_c + \alpha_p p)\,dp. \quad (12)$$

These terms have the analogous interpretations as those in (11).

Unlike utility maximization, when maximizing revenue, there may be both excess power and codes in the cell even when there are inactive users who could generate positive net revenue. Another difference is that for revenue maximization, there are no explicit relations between the optimal prices and shadow costs associated with the code and power constraints. The optimal prices can be computed for some specific utility and power distributions by using the following alternative revenue expression:

$$R_{\rm ave} = \int_0^1 [\alpha_c + (\alpha_p - \beta)p(r)]\, f_r(r) \overline{F}_U(\alpha_c + \alpha_p p(r))\,dr \quad (13)$$

where $\overline{F}_U(u) = \int_u^\infty f_U(u')\,du'$ is the complementary distribution function for the step utility and $p(r)$ is the power required by user at distance r to just achieve γ^*.

VIII. Numerical Study of Revenue Maximization

In this section, we present numerical results to show how the system phases (DL, IL, PL, and CL) and optimal prices vary with the offered load, power limit, and the transfer price. In the following subsections, we consider utility distributions given by uniform and delta densities.

A. Uniform Utility Distribution

As in the utility maximization problem, we assume the height of the step in the user's utility function is uniformly distributed between $u_1 = 5$ and $u_2 = 25$. We start with the same first scenario as in previous sections, in which $\mathcal{P}/\sigma^2 = 40$ dB and $\beta = 16$. The optimal price per code α_c and price per unit transmitted power α_p are shown in Fig. 6(a).

At loads less than two, the system is only IL, meaning that there are excess power and codes in the cell, but that the transmitted power in this cell negatively affects the revenue in neighboring cells. In the utility-maximization case, the optimal prices for loads less than one were $\alpha_c = 0$ and $\alpha_p = \beta$. In the revenue-maximization case, it can be shown using (13) that the optimal prices in this region are $\alpha_c = u_2/2$ and $\alpha_p = \beta/2$ (when $u_2/2 > u_1$). When maximizing utility, if the cell is not CL or PL, the goal is to activate all users below some power threshold. This is accomplished by setting the power price to β, which matches the shadow cost on interference. When maximizing revenue, however, additional revenue can be generated from all active users by raising the code price and correspondingly lowering the power price. This price set selects users with higher utilities by decreasing the number of active users at small r and increasing the number of active users at large r.

At loads greater than two, the system is both IL and CL. The transition to CL happens at about twice the load as in the utility-maximization case. This is because the prices at lower loads admit only about 1/2 of the offered traffic. The number of active users is shown in Fig. 6(b).

We now turn to the second scenario, in which $\mathcal{P}/\sigma^2 = 37$ dB and $\beta = 10$. The optimal prices are shown in Fig. 7. As in the first scenario, at low loads, the optimal prices are $\alpha_c = u_2/2$ and $\alpha_p = \beta/2$. The transition to PL again occurs at approximately twice the load at which the analogous transition occurs in the utility-maximization case.

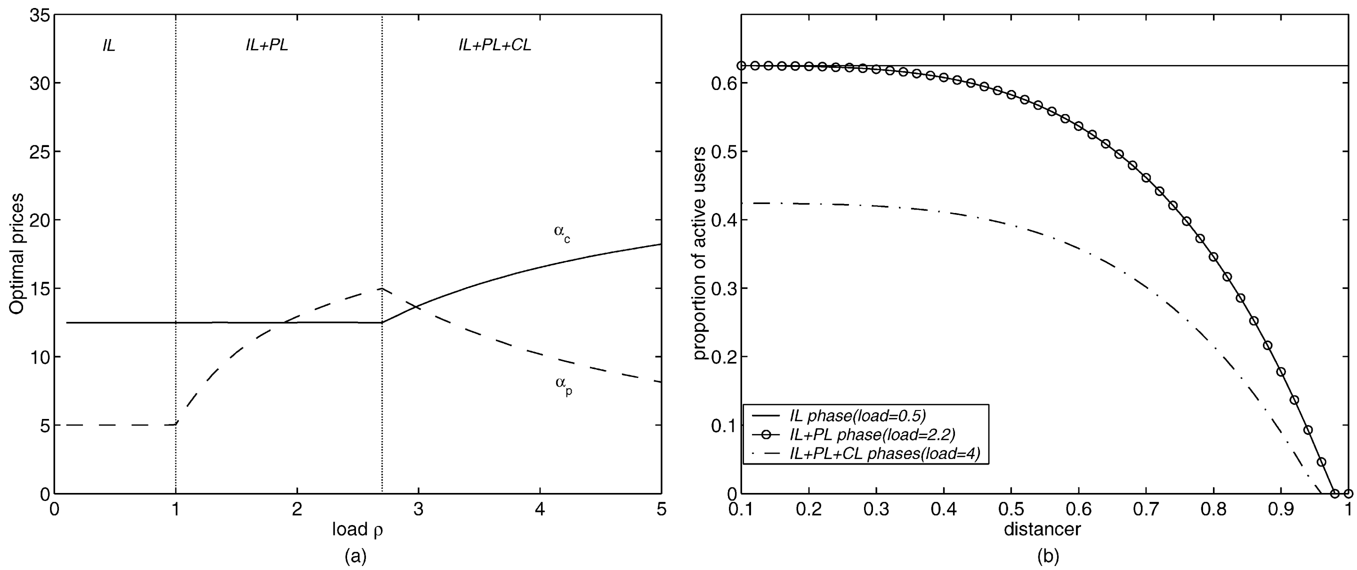

Fig. 7. Revenue maximization with uniform distribution at $\mathcal{P}/\sigma^2 = 37$ dB, $\beta = 10$. (a) Optimal prices versus load. (b) Proportion of users at distance r who are active.

B. Delta Utility Distribution

We now assume that all users have the same utility function, a step with a height of $u_0 = 15$. It can be proven that the optimal set of active users (cutoff distance) is exactly the same as that in the utility-maximizing case, while the optimal prices are now unique, shown in Fig. 5. More discussion can be found in [15].

IX. Conclusion

We have studied forward link power allocation for voice users in a single isolated cell. We proposed admission control policies that base a new call admission decision not only upon available capacity, but also upon the required forward link transmit power, and upon the user's willingness to pay. We have shown how pricing can achieve the optimal resource allocation in a distributed fashion. When the network objective is to maximize the total utility of all users, the optimal prices per code and per unit transmitted power can be set equal to the shadow costs corresponding to the resource constraints. When the network objective is to maximize total revenue, the optimal resource allocation may result in fewer active users; even if resources are available, the network may discourage users who have relatively low utilities.

Appendix
Proof of Theorem 5.5

We use the duality property in nonlinear programming [14]. Problem LU1 is the *primal problem*. The Lagrangian is

$$L(p,\alpha_c,\alpha_p') = \rho \iint_Q (u-\beta p) f_U f_P du dp + \alpha_p'\left(\mathcal{P} - \rho\iint_Q p f_U f_P du dp\right) + \alpha_c\left(1 - \rho\iint_Q f_U f_P du dp\right). \tag{14}$$

Let $q(\alpha_c,\alpha_p') = \sup_{\{p\ge 0\}} L(p,\alpha_c,\alpha_p')$. It is easy to show that $q(\alpha_c,\alpha_p')$ is achieved when each user chooses p to maximize surplus, which is given by $\left(u - \alpha_c \mathbf{1}_p - (\alpha_p' + \beta)p\right)$, where $\mathbf{1}_p$ is an indicator function for $p > 0$. For the step utility function [shown in Fig. 1(a)], the surplus is maximized either at ($u = 0$, $p = 0$), corresponding to an inactive user, or at ($u > 0$, $p^* > 0$), where p^* is the transmit power required to obtain the SINR target γ^*. The active user set is, therefore, $Q = \{(u,p^*) : u - \alpha_c \mathbf{1}_p - (\alpha_p' + \beta)p^* > 0\}$.

The *dual problem* is

$$\min_{\{\alpha_c,\alpha_p'\}} q(\alpha_c,\alpha_p'), \qquad \text{such that } \alpha_c, \alpha_p \ge 0.$$

Since $\alpha_c \ge 0$ and $\alpha_p' \ge 0$, to solve the dual problem, we must have

$$\alpha_p'\left(\mathcal{P} - \rho\iint_Q p f_U f_P du dp\right) = 0$$

and

$$\alpha_c\left(1 - \rho\iint_Q f_U f_P du dp\right) = 0. \tag{15}$$

These conditions state that the code or power constraint is not binding if and only if the corresponding price is zero. Let $U^* = \sup_{\Pi} U_{\text{ave}}$, and $q^* = \inf_{\{\alpha_c\ge 0,\alpha_p'\ge 0\}} q(\alpha_c,\alpha_p')$. We observe that when the users maximize their surplus, the average power per code ($\rho\iint_Q p f_U f_P du dp$) and fraction of active codes ($\rho\iint_Q f_U f_P du dp$) are continuous decreasing functions of α_p' and α_c, respectively. Furthermore, by varying these shadow costs, $\rho\iint_Q p f_U f_P du dp$ can be set to any value between zero and $\mathcal{P}$, and $\rho\iint_Q f_U f_P du dp$ can be set to any value between zero and one. This establishes the complementary slackness conditions (15). In other words, $U^* = q^*$, and there is no duality gap. By setting $\alpha_p = \alpha_p' + \beta$, the theorem follows from [14, Proposition 5.1.5]. Q.E.D.

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