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ACCORDING TO THE STRIP APPROXIMATION

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I. INTRODUCTION

The strip approximation to strong interaction dynamics assumes a close connection to exist between small momentum transfer reactions $(\leq 2 \text{ GeV})$ at high energies $(\geq 10 \text{ GeV})$ and the bound states and resonances observed at low energies $(\leq 2 \text{ GeV})$.¹ In the most up to date formulation both phenomena are supposed to be controlled by a relatively small number of high ranking Regge trajectories.² An attempt is being made by the authors to solve bootstrap equations that will generate these trajectories and the associated residues. Although this program is still in an early stage, we wish here to report indications that the observed high-energy diffraction scattering is in fact dominated by the top ranking Regge pole.

II. THE EXPERIMENTAL SITUATION

What are the facts to be explained? At the simplest level they are the magnitudes of high energy total cross sections and the widths of the forward peaks in elastic scattering.³ Unitarity, as expressed for example through the optical model, constrains the total cross section to be $\leq 2\pi R^2$, where $R^2 \sim (\Delta t)^{-1}$, Δt being the width of the forward peak. (Precisely, we define $(\Delta t)^{-1}$ as the logarithmic derivative at t = 0 of dg/dt, if t is the negative square of momentum transfer.) It turns out that this limit is closely approached in all the systems experimentally studied. For nucleon-nucleon scattering near 20 GeV lab energy $(\Delta t)_{NN} \approx \frac{1}{10} \text{ GeV}^2$, with $\sigma_{NN} \approx 40$ mb, while for pion-nucleon scattering^{*} the total cross section is half as large and the peak slightly broader; $(\Delta t)_{\pi N} \approx \frac{1}{7} \text{ GeV}^2$.[†] There are of course differences between $\pi^+ p$ and $\pi^- p$ and between pp and $p\overline{p}$, but these differences appear to be diminishing as the energy increases and may be ignored in a first approximation. Similarly we may temporarily ignore the small and erratic variations observed for the peak widths.

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It is unfortunate that $\pi\pi$ scattering cannot directly be measured because the dynamical equations here are the simplest. Nevertheless if the Regge pole representation is tentatively accepted then the factorizability of the residues, as pointed out by Gell-Mann and by Gribov and Pomeranchuk, allows the inference of the following high energy $\pi\pi$ total cross section and forward peak width:

> $\sigma_{\pi\pi}^{\text{tot}} \approx 10 \text{ mb},$ $(\Delta t)_{\pi\pi} \approx \frac{1}{4} \text{ GeV}^2$

a combination which again is near the unitarity limit. The first task of the theory is to explain these two numbers, and it is in this connection that we have results to report.

In view of the qualitative success of the eightfold way it seems safe to assume that KN scattering will be understandable if success is achieved for the πN system.

T This number contains a correction by Ahmadzadeh and Sakmar (Phys. Rev. Letters <u>11</u>, 429 (1963)) and by W. Rarita (private communication, Berkeley (1964)) to remove the effect of secondary trajectories. The corresponding correction in the NN case appears to be negligible.

(2)

III. AN APPROXIMATE FORMULA FOR An RESIDUES

We proceed immediately to derive an approximate formula for $\pi\pi$ residues, appropriate to both the Pomeranchuk and ρ trajectories. The derivation employs: the strip concept but does not neglect inelastic scattering, even in the low energy resonance region.

Let $A_{\ell}^{I}(s)$ be the partial wave amplitude for elastic $\pi\pi$ scattering with isotopic spin I at energy squared s, normalized to

 $\left[\frac{s}{s-4m_{\pi}^{2}}\right]^{1/2} \exp(i \delta_{\ell}^{I}) \sin \delta_{\ell}^{I}, \text{ where } \delta_{\ell}^{I} \text{ is the (complex), phase}$ shift. Following Ref. 2 we write

$$\frac{A_{\ell}^{I}(s)}{q_{s}^{2\ell}} = \frac{N_{\ell}^{I}(s)}{D_{\ell}^{I}(s)} = B_{\ell}^{I}(s) + \frac{1}{\pi} \int_{s}^{s} \frac{ds'}{s'-s} \frac{Im A_{\ell}^{I}(s')}{q_{s}'^{2}},$$

where $D_{\ell}^{I}(s)$ is cut only across the strip between $s_{0} = 4m_{\pi}^{2}$ and s_{1} , $N_{\ell}^{I}(s)$ carrying all the remaining cuts. As explained in Ref. (2) the I(V) term B_{ℓ} plays somewhat the role of a "potential" and may be

calculated from the trajectories and residues of the leading Regge poles. A Regge pole occurs at a zero of $D_g^{I}(s)$, that is,

the residue in
$$l$$
 of $A_{\rho}^{\perp}(s)$ being given b

 $\overset{\mathbf{D}_{\alpha_{1}}}{(\mathbf{s})}$ = 0,

$$\beta_{1}(s) = (q_{s}^{2})^{\alpha_{1}(s)} \gamma_{1}(s) = (q_{s}^{2})^{\alpha_{1}(s)} \frac{\alpha_{1}(s)}{\left[\frac{\partial}{\partial s} D_{\ell}^{I}(s)\right]_{\ell=\alpha_{1}(s)}} \left(\frac{d\alpha_{1}(s)}{ds}\right).$$
(3)

From formula 1) it is possible to show that

(4)

$$N_{\alpha_{1}(s)}^{I}(s) = -\frac{1}{\pi} \int_{s_{0}}^{s_{1}} \frac{ds'}{s'-s} B_{\alpha_{1}(s)}^{I(v)}(s') \operatorname{Im} D_{\alpha_{1}(s)}^{I(s')},$$

while

$$\left[\frac{\partial}{\partial s} D_{\ell}^{\mathrm{I}}(s)\right]_{\ell=\alpha_{i}(s)} = \frac{1}{\pi} \int_{s_{0}}^{s_{1}} \frac{\mathrm{d}s'}{(s'-s)^{2}} \operatorname{Im} D_{\alpha_{i}(s)}^{\mathrm{I}}(s') .$$
(5)

Note that inelastic scattering inside the strip has not been neglected. Now it turns out in all approximations studied so far that for both I = 0 and I = 1, when ℓ is near 1, the functions $B_{\ell}^{I(V)}$ and $(-\text{Im } D_{\ell}^{I})$ are all positive across the strip. Furthermore the "potential" $B_{\ell}^{I(V)}(s)$ varies slowly. Thus we are led to the basic approximation

$$\frac{\gamma_{i}(s)}{\alpha_{i}'(s)} \approx (\overline{s}_{i} - s) B_{\alpha_{i}(s)}^{I(V)}(\overline{s}_{i}), \qquad s \ll \overline{s}_{i} , \qquad (6)$$

where $\overline{s_i}$ is some average energy inside the strip. Calculations with plausible choices for B_{ℓ} support formula (6) and suggest that $\overline{s_i} \ge 2^{i} \text{GeV}^2$, but a precise value for $\overline{s_i}$ will not be required immediately.

IV. THE TOTAL AA CROSS SECTION

Many applications of formula (6) are possible. Our first application does not require the form of $B_{\ell}^{I(V)}(s)$ but only the relation

$$B_{g}^{I=O(V)}(s) \approx 2 B_{g}^{I=1(V)}(s)$$
, $s \lesssim s_{1}$, (

a result that follows from the crossing matrix if the high energy "potential" is dominated by I = 1 exchange. A study of I = 0 exchange, to be published elsewhere, confirms the usual assumption that this

-4-

contribution to the force for $s \leq s_1$ is less important than that of I = 1. It is taken for granted that I = 2 exchange is negligible. Assumption (7) leads to the circumstance that for the Pomeranchuk trajectory and the ρ trajectory the values of $\overline{s_1}$ are roughly the same. Noting that $\alpha_p(0) = 1$ while $\alpha_p(m_p^2) = 1$, we may combine (6) and (7) to obtain

$$\frac{\gamma r_{p}(0)}{\alpha_{p}'(0)} \frac{\alpha_{p}'(m_{p}^{2})}{\gamma_{p}(m_{p}^{2})} \approx 2 \frac{\overline{s}}{\overline{s} - m_{p}^{2}} \approx 2$$
(8)

if $\bar{s} \gg m_{\rho}^2$. It is easy to verify that

$$\frac{\gamma_{\rho}(m_{\rho}^{2})}{\alpha_{\rho}'(m_{\rho}^{2})} \approx \frac{4 \Gamma_{\rho}}{m_{\rho}}, \qquad (1)$$

where Γ_{ρ} is the full width of the ρ , so we find

$$\gamma_{\mathbf{p}}(0) \approx 8 \frac{\Gamma}{m_{\mathbf{p}}} \alpha_{\mathbf{p}}'(0) .$$
(10)

The high energy $\pi\pi$ total cross section, if it is in fact controlled by the Pomeranchuk-Regge pole. is given by

$$\sigma_{\pi\pi} = 8\pi^2 \gamma_{\rm p}(0)$$
, (11)

so we predict finally that

$$\sigma_{\pi\pi} \approx 64 \pi^2 \frac{1}{m_{\rho}} \alpha_{\rho}'(0) . \qquad (12)$$

The slope of the Pomeranchuk trajectory, assuming that it passes through l = 2 at the mass of the f^{0} , has been estimated by (Ahmadzadeh and Sakmar as $\frac{1}{3}$ GeV⁻². Taking $\Gamma_{\rho} = 100$ MeV and $m_{\rho} = 750$ MeV, we

then find from formula (12):

(14)

V. THE WIDTH OF THE AA DIFFRACTION PEAK

To estimate the width of the diffraction peak, a more specific assumption must be made about the "potential" $B_{\ell}^{I=O(V)}$. In general,

high energy dominance of the Pomeranchuk-Regge pole leads to

$$\operatorname{Im} A_{\pi\pi}(s,t) \xrightarrow{\pi} (2\alpha_{p}(t) + 1)\gamma_{p}(t)(q_{t}^{2}) \overset{\alpha_{p}(t)}{P}_{\alpha_{p}}(t)\left(\frac{s}{2q_{t}^{2}}\right),$$
$$\approx \frac{\pi}{3} (2\alpha_{p}(t) + 1) \gamma_{p}(t) \left(\frac{s}{2}\right) \overset{\alpha_{p}(t)}{, , , , , , , , , , }$$
(13)

for $\alpha_p(t)$ near 1. Using formula (6), we then have

$$\operatorname{Im} A_{\pi\pi}(s,t) \propto (2\alpha_{p}(t)+1)\alpha_{p}'(t)(\overline{s}-t)B_{\alpha_{p}(t)}^{I=0(V)}(\overline{s})\left(\frac{s}{2}\right)^{\alpha_{p}'(t)}.$$

The form most commonly used for $B_{\ell}^{I=O(V)}(s)$ is that based on exchange of a (fixed spin) ρ : $B_{\ell}^{I=O(V)}(s) \propto (1 + \frac{s}{2q_{\rho}^2}) \qquad \frac{q_{\ell}(1 + \frac{m_{\rho}^2}{2q_{g}^2})}{(q_{g}^2)^{\ell+1}}$ (15)

Although we expect important deviations from this behavior when the potential is carefully calculated, the form (15) may serve to indicate the ℓ dependence of the potential, which is all we need for the shape of the diffraction peak. For $s \sim 2 \text{ GeV}^2$, most of the ℓ dependence of (15) resides in the factor $(q_s^2)^\ell$, and the weak remaining ℓ dependence in Q_ℓ near $\ell = 1$ is conveniently almost proportional to $(2\ell + 1)^{-1}$. Thus (14) becomes

Im
$$A_{\pi\pi}(s,t) \propto \alpha_{p}'(t)(\overline{s}-t)\left(\frac{s}{2q_{2}}\right)^{p}$$
, (16)

The real part of the amplitude is small near t = 0.

(17)

(18)

(19)

At this point, evidently, an estimate of \overline{s} is required as well as an estimate of the shape of the Pomeranchuk trajectory near t = 0. From preliminary calculations of trajectories and residues with a variety of "potentials" and strip widths, when the potential and strip width are adjusted to give $\alpha_p(0) = 1$, $\alpha_p' \approx \frac{1}{3} \text{ GeV}^2$, we find $\overline{s} \sim 2 \text{ GeV}^2$. Furthermore the trajectory for $t \ll \overline{s}$, is represented roughly by the form

$$\alpha_{p}(t) \approx c + \frac{1-c}{1-\frac{t}{\overline{s}}},$$
$$\alpha_{p}'(t) \approx \frac{\alpha_{p}'(0)}{(1-\frac{t}{\overline{s}})^{2}}.$$

Taking the logarithmic derivative of (16) at t = 0 we then calculate

$$\approx (\overline{s})^{-1} + \alpha_{p}'(0) \ln(\frac{2s}{s})$$

where $a_{\pi\pi} = \frac{1}{2} (\Delta t_{\pi\pi})^{-1}$.

αππ

BOX

As explained in section II above we expect $(\Delta t_{\pi\pi})^{-1}$ to be ~ 4 GeV⁻² at an s corresponding to 20 GeV lab energy for NN scattering. This is $s \sim 40 \text{ GeV}^2$, so taking $\overline{s} = 2 \text{ GeV}^2$ we have from (19)

not far from the expected 2 GeV-2

ing the masses and widths of the ρ and f mesons it will

correctly predict both the high energy $\pi\pi$ total cross section and the width of the diffraction peak.

Note that even to get the correct sign for $\sigma_{\pi\pi}$ and $a_{\pi\pi}$ from a dynamical calculation of the Regge parameters is a non-trivial achievement.

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