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TOTAL CROSS SECTIONS AND DIFFRACTION SCATTERING IN THE 10-100 GeV RANGE

ACCORDING TO THE STRIP APPROXIMATION[*]

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I. INTRODUCTION

The strip approximation to strong interaction dynamics assumes
a close connection to exist between small momentum transfer reactions
($\lesssim$ 2 GeV) at high energies ($\gtrsim$ 10 GeV) and the bound states and resonances
observed at low energies ($\lesssim$ 2 GeV).[1] In the most up to date formulation
both phenomena are supposed to be controlled by a relatively small
number of high ranking Regge trajectories.[2] An attempt is being made
by the authors to solve bootstrap equations that will generate these
trajectories and the associated residues. Although this program is
still in an early stage, we wish here to report indications that the
observed high-energy diffraction scattering is in fact dominated by
the top ranking Regge pole.

II. THE EXPERIMENTAL SITUATION

What are the facts to be explained? At the simplest level they
are the magnitudes of high energy total cross sections and the widths
of the forward peaks in elastic scattering.[3] Unitarity, as expressed
for example through the optical model, constrains the total cross
section to be $\lesssim 2\pi R^2$, where $R^2 \sim (\Delta t)^{-1}$, Δt being the width of
the forward peak. (Precisely, we define $(\Delta t)^{-1}$ as the logarithmic
derivative at $t = 0$ of $d\sigma/dt$, if t is the negative square of

momentum transfer.) It turns out that this limit is closely approached in all the systems experimentally studied. For nucleon-nucleon scattering near 20 GeV lab energy $(\Delta t)_{NN} \approx \frac{1}{10} \text{ GeV}^2$, with $\sigma_{NN}{}^{tot} \approx 40$ mb, while for pion-nucleon scattering* the total cross section is half as large and the peak slightly broader; $(\Delta t)_{\pi N} \approx \frac{1}{7} \text{ GeV}^2$.† There are of course differences between $\pi^+ p$ and $\pi^- p$ and between pp and $\bar{p}p$, but these differences appear to be diminishing as the energy increases and may be ignored in a first approximation. Similarly we may temporarily ignore the small and erratic variations observed for the peak widths.

It is unfortunate that $\pi\pi$ scattering cannot directly be measured because the dynamical equations here are the simplest.. Nevertheless if the Regge pole representation is tentatively accepted then the factorizability of the residues, as pointed out by Gell-Mann and by Gribov and Pomeranchuk,[4] allows the inference of the following high energy $\pi\pi$ total cross section and forward peak width:

$$\sigma_{\pi\pi}{}^{tot} \approx 10 \text{ mb},$$

$$(\Delta t)_{\pi\pi} \approx \frac{1}{4} \text{ GeV}^2,$$

a combination which again is near the unitarity limit. The first task of the theory is to explain these two numbers, and it is in this connection that we have results to report.

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* In view of the qualitative success of the eightfold way it seems safe to assume that KN scattering will be understandable if success is achieved for the πN system.

† This number contains a correction by Ahmadzadeh and Sakmar (Phys. Rev. Letters 11, 429 (1963)) and by W. Rarita (private communication, Berkeley (1964)) to remove the effect of secondary trajectories. The corresponding correction in the NN case appears to be negligible.

III. AN APPROXIMATE FORMULA FOR $\pi\pi$ RESIDUES

We proceed immediately to derive an approximate formula for $\pi\pi$ residues, appropriate to both the Pomeranchuk and ρ trajectories. The derivation employs the strip concept but does not neglect inelastic scattering, even in the low energy resonance region.

Let $A_\ell^I(s)$ be the partial wave amplitude for elastic $\pi\pi$ scattering with isotopic spin I at energy squared s, normalized to

$$\left[\frac{s}{s-4m_\pi^2}\right]^{1/2} \exp(i\,\delta_\ell^I)\sin\delta_\ell^I ,$$

where δ_ℓ^I is the (complex) phase shift. Following Ref. 2 we write

$$\frac{A_\ell^I(s)}{q_s^{2\ell}} = \frac{N_\ell^I(s)}{D_\ell^I(s)} = B_\ell^{I(V)}(s) + \frac{1}{\pi}\int_{s_0}^{s_1}\frac{ds'}{s'-s}\frac{\mathrm{Im}\,A_\ell^I(s')}{q_s'^{2}}, \quad (1)$$

where $D_\ell^I(s)$ is cut only across the strip between $s_0 = 4m_\pi^2$ and s_1, $N_\ell^I(s)$ carrying all the remaining cuts. As explained in Ref. (2) the term $B_\ell^{I(V)}$ plays somewhat the role of a "potential" and may be calculated from the trajectories and residues of the leading Regge poles.

A Regge pole occurs at a zero of $D_\ell^I(s)$, that is

$$D_{\alpha_1(s)}^I = 0 , \quad (2)$$

the residue in ℓ of $A_\ell^I(s)$ being given by

$$\beta_1(s) = (q_s^2)^{\alpha_1(s)} \quad \gamma_1(s) = (q_s^2)^{\alpha_1(s)} \frac{N_{\alpha_1(s)}(s)}{\left[\frac{\partial}{\partial s}D_\ell^I(s)\right]_{\ell=\alpha_1(s)}}\left(-\frac{d\alpha_1(s)}{ds}\right). \quad (3)$$

From formula 1) it is possible to show that

$$N^I_{\alpha_i(s)}(s) = -\frac{1}{\pi} \int_{s_0}^{s_1} \frac{ds'}{s' - s} B^{I(V)}_{\alpha_i(s)}(s') \operatorname{Im} D^I_{\alpha_i(s)}(s'), \qquad (4)$$

while

$$\left[\frac{\partial}{\partial s} D^I_\ell(s) \right]_{\ell = \alpha_i(s)} = \frac{1}{\pi} \int_{s_0}^{s_1} \frac{ds'}{(s' - s)^2} \operatorname{Im} D^I_{\alpha_i(s)}(s'). \qquad (5)$$

Note that inelastic scattering inside the strip has not been neglected.

Now it turns out in all approximations studied so far that for both $I = 0$ and $I = 1$, when ℓ is near 1, the functions $B^{I(V)}_\ell$ and $(-\operatorname{Im} D^I_\ell)$ are all positive across the strip. Furthermore the "potential" $B^{I(V)}_\ell(s)$ varies slowly. Thus we are led to the basic approximation

$$\frac{\gamma_i(s)}{\alpha_i'(s)} \approx (\bar{s}_1 - s) B^{I(V)}_{\alpha_i(s)}(\bar{s}_1), \qquad s \ll \bar{s}_1, \qquad (6)$$

where $\bar{s}_1$ is some average energy inside the strip. Calculations with plausible choices for $B^{I(V)}_\ell$ support formula (6) and suggest that $\bar{s}_1 \gtrsim 2\,\text{GeV}^2$, but a precise value for $\bar{s}_1$ will not be required immediately.

IV. THE TOTAL $\pi\pi$ CROSS SECTION

Many applications of formula (6) are possible. Our first application does not require the form of $B^{I(V)}_\ell(s)$ but only the relation

$$B^{I=0(V)}_\ell(s) \approx 2 B^{I=1(V)}_\ell(s), \qquad s \lesssim s_1, \qquad (7)$$

a result that follows from the crossing matrix if the high energy "potential" is dominated by $I = 1$ exchange. A study of $I = 0$ exchange, to be published elsewhere, confirms the usual assumption that this

contribution to the force for $s \lesssim s_1$ is less important than that of $I = 1$. It is taken for granted that $I = 2$ exchange is negligible.

Assumption (7) leads to the circumstance that for the Pomeranchuk trajectory and the ρ trajectory the values of $\bar{s}_1$ are roughly the same. Noting that $\alpha_P(0) = 1$ while $\alpha_\rho(m_\rho^2) = 1$, we may combine (6) and (7) to obtain

$$\frac{\gamma_P(0)}{\alpha_P'(0)} \frac{\alpha_\rho'(m_\rho^2)}{\gamma_\rho(m_\rho^2)} \approx 2 \frac{\bar{s}}{\bar{s} - m_\rho^2} \approx 2 \tag{8}$$

if $\bar{s} \gg m_\rho^2$. It is easy to verify that

$$\frac{\gamma_\rho(m_\rho^2)}{\alpha_\rho'(m_\rho^2)} \approx \frac{4 \Gamma_\rho}{m_\rho}, \tag{9}$$

where Γ_ρ is the full width of the ρ, so we find

$$\gamma_P(0) \approx 8 \frac{\Gamma_\rho}{m_\rho} \alpha_P'(0). \tag{10}$$

The high energy $\pi\pi$ total cross section, if it is in fact controlled by the Pomeranchuk-Regge pole, is given by

$$\sigma_{\pi\pi} = 8\pi^2 \gamma_P(0), \tag{11}$$

so we predict finally that

$$\sigma_{\pi\pi} \approx 64 \pi^2 \frac{\Gamma_\rho}{m_\rho} \alpha_P'(0). \tag{12}$$

The slope of the Pomeranchuk trajectory, assuming that it passes through $\ell = 2$ at the mass of the f^o, has been estimated by Ahmadzadeh and Sakmar as $\frac{1}{3}$ GeV^{-2}.[5] Taking $\Gamma_\rho = 100$ MeV and $m_\rho = 750$ MeV, we then find from formula (12):

$$\sigma_{\pi\pi} \approx 11 \text{ mb.}$$

V. THE WIDTH OF THE $\pi\pi$ DIFFRACTION PEAK

To estimate the width of the diffraction peak, a more specific assumption must be made about the "potential" $B_\ell^{I=0(V)}$. In general, high energy dominance of the Pomeranchuk-Regge pole leads to

$$\text{Im } A_{\pi\pi}(s,t) \xrightarrow[s \to \infty]{} \frac{\pi}{3} (2\alpha_P(t) + 1)\gamma_P(t)(q_t^2)^{\alpha_P(t)} P_{\alpha_P(t)}\left(\frac{s}{2q_t^2}\right),$$

$$\approx \frac{\pi}{3} (2\alpha_P(t) + 1) \gamma_P(t) \left(\frac{s}{2}\right)^{\alpha_P(t)} , \qquad (13)$$

for $\alpha_P(t)$ near 1 .* Using formula (6), we then have

$$\text{Im } A_{\pi\pi}(s,t) \propto (2\alpha_P(t) + 1)\alpha_P'(t)(\bar{s} - t)B_{\alpha_P(t)}^{I=0(V)}(\bar{s}) \left(\frac{s}{2}\right)^{\alpha_P(t)} . \qquad (14)$$

The form most commonly used for $B_\ell^{I=0(V)}(s)$ is that based on exchange of a (fixed spin) ρ :

$$B_\ell^{I=0(V)}(s) \propto (1 + \frac{s}{2q_\rho^2}) \frac{Q_\ell\left(1 + \frac{m_\rho^2}{2q_s^2}\right)}{(q_s^2)^{\ell+1}} . \qquad (15)$$

Although we expect important deviations from this behavior when the potential is carefully calculated, the form (15) may serve to indicate the ℓ dependence of the potential, which is all we need for the shape of the diffraction peak. For $s \sim 2$ GeV^2, most of the ℓ dependence of (15) resides in the factor $(q_s^2)^\ell$, and the weak remaining ℓ dependence in Q_ℓ near $\ell = 1$ is conveniently almost proportional to $(2\ell + 1)^{-1}$. Thus (14) becomes

$$\text{Im } A_{\pi\pi}(s,t) \propto \alpha_P'(t)(\bar{s} - t)\left(\frac{s}{2q_s^2}\right)^{\alpha_P(t)} . \qquad (16)$$

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* The real part of the amplitude is small near $t = 0$.

At this point, evidently, an estimate of $\bar{s}$ is required as well
as an estimate of the shape of the Pomeranchuk trajectory near $t = 0$.
From preliminary calculations of trajectories and residues with a variety
of "potentials" and strip widths, when the potential and strip width are
adjusted to give $\alpha_p(0) = 1$, $\alpha_p' \approx \frac{1}{3}$ GeV^{-2} , we find $\bar{s} \sim 2$ GeV^2 .
Furthermore the trajectory for $t \ll \bar{s}$, is represented roughly by
the form

$$\alpha_p(t) \approx c + \frac{1 - c}{1 - \frac{t}{\bar{s}}} , \qquad (17)$$

so

$$\alpha_p'(t) \approx \frac{\alpha_p'(0)}{(1 - \frac{t}{\bar{s}})^2} . \qquad (18)$$

Taking the logarithmic derivative of (16) at $t = 0$ we then calculate

$$a_{\pi\pi} \approx (\bar{s})^{-1} + \alpha_p'(0) \ln(\frac{2s}{\bar{s}}) , \qquad (19)$$

where $a_{\pi\pi} = \frac{1}{2} (\Delta t_{\pi\pi})^{-1}$.

As explained in section II above we expect $(\Delta t_{\pi\pi})^{-1}$ to be
~ 4 GeV^{-2} at an s corresponding to 20 GeV lab energy for NN scattering.
This is $s \sim 40$ GeV^2, so taking $\bar{s} = 2$ GeV^2 we have from (19)

$$a_{\pi\pi} \approx (0.5 + \frac{1}{3} \ln 40) \text{GeV}^{-2}$$

$$\approx 1.7 \text{ GeV}^{-2} , \qquad (20)$$

not far from the expected 2 GeV^{-2}.

It thus appears that if the strip approximation succeeds in
calculating the masses and widths of the ρ and f^o mesons it will

correctly predict both the high energy $\pi\pi$ total cross section and the width of the diffraction peak.[*]

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[*] Note that even to get the correct sign for $\sigma_{\pi\pi}$ and $a_{\pi\pi}$ from a dynamical calculation of the Regge parameters is a non-trivial achievement.

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