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Abstract

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The purpose of this report is to investigate the effect of selected parameter variations on the response of a platoon. The response of a platoon under different control algorithms is also examined. A four-car platoon with prescribed lead car dynamics was used for simulations and vehicles in the platoon were assumed to be identical. Convergence studies of the numerical integration were undertaken and convergence was verified. A wide range of system parameters, such as the engine response time lag, transport lag, and communication delay was considered and nonlinearities were shown to have a strong effect on the simulation results. In addition, four controllers were implemented to evaluate the response under different types of control logic. The purpose of these simulations was to build a knowledge base with regard to nonlinear platoon responses, to be used as an initial guide in developing simpler analytical models and to aid in identifying those parameters that most significantly affect platooning operations.

Nomenclature

c_a	gain of the acceleration difference for Controllers I and II
c_{a0}	gain of the preceding vehicle's acceleration for Controllers I and II
c_{ij}	gain of the velocity difference between vehicles i and j for Controllers III and IV
c_p	gain of the position difference for Controllers I and II
c_v	gain of the velocity difference for Controller I and II
dt	integration time step, sec
F_a	aerodynamic force, N
F_r	rolling resistance force, N
F_t	force combining the traction and braking force, N
H_i	height of vehicle i, m
k_{ij}	gain of the position difference between vehicles i and j for Controllers III and IV
m_i	mass of vehicle i, kg
x_i	position of vehicle i, m
$\dot{x}_i$	velocity of vehicle i, m/s
$\ddot{x}_i$	acceleration of vehicle i, m/sec^2
Δ_{des}	desired headway spacing, m
τ_c	communication delay, sec
τ_e	engine response time lag, sec
τ_t	transport lag, sec

1 INTRODUCTION

The purpose of this report is to evaluate the qualitative behavior of a platoon under small system parameter variations and under the application of different control algorithms. These preliminary simulations will be used to guide the direction of future investigations for large scale simulations and to provide physical insight into platoon behavior.

Clearly, any realistic automobile model will be both complex and nonlinear. However, as discussed in a previous report [1], a reduced order vehicle model (ROM), which is implemented with several curve-fit expressions, can accurately capture the performance of a relatively sophisticated model. This computationally efficient approach allows extensive simulations of platoon dynamics to be run, simulations that would otherwise take too long to be reasonably undertaken. However, before extensive simulations can begin, the effects of parameter variations around nominal values should be examined in order to identify the most important factors that affect platooning operations. Among the parameters of interest are the integration time step lengths, engine response time lags, transport lags, and communication delays.

Two kinds of control algorithm were considered in this work. The first of these [2] presupposes complete knowledge of the vehicle dynamics and supplies a control action that depends upon the state errors between it and the front vehicle. The second was designed such that the controller took account of both the preceding and the following vehicles. This approach was meant to more closely mimic the response of human drivers, who observe vehicles both ahead of and behind them when controlling their own vehicles in emergency situations. Simulations were carried out on different combinations of platoons and controllers and the response due to various system parameters was investigated qualitatively.

An important finding from the simulation results is that strong couplings exist in the system. Although the nominal performance was similar under the different controllers, small unmodeled perturbations engendered large differences in overall behavior. Attention will focus on the study of platoon collision dynamics iii the next phase of this study. Criteria for determining the impact damage and definitions regarding operational situations will be clarified. Based on the degree of impact severity that leads to a loss of stable platoon operation, qualitative bounds for allowable parameter variations will also be established.

2 SYSTEMMODELS

A platoon consists of a longitudinal series of vehicles, coupled together by means of electronic sensors and controlled through onboard and external computers. The vehicle's behavior is determined by a controller installed in the vehicle. The information that a vehicle can detect is restricted to that which is associated with adjacent vehicles (and its own state, of course). Lead vehicle information is not used by any vehicle except the one immediately following it. Although such lead information could be included in the controller, (this study utilizes Sheikholeslam and Desoer's controller [2]), no lead information is transmitted.

2.1 Dynamics of vehicles

To simplify the vehicle model, a reduced-order system, defined in [1], has been used. Due to the projected rapid-response capability of future throttle and braking actuators, the first time constant has been neglected. Engine thrust is limited by a saturation function. The rolling resistance coefficient is assumed to be .01, and the aerodynamic forces, adopted from the previous report, are a function of the headway spacing and vehicle velocity.

Basically, the dynamics of each vehicle can be expressed as follows.

$$m_i\ddot{x}_i = F_t(x_{i-1}, x_i, x_{i+1}, \dot{x}_{i-1}, \dot{x}_i, \dot{x}_{i+1}, \ddot{x}_{i-1}, \ddot{x}_i, \ddot{x}_{i+1}) - F_a(x_{i-1}, x_i, \dot{x}_i, H_{i-1}) - F_r(m_i) \quad (1)$$

where m_i is the mass of vehicle i,

F_t is the combined traction and braking force,

F_a is the aerodynamic force,

F_r is the rolling resistance force,

x_i is the position of vehicle i,

$\dot{x}_i$ is the velocity of vehicle i,

$\ddot{x}_i$ is the acceleration of vehicle i,

H_{i-1} is the height of vehicle $i-1$.

Nonlinearities are present in the control force, F_t, and the aerodynamic force, F_a. F_t is determined by the preceding and the following state errors and it includes a saturation function [1] and an engine response time lag. F_a scales with the square of the velocity and has a time-varying coefficient, which is determined by the headway spacing and the height of the front vehicle. This represents a first order representation of the drafting effect induced by the preceding vehicle.

2.2 Platoon models

Two four-car platoon models based on the different communication policies have been investigated. Figure 1 shows a platoon composed of vehicles that can only detect information from the preceding car. Figure 2 illustrates a platoon consisting of vehicles for which information from adjacent vehicles is available. They are designated as Platoon I and Platoon II, respectively. Position, velocity, and acceleration data are assumed to be detectable. In addition, the behavior of the lead car is presumed for all the simulations.

2.3 Controller models

Four control algorithms have been applied to the platoon models. The corresponding SIMULAB programs are shown in Figures 3, 4, 5, and 6, respectively.

Controller I: In this controller, a complete knowledge of the vehicle and all forces acting on it has been assumed. These include the engine time lag, the mechanical drag, and the aerodynamic forces. Only the information of the preceding cas is presumed to be available. Controller gains with respect to each state error are assumed to be constant. An additional negative gain has been set to the acceleration of the preceding vehicle. Note that a constant aerodynamical coefficient has been used. This controller approach is directly adopted from Sheikholeslam and Desoer's work [2], except that in our case the traction and braking forces are restrained by the saturation function proposed in an earlier report [1].

Controller II: This control algorithm is similar to that of Controller I, except that the aerodynamics mechanism proposed in [1], in which the drag coefficient is spacing-dependent, has been included. In this model, the aerodynamic drag is reduced linearly as a vehicle approaches a preceding vehicle, thus approximating the drafting effect that is felt between closely spaced vehicles.

Controller III: States of both the preceding and the following vehicles are assumed to be available. Control gains for each state error are presumed to be constant arid equal-weighting gains are assigned to the front and rear tracking errors. Both the aerodynamic and rolling resistance forces are known. Space-depending aerodynamics have been used and knowledge of the engine time lag is assumed to be inaccessible to the controller, iii order to observe the effects that loss of this particular channel of information would induce.

Controller IV: The control algorithm is similar to that of Controller III, except that knowledge of the engine time lag has been included.

Note that delays of the communication and control signals are assumed to be possible for all controllers.

3 SIMULATION RESULTS

For preliminary simulations of platoon dynamics, a four-car platoon was selected. The goal of these simulations was to evaluate the effects of system parameter variations and to compare the results obtained from different combinations of platoons and controllers to command inputs. All the vehicles in the platoon were assumed to be identical, with mass equal to 1800 kg, a height of 1 .O m, **it** length of 3.0 m, and a maximum aerodynamical drag coefficient of 0.4295. To simplify the notation, the following abbreviations are assigned to denote different combinations of platoons and controllers.

P1C1	Platoon I composed of vehicles with Controller I
P1C2	Platoon I composed of vehicles with Controller II
P2C3	Platoon II composed of vehicles with Controller III
P2C4	Platoon II composed of vehicles with Controller IV

The behavior of the lead car is unchanged for all simulations. The configurations of lead acceleration and velocity are shown in Figures 7 and 8, respectively.

Note: Fixed line types have been used in the following plots. For all the spacing-error plots, the solid line represents the spacing error between the lead and vehicle one; the dashed line symbolizes the spacing error between vehicle one and vehicle two; and the dotted line stands for the spacing error between vehicle two and vehicle three. For all the acceleration plots, accelerations of the lead vehicle, vehicle one, vehicle two, and vehicle three are represented by the solid, dashed, dotted, and dashdot lines, respectively.

3.1 Variation of the integration time step, dt

For a well-posed system, the effect of time discretization is usually not a problem. However, the embedded nonlinearities make this problem a stiff one. The implementation of the controller also complicates the overall dynamics. Therefore, it is prudent to see how the choice of an integration time step affects the platoon dynamics.

P1C1 and P2C3 were used to evaluate the effects of the integration time step. The integration algorithm applied was the Runge-Kutta 4-th and 5-th order method. The engine response time lag, τ_e, was set to 0.2 sec. The controllers' gains were c_p = 91.99, c_v = 80.96, c_a = 17.56, c_{a0} = -5.15 (refer to Figure 3), k_{ij} = 100,000, and c_{ij} = 100,000 (refer to Figure 5), and four integration time steps, dt = 0.01, 0.05, 0.1, and 0.2 sec were applied.

Figures 9, 10, 11, and 12 show the spacing error and acceleration profiles of the P1C1 platoon corresponding to dt = 0.01, 0.05, 0.1, and 0.2 sec. As can be observed in Figures 9-1 1, the rear cars have slightly larger maximum spacing error and acceleration/deceleration magnitudes. Moreover, the performance of this system for dt < 0.2 sec will not cause a convergence problem. However, the case for dt = 0.2 sec (dt equal to τ_e) is highly degraded, especially for the rear cars. These fluctuations are attributable to the rapid switching of the controller and the saturation of the engine force.

Figures 13, 14, 15, and 16 illustrate the spacing error and acceleration profiles of the P2C3 platoon corresponding to dt = 0.01, 0.05, 0.1, and 0.2 sec. Contrary to the previous case, the spacing errors are wide-spread, but the rear car has a slightly larger acceleration/deceleration magnitude. Similar to the case P 1 C1, the tracking becomes unacceptable when the integration time step is equal to the engine response time lag. These fluctuations occur for the same reasons as in the previous case. One can also note that the switching rate is larger.

The conclusion from the simulation results is that we can have confidence in the simulations' fidelity as long as the time discretization is less than or equal to half the engine's own inherent delay. It should be noted from these results that while the forward looking controller keeps the vehicles close to a predetermined spacing, the forward/backward controller keeps each vehicle centered between the preceding and following vehicles. The error in spacing for each vehicle from dead center is shown in Figure 17. It can be seen that the controller does an excellent job of maintaining equal spacing to the front and rear.

3.2 Variation of the engine response time lag, τ_e

Engine time lag, a measure of the engine force's response delay, depends upon the engine model, the vehicle's inertia, vehicle's profile, etc. To investigate the effects of engine time lag on the platoon dynamics, P1C1 and P2C3 were used with an integration time step of 0.2 sec.

For P1C1, Figures 18, 19, 12, and 20 show the platoon response with respect to τ_e = 0.01, 0.1, 0.2, and 0.3 sec. Contrary to what one might expect, the system behaves well for the cases of $\tau_e < dt$. Moreover, for τ_e = 0.2 and 0.3 sec, the response profiles degrade markedly, especially for car three. Focusing on Figure 20, it is found that the tracking of car one is unchanged. Nevertheless, the spacing error of car two shifts to the negative side, and the response of car three becomes ragged.

For P2C3, the response is even worse. As observed from Figures 21 and 22, which correspond to $\tau_e =$ 0.1 sec. and $\tau_e =$ 0.3 sec. respectively, the spacing errors become larger and more oscillatory than those of previous cases in Figures 13, 14, and 15. One of the reasons for fluctuation can be attributed to the control algoritliin. Apparently, the tracking error is affected not only by the state errors with respect to the preceding car, but also by those of the following car. The propagation of a rebounding error wave degrades the tracking performance. This phenomenon can be observed from Figure 23, which shows the impulse response corresponding to the P1C1 arid P2C3.

For these two nonlinear platoon models, the interaction between the integration time step and the engine response time lag has been shown to be more complicated than initially expected. Due to the complete knowledge of engine response time lag in Controller I, the response of the P1C1 platoon seems to be more robust than that of the P2C3 platoon, which applies the back control algorithm. However, since the optimal control gains of the back controller have not been investigated, the real relationship between these parameters arid the control algorithms needs to be determined by further work.

3.3 Variation of the detection delay

It is possible that time delays exist in the detection system. Such transportation lags will cause the controller to take actions that are based on past data. To investigate the platoon response under a detection delay environment, two kinds of delays, transportation lag and communication delay, have been considered.

3.3.1 Transportation delay, τ_t

The existence of transportation lag implies that both the information with regard to the adjacent vehicles as well as the controlled vehicle's state information have been retarded by a constant time. P1C1 and P2C3 have been used to examine the effects of such a delay. In both systems, the integration time step was set to 0.01 sec. and the engine time lag to 0.1 sec.

Figures 24, 25, and 26 show the P1C1 platoon response corresponding to τ_t = 0.05, 0.1, and 0.15 sec.. It is noted that the rearward vehicles accelerations become oscillatory and saturated, as the transportation lag increases. At the same time, the spacing errors increase. The response of the P2C3 platoon with respect to τ_t = 0.05 sec. is shown in Figure 27. Compared to the performance of the P1C1 platoon, it would appear that the robustness of Controller III is inferior. It seems reasonable that this phenomenon is attributed to the characteristics of Controller I, which has a complete knowledge of the engine response time lag. However, the interaction between τ_e and τ_t plays an important role in Controller III, which treats the engine response time lag as an unmodeled parameter.

3.3.2 Communication delay, τ_c

Communication delay means that the controller knows its own states, but the information transmitted from other vehicles is delayed by a constant time. PlC2 and P2C4 are chosen to demonstrate the effects of the communication delay. dt = 0.01 sec. and τ_e = 0.1 sec. have been used in these simulations.

Figures 28, 29, and 30 illustrate the response of the P1C2 platoon with respect to τ_c = 0.0, 0.01, and 0.05 sec. Due to the reaction delay characterized by τ_e, steady state spacing errors exist in Figures 29 and 30. The spacing error increases dramatically as τ_c increases, and the steady state spacing error is exactly equal to the value of the steady state velocity multiplied by τ_c. Moreover, the acceleration satiirates and thus the system requires more time to achieve steady state.

Figures 31 and 32 illustrate the P2C4 platoon response with respect to τ_c = 0.0 and 0.05 sec. The behavior of the P2C4 system is different from that of PlC2. As observed from Figure 32, the spacing errors increase even more dramatically than those of the P1C2 platoon for the same τ_c. Because the front vehicle accumulates the state errors not only from the preceding vehicle, but also from the following vehicle, the engine force will become saturated for a smaller τ_c and oscillate during the steady state stage. Apparently, the error waves, which propagate back and forth due to the back control algorithm (as observed from Figure 23), are responsible for this response. This is supported by the character of the spacing error and the small-scale acceleration chattering shown in Figure 32 during the steady state stage. An interesting fact is that the steady state spacing error for car three is the same as the case in Figure 30; the steady state spacing error for car two is three times of that for car three, and the steady state spacing error for car one is five times of that for car three. In addition, the steady state spacing error corresponding to the center point is 1.095 m for cars two and three, which is equal to the value of the steady state velocity multiplied by τ_c.

4 SUMMARY

This report has documented the qualitative results of platoon simulations under several control configurations and for it variety of system parameters. The following qualitative observations can be made.

4.1 Controllers and the propagation of the disturbance wave

It seems that the controllers I and II, which only take notice of state errors between themselves and preceding cars have superior performance as compared with controllers using back control. One reason for this is the reflection of an internal disturbance wave. As mentioned previously, the disturbance wave will propagate back and forth, thus degrading the platoon's performance. As a result, the spacing errors of the front vehicles will be larger than those of the rear vehicles. This will become worse when the platoon is composed of a large number of vehicles. On the other hand, the one-way controller cuts off the rebound wave and seems to be more robust. However it should be noted that the comparison used a back controller for which no optimization had been clone. It may well be that a redesigned back controller, which explicitly damps out internal reflections, will show significantly improved performance. It is also believed that the assumption of an equal weighting for the front and rear state errors in the back control algorithm played an important role. It seems that information regarding the following vehicles is important while operating under emergency situations or exiting from a platoon. A modification of the control algorithm, which uses only front control for the nominal operation and applies the back control logic to the case of emergency maneuvering, is being conducted. The investigation of this smart controller will be discussed in the next report.

4.2 Integration time step, engine response time lag, and detection delay

It has been noted that the interrelationship among the integration time step, engine time lag, and detection delays is quite complicated. As observed from the simulation results, the system response becomes unacceptable when the engine force saturates. However, specific values of such parameters which allow satisfactory tracking have not been determined. As expected, the best policy towards maintaining good platoon operation is to minimize any detection delays and to keep integration time steps as small as possible for simulations.

5 FUTURE WORK

A primary aim of this project is to examine platoon collision dynamics. Before this can occur, several parameters regarding realistic- collision simulations must be determined. Among these are the physical characteristics of vehicle bumpers and their response under loading, the impact severity that leads to bumper failure and subsequently to vehicle damage, and criteria for determining when sensor failure occurs as a result of a collision. The end result will he a nominal vehicle characterization which includes a realistic range of shock absorption capability, structural integrity and sensor ruggedness.

Once these parameters have been defined, extensive platoon simulations under finite disturbance conditions can begin. In keeping with the observations of this report, it is believed that the collision propagation dynamic-s will be strongly dependent upon the control algorithms in use with the vehicles. Moreover, the intrinsic response dynamics of the vehicles arid the dynamic behavior of the vehicles' bumpers in absorbing and transmitting the energy of a collision will play very important roles. These topics will be ***examined*** and bounds for the relative parameters will be established for the degree of impact severity that leads to a loss of stable platoon operation. Maneuvering in the face of internal collisions and vehicle entry and exit from the platoon will be examined.

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A Numerical Engine Model

According to a previous research [1], which reduced the engine dynamics by a set of curve-fit equations, the maximum traction, F_{tfm}, and the engine response delay, τ, can be expressed as follows.

$$F_{tfm}(\phi, v) = \alpha(1 - e^{-\beta\phi})^{\gamma} \tag{2}$$

$$\tau(\dot{\phi}) = 2.0905\ \dot{\phi}^{-0.7033} \tag{3}$$

where

$$\alpha = 1.0 \times 10^{3}(-0.0053v + 2.7404)$$

$$\beta = 1.0 \times 10^{-3}(0.0613\ v + 101.9315)$$

$$\gamma = 1.0 \times 10^{-3}(18.8640\ v + 855.0600)$$

F_{tfm} is a function of throttle angle, ϕ, and vehicle velocity, v, and the engine response delay depends on throttle rate only.

B Longitudinal Vehicle Dynamics

The longitudinal dynamics of a vehicle can be simply expressed by Newton's second law:

$$\ddot{x}_i = \frac{1}{M_i}(F_{tf} - F_b - F_r - F_a - F_g) \tag{4}$$

where $\ddot{x}_i$ is the acceleration of vehicle i,

M_i is the vehicle mass,

F_{tf} is the traction force.

F_b is the braking force,

F_r is the rolling resistance force,

F_a is the aerodynamic: force,

F_g is the gravitational force caused by tlir road grade.

Equations for related forces terms are following.

$$F_b = \alpha \ F_{b,max} (1 - e^{-\frac{t}{\tau_b}}) \quad (5)$$

$$F_r = M \ ;g \ f_r \ \cos(\lambda) \quad (6)$$

$$\begin{aligned} f_r = & (0.4864 \text{ x } 10^{-3} G_0 - 0.0103 \text{ x } 10^{-6}) v^3 + \\ & (-0.0952 \ G_0 + 1.1425 \text{ x } 10^{-6}) \ v^2 + \\ & (7.0982 \ G_0 - 0.0310 \text{ x } 10^{-3}) v + 0.01 \end{aligned} \quad (7)$$

$$F_a = \begin{cases} 0 & \text{if } \Delta \leq .5H \\ 0.4 \, (\frac{\Delta}{H} - .5) \, C_a \ v^2 & \text{if } .5H < \Delta \leq 3H \\ C_a \ v^2 & \text{if } 3H < \Delta \end{cases} \quad (8)$$

$$F_g = M \ g \quad \sin(X) \quad (9)$$

where α is the braking percentage,

$F_{b,max}$ is the maximum braking force (11723.2 N),

t is the braking time,

τ_b is the time constant of the braking system,

f_r is the rolling resistance coefficient,

λ is the road grade,

C_a is the aerodynamical coefficient (0.4295),

H is the height of the front vehicle (1 m),

Δ is the headway spacing of vehicle,

G_0 is the road roughness coefficient,

$0.4050 \text{ x } 10^{-6} \leq G_0 \leq 6.400 \text{ x } 10^{-6}$, for highways.

C Control Laws

C.1 Platoons with Lead Vehicle Information

The longitudinal control laws proposed by Sheikholeslam and Desoer, for the platoons with lead vehicle information, are as follows.

$$\begin{aligned} c_1 &= c_{p1}\Delta_1^f(t) + c_{v1}\dot{\Delta}_1^f(t) + c_{a1}\ddot{\Delta}_1^f(t) + k_{v1}(v_l(t) - v_0) + k_{al}a_l(t) \quad (10) \\ c_i &= c_p\Delta_i^f(t) + c_v\dot{\Delta}_i^f(t) + c_a\ddot{\Delta}_i^f(t) + k_v(v_l(t) - v_i(t)) \\ &+ k_a(a_l(t) - a_i(t)) \quad (11) \end{aligned}$$

And the control laws for Back controller are following.

$$\begin{aligned} c_1 &= \Gamma_{1p}(\Delta_1^f, \Delta_1^f, \Delta_1^r, t) + \Gamma_{1v}(\dot{\Delta}_1^f, \dot{\Delta}_1^r, t) + \Gamma_{1a}(\ddot{\Delta}_1^f, \ddot{\Delta}_1^r, t) + k_{v1}(v_l(t) - v_0) + k_{al}a_l(t) \quad (12) \\ c_i &= \Gamma_p(\Delta_i^f, \Delta_i^r, t) + \Gamma_v(\dot{\Delta}_i^f, \dot{\Delta}_i^r, t) + \Gamma_a(\ddot{\Delta}_i^f, \ddot{\Delta}_i^r, t) + k_v(v_l(t) - v_i(t)) \\ &+ k_a(a_l(t) - a_i(t)) \quad (13) \end{aligned}$$

where

$$\Delta_i^f = x_{i-1}(t) - x_i(t) - L \quad (14)$$

$$\Delta_i^r = x_i(t) - x_{i+1}(t) - L \quad (15)$$

$$\Gamma_{1p}(\Delta_1^f, \Delta_1^r, t) = c_{p1}(1 - \frac{2}{2+\varphi_1(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\varphi_1(t)}))) \quad (16)$$

$$\Gamma_{1v}(\dot{\Delta}_1^f, \dot{\Delta}_1^r, t) = c_{v1}(1 - \frac{2}{2+\dot{\varphi}_1(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\dot{\varphi}_1(t)}))) \quad (17)$$

$$\Gamma_{1a}(\ddot{\Delta}_1^f, \ddot{\Delta}_1^r, t) = c_{a1}(1 - \frac{2}{2+\ddot{\varphi}_1(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\ddot{\varphi}_1(t)}))) \quad (18)$$

$$\Gamma_p(\Delta_i^f, \Delta_i^r, t) = c_p(1 - \frac{2}{2+\varphi_i(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\varphi_i(t)}))) \quad (19)$$

$$\Gamma_v(\dot{\Delta}_i^f, \dot{\Delta}_i^r, t) = c_v(1 - \frac{2}{2+\dot{\varphi}_i(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\dot{\varphi}_i(t)}))) \quad (20)$$

$$\Gamma_a(\ddot{\Delta}_i^f, \ddot{\Delta}_i^r, t) = c_a(1 - \frac{2}{2+\ddot{\varphi}_i(t)})(1 + 2(1 - sgn(1 - sgn(1 - \frac{2}{2+\ddot{\varphi}_i(t)}))) \quad (21)$$

and

$$\varphi_i(t) = \Delta_i^f - .5\Delta_i^r(1-sgn(\Delta_i^r)) \tag{22}$$

$$\dot{\varphi}_i(t) = \dot{\Delta}_i^f - .5\dot{\Delta}_i^r(1-sgn(\dot{\Delta}_i^r)) \tag{23}$$

$$\ddot{\varphi}_i(t) = \ddot{\Delta}_i^f - .5\ddot{\Delta}_i^r(1-sgn(\ddot{\Delta}_i^r) \tag{24}$$

where subscript i is refered to Car i (l for the lead car), c_i is the control law for Car i, L is the desired headway spacing, $x_i(t)$ is the position of Car i, A!(t) and $\Delta_i^r(t)$ are corresponding to the front and rear state errors of Car i, respectively. Control gains are following.

$(c_{p1}, c_{v1}, c_{a1}, k_{v1}, k_{a1})$	$(120, 74, 15,$ -0.05, -3.03)
$(c_p, c_v, c_a, k_v, k_a)$	$(120, 49, 5, 25, 10)$

C.2 Platoons without Lead Vehicle Information

The longitudinal control laws proposed by Sheikholeslam and Desoer, for platoons without lead vehicle information, are as follows.

$$c_i = c_p\Delta_i^f(t) + c_v\dot{\Delta}_i^f(t) + c_a\ddot{\Delta}_i^f(t) + k_c a_{i-1}(t) \tag{25}$$

And the control laws for Back controller are following.

$$c_i = \Gamma_p(\Delta_i^f, \Delta_i^r, t) + \Gamma_v(\dot{\Delta}_i^f, \dot{\Delta}_i^r, t) + \Gamma_a(\ddot{\Delta}_i^f, \ddot{\Delta}_i^r, t) + k_c a_{i-1}(t) \tag{26}$$

where

$$\Delta_i^f = x_{i-1}(t) - x_i(t) - L \tag{27}$$

$$\Delta_i^r = x_i(t) - x_{i+1}(t) - L \tag{28}$$

$$\Gamma_p(\Delta_i^f, \Delta_i^r, t) = c_p(1 - \frac{2}{2+\varphi_i(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\varphi_i(t)}))) \tag{29}$$

$$\Gamma_v(\dot{\Delta}_i^f, \dot{\Delta}_i^r, t) = c_v(1 - \frac{2}{2+\dot{\varphi}_i(t)})(1 + 2(1 - sgn(1 - \frac{2}{2+\dot{\varphi}_i(t)}))) \tag{30}$$

$$\Gamma_a(\ddot{\Delta}_i^f, \ddot{\Delta}_i^r, t) = c_a(1 \frac{2}{2+\ddot{\varphi}_i(t)})(1 + 2(1 - sgn(1 - sgn(1 - \frac{2}{2+\ddot{\varphi}_i(t)}))) \tag{31}$$

and

$$\varphi_i(t) = \Delta_i^f - .5\Delta_i^r(1 - sgn(\Delta_i^r)) \quad (32)$$

$$\dot{\varphi}_i(t) = \dot{\Delta}_i^f - .5\dot{\Delta}_i^r(1 - sgn(\dot{\Delta}_i^r)) \quad (33)$$

$$\ddot{\varphi}_i(t) = \ddot{\Delta}_i^f - .5\ddot{\Delta}_i^r(1 - sgn(\ddot{\Delta}_i^r) \quad (34)$$

where subscript i is refered to Car i (0 for the lead car), c_i is the control law for Car i, L is the desired headway spacing, $x_i(t)$ is the position of Car i, $\Delta_i^f(t)$ and $\Delta_i^r(t)$ are corresponding to the front and rear state errors of Car i, respectively. Control gains are following.

$$(c_p, c_v, c_a, k_c) \qquad (91.99, 80.96, 17.56, -5.15)$$

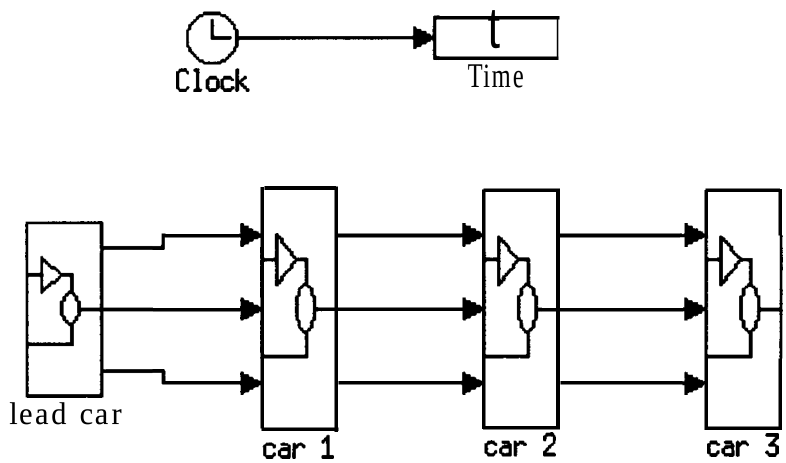


Figure 1. Platoon Model I

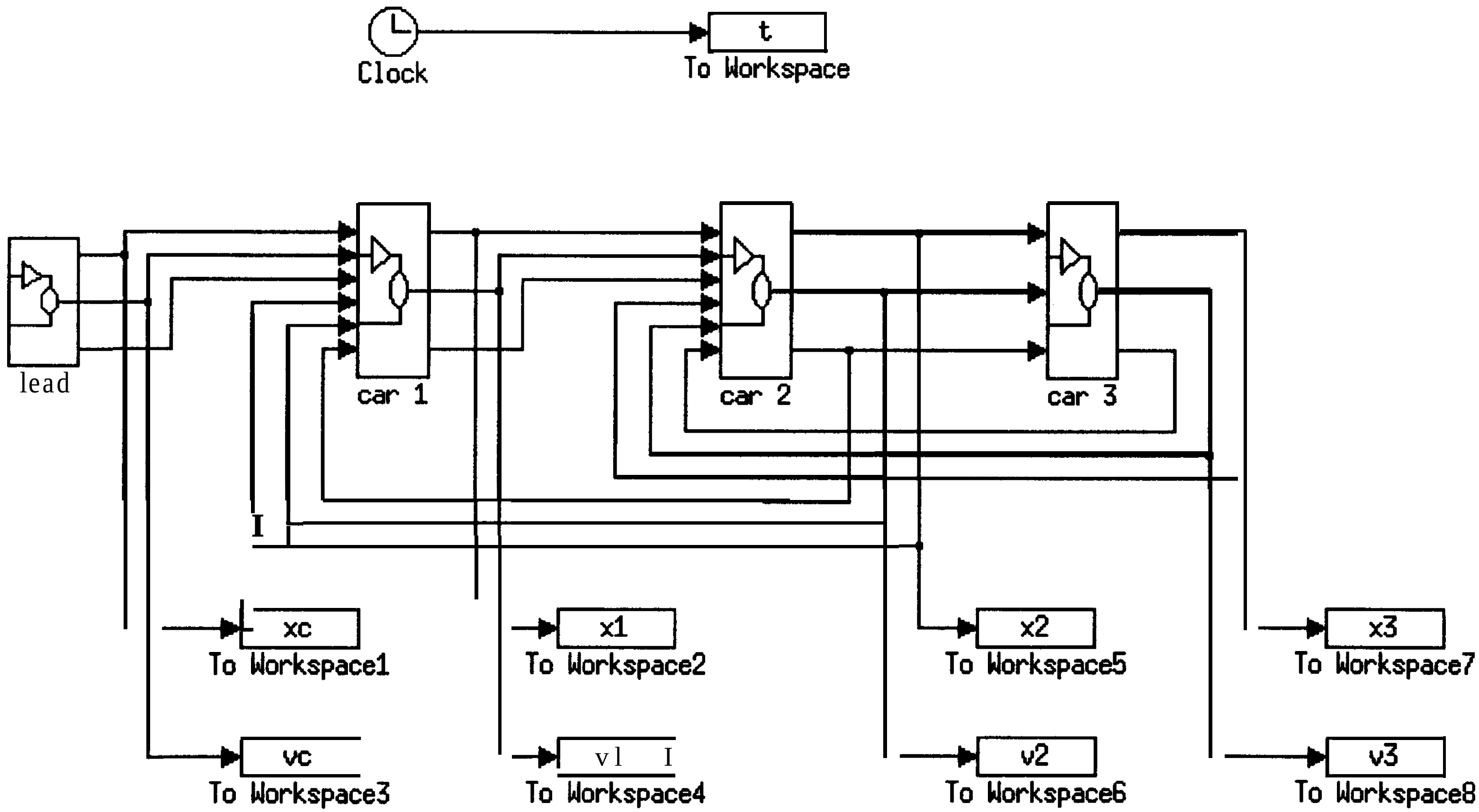


Figure 2. Platoon Model II

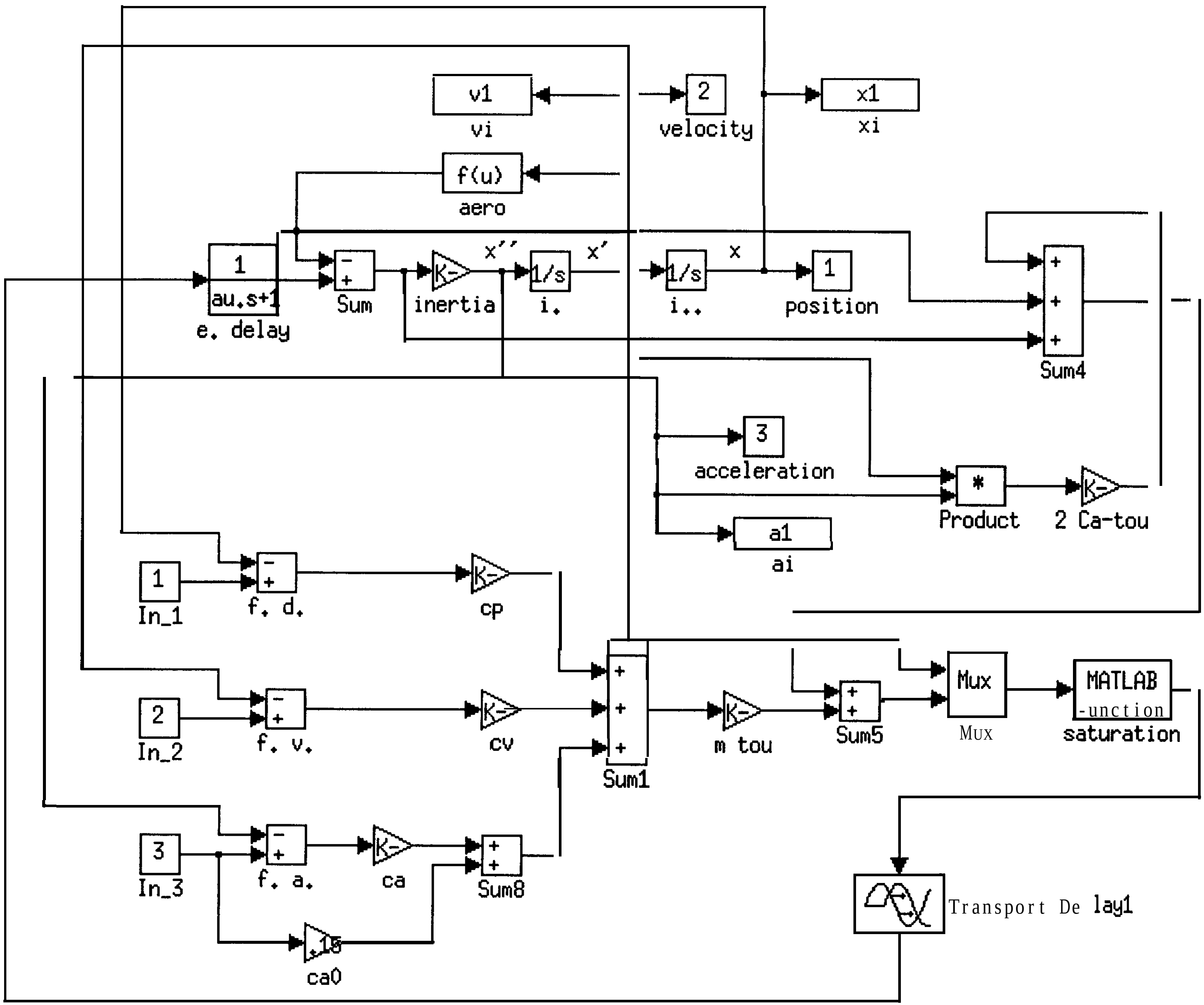


Figure 3. Controller I

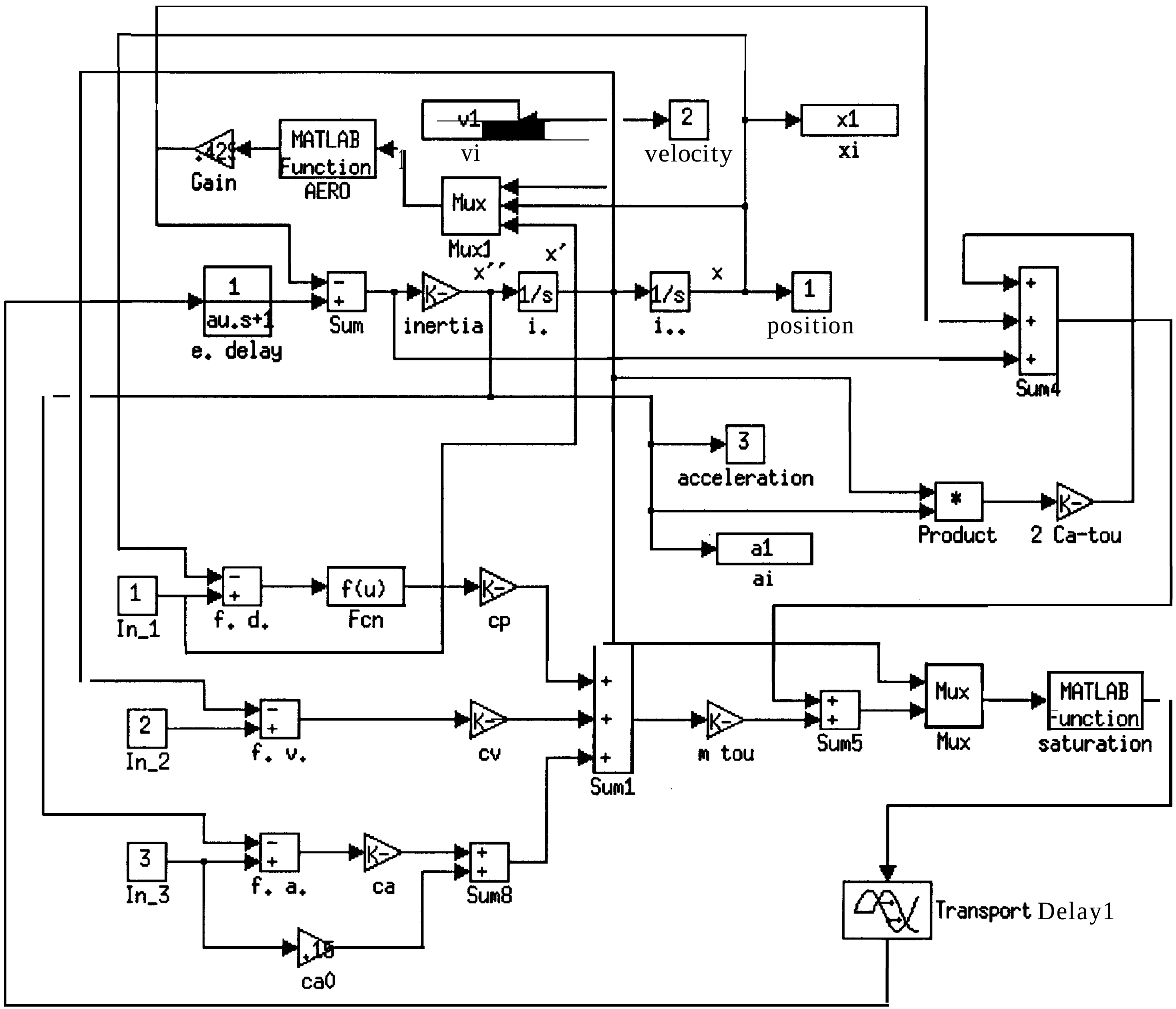


Figure 4. Controller I I

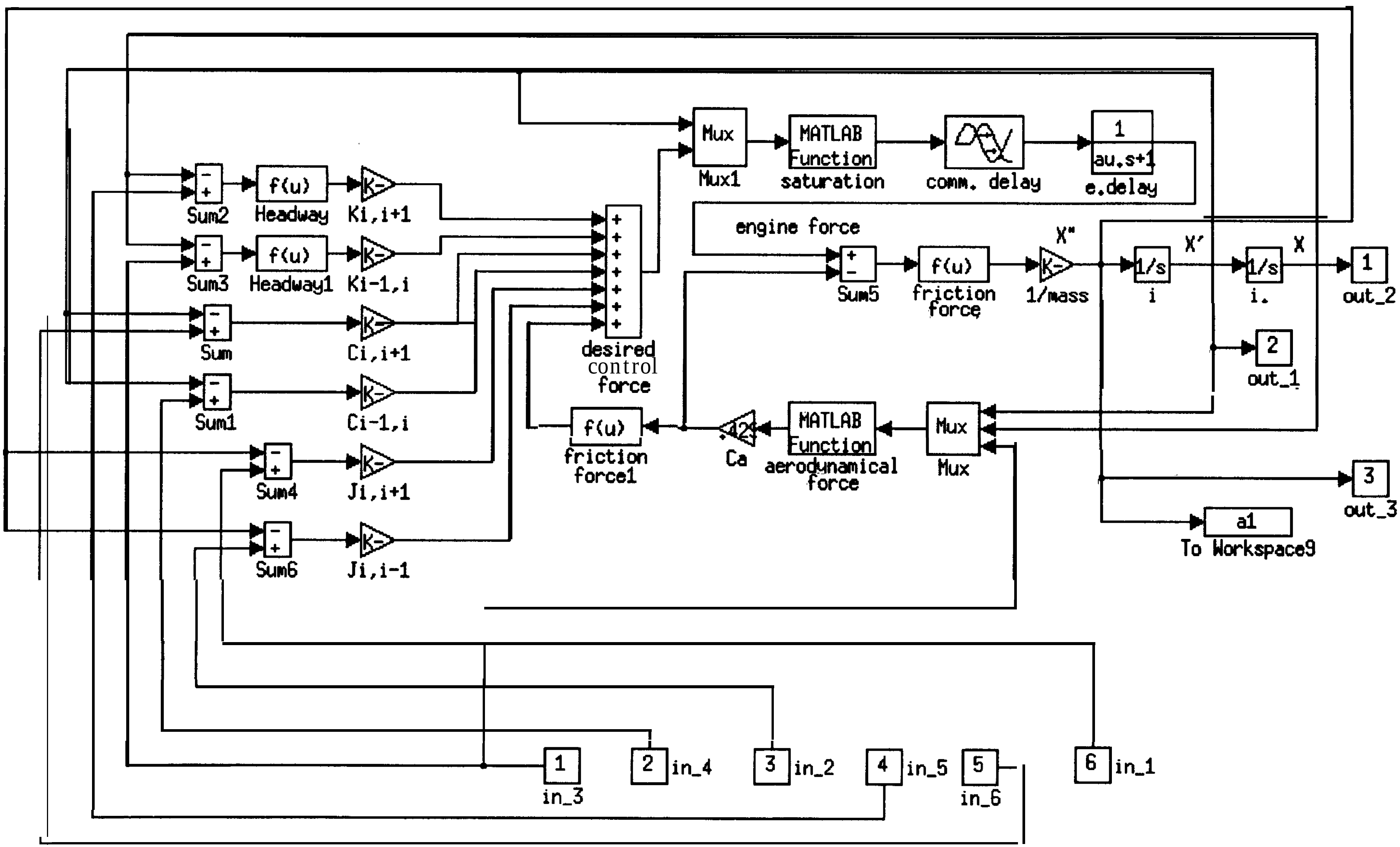


Figure 5. Controller III

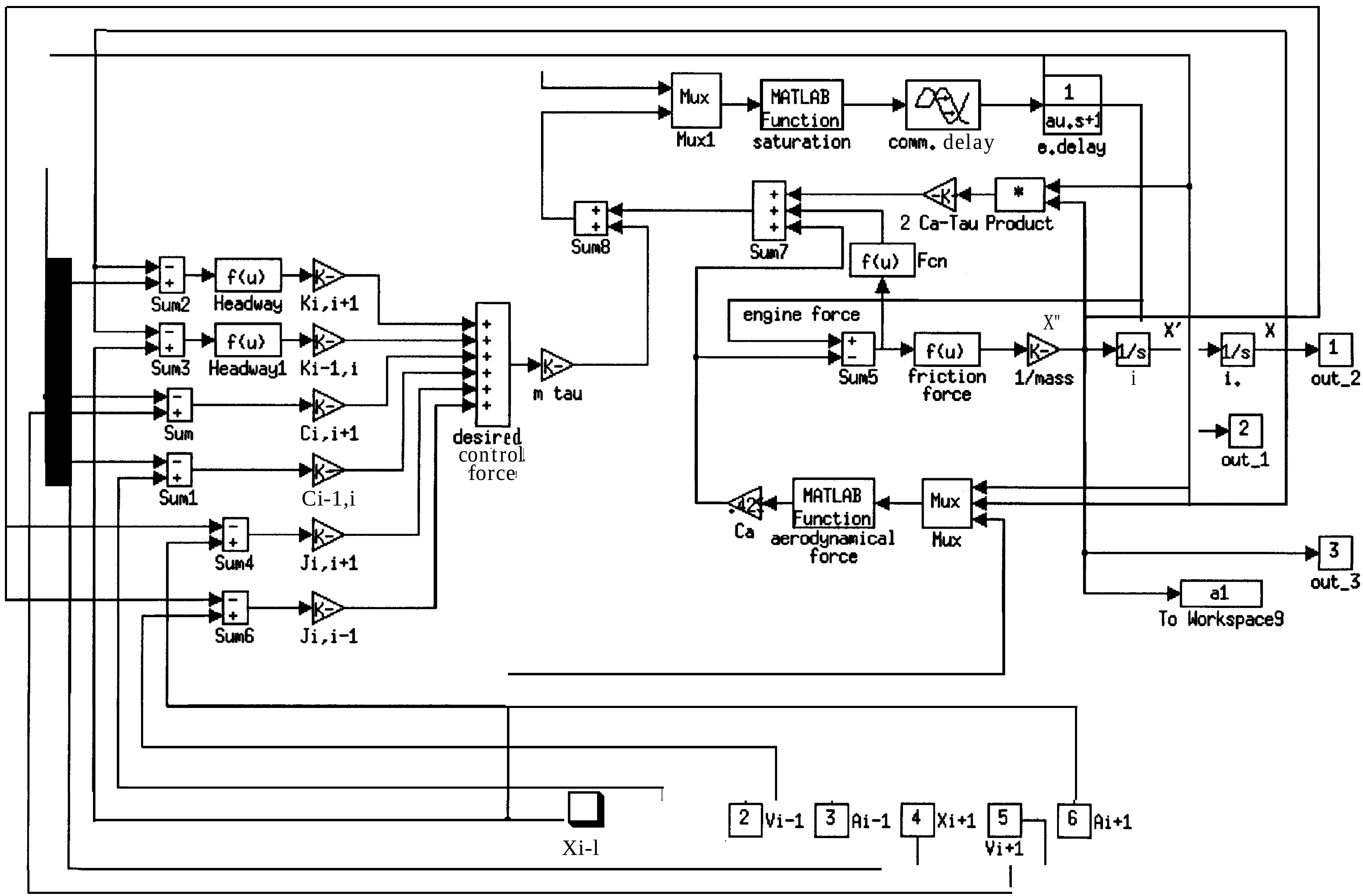


Figure 6. Controller IV

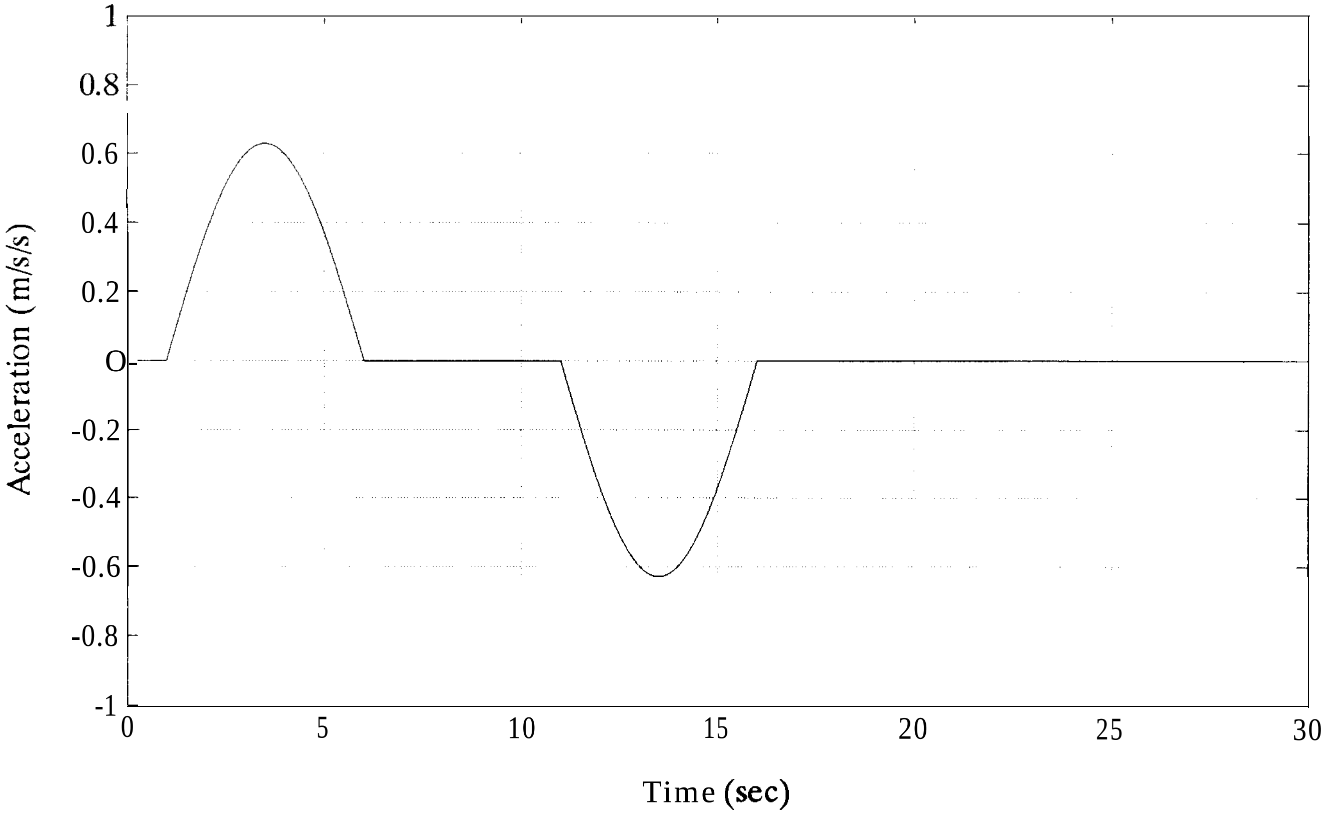
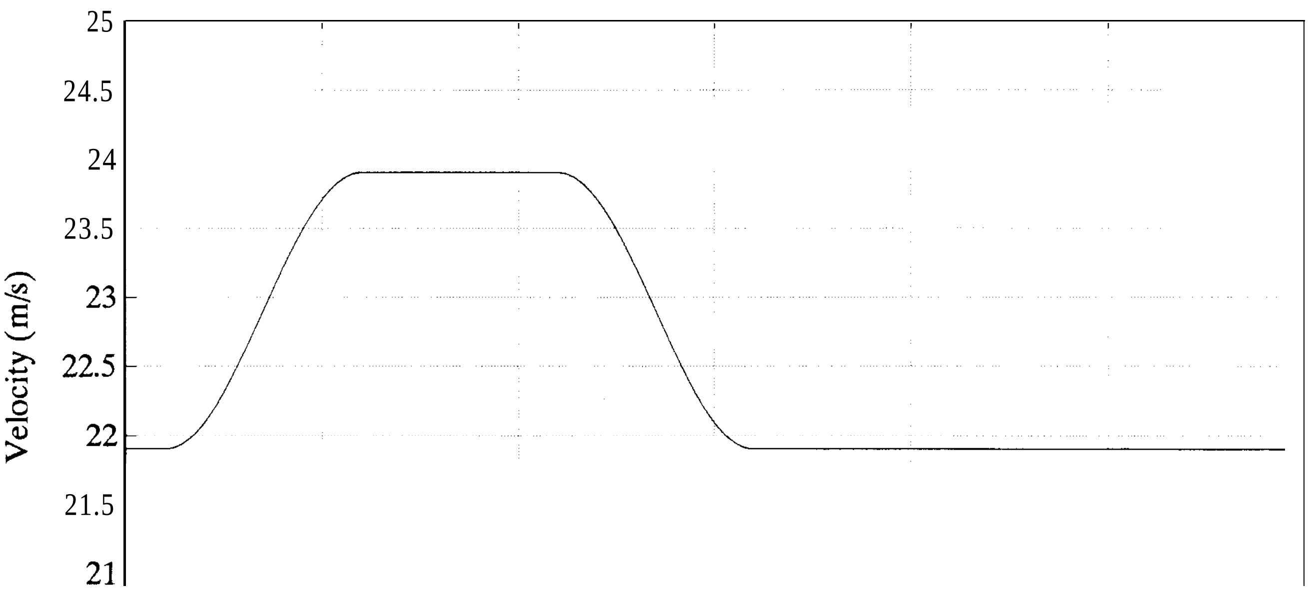


Figure 7. Acceleration Profile of Lead Vehicle



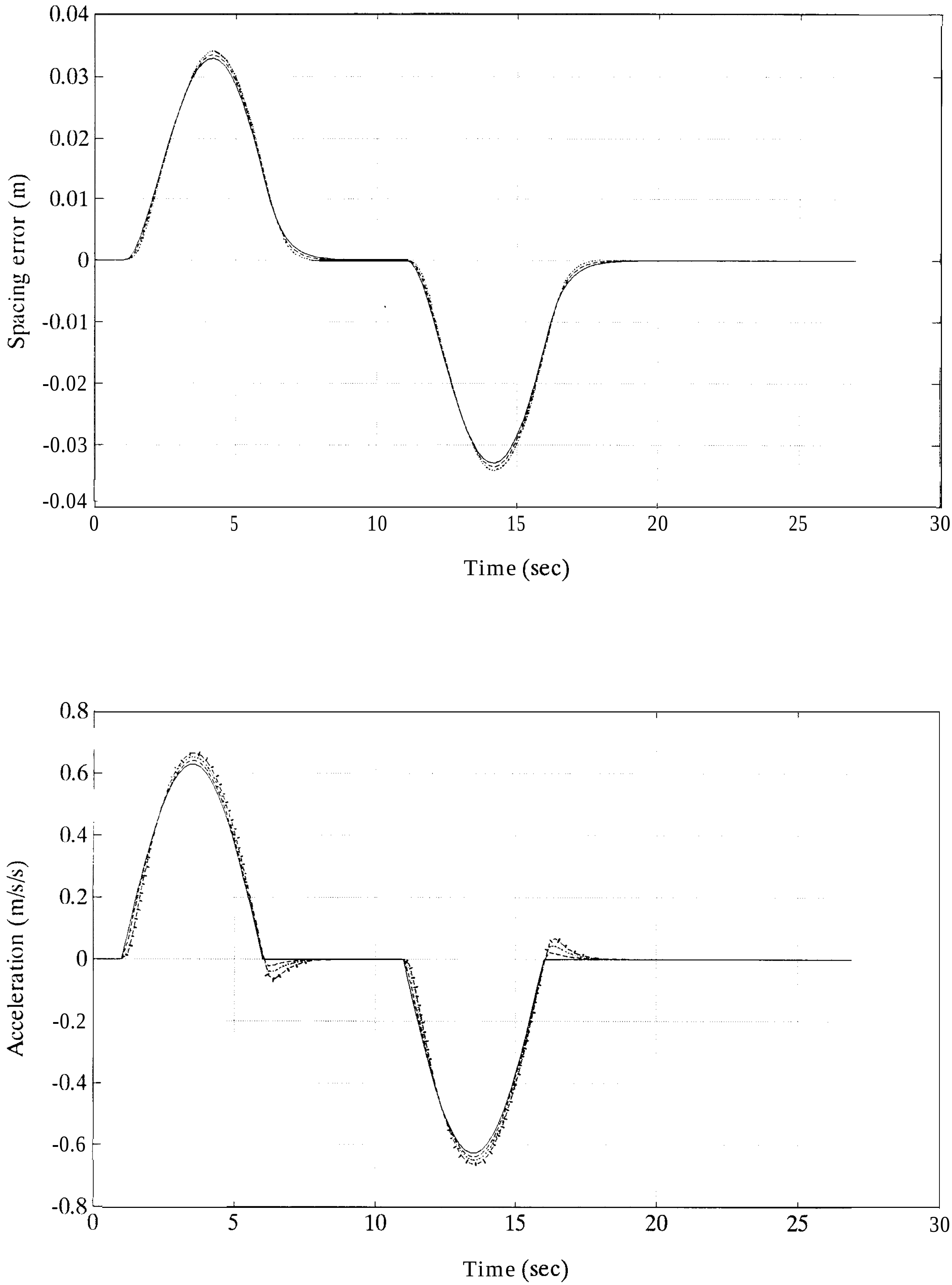


Figure 9. P1C1 Platoon Response for $dt = 0.01\,sec$ ($\tau_e = 0.2\ sec$)

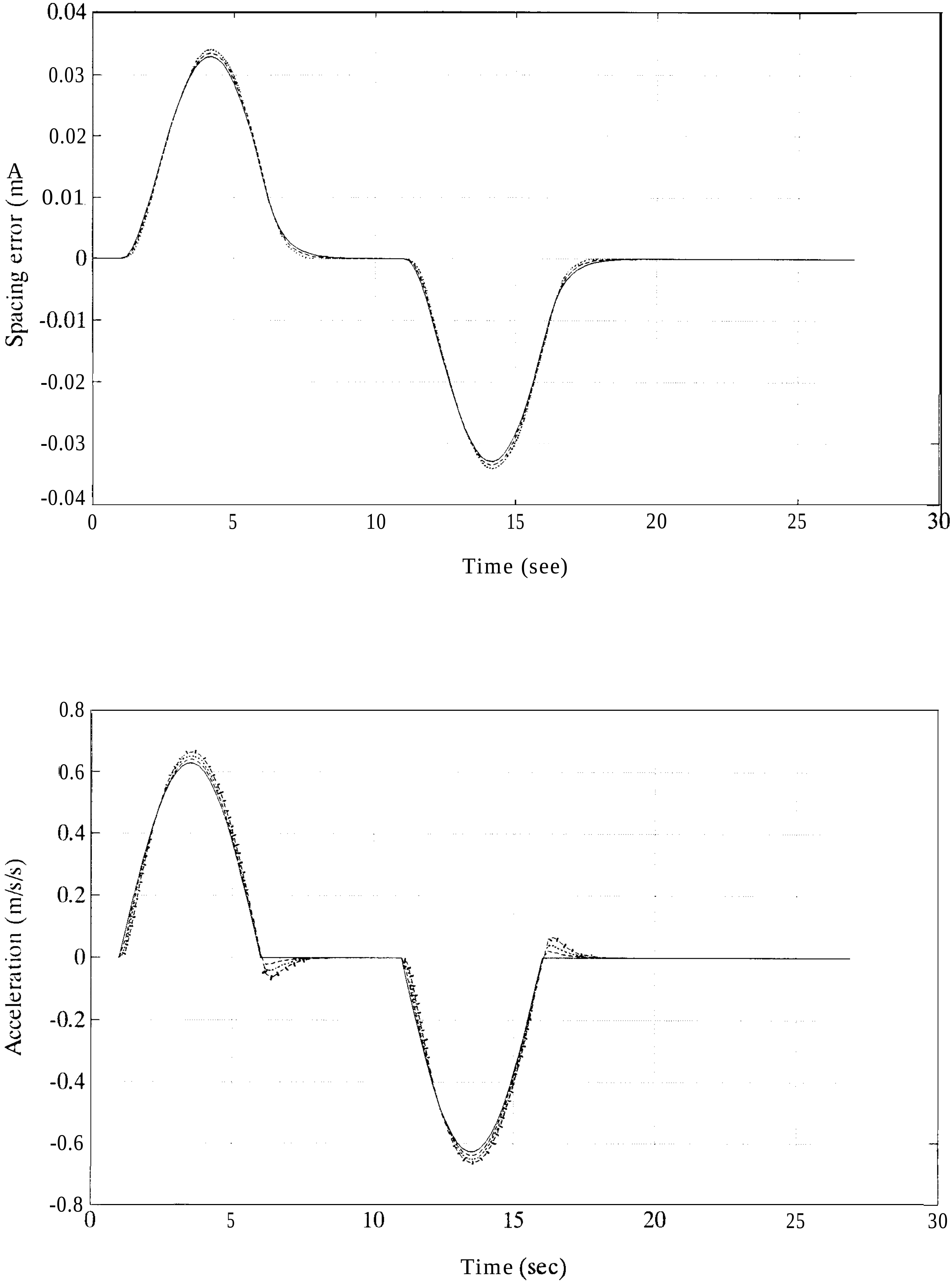


Figure 10. P1C1 Platoon Response for dt = 0.05 sec (τ_e = 0.2 sec)

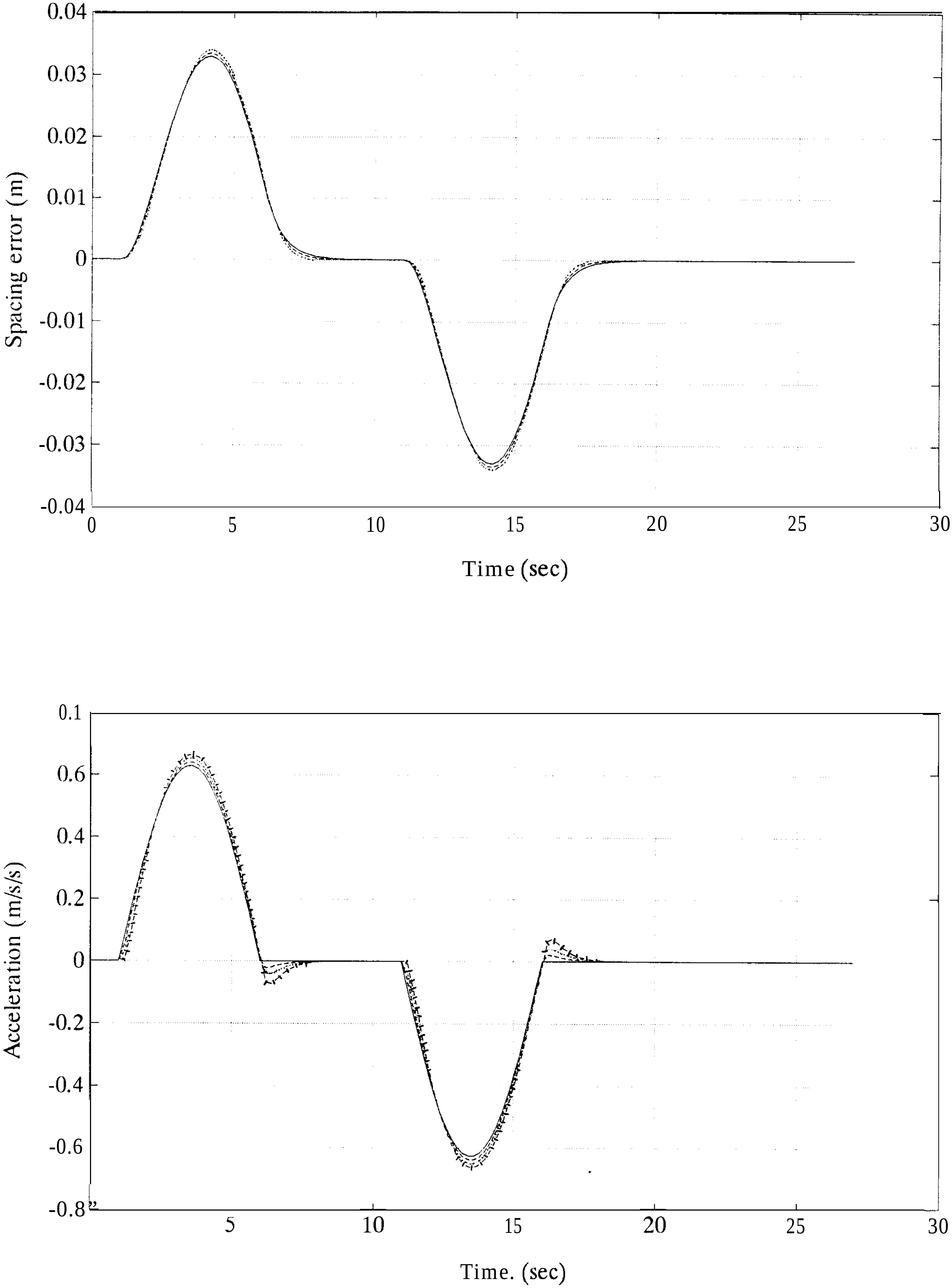


Figure 11. P1C1 Platoon Response for dt = 0.1 sec (τ_e = 0.2 sec)

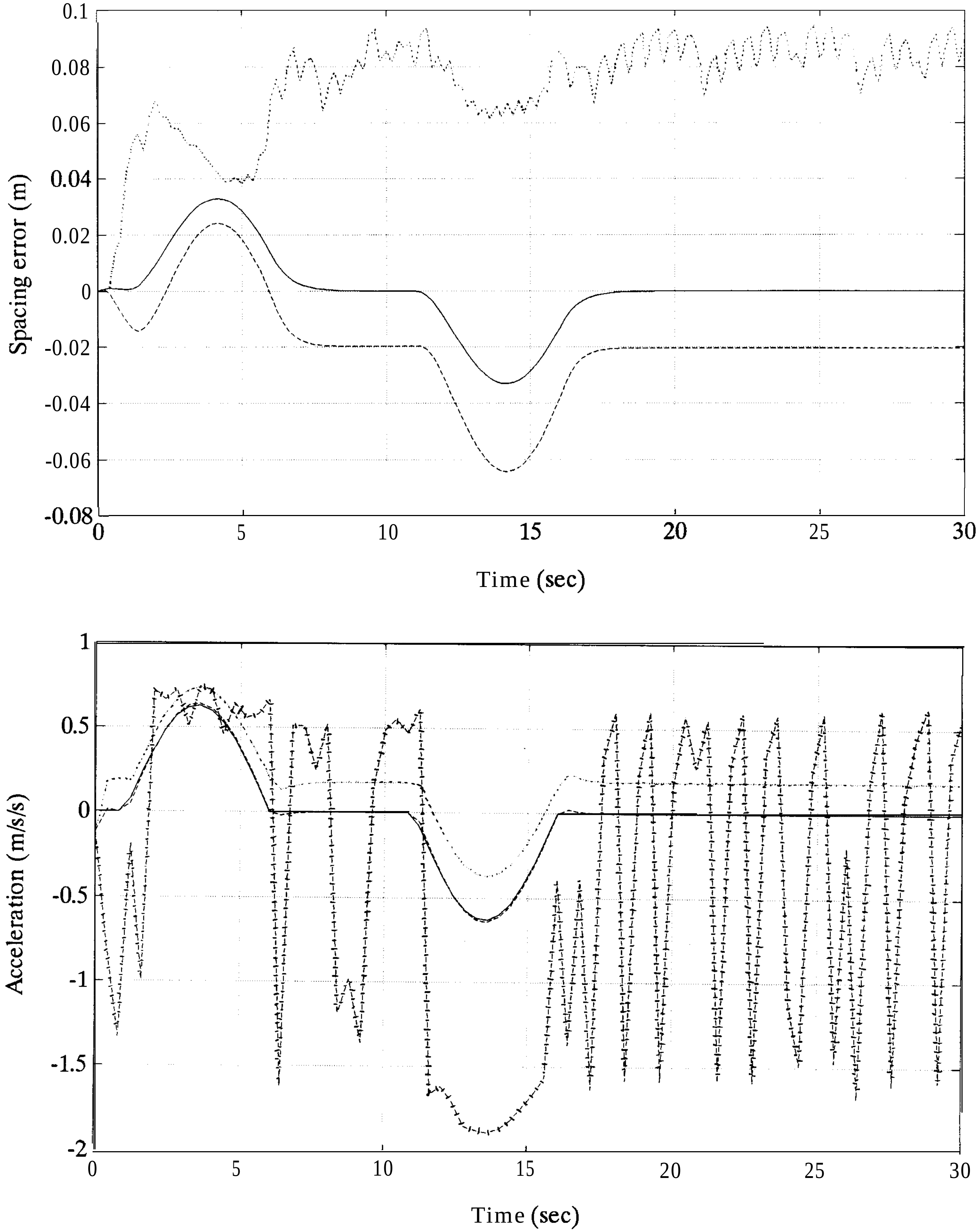
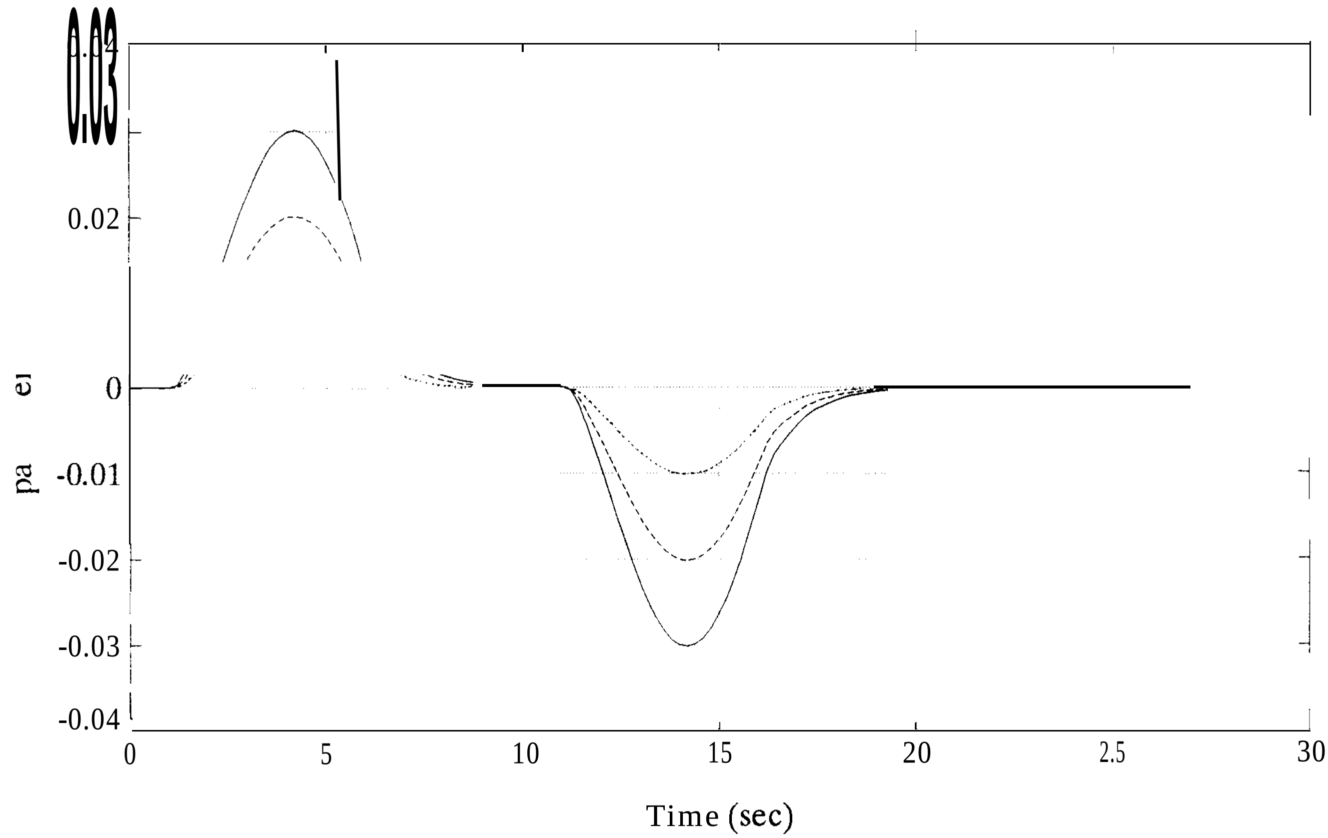


Figure 12. P1C1 Platoon Response for $dt = 0.2\,sec$ ($\tau_e = 0.2\ sec$)



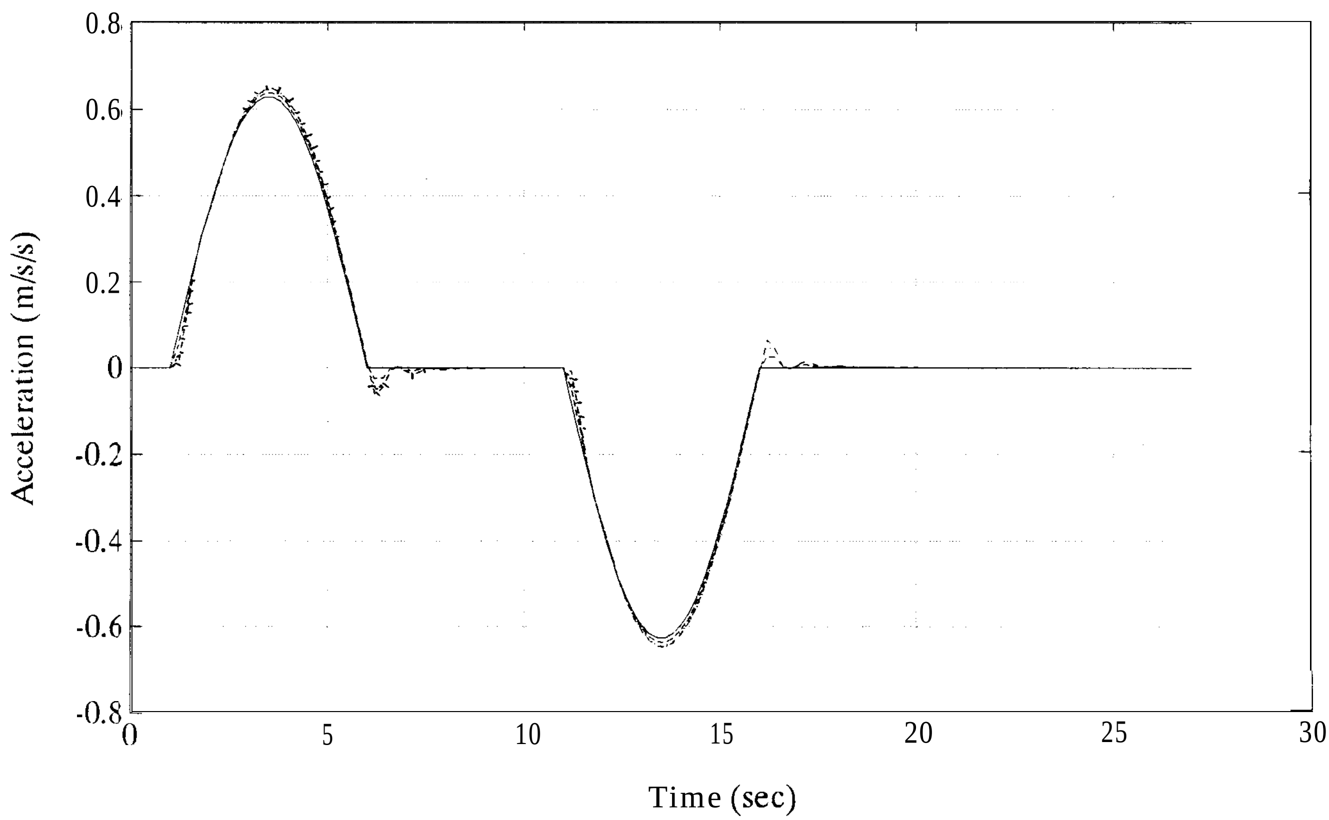


Figure 13. P2C3 Platoon Response for dt = 0.01 sec (τ_e = 0.2 sec)

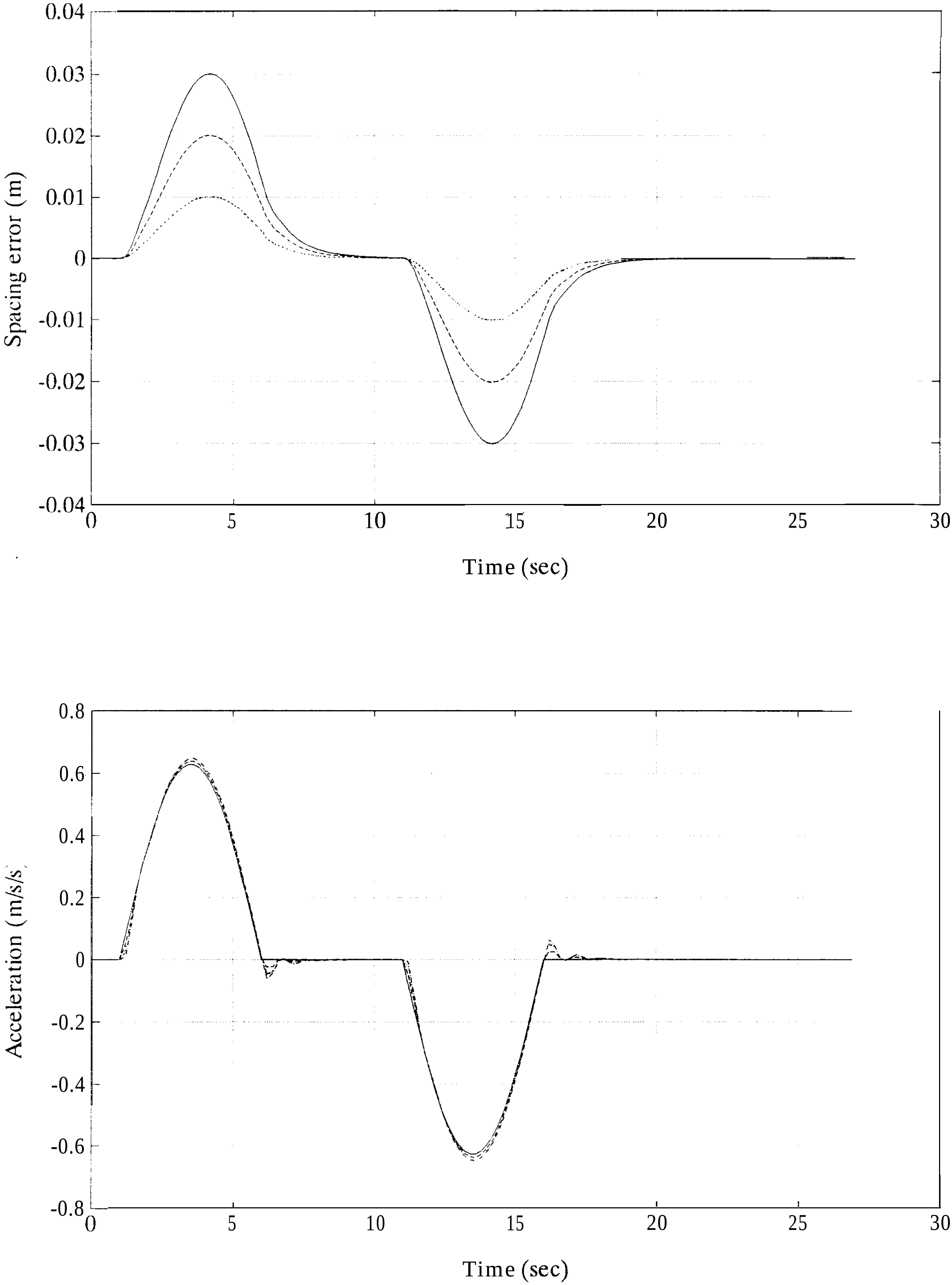


Figure 14. P2C3 Platoon Response for dt = 0.05 sec (τ_e = 0.2 sec)

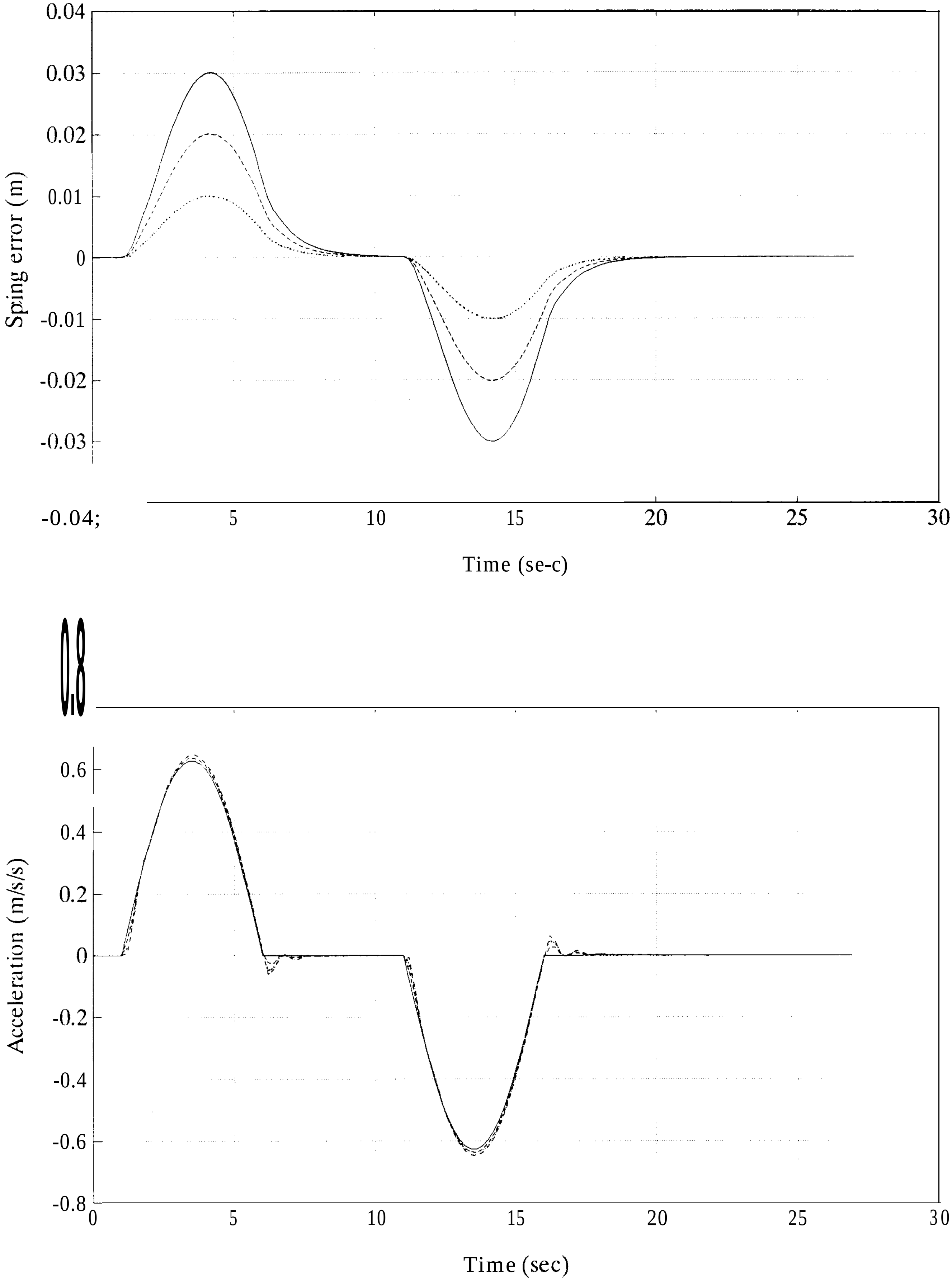


Figure 15. P2C3 Platoon Response for dt = 0.1 sec (τ_e = 0.2 sec)

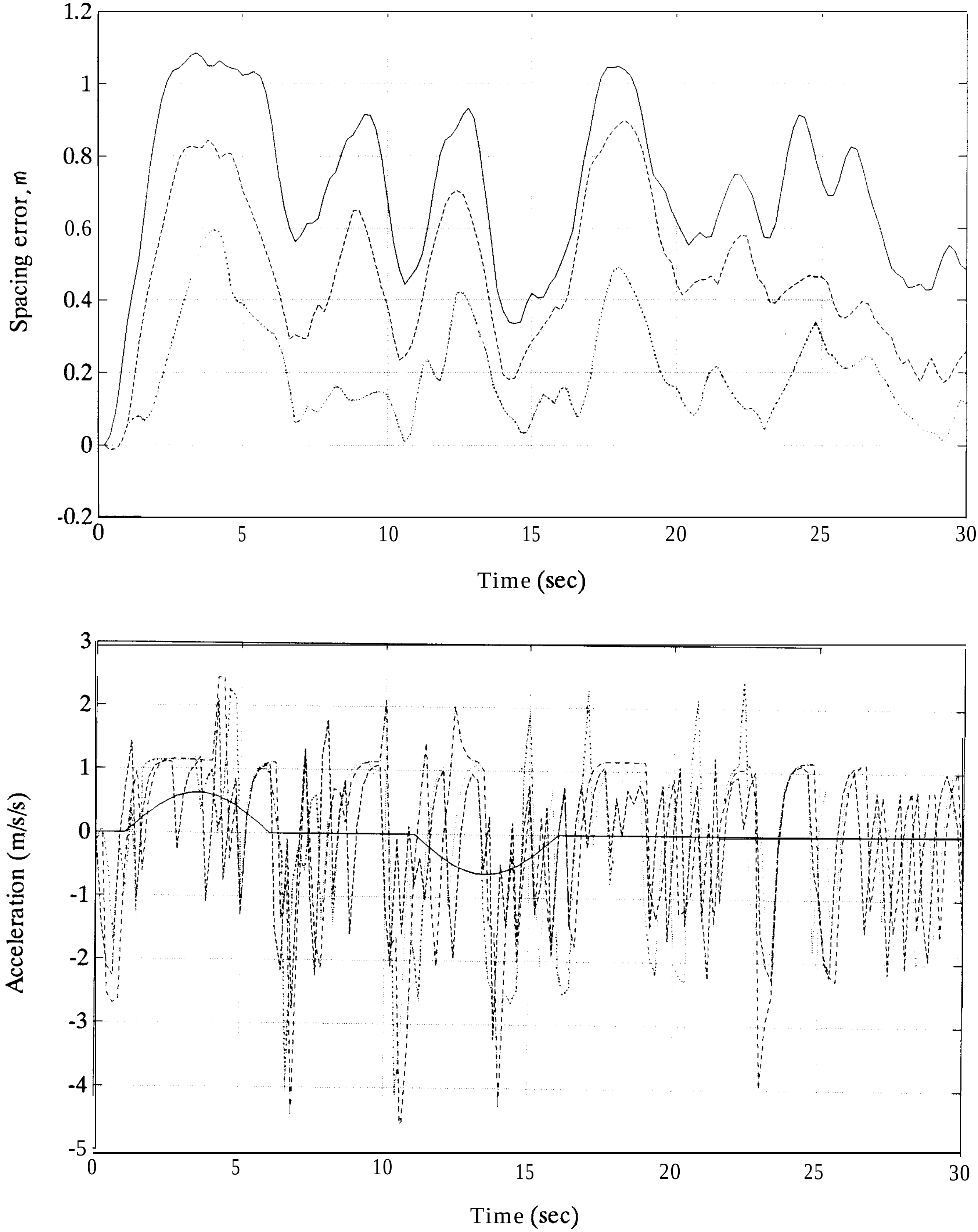


Figure 16. P2C3 Platoon Response for $dt = 0.2$ *sec* ($\tau_e = 0.2$ *sec*)

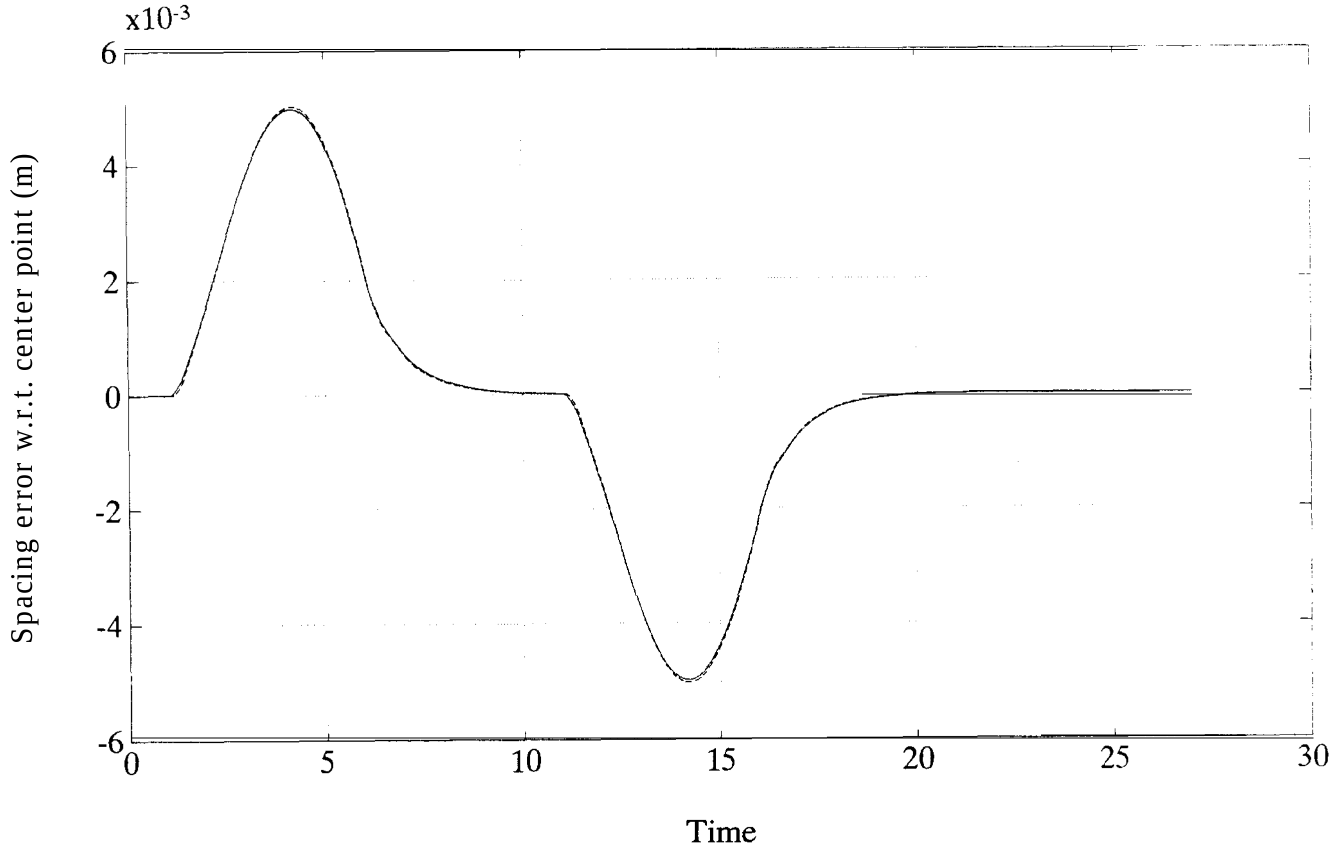
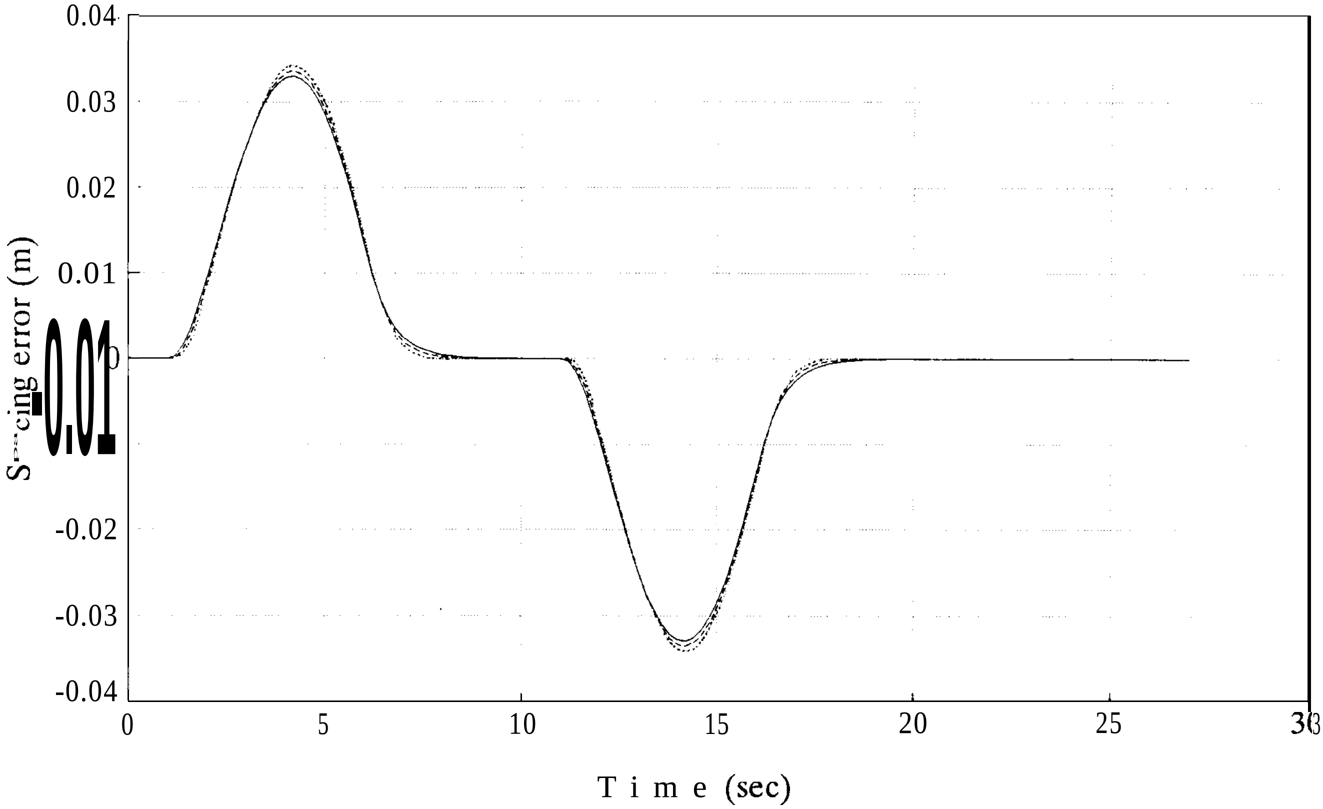


Figure 17. P2C3 Platoon Spacing Error of Cars 2 and 3 w.r.t. Center Point for $dt = 0.01$ *sec* ($\tau_e = 0.2$ *sec*)



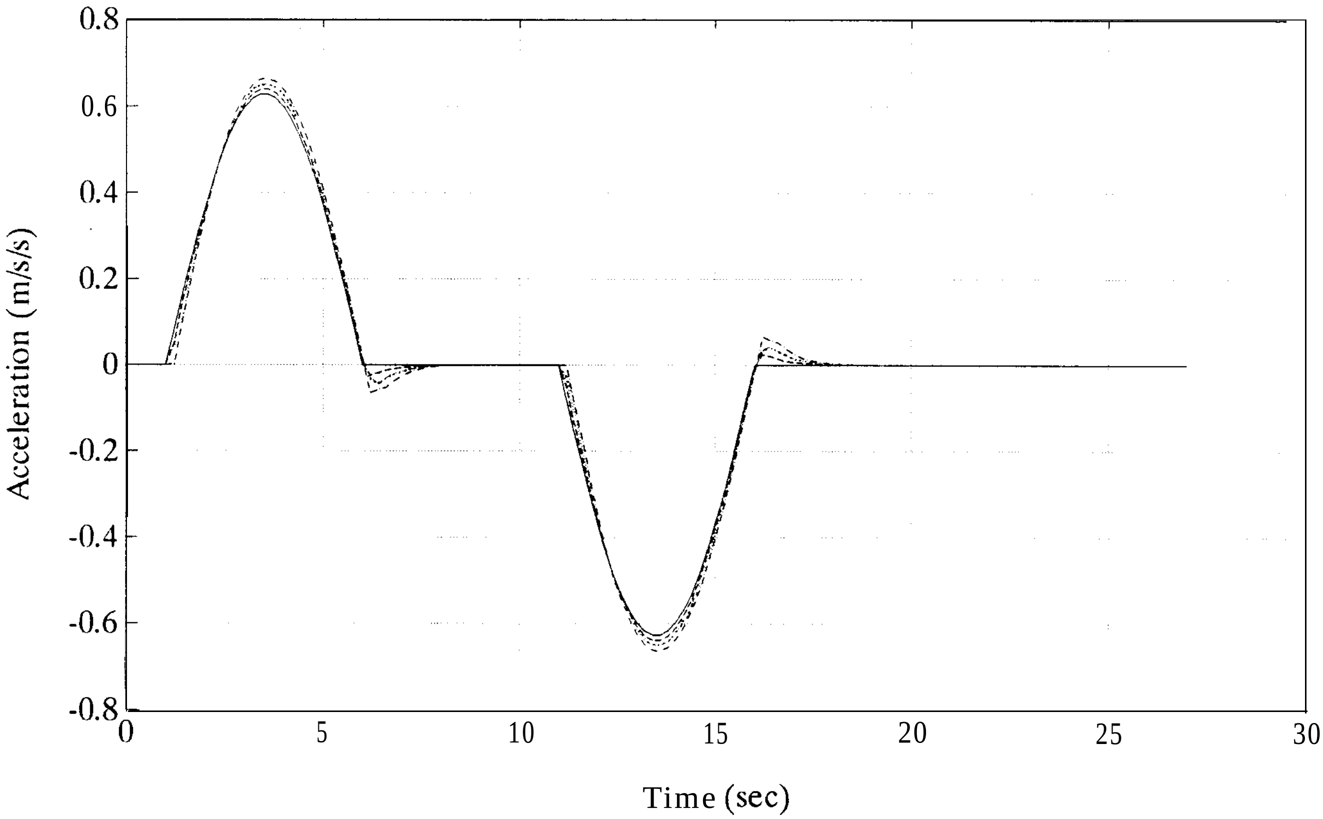


Figure 18. P1C1 Platoon Response for $\tau_e = 0.01$ *sec* ($dt = 0.2$ *sec*)

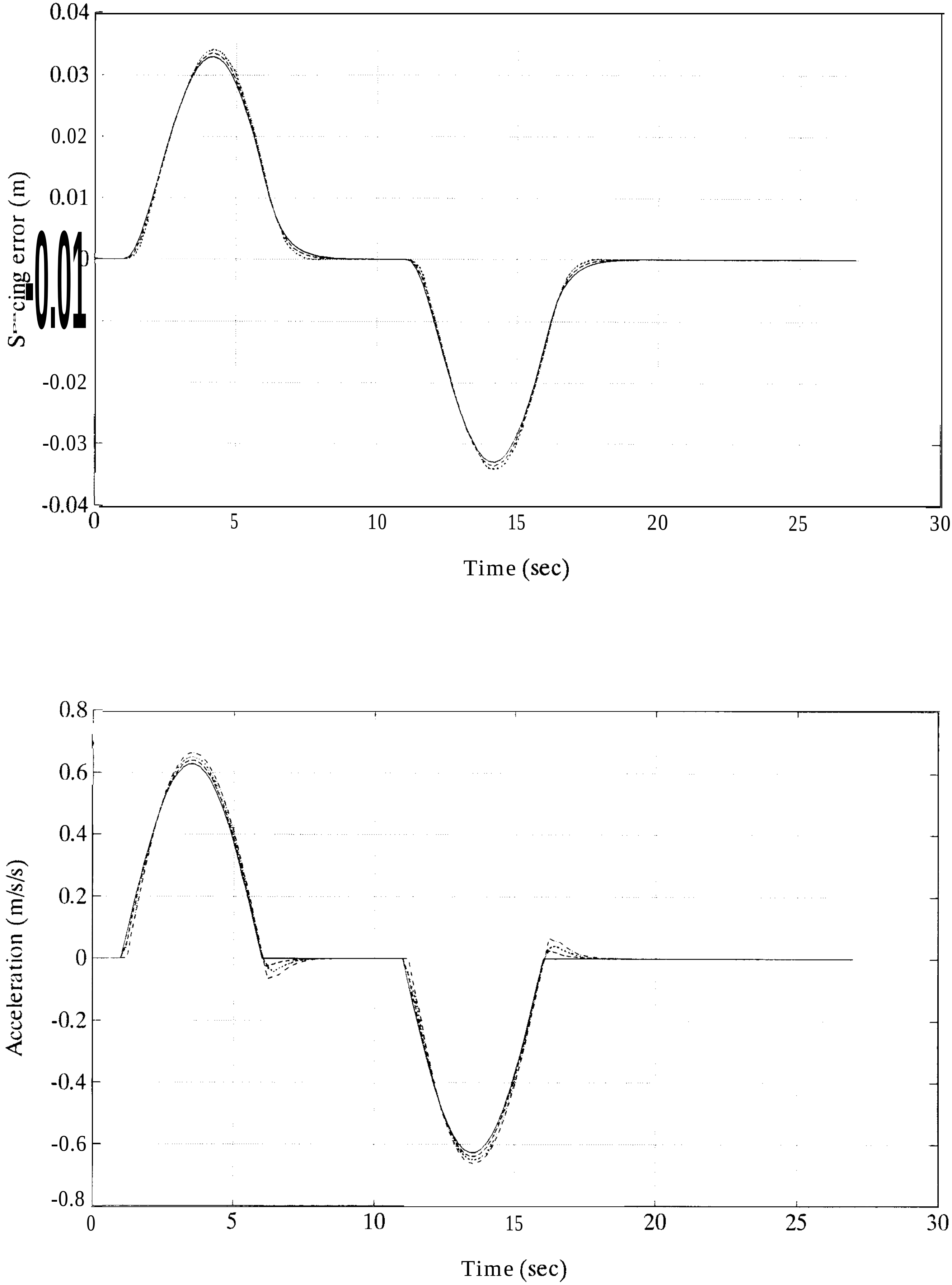


Figure 19. P1C1 Platoon Response for $\tau_e = 0.1$ *sec* ($dt = 0.2$ *sec*)

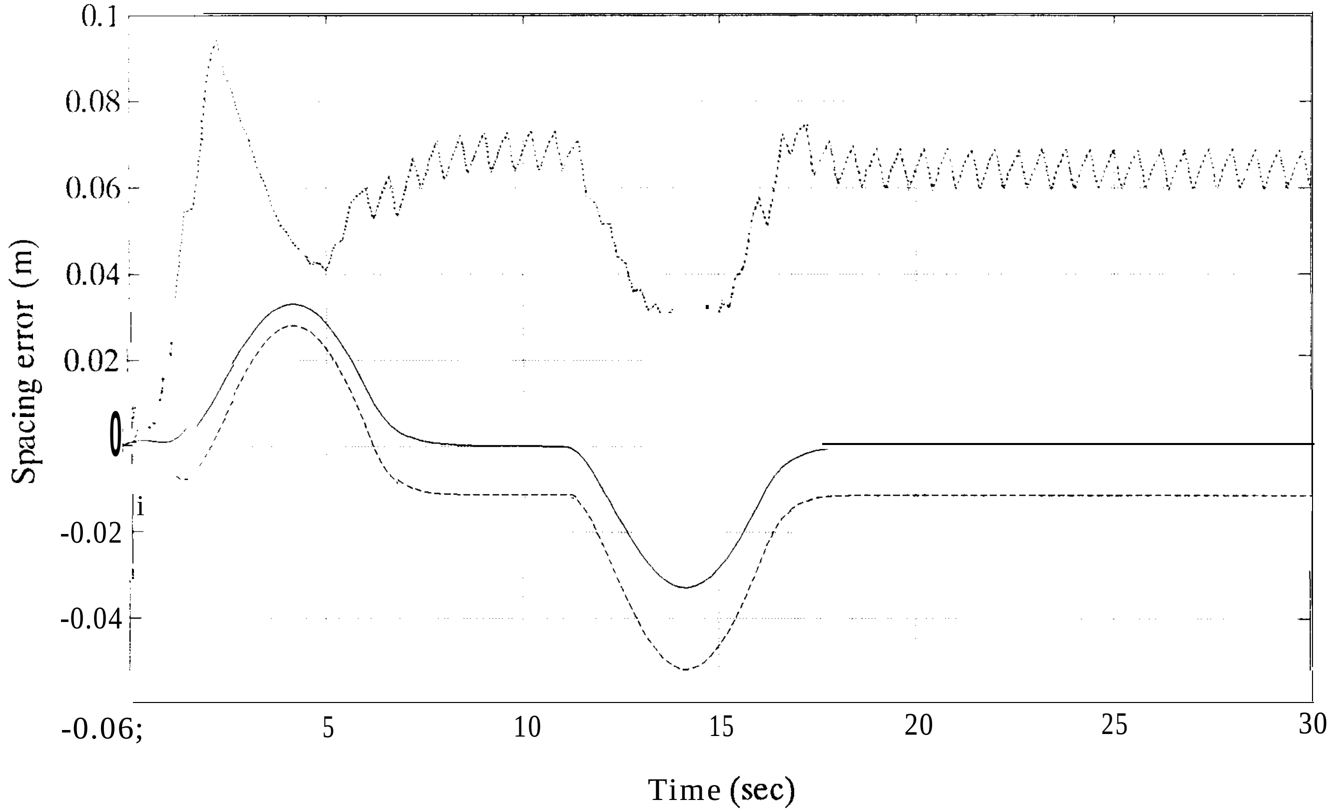


Figure 20. P1C1 Platoon Response for τ_e = 0.3 *sec* (dt = 0.2 *sec*)

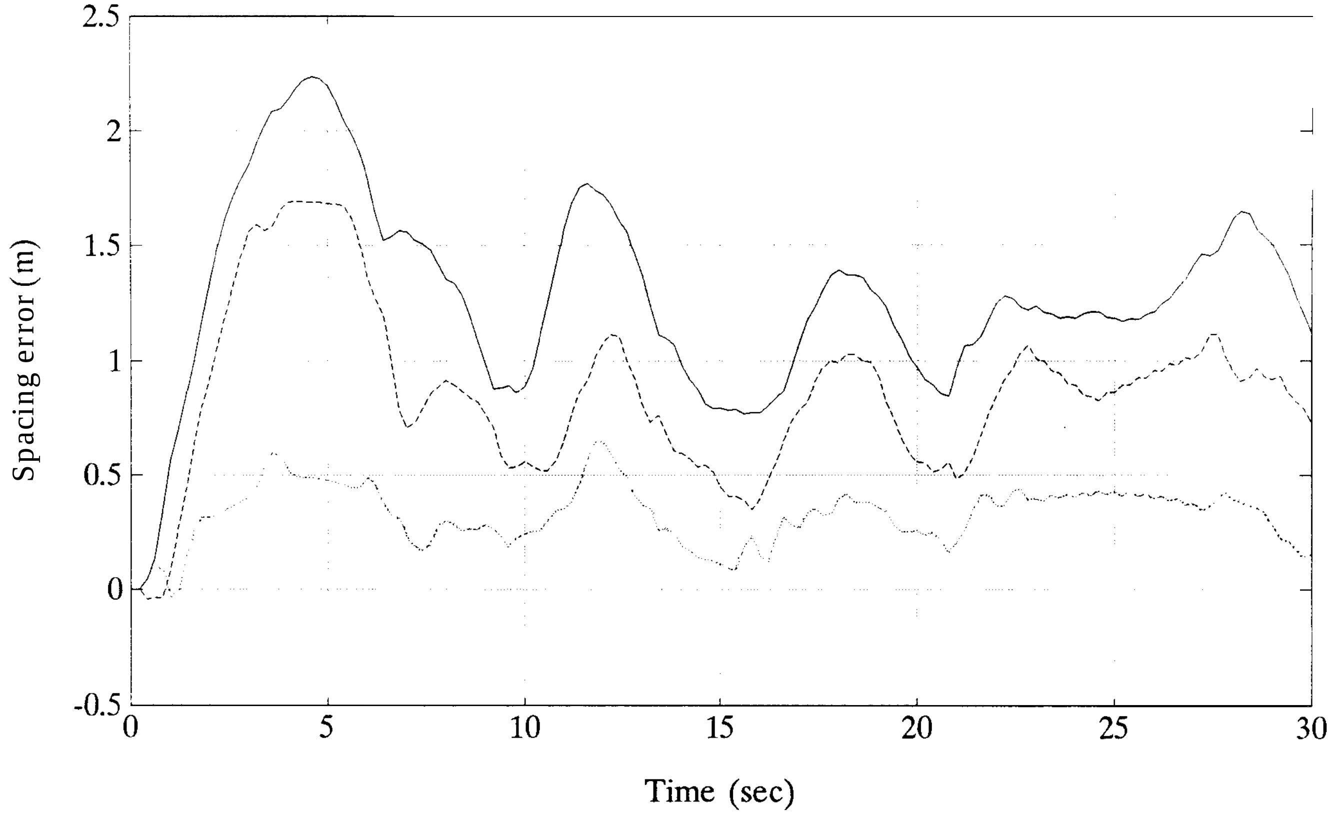


Figure 21. P2C3 Platoon Response for $\tau_e = 0.1\ sec$ ($dt = 0.2\ sec$)

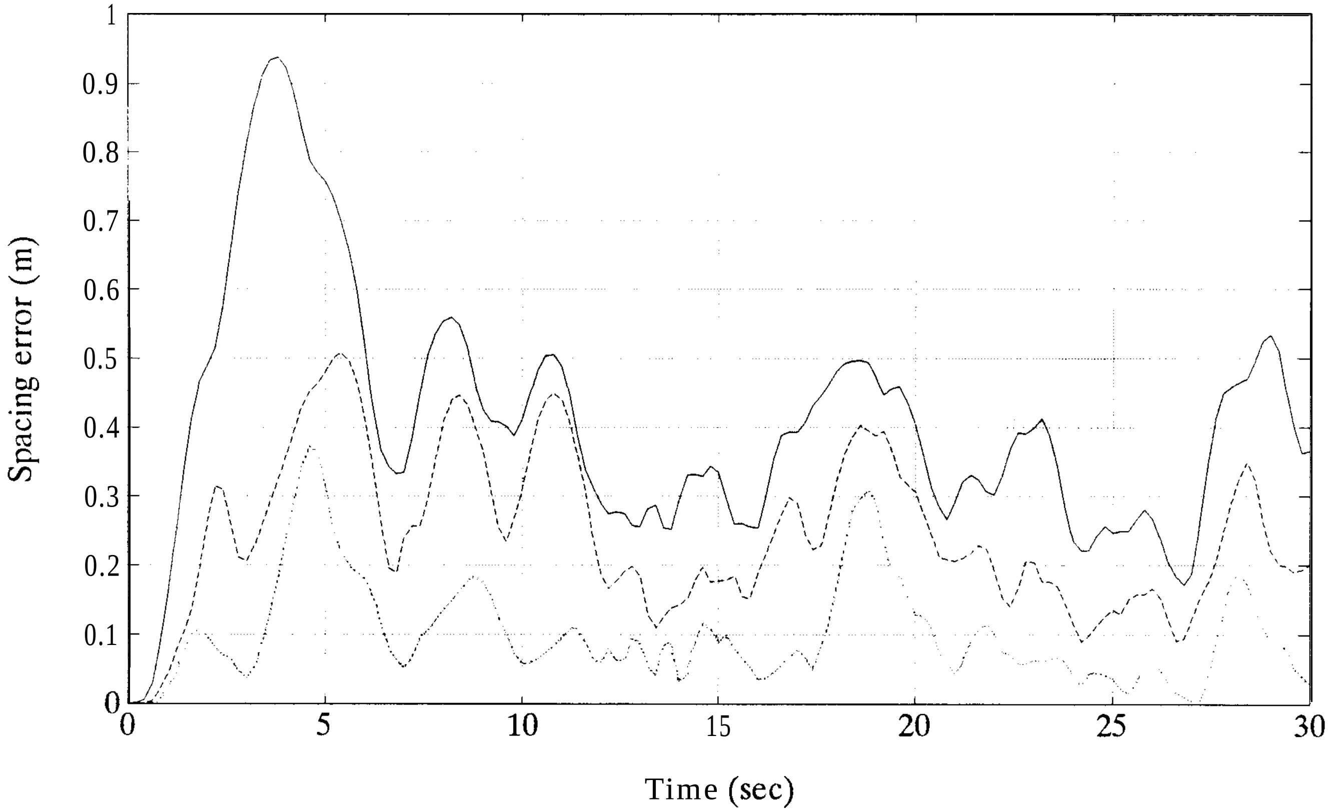
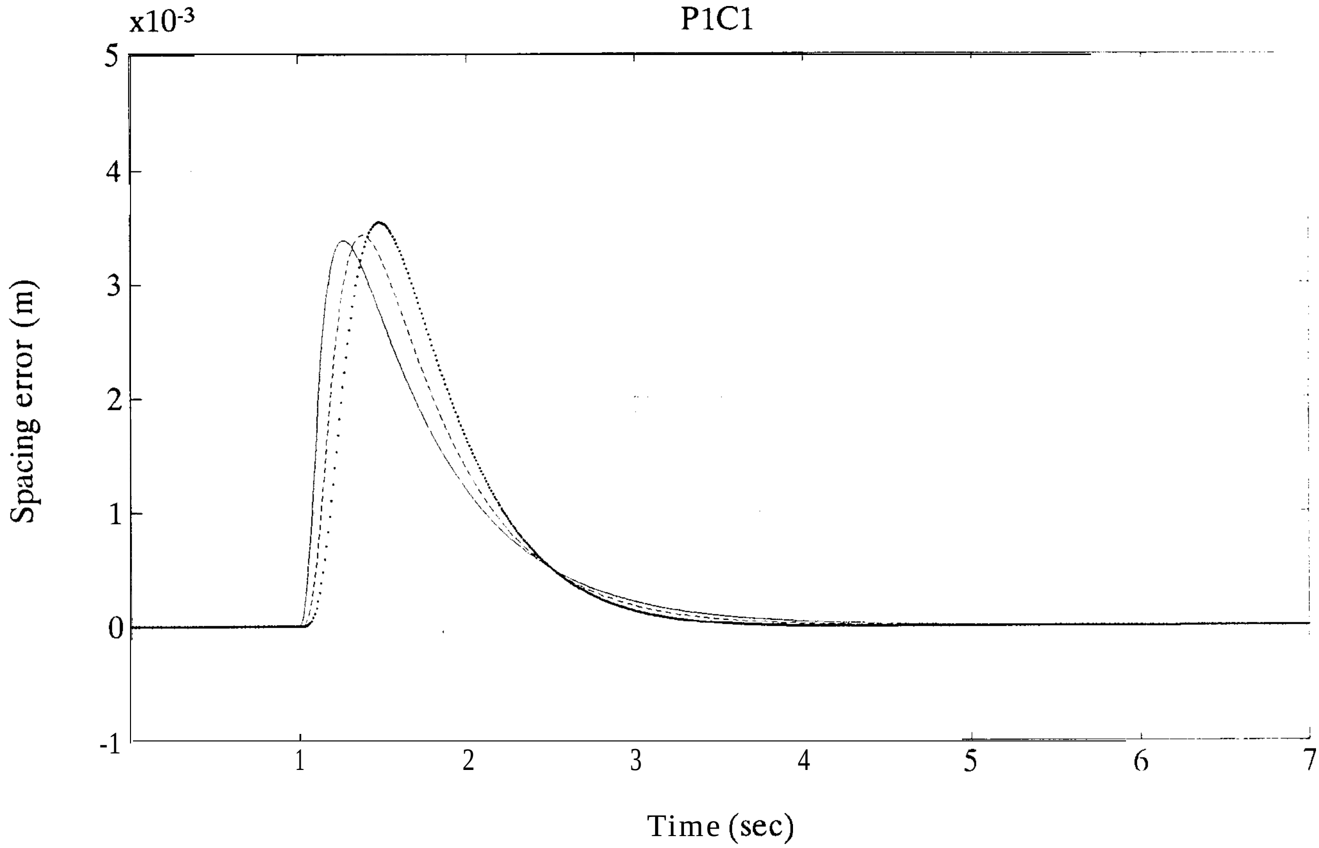


Figure 22. P2C3 Platoon Response for τ_e = 0.3 sec (dt = 0.2 sec)



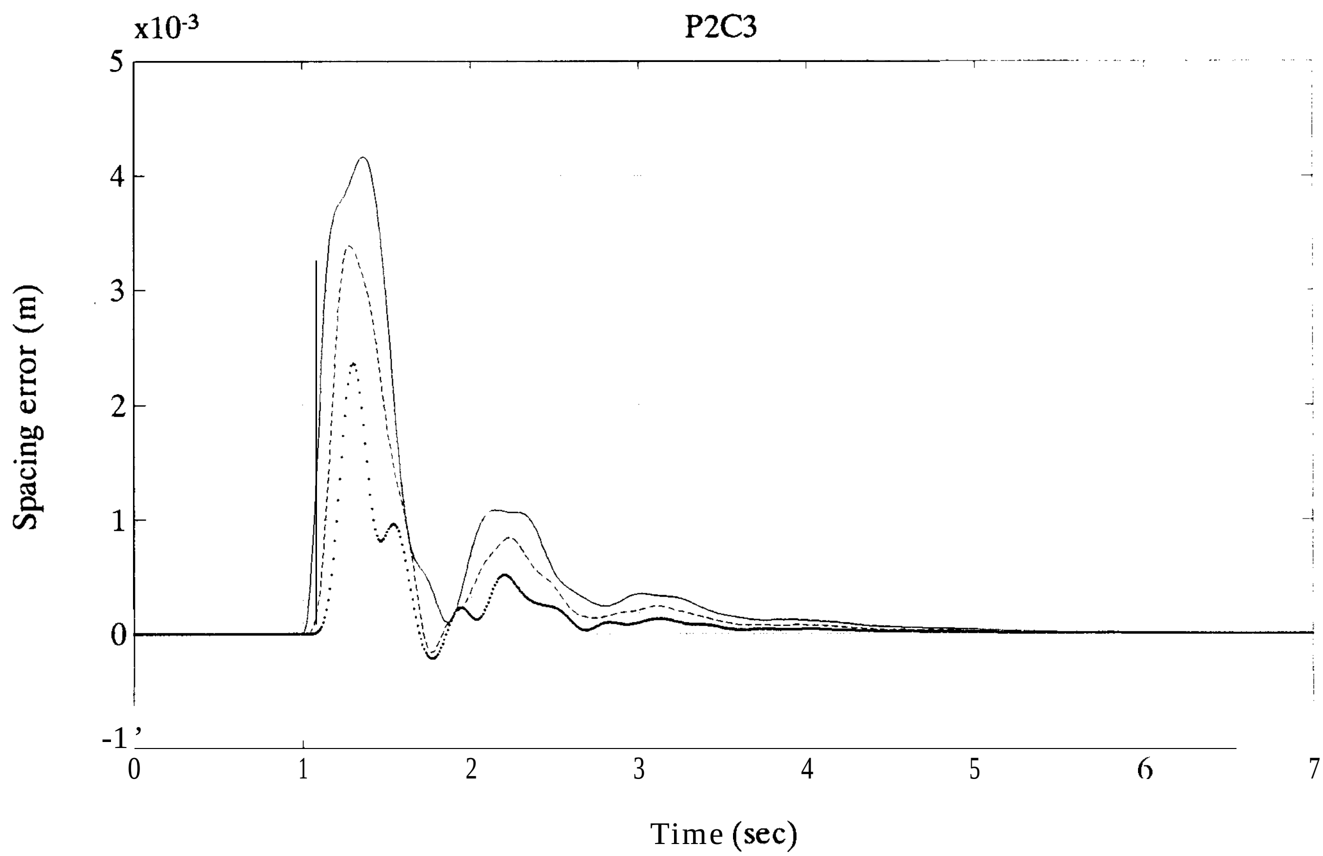


Figure 23. Impulse Response of P1C1 and P2C3 Platoons for τ_e = 0.2 *sec* (dt = 0.01 *sec*)

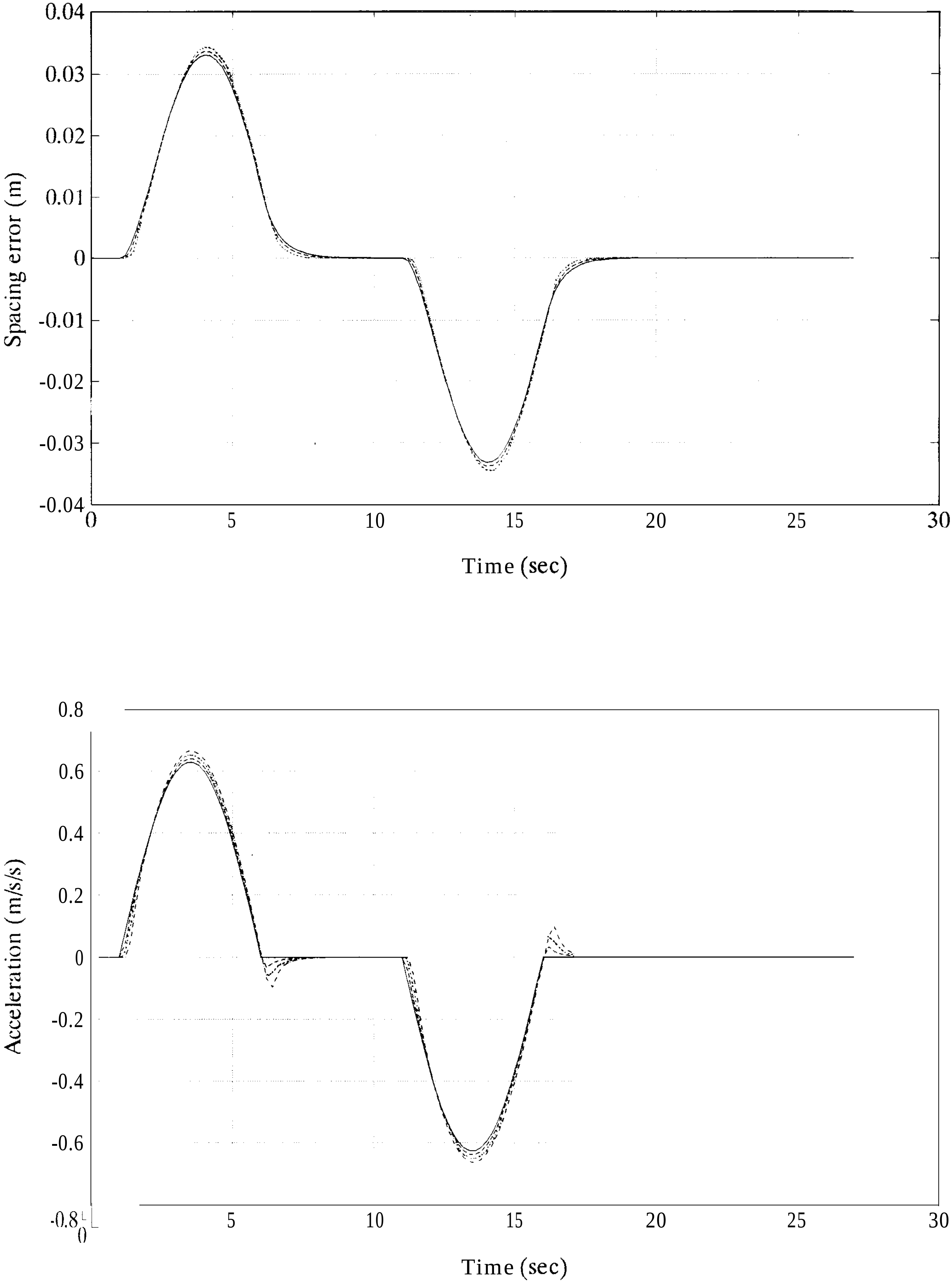


Figure 24. P1C1 Platoon Response for τ_t = 0.05 sec (dt = 0.01 sec and τ_e = 0.1 sec)

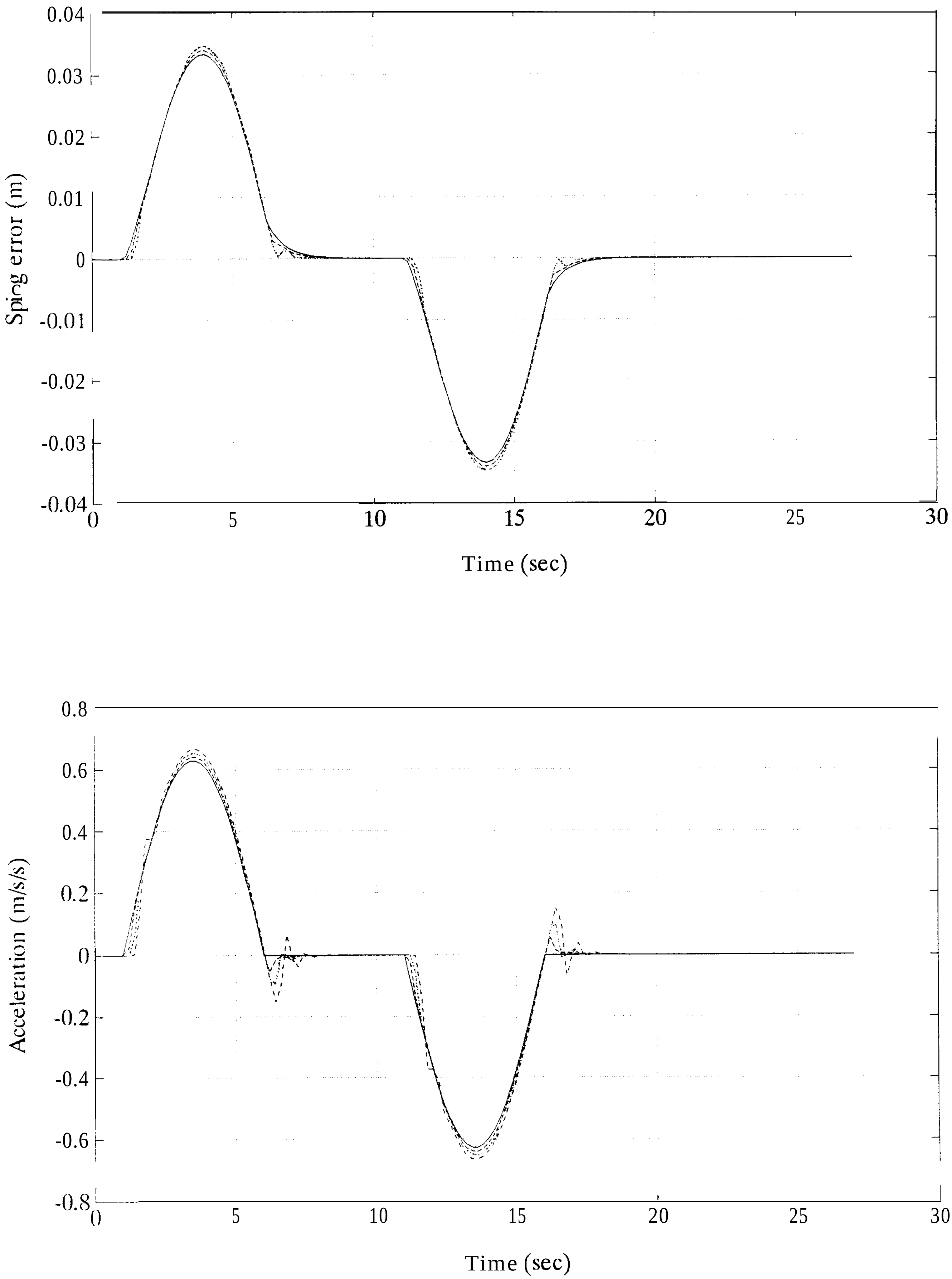


Figure 25. P1C1 Platoon Response for τ_t = 0.1 *sec* (dt = 0.01 *sec* and τ_e = 0.1 *sec*)

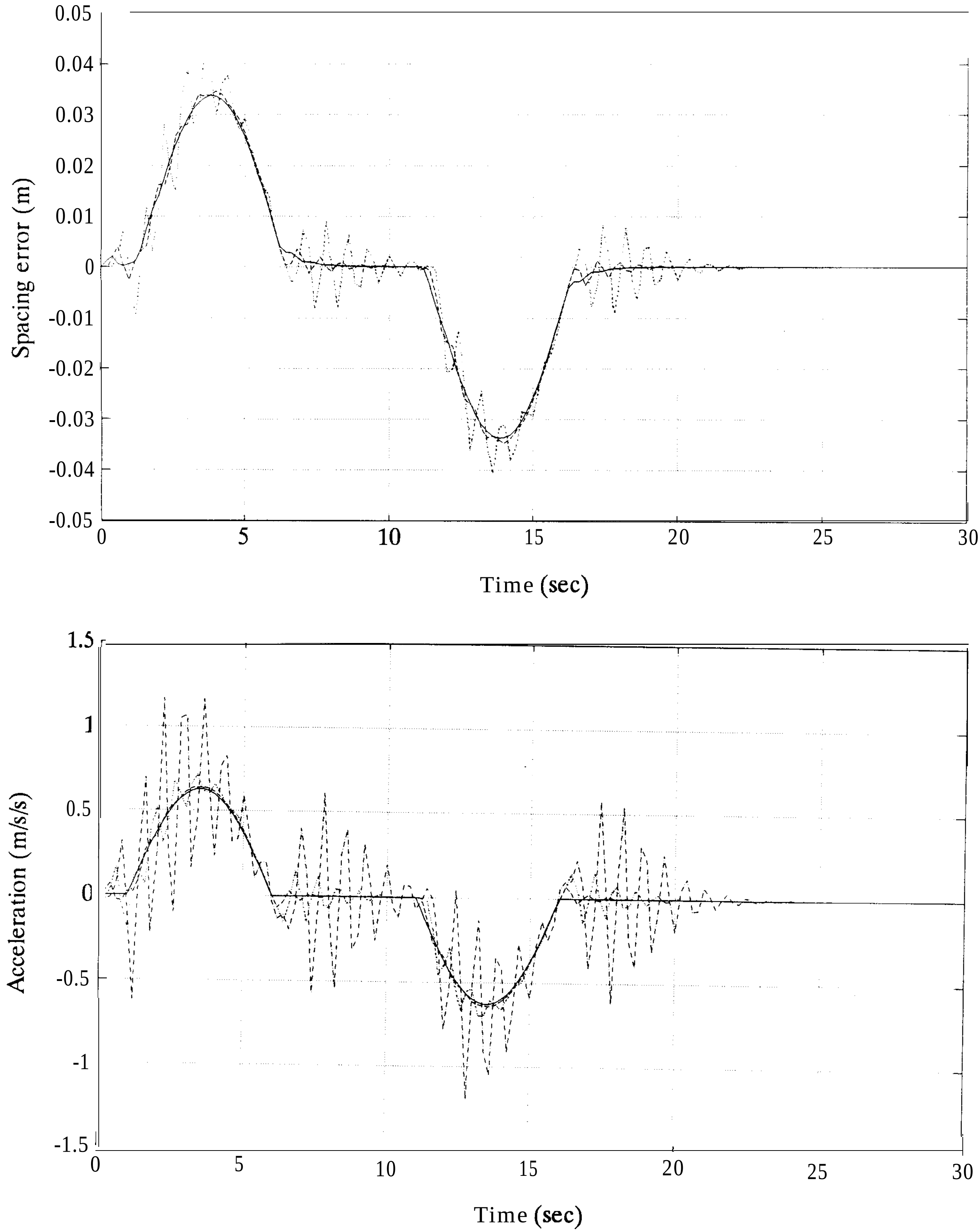


Figure 26. P1 Cl Platoon Response for $\tau_t = 0.15\ sec$ ($dt = 0.01\ sec$ and $\tau_e = 0.1\ sec$)

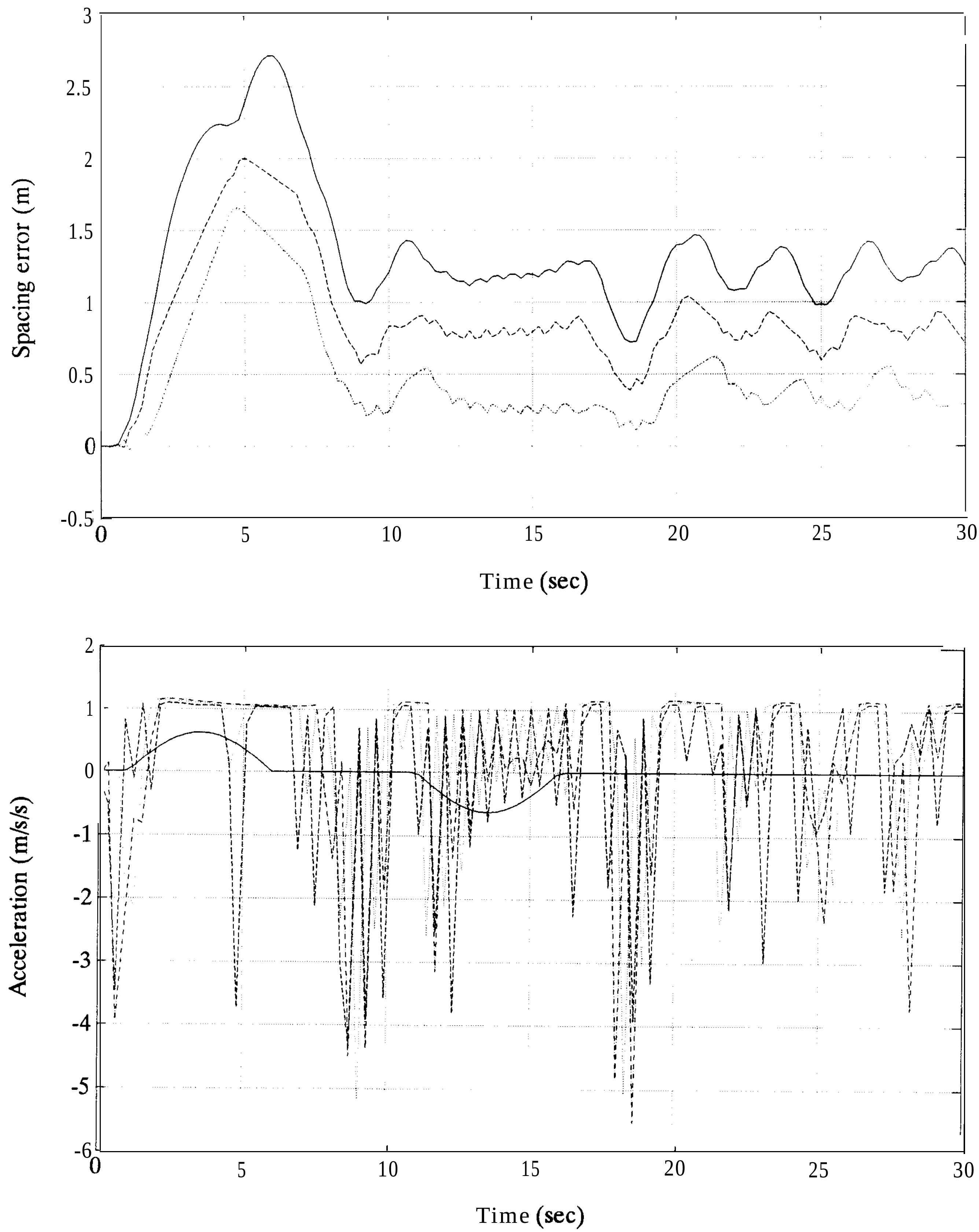
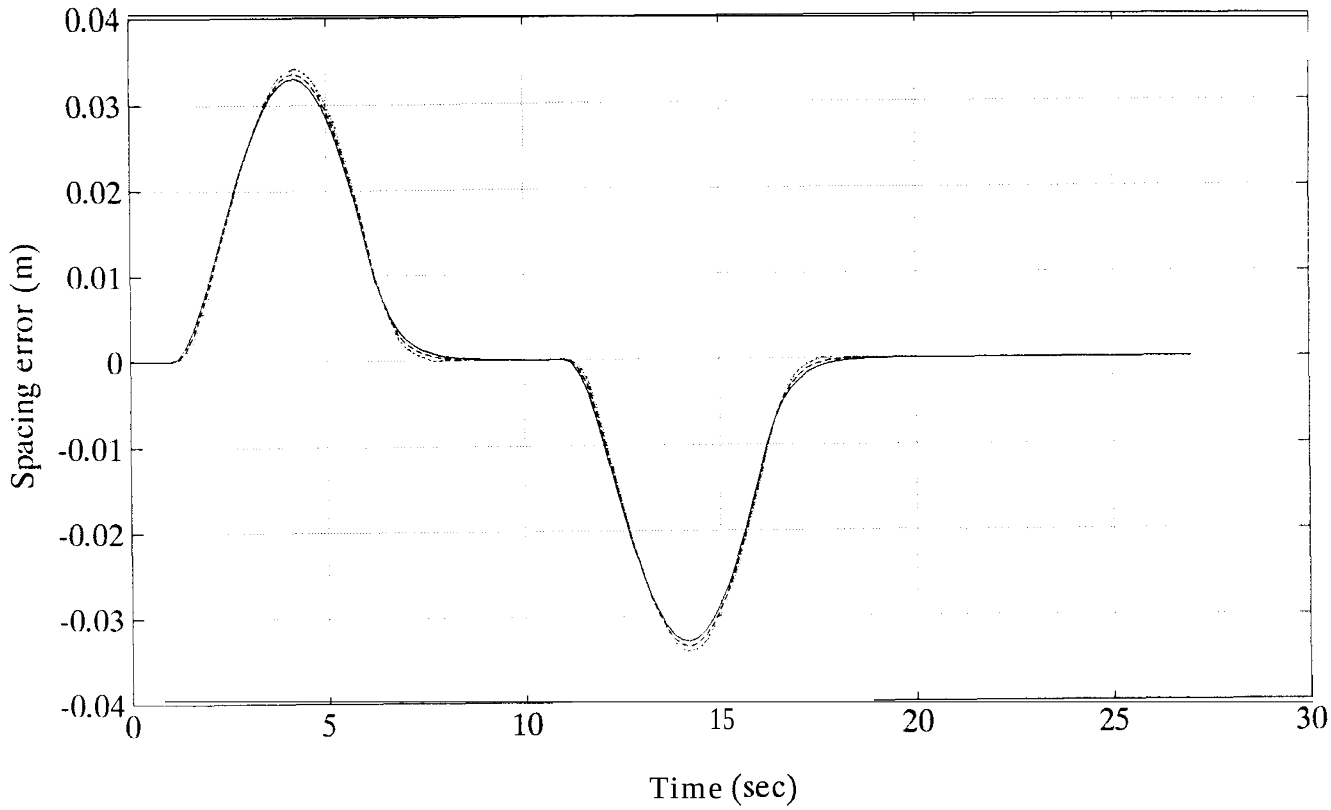


Figure 27. P2C3 Platoon Response for τ_t = 0.05 *sec* (dt = 0.01 *sec* and τ_e = 0.1 *sec*)



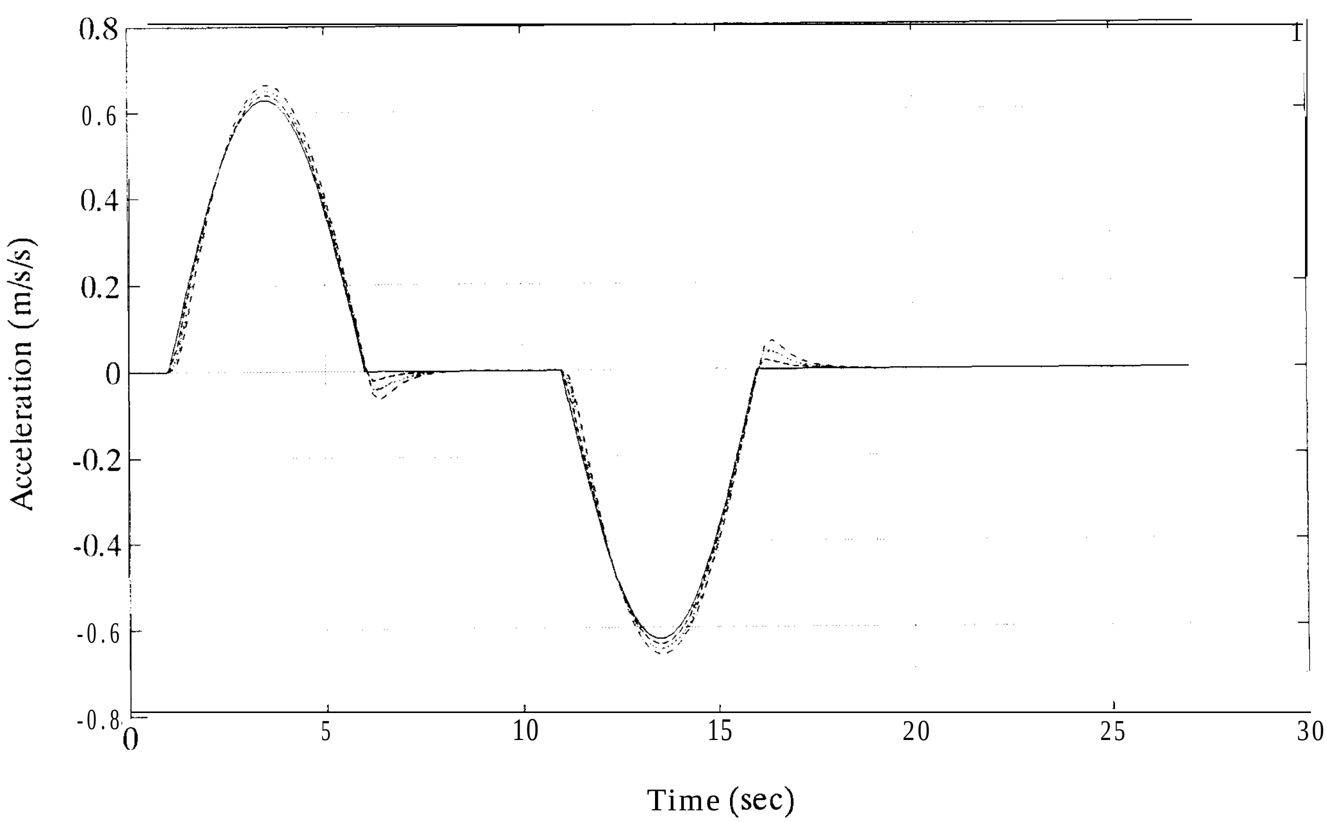
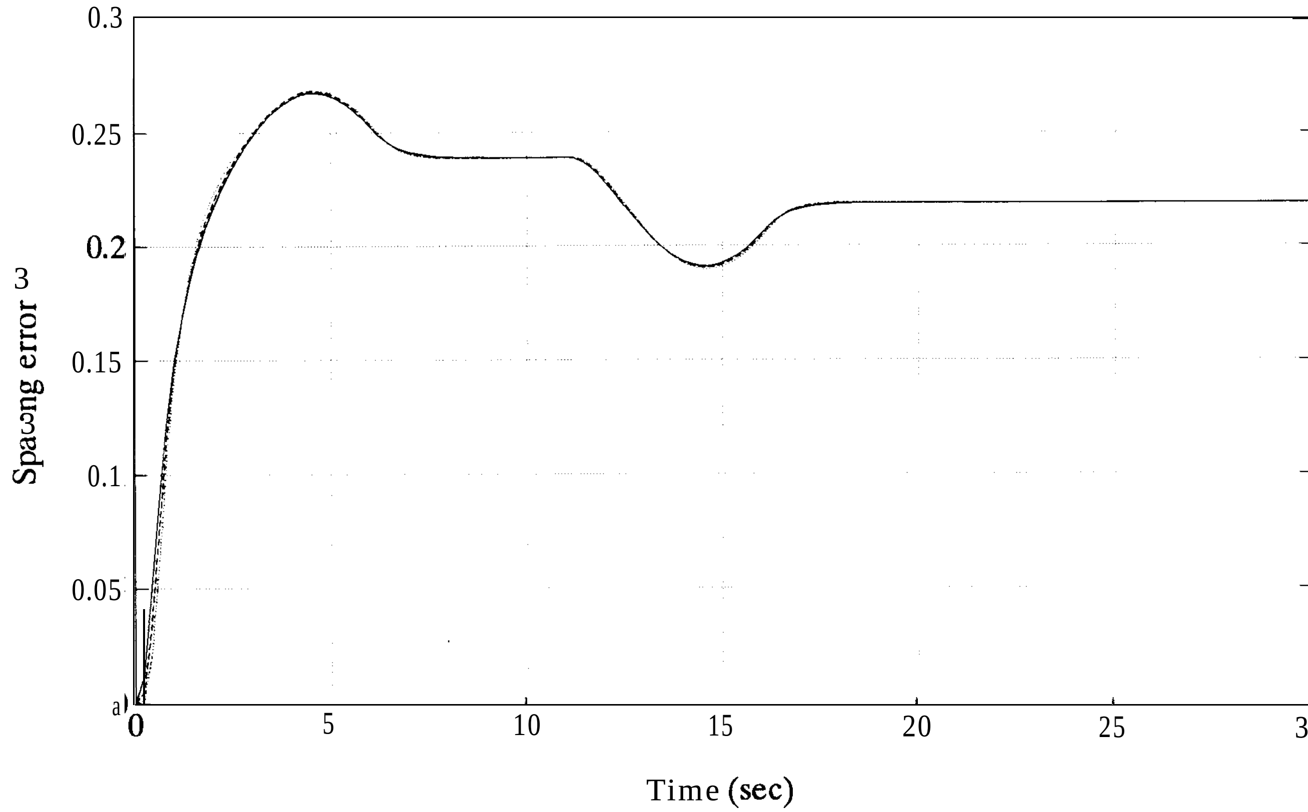


Figure 28. P1C2 Platoon Response for $\tau_c = 0$ *sec* ($dt = 0.01$ *sec* and $\tau_e = 0.1$ *sec*)



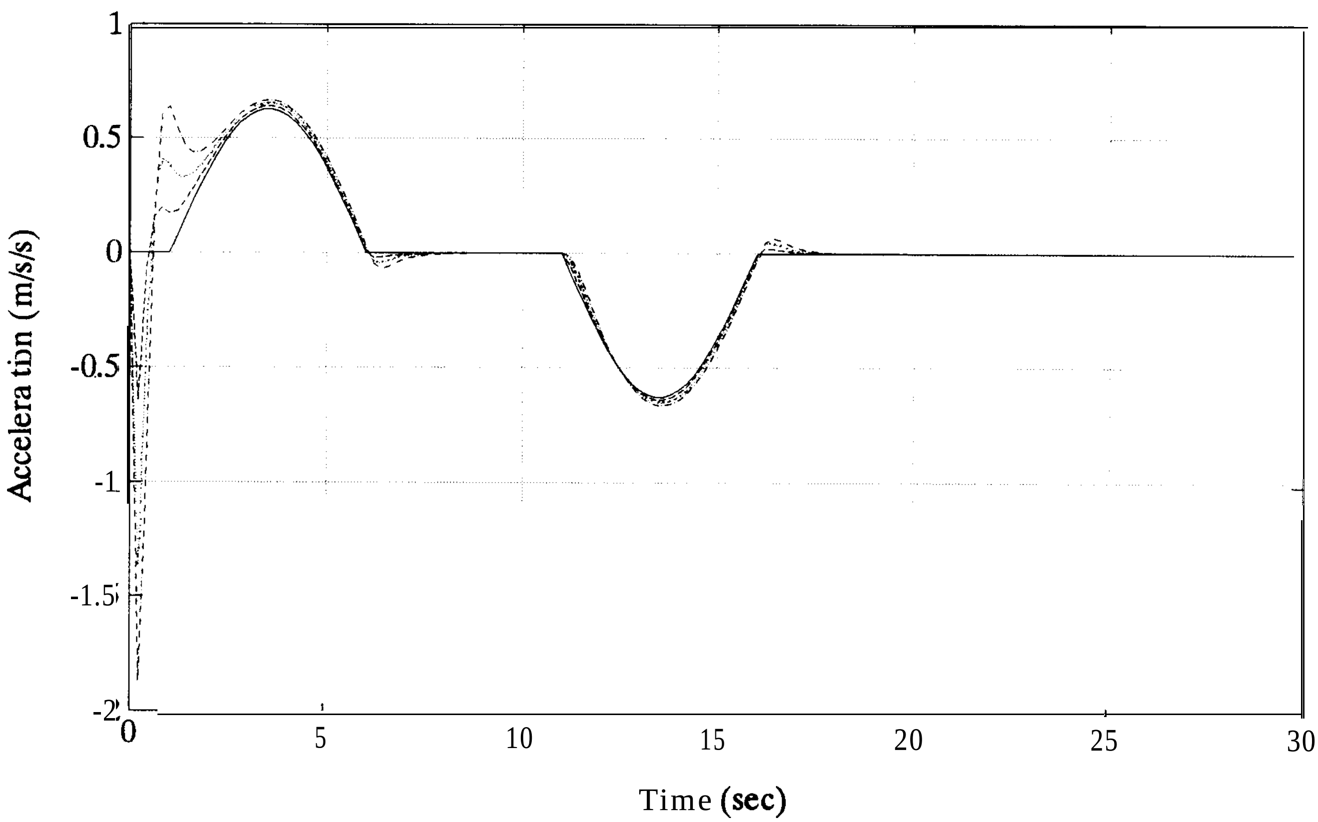


Figure 29. P1C2 Platoon Response for τ_c = 0.01 *sec* (dt = 0.01 *sec* and τ_e = 0.1 *sec*)

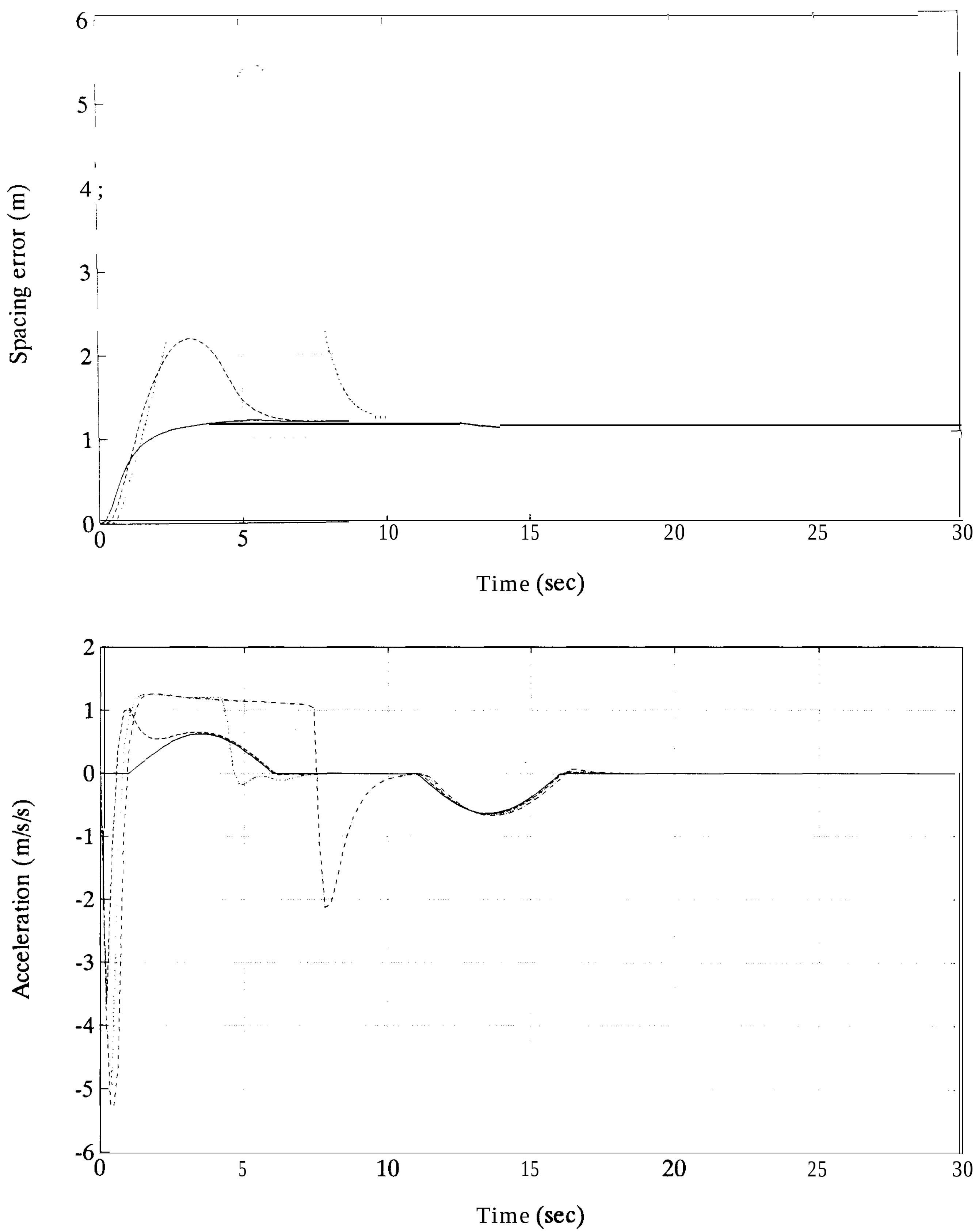


Figure 30. P1C2 Platoon Response for $\tau_c = 0.05$ *sec* (*dr* = 0.01 *sec* and $\tau_\epsilon = 0.1$ *sec*)

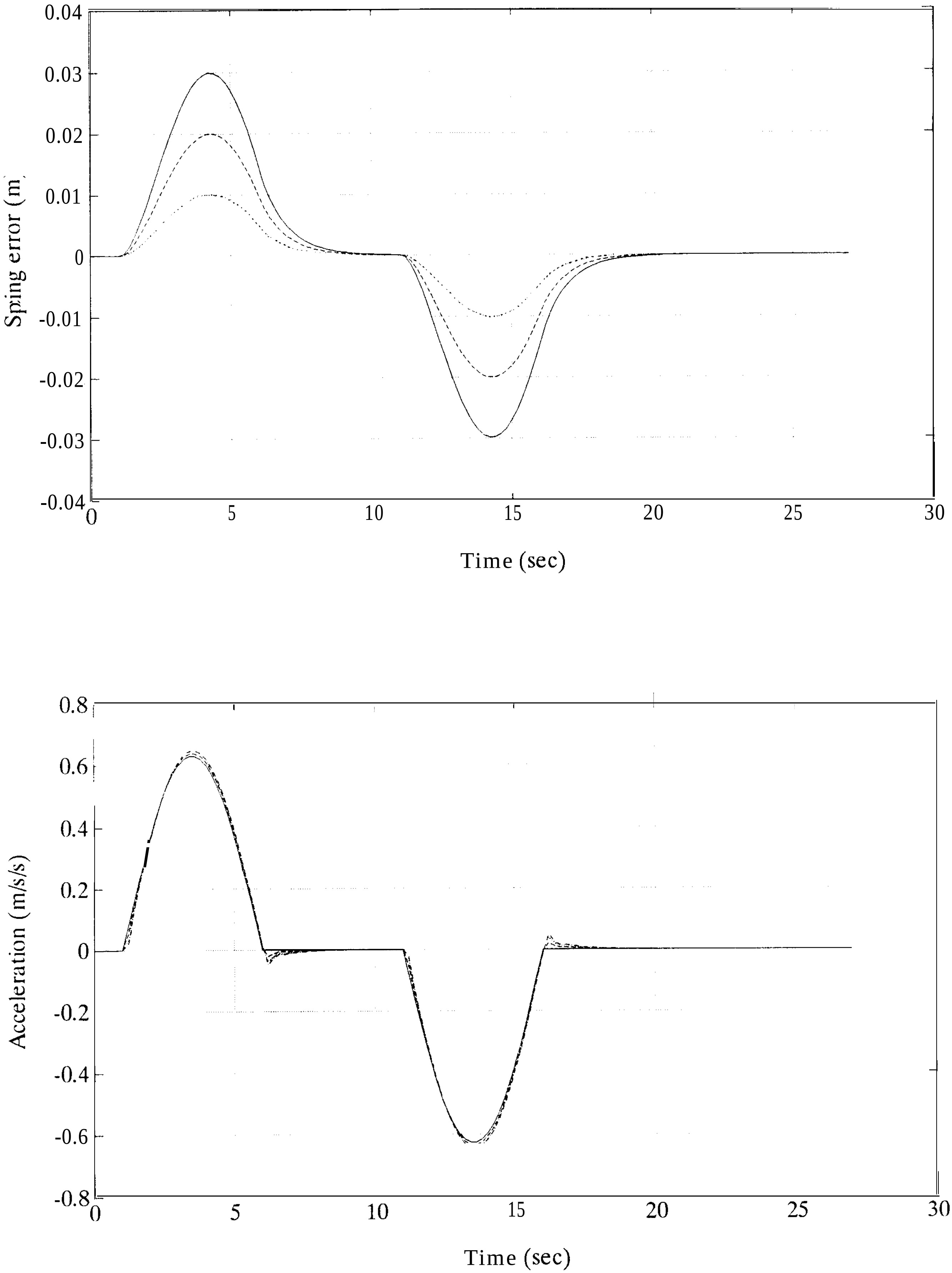


Figure 31. P2C4 Platoon Response for $\tau_c = 0$ *sec*
($dt = 0.01$ *sec* and $\tau_e = 0.1$ *sec*)

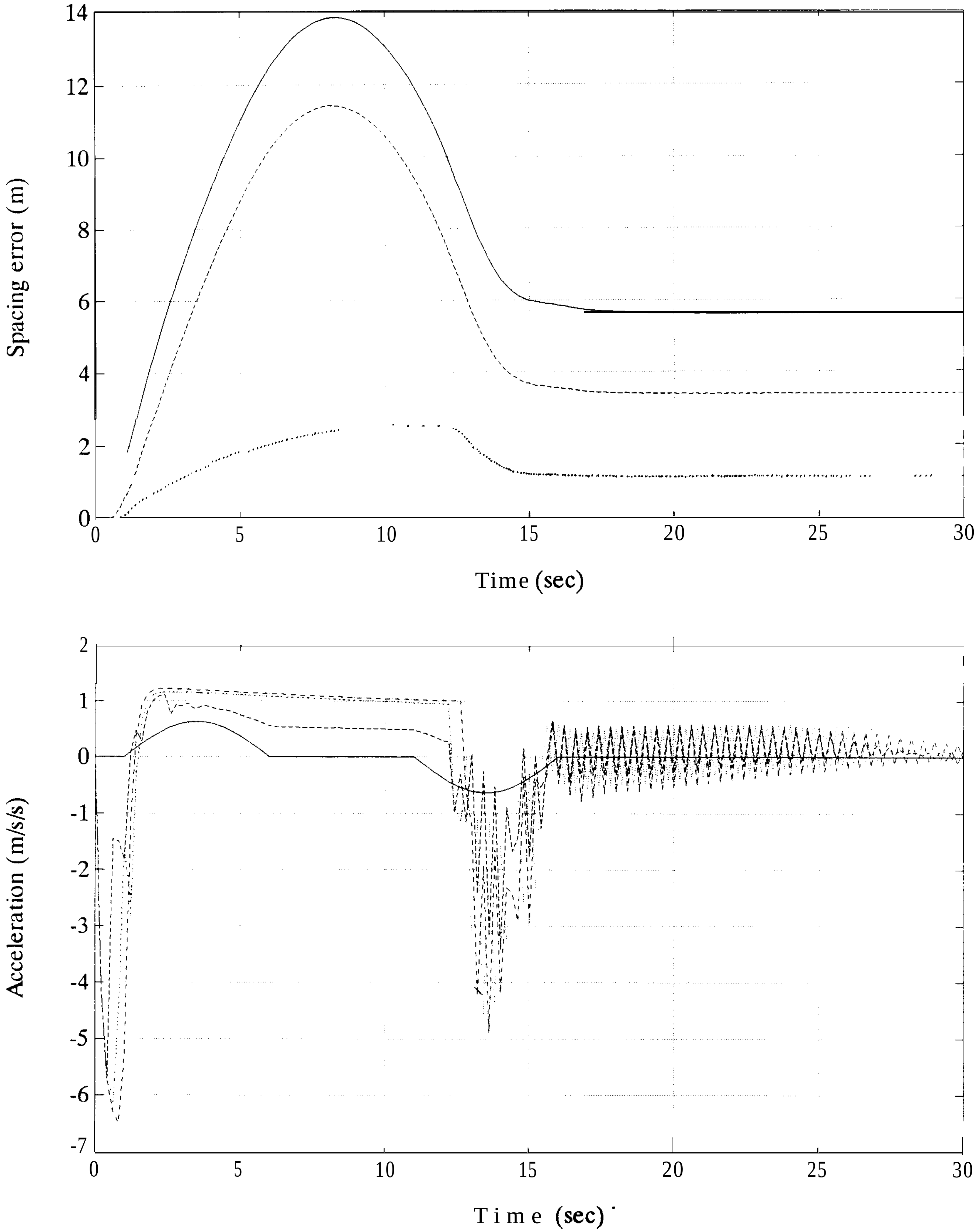
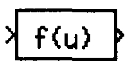
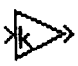
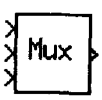
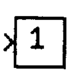
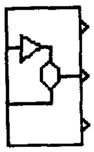
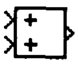
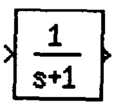


Figure 32. P2C4 Platoon Response for τ_c = *0.05 sec*
(dt = 0.01 *sec* and τ_e = 0.1 *sec)*

Appendix: Notations of SIMULAB Program

	Clock	To provide and display the system time.
	Fcn	To calculate the output by a. purpose function of the input.
	Gain	To multiply its input by a constant.
	Inport	To provide a link to an external input.
	Integrator	To integrate a signal.
	MATLAB Fcn	To apply a MATLAB function to the input.
	Mux	To group several scalar lines into a vector line.
	Outport	To provide a link to an external output.
	Product	To multiply inputs together.

	Subsystem	To group blocks into a subsystem.
	Sum	To sum outputs together.
	To Workspace	To write data to a matrix.
	Transfer Fcn	To implement a linear transfer function.
	Transportation Delay	To delay the input by a given amount of time.