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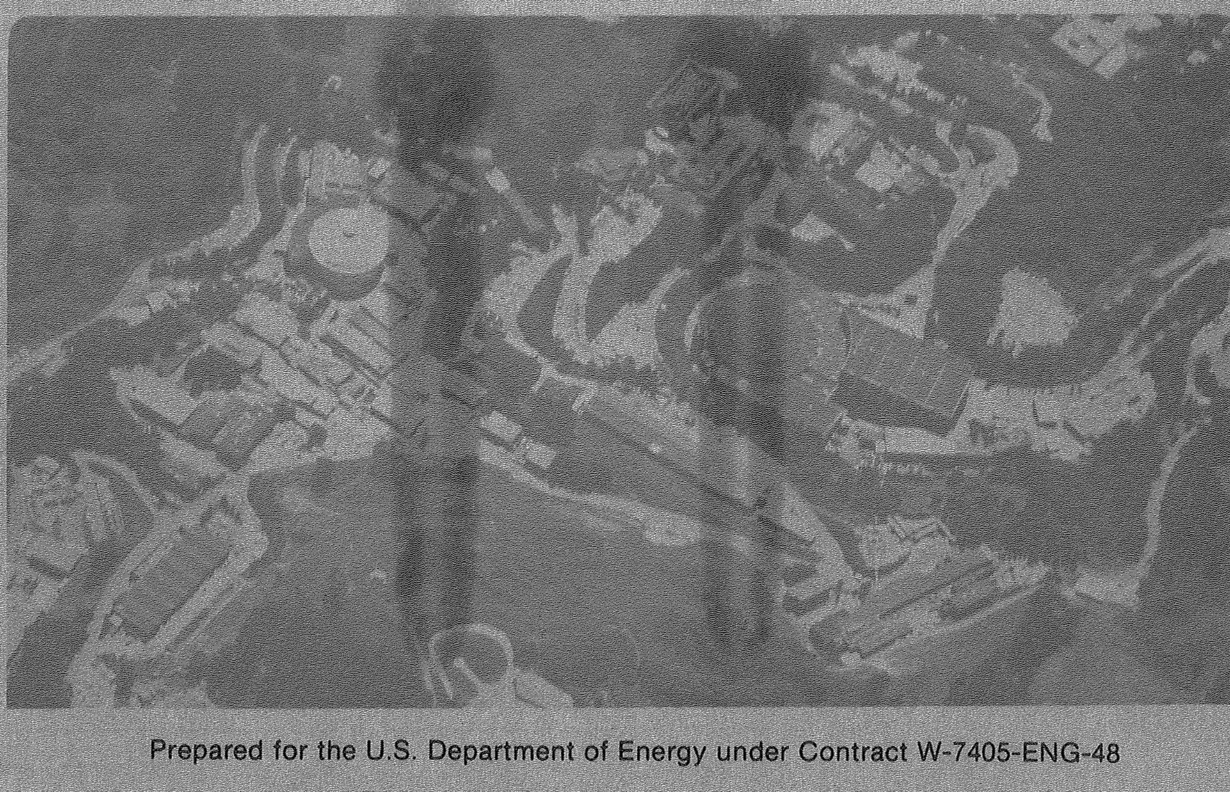

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ON THE WIDTH $\chi(3.55) \to \psi 3\pi$

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ABSTRACT

In view of a recent experiment, the width $\chi(3.55) \to \psi 3\pi$ is investigated. If the data are confirmed there is a serious difficulty with the conventional understanding of hadronic transitions between charmonium states.

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In this paper we present an estimate of the width $\chi(3.55) \to \psi\pi^+\pi^-\pi^o$. A recent experiment [1] using a pion beam at FNAL reported evidence for this decay mode with a substantial branching ratio, $BR\left(\frac{\chi \to \psi 3\pi}{\chi \to \text{all}}\right) \sim 45\% \pm 15\%$. This seems surprisingly large in view of the limited phase space, a Q value of 40 MeV, for the decay.

We will use multipole techniques [2] to estimate the width. These techniques involve not an expansion in powers of α_s but rather in powers of (ka_o) and $(\frac{k}{\Delta\varepsilon})$, where k is the energy of a gluon emitted in the decay, a_o is the size of the ψ state, and $\Delta\varepsilon$ is the binding energy. These methods have been applied with some success to the decay $\psi' \to \psi X$ [3], but it should be emphasized that the expansion parameters are not very small for this decay $((ka_o) \sim .25$ and $(\frac{k}{\Delta\varepsilon}) \sim .3)$. In view of this and other uncertainties it seems unlikely that one should expect to get a result accurate to more than a factor of 10 or so. We shall see that this is sufficient for us to make a strong statement about $\chi \to \psi 3\pi$.

To illustrate the method and to ensure that our approximations are reasonable, we will first discuss $\psi' \to \psi$ + hadrons. The hadronic system will contain at least two gluons; in the multipole expansion the transition is of order $(E1)^2$ and the gauge invariant matrix element is

$$\sum_X \frac{g^2}{6} < \psi g_1 | \underset{\sim}{E}^a \cdot r | \underset{\sim}{X} > \frac{1}{E_{\psi'} - E_X - k_2} < X g_2 | \underset{\sim}{E}^b \cdot \underset{\sim}{r} | \psi' >$$

$$+ (1 \leftrightarrow 2)$$

where $\underset{\sim}{E}^a$ is the color electric field, $|X>$ is a quark-antiquark color octet state of energy E_X, and k_2 is the energy of the emitted gluon g_2. The evaluation of this matrix element requires a knowledge of the singlet-octet energy difference $E_X - E_\psi$, and of the wavefunctions. For simplicity we will use Coulomb wave-functions for the ψ and ψ' states, and plane waves for the octet states, $|X>$. In addition, it is a very good approximation to neglect k_2 in the energy denominator (better than 5%). Summing over final states we have for the width $\psi' \to \psi$ + hadrons

$$\Gamma(\psi' \to \psi + \text{hadrons}) = 2.6 \times 10^{-3}\, \alpha_s^2 \frac{a_o^4}{(\Delta\varepsilon)^2} (m_{\psi'} - m_\psi)^7$$

where a_o is the Bohr radius, α_s is the strong coupling constant, and $\Delta\varepsilon$ is the singlet-octet energy difference. Making a fit to the ψ' and ψ with Coulomb wavefunctions we have $a_o = .81\ \text{GeV}^{-1}$. For $\Delta\varepsilon$ we use $m_8 - \frac{(m_\psi + m_{\psi'})}{2}$ where m_8 is the energy of the lowest octet state in the Coulomb approximation; $\Delta\varepsilon = .9$ GeV. Since α_s should be evaluated at a scale characteristic of the gluon momentum, we do not use the value obtained by fitting the mass spectrum ($\alpha_s = .9$). We chose to use $\alpha_s = .5$. With these approximations we have $\Gamma = 25$ KeV. We now invoke the concept of duality and assume that this gluonic final state materializes into hadrons. In this case, phase space and quantum number restrictions imply that the hadronic system is $\pi\pi$ or η. Experimentally $\Gamma(\psi' \to \psi\pi\pi) + \Gamma(\psi' \to \psi\eta) =$ 130 KeV[4]. In view of the approximations made the agreement is satisfactory. (The authors of Ref. 3 obtained a larger answer

primarily because they used $\Delta\varepsilon = 2m_Q - m_{\psi'}$. (Using this value increases our result by a factor of eight.)

Having established that our approximations are satisfactory we now turn to the decay $\chi \to \psi$ + hadrons. Here the transition is $(E1)^3$, so there are at least three gluons in the final state. The matrix element is

$$\sum_{X,X'} \frac{g^3}{12} d_{abc} < \chi g_1|\underset{\sim}{E}^a \cdot \underset{\sim}{r}|X > \frac{1}{(E_\chi + k_1 - E_X)} < X g_2|\underset{\sim}{E}^b \cdot r|X' >$$

$$\times \frac{1}{(E_\psi - E_{X'} - k_3)} < X' g_3|\underset{\sim}{E}^c \cdot \underset{\sim}{r}|\psi > .$$

Making the same approximation as before we now have

$$\Gamma(\chi \to \psi + \text{hadrons}) \simeq 1 \text{ ev}.$$

The final hadronic state can only consist of 3π, indeed since we are close to the 3π threshold we might expect that the width that we calculate would be suppressed further by phase space considerations. Let us now compare this to the experimental result. The total width of the $\chi(3.55)$ is estimated in potential models to be of order 1 MeV [5]. Using the measured branching ratio, this implies that $\Gamma(\chi \to \psi 3\pi)$ $\sim .1 - .5$ MeV. We see that there is a discrepancy of many orders of magnitude between this value and our calculated value.

We would conclude that in the context of the QCD multipole expansion it is impossible to understand the large width reported for $\chi(3.55) \to \psi 3\pi$. If this experimental result is confirmed it would cast grave doubts on the relevance of QCD multipole expansions for charmonium cascades.

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REFERENCES

1. S. Pordes, FNAL-Tufts-Penn-Illinois-Purdue collaboration reported at APS meeting, Washington D. C., April 1981.
2. K. Gottfried, Phys. Rev. Lett. 40 (1978) 598.
   T. M. Yan, Phys. Rev. D22 (1980) 1652.
   M. Peskin, Nucl. Phys. B156 (1979) 365.
   K. Shizuya, Phys. Rev. D23 1180 (1981).
3. A. Billoire et. al., Nucl. Phys. B155 (1979), 493.
4. G. Abrams et. al., Phys. Rev. Lett. 34 (1975) 1181.
5. A. Novikov et. al., Phys. Rep. 49C (1980) 1.