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FINITE TEMPERATURE CALCULATION OF ANGULAR VELOCITIES AND MOMENTS
OF INERTIA IN ROTATING NUCLEI[*]

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The rotational properties of nuclei in the statistical region are
predicted on the basis of a simple BCS model. The model can be described
in terms of three reduced variables. A back-bending, which disappears as the
temperature increases, is observed in the plot of the moment of inertia
versus the square of the angular velocity.

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The experimental discovery of strongly varying moments of inertia
in the ground state rotational bands of deformed even-even nuclei [1,2] has
prompted a variety of theoretical developments [3-5] for the interpretation of
this feature.   A rather appealing model explains the observed increase and
back-bending of the moment of inertia versus the square of the angular
velocity in terms of the disappearance of the pairing superfluidity due to
the increasing angular momentum [3,4].

In this work a very simple pairing model, introduced in detail in a
previous paper for different purposes [6], is used to calculate the zero and
finite temperature behavior of the angular velocity and of the moment of
inertia as a function of angular momentum.

The model is characterized by three parameters: the density $\underline{g}$ of the equidistant, doubly degenerate, single particle levels; the average z projection $\underline{m}$ of the single particle angular momentum; the gap parameter Δ_o corresponding to zero temperature and zero angular momentum.

The properties of the system are contained in the grand potential Ω:

$$\Omega = -\beta g \int_{-S}^{+S} d\varepsilon \left[\varepsilon - E + \frac{1}{\beta}\{\ln[1 + \exp - \beta(E - \gamma m)] + \ln[1 + \exp - \beta(E + \gamma m)]\}\right] - \frac{\beta\Delta^2}{G}$$

(1)

where $\beta = \frac{1}{T}$ is the reciprocal of the temperature, ε is the energy of the single particle levels measured with respect to the constant chemical potential, $E = [\varepsilon^2 + \Delta^2]^{1/2}$, γ is the angular velocity and G is the pairing strength. The pairing cut-off S is taken to be much larger than the gap parameter throughout the calculation: $S \gg \Delta_o$.

By suitable differentiation of the grand potential, all the other thermodynamical equations are obtained:

The gap equation:

$$\frac{\partial\Omega}{\partial\Delta} = 2g \int_{o}^{S} \frac{d\varepsilon}{2E} \left[\tanh\frac{1}{2}\beta(E - \gamma m) + \tanh\frac{1}{2}\beta(E + \gamma m)\right] = 0 \quad ; \quad (2)$$

the angular momentum:

$$I = \frac{1}{\beta}\frac{\partial\Omega}{\partial\gamma} = 2mg \int_{o}^{S} d\varepsilon \left[\frac{1}{1 + \exp\,\beta(E - \gamma m)} - \frac{1}{1 + \exp\,\beta(E + \gamma m)}\right]; \quad (3)$$

the energy

$$E = - \frac{\partial \Omega}{\partial \beta} = g \int_{-S}^{+S} d\epsilon \; \epsilon \left[1 - \frac{\epsilon}{2E} \{ \tanh \tfrac{1}{2} \beta(E - \gamma m) + \tanh \tfrac{1}{2} \beta(E + \gamma m) \} \right] - \frac{\Delta^2}{G} \quad . \quad (4)$$

The case in which $T = 0$ can be worked out analytically and is considered first. The angular momentum equation yields:

$$I = 2mg \; [(\gamma m)^2 - \Delta^2]^{1/2} \quad . \quad (5)$$

The gap equation can also be integrated explicitly, producing the following relationship between Δ and γ:

$$\frac{1}{2} \frac{\Delta_o^2 - \Delta^2}{\Delta_o} = [(\gamma m)^2 - \Delta^2]^{1/2} \quad . \quad (6)$$

By means of eqs. (5) and (6) one obtains the following important relations:

$$\frac{\Delta}{\Delta_o} = (1 - \frac{I}{I_{cr}})^{1/2} \text{ for } I \leqslant I_{cr} \quad \Delta = 0 \text{ for } I \geqslant I_{cr} \quad (7)$$

$$\gamma = \frac{I_{cr}}{\mathscr{J}_R} (2 - \frac{I}{I_{cr}}) \text{ for } I \leqslant I_{cr} \quad \gamma = \frac{I}{\mathscr{J}_R} \text{ for } I \geqslant I_{cr} \quad (8)$$

$$\mathscr{J} = \frac{I}{\gamma} = \mathscr{J}_R \frac{I/I_{cr}}{2 - \frac{I}{I_{cr}}} \text{ for } I \leqslant I_{cr} \quad = \mathscr{J}_R \text{ for } I \geqslant I_{cr} \quad (9)$$

where $I_{cr} = g \underline{m} \Delta_o$, $\mathscr{J}_R = 2 \underline{m}^2 g$.

The eq. (7) gives the dependence of the gap parameter Δ upon angular momentum. The gap parameter decreases as the angular momentum increases and vanishes at the critical angular momentum I_{cr}. The effect of superfluidity

on the moment of inertia is quite visible in eq. (9). The moment of inertia

vanishes at zero angular momentum, and it increases monotonically as the

angular momentum increases from zero to its critical value. Above the critical

angular momentum the superfluidity disappears and the moment of inertia attains

its rigid value. More peculiarly, the angular velocity γ, given by eq. (8),

decreases with increasing angular momentum in the interval $0 \leqslant I \leqslant I_{cr}$. For

values of the angular momentum larger than I_{cr} the angular velocity increases

proportionally to the angular momentum. This behavior generates a very

pronounced back-bend in the plot of the moment of inertia versus the square

of the angular velocity.

By integrating eq. (4) and subtracting the ground state energy one

obtains the yrast function, namely the region of the highest angular momentum

states at a given energy, or conversely, the region of the lowest energy states

at a given angular momentum:

$$E_y = \begin{cases} \dfrac{I_{cr}^2}{\mathscr{J}_R} \dfrac{I}{I_{cr}} \left(2 - \dfrac{I}{2I_{cr}}\right) & \text{for } I \leqslant I_{cr} \\[3mm] \dfrac{I_{cr}^2}{\mathscr{J}_R} + \dfrac{I^2}{2\mathscr{J}_R} & \text{for } I \geqslant I_{cr} \end{cases} \qquad (10)$$

Above the critical angular momentum, the yrast line is a parabola typical of

an object rotating with a rigid moment of inertia. Below the critical angular

momentum the yrast line is also a parabola but it has a negative instead of a

positive second derivative. The consistency of eq. (10) and eq. (8) can be

tested by remembering that:

$$\gamma = \frac{\partial E}{\partial I} \tag{11}$$

If the system is characterized by a moment of inertia depending upon a variable which is allowed to take its equilibrium value, one obtains:

$$\gamma = \frac{\partial E}{\partial I} = \frac{dE}{dI} \tag{12}$$

It is easy to verify that by applying eq. (12) to eq. (10) one does indeed obtain eq. (8).

The case in which $T > 0$ will now be considered. It is well known that at zero angular momentum the gap parameter decreases with increasing temperature and vanishes at the critical temperature T_{cr} given by:

$$T_{cr} = 2\Delta_o/3.5 \tag{13}$$

It follows that the system is characterized by three critical quantities I_{cr}, T_{cr}, $\mathfrak{J}_R$ which can be related to the physical parameters of the system as follows:

$$I_{cr} = g \underline{m} \Delta_o \; ; \quad T_{cr} = 2\Delta_o/3.5 \; ; \quad \mathfrak{J}_R = 2 g \underline{m}^2 \; ; \tag{14}$$

Since these three critical quantities contain all the physical information on the model, it is possible to scale every specific physical situation by means of the following three reduced variables: the reduced angular momentum I/I_{cr}, the reduced temperature T/T_{cr} and the reduced moment of inertia $\mathfrak{J}/\mathfrak{J}_R$.

The dependence of the critical temperature upon angular momentum as well as the overall dependence of the gap parameter upon angular momentum and

temperature are shown in fig. 1. It can be observed that at large angular

momenta, an increase in temperature has an antiblocking effect resulting in an

increase of the pairing correlation. The same effect is responsible for

an increase of the critical angular momentum up to a value $\frac{I}{I_{cr}} = 1.22$ at a

temperature $\frac{T}{T_{cr}} = 0.47$. This effect, called thermally assisted pairing

correlation, has been discussed in detail elsewhere [6].

In fig. 2 the yrast line is shown together with higher energy-angular

momentum isotherms and with the line of the critical energies. One can see that

the yrast line has negative second derivative everywhere below the critical

angular momentum; the lower isotherms have positive second derivatives at low

angular momentum and negative second derivatives at high angular momentum; the

higher isotherms have positive second derivatives everywhere. This behavior

is better appreciated in fig. 3 where the angular velocity is plotted as a

function of angular momentum for various temperatures. At T = 0, as predicted

by eq. (8), the angular velocity decreases as the angular momentum increases.

At higher temperatures the angular velocity goes from zero to a maximum and

then decreases. At the highest temperatures the angular velocity increases

monotonically with angular momentum. At and above the critical angular

momenta, the angular velocity becomes independent of temperature and increases

proportionally to the angular momentum.

The dependence of the moment of inertia on angular momentum is shown

in fig. 4. At zero temperature such a dependence is given by eq. (9). At

larger temperatures the moment of inertia starts with a finite value and

increases until it reaches the rigid value at the corresponding critical

angular momenta.

More impressive is the dependence of the moment of inertia on the square

of the angular velocity shown in fig. 5. The back-bending is extremely pronounced

at T = 0, it is attenuated as the temperature increases until, at a temperature

$\frac{T}{T_{cr}} \approx 0.53$, it disappears. Above this temperature and below the critical

temperature the moment of inertia increases regularly with the square of the

angular velocity. The back bending is associated with the presence of negative

second derivatives in the energy angular momentum isotherms and with the presence

of negative first derivatives in the angular velocity angular momentum isotherms.

The kinks appearing in figs. 3, 4, 5 at the critical angular momenta are due to

a phase transition which should be real for macroscopic systems, but which

becomes an artifact of the theory for small systems where fluctuations are

large. This difficulty can be eliminated by accounting for the fluctuations

as it has been done in ref. 7. Calculations which allow for fluctuations

and which are based on a more realistic single particle model are in progress.

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FIGURE CAPTIONS

Fig. 1. Contour map of the reduced gap parameter as a function of the reduced temperature and of the reduced angular momentum. The lines of constant gap parameter are spaced 0.05 Δ_o.

Fig. 2. Yrast line and higher isotherms. The energy scale is expressed in units of condensation energy. The isotherm next to the yrast line is at T/T_{cr} = 0.158 and the following isotherms are in steps of 0.0526 T_{cr}. The dashed line represents the yrast line of the unpaired system. The upper line crossing the various isotherms represents the boundaries between the paired and unpaired region.

Fig. 3. Reduced angular velocity as a function of the reduced angular momentum for various temperatures. The rising diagonal line represents the angular velocity of an unpaired system. The upper line corresponds to T/T_{cr} = 0. The next isotherm is at T/T_{cr} = 0.158 and the following isotherms are in steps of 0.04 T_{cr}.

Fig. 4. Moment of inertia as a function of angular momentum for various temperatures. The T/T_{cr} = 0 curve reaches the rigid value at I/I_{cr} = 1. The following isotherm is at T/T_{cr} = 0.158 and the spacing between the remaining isotherms is 0.08 T_{cr}.

Fig. 5. Moment of inertia versus the square of the angular velocity for various temperatures (solid lines). The T/T_{cr} = 0 isotherm is incomplete on the right. The next isotherm is at T/T_{cr} = 0.158 and the following isotherms are in steps of 0.04 T_{cr}. The dashed lines are lines of constant angular momentum in steps of 1/7 I_{cr}.
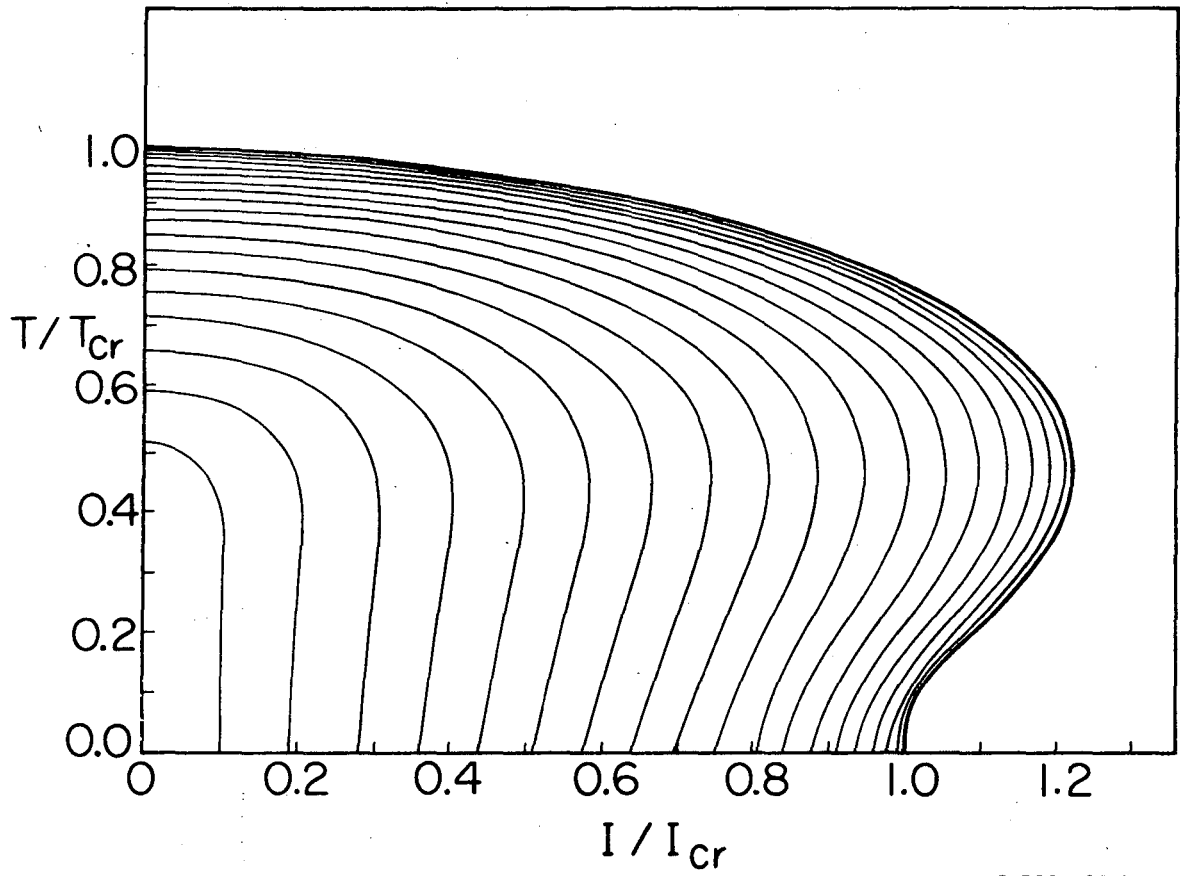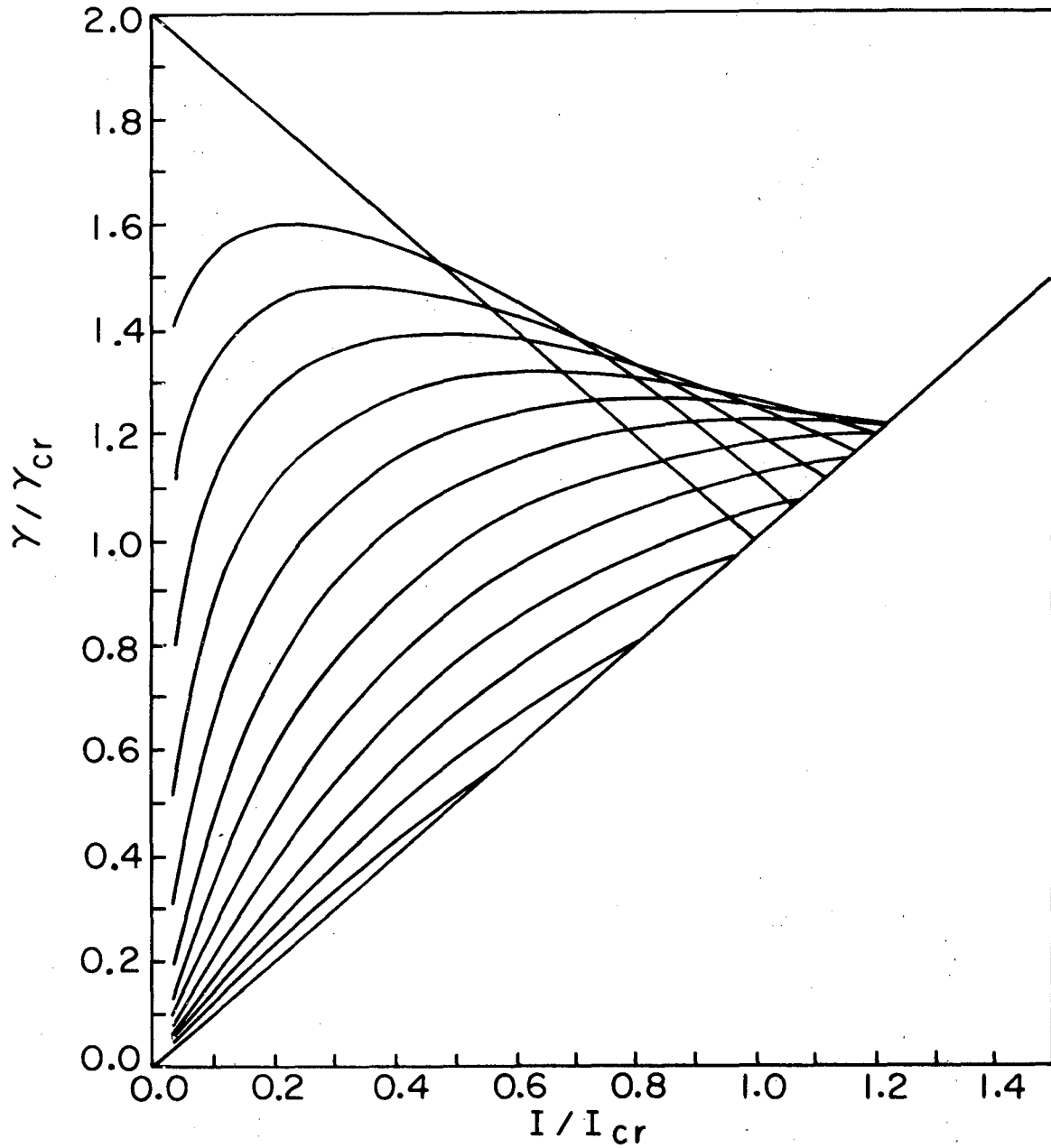
XBL733 - 2510

Fig. 1
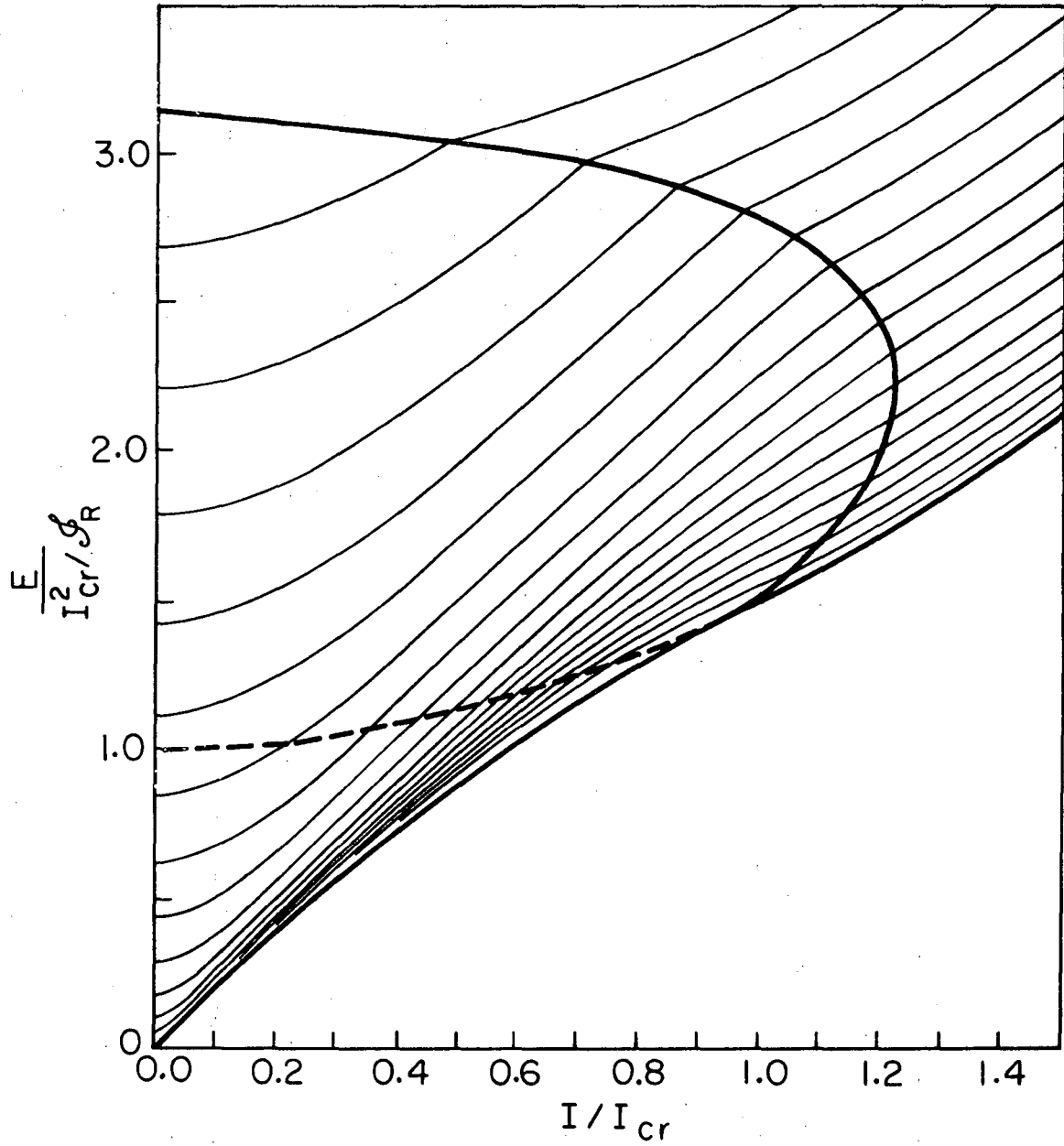
Fig. 2

XBL733-2481

Fig. 3
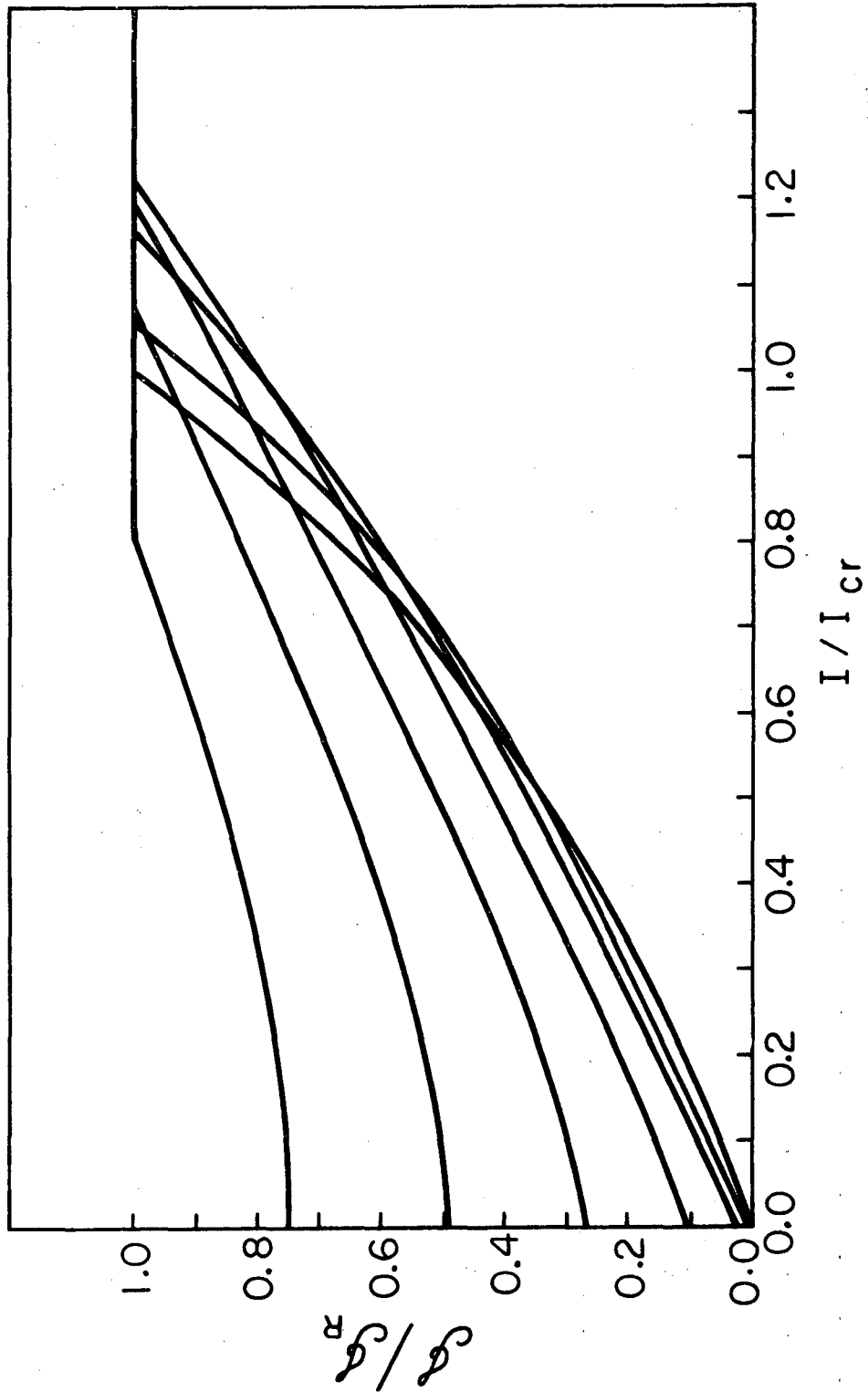
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Fig. 4

Fig. 5

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