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Title
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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 35(35)

ISSN
1069-7977

Author
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Publication Date
2013

Peer reviewed

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A Dynamic Dual-Process Model of Decision-making Under Uncertainty

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Abstract

Current dynamic models of decision-making assume that a unitary system is responsible for forming preferences. However, extensive research has shown that decision-making and behavior result from the interaction of two separate systems of reasoning - one that is fast, automatic, and experiential and one that is slow, deliberative and rational. This paper develops the first dynamic dual-process model of decision-making that can account for choice, response times, and prices. The model is applied to several phenomena from the risky decision-making literature including enhancements in preference by small losses, preference reversals due to response mode, and the influence of price and affect on preference.

Keywords: Decision-making, dual-process theory, preference reversal, dynamic models

Introduction

Existing dynamic models of decision-making such as Decision Field Theory (Busemeyer & Townsend, 1993) assume there is a single system of thought that produces preferences. However, there is substantial research supporting the idea that preferences are formed from a dual process of reasoning. This paper introduces a dynamic dual-process model of risky decision-making. The model generalizes a previous static two systems model developed by Mukherjee (2010) to account for choice, response times, and prices.

Dual-process theory postulates that there are two fundamentally different systems that can process information. One system is described as automatic, intuitive, fast, and experiential. The other is labeled as deliberative, analytical, slow, and rational. The former system is typically referred to as System 1 and the latter System 2 (Stanovich & West, 2000). (In this paper, System 1 is labeled the affective system and System 2 the deliberative system following the terminology of Mukherjee (2010).) Research has shown that dual-process accounts are often more successful at explaining behavioral phenomena than unitary approaches (Hogarth, 2001; Kahneman, 2003; Sanfey, Loewenstein, McClure, Cohen, et al., 2006). There is also evidence from the neuroscience community for two separable systems in the brain that contribute to decision-making (Damasio, 1994; Sanfey et al., 2006).

The paper begins by describing the Dynamic Dual-Process (DDP) model for choice and response times which is later extended to also account for pricing elicitation methods. The model is used to make new predictions about the relationship between response time and the involvement of the affective system. It is shown that in some situations greater involvement of the affective system leads to faster decisions as expected. However, the model also predicts that in other situations the two systems can compete with one another resulting in longer response times. DDP is also applied to three phenomena from the risky decision-making literature: the enhancement effect which occurs when a small loss is added to a positive gamble, preference reversals due to response mode, and the influence of price and affect on preference.

A Static Two Systems Model

The DDP model generalizes the Dual System Model (DSM) by Mukherjee (2010). DSM is a utility model of risky decision-making where the overall utility for a gamble is composed of two components: the utility of the gamble with respect to the deliberative system and the utility of the gamble with respect to the affective system. Mathematically, the overall utility can be written as $V(G) = V_A(G) \oplus V_D(G)$ where $V_A(G)$ is the evaluation due to the affective system and $V_D(G)$ is the evaluation due to the deliberative system.

Based on experimental results by Hsee and Rottenstreich (2004), Mukherjee made two assumptions about the evaluation of outcomes. The first assumption was that the deliberative system evaluates outcomes linearly so that $V_D(x) = kx$ where x is an outcome and k is a free parameter. The second assumption was that the affective system evaluates outcomes with respect to a concave value function in the gain domain and a convex value function in the loss domain similar to the value function in prospect theory (Kahneman & Tversky, 1979). Mukherjee postulates that for positive outcomes this value function can be approximated by $V_A(x) = x^m$ for $m < 1$.

Mukherjee made two additional assumptions about the perception of probabilities based on experimental work by Rottenstreich and Hsee (2001). He assumed that the deliberative system perceives probabilities directly without distortion so that the probability weighting function is $w(p) = p$ as in Expected Utility theory. He also assumed that the affective system is insensitive to probabilities and only recognizes whether or not an outcome is possible. Thus, each possible outcome receives equal weight so that for n outcomes the probability of any single outcome is $1/n$.

Let $(p_1, x_1; ...; p_n, x_n)$ be the gamble G with n positive outcomes x_i. Using the four assumptions above, Mukherjee defined the utility for the deliberative system as $V_D(G) = \sum_i^n p_i(kx_i)$ and the utility for the affective system as $V_A(G) = 1/n \sum_i^n x_i^m$. The overall utility $V(G)$ is simply the convex combination of the two utilities for the two different systems:

$$\begin{aligned} V(G) &= \gamma V_A(G) + (1-\gamma) V_D(G) \\ &= \gamma \frac{1}{n} \sum_i^n x_i^m + (1-\gamma) k \sum_i^n p_i(x_i) \end{aligned} \quad (1)$$

where γ is the weight given to the affective system. DSM can account for a wide range of choice phenomena including

violations of nontransparent stochastic dominance, ambiguity aversion, common consequence effect, and the common ratio effect (to name a few).

The Dynamic Dual-Process Model

While DSM has been very successful in accounting for a variety of phenomena, it does not describe the dynamic process underlying decisions. Like most models of judgment and decision-making, DSM is a descriptive model concerned with theorizing at the highest level. Because DSM is a static utility model, it cannot make predictions about response times. On the other hand, DDP is a processing model aimed at explaining the mechanisms that produce behavior. In this way, it is similar to other dynamic models of decision-making such as Decision Field Theory (Busemeyer & Townsend, 1993). DDP is the first two systems model that can account for both choice and response times.

Another drawback to DSM is that it has not been applied to experiments using pricing elicitation methods such as asking how much money one is willing to pay (WTP) for an option or how much money one is willing to accept (WTA) to sell an option. Many of the experiments in affective decision-making use these pricing procedures including the majority of the experiments by Rottenstreich and Hsee (2001) and Hsee and Rottenstreich (2004) that served as motivation for DSM. The DDP model can easily be extended to account for pricing elicitation methods as described in a later section.

The DDP model is formulated with respect to the typical risky decision task of choosing between two gambles. Let $G_1 = (p_1, x_1; ...; p_n, x_n)$ and $G_2 = (q_1, y_1; ...; q_m, y_m)$ be gambles with outcomes x_i and y_i and probabilities p_i and q_i respectively. As a decision-maker considers the two gambles, his or her preference evolves across time. Let $P(t)$ be the individual's preference at time t where positive preference states represent momentary preference for gamble G_1 and negative preference states represent momentary preference for gamble G_2. A new preference state $P(t+1)$ is formed at each moment in time from the previous preference state according the linear stochastic difference equation:

$$P(t+1) = P(t) + d + \varepsilon(t) \tag{2}$$

where $\varepsilon(t)$ is the stochastic error term and d is the difference in the evaluations of the gambles. The evaluation of each gamble is determined by evaluations from the affective and deliberative systems as in DSM. The difference d is given by

$$d = V(G_1) - V(G_2) \tag{3}$$

where $V(G_1)$ and $V(G_2)$ are calculated as in equation 1.

The preference state starts at an initial state $P(0) = z$ reflecting an initial bias for one gamble over the other. Specifically, if $z > 0$, then there is an initial bias for G_1 and if $z < 0$, then there is an initial bias for G_2. The preference state evolves until it reaches a threshold. There are two thresholds for the model, a positive threshold θ associated with G_1 and a negative threshold $-\theta$ associated with G_2. When the preference state reaches the positive threshold, G_1 is selected. When it reaches the negative threshold, G_2 is selected.

In total, DDP has six parameters. Three parameters, k, m, and γ, are used in the evaluation of the gambles given in equation 1. Parameters θ and z define the threshold and initial bias respectively. There is an additional variance parameter s used to define the amount of noise in the accumulation process.

DDP is a Wiener diffusion process (the continuous-time, continuous-state version of the random walk). Link and Heath (1975) derived equations for choice probabilities and the conditional mean response time for the Wiener process. Thus, DDP is computationally tractable and easy to apply.

Response Time and Affect

The affective system is typically characterized as automatic and fast as compared to the deliberative system which is analytic and slow. As such, when the affective system plays a larger role in the decision-making process, decisions should be quick. In terms of the DDP model, this implies that as γ increases, response times should decrease.

To test the relationship between response time and the involvement of the affective system, two gambles of equal expected value were analyzed: $G_1 = (4/10, \$9; 6/10, \$0)$ and $G_2 = (9/10, \$4; 1/10, \$0)$. Choice and response times from DDP were examined for all possible values of γ. The m parameter used to specify the curvature of the affective valuation function was also allowed to vary from $0.1 \leq m \leq 0.9$. The scaling parameter k in the deliberative value function was fixed to $k = 1$ so that $V_D(x) = x$ without distortion. The initial bias was fixed to $z = 0$ reflecting no bias towards one gamble over the other. The threshold parameter was fixed to $\theta = 10$, and the variance parameter was fixed to $s = 1$ as is common in response time modeling.

The top left panel of Figure 1 shows the choice probability for gamble G_1 over G_2 for different values of γ and m. As γ increases, the probability of selecting G_1 increases. This reflects the assumption that the affective system ignores probabilities when evaluating gambles. As the affective system becomes more involved, the gamble with the highest payoff is viewed more favorably. As m decreases and the value function for the affective system becomes more concave, the two gambles are viewed as indifferent. Thus, favorability for G_1 by the affective system is moderated by increasing risk aversion (i.e., decreasing m). The top right panel of Figure 1 shows the mean response time conditional on selecting G_1. As predicted, increases in γ lead to faster response times. In general, for gambles of equal expected value, as the affective system becomes more involved, decisions become quick and high payoff options are preferred.

When gambles have unequal expected value, it is not necessarily the case that response times decrease with increased involvement of the affective system. The bottom panels of Figure 1 show choice probabilities and response times for the gamble $G_1^* = (3/10, \$9; 7/10, \$0)$ as compared to G_2. In this situation, G_2 has a greater expected value than G_1^*. When the

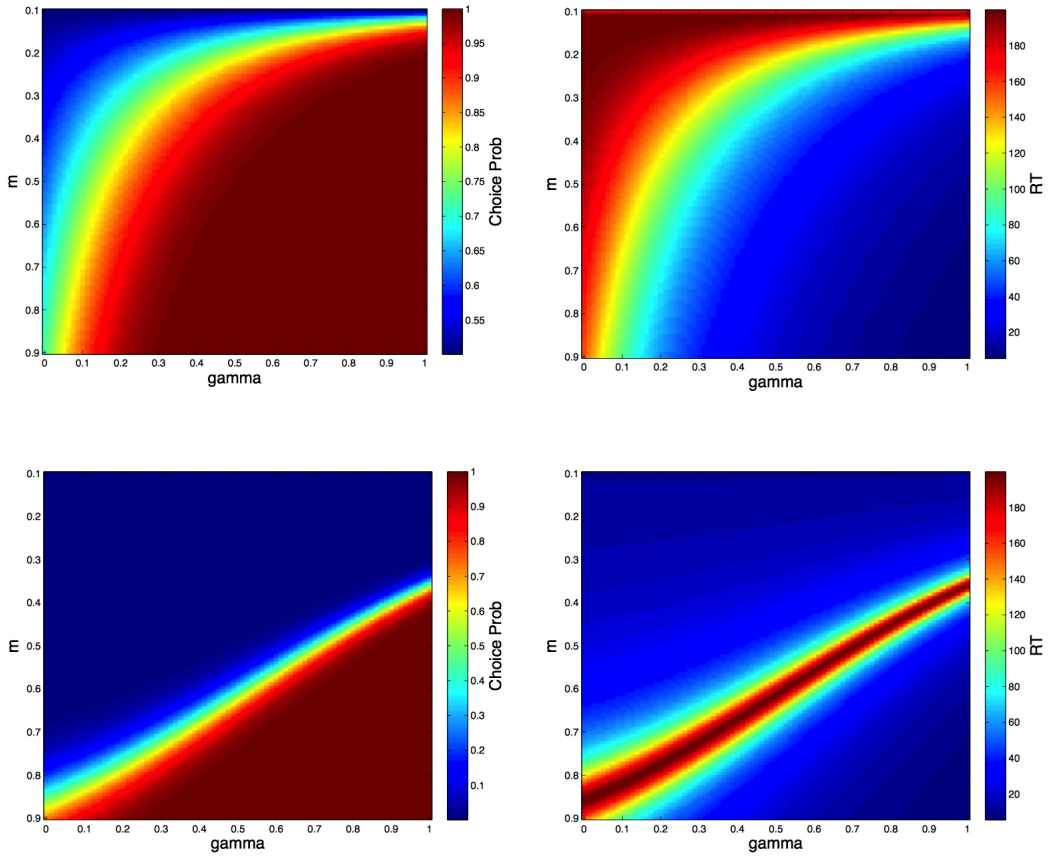


Figure 1: Top panel: choice probability (left) and conditional mean response time (right) for gambles with equal expected value for various values of γ and m. Bottom panels: choice probability (left) and conditional mean response time (right) for gambles with unequal expected value. Red values indicate larger choice probabilities and longer response times.

deliberative system is more involved as indicated by small values of γ, G_2 is preferred. However, as γ increases and the affective value function becomes less concave (corresponding to an increase in m), G_1^* is preferred. Response times are fast for small values of γ paired with small values m and for large values of γ paired with large values of m. In other words, quick decisions can be made for G_2 by the deliberative system and for G_1^* by the affective system, but response times increase when there is conflict between the two systems. The influence of system conflict on response time is a new prediction by DDP which could be tested in future experiments.

Enhancement by Small Loss

Slovic, Finucane, Peters, and MacGregor (2002) found that adding a small loss to a positive gamble can increase its attractiveness rating and choice probability. When asked to rate the attractiveness of gamble $G_1 = (7/36, \$9; 29/36, \$0)$ and $G_2 = (7/36, \$9; 29/36, -\$0.05)$, participants rated gamble G_1 with no loss lower (mean = 9.4 on a 0-20 scale) than gamble G_2 (mean = 14.9). The gambles were then each paired with a sure gain of \$2 (denoted by S). Half of the participants were asked to choose between G_1 and S and the other half were asked to choose between G_2 and S. Only 33.3% chose G_1 over the gain whereas 60.8% chose G_2 over the gain. Slovic et al. (2002) explained these findings by the affect heuristic. The inclusion of a small loss enhances the perceived benefit of \$9 producing a positive affective feeling for G_1 leading to higher attractiveness ratings and choice probabilities.

The enhancement in choice probability by the inclusion of a small loss can also be explained by DDP. Based on the idea that G_2 produces a more affective response than G_1, it is assumed that the affective system is more activated by G_2 than G_1. Mathematically, this implies that $\gamma_2 > \gamma_1$ where γ_1 is associated with the choice between G_1 and S and γ_2 is associated with the choice between G_2 and S.

To test the enhancement effect, the difference in probabilities $Pr(G_2|\{G_2,S\}) - Pr(G_1|\{G_1,S\})$ was examined for different values of γ_1, γ_2 and m. For this analysis, γ_1 was allowed to vary from 0.1 to 0.5 and γ_2 was defined in terms of γ_1 by

the equation $\gamma_2 = \gamma_1 + \alpha$ where α varied from 0.1 to 0.3 in increments of 0.05. As in the previous demonstration, $k = 1$, $\theta = 10$, $z = 0$, and $s = 1$. Because the affective value function postulated by Mukherjee (2010) only applies to positive outcomes, the function was generalized to $V_A(x) = -|x|^m$ for negative outcomes. Figure 2 plots the difference in probabilities for different values of γ_1 given along the x-axis. The different curves in the figure are associated with different values of γ_2 The m parameter was fixed to 0.3 in the top panel and 0.5 in the bottom panel.

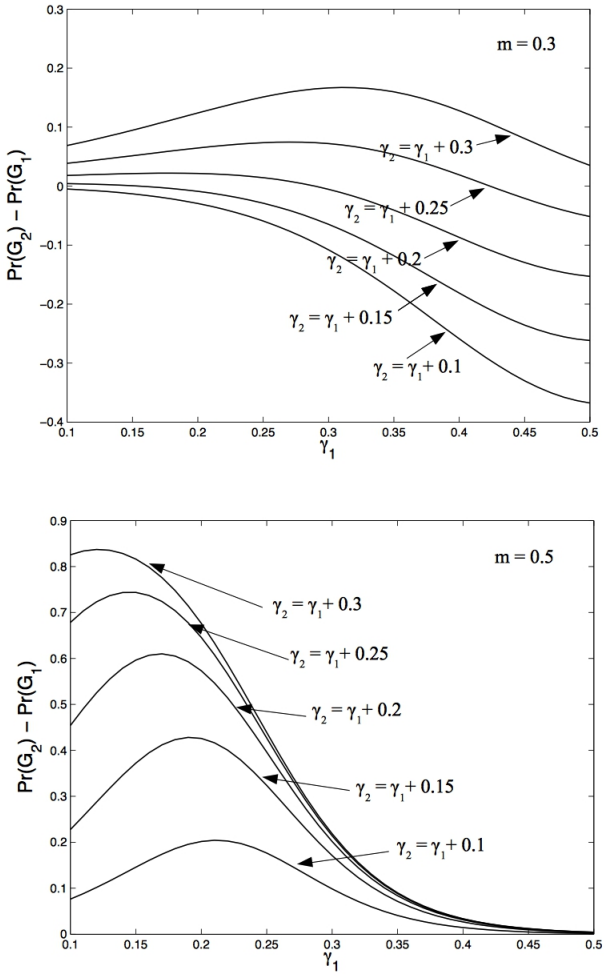


Figure 2: Enhancement effect shown as the difference in choice probabilities for G_2 and G_1 for different values of γ_1. The top panel shows the effect when $m = 0.3$ and the bottom panel shows the effect when $m = 0.5$. An enhancement effect occurs when the difference is greater than zero.

In the figure, the enhancement effect occurs when the difference in probabilities is greater than zero. When $m = 0.5$, this happens for all values of γ_2. When $m = 0.3$ implying greater risk aversion, the enhancement effect only occurs for large values of γ_2 (when $\alpha > 0.25$) suggesting greater involvement in the affective system is needed in order to produce the effect. In sum, the figure shows that the DDP model can easily account for the enhancement in choice probability by small losses. Further, the model makes new predictions about the magnitude of enhancement with respect to risk aversion and affect.

Extending DDP for Pricing Elicitation Methods

In many decision tasks, participants are asked to report a value such as a price that they are willing to assign to a particular option. For example, participants might be asked how much they are willing to pay (WTP) to play a certain gamble. DDP can be extended to account for such elicitation methods. The approach taken is similar to the one developed by Busemeyer and Goldstein (1992) and Johnson and Busemeyer (2005). The basic idea is that when an individual is determining price equivalence, they search through a range of possible prices. When a particular price is being considered, the individual can decide that it is too low, too high, or equivalent to the gamble. In the case when the price is too low, the individual increases the price. When the price is too high, the individual decreases the prices. If the price is equivalent, then it is reported.

Mathematically, this search process can be formulated as a discrete Markov chain as illustrated in Figure 3. The states in the chain correspond to possible prices increasing from left to right. The range of possible states is determined by the problem. For example, if an individual is asked how much they would pay to play the gamble $(7/36, \$9; 29/36, \$0)$, the the range of possible prices would be \$0 to \$9. The search process is assumed to begin near the middle of the candidate prices. A step to the right in the chain corresponds to increasing the price. The probability p of stepping to the right is the choice probability from DDP of choosing the gamble over a sure gain of $\$x$ where $\$x$ is the candidate price. A step to the left in the chain corresponds to decreasing the price. The probability q of stepping to the left is the choice probability from DDP of choosing a sure gain of $\$x$ over the gamble. The probability of exiting the search process and reporting a price occurs with probability r whenever DDP enters a neutral state. The neutral state is the point of indifference between the gamble and a sure gain and corresponds to $P(t) = 0$ in DDP. Details about implementing Markov chain models can be found in a Diederich and Busemeyer (2003) and Johnson and Busemeyer (2005).

Response Mode Preference Reversals

A puzzling phenomenon in decision-making is the occurrence of preference reversals with changes in response mode (Lichtenstein & Slovic, 1971, 1973). For example, Slovic et al. (2002) asked subjects to rate the attractiveness of gambles $G_1 = (29/36, \$2; 7/36, \$0)$ and $G_2 = (7/36, \$9; 29/36, \$0)$ on a 0=20 scale. On average, participants rated G_1 (mean 13.2) as more attractive than G_2 (mean 7.5). Yet, when asked how much they would be willing to pay to play the gambles,

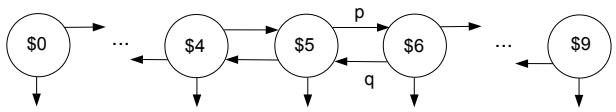


Figure 3: Markov chain model for pricing elicitation methods. Transition probabilities p and q are determined by the DDP model.

participants were willing to pay more to play G_2 (mean \$2.11) than G_1 (mean \$1.25).

The DDP model explains this preference reversal by assuming that a high attractiveness rating is associated with a strong affective response. Because G_1 had a higher attractiveness rating than G_2, it is hypothesized that the affective system is more involved with decisions about G_1 than G_2. Mathematically, this implies that the γ parameter for G_1 should be greater than the γ parameter for G_2. To test this hypothesis, a grid search was performed over the γ and m parameters to find the ranges of these parameters that produce price equivalences similar to those in the experiment. Specifically, parameter pairs that produced prices within \$0.20 of the mean prices from the experiment were examined. For this analysis, $k = 1$, $z = 0$, and $s = 1$ as before. Matrix methods (Diederich & Busemeyer, 2003) were used to determine the transition probabilities from DDP rather than using analytical solutions. This was done to accommodate the inclusion of the exit probability r. Because the matrix methods only provide an approximation to the choice probabilities, the threshold was fixed to $\theta = 50$ to improve the estimates. For gamble G_1, the states of the Markov chain ranged from \$0 to \$2 in increments of \$0.10. Similarly, for gamble G_2, the states of the Markov chain ranged from \$0 to \$9 in increments of \$0.10. The exit probability was set to $r = 0.01$.

Figure 4 plots the γ and m parameter pairs that produce prices in the given ranges. The blue region shows the parameters that yield prices between \$1.05 and \$1.45 for G_1 and the red region shows the parameters that yield prices between \$1.91 and \$2.31 for G_2. From the figure, it is clear that the γ parameter for G_1 must be greater than the γ parameter for G_2 to produce prices in these ranges. Thus, the DDP model can explain preference reversals by greater involvement of the affective system for more attractive gambles.

Influence of Probability and Affect on Price

Rottenstreich and Hsee (2001) found that the amount of money participants were willing to pay to play a gamble depends on both the probability of winning and whether the outcome is affect-rich or affect-poor. In their experiment, participants were asked how much they were willing to pay to play a gamble offering a \$500 coupon for a European vacation or a \$500 coupon for tuition at their university. The European vacation coupon was designed to be affect-rich whereas tuition coupon was designed to affect-poor. Rottenstreich and Hsee (2001) also manipulated the probability of winning the coupons. Some participants were told they had a 1% chance of winning and others were told that they had a 99% chance of winning. They found that even though the coupons were worth the same redemption value, the median price for the European coupon was \$20 as compared to \$5 for the tuition coupon when there was only a 1% chance of winning. However, when there was a 99% chance or winning, participants were willing to pay more for the tuition coupon (median price \$478) than for the European coupon (median price \$450).

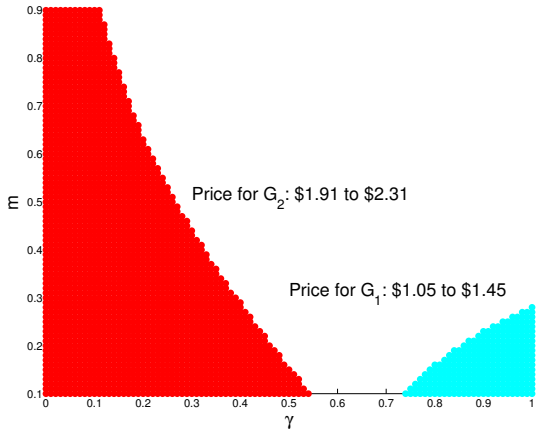


Figure 4: Results of a grid search over γ and m for WTP prices. The blue region shows the parameters that produce prices for G_1 within \$0.20 of the mean price of \$1.25. The red region shows the parameters that produce prices for G_2 within \$0.20 of the mean price of \$2.11. As predicted, γ for G_1 is greater than γ for G_2.

Rottenstreich and Hsee (2001) explained this preference reversal by greater involvement of the affective system in the European coupon gamble than the tuition coupon gamble. For the low probability gamble, the affective system overweights the probability of winning and thus the affect-rich European coupon is valued more than then affect-poor tuition coupon. However, for the high probability gamble, the affective system underweights the probability of winning and the European coupon is valued less than the tuition coupon.

Following the intuition of Rottenstreich and Hsee (2001), the DDP model explains this preference reversal by using a larger γ parameter for the affect-rich European coupon as compared to the affect-poor tuition coupon. For this analysis, the γ parameter for the European coupon gamble was set to $\gamma = 1$ implying complete involvement of the affective system and no involvement of the deliberative system. For the tuition coupon gamble, $\gamma = 0$ implying complete involvement of the deliberative system and no involvement of the affective system. These parameter settings reflect the extreme case when only one system is involved in the decision-making process. In reality, it is more likely that both systems are involved in

both gambles with $0 < \gamma_T < \gamma_E < 1$ where γ_T is associated with the tuition coupon and γ_E is associated with the European coupon. For ease of demonstration, the extreme γ values were used, but the results also hold for more intermediate values of γ.

As before, $k = 1$, $z = 0$, $s = 1$, $\theta = 50$, and $r = 0.01$. For the European coupon, the m parameter was allowed to range from 0.1 to 0.9. For the tuition coupon, m has no impact because $\gamma = 0$. Table 1 shows the WTP prices from DDP for the two different coupons and two different chances of winning. In the table, a range of prices is given for the European coupon showing the maximum and minimum prices as m is varied. Because m does not play a role in the tuition coupon, a single price is shown. From the table, the DDP model produces the same pattern of results as Rottenstreich and Hsee's experiment. Namely, the price for the European coupon is greater than the price for the tuition coupon when there is a low probability of winning and the price for the tuition coupon is greater than the price for the European coupon when there is a high probability of winning.

Table 1: WTP prices from the DDP model for the European and tuition coupons for two different probabilities of winning.

Coupon	1% chance	99% chance
European ($\gamma = 1$)	\$32.59-\$39.38	\$303.48-\$491.94
Tuition ($\gamma = 0$)	\$5.14	\$492.97

Discussion

The DDP model synthesizes ideas from several lines of research in decision-making and cognitive modeling. DDP draws upon the static DSM model developed by Mukherjee (2010) to explain how dual systems of reasoning evaluate options. DDP formalizes the the formation of preferences as an accumulation of information over time similar to other dynamic models such as Decision Field Theory (Busemeyer & Townsend, 1993). DDP also employs a Markov chain model to account for pricing elicitation methods as in Busemeyer and Goldstein (1992) and Johnson and Busemeyer (2005). In sum, DDP provides a unified theory of how dual systems interact to produce choices, response times, and prices that is grounded in decades of research.

Future work will develop new experiments to rigorously test DDP and to investigate novel predictions from the model. In particular, DDP makes new predictions about the interaction between systems and response times. The affective system is typically conceived as being fast and automatic. Thus, when it is engaged in a task, responses should be quick. DDP suggests that the relationship between response time and the affective system is not this simple. It predicts that response times are influenced by many factors including conflict between the two systems, risk attitudes, and the options themselves.

References

Busemeyer, J. R., & Goldstein, W. M. (1992). Linking together different measures of preference: A dynamic model of matching derived from decision field theory. *Organizational Behavior and Human Decision Processes*, *52*(3), 370–396.

Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic cognition approach to decision making. *Psychological Review*, *100*, 432-459.

Damasio, A. (1994). *Descartes' error: Emotion, reason, and the human brain*. Putnam New York.

Diederich, A., & Busemeyer, J. R. (2003). Simple matrix methods for analyzing diffusion models of choice probability, choice response time, and simple response time. *Journal of Mathematical Psychology*, *47*(3), 304–322.

Hogarth, R. (2001). *Educating intuition*. University of Chicago Press.

Hsee, C., & Rottenstreich, Y. (2004). Music, pandas, and muggers: on the affective psychology of value. *Journal of Experimental Psychology: General*, *133*(1), 23.

Johnson, J., & Busemeyer, J. (2005). A dynamic, stochastic, computational model of preference reversal phenomena. *Psychological Review*, *112*(4), 841.

Kahneman, D. (2003). Maps of bounded rationality: Psychology for behavioral economics. *American economic review*, 1449–1475.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision making under risk. *Econometrica*, *XLVII*, 263-291.

Lichtenstein, S., & Slovic, P. (1971). Reversals of preference between bids and choices in gambling decisions. *Journal of experimental psychology*, *89*(1), 46.

Lichtenstein, S., & Slovic, P. (1973). Response-induced reversals of preference in gambling: An extended replication in las vegas. *Journal of Experimental Psychology*, *101*(1), 16.

Link, S., & Heath, R. (1975). A sequential theory of psychological discrimination. *Psychometrika*, *40*(1), 77–105.

Mukherjee, K. (2010). A dual system model of preferences under risk. *Psychological review*, *117*(1), 243.

Rottenstreich, Y., & Hsee, C. (2001). Money, kisses, and electric shocks: On the affective psychology of risk. *Psychological Science*, *12*(3), 185–190.

Sanfey, A., Loewenstein, G., McClure, S., Cohen, J., et al. (2006). Neuroeconomics: cross-currents in research on decision-making. *Trends in cognitive sciences*, *10*(3), 108–116.

Slovic, P., Finucane, M., Peters, E., & MacGregor, D. G. (2002). The affect heuristic. In T. Gilovich, D. Griffin, & D. Kahneman (Eds.), *Heuristics and biases: The psychology of intuitive judgment* (p. 397-420). Cambridge University Press.

Stanovich, K., & West, R. (2000). Individual differences in reasoning: Implications for the rationality debate? *Behavioral and brain sciences*, *23*(5), 645–665.