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## HIGH ENERGY POLARIZATION EXPERIMENTS AND REGGE POLES

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TO: All recipients of UCRL-16185
FROM: Technical Information Division
Subject: UCRL-16185, High Energy Polarization Experiments and Regge Poles, Roger J. N. Phillips and William Rarita, June 10, 1965

Please make the following corrections on subject report.

Abstract page reads: 'William Rarita!" It should read "William Rarita ${ }^{\ddagger}$ ",
Page 16, line 10 from bottom reads: "... $\rho$ and $A_{z}$..." It should read
"... $\rho$ and $A_{2} \ldots$ "
Page 17, line 2 reads: " $\rho: A_{z} .$. " It should read " $\rho_{:} A_{2} . . . "$
Page 18: line 4 reads: ". .useful for measing..." It should read ". . useful for measuring..."

# Research and Development 

# UNIVERSITY OF CALIFORNIA <br> Lawrence Radiation Laboratory Berkeley, California <br> AEC Contract No. W-7405-eng-48 

HIGH ENERGY POLARIZATION EXPERIMENTS AND REGGE POLES

Roger J. N. Phillips and William Rarita

June 10, 1965

HIGH ENERGY POLARIZATION EXPERIMENIS AND REGGE POLES*<br>Roger J. N. Phillips ${ }^{+}$<br>-<br>Lawrence Radiation Laboratory<br>University of California<br>Berkeley, California<br>and<br>William Rarita<br>Department of Physics<br>University of California<br>Berkeley, California

June 10, 1965

ABSIRACT

This report reviews various types of higheenergy polarization measurement, and their relation to Regge pole models. Illustrations are given.

## I. INIRODUCITON

This is a review of various types of polarization experiment at high energy, that may be performed, now or in the future, and the types of prediction that Regge pole models make. We hope it will focus attention on certain properties of these models and stimulate experiments.

Specifically we consider $\pi N$, $K N, \bar{K} N$, NN and $\bar{N} N$ scattering, and polarization tensors of the first and second ranks only. The experiments may not all be feasible in the near future, but at least they are not unthinkable.

The polarization formalism is given in §2. General qualitative properties of Regge pole models and their experimental implications are described in \$3. Illustrations are given. §. 4 contains some conclusions.

## 2. POLARIZATION FORMALISM

We shall be concerned with spin-0 and spin- $\frac{1}{2}$ particles only. ${ }^{1}$
The spin state of a spin $-\frac{1}{2}$ particle is conveniently described by going to its rest-frame, where a two-dimensional spinor is sufficient. This approach leads to a formalism that looks nonorelativistic, in the sense that no $\gamma$-matrices appear; however, $\operatorname{Stapp}^{2}$ has shown that it is completely general. Relativistic effects enter explicitly at two points; in the relations between lab and c.m. angles, and in a rotation of spin co-ordinates when the appropriate rest frame Is redefined between two scatterings. ${ }^{2}$

The c.m. scattering amplitude $M$ is a matrix in spin space. For $\pi \mathbb{N}, \mathrm{KN}$ or $\bar{K} \mathbb{N}$ scattering with given isospin, $M$ is a $2 \times 2$ matrix ; the most general form consistent with parity conservation is

$$
\begin{equation*}
M=g+1 h g \cdot N \tag{1}
\end{equation*}
$$

where $\underset{\sim}{N}$ is a unit vector normal to the scattering plane; $\underset{\sim}{\sigma}$ is the Paull spin operator for the nucleon; $g$ and $h$ are scalar coefficients depending on energy and scattering angle (but not on azimuth). It is implied that $g$ is multiplied by the unit. operator in spin space.

For $N \mathbb{N}$ or $\bar{N} N$ scattering with given isospin; $M$ is a $4 \times 4$ matrix. The most general form consistent with parity
conservation, charge conjugation and time-reversal Invariance, and charge independence, may be written.

$$
\begin{align*}
& +(g \omega h) g^{(1)} \cdot K_{n} \sigma^{(2)} \cdot K_{\sim} \tag{2}
\end{align*}
$$

Here ${\underset{\sim}{\sigma}}^{(1)}$ and $\underset{\sim}{\underset{\sim}{r}}(2)$ are the Pauli spin operators for the two particles ; a, c, $m, g$ and $h$ are scalar functions of energy and scattering angle; $\underset{\sim}{\mathbb{N}} \underset{\sim}{P}$ and $\underset{\sim}{K}$ are a right-handed trisd bf unit
 where $\underset{\sim}{p}{ }_{i}$ and $\underset{\sim}{p} \underset{f}{ }$ are the initial and final c.m. momenta of particle 1.

In what follows it will be convenient to use ${\underset{\sim}{\sim}}_{\underset{\sim}{p}}^{\underset{\sim}{P}}$ and $\underset{\sim}{K}$ as axes of reference, and denote components with respect to these axes by subscripts: $\quad \underset{\sim}{(1)} \cdot \underset{\sim}{N}=a_{N}^{(1)}$, etc

Consider a monoenergetic beam incident upon a target, and let the initial polarization of the system be described by a density matrix $\rho_{1}$. After scattering through some angle the final density matrix $\rho_{f}$ depends on $M$ and is given by

$$
\begin{equation*}
\rho_{f} \doteq M \rho_{1} \hat{M}^{+} \tag{3}
\end{equation*}
$$

If intensity alone is measured after scattering, we have the differentidl cross section

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{f}=\operatorname{trace} \rho_{f} / \text { trace } \rho_{1}=\operatorname{trace} / M \rho_{1} M^{+} / \text {trace } \rho_{1} \tag{4}
\end{equation*}
$$

If the expectation value of some spin variable $\xi$ : 1 measured, we have

$$
\begin{align*}
\langle\xi\rangle_{f} & =\text { trace } \rho_{f} / / \text { trace } \rho_{f}  \tag{5}\\
\therefore\langle\xi\rangle_{f}\left(\frac{d \sigma}{d \delta}\right)_{f} & =\text { trace } M \rho_{1} M^{+} \xi / \text { trace } \rho_{1} \tag{6}
\end{align*}
$$

Thus experiments essentially measure quantities of the form

$$
\begin{equation*}
\text { trace } M \times M^{+} y, \tag{7}
\end{equation*}
$$

where $x$ and $y$ are spin operators (Including the unit operator) referring to some aspect of initial polarization and final analysis, respectively.

Let us take some practical examples.
i) No initial polarization, final intensity measured; $\rho_{i}=1, \xi=1$. This gives the unpolarized cross section which we shall denote by $I_{0}:$

$$
\begin{equation*}
I_{0}=\left(\frac{d \sigma}{\partial \Omega}\right)_{0}=\text { trace } M M^{+} / \text {trace } 1 \tag{8}
\end{equation*}
$$

(ii) No initial polarization, final polarization of particle 1 measured; $\rho_{i}=1, \xi={\underset{\sim}{j}}^{(1)}$

[^0]\[

$$
\begin{equation*}
I_{0}\left\langle{\underset{\sim}{0}}^{(1)}\right\rangle_{f}=\text { trace } M^{+} \sigma^{(1)} / \text { trace } 1 \tag{9}
\end{equation*}
$$

\]

Parity conservation, incorporated in $M$, constrains $\left\langle\dot{\sim}^{(1)}\right\rangle$, to be normal to the scattering plane: $\left\langle{\underset{\sim}{d}}^{(1)}\right\rangle_{f}=P(\theta) \underset{\sim}{\mathbb{N}}, P(\theta)$ is known as the polarization parameter; it can also be measured in an experiment where one particle is initially polarized and an asymmetry in the cross section is measured, essentially the time-reverse of the present case.
iii) Particle 1 has initial polarization $\underset{\sim}{P}$, and its final polarization is measured in some direction $j ; \rho_{1}=1+{\underset{\sim}{a}}^{(1)} \cdot \underset{\sim}{p}, \quad \xi^{1}=\sigma_{j}{ }^{(1)}$. The final polarization contains a term depending on the initial polarization through the "depolarization tensor" $D_{k j}(\theta)$ :

$$
\begin{align*}
\left\langle\sigma_{j}^{(1)}\right\rangle_{f}\left(\frac{d \sigma}{d \Omega}\right)_{P} & =\operatorname{trace} M\left(1+{\underset{\sim}{o}}^{(1)} \cdot \underline{2}\right) \mathrm{M}_{\sigma_{j}^{+}}^{(I)} / \text { trace } 1 \text { (10) } \\
& =I_{0}\left\{P(\theta) \mathbb{N}_{j}+D_{k j}(\theta) P_{k}\right\} \quad \text { (II) } \tag{그}
\end{align*}
$$

With respect to the axes ${\underset{\sim}{N}}_{\mathbb{N}}^{\sim} P$ and ${ }_{\sim} K$ the tensor $D_{k j}$ simplifies: $D_{\mathrm{NK}}=D_{\mathrm{KN}}=D_{\mathrm{NP}}=, D_{\mathrm{PN}}=0$ and $D_{\mathrm{PK}}=-D_{\mathrm{KP}} \cdot$. The component $D_{\mathrm{NN}}$ is the so-called "depolarization parameter" $D(\theta)$; the other components appear in various combinations in the so-called "rotation parameters" $R(\theta), A(\theta), R^{\prime}(\theta), A^{\prime}(\theta)$ 。

For the " $R$ " experiment, the incident beam is polarized transversely and in the plane of scattering, while the scattered beam is also analyzed transversely, in the scattering plane. For the "A" experiment, the incident beam is polarized longitudinally, but is analyzed as before. Thus the initial polarizations are in the directions $\underset{\sim}{P} \sin \frac{\theta}{2}+K \cos \frac{\theta}{2}$ and $\underset{\sim}{P} \cos \frac{\theta}{2}-K \sin \frac{\theta}{2}$, respectively, where $\theta$ is the com. scattering angle, (See Fig. 1). However, because of the two relativistic effects mentioned earlier, ${ }^{2}$ the effective direction of analysis is not $K$ but rather ${ }_{\sim}^{K} \cos \left(\frac{\theta}{2}-\theta_{L}\right)-\underset{\sim}{P} \sin \left(\frac{\theta}{2}-\theta_{L}\right)$, where $\theta_{L}$ is the $I_{a b}$ scattering angle. Hence

$$
\begin{equation*}
R=\frac{1}{2}\left(D_{K K}+D_{P P}\right) \cos \left(\theta-\theta_{\mathrm{I}}\right)+\frac{1}{2}\left(D_{K K}-D_{P P}\right) \cos \theta_{\mathrm{L}}+D_{\mathrm{PK}} \sin \left(\theta-\theta_{\mathrm{L}}\right) \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
A=-\frac{1}{2}\left(D_{K K}+D_{P P}\right) \sin \left(\theta-\theta_{L}\right)-\frac{1}{2}\left(D_{K K}-D_{P P}\right) \sin \theta_{L}+D_{T K} \cos \left(\theta-\theta_{L}\right) \tag{13}
\end{equation*}
$$

For these experiments it may be more practical to polarize the target rather than the beam, and to analyze the recoil polarization. Let ${\underset{\sim}{p}}_{1}, \underset{\sim}{p}, \underset{\sim}{P}, \underset{\sim}{K}$ and $\theta$ be defined relative to the fast particle, and label the c.m. and Lab recoil angles by $\varnothing$ and $\phi_{I}$, as shown in Figure 2. Then analogous $R$ and $A$ experiments can be defined, with initial polarizations in the directions $-\underset{\sim}{X} \sin \frac{\varnothing}{2}-\underset{\sim}{P} \cos \frac{\varnothing}{2}$ and $\underset{\sim}{K} \cos \sum_{2}^{\varnothing}-\underset{\sim}{p} \sin \frac{\varnothing}{2}$ respectively. With the geometry shown in Fig. 2, the effective direction of polarization analysis is $\underset{\sim}{K} \sin \left(\frac{\varnothing}{2}-\phi_{I}\right)-\underset{\sim}{P} \cos \left(\frac{\varnothing}{2}-\phi_{I}\right)$. Hence

$$
\begin{equation*}
R_{\text {recoit }}=\frac{1}{2}\left(D_{K K}+D_{P P}\right) \cos \left(\phi-\phi_{L}\right)+\frac{1}{2}\left(D_{P P}-D_{K K}\right) \cos \phi_{L}-D_{P K} \sin \left(\phi-\phi_{L}\right) \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
A_{\text {recoil }}=\frac{1}{2}\left(D_{K K}+D_{P P}\right) \sin \left(\phi-\phi_{L}\right)+\frac{1}{2}\left(D_{P P}-D_{K K}\right) \sin \phi_{L}+D_{\mathrm{FK}} \cos \left(\phi-\phi_{L}\right) \tag{15}
\end{equation*}
$$

Experiments to find $R^{\prime}$ and $A^{\prime}$ require the measurement of longitudinal polarization: we shall not discuss them here.

1v) Suppose particle 1 has initial polarization $\underset{\sim}{\underset{\sim}{P} \text {, and the final polari- }}$ zation of the other particle is measured in some direction $j$. This is like case (iii) above, except that $D_{k j}$ is now replaced by a "polarization transfer" tensor $K_{k j}$ :

$$
\begin{align*}
\left\langle\sigma_{j}^{(1)_{P}\left(\frac{\partial \sigma}{d \Omega}\right)_{f}}\right. & =\operatorname{trace} M\left(1+\sigma_{\sim}^{(1)} \cdot P\right) M^{+} \sigma_{j}(2) / \text { trace } 1  \tag{16}\\
& =I_{0}\left\{P(\theta) N_{j}+K_{k j}(\theta) \rho_{k}\right\} \tag{17}
\end{align*}
$$

The tensor $K_{k j}$ simplifies, just like $D_{k j}$, with respect to the axes $\underset{\sim}{\mathbb{N}}, \underset{\sim}{P}$ and $\underset{\sim}{K}$. The component $K_{N N}$ is directly measureable. by polarizing and analyzing normal to the scattering plane. The other components appear in various analogues of the $R, A, R^{\prime}$ and $A^{\prime}$. experiments that may be defined. We shall not discuss them in detail, however, since they prove to have no interest in the present context.
v) Suppose there is no initial polarization, and the polarization of particles 1 and 2 are analyzed in coincidence in directions $k$ and $j, b_{1}=1, \xi=\sigma_{k}^{(1)} \sigma_{j}^{(2)}$. This experiment measures the spin correlation tensor $C_{k j}$ :

$$
\begin{gather*}
I_{0}\left(\sigma_{k}(1)_{\sigma_{j}}(2) \quad=\operatorname{trace} M M_{k}^{+}(1)_{\sigma_{j}}(2) / \text { trace } 1\right.  \tag{18}\\
\therefore \quad=I_{0} C_{k j}(\theta) \tag{19}
\end{gather*}
$$

With respect to the axes $\mathbb{N}_{\sim} P_{\sim}$ and $K_{\alpha}, C_{k J}$ simplifies ; $\mathrm{C}_{\mathrm{as}}$ $C_{N P}=C_{P N}=C_{N K}=C_{K N}=0 ; C_{K P}=C_{P K} \because$ The component $C_{N N}$ is immediately measureable. The others normally occur in various combinations; however, they prove to have no interest in the present context. The spin-correlation tensor can also be found by polarizing both particles initially and measuring the final intensity- .essentially the time reverse of case (v).

The experiments described above are these in which at most two polarizations occur (either initially or finally). We shall not discuss more complicated possibilities.

For spin-0 scattering on spin- $\frac{1}{2}$ particles, the expertmental quantities can be written in terms of the $M$-matrix coefficients as follows (see Eq. (1)):

$$
\begin{aligned}
& I_{0}(\theta)=|g|^{2}+|h|^{2} \\
& I_{0} P(\theta)=2 \operatorname{Im}\left(h^{*} g\right) \\
& I_{0} D_{K K}(\theta)=I_{0} D_{P P}(\theta)=|g|^{2}|h|^{2} \\
& I_{0} D_{K P}(\theta)=-I_{0} D_{P K}(\theta)=2 \operatorname{Re}\left(g^{*} h\right) \\
& D_{N N}(\theta)=1,
\end{aligned}
$$

All other components vanish. The tensors $K_{k j}$ and $C_{\dot{k j}}$ are of course not defined when only one particle has spin. The corresponding relations for two spin- $\frac{1}{2}$ particles are more complicated. However, it turns out that Rage pole models based on the leading trajectories give a simpler form for the M-matrix than Eq. (2): in fact, only the coefficients $a, c$ and $m$ remain. Using this special form, the expressions for physical quantities simplify enormously, and we find

$$
\begin{align*}
& I_{0}(\theta)=|a|^{2}+2|c|^{2}+|m|^{2} \\
& I_{0} P(\theta)=2 \operatorname{Im} c^{*}(a+m)  \tag{21}\\
& D_{N N V}(\theta)
\end{align*}
$$

$$
I_{0} D_{K K}(\theta)=I_{0} D_{P P}(\theta)=|a|^{2}-|m|^{2}
$$

$$
I_{0} D_{K P}(\theta)=-I_{0} D_{P K}(\theta)=2 \operatorname{Re} c^{*}(a-m)
$$

$$
I_{0} K_{N N}(\theta)=I_{0} C_{N N}(\theta)=2 \operatorname{Re} \mathrm{am}^{*}+2|c|^{2}
$$

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III. REGGE POLE MODELS

For the scattering of two spinless particles, a Regge pole In the crossed channel gives asymptotically a contribution to the scattering amplitude of the form, ${ }^{3}$

$$
\begin{equation*}
M=-\sqrt{s} \beta(t)\left[\frac{1 \pm \exp (-1 \pi \alpha(t))}{2 \sin \pi \alpha(t)}\right]\left(\frac{s}{s_{0}}\right)^{\alpha(t)-1} \tag{22}
\end{equation*}
$$

Here $s$ is the invariant square of total energy, $t$ is the invariant square of momentum transfer, $s_{o}$ is a scale constant, $\alpha(t)$ is the trajectory of the Regge pole and $\beta(t)$ is related to the residue at the pole. Both $\alpha$ and $\beta$ are supposed to be real in the scattering region; thus the phase of the amplitude comes from the "signature factor" (in square braces). The signature $\pm$ determines whether the particles associated with the trajectory have even ( + ) or odd ( - ) spin. Contributions like Eq. (22) are additive.

For forward $\pi N$ scattering (and for $\operatorname{spin}-0$ on $\operatorname{spin}-\frac{1}{2}$ in general) there are similar contributions to both the scalar amplitudes $g$ and $h$ which appear in $M$. In the high-energy approximation of Wagner, 4 and using his notation, we find the following contribution from a Regge pole labelled 1 :

$$
\begin{equation*}
M^{1}=-\frac{\sqrt{s}}{4 \pi} \zeta_{1}\left(\frac{s}{s}\right)^{\alpha_{1}-1} \eta_{\pi 1}\left(\eta_{N 1} 110 \cdot N_{\sim}^{N} \phi_{N 1}\right) \tag{23}
\end{equation*}
$$

Here $\zeta_{1}$ is an abbreviation for the signature factor. The residue function is explicitly factored into a part $\eta_{\pi i}$ refering to the pion and a part, $\eta_{N i}$ or ффNi, referring to the nucleon. We have normalized $M$ to correspond to the formulae in $§ 2$.

For forward $N a \mathbb{N}$ scattering (and for $\operatorname{spin}-\frac{1}{2}$ on $\operatorname{spin}-\frac{1}{2}$ in general), the form of the contribution depends on the quantum numbers of the Regge pole. However, for the trajectories which correspond to $0^{+}, 2^{+} \ldots$ and $1^{-}, 3^{-}, \therefore \because$ particles (which appear to include all the leading trajectories of most physical importance), each contribution to $M$ has the form

$$
\begin{equation*}
M^{1}=-\frac{\sqrt{s}}{4 \pi} \zeta_{1}\left(\frac{s}{s_{01}}\right)^{\alpha_{1}-1}\left[\eta_{N 1}-1 \sigma_{\sim} N \phi_{N 1}\right]^{2} \tag{24}
\end{equation*}
$$

Here again the residue function factors into two partis (in this case equal). Note that only the coefficients $a, c$ and mappear, so that the simplified expressions in Eq. (21) are relevant.

Strictly speaking, Eqs. (23) and (24) describe the contributions of a Regge pole with vacuum quantum numbers. In general, there is a $\pm$ sign and sometimes a ClebschoGordon coefficient, depending on the particular process in question, the isospins of the particles, and the quantum numbers of the Regge pole. These details do not affect the present discussion. The Wagner formulae are asynptotic, in the sense that they retain only terms with the leading energy dependence. Thus it may be that in the 10-20 Gevi fegion some of the terms omitted axe not completely negligible.

However, we only require them as a qualitative guide; and as such they should not be misleading.

It is a simple matter to derive expressions for physical quantities in terms of Wagner's parameters, using the formulae above. We shall not: write them out. (Incidentally, it seems that reference 4 uses the opposite sign convention for P).

The Regge pole contributions (23) and (24) have two Important properties: 5 (i) the spin-dependent and spin-independent terms from a given pole have the same energy dependence; (ii) these terms also have the same phase.

Let us discuss some consequences for various processes.
$\pi N_{\text {, }} K N$ and $\overline{K N}$ elastic scattering.
Point (i) implies that there may exist non-trivial spin dependence even in the asymptotic region. In contrast, one does not expect such behavior in a diffraction picture of scattering. 6

We must be careful, however, what we mean by the absence of spin dependence. In the formalism we have been using, the natural definition seems to be the absence of $\sigma$-matrices in M. This definition, which we label (a), is equivalent to

$$
\begin{equation*}
\phi_{N 1}=0 \tag{25a}
\end{equation*}
$$

On the other hand, if we consider the relativistic r-matrix referred toipirac spinors, the natural definition is the absence of
$\gamma$-matrices; we label this (b); it is equivalent to.

$$
\begin{equation*}
\phi_{N 1}=\sqrt{-t / 4 m_{N N}^{2}} \tag{25b}
\end{equation*}
$$

Where $m_{N}$ is the nucleon mass. The latter form introduces a certain kinematic spin-dependence for the rest-frame spinors. We do not need to decide here whether (a) or (b) is more appropriate for a diffraction model: the point is that Regge pole models are not restricted in such a way.

Now asymptotically, where the Pomeranchuk polel alore is significant, the polarization parameter $P(\theta)$ vanishes because of point (ii) and the form of its definftion. To detect an asymptotic spin-dependence, therefore, we are driven to secondrank polarization tensors. In the present case only $D_{k j}(\theta)$ is available, and the measurements have to be made on a polarized target.

To illustrate the effects that may be found, Figure 3 shows predictions of $R_{\text {recoll }}$ and $A_{\text {recoll }}$ for $\pi^{+} p$ scattering at 5,40 and $320 \mathrm{GeV} / \mathrm{c}$ for a particular Regge pole model, number I in reference 7. (For the sake of illustration, we have extrapolated this model beyond the range in $s$ and $t$ where it was fitted to data). For comparision, Figure 4 shows the predictions at $20 \mathrm{GeV} / \mathrm{c}$ only, for the definitions (a) and (b) of spinIndependence above, which lead respectively to

$$
\begin{align*}
& D_{K K}=D_{P P}=1 \\
& D_{K P}=D_{P K}=0 \tag{26a}
\end{align*}
$$

$$
D_{K K}=D_{P P}=\left(1-x^{2}\right) /(1+x)
$$

$$
\begin{equation*}
D_{K P}=-D_{F K}=2 x /\left(1+x^{2}\right) \tag{26b}
\end{equation*}
$$

where $\left.x=p^{2} \sin \theta /\left(F_{N}+m_{N}\right)^{2}-p^{2} \cos \theta\right)$, and $F_{N}=\sqrt{p^{2}+m_{N}^{2}}$ is the $c$. $m$ n nucleon energy
The polarization parameter $P(\theta)$, on the other hand, depends on interference between at least two Regge poles, and behaves asymptotically as $\mathrm{s}^{\alpha_{2}-\alpha_{1}}$, where $\alpha_{1}$ and $\alpha_{2}$ are the highest and second-highest trajectories. For the particular example used above ( $\pi^{+} p$ scattering according to solution 1 of ref. 7), the maximum value of $|P|$ is about $0.12,0.04$ and 0.014 at 5,40 and $320 \mathrm{GeV} / \mathrm{c}$, respectively.

NN and $\bar{N} N$ elastic scattering.

The general remarks above about asymptotic spin dependence apply here also. There is nothing new to add about $P(\theta)$ or $D_{k j}(\theta)$, except that a polarized beam may now be considered, but there are two new second -rank tensors $K_{k g}(\theta)$ and $C_{k j}(\theta)$ available. However, the property of the leading Regge poles that $g=h=0$ in Eq. (2) effectively reduces these two tensors to a
single component, as we have already seen. Furthermore, the factorized form of Eq. (24) causes this remaining component to vanish for a single Regge pole. ${ }^{9}$ Thus $C_{\mathbb{N N}}$ and $K_{N N}$, like $P$, depend on interference between poles.

To exhibit asymptotic spin dependence, we are therefore left with the depolarization tensor as before. To illustrate the effects that may plausibly occur, we take an unpublished Regge pole model ${ }^{8}$ which approximately reproduces $p p$ and $\overline{p p}$ data for cross sections and $P(\theta)$ - Figure 5 shows the predictions for $A_{\text {recoil }}$ and $R_{\text {recoil }}$ in $p p$ scatterd ng at 5,20 and $200 \mathrm{GeV} / \mathrm{c}$. The two definitions of spin independence described above carry over to the present case; Eqs. (25 a,b) and (26,a;b) apply equally to the $\mathbb{N N}$ and $\bar{N} N$ case, and so does Figure 4 (ignoring the tiny: effect of different particle masses).

Charge-exchange scattering.

These cross sections are much smaller than elastic scattering, at high energy. In a Regge pole model, the Integrated cross section tends to zero asymptotically like $s^{2 \alpha_{0}-2} /$ ins, where $\alpha_{0}$ is the intercept at $t=0$ of the leading trajectory; for elastic scattering $\alpha_{0}$ is 1 , but for charge exchange it is around 0.5 or less. We therefore limit our discussion to $P(\theta)$ at nonasymptotic energies.

Consider first $\bar{\pi}+p \rightarrow \pi^{0}+n$. The present data are
compatible with the dominance of the $\rho$ Regge pole alone.

If this is true, $P=0$. However, other contributions may be present, and a measurement of $P$ can detect them. Even if these other terms give only. $20 \%$ of the omplitude at some energy and angle, they can still make $P$ as large as 0.4 under favorable circumstances.

Consider next $\pi^{\prime \prime}+p \rightarrow \eta^{\circ}+n$. The only known Regge pole that can contribute here is the one associated with the $A_{2}$ meson. Once more, a measurement of $P$ 'would be a sensitive test of other contributions.

In the case of $K^{\infty}+p \rightarrow \bar{K}^{o}+n$ and $K^{+}+n \rightarrow K^{b}+p$, the $\rho$ and $A_{2}$. Regge poles are believed to dominate. ${ }^{7}$ Since these trajectories lie close together, the polarlzation resulting from interference falls rather slowly with increasing energy. Since these trajectories have opposite signatures, the relative phase of their contributions is extremely favorable for $P$. The fits to data that have been made require large spin-dependences for both the $\rho$ and $A_{z}$ terms, leading to big polarization effects which ought to be measured. As an illustration, Figure 6 shows $P(\theta)$ for $K^{-}+p \rightarrow \bar{K}^{\circ}+n$ at. 5 and $40 \mathrm{GeV} / \mathrm{c}$; according to solution 1 of reference 7. This illustrates dramatically how in. such favorable circumstances, $P$ can be very large and show no sign of tending to zero. Note that. $P$ appears to be increasing with energy in some regions. It turns out that even at $1.100 \mathrm{GeV} / \mathrm{c}$ the asymptotic region where a single pole dominates has not been reached, and $P$ still comes close to 1 at some angles, with the model in question.

For $p+n \rightarrow n+p$ and $p+p \rightarrow \ddot{n}+n$, we expect contributions from $\rho, A_{z}$ and $\pi$ Regge poles at least. No really satisfactory explanation of the differential cross section has yet been given (either with or without Rage poles), but there is still a shortage of data. There seems to be every chance that $P$ may be substantial in the present accelerator range.

Relation between asymptotic spin-dependences
When the Pomeranchuk pole alone dominates, the depolarization tensor $D_{k j}$ becomes the same for all the elastic scatterings we have considered ( $\pi N, K N, \bar{K} N, N N, \bar{N} N$ ). This is because of factorization, and the trivial connection between $\mathbb{N}$ and $\overline{\mathbb{N}}$ vertices. 9 We then have

$$
\begin{align*}
& D_{N N}=0 \\
& D_{K K}=D_{P P}=\left(\eta_{N}^{2}-\phi_{N}^{2}\right) /\left(\eta_{N}^{2}+\phi_{N}^{2}\right)  \tag{27}\\
& D_{K P}=-D_{P K}=-2 \eta_{N} \phi_{N} /\left(\eta_{N}^{2}+\phi_{N}^{2}\right)
\end{align*}
$$

Incidentally, a comparison of Figures 3 and 5 shows that the two corresponding Rage pole models are not strictly compatible.

Regge pole models allow an asymptotic spin dependence, which cannot be measured by $P(\theta)$ because of the phase relations. It can however be measured through second-rank polarization tensors.

The parameter $P(\theta)$ is useful for measing the spacing between two leading trajectories, and for shedding light on their combined spin dependence. In favorable circumstances, such as $K^{\prime \prime} \mathrm{p}$ charge exchange, it may have large values up to surprisingly, high energies. $P$ is also a good test for secondary contribitions, when a single Regge pole is believed to dominate.

For $\mathbb{N N}$ and $\overline{\mathrm{N} N}$ scattering, Regge pole models introduce some further special restrictions. The quantum numbers of the leading trajectories are such that the Mmatrix simplifies. As a result, the polarization transfer tensor $K_{k j}(\theta)$ and spin correlation tensor $C_{k j}(\theta)$ have only one non-trivial component between them $\left(K_{N N}=C_{N N}\right)$. The depolarization tensor $D_{k j}(\theta)$ retains two independent components, measurable through $R$ and A.

The factored form of a single pole contribution is such that the single remaining element of $K_{k j}$ and $C_{k j}$ vanishes asymptotically. Only $D_{k j}$ is suitable for detecting the asymptotic spin dependence that is predicted.
$C_{N N}\left(K_{N N}\right)$ depends on Interference between different Regge poles, like .P, but is harder to measure. Perhaps the first interest in $C_{N N}$ and $K_{N N}$ is to see if they tend to zero asymptotically, as predicted by factorization.
H.
$-19{ }^{\circ}$

It would also be interesting to check the relation $C_{\text {NNN }}=K_{\mathrm{NN}}$, and some of the others (such as $D_{N N}=1, D_{P P}=D_{K K}$, $K_{\mathrm{PP}}=\mathrm{K}_{\mathrm{KK}}=0$, etc) which follow from the property $\mathrm{g}=\mathrm{h}=0$ of the leading terms of the leading trajectories. This could give some measure of the importance of other contributions, such as those of the $\pi$ and $A_{1}$ Regge poles, which do not have this restriction.

The relation $I_{0} P=2 \operatorname{Im} c^{*}(a+m)$ continues to hold even when $g$ and $h$ are present in Eq. (2), so we do not expect $P$ to be sensitive to these terms.

The predicted asymptotic equality of the depolarization tensors, for all the processes considered, offers an important check on factorization.

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## FOOTNOTES ARD REFERENCES

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5. These two properties are related, as shown by dispersion relations.
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of $R$ and $A$ have been made directly, without going through Eq. (23).
8. This model, constructed by W. Rarita, uses the $P, P^{\prime}$ and $\omega$ Regge poles and the Wagner approximation.
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## FIGURE CAPTIONS:

Figure 1. Geometry for $R$ and $A$ experiments on the beam particle.
Figure 2. Geometry for $R$ and $A$ experiments on the target particle.
Figure 3. $R_{\text {recoil }}$ and $A_{\text {recoil }}$, for $\pi^{+} p$ scattering at 5,40 and $320 \mathrm{GeV} / \mathrm{c}$, according to solution 1 of reference 7 .

Figure 4. $R_{\text {recoil }}$ and $A_{\text {recoil }}$, for meson-baryon and baryonbaryon scattering at $20 \mathrm{GeV} / \mathrm{c}$, according to the two alternative definitions of spin-independent scattering defined in the text. Case (a) is denoted by dotted lines, case (b) by solid lines.

Figure 5. $R_{\text {recoil }}$ and $A_{\text {recoil }}$ for $p p$ scattering at 5,20 and $200 \mathrm{GeV} / \mathrm{c}$, according to the model of Ref: 8 .

Figure 6. P for $K^{-}+\mathrm{p} \rightarrow \bar{K}^{0}+\mathrm{n}$ at 5 and $40 \mathrm{GeV} / \mathrm{c}$, according to solution 1 of reference 7.

Final analysis of

polarization


Laboratory


Fig. 1


Fig. 2


Fig. 3


Fig. 4


MUB-6579

Fig. 5


Fig. 6

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