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Relation among structure functions for massive currents*

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A relation among structure functions for processes involving massive photons (deep-inelastic electron scattering, $e^+ + e^-$ annihilation into hadrons and massive lepton pair production) is presented. This relation is valid in a variety of models. A rough comparison with experimental data shows this relation to be grossly violated. Some speculations on this violation are presented.

Reactions involving currents with large momentum squared are believed to probe fundamental constituents of hadronic physics. Among these we find not only deep-inelastic electron scattering, but likewise $e^+ + e^-$ annihilation into hadrons and the production of massive lepton pairs. This pointlike constituent structure supposedly manifests itself in scaling behavior of the rates for these processes.

Though the parton picture,¹ or to a lesser extent the operator production expansion on the light cone,² may be most directly tied to the concept of fundamental point constituents, the hypothesized scaling behavior is consistent with more prosaic views of hadronic reactions: multiperipheral, Regge-pole exchange, etc. In this note we obtain a relation between the above-mentioned current processes that, under specific assumptions, may be common to all models.

At this juncture we must point out that the entire scaling structure may be suspect in light of recent experiments on $e^+ + e^-$ annihilation,³ which (at least at present energies) do not indicate the desired scaling. We shall have more to say on this later; for the moment let us proceed on the assumption that all reactions will scale.

Before stating the relation central to this article we review and establish the necessary notation. $e^+ + e^-$ annihilation into hadrons is described in terms of $R(q^2)$, the ratio of the cross section for this process to the cross section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$. In the scaling limit $R(q^2)$ is taken to be independent of q^2 . Deep-inelastic electron scattering is presented in terms of the usual structure

functions $mW_1(\nu, q^2)$, $\nu W_2(\nu, q^2)$ which are to approach the scaling limits $F_1(x)$, $F_2(x)$ with $x = q^2/2m\nu$. The third process we study is inclusive lepton pair production, specifically $p + p \rightarrow \mu^+ + \mu^- + \dots$. Let q^μ be the four-momentum of the lepton pair and p_1, p_2 the momenta of the incident protons. This process is governed by the structure function

$$W = -(2\pi)^6 \frac{E_1 E_2}{m^2} \sum_n \langle p_1, p_2 | J_\mu | n \rangle \langle n | J^\mu | p_1, p_2 \rangle \times \delta^4(p_1 + p_2 - p_n - q). \quad (1)$$

Letting $x_1 = q^2/2p_1 \cdot q$, $s = 4p_1 \cdot p_2$, we obtain

$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{3s} \frac{V(s; x_1, x_2)}{(x_1 x_2)^2}, \quad (2)$$

with

$$\frac{V(s; x_1, x_2)}{x_1 x_2} = 4m^2 \int \frac{d^4 q}{q^2} \delta(x_1 - q^2/2p_1 \cdot q) \times \delta(x_2 - q^2/2p_2 \cdot q) W. \quad (3)$$

In the scaling limit $V(s; x_1, x_2)$ ceases to depend on s .

The following relation between the above structure functions is proposed:

$$\lim_{x_1, x_2 \rightarrow 0} \frac{[2x_1 F_1(x_1) + F_2(x_1)][2x_2 F_1(x_2) + F_2(x_2)]}{8V(x_1 x_2)} = R. \quad (4)$$

We shall briefly indicate, in two classes of models, the conditions necessary for the validity of this relation:

(i) *Parton models.*⁴ In the *wee* parton region all partons and antipartons are present with identical distributions.

(ii) *Multiperipheral and Regge-Mueller models.* The amplitude for any of these massive current processes factorizes into a hadronic component and a current component. The current component extends in rapidity of the produced hadrons over a range $\Delta y = \ln Q$ to either side of the current rapidity.⁵ It is the factorization assumption, rather than details of the models, that leads to Eq. (4). Details of such models are presented in a separate publication.⁶

A verification of Eq. (4) based on presently available data is difficult on two grounds. As mentioned earlier, R is growing with q^2 . Second, massive lepton pair production data exist for a uranium target, and the extraction of the required proton data is suspect.⁷ Taking this extraction at face value, we still have a difficulty, as only $d\sigma/dq^2$ has been measured. However, with some additional assumptions we may determine $V(0,0)$ from the data. Assuming Regge-pole dominance, we expect $V(x_1, x_2)$ to have the expansion⁸

$$V(x_1, x_2) = V(0, 0) + \sum C_{\alpha\beta} x^\alpha x^\beta, \quad \alpha, \beta > 0 \quad (5)$$

leading to

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{3q^4} [V(0, 0) \ln(s/q^2) + \dots]. \quad (6)$$

In obtaining the above we have set $q_\perp = 0$ in all kinematical calculations. Good data for s/q^2 large should yield $V(0, 0)$. In the case of deep-inelastic electroproduction we assumed $\sigma_L/\sigma_T = 0$ (Ref. 9),

and thus Eq. (4) reduces to

$$\frac{[F_2(0)]^2}{2V(0, 0)} = R. \quad (7)$$

For $s \sim 58 \text{ GeV}^2$, $1.5 < q < 2.1 \text{ GeV}$, the data of Ref. 7 and Eq. (6) yield a value of $V(0, 0)$ between 0.025 and 0.04. $F_2(0)$, on the other hand, varies between 0.2 and 0.3, from which we obtain

$$0.5 < R < 2. \quad (8)$$

Should R become constant in q^2 , at its present large value of ~ 6 , it would still be in disagreement with Eq. (4), even within the crude estimates we have made. One may take a more optimistic view and hope that the present experimental trend in R will reverse and R will decrease before leveling off at a value consistent with Eq. (8). In this case the value of R is consistent with those predicted by quark-parton models with $R = \frac{2}{3}$ or $R = 2$.

However, if scaling does break down for reactions involving timelike q^2 , we might envisage that this could be due to a parton form factor,¹⁰ $f(q^2)$. [$f(q^2) \sim 1$ for q^2 spacelike.] In such a situation Eq. (4) may be transformed into a measurement of $f(q^2)$:

$$\lim_{x_1, x_2 \rightarrow 0} \frac{[2x_1 F_1(x_1) + F_2(x_1)][2x_2 F_1(x_2) + F_2(x_2)]}{8R(q^2)V(q^2; x_1, x_2)} = \frac{1}{|f(q^2)|^4}. \quad (9)$$

Of course, for Eq. (9) to make sense R/V would have to scale.

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