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Author
Smith, A.

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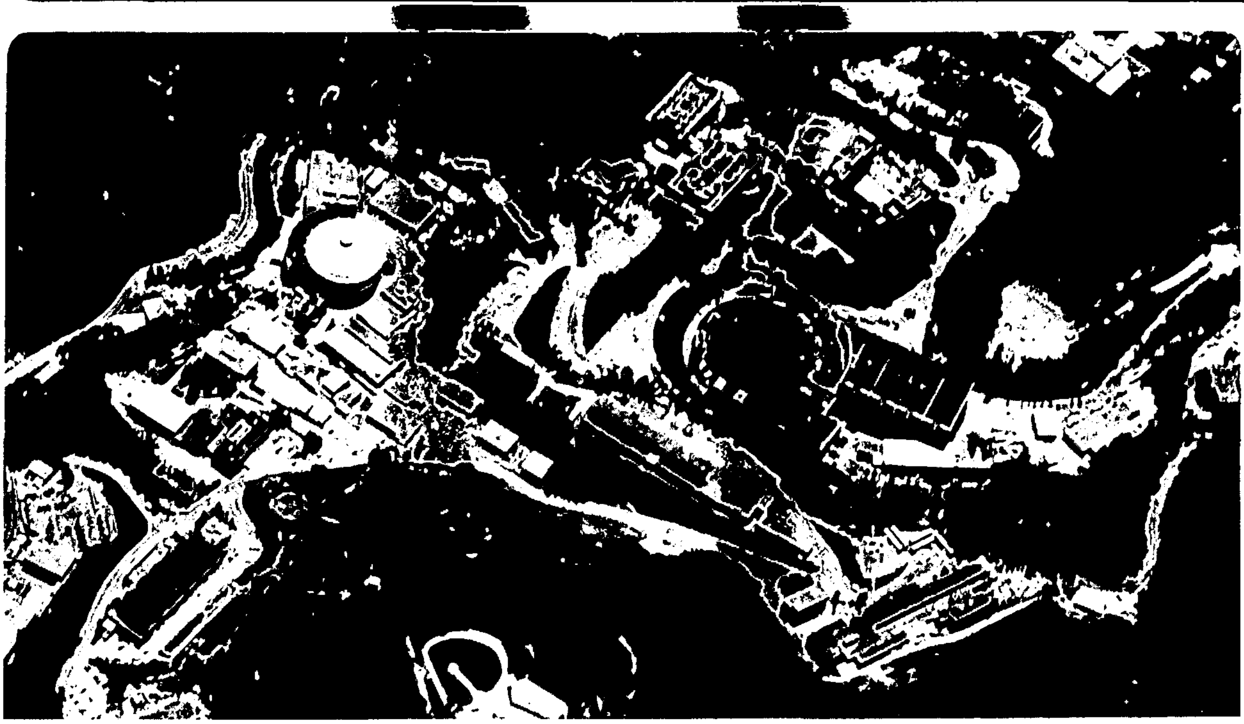

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TWO LOOP CALCULATION IN N=2 LIGHTCONE YANG MILLS

[1]

Adlai Smith

Lawrence Berkeley Laboratory

University of California

Berkeley, California 94720, U.S.A.

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1 Abstract

We calculate the two loop β function in the lightcone form of N=2 Yang Mills. The result is zero in agreement with other calculations using regularization by dimensional reduction. All integrals involved are infrared finite. Supersymmetry makes so many graphs finite that only three topologically distinct 2 loop graphs need be computed.

2 Introduction

An important issue for supersymmetric field theories is the choice of regulator. In the particular scheme of regularization by dimensional reduction (RDR) a breakdown is known to occur [1] for N=2 Yang Mills at the three loop level. While this breakdown does not stop us from discussing one and two loop finite theories it does interfere with finiteness proofs to all orders. While we do not offer a suitable regulator we can use some of the results developed here to prove the graph by graph finiteness of N=4 Yang Mills. This has already been done in the conventional formalism by Lindgren [2].

For N=2 Yang Mills, why do we need another computation of the two loop β function when it is already known to vanish [3,4] within RDR? Due to the difficulty of doing lightcone integrals and until recently the lack of a suitable $1/p_+$ prescription [5] no one has done any complete 2 loop calculation in the lightcone gauge [6]. Hopefully, this work will encourage more such efforts. Working in the lightcone gauge we encounter no offshell infrared infinities in any graph. As opposed to a previous calculation [3] there are no infrared divergent, ultraviolet finite integrals involved. Finally, the infinite part of the 2 loop counterterm is independent of the one loop subtraction prescription.

This paper is organized as follows. Section 2 reviews N=2 Yang Mills and the one loop results concerning it. In section 3 we classify the divergent graphs by their external leg configurations. It turns out that there is only 1

type of divergent four point function. Section 4 contains the computation of the two loop counterterm for the four point function. Possible extensions of this work are mentioned in section 5 and appendices contain notation, Feynman rules, some lightcone integrals and proof of the cancellation theorem.

3 Features of N=2 Yang Mills

The lightcone form of N=2 Yang Mills has been discussed by several authors [7,8]. Throughout we shall use the formulation of reference [7]. In this formulation there are 2 independent superfields ϕ , ϕ^* which depend on 2 θ's only and the lagrangian involves no explicit θ's only $\partial/\partial\theta$'s. As such we write the Feynman rules (fig. 1) in momentum space for both the x_μ's and the θ's. So in addition to the usual 4-momentum integral each loop carries an integral over the two Grassmann or θ momenta. We discuss this further in the next section.

This theory realizes Lorentz symmetry nonlinearly and therefore the wavefunction, three point, and four point renormalization constants are equal to one another provided we can regulate the theory in a covariant fashion. RDR at the one loop level produces equal renormalization constants as shown in [7].

Also the infinite part of the two loop counterterm is independent of the finite one loop subtraction constant. The one loop renormalized two and four point couplings are schematicaly $Z\phi\phi^*$ and $Z\phi^2\phi^{*2}$ where Z=1+$C_2/\pi^2 \cdot g^2/\epsilon$ +g^2B at one loop [7] and we have included the effect of adding an arbitrary finite subtraction constant B. Our subtraction scheme must respect Lorentz symmetry so B is the same for all four renormalization constants. So the four point coupling gets an additional factor Z and the propagator recieves a 1/Z.

As we will see in the next section the only infinite four point functions have external line connections as in figure 2. From figure 2 we see the only effect of one loop counterterms on the four point function is to multiply the one loop infinite graph by $Z^2 \cdot 1/Z^2=1$. So the infinite part of the two loop, four point counterterm will be independent of the finite part of the one loop counterterm. Similar considerations demonstrate the two loop scheme independence of the infinite parts of the propagator and three point counterterms.

4 Classification of Divergent Diagrams

What greatly simplifies calculation of the two loop counterterms is the small number of graphs contributing to the four point counterterm. Analysis of the loop integrals over θ momenta is central. Circulating in each loop along with ordinary four-momentum p_μ are 2 θ momenta $\overline{p}_1$, $\overline{p}_2$ and they are integrated over $\int d\overline{p}_2 d\overline{p}_1$. For purposes of power counting $\overline{p}_1 \sim m^{1/2}$. θ momenta appear in four point vertices and one type of three point vertex and then only in the combination (called brackets) $[\![p,q]\!] \equiv (p,q)_1 (p,q)_2$ where $(p,q)_i \equiv p_+\overline{q}_i - q_+\overline{p}_i$. One fact (cancellation theorem) we need about brackets is: if $I = [\![q_1,q_2]\!] \cdots [\![q_{2n-1},q_{2n}]\!]$ (n brackets in all) and the q_i's are linear combinations of $\leq$ n independent momenta then $I = 0$ identically. The proof is in the appendix. This theorem immediately implies the vanishing of all vacuum diagrams since an l loop diagram must have $\geq$l brackets to 'saturate' the θ momenta integrals but the brackets contain only l independent momenta.

We wish to determine which configurations of external legs produce divergent diagrams. First some general powercounting considerations. Each loop has a $d^4p d\overline{p}_1 d\overline{p}_2$ integration; this is 4-1=3 powers of p. Each vertex has 3 powers of p and each propagator -3 powers. So the total degree of divergence D of a diagram is D=3·loops -3·internal lines +3·vertices -external powers =3·(loops -internal lines + vertices) -external powers = 3 -external powers.

Examining the Feynman rules and considering all possible ways a line or pair of lines can be external we see that every external vertex (a vertex with at least one external line) carries at least one power of external momentum. So it suffices to consider graphs with three or fewer external vertices. Another easily proven topological relation is that the number of brackets in a graph is, l−1 + (number of outgoing external lines), where l= number of loops.

First we will classify the divergent four point functions with two incoming and two outgoing lines (fig 3). Such l loop graphs have l+1 brackets. Those graphs with 3 external vertices are listed in fig.4. They always contain a four point vertex with 2 external lines. The only configurations for external lines from this vertex which contribute only 1 external power of momentun are in fig.5. So among the graphs of fig.4 the divergent candidates have θ momenta integrals that are one of the two following forms:

$$\int d^2\overline{q}_1 \cdots d^2\overline{q}_l [\![X,q_1]\!][\![q_2,q_3]\!] \cdots [\![q_{2l},q_{2l+1}]\!] \tag{3.1}$$

$$\int d^2\overline{q}_1 \cdots d^2\overline{q}_l [\![X,Y]\!][\![q_1,q_2]\!] \cdots [\![q_{2l-1},q_{2l}]\!] \tag{3.2}$$

where the total number of brackets is l+1. X and Y are external momenta while the q_i are linear combinations of internal and external momenta. The most pessimistic analysis ,which we always use, would say expression 3.1 $=\overline{X}_1\overline{X}_2 q_+^{2l+2} + O\left(X_+^2\overline{X}_1\overline{X}_2\right)$ corresponding to D=0 for the diagram. In fact expression 3.1 equals $\overline{X}_1\overline{X}_2 X_+^2 q_+^{2l} + O\left(X_+^4\overline{X}_1\overline{X}_2\right)$ which implies D=-2. This follows from the cancellation theorem. Say the only external vertex factor we got from 3.1 was $\overline{X}_1\overline{X}_2$. This factor comes from the first bracket. The remaining terms are polynomial in X_+^2 (X_+ is any external momentum) and setting the external momentum equal to zero in the remaining l brackets gives the coefficent of the leading (X_+^0) term of this polynomial. But this coefficent consists of the product of l brackets depending on l independent momenta and by the cancellation theorem this product vanishes identically. So the form of 3.1 is as stated.

Applying the same sort of argument as above to expression 3.2 shows it to have the form $[\![X,Y]\!]X_+^2$+more optimistic terms. Therefore diagrams with θ momenta integrations as in 3.2 are also convergent.

Having shown the diagrams with 3 external vertices to be finite we examine those with 2 external vertices. Such diagrams have 2 external four point vertices, each vertex with 2 external legs. For such diagrams the θ momentum integral either vanishes or has the form

$$\int d^2\overline{q}_1\cdots d^2\overline{q}_l[\![Y,Z]\!][\![X,q_1]\!][\![q_2,q_3]\!]\cdots[\![q_{2l-2},q_{2l-1}]\!] \tag{3.3}$$

where there are l+1 brackets in all and X,Y,Z are external momenta. Using arguments similar to those before, 3.3 has the form $[\![Y,Z]\!]X_+^2q_+^{2l+2}$ providing 1 more external power than the most pessimistic estimate. So the only diagrams with two external vertices that diverge have external vertex configurations as in fig.5. Examining them we see that only diagrams of the type

in fig.6 are divergent.

So far we have only discussed four point functions with two incoming and two outgoing lines. Other variations of incoming and outgoing lines are possible, however their divergence would correspond to counterterms in the lagrangian of a form not present from the begining (e.g. $\phi^2\phi^{*2}$). Such terms can be ruled out on the basis of renormalizability. Furthermore an analysis like the one above shows them to be finite.

Examining the other Green functions in the theory we can compute their degree of divergence from their external line configurations. As expected, the only other divergent graphs are 2 and 3 point functions and their external line configurations are listed in figs.7 and 8. Every divergent graph has only 2 external vertices, therefore the corresponding Feynman integrals depend on only one external momentum. There are a large number of types of divergent two point functions.

We have discussed the divergence of diagrams as a whole implicitly assuming a negative D implies convergence of the corresponding diagram. This is false but there is only one exception, a certain one loop propogator diagram. This is discussed in [7] and presents no essential difficulty.

5 Calculation of the Two Loop Counterterms

As noted previously, Lorentz symmetry forces the four renormalization constants to be equal. This is only true so long as we have a regulator which obeys the symmetry. At one loop RDR produces a counterterm proportional to the lagrangian, as required [7]. If we were certain our regulator would produce equal renormalization constants the best calculational method is to evaluate the divergent part of the four point function at the required number of loops. At more than one loop this would involve the fewest number of graphs. So we will calculate $Z_{4-point}$ and assume the Lorentz Ward identities hold at two loops.

There are six graphs contributing at two loops to the infinite part of the four point function. They and their corresponding integrals are listed in fig.9 and table 1. Notice that each graph, especially those with a one loop propagator insertion, are offshell infrared finite in contrast to the calculation in the N=1 superfield formalism using the Fermi-Feynman gauge where the result is IR divergent [3]. RDR for lightcone theories consists of continuing the integrals in the transverse number of dimensions (e.g. in the directions other than p_+ and p_-). This has been done in ordinary Yang Mills [6,9] and gravity [10] and there the continuation is unambiguous since the theories themselves can be formulated in any number of dimensions. As a result we naturally get tensor integrals in the transverse dimensions; in the numer-

ator of a Feynman integral we have terms like, $p_i\,(p-q)_j$ with i,j running over transverse indices. Since as a theory N=2 Yang Mills is only defined in four dimensions the continuation of Feynman integrals in dimension does not come naturally. Mechanically, the difficulty for N=2 Yang Mills is in continuing the integrals of table 1 to arbitrary transverse dimensions. However this is possible since the infinite parts of integrals in table 1 involve in the numerators only the quantity $(p,q)^*\,(q,p) = -p_+^2 q_T^2 - q_+^2 p_T^2 + 2p_+q_+p_T\cdot q_T$, with q_T=transverse components of q_μ. The above quantity can obviously be continued in it's transverse dimension.

Before calculating the two loop graphs remember that as illustrated in fig.2, the one loop counterterms inserted in one loop infinite graphs make no contribution to the two loop counterterm. Also, since the renormalized coupling g_R and the bare coupling are related by $g_R^2{=}g^2/Z\,(g^2,\epsilon)$ there is no $1/\epsilon^2$ part in the sum of the six graphs, only (at most) a simple pole in ϵ.

Adding together the six graphs in fig.9 we get:

$$\frac{-g^4C_2^2}{4\pi^8}\cdot(BV)\cdot[2I+I_1+4I_2+I_3+I_5+2I_6] \tag{4.1}$$

where BV is the bare four point vertex. Using the identity $(p,q)\,(q,p)^* = p_+q_+\,(p+q)^2 - p_+\,(p_++q_+)\,q^2 - q_+\,(p_++q_+)\,p^2$ which is also valid for the extension in transverse dimension we get:

$$2I + I_1 + 4I_2 + I_3 + I_5 + 2I_6 =$$

$$\begin{aligned} &\int \frac{p_+}{q_+} \frac{1}{(p+q)^2 (p+q-X)^2 q^2} \left[\frac{1}{p^2} - \frac{1}{(p-X)^2} \right] \\ &+ \int \frac{(p_+ + q_+)^2}{p_+ q_+} \frac{1}{(p+q)^2 (p+q-X)^2 q^2} \left[\frac{1}{p^2} - \frac{1}{(p-X)^2} \right] \\ &-4 \int \frac{(p_+ + q_+)}{p_+} \frac{1}{p^2 (p+q)^4 (p+q-X)^2} \\ &+ \int \frac{p_+}{q_+} \frac{1}{p^2 (p+q)^2 (p+q-X)^2 q^2} \\ &+ \int \frac{p_+^2}{q_+^2} \frac{1}{p^2 (p-X)^2 (p+q)^2 (p+q-X)^2} \end{aligned} \tag{4.2}$$

In the above $\int \equiv \int d^{4-\epsilon}p d^{4-\epsilon}q$ and X= incoming momenta - outgoing momenta on one of the external vertices. By power counting the first two integrals in 4.2 are finite and after the shift of variable $q \to q - p$ the p integration in the third integral is $\int d^{4-\epsilon}p/p_+p^2 = 0$. Feynman parameterizing the denominators and using two integrals given in the appendix the RHS of equation 4.2 becomes

$$\frac{\pi^{4-\epsilon}}{X_\mu^{2\epsilon}} \left[\frac{2}{\epsilon^2} + \frac{4}{\epsilon} - \frac{2\gamma}{\epsilon} \right] - \frac{\pi^{4-\epsilon}}{X_\mu^{2\epsilon}} \left[\frac{2}{\epsilon^2} + \frac{4}{\epsilon} - \frac{2\gamma}{\epsilon} \right] = 0 + finite (4.3)$$

. The integrals were calculated using the $1/p_+$ prescription of reference [5]. It is interesting that the graphs with double pole parts (first, second and fifth graph in fig 9) cancel against each other when only their double pole part is expected to cancel. This is most easily seen by adding together the graphs with only single pole parts (third,fourth and sixth graphs in fig.9). We need $4I_2 + I_3 + I_6$. Using the formulas in the appendix we calculate

$$I_2 = -\frac{\pi^4}{2\epsilon} \qquad I_3 = \frac{2\pi^4}{\epsilon} \qquad I_6 = 0$$

which implies $4I_2 + I_3 + I_6 = 0$. Whether this generalizes to higher loops is unknown.

So in agreement with previous covariant calculations [3] utilizing RDR , the β function has zero contribution at two loops in the minimal subtraction scheme. In addition to the previously mentioned infrared finiteness the simplicity of the present calculation should encourage further work in multiloop lightcone computations.

6 Extensions and Conclusion

The real consistency check for RDR in the lightcone formalism at two loops would be to calculate the propagator and both three point counterterms. We have not done this but it is a prerequisite for the three loop calculation. At three loops there does not appear to be any problem in continuing the integrals in transverse dimensions for the divergent part of the four point function. However, since RDR breaks down in the conventional formalism at three loops [1] it is likely to break down in the lightcone formalism at three loops also. The vanishing of the infinite part of the two loop counterterm implies the infinite part of the three loop counterterm is independent of the finite one loop subtraction constant. Since the insertion of the one loop counterterm in the two loop graphs in all possible ways gives the contribution of the one loop counterterm to the three loop divergence, and this quantity is just $1/Z\cdot$(sum of two loop graphs) $= 1/Z\cdot$finite,the stated result follows.

In conclusion we have calculated the two loop β function in N=2 Yang Mills by considering the divergent part of the four point function . The computation is greatly simplified by the small number of graphs that are infinite. There are no UV finite, IR divergent integrals encountered in the computation. It is hoped that the classification of divergent graphs will be an aid to a lightcone proof of $\geq$2 loop finiteness. It is also hoped that this two loop β

function computation will encourage higher loop lightcone computations in other models where there is no problem continuing in transverse dimensions, like Yang Mills.

7 Acknowledgements

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8 Appendix on Notation

Throughout the metric is $(-+++)$. The notation used in the Feynman rules is

$$p = p_1 + ip_2 \qquad p_+ = p_0 + p_3$$

$$p^* = p_1 - ip_2 \qquad p_- = p_0 - p_3$$

$$(p,q) = pq_+ - qp_+ \qquad (p,q)^* = p^*q_+ - q^*p_+$$

$$[\![p,q]\!] = (p_+\overline{q}_1 - q_+\overline{p}_1)\,(p_+\overline{q}_2 - q_+\overline{p}_2)$$

$\overline{p}_1, \overline{p}_2, \overline{q}_1, \overline{q}_2$ are the Grassmann momenta.

9 Proof of the Cancellation Theorem

It suffices to prove $J \equiv (q_1, q_2)_1 \cdots (q_{2n-1}, q_{2n})_1 = 0$ when the q's are linear combinations of $\leq$ n momenta. Write the 2n q's in terms of n momenta p_i, $q_j = \sum_k A_{jk} p_k$ where some of the p's may vanish if there are less than n independent momenta. Abbrevating $(j_1, j_2) \equiv (p_{j_1}, p_{j_2})_1$ J becomes

$$J = \sum_{j_1 \cdots j_{2n}=1}^{n} A_{1j_1} \cdots A_{2nj_n} (j_1, j_2) \cdots (j_{2n-1}, j_{2n}) \qquad (a1)$$

. Each term in the sum a1 vanishes since every term contains a closed chain and closed chains vanish. For example, (1,2)(1,3)(3,4)(5,1)(1,4) contains the closed chain (1,3)(3,4)(4,1) (from their definition (a,b)=-(b,a)). Proving closed chains vanish is just a calculation utilizing the identity;

$$\left(p^{(1)}, p^{(2)}\right) \left(p^{(2)} p^{(3)}\right) \cdots \left(p^{(n-1)}, p^{(n)}\right) =$$

$$p_+^{(2)} \cdots p_+^{(n-1)} \left(p_+^{(1)} \overline{p}_1^{(2)} \cdots \overline{p}_1^{(n)} - \overline{p}_1^{(1)} p_+^{(2)} \overline{p}_1^{(3)} \cdots \overline{p}_1^{(n)} + \cdots + (-1)^{n+1} \overline{p}_1^{(1)} \cdots \overline{p}_1^{(n-1)} p_+^{(n)}\right)$$

with $p^{(1)} = p^{(n)}$. This identity is also very useful for computing the θ momenta integrals occuring in loop graphs.

So it suffices to show that if $j_1, \cdots j_{2n}$ are chosen in any way from the set 1,$\cdots$n then $(j_1, j_2)(j_3, j_4) \cdots (j_{2n-1}, j_{2n})$ contains a closed chain. This is obviously true for n=1 or 2. Assume it's true for $n \leq N-1$. There are two cases to consider. First, among the N brackets each number from the set $1 \cdots N$ does not occur exactly twice. If one number does not occur at

all, throw away any one bracket and the inductive hypothesis insures the existence of a closed chain. So some number $j \geq 1$ of the brackets contain the only instance of j numbers. In the previous example, j=2 and 2 and 5 occur once each. Forgetting about the j aforementioned brackets, we are left with N-j brackets chosen from among N-j numbers, and by the inductive hypothesis there exists a closed chain among them.

The second case is easier. If among the N brackets each of the numbers $1, \cdots N$ occurs exactly twice then we have the closed chain

$$(1, a_2)(a_2, a_3)(a_3, a_4) \cdots (a_{n-1}, a_n)(a_n, 1)$$

10 Light Cone Integrals

The only two integrals we need that are not special cases of tensor integrals evaluated in dimensional regularization are:

$$\int \frac{d^{4-\epsilon}q}{q_+ [\omega q^2 + 2l \cdot q + m^2]^\alpha} = i\pi^{2-\frac{\epsilon}{2}} \frac{\Gamma\left(\alpha - 2 + \frac{\epsilon}{2}\right)}{\Gamma(\alpha)} \frac{\omega^{\alpha-3+\epsilon}}{l_+}$$
$$\left[\frac{1}{[\omega m^2 - l_T^2]^{\alpha-2+\frac{\epsilon}{2}}} - \frac{1}{[\omega m^2 - l^2]^{\alpha-2+\frac{\epsilon}{2}}}\right]$$

$$\int \frac{d^{4-\epsilon}q}{q_+^2 [\omega q^2 + 2l \cdot q + m^2]^\alpha} = -i\pi^{2-\frac{\epsilon}{2}} \frac{\Gamma\left(\alpha - 2 + \frac{\epsilon}{2}\right)}{\Gamma(\alpha)} \frac{\omega^{\alpha-2+\epsilon}}{l_+^2}$$
$$\cdot \left[\frac{1}{[\omega m^2 - l_T^2]^{\alpha-2+\frac{\epsilon}{2}}} - \frac{1}{[\omega m^2 - l^2]^{\alpha-2+\frac{\epsilon}{2}}}\right]$$
$$+ i\pi^{2-\frac{\epsilon}{2}} \frac{\Gamma\left(\alpha - 1 + \frac{\epsilon}{2}\right)}{\Gamma(\alpha)} \omega^{\alpha-2+\epsilon} \frac{l_-}{l_+} \frac{1}{[\omega m^2 - l_T^2]^{\alpha-1+\frac{\epsilon}{2}}}$$

.The metric is (- + + +) and both results were obtained by parameterizing denominators, Wick rotating and then doing the transverse and longitudinal integrals in sucession. Here l_T= transverse components of l_μ.

11 References

[1] L.V. Avdeev, A.A.Vladimirov, Nucl.Phys. B219,262 (1983)

[2] O.Lindgren, Proc. XIX Winter School and Workshop of Theoretical Physics, pg.305

[3] P.S.Howe, P.C.West, Nucl. Phys. B242, 364 (1984)

[4] G.Curci, G.Paffuti, Phys.Lett. 148B,78(1984)

I.Jack, Phys.Lett.147B, 405 (1984)

[5] S.Mandelstam, Nucl.Phys. B213, 149 (1983)

[6] G.Leibbrandt, S.Nyeo U.of Guelph preprint (1985-99)

[7] A.Smith,LBL-18668(revised), to be published in Nucl.Phys. B

[8] J.G.Taylor, King's College preprint (unpublished)1982 and Proc. XIX Winter School and Workshop of Theoretical Physics, pg.211

A.Tollsten, I.T.P.(Goteburg) preprint 83-13

[9] A.Andrasi, G.Leibbrandt,S.Nyeo, U.of Guelph preprint (1985-100)

H.Lee, M.Milgram, CRNL-TP-85-II-11

[10] M.Ogren, U.of Goteburg (85-9)

12 Figure and Table Captions

Table 1. $\int \equiv \int d^2 p_L d^{2-\epsilon} p_T \int d^2 q_L d^{2-\epsilon} q_T$

Figure 1. Feynman rules, momentum flows in arrow's direction and the four point vertex is represented as a sum of asymmetric four point vertices.

Figure 2. One loop counterterm contributions to the two loop β function. Only this configuration of external legs will produce an overall divergent four point function at any number of loops.

Figure 3. A class of four point functions which are analyzed for divergences.

Figure 4. Four point function with three external vertices. Arrows on the external lines and the adjoining lines have been omitted as they can be labeled in many different ways.

Figure 5. The only configurations of four point vertices with 2 external lines that contribute 1 external power. At right is the form of the corresponding bracket, X,Y= external momenta q= internal momenta.

Figure 6. The only configuration of external lines producing a divergent four point function.

Figure 7. General form for the divergent three point functions.

Figure 8. The external vertex configurations for the divergent propagator graphs.

Figure 9. Divergent graphs and their corresponding expression. The I's are listed in table 1, BV= bare four point vertex (unsymmetrized).

13 Table 1

$$I = \int \frac{1}{q_+^2} \frac{(q,p)\,(p,q)^*}{p^2\,(p+q)^4\,(p+q-X)^2\,q^2}, \qquad I_1 = \int \frac{(p_+ + q_+)^2}{q_+^2 p_+^2} \frac{(q,p)\,(p,q)^*}{p^2\,(p+q)^4\,(p+q-X)^2\,q^2}$$

$$I_2 = \int \frac{(p_+ + q_+)^2}{q_+^2} \frac{1}{p^2\,(p+q)^4\,(p+q-X)^2}, I_3 = \int \frac{(p_+ + q_+)}{q_+^2 p_+} \frac{(p-X,q)^*\,(q,p)}{p^2\,(p-X)^2\,(p+q)^2\,(p+q-X)^2 q^2}$$

$$I_5 = \int \frac{1}{p^2\,(p-X)^2\,q^2\,(q-X)^2}, \qquad I_6 = \int \frac{p_+ q_+}{(p_+ - q_+)^2} \frac{1}{p^2\,(p-X)^2\,q^2\,(q-X)^2}$$

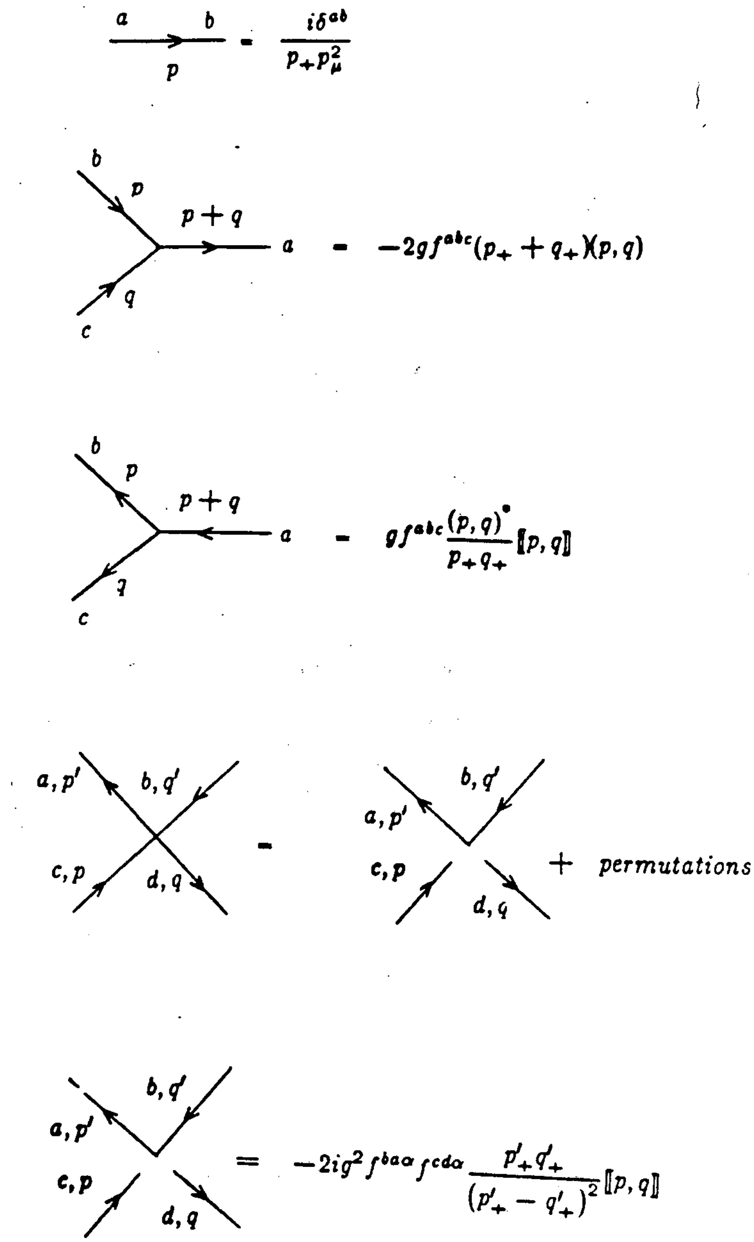

a
b
p
$= \frac{i\delta^{ab}}{p_+ p_\mu^2}$
b
p
$p+q$
a
q
c
$= -2gf^{abc}(p_+ + q_+)(p,q)$
b
p
$p+q$
a
q
c
$= gf^{abc}\frac{(p,q)^*}{p_+ q_+}[\![p,q]\!]$
a, p′
b, q′
c, p
d, q
=
b, q′
a, p′
c, p
d, q
+ permutations
b, q′
a, p′
c, p
d, q
$= -2ig^2 f^{ba\alpha} f^{cd\alpha} \frac{p'_+ q'_+}{(p'_+ - q'_+)^2}[\![p,q]\!]$

Figure 1

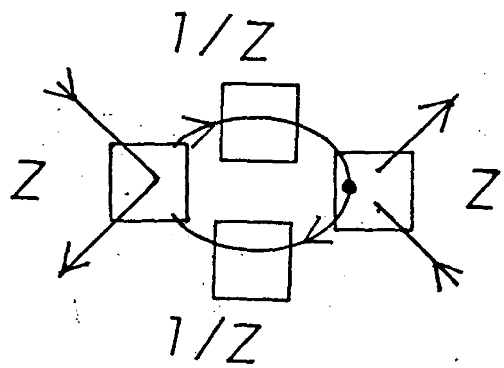


Fig.2

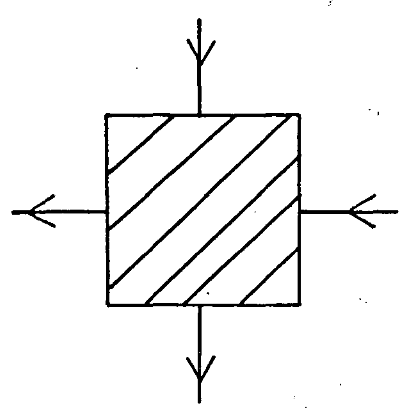


Fig.3

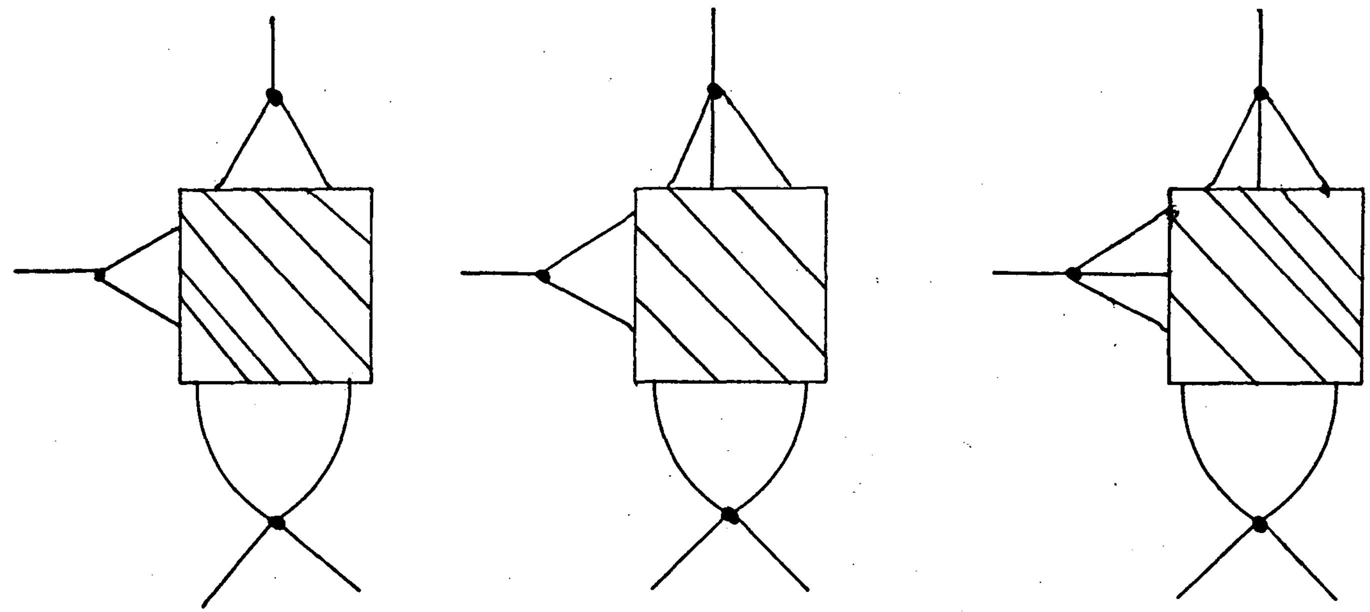


Fig.4

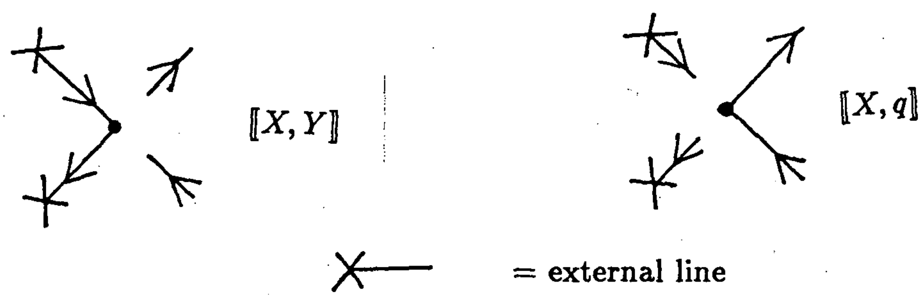


Fig.5

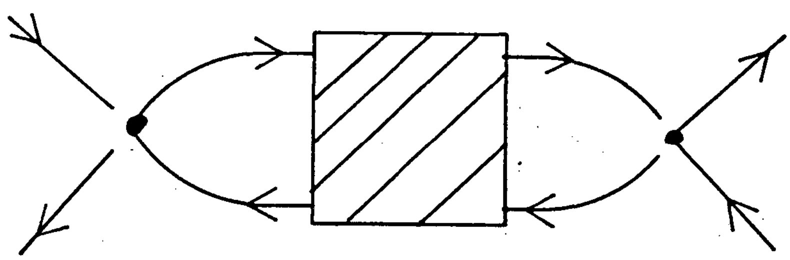


Fig.6

Figure 7

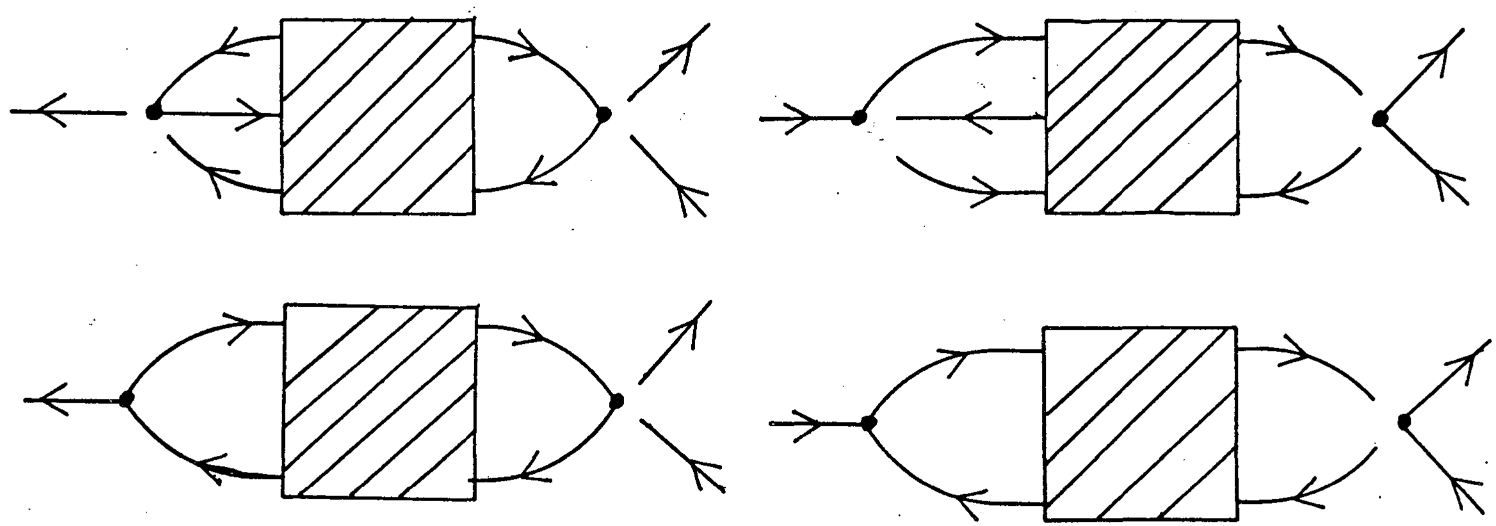
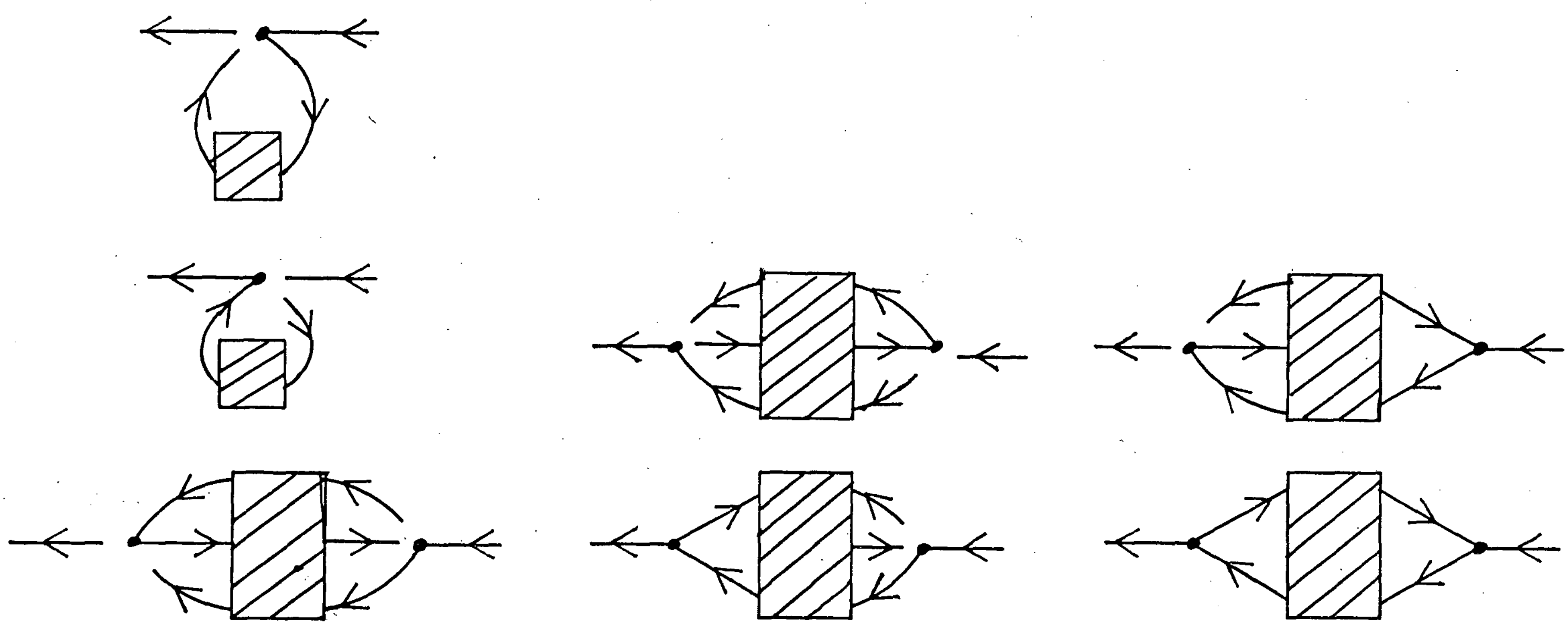
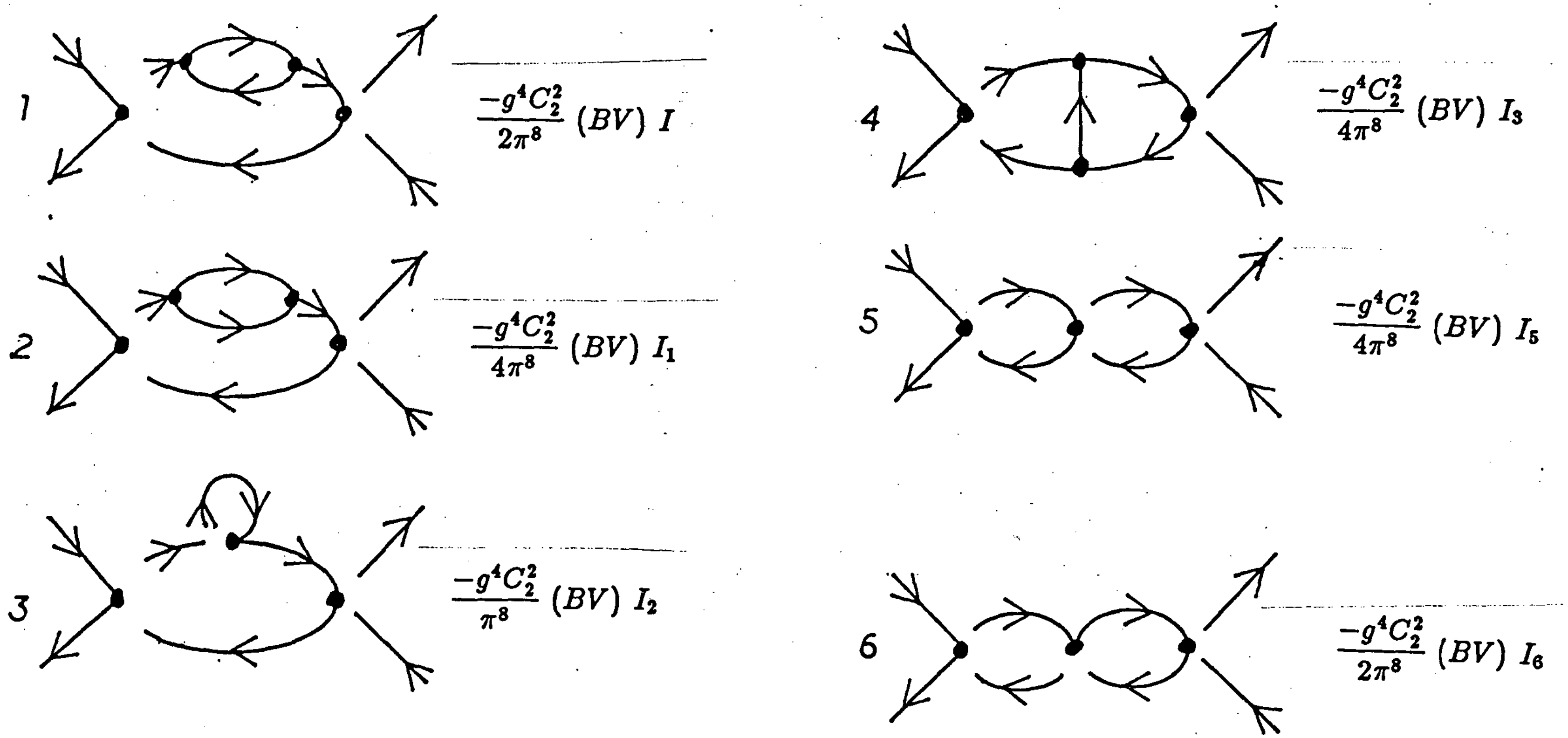


Figure 8





1
$\frac{-g^4 C_2^2}{2\pi^8}$ (BV) I
2
$\frac{-g^4 C_2^2}{4\pi^8}$ (BV) I_1
3
$\frac{-g^4 C_2^2}{\pi^8}$ (BV) I_2
4
$\frac{-g^4 C_2^2}{4\pi^8}$ (BV) I_3
5
$\frac{-g^4 C_2^2}{4\pi^8}$ (BV) I_5
6
$\frac{-g^4 C_2^2}{2\pi^8}$ (BV) I_6

Figure 9

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720