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UNIVERSITY OF CALIFORNIA
SANTA CRUZ

**MAGNETIC FLUX TUBE DYNAMICS IN THE DEEP SOLAR
INTERIOR**

A dissertation submitted in partial satisfaction of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

APPLIED MATHEMATICS

by

Bhishek Manek

December 2021

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2021

Table of Contents

List of Figures	v
List of Tables	xvii
Abstract	xviii
Dedication	xxi
Acknowledgments	xxii
1 Introduction	1
1.1 Solar Observations	1
1.2 Solar Dynamo Theory	2
1.3 Modeling of Magnetic Flux Tubes	5
1.3.1 Rise of Pre-conceived Flux Tubes	5
1.3.2 Magnetic Flux Sheet Instabilities	6
1.3.3 Alternative studies	10
1.4 Solar Hemispheric Helicity Rules	11
1.5 Modeling flux tubes as concentrations	16
2 Magnetic Flux Concentrations in a Background Field: Non-convective simulations	18
2.1 Introduction	18
2.2 Model setup	19
2.3 Flux tube dynamics	26
2.3.1 Suppression of magnetic concentration rise by a background field	28
2.3.2 Causes of the dynamics	31
2.3.3 Implications of selection mechanism to helicity observations	34
2.3.4 A broader survey of $q - B_s$ space at fixed H_B	36
2.3.5 An analytical model of the dynamics	41
2.3.6 Dependence on the scale-height of the background field . .	58

2.3.7	Multiple flux concentrations	63
2.4	Discussion	74
3	Magnetic Flux Concentrations in a Background Field: Convective simulations	82
3.1	Introduction	82
3.2	Model and Methods	86
3.3	Results	91
3.3.1	Convection in 2D	93
3.3.2	Rise of Flux tubes in Pure Convection	98
3.3.3	Dynamical formation of Background Magnetic Field	106
3.3.4	Rise of Flux tubes in the presence of a Background Field .	113
3.3.5	Dependence on B_s or vertical location	121
3.3.6	Dependence on horizontal location and initial timestep . .	123
3.3.7	Tension forces	128
3.3.8	Variation with S	134
3.4	Discussion	142
4	Conclusion	152
5	Appendix	158
5.1	Robustness of flux tube dynamics	158
5.2	Gaussian flux tube dynamics	163
5.3	Three-dimensional flux tube dynamics: Non-convective simulations	169
5.4	Three-dimensional flux tube dynamics: Convective simulations . .	173
5.5	Magnetic Flux Emergence	178
	Bibliography	184

List of Figures

1.1	(i)a Interconnecting loops from X-ray observations of two active regions in solar corona, (i)b line-of-sight magnetic field measurements with positive polarity in white and negative polarity in black, (i)c 3D magnetic field line from mathematical extrapolation of surface fields in panel a, (i)d Side view of field lines in c. (ii) Cyclic activity of sunspots as butterfly diagram. Credits: NASA	2
1.2	Self sustaining turbulent dynamo action. Credits: Parker, E.N. . .	3
1.3	(a) Rise of twisted flux tube through a stratification. The gray scale and contours correspond to the longitudinal magnetic field intensity. Figure adapted from [39], (b) Evolution of magnetic field from a flux sheet instability. Figure adapted from [21].	7
1.4	Isosurfaces of $ B $ at three different times. Adapted from [82]. . . .	8
1.5	Volume renderings of magnetic field magnitude, $ B $, at different time, showing the formation and evolution of magnetic layer to form flux tube-like structures. Adapted from [112].	9
1.6	Mean density of the current helicity parameter $B_{ }(\nabla \times B_{ })$ of active regions plotted as a function of latitude and time. Open circles denote negative helicity and filled circles denote positive helicity active regions. Figure adapted from: [119].	13

2.1	Cartoon sketch of the relation of the model to the Sun (adapted from Section 2.3). Our 2.5 dimensional model relates the Cartesian directions x to latitude, y to height, and z to longitude. Vectors in the z direction exist but the model is independent of this direction. The domain spans the lower convection zone and the transition to the radiative zone. The model horizontal background field is related to deep, large-scale poloidal field. The magnetic concentration is identified with twisted toroidal field structures, and is initially placed deep in the stratification.	22
2.2	Vertical profiles of the initial thermodynamic conditions through the center of the magnetic flux concentration, along $x = 0$	25
2.3	Intensity plots of normalised $B_z(x, y, t)$ for $B_s = 0, 0.1, 0.2, -0.02, -0.06$ (a,b,c,d,e respectively) and times $(t_1, t_2, t_3) = (5, 10, 15)$ except for (c) where $(t_1, t_2, t_3) = (2.5, 5, 8.75)$. Contours of A_z have been added to some panels. Panels (a,b,d) represent successful rises, whereas in panels (c,e) the rise is suppressed.	27
2.4	Rising flux fraction $f_{flux}(t) = \int \int B_{z,t}^* dx dy / \int \int B_{z,0} dx dy$ where $B_{z,0} = B_z(x, y, t = 0)$ and $B_{z,t}^* = B_z(\{(x, y) \text{ where } v_y(x, y) > v_{y,threshold}\}, t)$ for the same cases as in Fig. 2.3. Here, $v_{y,threshold} = 0.03$ is a judiciously-chosen threshold vertical velocity that filters out unimportant motions. (a) $B_s \geq 0$. (b) $B_s \leq 0$	28
2.5	Simulation data and cartoons for the forces acting in y -direction: (i) tension, $F_T = (B_x \partial_x + B_y \partial_y) B_y$; (ii) buoyancy force $F_B = -\partial_y B_y^2 / 2 - \rho \theta (m + 1) - \partial_y P_{gas}$ evaluated at $x = 0, t = 0$. Panels (a,b,c) correspond to $B_{back} = 0\%, 6\%, -6\%$ respectively. The cartoons show end-on views of rising tubes with black net force arrows for the respective forces (tension or buoyancy) on representative field lines inside and outside the tube for the (b) and (c) cases. The green arrows show the twist of the tube. (iii) Time series of the vertical flux of axial field, $F_A = v_y B_z$ for $B_s = 0.1$	32

- 2.6 This figure shows how to translate our model results into the solar case. Panels (a) and (b) show a pictorial representation of the first and second half of an arbitrary full 22-year solar cycle, respectively. The cartoon spheres depict the action of differential rotation on existing dynamo poloidal field (i.e. the mean-field Ω effect). The dynamo poloidal field is represented with red arrow markers and toroidal field with blue markers. The differential rotation (exhibited as black arrow markers) stretches the poloidal field into toroidal field, leading to a certain orientation of the directions of the two field in each hemisphere (for each half of the cycle). The cylindrical cartoons show how the above oriented fields relate to the axial (blue, toroidal) and background field (red, poloidal) of our model. Furthermore, the tube cartoons show the twist (azimuthal, non-axial field, green markers) of structures that are predicted to rise preferentially by our model. The resultant correlation of axial (blue) and twist (green) is commensurate with the SHHR. Figure adapted from Section 2.3. 36
- 2.7 A map of rise success/failure in $q - B_s$ space for fixed $H_B = 0.125$. Background field strengths B_s are given as percentages of the axial field strength of the magnetic flux structure. Background field and flux tube twist orientation by sign are shown schematically for clarity. Red (darkest) color signifies a definite failed rise of the flux structure, whereas the green (lightest) color signifies a definite successful rise. Orange (intermediate shade) signifies an intermediate result, where the structure may rise but the dynamics are noticeably different than other rising cases. Simulations lying inside the white dotted lines are either cases with negligible background field $B_s \sim 0$ or cases where the twist is so low $q \sim 0$ that structures would not rise coherently. 39

2.8	Shown are forces acting on the flux tube in vertical direction calculated from the theoretical model: Area-integrated net buoyancy and tension forces internal to the tube; the upper bound estimation of the opposing tension force induced in the background field by the rise; and, the total upwards vertical force (the sum of internal buoyancy, internal tension and induced external tension). These are plotted as a function of the flux tube twist, q , for $B_s = (a)$ 0.03, (b) 0.10 and (c) 0.16, for the canonical value of $H_B = 0.125$. The symbols show the corresponding integrated forces calculated using the simulation data.	49
2.9	The SRR given by the model as a function of B_s for $H_B = 0.125$. Black circles and red diamonds represent the absolute value of roots of Equation (2.29). For these cases with positive B_s , q_1 is negative and is the root with larger modulus (black circles). The other root q_2 has smaller modulus (red diamonds) and changes sign from positive to negative at around $B_s \sim 0.09$. Blue dashed vertical lines in the plot are the cases studied using numerical simulations and the black dashed line represents the highest allowable twist that keeps the thermodynamic conditions inside the flux concentration physical (i.e. such that the density remains positive). The green circles are the limits of the SRR found from the numerical simulations.	51
2.10	Variation of background field as a function of height for different scale heights H_B for $B_s = 0.1$. Dotted horizontal lines indicate the vertical location of the flux tube.	55
2.11	Effect of the background field scale-height variation on the dynamics. Intensity plots of normalized $B_z(x, y, t)$ for $H_B = 0.125, 0.175, 0.300$ (a, b and c respectively) when $B_s = 0.10$ and $q = 2.5$ in all cases. All the subplots are scaled with respective $\max(B_z)$	56

2.12	Absolute value of the roots of Equation (2.29). The region between the black circles and red diamonds is the SRR. (a) Variation of SRR for a fixed H_B as a function of B_s . Blue vertical dashed lines correspond to $B_s = 0.05, 0.10, 0.16$, which are values where we have comparable simulations. (b) Variation of SRR for a fixed B_s as a function of H_B , Blue vertical dashed lines correspond to $H_B = 0.125, 0.175, 0.300$, which are values where we have comparable simulations.	57
2.13	Width of the SRR, $ (q_1 - q_2) $ plotted as a function of (a) B_s for different H_B and (b) H_B for different B_s	58
2.14	Synthetic SHHR. Each dot shows the sign (solid circle = positive; open circle = negative) and aggregated strength (vertical axis) of the horizontal component of the current helicity that passes through $y = 3$ during a the interval $t = 0$ to $t = 70$ from a simulation involving five flux structures embedded within the background field of strength B_s shown for each panel. The flux structures in each simulation have randomly assigned twist and separation from a distribution. Each panel contains 50 of such MC simulations, and represents a particular time in the solar cycle, with low B_s representing times around solar minimum, and higher B_s being associated with solar maximum. Percentages show the relative distribution of positive and negative helicity. Positive helicity is expected to dominate for the configuration.	66

2.15	Solar Hemispheric Helicity rule violations. Both panels show intensity plots of normalized $B_z(x, y, t)$ at $(t_1, t_2, t_3, t_4, t_5, t_6) = (1, 11, 21, 31, 41, 51)$. Panels a and b are independent instances of the MC simulations at $B_s = 0.05$, chosen to depict particular SHHR violation mechanisms. With the axial field, B_z , out of the plane and poloidal background field aligned in the positive x-direction, this setup is representative of the solar southern hemisphere (see Fig. 2.6a). The sign and strength of the field line twist (q) at t_1 is indicated for all the concentrations, with the subscript corresponding to the numbering of the concentrations along the x-axis. Panel (a) shows violations occurring due to the rise of strongly twisted tube with negative twist. Panel (b) shows a violation occurring due to the interaction of two negatively twisted tubes.	69
2.16	Violation of the SHHR via restructuring of the twist. Panels show normalized $B_x(x, y, t)$ for $B_s = 0.1$ at $(t_1, t_2, t_3, t_4, t_5, t_6) = (0, 5, 9, 15, 20, 40)$. Tube 5 has anti-clockwise (positive) twist at $t = t_1$ but gets rearranged to a clockwise (negative) twist by $t = t_5$. This mechanism is quite common for strong background fields and weakly twisted structures.	71
3.1	Profiles of polytropic thermodynamic initial conditions, a) T , and b) ρ , as a function of depth z at $x = 0$ for $S = 3, 7$ and 15 . The dashed line marks the transition between the convection zone and the radiative zone.	92
3.2	Kinetic energy as a function of time for the case with $S = 7$, $Ra = 4 \times 10^4$ and $Pr = 0.1$	94
3.3	Vertical velocity (w) snapshots plotted at 4 different times for the case with $S = 7$, $Ra = 4 \times 10^4$ and $Pr = 0.1$. Blue color indicates downflows whereas upflows are red. The dashed line marks the transition between the convection zone and the radiative zone. . .	95

3.4	Time- and horizontally-averaged kinetic energy flux as a function of z for three different cases, $S = 3, 7$, and 15 . The horizontal dashed line at $z = 1$ marks the transition between the convection zone and the radiative zone. Overshooting depth is the z -location at which the curves meet the y -axis in the stable region.	97
3.5	Vertical velocity (w) snapshots for (a) $S = 3$ and (b) $S = 15$. . .	98
3.6	Time- and horizontally-averaged entropy as a function of height for $S = 3, 7$, and 15	99
3.7	Intensity plots of normalized B_y , overplotted with the normalized vertical velocity, w , as a function of time for the case with $S = 7$, $Ra = 4 \times 10^4$, $Q = 2 \times 10^8$, $Pr = 0.1$ and $Pm = 100$. The initial location of the flux tube is $(x_c, z_c) = (3.0, 2.0)$. The dashed horizontal line indicates the transition between the convection zone and the radiative zone.	103
3.8	Same as Figure 3.7 but for $Q = 5 \times 10^6$	104
3.9	The maximum of B_y as a function of time z_{ft} for different Q . Other parameters are $Ra = 4 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, $q = 0.5$, and $S = 7$	106
3.10	Comparison of the dynamics of positively- and negatively-twisted tubes for $Q = 2 \times 10^8$ (a and b) and $Q = 5 \times 10^6$ (c and d): (a) z_{ft} plotted as a function of time for both $q = 0.5$ and $q = -0.5$, (b) The equivalent of Figure 3.7d but for $q = -0.5$, (c) z_{ft} plotted as a function of time for both $q = 0.5$ and $q = -0.5$, (d) The equivalent of Figure 3.8d but for $q = -0.5$	107
3.11	Snapshots of normalized horizontal magnetic field, B_x , as a function of time. (a)-(f) have $Q = 4 \times 10^6$ whereas (g)-(h) have $Q = 2 \times 10^8$	114

3.12	Time- and horizontally-averaged pumped horizontal field as a function of height (z) for the case with $S = 7$. Initial and final profiles are plotted with a thicker line in each of the subplots. (a) shows the initial pumping of the imposed magnetic layer. The final profile of the pumped field is multiplied by a factor of 20 and its evolution is shown in (d).	115
3.13	Time averaged pumped horizontal field as a function of height (z) for the case with $S = 7$. Black diamond markers indicate the different z -coordinates centred at which flux tubes are introduced. These z -coordinates correspond to 1, 2.5, 5, 10, 15 and 20% strength of the pumped horizontal field as compared to the flux tube strength. The dashed line marks the transition between the convection zone and the radiative zone.	116
3.14	Intensity plots of normalized B_y , overplotted with normalized vertical velocity, w , for (a) $q = 0.5$ and (b) $q = -0.5$ at four different times. The time-averaged pumped background field strength, B_s , at the center of flux tube in both cases is 0.10, i.e., 10% of the initial strength of the flux tube.	120
3.15	Plots from the canonical case at $S = 7$ (from Section 3.3.4) showing the tension force acting in the z -direction, $F_{z,tens} = (B_x\partial_x + B_z\partial_z)B_z$, evaluated at $x = 3.0$ and $t = 0.0$ for both positive ($q = 0.5$) and negative ($q = -0.5$) twisted tube. The dashed line shows the profile from the symmetrical tube from Section 3.3.2 where no pumped background field is present.	121
3.16	Plot of z_{ft} as a function of time for (a) $q = 0.5$ and (b) $q = -0.5$. Each subplot shows five cases with different B_s (namely, $B_s = 1, 2.5, 5, 10, 15, 20\%$ of the initial flux tube strength) at the center of the flux tube.	123

3.17	Plots of z_{ft} as a function of time showing the successful/unsuccessful rise of the flux tube in the presence of a pumped background field of strength, $B_s = 0.10$. The first column (subplots a, c and e) shows the cases with $q = 0.5$ whereas the second column (subplots b, d and f) has $q = -0.5$. Each of the three rows represent different initial conditions chosen from the results of Section 3.3.3 at times, $t_1 < t_2 < t_3$. Each subplot shows five different horizontal locations of the flux tube, x_c	129
3.18	Same as Figure 3.17 with $B_s = 0.01$	130
3.19	Same as Figure 3.17 with $B_s = 0.20$	131
3.20	Plots of the tension force acting in the z -direction, $F_{z,tens} = (B_x \partial_x + B_z \partial_z) B_z$, from the MC data in Section 3.3.6). The data in this Figure corresponds to that in Figure 3.17e and f. The line cuts of tension force are evaluated at $t = 0.0$ and the center of each tube, for both positively-twisted ($q = 0.5$) and negatively-twisted ($q = -0.5$) tubes. The dashed line shows the profile from the symmetric tube of Section 3.3.2 where no pumped background field is present.	132
3.21	Same as Figure 3.9 but for (a) $S = 3$ and (b) $S = 15$	136
3.22	Snapshots of normalized horizontal magnetic field, B_x , for (a) $S = 3$ and (b) $S = 15$, at time $t = t_1$. Note that t_1 represents different times in both the subplots. Here, we have $Q = 4 \times 10^7$	137
3.23	Time and horizontally averaged pumped field as a function of z for $S = 3$ and 15. Diamond markers indicate the vertical location corresponding to $\langle B_x \rangle = B_s = 0.10$	137
3.24	z_{ft} plotted as a function of time, at different x_c , for twists, (a) $q = 0.5$, and (b) $q = -0.5$. Here, $t = t_1$ indicated the time at which the pumped background field was inserted with a flux tube. Note that $S = 3$ for this case. Here, $Q = 4 \times 10^7$	141
3.25	Same as Figure 3.24 but for $S = 15$	142

3.26	Plots of the tension force acting in the z-direction, $[F_{tens}]_z$ on line cuts through the middle of the tubes at $t = 0$ for various different x_c and both signs of q : $q = 0.5, -0.5$. This case corresponds to the $S = 3$ MC cases of Figure 3.24a and b.	143
3.27	Same as Figure 3.26 but for MC cases at $S = 15$ that correspond to those in Figure 3.25.	144
5.1	Thermodynamic initial conditions for (i) Case 1 and (ii) Case 2: (a) gas pressure, (b) density, (c) temperature, and (d) total pressure. A subset of the domain is shown for clarity.	161
5.2	Comparison of (a) vertical position of the head of the rising flux tube and (b) rising flux fractions for two different initial thermodynamic conditions for $B_s = 0$. Flux fraction (calculated with $v_{y,threshold} = 0.03$) in (b) has been curtailed at the time when the flux tube structure reaches the top of the simulation box.	162
5.3	Line cuts of B_z against x at the height of the center of the magnetic concentration, $y = 0.5$, for simulations started with both top hat and Gaussian profiles inside the magnetic structure. (a) The initial condition at $t = 0$. (b) A short time later, at $t = 1$	164
5.4	(a) Rise time and (b) flux fraction for top-hat and Gaussian magnetic field profiles at $B_s = 0$	165
5.5	Intensity plots showing the evolution of (normalized) $B_z(x, y, t)$ for various background field strengths. Shown in rows (a, b, c, d and e respectively) are $B_s = 0, 0.1, 0.2, -0.02, -0.06$. For all cases, $H_B = 0.125$. Three times are shown for each case. In (a, b, d, e) the times are $(t_1, t_2, t_3) = (5, 10, 15)$ but for (c) the times are $(t_1, t_2, t_3) = (2, 5, 8)$. Contours of A_z have been added to panels (b) and (d). Panels (a, b and d) demonstrate cases of successful rise, whereas in panels (c and e) the rise is suppressed. The initial axial magnetic field profile in each of these cases is given by Equation (5.8).	167

5.6	Rising flux fraction, $f_{flux}(t)$ for the initial Gaussian magnetic axial field profile. (a) $B_s \geq 0$, (b) $B_s \leq 0$ with $H_B = 0.125$ and $v_{y,threshold} = 0.02$	169
5.7	Side-on perspective of 3D magnetic field lines for the case with $B_s = 0.10$ at two different times.	170
5.8	Volume rendering of B^2 : (a) $B_s = 0.0$, (b) $B_s = 0.05$, and (c) $B_s = -0.05$. The twist of the flux tube is fixed at $q = 2.5$. Red, green, and blue arrows at the edge of the box (clearly visible in (a)) correspond to the x, y, and z axes.	172
5.9	Volume renderings of vertical velocity (w) at four different times for $Ra = 1 \times 10^4$ and $Pr = 0.1$. A faintly marked surface separates the convective layer on the top with the stable layer at the bottom. Red and green arrows indicate the x and y directions.	174
5.10	Volume renderings of B^2 at four different times for $Ra = 1 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, and $Q = 4 \times 10^6$. The plane layer separates the convective layer at the top from the stable layer at the bottom. Red and green arrows indicate the x and y directions.	176
5.11	Time- and horizontally-averaged pumped horizontal field as a function of height (z) for the case with $S = 3$	177
5.12	(a) The initial set up of the simulation domain. The gray plane corresponds to the photosphere. The color image in the vertical cross-section shows the temperature of the initial background equilibrium plane-parallel atmosphere permeated with a uniform open magnetic field represented by the field lines in the vertical cross-section. A twisted magnetic flux tube, as represented by the twisted field lines, is inserted into the interior layer below the photosphere. (b) The temperature $T_0(z)$ (black curve) and density $\rho_0(z)$ (blue curve) profiles of the initial background equilibrium plane-parallel atmosphere. See text for detailed description.	180

5.13	Intensity plots of (a) density and (b) temperature (in log scale) overplotted with velocity and magnetic field vectors respectively. For this case, $B_s > 0$	182
5.14	Same as 5.13 but for $B_s < 0$	183

List of Tables

2.1	Roots of the quadratic equation 2.29 for some example values of B_s and H_B	48
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Abstract

Magnetic Flux tube dynamics in the deep solar interior

by

Bhishek Manek

This doctoral work is motivated by magnetic field transport processes in the deep interior of the Sun and their eventual observational signatures at the solar surface. In particular, we are inspired by a combination of previous computational and observational work. First of all, based on observations at the solar surface of sunspots embedded in active regions, it is widely believed that large-scale, strong magnetic flux emerges from the Sun’s deep interior in the form of arched, cylindrical tube-like structures often known simply as ‘magnetic flux tubes.’ The buoyant transport of these flux tubes from the solar interior towards the surface plays an essential role in the emergence of active regions, the formation of sunspots, and the overall solar dynamo. A range of two- and three-dimensional computational work in the past has focused on the formation and transport of magnetic flux tubes from the deep solar interior. Many such studies have assumed the existence of such magnetic structures and studied highly simplified models like the buoyant rise of an isolated flux tube in a quiescent, field-free environment. Here, motivated by their formation, e.g. [33, 18], we systematically attempt to remove some of these restrictive assumptions and study the rise of a toroidal flux tube embedded in a large-scale poloidal background magnetic field, with and without the presence of turbulent convection.

In this thesis, we have divided our work into two major parts. In the first part, we investigate the rise of a *non-isolated* toroidal flux tube through a volume-filling large-scale magnetic field in a quiescent background state. We find that positive

and negative twisted flux tubes show starkly different dynamics. In particular, for a given large-scale magnetic field, flux tubes of one sign of the twist are more likely to rise than the other. We have created a mathematical model based on the forces acting on the flux tube that can explain and even predict the observed asymmetric rise dynamics for a given flux tube twist and background field orientation. This reveals a filtering region in the parameter space that we refer to as Selective Rise Regime (SRR). To better understand the statistical relevance of the SRR, we also carry out Monte Carlo (MC) simulations of multiple flux tubes rising through the background field. This study further shows that the SRR, along with the MC simulations, plausibly explains the broad range of solar helicity observations that are collectively known as the Solar Hemispheric Helicity rules (SHHR). The SHHR declare primarily that there is a preferred helicity of the emerging flux in each hemisphere (negative for Northern; positive for Southern). Our model provides an explanation for this major bias. Furthermore, the SHHR is a weak rule, obeyed only 60-80% of the time. Our models also provide good physical reasons for statistical violations of the rule, and further makes predictions about the cycle dependence of the rule that could be tested.

In the second part, we examine a more realistic and therefore more complex version of the same thing. Specifically, we consider the buoyant rise of magnetic flux tubes from their formation in a deep radiative zone, through a turbulent overshooting convection zone that self-consistently arranges a volume-filling large-scale background field. Despite the presence of much more complicated dynamics, we still find the same preferential rise of flux tubes of a particular twist, thus establishing the robustness of SRR mechanism. The presence of convection does however add another layer of statistical fluctuations to the observed asymmetry in the flux tube dynamics, which we investigate by Monte Carlo-like suites of

simulations to again confront variations in the SHHR.

This Ph.D. work is dedicated to the loving memory of my grandfather, Shri Ishwarlal Manek, my academic mentor, Prof. Shashikumar M. Chitre, and two of my beloved friends, Abhijith Varma and Dr. Ruchika Seth.

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Chapter 1

Introduction

1.1 Solar Observations

Observations of the Sun show the ubiquitous presence of magnetic fields. Detailed observations of these magnetic fields have revealed the wide-ranging length scales that they display. Intergranular bright points, believed to be the foot-points of the flux tube, have diameters of about a few hundred km. Solar pores, also known as naked sunspots, have diameters of about 1000 – 6000 km. The largest scales observed can cover significant fractions of a hemisphere. However, the most easily and therefore most commonly observed magnetic features on the solar surface are sunspots embedded in active regions (see Fig. 1.1(i)), with their length scales ranging from a few thousand km to tens of thousands km.

While small-scale fields (on the scale of observable solar surface velocities) exhibit very turbulent behavior, the active region-scale fields appear to have surprisingly ordered dynamics. Observational studies have clearly established the cyclic nature of such large-scale solar magnetic fields, often exhibited as the butterfly pattern of the emergence of active regions and surface sunspots (Fig 1.1(ii)). These bi-polar sunspots follow an 11-year cycle and occur in pairs with opposite

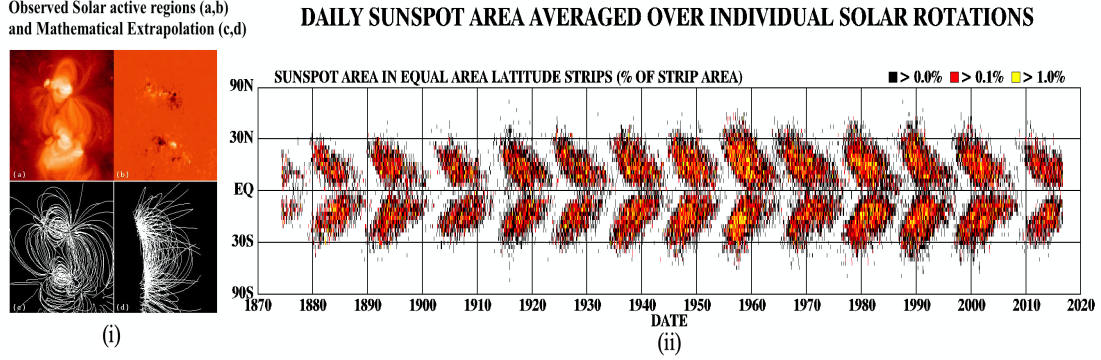


Figure 1.1: (i)a Interconnecting loops from X-ray observations of two active regions in solar corona, (i)b line-of-sight magnetic field measurements with positive polarity in white and negative polarity in black, (i)c 3D magnetic field line from mathematical extrapolation of surface fields in panel a, (i)d Side view of field lines in c. (ii) Cyclic activity of sunspots as butterfly diagram. Credits: NASA

polarities in either hemisphere, and these polarities are reversed every solar cycle (Hale’s Polarity law). Joy’s law [55], another such regularity rule, states that the longitudinal tilt angle of the bipolar sunspots increases with increasing latitude.

1.2 Solar Dynamo Theory

The current understanding of the formation of large-scale solar magnetic fields revolves around a self-sustaining dynamo mechanism operating in the solar interior. Observations and modeling have led to a simple picture of how the dynamo mechanism might work. The cyclic amplification of the magnetic field is essentially described as a feedback loop where the toroidal field is generated from the poloidal field by differential rotation of the gas, and the poloidal field is regenerated from the toroidal by helical turbulence. In mean-field theory these are parametrized as the Ω and α effect respectively (see Figure 1.2).

Helioseismic studies of the solar interior have established that the outer 30% of the Sun (convection zone) is differentially rotating, faster near the equator (25

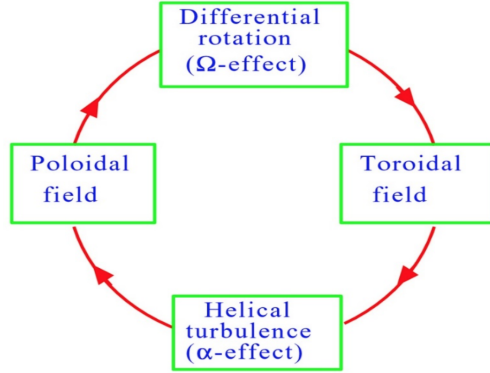


Figure 1.2: Self sustaining turbulent dynamo action. Credits: Parker, E.N.

days period) and slower near the poles (33 days). The innermost 70% of the Sun (the radiative zone and core) rotates as a solid body. Thus there exists a thin strong shearing region between the radiative zone and convection zone known as the tachocline. This is believed to be the region of toroidal field formation (mean field Ω effect) and storage.

The formation of a poloidal field from a pre-existing toroidal field is more complicated and not very well understood. An issue arises of how to avoid the fact that an axisymmetric flow cannot sustain an axisymmetric magnetic field via dynamo action (Cowling's anti-dynamo theorem). Mean-field dynamo theory circumvents this by introducing a scale separation between the large-scale axisymmetric flows and fields and their small-scale turbulent counterparts. If it is the small-scale components that break symmetry then the large scale components can be left intact. The small-scale velocity can easily have non-symmetric (chiral or handed) statistics, enforced by the presence of rotation thereby explain the toroidal to poloidal conversion (see [23]). However, the length scales in which this happens are unclear. Parker [95] suggested that small-scale helical motions (arising from the presence of convection and rotation) acting on the rising toroidal field could

form the poloidal magnetic field component. On the other hand, an alternative line of thinking, the Babcock-Leighton mechanism [7] (and “flux transport” dynamos in general), proposes that the decay of bipolar active regions (larger scale) on the solar surface can produce the poloidal component. Even though the workings and location of both the mechanisms are strikingly different, they are loosely referred to as α effect in solar dynamo theory.

With the basic mechanisms governing the solar dynamo theory discussed above, it is important to note that the spatial location of the formation of toroidal fields and their surface emergence are widely separated. Clearly, a mechanism is required to transport magnetic flux from the solar interior towards the surface. Parker (1975) [94] proposed the concept of magnetic buoyancy, which states that a magnetic concentration embedded in a stratified background will be endowed with a density deficit owing to total pressure and thermal equilibrium (see Equation 1.1). The total pressure balance between a magnetic flux concentration and the background stratification can be expressed as

$$p_{gas,int} + \frac{B^2}{2\mu_o} = p_{gas,ext} \quad (1.1)$$

where $p_{gas,int}$ is the gas pressure inside the magnetic concentration, $p_{gas,ext}$ is the gas pressure of the background stratification, and $B^2/(2\mu_o)$ is the magnetic pressure of the magnetic concentration. Under the assumption of thermal equilibrium and using the ideal gas law, we can quickly realize that the density of the interior is decreased ($\rho_{gas,int} < \rho_{gas,ext}$) and a buoyancy force is experienced by the tube.

1.3 Modeling of Magnetic Flux Tubes

The bipolar nature of sunspots observed on the solar surface suggests a tubular emergence of magnetic fields from the solar interior. Motivated by these observations, the theoretical studies on magnetic flux emergence usually deal with idealized cylindrical magnetic flux concentrations in the solar interior, often simply known as “magnetic flux tubes”. In this section, we now focus on the wide range of modeling efforts of magnetic flux tubes that have been studied in the past. The modeling of rising magnetic flux based solely on magnetic buoyancy (therefore generally ignoring convection and dynamo processes) can be broadly divided into two classes: (a) studies of the rise of preconceived magnetic flux tubes; and (b) studies of magnetic buoyancy instabilities.

1.3.1 Rise of Pre-conceived Flux Tubes

The first class of studies assumes the existence of a magnetic flux-tube-like structure without any dynamical inclusion of its originating process or field. It is important to note that, in this case, the “flux tube” is a purely arbitrary geometrical construction that is placed in a field-free environment. The environment may mimic certain solar-like conditions, such as the stratification, but the magnetic structure is totally isolated and disconnected from any larger-scale field (and the environment is often initially quiescent). Initial studies in this class (e.g. [86, 87, 28, 36, 43, 44, 20, 76]) focused on the “thin flux tube approximation” [104] where a flux tube is purely a line with no cross-sectional area but has ascribed buoyancy, tension, and drag forces. Using appropriately-chosen parameters, these thin flux tube models can exhibit many of the characteristics of the solar observations, such as the correct latitudes of emergence and Joy’s law, for example. Such agreements with solar observations have then been used to infer unobservable so-

lar characteristics, such as the strength of the magnetic field at their region of formation [31].

Even though extremely useful in building initial intuition, these studies do not capture more complete dynamics, as was quickly discovered when finite cross-sectional flux tubes were modeled. For example, it was soon found via two-dimensional numerical simulations that, in order to maintain a coherent rise, finite-size flux tubes need to have a significant amount of magnetic field line twist in order to avoid being ripped apart by the trailing vortices generated in their wake [88] (see Figure 1.3a). This revealed the essential role of the twist, in this case, providing the necessary centrally-directed tension force required for a flux tube to remain cohesive. The rise of finite-size flux tubes in these models is still driven by magnetic buoyancy, either from a pre-assigned non-equilibrium initial condition or via an instability (e.g. [100, 101, 88, 74, 46, 39]), although, as mentioned above, other dynamics, such as the wake dynamics, quickly play a significant role. In three-dimensional numerical models of finite cross-sectional tubes, secondary instabilities were revealed that lead to kinking or arching structure [71, 47, 72, 48, 40, 41], very much reminiscent of the Ω -loops envisaged as necessary to match the solar surface emergence observations.

1.3.2 Magnetic Flux Sheet Instabilities

The second general class of modeling studies has focused on the formation of magnetic flux structures from the instability of large-scale flux sheets. The parcel argument for the concept of magnetic buoyancy briefly mentioned above can be rigorously formulated into an instability problem that generally reveals that instability can occur when horizontal magnetic field increases sufficiently rapidly with depth (compared to the background entropy gradient)[95, 3]. Two- and three-

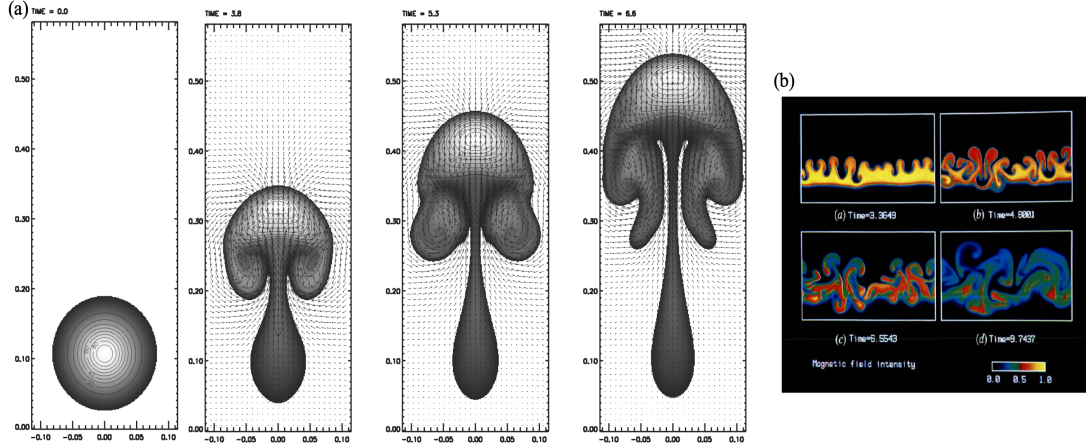


Figure 1.3: (a) Rise of twisted flux tube through a stratification. The gray scale and contours correspond to the longitudinal magnetic field intensity. Figure adapted from [39], (b) Evolution of magnetic field from a flux sheet instability. Figure adapted from [21].

dimensional simulations of horizontal magnetic flux sheets (e.g. [21, 82, 61, 117, 112, 53]) have exhibited that magnetic buoyancy instabilities evolve to form strong flux concentrations that are akin to the conceptual geometry of the isolated flux tubes described for the type (a) studies above (see Figure 1.4). The concentrations are pseudo-cylindrical, in the sense that they have a “mushroom-like” cross-section perpendicular to the field lines that is substantially narrower than any variation along the field lines (see Figure 1.3b). The long-wavelength variation down the initially-horizontal fieldlines corresponding to the most unstable modes naturally leads to Ω -like rising structures, as required by observations. Only a few studies have included processes related to the origin and formation of the magnetic flux sheets that then subsequently give rise to these instabilities (e.g. [18, 33, 32, 34, 22, 65, 112]). For example, in [112], a forced, vertically-sheared, horizontal flow first creates a localized horizontal (“toroidal”) magnetic sheet from seed initial vertical (“poloidal”) field (see Figure 1.5). This sheet subsequently goes unstable to magnetic buoyancy instabilities [113] that again show the formation of strong

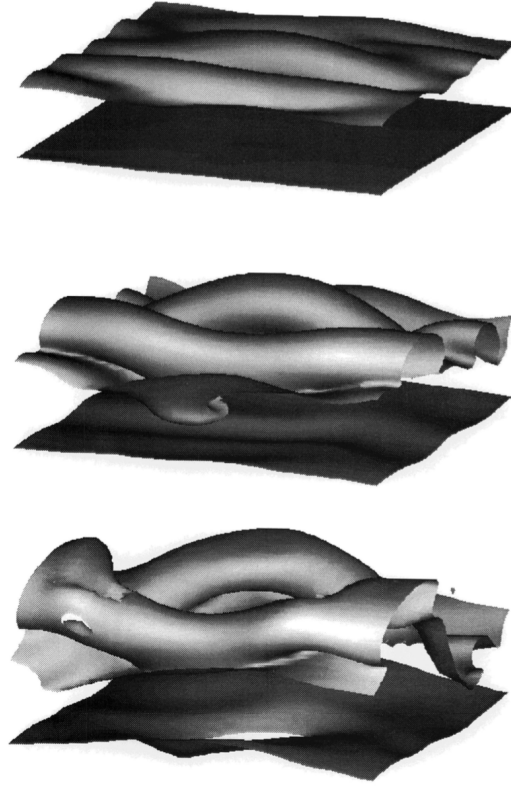


Figure 1.4: Isosurfaces of $|B|$ at three different times. Adapted from [82].

magnetic flux-tube-like structures.

It is crucial to realize that, in the second class of magnetic buoyancy investigations (and especially the ones that include the layer formation process), the flux structures formed are not isolated magnetic entities, as is assumed in the first class. Instead, any magnetic structures formed are embedded in a large-scale background field. Therefore such structures may be better referred to as magnetic flux concentrations. The rise of such concentrations may be significantly different from the rise of isolated flux tubes in a field-free environment, as discussed in some detail in [33]. For example, whether the enhanced connectivity of an embedded concentration in a large-scale background field helps or hinders rise is not understood. Connectivity to the background field during the rise may create

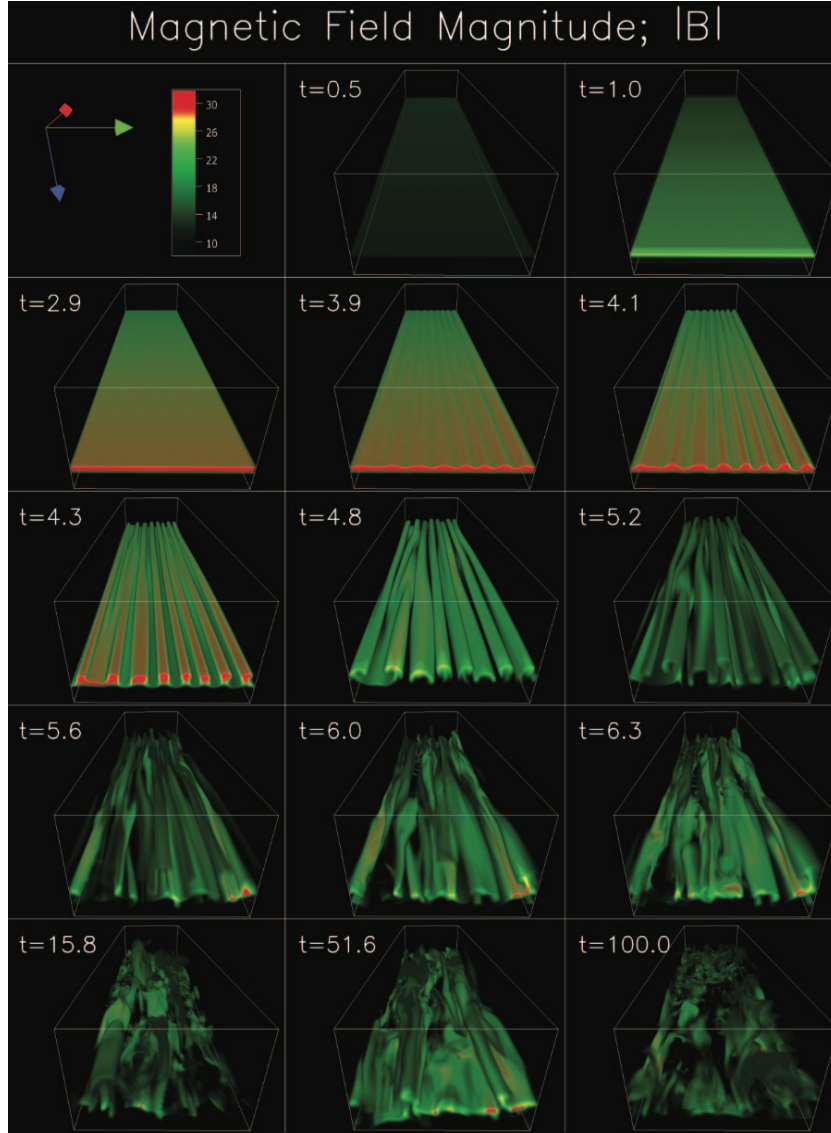


Figure 1.5: Volume renderings of magnetic field magnitude, $|B|$, at different time, showing the formation and evolution of magnetic layer to form flux tube-like structures. Adapted from [112].

tension that opposes the rise. On the other hand, connectivity could potentially also alleviate some of the issues with the rise of isolated structures, such as their need to conserve flux as the structure rises through a strong density stratification leading to “ballooning” of the structure. Interaction between the background field and the rising concentration could also certainly rearrange observable quantities of interest, such as the helicity of the structure. We discuss the solar observations of magnetic and current helicity in detail later in this section.

1.3.3 Alternative studies

The previous two classes of studies focused solely on the deep origin of magnetic flux tubes. With no direct observations available, the issue of where the flux tubes are formed is debatable. Various alternative theories have been proposed that attempt to explain this.

One of the significant ones arises from full global spherical convective dynamo models. In these models, strong bands of toroidal magnetic fields that potentially contain buoyant loop-like elements can be seen [17, 91, 90, 89]. However, since the scale of these structures is very large, it is hard to relate these directly to “flux tubes”. Such structures could be re-organized by near-surface processes into smaller-scale structures or could be the origin of the field for the other two types of investigations just mentioned.

With the presence of strong shear, tachocline has been an obvious candidate for the formation of flux tube-like structures. Based on the differential rotation profile of the Sun, an alternative candidate for flux tube formation is the near-surface shear layer [59] (NSSL) where there is an outward decrease in the angular velocity. NSSL is a region extending below the solar photosphere to about $35Mm$. Consequently, NSSL is relatively easy to “observe” using helioseismology. The

sharp drop in angular velocity with increasing radius can possibly create an ideal situation for the stretching of large-scale magnetic fields below the photosphere to form strong toroidal bands of magnetic fields which can rise due to buoyancy. However, this conversion of the poloidal magnetic field to the toroidal field in the NSSL comes with a caveat. The time taken by magnetic flux to emerge from the NSSL to the solar photosphere due to buoyancy is much shorter than the timescale in which the near-surface shear can act. This could lead to significant flux loss, preventing the formation of strong toroidal magnetic structures that are typically observed in flux emergence.

While both these alternative models are plausible, work conducted in this thesis is based on the premise that the flux tubes, once formed, are naturally embedded in a large-scale magnetic field. The subsequent rise of flux tube then has to transit through this large-scale field. This seems to be a consistent theme through all the different classes of flux tube modeling studies discussed above. With that said, the work in this thesis also assumes that the likely location of flux tube formation is in the tachocline. In that sense, our work aligns more closely with the ideas discussed above in Sections 1.3.1 and 1.3.2.

1.4 Solar Hemispheric Helicity Rules

Significant attention has been paid recently (see [96]) to observations of the magnetic helicity, $H_m = \int_V \mathbf{A} \cdot (\nabla \times \mathbf{A}) dV$ (where $\mathbf{A}$ is the magnetic potential, $\nabla \times \mathbf{A} = \mathbf{B}$), and the current helicity, $H_c = \int_V \mathbf{B} \cdot (\nabla \times \mathbf{B}) dV$ at the solar surface. These quantities measure the total twist, kinking and linking in a volume, and give information about the topology of the field that might be important to subsequent processes [13, 99]. Magnetic helicity is an important dynamical quantity for a number of reasons. For example, in ideal MHD, H_M is conserved and is, therefore,

a significant constraint for dynamo theory [11, 12, 14]. Furthermore, the release of energy stored in helical fields by reconnection (requiring some resistivity) in the solar atmosphere is thought to be a driver of energetic events, such as flares, jets, and coronal mass ejections (CMEs) (see e.g. [79, 4, 92]). Observationally, calculation of the magnetic helicity is complicated since the vector potential $\mathbf{A}$ is not directly observable and is even not uniquely defined in terms of the directly-observable $\mathbf{B}$ field. However, the current helicity, $H_C = \int_V \mathbf{B} \cdot (\nabla \times \mathbf{B}) dV$, is unique and its key components can be constructed from observations, and, therefore, H_C has been extensively used in place of, or as a proxy for, H_M .

Detailed observations of the current helicity have once again exhibited a remarkable degree of temporal and structural coherency in the large-scale field [102, 98, 2, 10]. These observations together have established the “Solar Hemispheric Helicity Rule(s) (SHHR)”. The SHHR primarily states that active regions in the northern hemisphere possess predominantly negative helicity, whereas active regions in the southern hemisphere have predominantly positive helicity (see Figure 1.6). This bias is cycle-independent but is not an absolute rule since it is obeyed by only 60 – 80% active regions. Although the detailed temporal variation of the adherence to the rule has not been established yet, various observational studies [8, 9, 54, 56] at least seem to agree that the SHHR is violated more strongly at the transition between the cycles. Modeling efforts that might contribute to the explanation of the origin of the SHHR are the subject of this thesis.

Theories for the origin of the SHHR abound, but few are very complete dynamically or fit the observations in a highly compelling manner. One might expect solar magnetic fields, in general, to have a handedness or chirality since much of our understanding of solar dynamo theory is based around the necessity of the existence of turbulent flows that are chiral in order to produce any mechanism

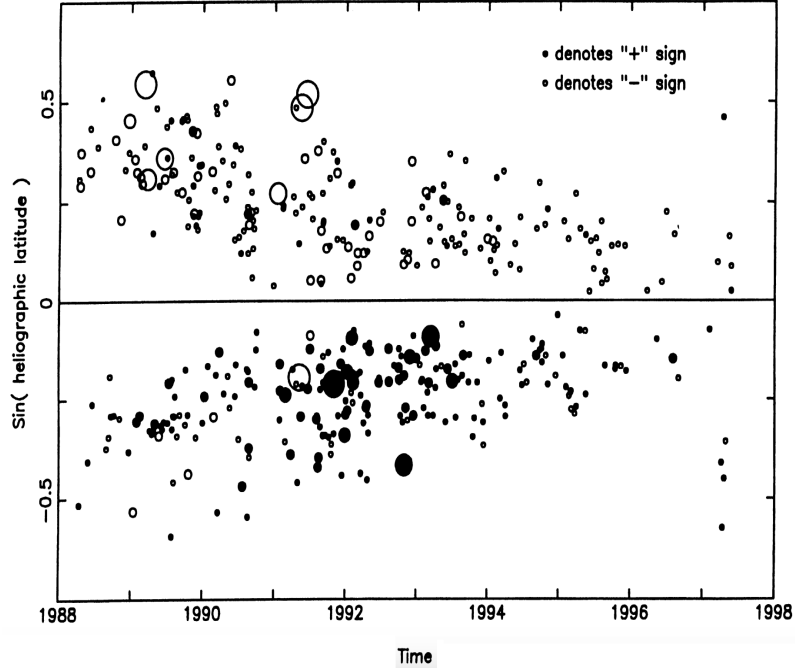


Figure 1.6: Mean density of the current helicity parameter $B_{\parallel} \cdot (\nabla \times B_{\parallel})$ of active regions plotted as a function of latitude and time. Open circles denote negative helicity and filled circles denote positive helicity active regions. Figure adapted from: [119].

that resembles a mean-field α effect [105]. Of course, the Coriolis force is readily available to break symmetry to achieve this. Many theories for the SHHR therefore rely on the existence of the Coriolis force either directly or indirectly. Some authors have argued that the helicity in the SHHR is a direct result of the correlation of the fields in a mean field dynamo. For example, [51] explore the relationship between different mean-field dynamo models and the subsequently observed current helicity. Unfortunately, the results are very dependent on the exact formulation of the α effect and can produce mean fields that have the same correlations as the SHHR or the opposite, depending on model choices. Other models loosen the direct correlation with the α effect [52] somewhat by invoking scale-dependence, whereby the small-scales have the opposite sign of α to the large scales. Overall, however, such mean-field models can only provide the overall

helicity of global axisymmetric fields and struggle to say something meaningful about anything that looks like an active region.

Most other investigations therefore fall into the first category of magnetic buoyancy modeling (described as type (a) above), and have centered around how isolated, preconceived, rising flux tubes of some sort can ‘acquire’ helicity during their transit to the surface¹. Rising isolated flux tubes writhe in response to the Coriolis force to create a tilt of the Ω loop in accordance with Joy’s Law [115, 36, 44]. Since the tilt corresponds to the writhe of a flux tube, in order to conserve helicity, the tube acquires the opposite-signed twist. Since the tilt was right-handed (left-handed) in the Northern (Southern) hemisphere, the twist is left-handed (right-handed) in accordance with the SHHR. Unfortunately, Joy’s Law tilts do not provide sufficiently strong enough twists to explain observed surface helicity values [78, 45]. Furthermore, the latitudinal dependence expected in this case is not seen in observations [97, 57]. Wang (2013) [114] attempted to circumvent this constraint by allowing the twist to be generated solely by the action of the Coriolis force on the expansion of legs of the Ω loop as it rises, concluding that sufficient helicity could be generated as long as the rise were slow enough.

Choudhuri [29] postulated a different, highly conceptual model whereby the twist is acquired when an untwisted tube rises into a near-surface large-scale background poloidal field. In their “mean-field circulation-dominated solar dynamo (CDSD)” model, this poloidal field originates from the decay of tilted active regions in a Babcock-Leighton type mechanism [7, 70]. The acquired twist is thus ultimately a result of the interaction of the dynamo field with rotation again. Inspired by the [29] model, [24] and [58] have studied (via models and simulations

¹Note that most authors cited now refer to mechanisms for generating twist not helicity, leaving the direction of the toroidal field connecting the two to be understood from the underlying dynamo fields. We refer to authors’ results here in their language.

respectively) the accretion of twist onto an initially untwisted flux tube in the presence of specific formulations of background field.

A general drawback of the above models is that they predict a fairly strictly-enforced SHHR and do not easily explain the observed weak (60-80%) adherence to the rule and the significant inherent scatter therein (although [30] does show some level of violation during the transition between cycles in the Choudhuri model, thanks to the correlation of the dynamo fields in the CDSD model). One theory that does account for this, and is therefore perhaps the leading theory of the SHHR to date, is that of Longcope, Fisher and Pevtsov [75], hereafter referred to as LFP. LFP invoke the action of rotationally-influenced convective turbulence on the transit of thin flux tubes through the convection zone. In some sense, their “ Σ ” effect is a small-scale version of the previous ideas, whereby the buffeting by Coriolis-influenced helical turbulence imparts net writhe to a thin flux tube that must be compensated by a net twist of opposite sign. In this manner, the sign is again in accordance with the SHHR, and the turbulent nature leads to significant fluctuations and scatter, as required. However, this model has no obvious basis for temporal variations of the adherence to the rule over the solar cycle which becomes even more pronounced towards its end as found by various observations [83].

Having discussed all the salient features of the SHHR observations and raised some caveats regarding the prevailing theories, the work in this thesis will attempt to address many of these issues in the coming chapters. At this point, we now focus on the the major conceptual difference regarding flux tubes that forms the basis of our numerical studies in this thesis.

1.5 Modeling flux tubes as concentrations

In the discussion above, we have distinguished between magnetic structures known as flux tubes and concentrations. While the flux tubes have been generally been studied as isolated flux structures, concentrations are tube-like entities embedded in a large-scale volume-filling field. This idea forms the very essence of the work done in this thesis. Specifically, we study the dynamics of these magnetic concentrations in setups intended to model the deep interior of the Sun. The initial setup lies somewhere in between the models discussed in Section 1.3.1 and 1.3.2. In particular, we ignore the origins of these magnetic flux concentrations, assuming they have already been formed by dynamo and instability processes, and study their evolution. The first set of studies (see Chapter 2) that we carry out are in a quiescent background state, i.e., in the absence of convection, as has been typical of the vast majority of magnetic buoyancy studies. Our initial setup consists of an adiabatically stratified background state (representing well-mixed convection) with a magnetic flux tube embedded in an artificially imposed large-scale magnetic field. The aim is to examine purely the influence of the background field on the evolution of the flux tubes.

The second set of studies (see Chapter 3) that we carry out simulates the deep solar interior more realistically by using a more complex two-layer system where a convectively unstable layer sits atop a stable region. This stable region represents the upper solar radiative zone (including the tachocline, although we do not include the tachocline shear), and the unstable region is representative of the deep convection zone. An essential distinction in this second part is that the large-scale volume-filling background magnetic field is not artificially imposed but forms self-consistently. The details of the self-consistent formation of the background field layer are discussed in later chapters. In effect, this setup simulates the rise

of a magnetic flux concentration from the tachocline (or radiative zone) through the convection zone without considering its formation by the action of shear. We discuss the specific details of the setup in-depth in the relevant chapters.

Chapter 2

Magnetic Flux Concentrations in a Background Field:

Non-convective simulations

The following chapter contains a combination of work and text from two papers, Manek, Brummell & Lee (2018) [81] and Manek & Brummell (2021) [80], both published in The Astrophysical Journal.

2.1 Introduction

This chapter discusses two-dimensional numerical simulations of the rise of magnetic flux tubes through an artificially imposed large-scale background magnetic field in an adiabatic stratification. The simulation setup consists of a single, convectively-neutral layer. We organize the chapter as follows: Section 2.2 describes the model setup. Section 2.3.1 presents the results from the numerical simulations for a top-hat initial magnetic field condition of the flux tube. The reasons behind the observed dynamics in these sections are briefly discussed in

Section 2.3.2 with the help of numerical data. Section 2.3.3 presents the simulation results in a broader context and connects the numerical model and realistic solar scenario. A link between the results and observations of SHHR is established in this section.

Subsequently, a predictive mathematical model based on the forces acting on the flux tube is developed in Sections 2.3.4 and 2.3.5. Section 2.3.6 checks the robustness of the results with changing background magnetic field configuration. Section 2.3.7 presents Monte Carlo (MC) simulations of multiple flux tubes, with random twists and random horizontal locations, rising through a large-scale background field, in order to make direct contact with the observations of the SHHR. We explicitly discuss instances from MC simulations that show non-trivial dynamics, accounting for violations of the rule. Finally, in Section 2.4, we summarize our results and discuss the shortcomings of the numerical simulations as well as the mathematical model developed in this chapter.

2.2 Model setup

Our model essentially evolves a cylindrical flux structure (comprised of both axial and a locally-azimuthal magnetic field component so that the field is helical) embedded in a large-scale background magnetic field oriented horizontally and perpendicularly to the tube axis. When comparing to the motivating solar application, the cylindrical flux structure should be thought of as toroidal (and therefore represents the typical idea of a twisted magnetic flux tube, soon after formation, in the deep interior), and the large-scale background field should be thought of as poloidal, representing the deep interior poloidal component of the dynamo field (see Fig. 2.1). The flux structure setup is very similar to many previous studies (e.g. [88, 60], hereafter referred to as HFJ) but our model has

the crucial addition of the large-scale background field, making the flux structure a concentration rather than an isolated tube. We choose an adiabatically stratified fluid layer covering 1.2 density scale heights so that it mimics roughly the region containing the upper tachocline and the lower 40% of the convection zone. Even though no convection is present in our current simulation setup, we have chosen the initial background field profile to mimic what might be expected if indeed convection were to be present. That is, we concentrate the horizontal field near the bottom of the domain as if it had undergone magnetic pumping into the overshoot region at the top of the tachocline by the turbulent convection (e.g. [110]). Ultimately, however, our results turn out to be relatively insensitive to this choice of configuration. The more complicated problem with convection present will be addressed in Chapter 3. We ignore the origins of these initial fields, assuming that they have already been formed by dynamo and instability processes, and study their evolution. Note that these are not instability calculations, as the initial conditions are not in any equilibrium by choice; we imagine the start of our simulations to represent a portion of the later stages of a nonlinear instability simulation (e.g. [112]). Our aim is very simply to understand the effect of an volume-filling source background field on the rise of a pre-formed concentration under conditions akin to the deeper solar interior. Other models (e.g. [26]) examine the emergence of field at the photosphere given initial conditions from the interior and it is one of our goals to build towards an understanding of the origin of the initial conditions needed for those simulations.

In order to study the above-described model, we solve the equations of compressible magnetohydrodynamics (MHD) using the publicly-available FLASH code [50, 69, 68, 37] in a Cartesian domain. The equations solved, in non-dimensional conservation form, are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (2.1a)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B}) + \nabla p_* = \rho \mathbf{g} + \nabla \cdot \tau, \quad (2.1b)$$

$$\frac{\partial \rho E}{\partial t} + \nabla \cdot (\mathbf{v}(\rho E + p_*) - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})) = \rho \mathbf{g} \cdot \mathbf{v} + \nabla \cdot (\mathbf{v} \cdot \tau + \sigma \nabla T) + \nabla \cdot (\mathbf{B} \times (\eta \nabla \times \mathbf{B})), \quad (2.1c)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) = -\nabla \times (\eta \nabla \times \mathbf{B}), \quad (2.1d)$$

where

$$p_* = p + \frac{\mathbf{B}^2}{2}, \quad (2.1e)$$

$$E = \frac{1}{2} \mathbf{v}^2 + \epsilon + \frac{1}{2} \frac{\mathbf{B}^2}{\rho}, \quad (2.1f)$$

$$\tau = \mu \left((\nabla \mathbf{v}) + (\nabla \mathbf{v})^T - \frac{2}{3} (\nabla \cdot \mathbf{v}) \mathbf{I} \right). \quad (2.1g)$$

Here, ρ is the density of a magnetized fluid, $\mathbf{v} = (v_x, v_y, v_z)$ is the fluid velocity, $\mathbf{B} = (B_x, B_y, B_z)$ is the magnetic field, p is the fluid thermal pressure, p_* is the total pressure (fluid and magnetic), g is the body force per unit mass, τ is the viscous stress tensor, E is the specific total energy, T is the temperature, σ is the heat conductivity, η is the magnetic resistivity, ϵ is the specific internal energy, μ is the coefficient of viscosity (dynamic viscosity), and $\mathbf{I}$ is the unit (identity) tensor. Here, μ , σ and η are chosen such that magnetic Prandtl number, $Pm = \frac{\sigma}{\eta} = 1$

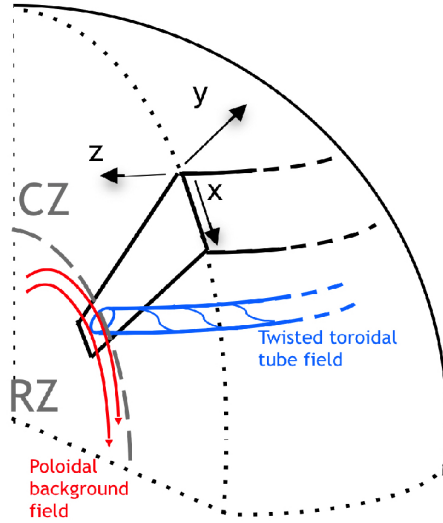


Figure 2.1: Cartoon sketch of the relation of the model to the Sun (adapted from Section 2.3). Our 2.5 dimensional model relates the Cartesian directions x to latitude, y to height, and z to longitude. Vectors in the z direction exist but the model is independent of this direction. The domain spans the lower convection zone and the transition to the radiative zone. The model horizontal background field is related to deep, large-scale poloidal field. The magnetic concentration is identified with twisted toroidal field structures, and is initially placed deep in the stratification.

and Prandtl number, $Pr = \frac{\sigma}{\mu} = 0.1$.

These MHD equations are solved in a local Cartesian 2D simulation box of dimensions $x \in [-1, 1]$ and $y \in [0, 4]$, whose geometry relative to a spherical star is shown in Fig. 2.1. The simulations are 2.5-dimensional, in the sense that fields in the third (z) direction are included but that the dynamics are independent of this direction. The non-dimensionalization can be thought of as specified by the relative size of the tube compared to the variation of the background hydrostatic state, which is a weak adiabatic polytropic stratification

$$T = 1 + \theta y'; \quad \rho = (1 + \theta y')^m; \quad p = (1 + \theta y')^{m+1}, \quad (2.2)$$

where $y' = 4 - y$. Throughout this chapter, we set non-dimensional temperature

gradient $\theta = 0.25$, and the polytropic index $m = 1.5$ (the adiabatic value for a perfect gas with $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$, where C_p and C_v are the specific heats at constant pressure and volume respectively). The polytropic hydrostatic model dictates that $\mathbf{g} = -g\hat{\mathbf{y}} = -\theta(m+1)\hat{\mathbf{y}}$. Note that our model is essentially at a fixed plasma β . The variation of the the magnetic field configuration does alter β somewhat, but for the parameters that we have surveyed, the variation is not substantial.

The magnetic initial conditions consist of a cylindrical concentration (which we still often colloquially refer to as “the tube”) of radius $r \leq R$ centered at (x_c, y_c) (where r is the local radius of the tube relative to its center: $r = \sqrt{(x - x_c)^2 + (y - y_c)^2}$), embedded within a background horizontal (but vertically-varying) field. Throughout this study, we choose to use $R = 0.125$ and take the centre of the tube to be at $(x_c, y_c) = (0, 0.5)$. The axial (z direction) field inside the concentration is constant and defines the scale for the amplitude of the magnetic field, so that $B_z = 1$ there (although note that a Gaussian cross-section of axial field is used instead of this top hat profile to test robustness in Section 5.2). To define the azimuthal field in the concentration, we use a potential function A_z to ensure the solenoidal condition is satisfied:

$$A_z = -q(x^2 + (y - y_c)^2) + K, \quad (2.3)$$

where we choose K such that the continuity of $A_z = 0$ is ensured at the edge of the tube $r = R$. The parameter q defines the twist of the tube fieldlines and is a key parameter of the simulations. The magnetic field defining the concentration is therefore given by

$$\mathbf{B}_{\text{tube}} = (-2q(y - y_c), 2qx, 1), \quad r \leq R. \quad (2.4)$$

The large-scale overlying background field in which the flux tube is embedded is aligned horizontally (in x) and we choose an exponential variation in y to represent poloidal field perhaps more confined to the base of the convection or upper tachocline:

$$\mathbf{B}_{\text{back}} = (B_{\text{back}}, 0, 0) = \left(B_s \exp\left(\frac{y_c - y}{2H_B}\right), 0, 0 \right). \quad (2.5)$$

Here, B_s is therefore the strength of the background field relative to the initial magnetic field strength at the center of the flux tube, and H_b is the scale-height of the exponentially-decreasing field. We vary both these parameters to explore the effect of the configuration of overlying field, from weak to strong, and from significantly confined to almost constant in the vertical. The total initial magnetic field in the concentration is therefore given by

$$\mathbf{B}_{\text{in}} = \mathbf{B}_{\text{tube}} + \mathbf{B}_{\text{back}} = \left(-2q(y - y_c) + B_s \exp\left(\frac{y_c - y}{2H_B}\right), 2qx, 1 \right), \quad r \leq R, \quad (2.6)$$

and outside the tube is simply $\mathbf{B}_{\text{out}} = \mathbf{B}_{\text{back}}$. This setup with no background field ($B_s = 0$) is similar to the case studied in HFJ with the parameter $\alpha = 0$ (with the only differences being that we do not assume symmetry of the rising flux tube about the mid-plane, and we choose a slightly larger tube cross-section).

On insertion of a magnetic field into a stratification, it is generally assumed that the total pressure equilibrates quickly making the total pressure continuous with the external conditions. The thermodynamics of the tube can then be specified such that there is a density anomaly with the tube less dense than the surroundings. However, there is no unique way of specifying these initial conditions, as the effect of extra magnetic pressure can be accounted for by the density,

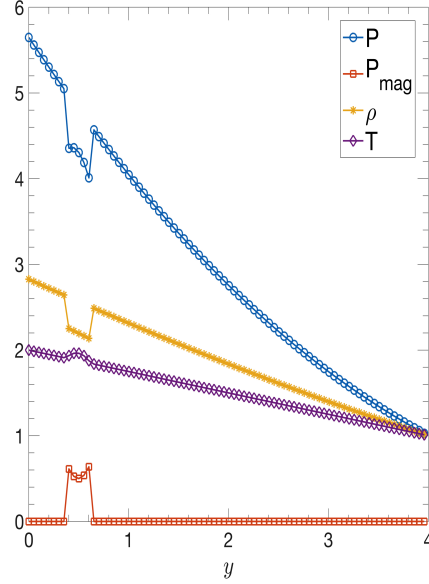


Figure 2.2: Vertical profiles of the initial thermodynamic conditions through the center of the magnetic flux concentration, along $x = 0$.

the temperature, or both combined. One formulation of the initial conditions that has been used by others (e.g. HFJ, [88]), and was used in Section 2.3, is to have the temperature continuous at the edge but varying inside the tube such that density and total pressure are merely a function of height (Fig. 2.2). We adopt this again here, but also investigate the effects of varying the form of these initial conditions briefly as part of the robustness tests. The details of this can be found in Appendix 5.1. The major conclusion from this robustness test is that they have little bearing on the major results discussed in this chapter.

We choose the top and bottom boundaries of the simulation box to be fixed temperature, stress-free and impermeable to flow but not to magnetic fields, given respectively by

$$T(y = 0) = 1 + \theta * 4 = 2; \quad T(y = 4) = 1 \quad (2.7)$$

$$v_y = \frac{\partial v_x}{\partial y} = 0 \quad (2.8)$$

$$\frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial y} = 0 \quad (2.9)$$

At the right and left boundaries of the simulation box, we implement Neumann boundary conditions

$$\frac{\partial T}{\partial x} = \frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial x} = \frac{\partial B_x}{\partial x} = \frac{\partial B_z}{\partial x} = 0 \quad (2.10)$$

that allow outflow of the fields. Whilst outflow magnetic boundary conditions are somewhat controversial [49], these conditions are commensurate with earlier studies of HFJ. In the current setup, the diameter of the tube is $1/8^{th}$ of the horizontal domain, and can rise vertically through 16 times its size thereby allowing significant room for unconstrained dynamics of a single tube. Hence, in this work, we only present dynamics far from the boundaries, and we have further checked that the results are robust to alternative choices of boundary conditions and other reasonable domain sizes.

2.3 Flux tube dynamics

Our first set of studies focuses on two-dimensional (x, y) simulations of the flux tube in the presence of a background field. Technically, our simulations can be classified as 2.5D as we have all the variables expressed in the y -direction too. However, the spatial derivatives in y -direction of all physical quantities is set to zero in this formalism.

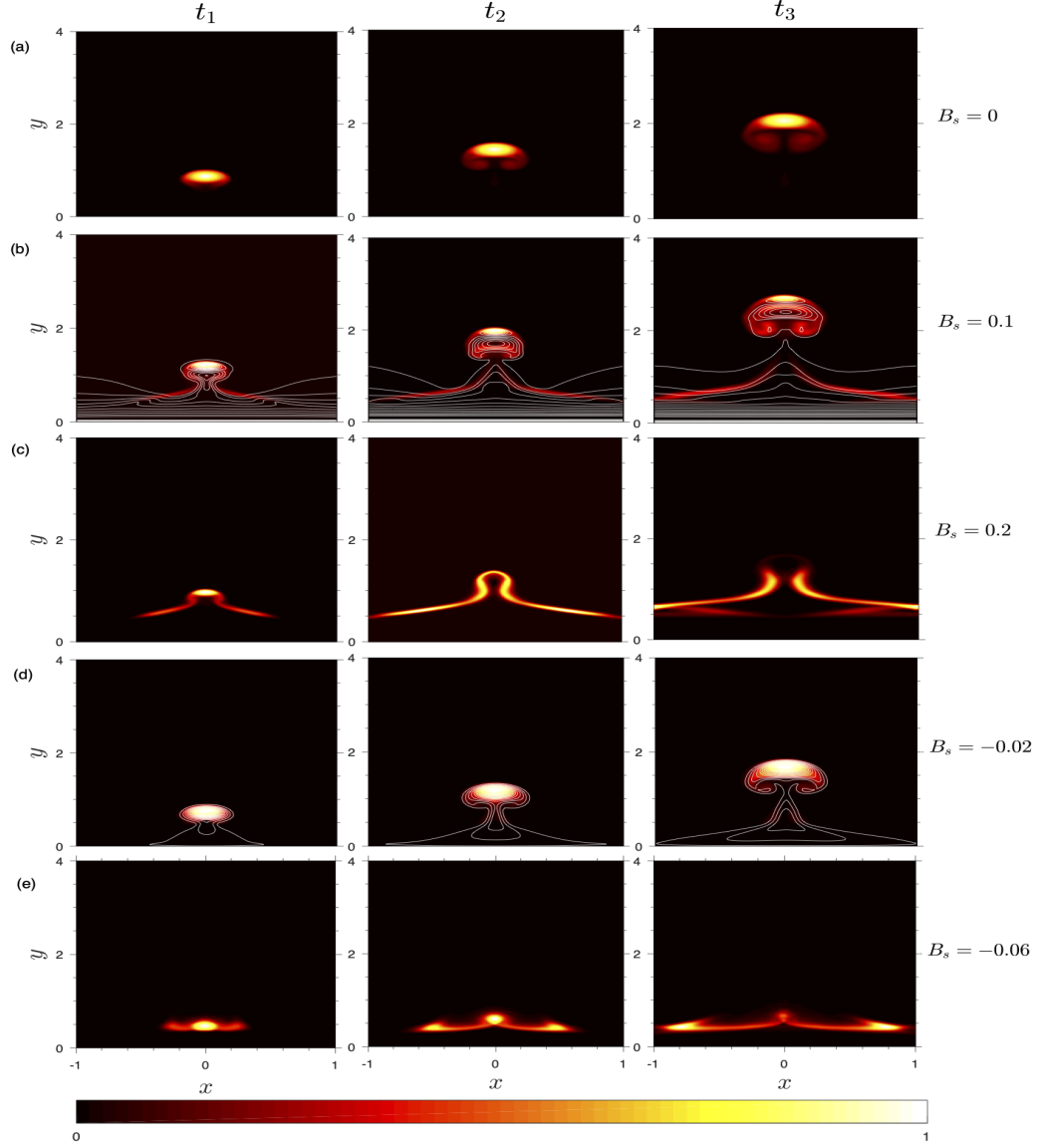


Figure 2.3: Intensity plots of normalised $B_z(x, y, t)$ for $B_s = 0, 0.1, 0.2, -0.02, -0.06$ (a,b,c,d,e respectively) and times $(t_1, t_2, t_3) = (5, 10, 15)$ except for (c) where $(t_1, t_2, t_3) = (2.5, 5, 8.75)$. Contours of A_z have been added to some panels. Panels (a,b,d) represent successful rises, whereas in panels (c,e) the rise is suppressed.

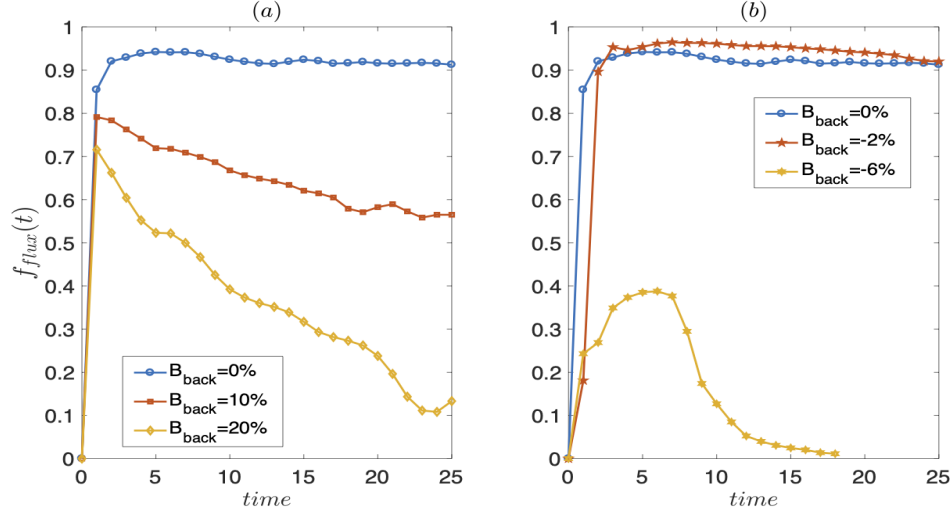


Figure 2.4: Rising flux fraction $f_{flux}(t) = \iint B_{z,t}^* dx dy / \iint B_{z,0} dx dy$ where $B_{z,0} = B_z(x, y, t = 0)$ and $B_{z,t}^* = B_z(\{(x, y) \text{ where } v_y(x, y) > v_{y,threshold}\}, t)$ for the same cases as in Fig. 2.3. Here, $v_{y,threshold} = 0.03$ is a judiciously-chosen threshold vertical velocity that filters out unimportant motions. (a) $B_s \geq 0$. (b) $B_s \leq 0$.

2.3.1 Suppression of magnetic concentration rise by a background field

Figures 2.3 and 2.4 exhibit the main results of this section. Figure 2.3 shows intensity plots of the evolution of the axial field, B_z , as a tracer of the magnetic structure, for five different values of B_s . Figure 2.4 exhibits $f_{flux}(t)$, a time trace of the fraction of the initial axial flux that is rising faster than a chosen threshold velocity. If a simulation exhibits the coherent rise of a magnetic structure, we would expect this measure to be close to unity and relatively constant.

Figure 2.3(a) shows the results for no background field, $B_s = 0$. This canonical case exhibits the steady rise of a coherent magnetic structure. The rise of the tube is initially driven by magnetic buoyancy arising from magnetic perturbations to the density of the initial conditions. After some time, a pair of counter-rotating trailing vortices develop behind the initial tube, as seen in the tracer B_z . The later

rise then becomes dominated by a self-advection of the whole structure driven by the vortices, with only diffusion acting to spread the structure very slowly compared to the rise time. This result has been well-established by HFJ and others. The coherent rise, in this case, is more quantitatively seen as the blue line in Figure 2.4(a). After an initial acceleration, most of the initial B_z flux ($\sim 95\%$) rises at a speed above the threshold velocity, indicating that the whole magnetic structure is rising, with only a slight loss of flux (likely due to diffusion).

Figures 2.3(b) and (c) shows cases where we have included positively-oriented background field with strengths $B_s = 0.1, 0.2$ (i.e., 10% and 20% of the axial field). Figure 2.3(b) exhibits the rise of a still distinct but different tube-like head magnetic structure, but with some axial flux loss to the trailing environment (along A_z contour lines). The head appears to eventually leave the trailing field behind, continuing on to rise seemingly independently as in the no background field case, albeit with a different geometry. The red line corresponding to this case in Figure 2.4(a) corroborates these impressions. The initial magnetically-buoyant acceleration moves a significant (but smaller, ~ 0.8) fraction of the flux, but then f_{flux} drops steadily, indicating a regular drainage of axial flux from the rising structure to the non-rising background. The rate of loss lessens at $t \sim 20$ corresponding to the “separation” of the structure from the trailing field (although this behavior is seen more clearly in more diffusive simulations, not shown). These dynamics may be deemed a successful rise, but just barely.

Figure 2.3(c), where the background field strength is 20% of the axial tube field, shows a total suppression of the coherent rise of any structure. Axial flux in the initial attempted rise is very quickly drained into the trailing environment. This is confirmed by the yellow line in Figure 2.4(a) which shows a rapid flux fraction decrease after initiation to low levels (~ 0.1) likely associated solely with

waves in the trailing field. With other simulations, we find that rise is suppressed for a B_s somewhere between 16% and 20%. An immediate conclusion is that a relatively weak ($\gtrsim 20\%$) background field will suppress the rise of a twisted tube that would otherwise rise coherently in a non-magnetic background.

Somewhat surprisingly, this conclusion is not independent of the orientation of the background field (relative to the fixed, positive twist (anti-clockwise) of the tube). Figures 2.3(d)-(e) and Figure 3(b) show results for $B_s = -0.02, -0.06$ (i.e. 2% and 6% of the axial field strength but oriented in the negative direction). Figure 2.3(d) and the red line in Figure 2.3(b) indicate a successful rise with very little flux loss amongst the -2% background field. However, Figure 2.3(e) and the yellow line in Figure 2.4(b) show a very conclusive failure to rise in the -6% background field. It appears that a significantly weaker negatively-oriented field ($|B_s| \gtrsim 6\%$) can suppress the rise of this twisted tube (compared to $|B_s| \gtrsim 20\%$ for a positively-oriented background).

Note that, in the investigations described over here and in Section 5.2, the twist is always positive (i.e. the azimuthal field in the concentration is oriented in a counterclockwise fashion) and we then experimented by flipping the orientation of the background field. The symmetry of the problem allows us to make analogous conclusions for a case where the flux tube twist is flipped while keeping the orientation of the background field constant. That is, keeping the background field always positive, for example, then tubes with positive twist (where the azimuthal field forming the twist is aligned with background field at the bottom of the tube and counter to the background field at the top) are quenched when fields above $B_s = 0.16$ are present, whereas tubes with negative twist (azimuthal field aligned with the background field at the top of the tube and counter to the background field at the bottom) are quenched at strengths above $B_s = 0.06$. There

are, therefore, certain intermediate background field strengths where one orientation of twist (relative to the background field) is suppressed whereas the other is not. That is, there exists a “selective rise regime (SRR)” in background field strength where a selection mechanism operates allowing structures of one twist to rise whilst suppressing the other. It so happens that this selection mechanism of tubes is remarkably commensurate with the SHHR in many respects, as was originally explained in Section 2.3 and as we will outline again shortly. In this chapter, and in particular in Section 2.3.4, we will investigate the mechanism, parametric dependencies and properties of this SRR in far greater detail.

Various studies in the past have explored different initial magnetic field profiles for the flux tube. As the initial setup was motivated by HFJ, we used a top-hat profile in this section. In addition to this, we have also carried out a robustness test with a Gaussian magnetic field profile for the flux tube. The main conclusion is that the results discussed over here are valid for the Gaussian flux tube too. The details of this are available in Appendix 5.2.

2.3.2 Causes of the dynamics

Numerical evidence and cartoon explanations for the causes of these effects are supplied in Figure 2.5.

A tube rising upward through an overlying field induces a downward tension force in the background field due to the wrapping of the field lines around the structure (see the A_z contours in Fig. 2.3). The wrapping also leaves stretched and therefore strong, high tension field in the wake that both resists the formation of the trailing vortices (that would eventually drive the steady coherent rise in the case lacking a background field) and also provides a channel for rapid advective drainage of the axial flux out of the tube thereby reducing its buoyant driving

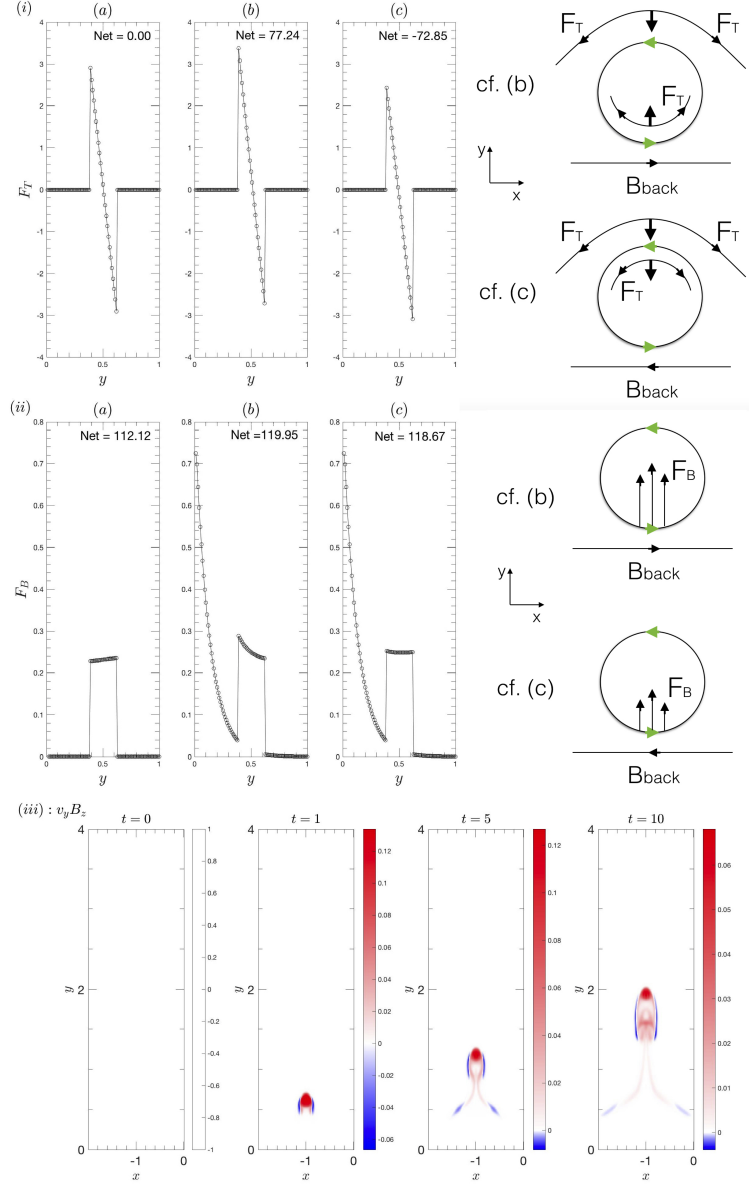


Figure 2.5: Simulation data and cartoons for the forces acting in y -direction: (i) tension, $F_T = (B_x \partial_x + B_y \partial_y) B_y$; (ii) buoyancy force $F_B = -\partial_y B_y^2 / 2 - \rho \theta (m + 1) - \partial_y P_{gas}$ evaluated at $x = 0, t = 0$. Panels (a,b,c) correspond to $B_{back} = 0\%, 6\%, -6\%$ respectively. The cartoons show end-on views of rising tubes with black net force arrows for the respective forces (tension or buoyancy) on representative field lines inside and outside the tube for the (b) and (c) cases. The green arrows show the twist of the tube. (iii) Time series of the vertical flux of axial field, $F_A = v_y B_z$ for $B_s = 0.1$.

(seen as the negative fluxes in Fig. 2.5(iii)). These effects all counter or reduce the upward forces on the structure, and can completely suppress the rise, depending on the relative strengths of the overlying and tube magnetic fields (and the effects of magnetic diffusivity, fixed here).

Why one orientation of the background field is more efficient at suppressing the rise than the other depends on the contribution of the background field to the internal forces in the tube. Figure 2.5 also shows (i) the vertical component of the magnetic tension force F_T , and, (ii) the vertical component of the total buoyancy force F_B on cuts through the initial tube at $x = 0$ for relative background fields strengths of 0%, 6% and -6%. The vertical tension force in the 0% case is symmetrical about the tube center and thus has no net value: the only role of tension here is to act radially inwards to maintain coherency of the structure. When background field is present, for our chosen (positively) twisted tube, a positively-oriented background field skews the total field inducing stronger positive (upward) magnetic tension at the bottom of the tube and weaker negative (downward) forces at the top, thus inducing a net upward tension force in the tube itself. Reversing the background field (for our fixed tube twist) reverses the asymmetry and induces downward net internal tension. The cartoons on the right of Figure 2.5 (i) show this pictorially. The buoyancy force, on the other hand, is enhanced to a similar net value by the presence of either background field direction, although the bottom of the tube is emphasized for $B_s > 0$. The incorporation of a positive background field, therefore, produces net tension and buoyancy forces in the tube that counter the detrimental effect of the magnetic tension induced by the background field during the rise, leading to an enhanced ability to rise. Conversely, a negative background field creates internal tension forces that act in concert with those induced by the overlying field, to enhance

suppression.

2.3.3 Implications of selection mechanism to helicity observations

The selection mechanism discussed above, when applied to a solar context, has many qualities that agrees with the SHHR, but in particular the correct helicity parity. Figure 2.6 shows how the model results translate to the solar scenario for a complete 22-year solar cycle where the fields reverse after 11 years. The azimuthal field direction (twist) of the flux concentration along with its axial field direction gives a certain sign of current helicity to the initial flux concentration. Note that our selection rule depends only on the relative orientation of the twist and the background field, and so the axial field direction, and therefore the current helicity, must be determined by the particular circumstances of the solar situation. Fig 2.6a shows the first half of a sample solar cycle defined by the N-S orientation of the large-scale poloidal field (red arrows). We assume, as is commonly the case in the current understanding of the generation of the solar large-scale fields, that deep in the solar interior, the action of differential rotation on the large-scale background poloidal field leads to the generation of strong toroidal flux sheets, which subsequently leads to more localised toroidal flux concentrations (likely via magnetic buoyancy instabilities). This assumption ensures that the orientations of toroidal axial field (blue arrows) of a flux concentration and the large-scale background poloidal field are correlated with each other. For example, for the first half of our sample cycle shown in Fig. 2.6a, the N-S orientation of the poloidal field leads to eastward toroidal field in the northern hemisphere and westward in the southern. In the second half of our sample cycle (Fig. 2.6b), the poloidal field (red arrows) reverses (S-N) and the action of the differential rotation therefore

produces westward toroidal field in the northern hemisphere and eastward in the southern. Based on the N-S orientation of the poloidal background field for the first half of the sample solar cycle in Fig. 2.6a, the selection mechanism of Section 2.3 then requires that flux concentrations twisted in anticlockwise sense are more likely to rise (in either hemisphere) as they require a higher threshold of poloidal field strength to disrupt their dynamics. This anticlockwise twist then paired with the different axial toroidal field direction in each hemisphere then dictates which helicity has a preferential rise. A positive correlation between the twist and axial toroidal field direction would lead to a positive helicity flux concentration and similarly a negative correlation would lead to a negative helicity flux concentration. It can be seen in Fig. 2.6a that flux structures with negative helicity in the northern hemisphere and positive helicity in the southern hemisphere are the ones that are more likely to rise. For simplicity of understanding, Figure 2.6 shows these cases – the ones that require a higher threshold of background field to halt/breakup the rise of a flux concentration, i.e., the configurations that are preferred in the sense that they are more likely to rise. It can be seen that Figure 2.6a is in agreement with the “Solar Hemispheric Helicity Rule”. Furthermore, in the second half of the cycle, shown in Figure 2.6b, both the poloidal field and the toroidal field flip direction (sign) in each hemisphere, thereby preserving the helicity of the structure that has preferential rise. This selection mechanism is therefore commensurate with the parity rules of the SHHR and its invariance over the full solar cycle.

The mechanism discovered also points to a potential explanation for the large scatter found in the SHHR observations, that was not examined in Section 2.3.2 but is explored below in detail. The selection only happens for a range of the relative strengths of the twist of the structure and background field strengths (which we call the SRR). We might expect either of these contributions to vary

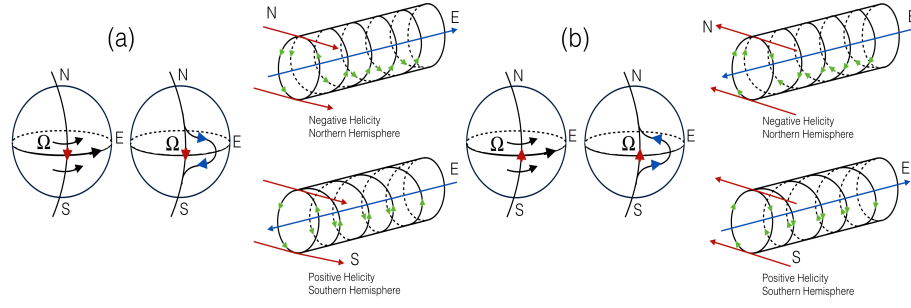


Figure 2.6: This figure shows how to translate our model results into the solar case. Panels (a) and (b) show a pictorial representation of the first and second half of an arbitrary full 22-year solar cycle, respectively. The cartoon spheres depict the action of differential rotation on existing dynamo poloidal field (i.e. the mean-field Ω effect). The dynamo poloidal field is represented with red arrow markers and toroidal field with blue markers. The differential rotation (exhibited as black arrow markers) stretches the poloidal field into toroidal field, leading to a certain orientation of the directions of the two field in each hemisphere (for each half of the cycle). The cylindrical cartoons show how the above oriented fields relate to the axial (blue, toroidal) and background field (red, poloidal) of our model. Furthermore, the tube cartoons show the twist (azimuthal, non-axial field, green markers) of structures that are predicted to rise preferentially by our model. The resultant correlation of axial (blue) and twist (green) is commensurate with the SHHR. Figure adapted from Section 2.3.

outside of this range thereby allowing violations to the rule. We investigate, and also synthetically reproduce, this scatter in more detail later in this chapter.

The results discussed in this Section and in Section 2.3.1 are obtained from a small number of experiments based around a canonical setup. We now proceed to quantify the robustness and parametric dependence of the SRR in detail through a much broader range of simulations.

2.3.4 A broader survey of $q - B_s$ space at fixed H_B

We define our SRR as the region in parameter space where one relative orientation of twist and background field rises successfully but the reverse orientation does not. Seeing as the dynamics of the selection mechanism depend on the in-

teraction between the background field and the twist of the tube, it might be expected that the SRR is most strongly dependent on q and B_s , and we concentrate on this first, fixing for now the other main parameter, the scale height of the background field configuration, at $H_B = 0.125$ (and, as always, keeping the other parameters at their canonical values: $m = 1.5$, $R = 0.125$, $\theta = 0.25$). It is perhaps easiest to think of the SRR as the parameter regime where one sign of twist rises but the other does not at fixed background field strength and orientation. Note, however, that our original simulations fixed the twist and examined the effect of reversing the field. The SRR is, therefore, really a two-dimensional region of $q - B_s$ parameter space (for fixed H_B) and we investigate this here.

Figure 2.7 shows a much broader survey of results in the $q - B_s$ parameter space than was shown in Section 2.3.2. The work of Section 2.3.2 would correspond to a single row at $q = 2.5$ (not actually shown in this new figure). Schematic indications of the crucial twist and background field orientations are shown in the border of the table, along with their values, for ease of interpretation of the signs. The figure shows a matrix colored at each $q - B_s$ value according to whether a simulation at those values shows that the flux concentration clearly rises (green), clearly fails to rise (red), or something less easily determined (orange). The determination in each simulation is done using visualisations and the flux fractions, as exemplified in Figures 5.5 and 5.6. The relatively complicated intermediate dynamics represented by the orange colored cases are examined in more detail in Section 2.3.7. Swapping both the sign of q and B_s results in the same dynamics (equivalent to just viewing the same system from the other end of the flux tube), and so the table is diagonally-symmetric across the origin. All four quadrants of the matrix have been included for ease of using the table, but this symmetry emphasizes that the selection mechanism only depends on the *relative* direction

and strength of B_s and the twist, q . We, therefore, explain one half of the survey chart ($B_s > 0$) since analogous interpretations can be drawn for the other half. Note that in our model setup, when q changes sign, the current helicity of the tube changes sign, since B_z remains fixed in the positive z direction. This is irrelevant to the selection mechanism, since it depends solely on twist, not helicity. For relevance to the solar case, however, the selection mechanism can be cast in terms of the helicity, since, there, the sign of the background field and the tubes axial field are correlated, according to our understanding of the dynamics of the solar dynamo. The cases within the white dotted lines are special cases. For $q \lesssim 0.25$, all cases fail to rise coherently regardless of the background field strength because some initial twist is required to maintain the coherency of the tube [88]. For all other $q \gtrsim 0.25$, but zero background field, the tube rises successfully.

For any fixed B_s , the table in Figure 2.7 clearly shows that an asymmetry exists between positive and negative q . For $B_s > 0$, the transition from red (unsuccessful rise) to green (successful rise) for negative q occurs (if it occurs at all in the cases surveyed) at far larger $|q|$ than for positive q . By symmetry, for $B_s < 0$, the roles are reversed with the $q > 0$ boundary occurring at much larger $|q|$. The different $|q|$ at which the red-green (unsuccessful-successful) transition occurs (say $|q_-|$ for the negative q and $|q_+|$ for the positive q) delineate the SRR at that B_s . The SRR can therefore be defined from the simulation data at any B_s as the set

$$\text{SRR} = \{q \text{ s.t. } |q| \in [\min(|q_-|, |q_+|), \max(|q_-|, |q_+|)]\} \quad (2.11)$$

Within this set, q of one sign rises and the other does not. It can easily be verified from Figure 2.7 that, for $B_s > 0$, it is structures with $q > 0$ that rise if their q lies in the SRR found at that B_s , whereas for $B_s < 0$, it is the structures with $q < 0$ that are the ones that rise.

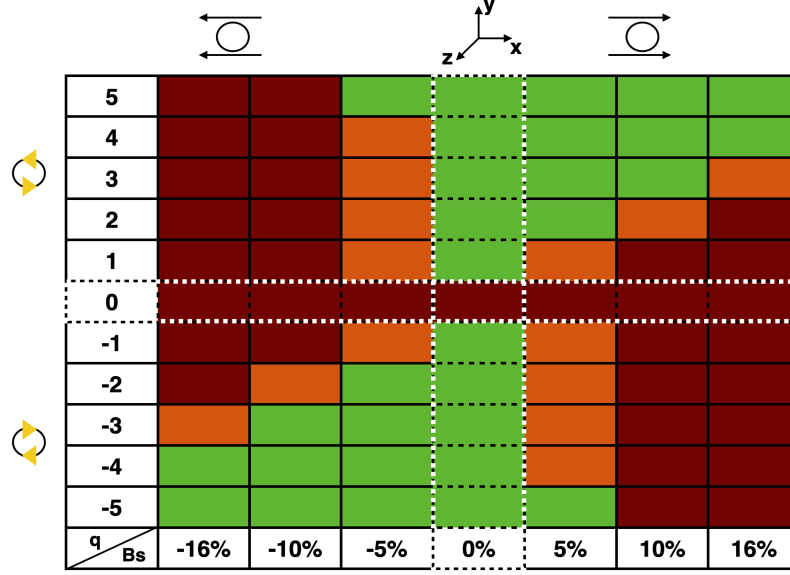


Figure 2.7: A map of rise success/failure in $q - B_s$ space for fixed $H_B = 0.125$. Background field strengths B_s are given as percentages of the axial field strength of the magnetic flux structure. Background field and flux tube twist orientation by sign are shown schematically for clarity. Red (darkest) color signifies a definite failed rise of the flux structure, whereas the green (lightest) color signifies a definite successful rise. Orange (intermediate shade) signifies an intermediate result, where the structure may rise but the dynamics are noticeably different than other rising cases. Simulations lying inside the white dotted lines are either cases with negligible background field $B_s \sim 0$ or cases where the twist is so low $q \sim 0$ that structures would not rise coherently.

Figure 2.7 shows that the boundaries of the SRR, given by $|q_-|$ and $|q_+|$, vary with B_s . For higher $|B_s|$, stronger q is clearly required to overcome the background field in order to rise, and the SRR becomes defined by higher $|q|$ values and appears to widen in extent of $|q|$. These issues are investigated in detail via a numerical model in the next section.

At this point, it is worth noting that there are some physical limits on what we might expect for reasonable values of $|q|$, despite the fact that we have no direct observational evidence from the solar interior to help us out in this matter. Firstly, in our model, structures with $|q| \gtrsim 12$ are unphysical, since then the azimuthal field is sufficiently strong that the associated magnetic pressure induces a negative density in the initial conditions for the structure ¹. Secondly, structures with $|q| \gtrsim 8$ are likely unstable to kink instabilities (according to [73], whose three-dimensional model has a different field configuration from our two-dimensional model, but is likely a reasonable guide). These issues informed our choice of range in q in these simulations. Similarly, we have no direct observational data for the strength of solar interior magnetic fields, but it seems reasonable to presume that the background poloidal field strength is only a fraction of the strength of any rising flux structure.

Ultimately, the $q - B_s$ diagram in Figure 2.7 exhibits compactly the results of a selection mechanism and therefore illustrates clearly what we have referred to as the SRR, the parameter regime in which the selection occurs. The overall conclusions from the simulation data presented in Figure 2.7 may be that an SRR of some extent occurs for almost all reasonably physical parameter values. This seems to imply the existence of an overall bias for the rise of flux structures, even if they have significant variance in their magnetic composition. In general, a prevalence

¹This limit is actually not constant but instead a very weakly decreasing function of B_s , and is shown later as the dashed line in Fig. 2.9

of positively twisted structures might be expected to emerge from a distribution of structures in the presence of a positive background field, and a prevalence of negatively twisted structures might be expected in a negative background field. We verify these ideas in Section 2.3.7, but first we turn to an analytical model that defines the SRR explicitly in terms of the parameters, allowing a more fine-grained exploration and understanding of the parameter space.

2.3.5 An analytical model of the dynamics

Section 2.3.2 provided some brief insight into the reasons for the varying dynamics associated with different orientations of the fields and tubes. Here, we extend those ideas into a detailed model that can explain and predict the $q - B_s$ behaviour discussed above, and the dependence on other parameters. To formulate the model, we carry out a theoretical analysis of the key magnetic forces acting on the flux concentration in the vertical (y) direction as it initially begins to rise. Our model is based on the following ideas. Firstly, we have in mind that the rise of a tube-like concentration through the overlying background field is generally hindered by tension induced in the background field as it is wrapped around the structure and gets lifted and stretched upwards. Secondly, originating from the ideas in Section 2.3.2, we expect that the introduction of the background field induces differential values of the internal tension and buoyancy forces across the tube-like concentration which can lead to net forces that can help or hinder the rise. To create a model that predicts whether rise is successful or not, we calculate the differential buoyancy and tension forces from the initial conditions at $t = 0$ and then estimate the impeding tension forces from the overlying field that are induced as the structure rises. A balance of these forces should delimit the bounding case between successful and unsuccessful rise. Note that, since the

selection mechanism and all the results discussed so far are robust to different formulations the magnetic initial conditions (see Section 2.3.1 and 5.2), in order to make analytic integration of the net forces easier, we revert back to the “top-hat” magnetic initial conditions ($B_z = 1$ for $r \leq R$ in the concentration; see Eqn 2.6) as used in Section 2.3.2 rather than the Gaussian cross-section used in Section 5.2.

The buoyancy force in the y-direction, $F_{buoyancy,y}$, including its magnetic contribution, is generally thought of as the driving force for the rise of the concentration. We have deliberately set up our problem so that the presence of the magnetic field of the concentration produces a density (and temperature) perturbation (under the assumption of fast equilibrium of the total pressure; see Appendix 5.1, Case 2) that will drive the rise. The addition of a background field that varies with height can create a perturbation to the vertical buoyancy force that is asymmetric about the center of the tube and therefore has a net contribution. Ultimately, however, we will find in the following calculation that the magnetic contribution to the buoyancy force is dominated by the axial field of the tube, and that this differential perturbation is unimportant (for the buoyancy force, although it will be crucial for the tension force). The net buoyancy force in the vertical (y) direction over the circular region of the tube-like concentration in our Cartesian coordinate system is given by

$$F_{buoyancy,y} = \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y'^2}}^{\sqrt{R^2-y'^2}} \left(-\frac{\partial}{\partial y'} \left(p_{gas,in} + \frac{B_{eff}^2}{2} \right) + \rho_{gas,in} g \right) dx \right] dy'. \quad (2.12)$$

where $\mathbf{g} = \theta(m+1)\hat{\mathbf{y}}$ from equation (2.2) and $y' = y - y_c$. This latter translation does not affect the analytical calculations mentioned below and hence for clarity, we drop the dashes in the subsequent calculations from y' and simply use y . Here,

B_{eff} is the total magnetic field inside the flux tube accounting for both the field of the magnetic flux structure and the background field; $p_{gas,in}$ and $\rho_{gas,in}$ are the gas pressure and density inside the tube. Assuming total pressure balance between the inside of the tube and the outside, we have

$$p_{gas,in} + \frac{B_{eff}^2}{2} = p_{gas,out} + \frac{B_{back}^2}{2}, \quad (2.13)$$

where $p_{gas,out}$ and the corresponding $\rho_{gas,out}$ are the gas pressure and density outside the flux tube in the stratified background given by the polytropic model (Equation 2.2). The density inside the flux tube, $\rho_{gas,in}$, is then given in terms of $\rho_{gas,out}$ adjusted by the magnetic field (as described in more detail by the initial conditions outlined in Appendix 5.1, Case 2):

$$\rho_{gas,in}(y) = \rho_{gas,out}(y) - \frac{1 + 4q^2 R^2 - 4qyB_{back}}{2(1 + \theta(4 - (y + y_c)))}. \quad (2.14)$$

The factor in the denominator related to the initial polytropic temperature profile above, $T_{gas,out} = 1 + \theta(4 - (y + y_c))$, varies only by a small fraction across the vertical extent of the flux tube and can be assumed to be roughly constant, equal to the average value, T_{avg} . This makes the analytical calculation of the integrals in Equation (2.13) more tractable. Plugging (2.13) and (2.14) into (2.12) and using the polytropic nature of the outside gas, the net buoyancy force equation reduces to

$$F_{buoyancy,y} = \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left(-\frac{\partial}{\partial y} \left(\frac{B_{back}^2}{2} \right) - \theta(m+1) \left(\frac{1 + 4q^2 R^2 - 4qyB_{back}}{2T_{avg}} \right) \right) dx \right] dy. \quad (2.15)$$

The final term in the integral is related to the cross term ($\sim B_x B_{back}$) and yields

a difficult term in the integration. We drop this term here, expecting it to be relatively insignificant, and indeed verify that this is the case later. In order to integrate the first term in equation (2.15), we use Equation (2.5) to substitute

$$\frac{\partial}{\partial y} \left(\frac{B_{back}^2}{2} \right) = -\frac{H_B}{2} B_{back}^2 = -\frac{H_B}{2} B_s^2 \exp \left(\frac{-y}{H_B} \right), \quad (2.16)$$

and then, after integrating equation (2.15) once with respect to x , we further use the definition of modified Bessel function of first order,

$$I_1(z) = \frac{z}{\pi} \int_{-1}^1 (1-t^2)^{1/2} e^{-zt} dt, \quad (2.17)$$

with $t = y/R$ and $z = R/H_B$. The result of the overall integration of Equation (2.15) (excluding the cross term) is then

$$F_{buoyancy,y} = \pi B_s^2 H_B^2 R I_1(2\tau) + \frac{\theta(m+1)\pi R^2}{2T_{avg}} (4q^2 R^2 + 1), \quad (2.18)$$

where $\tau = R/(2H_B)$. Equation (2.18) shows that the net buoyancy force has components that depend on the strength of the background field (through the first term and B_s^2) and the tube field strength (through the second term, dependent on q^2 and a constant value for the axial field). When evaluating this expression for our parameters later, we will see that the second term clearly dominates.

When a background field is included, the role of the internal tension force in the overall dynamics can be equally as important as the buoyancy force in determining the net upwards driving force. When there is no background field ($B_s = 0$), the internal tension force in the tube is symmetric and acts towards the center of the tube. Hence, there is no net force in any direction at $t = 0$ and tension serves only to maintain the coherency of the tube in this case. On the introduction of a background field that varies in the vertical, this internal tension force becomes

asymmetrical around the horizontal mid-line of the tube, and, as pointed out in Section 2.3.2, the net vertical force induced can be upwards, thereby aiding the rise, or downwards, thereby hindering the rise. We denote this internal tension force in the vertical (y) direction by $F_{tension,y}$, given by

$$F_{tension,y} = \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} (\mathbf{B} \cdot \nabla) B_y dx \right] dy = \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left(B_x \frac{\partial B_y}{\partial x} + B_y \frac{\partial B_y}{\partial y} \right) dx \right] dy, \quad (2.19)$$

where y has been translated as before. With the chosen magnetic initial conditions given in equation (2.6), $\frac{\partial B_y}{\partial y} = 0$ and so, at $t = 0$,

$$F_{tension,y} = \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} B_x \frac{\partial B_y}{\partial x} dx \right] dy = \int_{-R}^{+R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left[(-2qy + B_{back})(2q) dx \right] dy \quad (2.20)$$

$$= \int_{-R}^{+R} \left[\int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \left(-4q^2y + 2qB_s \exp\left(\frac{-y}{2H_B}\right) \right) dx \right] dy. \quad (2.21)$$

Integrating this, using again the modified Bessel function in (2.17) (with $t = y/R$ and $z = R/2H_B$), gives

$$F_{tension,y} = 8\pi q R B_s H_B I_1(\tau). \quad (2.22)$$

Equation (2.22) confirms that the net internal tension force is zero when the large-scale background field is not present ($B_s = 0$). Moreover, it is now clear that the introduction of background field ($B_s \neq 0$) creates a net internal tension force that can either support the rise of the flux tube ($F_{tension,y} > 0$) or hinder its rise ($F_{tension,y} < 0$) based on the sign of the product of twist and background field. This is commensurate with our earlier findings from the simulations that it is the alignment of the twist and the background field that dictate the behaviour. If q

and B_s have the same sign (i.e. the azimuthal field of the tube and the background field are aligned at the bottom of the tube), then the product qB_s is positive and the tension force is upwards, supporting rise. Note that for a fixed B_s , the net tension force varies linearly with q .

The third part of the force budget that we need to consider is the tension force induced in the large-scale background field by the rise of the structure upwards into this field, which we denote by $F_{back,y}$. This opposes the rise of the tube. To estimate $F_{back,y}$, we examine the tension force induced in a rise that proceeds to a general height, H . For a successful rise in our simulations, H will be approximately equal to or greater than the maximum vertical size of the simulation box, $y_{max} = 4$. Tension forces in curved field lines can be written as square of the magnitude of the field divided by the radius of curvature, pointing inwards to the curve. If we assume that a rising tube lifts all of the field it encounters during the rise and concentrates it into a narrow region wrapped around the tube, then a description of the net tension force (in the y -translated coordinate system used earlier) opposing the vertical rise induced during the rise might be reasonably given by the integrated square amplitude of the field divided by the radius of the tube:

$$F_{back,y} = -\frac{1}{R} \int_0^H B_{back}^2 dy \quad (2.23)$$

$$= -\frac{B_s^2}{R} \int_0^H \exp\left(\frac{-y}{H_B}\right) dy \quad (2.24)$$

$$= -\frac{H_B B_s^2}{R} \left(1 - \exp\left(\frac{-H}{H_B}\right)\right). \quad (2.25)$$

For small H , i.e. $H \ll H_B$, we can rewrite the exponential function using series

expansion and keep only terms up to first order, giving

$$F_{back,y} = -\frac{HB_s^2}{R}. \quad (2.26)$$

Clearly then the tension force induced in the background field is linear in H and will be negligible when the height to which the flux tube rises, H , is very small, as expected. When the flux tube rises more significantly, i.e. $H \gg H_B$, the net induced tension force tends to a constant value

$$F_{back,y} = \frac{-H_B B_s^2}{R}. \quad (2.27)$$

The tension therefore only varies rapidly over a height of about a scale height H_B and its role can be bounded above nicely by equation (2.27).

If we make the assumption that the internal buoyancy and tension forces in the tube-like concentration do not change much from their initial values, then the marginal case that defines the transition from a failed rise to a successful rise is given by a balance of the three elements described above, i.e.

$$F_{tension,y} + F_{buoyancy,y} + F_{back,y} = 0. \quad (2.28)$$

Assuming some non-negligible rise, $H \gg H_B$, using Equations (2.18), (2.27) and (2.22), this becomes

$$\left[\frac{2\theta(m+1)\pi R^4}{T_{avg}} \right] q^2 + \left[8\pi R B_s H_B I_1(\tau) \right] q + \left[\frac{\theta(m+1)\pi R^2}{2T_{avg}} + \pi B_s^2 H_B^2 R I_1(2\tau) - \frac{H_B B_s^2}{R} \right] = 0. \quad (2.29)$$

This equation is quadratic in q and we can calculate its roots (q_1, q_2) for given H_B and B_s (given also θ, m, T_{avg} and R which are assumed to have fixed values

B_s	H_B	q_1	q_2
0.03	0.075	$-3.14 + 2.26i$	$-3.14 - 2.26i$
	0.125	$-2.97 + 2.32i$	$-2.97 - 2.32i$
	0.175	$-2.93 + 2.23i$	$-2.93 - 2.23i$
0.05	0.075	-9.01	-1.45
	0.125	-8.62	-1.29
	0.175	-8.71	-1.05
0.10	0.075	-20.72	-0.21
	0.125	-19.99	0.17
	0.175	-20.08	0.56
0.16	0.075	-33.90	0.41
	0.125	-32.75	1.04
	0.175	-32.87	1.64

Table 2.1: Roots of the quadratic equation 2.29 for some example values of B_s and H_B

throughout this chapter). These roots (q_1, q_2) are the model estimates of the (q_-, q_+) from the simulation data in Figure 2.7 earlier.

Table 2.1 shows examples of the numerical values of these roots for some selected B_s and H_B . Figure 2.8 accompanies the table and shows the relative contributions of the individual components of magnetic buoyancy, internal tension and background field tension forces, plus their total, from the model equations (2.18), (2.22) and (2.27) respectively, as a function of q for the canonical background field configuration with $H_B = 0.125$ and the B_s indicated. We will use these cases as illustrative examples. A more complete evaluation of the roots of the canonical case is given later in Figure 2.9, and other H_B values are addressed in the next section. In Table 2.1, we see that for very weak values of B_s ($B_s \leq 0.03$ at $H_B = 0.125$), there exists no real roots for q . This is consistent with the fact that all values of twists, whether positive or negative, will rise successfully when only a very weak background field is present (assuming that q is above the minimum threshold necessary to maintain coherency of the structure)². Figure 2.8a illustrates this result,

²This is likely similar to the $q \gtrsim 0.25$ threshold for the $B_s = 0$ case, but note that the

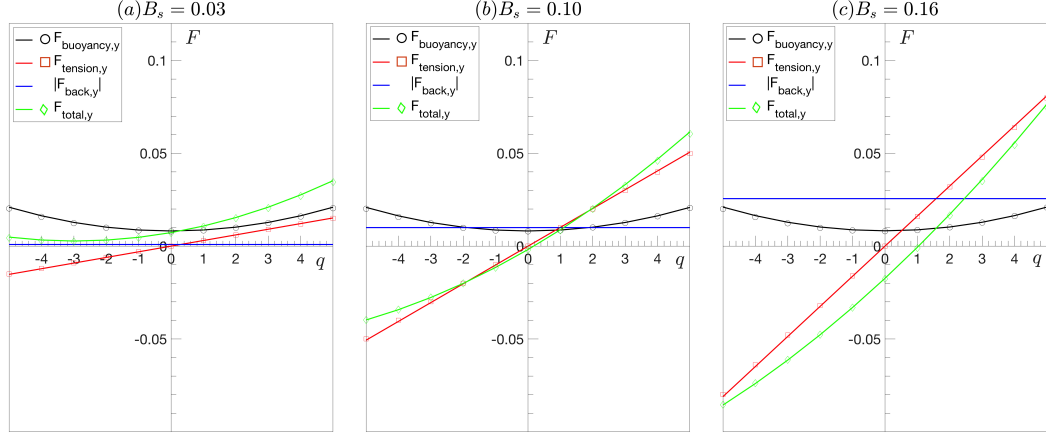


Figure 2.8: Shown are forces acting on the flux tube in vertical direction calculated from the theoretical model: Area-integrated net buoyancy and tension forces internal to the tube; the upper bound estimation of the opposing tension force induced in the background field by the rise; and, the total upwards vertical force (the sum of internal buoyancy, internal tension and induced external tension). These are plotted as a function of the flux tube twist, q , for $B_s = (a) 0.03$, $(b) 0.10$ and $(c) 0.16$, for the canonical value of $H_B = 0.125$. The symbols show the corresponding integrated forces calculated using the simulation data.

as $F_{total,y}$, the sum of the forces on the left hand side of equation (2.29), does not cross the horizontal axis for any q at this B_s . For intermediate and high values of B_s , we get two real roots and these are asymmetric in q . For example, we find that the roots for $B_s = 0.10$ are $q = (q_1, q_2) \approx (-19.99, 0.17)$ and for $B_s = 0.16$ are $q = (q_1, q_2) \approx (-32.75, 1.04)$. This asymmetry in the roots defines the model definition of the SRR. For example, if $q_1 < 0$ and $q_2 > 0$, then rise occurs for structures with $q < q_1$ and $q > q_2$ and there is a region of $|q|$ between $|q_1|$ and $|q_2|$ where structures possessing $+|q|$ and $-|q|$ behave differently ($+|q|$ rises, $-|q|$ does not). The canonical examples from 2.3.2 for $B_s = 0.10$, showed that $q = 2.5$ rises but $q = -2.5$ did not, and so $|q| = 2.5$ lies in the SRR of this B_s , which we can see here is given by $0.17 < |q| < 19.99$. More generally, in these examples with positive B_s , positively-twisted flux tube that are fairly weak can rise whereas presence of background field may adjust this.

negatively-twisted flux structures of the same strength (or even relatively much stronger) cannot.

Figure 2.8 clarifies the origin of the asymmetrical roots and therefore the SRR in terms of the individual force contributions. The buoyancy force (black lines derived from Equation (2.18) with associated circles from simulation data – see later) is quadratic in q and can now be seen to be dominated by the contribution from the initial density perturbation due to the magnetic structure (the second term on the right hand side of Equation (2.18)), since the buoyancy force is very close to symmetric in q and only very weakly dependent on B_s . The asymmetry in the q dependence of the total upwards force on the tube (green line; diamonds) instead stems from the net vertical internal tension force in the tube (red line; squares). This is linear in q , and significant in size because it depends on B_s and H_B rather than these quantities squared. For the $B_s > 0$ shown here, negative q creates a negative tension force serving to reduce the effectiveness of the buoyancy force, retarding the rise of negative q structures. This effect can completely inhibit the rise unless the buoyancy force (proportional to q^2) is large enough to compensate. Where $q > 0$, the internal tension force is positive and aids in the rise of the structure. The tension in the background field (blue line; no symbols) is independent of q and only serves to reduce the chances of rise evenly across the range of q at fixed B_s (but is dependent on B_s^2). The net force, given by the left hand side of equation (2.29) (green line; diamonds) is therefore a shifted and tilted quadratic function. Where this function passes through zero are the roots q_1 and q_2 , and now it is clear that the effect of the background field on the internal tension of the tube is responsible for introducing the major asymmetry into this function and therefore is the culprit for creating the asymmetric roots leading to the existence of a SRR.

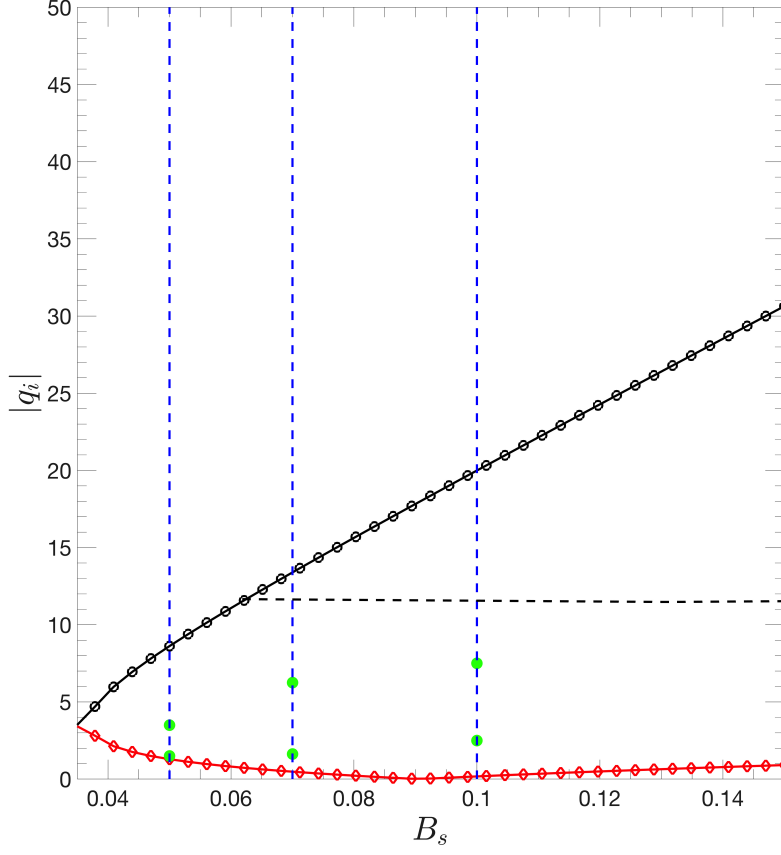


Figure 2.9: The SRR given by the model as a function of B_s for $H_B = 0.125$. Black circles and red diamonds represent the absolute value of roots of Equation (2.29). For these cases with positive B_s , q_1 is negative and is the root with larger modulus (black circles). The other root q_2 has smaller modulus (red diamonds) and changes sign from positive to negative at around $B_s \sim 0.09$. Blue dashed vertical lines in the plot are the cases studied using numerical simulations and the black dashed line represents the highest allowable twist that keeps the thermodynamic conditions inside the flux concentration physical (i.e. such that the density remains positive). The green circles are the limits of the SRR found from the numerical simulations.

The results have been discussed here only for $B_s > 0$. The results for $B_s < 0$ can be inferred by the symmetry in the $q - B_s$ space, as noted earlier with relation to the simulation data in Fig. 2.7. For example, in an equivalent of Fig. 2.8 for $B_s < 0$, the buoyancy force and the tension force induced in the background field would remain essentially the same, but the internal tube tension force would switch sign. The roles of q_1 and q_2 would therefore be reversed, with the asymmetry such that $|q_1| \ll |q_2|$. A general definition of the model SRR such that it is valid for any orientation of B_{back} in terms of the roots of the quadratic is given similarly to Equation 2.11 by

$$\text{SRR}_{\text{model}} = \{q \text{ s.t. } |q| \in [\min(|q_1|, |q_2|), \max(|q_1|, |q_2|)]\} \quad (2.30)$$

Figure 2.9 shows an example of the delineation of the SRR in $q - B_s$ space via this method (at the canonical parameter values, in particular $H_B = 0.125$; other H_B are done shortly). The red diamond points show $\min(|q_1|, |q_2|)$ values and the black circular points show $\max(|q_1|, |q_2|)$. For these cases with $B_s > 0$, the root of smaller modulus is q_2 . The SRR is the region between the lines. Structures with values of q of either sign such that $|q|$ is above the upper limit of the SRR (line of black points) all rise. For $B_s \gtrsim 0.09$, where q_1 and q_2 are of opposite sign, structures with both signs of q would fail to rise if $|q|$ is below the lower limit (red diamonds). For $0 < B_s \lesssim 0.09$, where q_1 and q_2 are both negative, both signs would rise for $|q|$ below the red line. Only within the SRR range do structures of one sign rise whereas structures with the opposite sign do not. Positive twist rises preferentially for the cases shown here with $B_s > 0$, but negative twist would rise preferentially if $B_s < 0$. The SRR definition given in Equation (2.30) is presented as a range of q at fixed B_s and is therefore a function of B_s . The SRR is actually a region in $q - B_s$ space and could equally well be presented as a range of B_s that

is a function of q , as is clear from both the data and the model, as shown in Figs. 2.7 and 2.9.

The model proposed therefore seems to explain well the nature of the dynamics that have been observed. However, a number of simplifications were introduced, namely: the assumption that the background temperature variation across the tube was reasonably represented by its average; that it is reasonable to omit the q dependent term in equation (2.15); and, that it is a good approximation to estimate the internal tube forces (buoyancy and internal tension) only at $t = 0$ and then use that to deduce the subsequent $t > 0$ rise behavior. Since we have all the data from the simulations, it is constructive to compare what we can in the model with the numerical simulations in order to verify our assumptions. We therefore start by crudely calculating the net internal forces in the structure at $t = 0$ by summing over the relevant quantities inside the tube area as follows:

$$F_{buoyancy,y,sim} = \sum_{i,j \text{ s.t. } r_{i,j} \leq R} \left[-\frac{\partial}{\partial y} \left(P_{gas} + \frac{B^2}{2} \right) + \rho g \right]_{i,j} \Delta x_i \Delta y_j \quad (2.31)$$

$$F_{tension,y,sim} = \sum_{i,j \text{ s.t. } r_{i,j} \leq R} \left[(\mathbf{B} \cdot \nabla) B_y \right]_{i,j} \Delta x_i \Delta y_j \quad (2.32)$$

$$F_{total,y,sim} = F_{buoyancy,y,sim} + F_{tension,y,sim} + F_{back,y}, \quad (2.33)$$

where $r_{i,j} = x_i^2 + (y_j - y_c)^2$ with i, j the indices of the discrete simulation grid in the x and y direction respectively. For each relevant simulation that we have performed, we plot the values of these net internal forces as the appropriately-colored symbols in Figure 2.8. Internal magnetic tension (red squares) calculated using the simulation data exactly matches with the model estimate. The black circles representing the internal tube buoyancy from the simulations do have a slight linear dependence on q added to the symmetrical quadratic term, but so

slight that it is not visually apparent in the figure, vindicating our omission of this term in the model.

The major source of uncertainty in our model then comes from the estimate of $F_{back,y}$. This quantity is a very rough approximation to one aspect of more complex dynamics. Omitted effects are the deformed geometry of the structure, effects of the wake and the overlaying field on the wake, any drainage from the main structure, and diffusive effects, at least. These other effects are hard to model and would be hard to check against data even if they were modeled, since they are not compactly spatially located to the structure and are time-evolving. The discrepancy associated with these errors shows up in comparisons of the roots that we obtain by solving the quadratic Equation (2.29) and the SRR range deduced from observing the simulations. The upper and lower bounds of the SRR deduced from the simulations are shown for a few sample B_s as the green circles in Figure 2.9. Although the simulation SRR boundaries are only visually extracted from a coarse set of simulations and are therefore relatively inaccurate, it is clear from this comparison that the simulated SRR is substantially narrower than the model predicts. It appears that our estimate of the rise-quenching effects from the background field is indeed quite a severe underestimation, as we imagined. We could attempt ad hoc changes to the model (e.g. increasing the radius of curvature of the retarding external tension force in the background field to reduce its effect and to account for the topology of the rising structure later in its rise), or try to calibrate the model to the data, but since the simulations are fairly coarsely spaced in parameter ($q - B_s$ space) and the boundaries between successful and failed rise are sometimes not entirely obvious, then this seems unlikely to be particularly accurate. Furthermore, the omitted effects are probably dependent on the parameters (such as B_s and H_B) too, and therefore a simple calibration

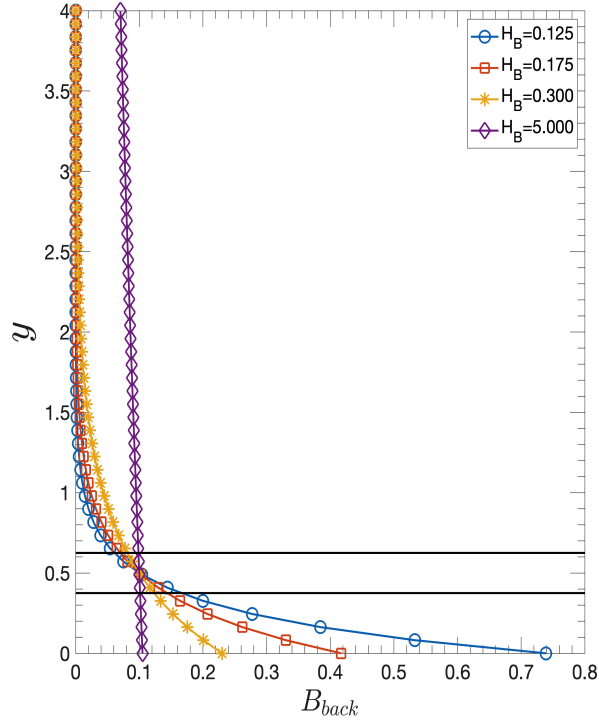


Figure 2.10: Variation of background field as a function of height for different scale heights H_B for $B_s = 0.1$. Dotted horizontal lines indicate the vertical location of the flux tube.

is unlikely to work universally. However, even with some significant uncertainty, the model still provides a good explanation of the dynamics behind the SRR, and gives useful predictions of the trends seen in the simulations, at least. In particular, it appears to be a robust result that the extent of the SRR grows and the root of lower modulus grows (in signed value) with increasing B_s (see also Fig. 2.13 later). These facts have a significant influence on the expected distribution of the signs of the emerging helicity, as will be discussed in detail in Section 2.3.7.

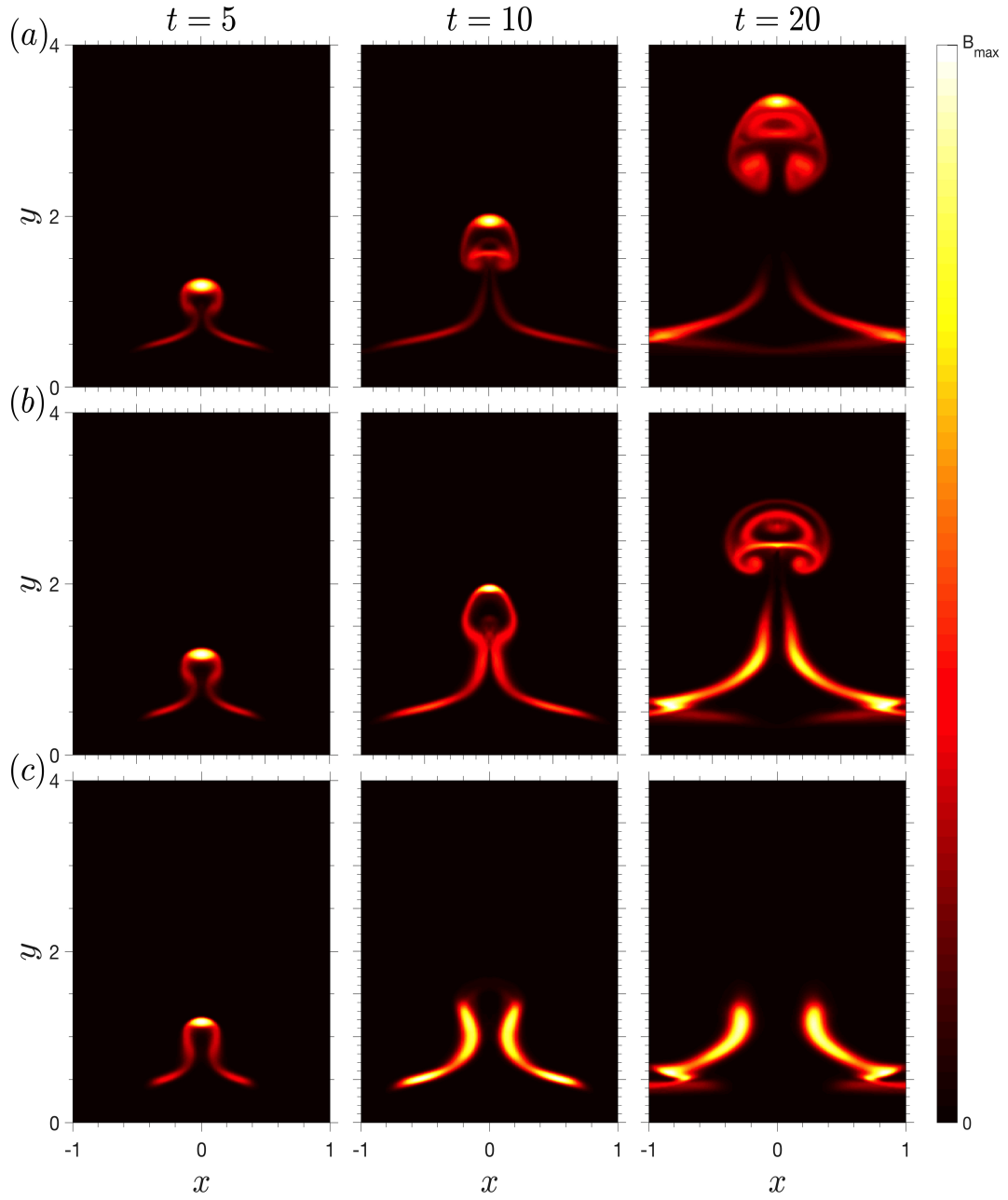


Figure 2.11: Effect of the background field scale-height variation on the dynamics. Intensity plots of normalized $B_z(x, y, t)$ for $H_B = 0.125, 0.175, 0.300$ (a, b and c respectively) when $B_s = 0.10$ and $q = 2.5$ in all cases. All the subplots are scaled with respective $\max(B_z)$.

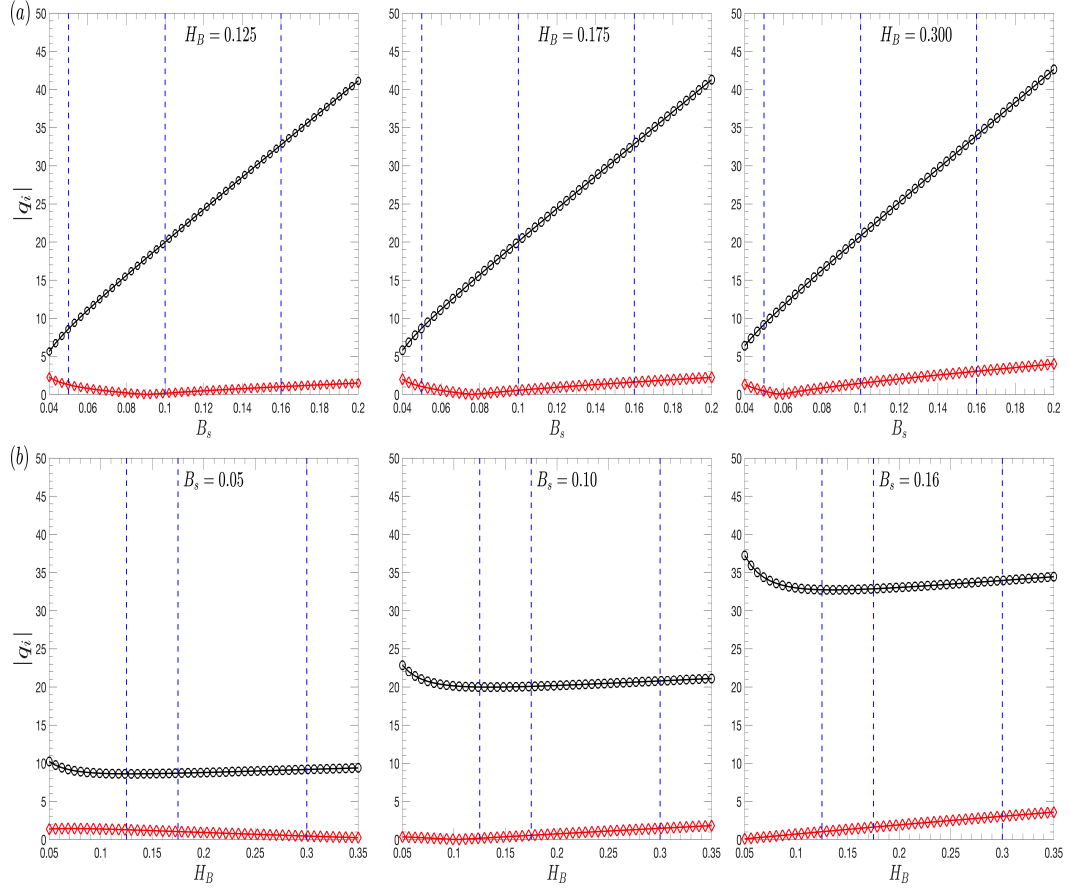


Figure 2.12: Absolute value of the roots of Equation (2.29). The region between the black circles and red diamonds is the SRR. (a) Variation of SRR for a fixed H_B as a function of B_s . Blue vertical dashed lines correspond to $B_s = 0.05, 0.10, 0.16$, which are values where we have comparable simulations. (b) Variation of SRR for a fixed B_s as a function of H_B . Blue vertical dashed lines correspond to $H_B = 0.125, 0.175, 0.300$, which are values where we have comparable simulations.

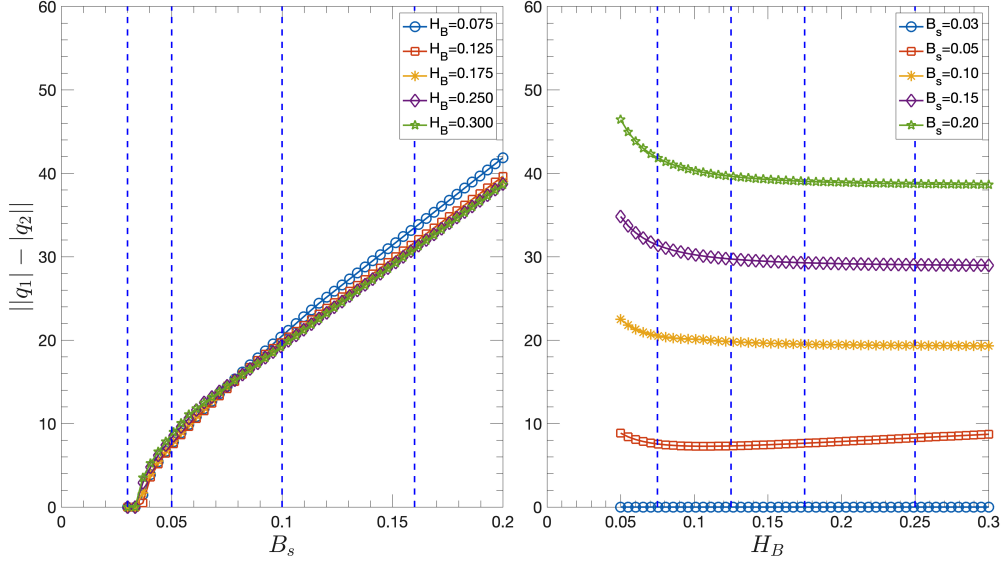


Figure 2.13: Width of the SRR, $||q_1|| - ||q_2||$ plotted as a function of (a) B_s for different H_B and (b) H_B for different B_s .

2.3.6 Dependence on the scale-height of the background field

We have now established the importance of the presence of the large-scale background field on the dynamics of a rising flux tube, concentrating so far on the relationship between the background field strength and the twist strength, and their relative orientations. We now examine the dependence on the configuration of the background field, which we have characterized as an exponential with scale height H_B . Figure 2.10 shows the variation of the background field, B_{Back} , as a function of height, y , for some example values of the scale-height, H_B , when $B_s = 0.1$. The dotted horizontal lines in the figure show the initial vertical location of the flux concentration. Recall that B_s specifies the field strength at the center of the tube so all these lines intersect at that point with value $B_s = 0.1$. This figure is intended to emphasize the fact that, when H_B is small, the background field variation across the flux tube is substantial, whereas for high values of H_B

the background field changes very little with height overall and hence its variation across the tube is small. Since this differential is key to the new dynamics discussed above, we might expect H_B to play a role.

Figure 2.11 shows intensity plots of the (normalized) axial field B_z at three different times in the evolution of system for three values of scale height, $H_B = 0.125, 0.175, 0.300$, whilst keeping the background field strength and twist of the flux tube constant (at $B_s = 0.10$ and $q = 2.5$). Overall, Figure 2.11 shows that the rise of the flux structure is quenched for higher H_B . Figure 2.11*a*, exhibiting the evolution for the canonical $H_B = 0.125$, shows that a typical flux structure – a “squashed” head with trailing vortices – rises through the background field towards the top of the simulation domain. Figure 2.11*b*, for $H_B = 0.175$, shows another overall successful rise, but this time there is considerable drainage of the axial flux B_z away from the tube head down the the overlaying background fieldlines, plus some significant change to the geometry of the head and the trailing vortices. Note that the flux tube is further from the top of the simulation domain at $t = 20$ in Fig. 2.11*b* and is therefore rising more slowly compared to the previous case, even though it appears to be still rising as a fairly independent structure. Figure 2.11*c* shows the case with an even larger scale height, $H_B = 0.3$. In this case, there does not appear to be any coherent rising flux structure.

The quenching of the rise of the flux structure with the increase in scale height of the background field can be understood from a physical perspective, with confirmation from our mathematical model. One can imagine that increasing the scale height of the background field could lead to three effects that could potentially affect the dynamics. Firstly, a larger H_B redistributes the background field in the tube and this affects the magnetic pressure that drives the rise of the tube. The analytical approximation in Equation (2.18) reveals that this dependence is quite

complex, being proportional to $H_B^2 I_1(R/H_B)$. This is a function that increases with H_B and therefore its contribution to the magnetic buoyancy is enhanced at larger H_B . However, as we discussed earlier, this magnetic pressure contribution is much smaller than the contribution from the initial density perturbation in the buoyancy term proportional to $g = \theta(m + 1)$. This first effect is therefore likely not very significant.

The second potential effect is that an increase in H_B leads to a decrease in the differential of the amplitude of the field across the tube vertically. It is this differential that drives the selection mechanism via its effect on the magnetic tension in the tube. Any effect (either increased likelihood of rise when B_s and q are of the same sign, or decreased likelihood when B_s and q are of opposite signs) is reduced for larger H_B . In the case shown in Figure 2.11, we might expect the tension-driven enhancement of the rise to be reduced by larger H_B . This expectation can again be examined with the analytical model, this time via Equation (2.22). There, it can be seen that the direction of the force depends on the sign of qB_s and also that the amplitude again has a complicated dependence on H_B : $F_{tension,y} \sim H_B I_1(R/2H_B)$. This dependence is almost constant, only very slightly decreasing over a significant range of H_B . This second effect is therefore again probably not very important.

The third effect remains as the likely major dynamical factor. Increasing H_B also serves to increase the integrated field that contributes to the retarding force of the overlaying background field. One might expect larger H_B to imply a larger value of the integrated field and therefore a large force countering the rise. Since this is an integral over the distance travelled upwards by the tube, this is probably a substantial effect. These ideas can again be verified by the analytical model, where this time the dependence can be seen from Equation (2.27) to be

straightforwardly linear: $F_{back,y} \sim H_B$. As mentioned previously, this estimate is also potentially an underestimate, with some significant factors related to other effects omitted. This also lends some credence to this being the dominant physical term here, although we do not know the H_B dependence of the other effects.

Having established some physical intuition, we can further employ the analytical model developed in Section 2.3.5 to analyze the broader dependence of the SRR on the scale height of the background field, H_B . Figure 2.12 plots the SRR from Equation (2.30) in $q - B_s$ space for fixed H_b (Figure 2.12a) and in $q - H_B$ space for fixed B_s (Figure 2.12b). The blue vertical dashed lines in each of the subplots show the values where numerical simulations exist for comparison with the model. For the values of H_B surveyed, by comparing the panels of 2.12a or examining an individual panel of Figure 2.12b at fixed B_s , we see that the demarcation (and therefore the extent) of the SRR (in $|q|$) only varies a very moderate amount. This is commensurate with our earlier conclusion that the tension effect that drives the existence of the SRR is only very weakly dependent on H_B . On the other hand, individual panels of Figure 2.12a or comparison of the panels in Figure 2.12b confirms that varying the background field strength, B_s , can significantly change the extent of the SRR for fixed values of H_B . This is all again clarified by the analytical model where the tube internal tension term in Equation (2.22) that establishes the SRR is linearly dependent on B_s , but is roughly constant with H_B (via the term $H_B I_1(R/2H_B)$). An important, and perhaps unexpected, result overall here is that an SRR still exists even for high H_B , although the q values at which it occurs become less physical.

It now becomes necessary to reconcile the above statements with the dramatic quenching of the rise of a structure with increasing H_B observed in Figure 2.11. First, it should be noted again that we expect realistic values of the twist q to be

of order $q < 10$. Higher values create unphysical (negative) values of the density (see horizontal dashed line in Fig. 2.9) and other work has shown that high q are unstable. For instance, the initial Gaussian profile for the axial field used in Section 5.2 would have a critical value of $q_{cr} = 1/R = 8$ for the flux tube to be kink unstable in three dimensions [73]. This restriction means that realistic flux structures probably lie much closer to the lower limit of the SRR than the upper. If q is such that it is close to the lower boundary, then even a small variation in H_B and a weak dependence of the SRR boundary on H_B can have a dramatic effect. This is essentially what was demonstrated through the simulations shown in Figure 2.11. The simulations in this figure are at $q = 2.5$ and $B_s = 0.10$, corresponding to the second blue dashed line in Figure 2.12a and the middle panel in Figure 2.12b. In the latter, for $q = 2.5$, the moderate upwards tilt of the lower boundary of the SRR (red diamonds) with increasing H_B shifts the solution from inside the SRR (for $H_B = 0.125$) to below the SRR (for $H_B = 0.3$), thereby switching it from a rising solution to a quenched solution. A small change in H_B can therefore have an effect due to the proximity of natural solutions to the lower boundary of the SRR.

To emphasize the above dependencies, Figure 2.13 plots the width of the SRR, $||q_1| - |q_2||$, as a function of (a) B_s , for a range of H_B , and, (b) H_B , for a wide range of B_s (as in the previous figure). These figures solidify our interpretation that the width of the SRR is only very weakly dependent on H_B but significantly dependent on B_s . Note again that at very low values of B_s (e.g. $B_s = 0.03$) there is no SRR since the differential tension force required to generate the necessary asymmetry for the SRR is negligible.

2.3.7 Multiple flux concentrations

With an understanding of the dynamics of a selection mechanism established for a single flux concentration, in this section we examine the collective effect that leads to the observations that are known as the SHHR. The SHHR observations accumulate the signs and strengths of the helicity of emerging active regions over the whole solar cycle (and eventually multiple cycles) to extract any net bias. The SHHR observations have a large “scatter”, in the sense that not all individual active regions agree with the rules – overall, only 60-80% of active regions appear to concur – and the degree of agreement appears to vary over the solar cycle [102, 98, 2, 10, 8, 9, 54, 56, 103]. The scatter and its temporal variation is not well understood. In this section, we therefore attempt to use our model to create synthetic SHHR observations with a view to potentially elucidating reasons for these effects.

To reproduce SHHR observations, we simulate the evolution of multiple (toroidal) flux structures through different background (poloidal) field strengths representing different parts of the solar cycle. We fix the values of the other parameters at the canonical values: $m = 1.5$, $H_B = 0.125$, $R = 0.125$, $\theta = 0.25$. For each field strength, we perform a series of Monte Carlo (MC) simulations where, in each simulation, a wide domain is initialised with a number of flux tubes with randomly assigned twist strength (both positive and negative) and horizontal location. We then examine whether the selection effects shown for single flux tubes leads to a persistent bias over the whole cycle (simulated by our range of background field strengths). It may seem likely that this is the case as we have identified a selection mechanism, but this needs to be confirmed, since this setup allows the possibility of new dynamics emerging from the interaction of tubes, and the randomness may potentially swamp any bias.

In order to have reasonably significant statistics for each case in this MC study, we perform fifty independent simulations (each containing multiple tubes) for each background field strength. Since we are not addressing the origin of the flux structures, our assumption is that they are formed from a sheet of toroidal field, likely by a magnetic buoyancy instability (see for e.g. [112, 53]). If this were to be the case, we might then expect multiple flux structures to be formed at the same time in a turbulent environment, and such structures would most likely possess random strengths, twists and separations drawn from some distribution. We therefore choose each of our simulations in the MC series for a particular background field strength to start with five flux concentrations with random initial twists ($-5 \leq q \leq 5$) and random distance, d , between them (with the constraint, $d \geq 2R$, so that they do not overlap), embedded initially at the same (canonical) height in the prescribed overlying large-scale background field. The random values are drawn from a uniform distribution over the range given. Our setup certainly has some significant degree of arbitrariness because of the limitations in our quantitative understanding of the amount of twist in flux tubes, the distance between them and their strengths, at their origination in the deep solar interior. For example, our choice of drawing the random values of twist from a uniform distribution might not be very realistic and a Gaussian with a larger range might be better (or perhaps even a skewed distribution to account for the effects in the formation of tubes not included in our model, e.g. rotation: see [25]). However, as mentioned earlier, there are some constraints on the value of $|q|$. Firstly, $|q| \gtrsim 12$ (the dashed line in Fig. 2.9, which is actually a very weakly decreasing function of B_s) is unphysical, since then the azimuthal field is sufficiently strong induce a negative density in the structure. Secondly, structures with $|q| \gtrsim 8$ are unstable to kink instabilities (according to [73], which does not fit our model exactly, but

is likely a reasonable guide). Therefore, our chosen distribution (uniform, $|q| \leq 5$) seems reasonable, in that it makes it likely that the initial simulation setup has a mixture of weak and reasonably strong concentrations of both helicities, and, if anything, overemphasizes the stronger twists (that are more likely to violate the SHHR). An MC series of this type (fifty simulations, five random tubes each) is reported for four different background field strengths, ranging from relatively weak to relatively influential ($0.01 \leq B_s \leq 0.10$). We identify the weak background field with the beginning and end of the cycle and the stronger background with the peak of the cycle, although we have no direct observational evidence from the deep solar interior of the relative strengths of these fields from which to set their values.

Fig. 2.14 shows the results of the MC simulations for each of the four different background field strengths. Each circle in the plot represents an accumulation over a specified time interval $[t_0, t_1]$ in the simulation of the absolute value of the z-component of the net horizontal current helicity measured along a line at $y = 3$. We call this quantity H_3 :

$$H_3 = \left| \int_{t_0}^{t_1} \int_{-1}^1 [B \cdot (\nabla \times B)]_z \, dx \, dt \right| = \left| \int_{t_0}^{t_1} \int_{-1}^1 B_z \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \, dx \, dt \right|. \quad (2.34)$$

This quantity is evaluated from the simulations as a sum over the x grid points at height $y = 3$ at a particular time, and over the data output in the time interval from $t_0 = 0$ to $t_1 = 70$. This quantity represents the magnitude of the net helicity of all the flux concentrations that successfully rise through 75% of the model solar interior in the simulation setup. The open and filled circles indicate the net sign, corresponding to net negative and positive helicity respectively. Each panel in the plot shows the 50 MC simulations for a given field strength, B_s , representing a given time in the solar cycle, plotted against the simulation number, which could

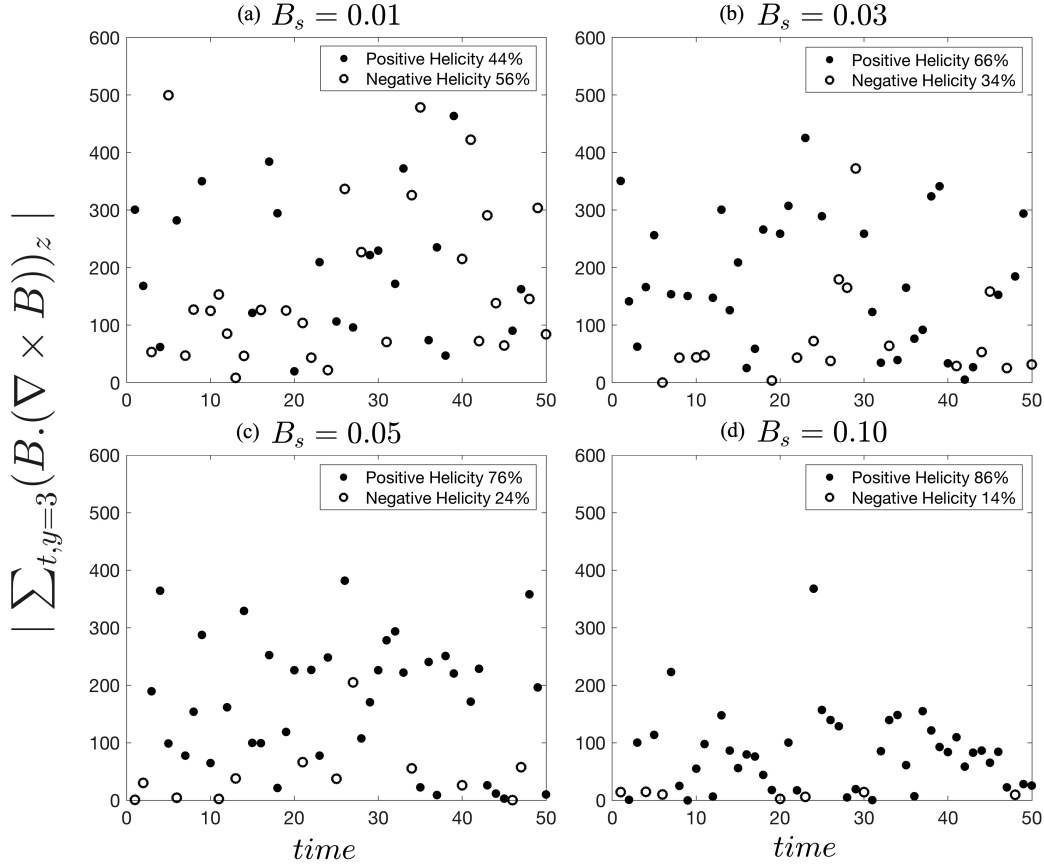


Figure 2.14: Synthetic SHHR. Each dot shows the sign (solid circle = positive; open circle = negative) and aggregated strength (vertical axis) of the horizontal component of the current helicity that passes through $y = 3$ during a the interval $t = 0$ to $t = 70$ from a simulation involving five flux structures embedded within the background field of strength B_s shown for each panel. The flux structures in each simulation have randomly assigned twist and separation from a distribution. Each panel contains 50 of such MC simulations, and represents a particular time in the solar cycle, with low B_s representing times around solar minimum, and higher B_s being associated with solar maximum. Percentages show the relative distribution of positive and negative helicity. Positive helicity is expected to dominate for the configuration.

be loosely interpreted as short time intervals at that point in the cycle, but are in reality just different realizations whose ensemble average becomes meaningful. The overall bias in the emerging helicity at any specific time in the solar cycle is therefore given by any statistically-significant deviation of the percentage of each sign of helicity away from a 50% – 50% balance. These percentage values are noted on the plots. Note that owing to the orientation of the tube’s axial field along the positive z direction and the background field along the the positive x direction, our setup corresponds to the the southern hemisphere in the first half of the cycle as shown in Fig. 5a, where positive helicity is expected to dominate by the SHHR. Furthermore, to relate to observations, we are assuming that some three dimensional process leads to arching of our simulated flux concentrations for emergence so that the axial helicity that we measure horizontally in 2D is strongly correlated with the vertical helicity component that emerges in the observations. We will test these assumptions in 3D in the future.

Fig. 2.14 displays a systematic trend in the relative percentages of the two signs of accumulated helicity with increasing background field strength. For a very weak background field strength of $B_s = 0.01$ in Figure 2.14a, we find that the surviving and accumulating helicity at $y = 3$ is roughly equally divided amongst both signs of helicity. There is a slight bias towards an anti-SHHR result (more negative helicity) as has actually been observed in the solar case, but we do not dwell on this as this seems fairly statistically insignificant here, since the number of MC simulations is only 50. At higher background field strengths (which we relate to closer to solar maximum in the the solar cycle), we find results that are definitely commensurate with the SHHR. At $B_s = 0.03$, we find that around 66% of the successfully rising flux concentrations possess positive helicity; at $B_s = 0.05$ we find that 78% are positive; and at $B_s = 0.1$, 86% are positive. These results are

in agreement with the overall SHHR trend, which is that positive helicity should generally be preferred in this Southern hemisphere setup (and, of course, the results for the Northern hemisphere can be inferred by symmetry). Furthermore, these results also agree with the observed trend that there are fewer violations to the SHHR (i.e. better agreement) around solar maximum and more violations (worse agreement) around solar minimum [54]. Note that we correlate a stronger deep interior background poloidal field in our model with solar maximum and a weaker poloidal field with solar minima. Since we do not know the deep interior fields, we rely upon mean-field dynamo models to justify this (see e.g. [23] and references there-in). Many such models exhibit that the poloidal and toroidal fields are reasonably in phase, and the latter are widely used as a proxy for the butterfly diagram.

These major results can be interpreted fairly straightforwardly in the light of the work in Sections 2.3.4 and 2.3.5, and, in particular, the $q - B_s$ diagrams from simulations (e.g. Fig. 2.7) and the model (e.g. Figs. 2.9, 2.12 and see also Fig. 2.13). In general, these plots show that there is only a narrow SRR at weak B_s but that the SRR grows wider in extent in $|q|$ as B_s increases (a robust trend, even though the data and the model do not agree precisely on location of the boundaries of the SRR due to the inaccuracies of the model, as discussed earlier). Violations of the SRR would mainly be detected when the twist of a structure is strong enough for it to lie above the SRR (although this is not the only possibility, as discussed shortly). Since there are good physical reasons for the value of $|q|$ to be capped above, clearly, where the SRR is narrower at lower B_s , more violations might be expected to occur, since our distribution of q in the MC simulations is more likely to place them above the SRR region (and below the cap). In the limit of the SRR vanishing, all structures lie outside the SRR

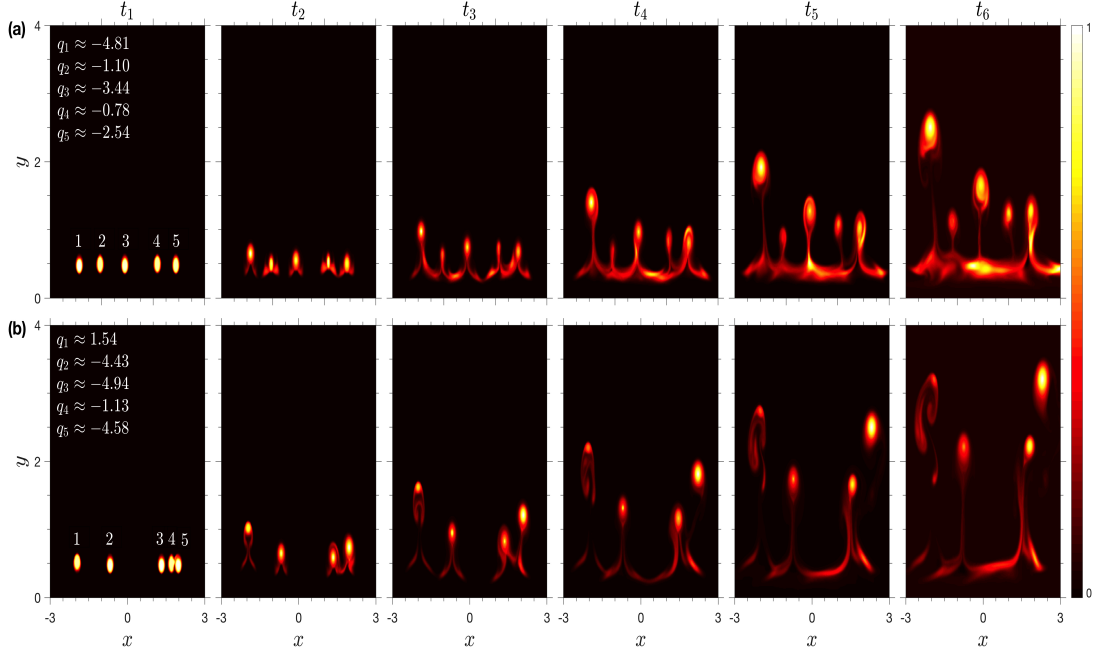


Figure 2.15: Solar Hemispheric Helicity rule violations. Both panels show intensity plots of normalized $B_z(x, y, t)$ at $(t_1, t_2, t_3, t_4, t_5, t_6) = (1, 11, 21, 31, 41, 51)$. Panels a and b are independent instances of the MC simulations at $B_s = 0.05$, chosen to depict particular SHHR violation mechanisms. With the axial field, B_z , out of the plane and poloidal background field aligned in the positive x-direction, this setup is representative of the solar southern hemisphere (see Fig. 2.6a). The sign and strength of the field line twist (q) at t_1 is indicated for all the concentrations, with the subscript corresponding to the numbering of the concentrations along the x-axis. Panel (a) shows violations occurring due to the rise of strongly twisted tube with negative twist. Panel (b) shows a violation occurring due to the interaction of two negatively twisted tubes.

and either don't rise, or are equally likely to rise, therefore yielding percentage values for the helicity sign distribution close to 50%-50%. As the SRR becomes wider at higher B_s , the number of violations drops as q values drawn from the distribution become less likely to lie above the SRR. This leads to more complete compliance with the SRR and a ratio of the emerging helicity signs that would tend towards 100%-0% in favor of the sign of helicity expected from the SHHR (positive in our example cases). Indeed, for higher B_s ($B_s \gtrsim 0.06$ in Fig. 2.9), the cap of our distribution ($|q| = 5$) lies below the upper bound of the SRR from both the model and data, and almost 100% agreement would be expected, potentially violated only by very low $|q|$ lying in the small region below the lower bound of the SRR (red diamonds in the model). Structures with such sub-SRR twist are only expected to rise when $B_s \lesssim 0.09$ (where both roots q_1 and q_2 are negative); violations are not expected for sub-SRR values of twist for $B_s \gtrsim 0.09$ since both signs should not rise, but interestingly in this region another method of violation (discussed shortly) operates, reducing the agreement with the SHHR from 100%. Regardless of these details, the trend to fewer violations for stronger B_s seems very robust and explainable by the selection mechanism found here.

The above result, related to the expected degree of scatter in the observations and its dependence on the solar cycle, is a reflection of how violations to the SHHR are achieved in the MC simulations. We therefore investigate the causes of these violations in more details now. To exhibit the mechanisms we find, we focus on the cases with $B_s = 0.05$ and 0.10 . We find three distinct, relevant reasons for violations which are discussed below:

1. Strongly twisted flux tube: The first, and main, reason, mentioned previously, is simply the existence of strongly twisted flux concentrations of the “wrong” sign (where, here, “wrong” means that the sign of helicity

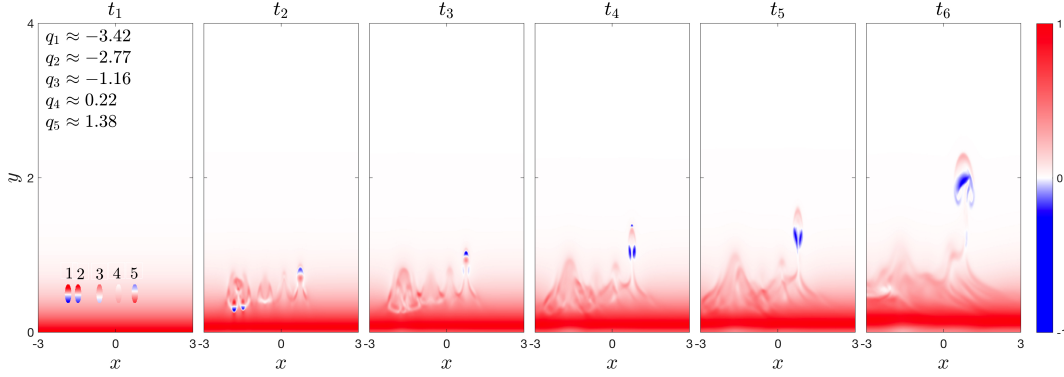


Figure 2.16: Violation of the SHHR via restructuring of the twist. Panels show normalized $B_x(x, y, t)$ for $B_s = 0.1$ at $(t_1, t_2, t_3, t_4, t_5, t_6) = (0, 5, 9, 15, 20, 40)$. Tube 5 has anti-clockwise (positive) twist at $t = t_1$ but gets rearranged to a clockwise (negative) twist by $t = t_5$. This mechanism is quite common for strong background fields and weakly twisted structures.

of the structure violates the SHHR; in the cases shown here, the “wrong” sign is negative since the SHHR would predict predominantly positive helicity). If the structures are sufficiently strongly twisted to lie above the SRR, then these structures will rise. As proof of this from the simulations, Figures 2.15*a, b* show examples of the time evolution of normalized axial magnetic field strength, B_z , from the MC simulations performed with $B_s = 0.05$. The twist sign and strength of individual flux concentrations are shown, numbered by their relative position along the horizontal axis at $t = t_1$. Fig. 2.15*a* shows a case where the randomly-assigned twists of all the flux concentrations in the simulation are negative (clockwise) and therefore not what would be expected from the SHHR. However, tube 1 has almost the maximum twist allowable from our distribution and we see that at $t = t_6$, this tube has risen significantly through the model solar interior and is approaching the $y = 3$ level where we measure the net helicity. Note that tubes 3 and 5 are behaving similarly, but to a lesser extent. From Figures 2.7 and 2.9 it can be seen that these high values of $|q|$ lie outside

the numerically-identified SRR (but inside the potentially inaccurate upper boundary of the model SRR) and therefore might be expected to rise. This is the main source of scatter, and was predicted from our single structure simulations.

2. Tube-tube interaction: Fig. 2.15*b* exhibits the second class of violation, which is a case where interactions between multiple structures do play an important role. Of the five flux concentrations in this figure, four of them have $q < 0$ and the remaining one (q_1) has $q > 0$. The first three, q_1, q_2, q_3 , behave as one might expect: q_1 is moderately strong and positive and therefore from being in the SRR, it is expected to rise; q_2 and q_3 are strong but negative and therefore may still rise since they are above the SRR (by the previous method of violation). The other two, q_4 and q_5 , are very close together, thanks to the use of random distances between concentrations in these MC simulations. The dynamics of the tubes then are no longer those of individual single tubes as interactions between structures become important. Since both these concentrations have the same twist, they coalesce very quickly and form a single, more strongly buoyant region. This structure is still negative but has effectively more twist and more buoyancy, and therefore rises more quickly, becoming a violation of the previous kind due to its strong effective twist. This shows that separation of the structures can be an important element of the dynamics, as the original twist distribution alone would not have predicted the resulting outcome.

3. Flux tube twist rearrangement: A third scenario that can lead to violations of the SHHR is particularly peculiar and results from the conversion of weak “correct” signed structures into the “wrong” sign during the rise. An example is shown from one of the MC simulations for the case of $B_s = 0.1$

in Fig. 2.16. These types of violations are quite common at higher B_s and account for the majority of the violations shown in Fig. 2.14*d*. Fig. 2.16 shows the evolution of B_x , a clear indicator of twist of the tube, in this example. The first three of the structures initially have negative twist, and the remaining two are positive. The first three tubes do not rise successfully since they have only moderate twist strength lying in the SRR and are therefore filtered out by the selection mechanism. Tube 4 has positive twist but is too weak to rise (lies below the SRR). Tube 5 is the peculiar case. This structure has an intermediate positive twist and is expected to rise from consideration of the SRR. However, this structure undergoes a topological arrangement early in its transit, flipping the sign of its twist (visible in the plot as a flip from blue over red in the B_x field to red over blue). The exact mechanism for the flip is a little difficult to discern, but appears to be facilitated by the interaction of the strong background field wrapping around into the trailing vortices of the original structure. The structure continues to rise despite this rearrangement, and therefore emerges with the “wrong” sign.

Overall, these MC simulations have revealed the perhaps expected result, that the SRR mechanism can be responsible for the aggregated helicity bias that is the observed SHHR, but has further identified good reasons for the scatter in the observations and its temporal variation, some even beyond explanation by our concept of the existence of an SRR.

2.4 Discussion

In this chapter, we have examined the rise of a tube-like concentration of magnetic flux in the presence of a large-scale background field. Our model is aimed at elucidating the processes which bring magnetic structures from the deep solar interior towards the solar surface, where observations are taken. The core of our work has been to relax some of the constraints of previous models. Much intuition has been gleaned from vastly simplified models using either the thin flux tube approximation or finite cross-sectional tubes, but in general, these studies have examined the evolution of preconceived cylindrical magnetic structures that are isolated, in the sense that they are embedded in a field-free background. These simulations do not address the origin of such structures. Studies of magnetic buoyancy instabilities that do try to examine this question show that the resultant structures are concentrations of strong magnetic field amongst a volume-filling, weaker, large-scale background field. Our premise has been that the dynamics of concentrations might be substantially different from isolated structures. At this stage, we have examined the most obvious extension of previous models, where we embed the preconceived tube-like structures from before in a large-scale background field, to convert it from an isolated structure into a concentration. We continue to ignore the origin of these fields and the dynamics of convection in the rise of the concentration, leaving these added complexities for later studies.

We indeed succeed in showing that relaxing these assumptions can lead to drastically different rise dynamics of the magnetic flux structure. In particular, our results reveal that the rise of the flux structure is (perhaps not surprisingly) impeded by the background field, although unexpectedly it can be completely quenched by a relatively weak background field (a factor of 10-20 weaker than the peak tube strength). This quenching is achieved by a combination of (i)

tension induced when stretching out the overlaying field as it is lifted by the rising structure, (ii) a drainage of the axial flux that supplies the buoyancy of the structure out along the overlaying background field lines, and, (iii) suppression of the trailing vortices that self-propel the structure after the initial buoyant rise.

Much more surprising is that the strength of the background field (B_s) required to quench the rise of the magnetic structure is dependent on the relative orientation of the twist of the structure (q) and the background field. When the background field is aligned with the azimuthal field (B_θ) at the bottom of the structure, a net tension force is created inside the tube that is directed upwards, complementing the buoyancy force driving the rise. When the background field is aligned with azimuthal field at the top of the structure, the net internal tension force points downwards, complementing the forces that are trying to retard the rise of the tube. Since this mechanism depends only on the relative orientation of the twist of the structure and background field, this mechanism can be thought of as one that either (a) for a structure of fixed twist, enhances the likelihood of the rise of the structure for one particular orientation of background field, or (b) for a fixed background field, enhances the likelihood for rise of structures with a certain twist. Note that, given the limited knowledge of the field strengths in the deep interior of the Sun, for this work, we have little guidance on our choice of B_s which sets the relative strengths of the background poloidal field to the axial field of the tube. However, our experience with instability simulations (e.g. [112]) suggests that rising structures are not many orders of magnitudes stronger than their originating field, providing hope that the range of B_s where we find interesting dynamics is realistic.

The latter point of view more easily leads to the conclusion that this selection mechanism is commensurate with the SHHR. In the solar context, the axial field

direction and the overlaying poloidal field are not independent, and this connection (via the differential rotation), allows our mechanism, which depends on twist only, to be cast as one that depends on helicity. When applied to the solar context, our mechanism selects the correct helicity as the preferred helicity in each hemisphere to concur with the SHHR (negative helicity in the Northern hemisphere, positive in the Southern). This preference is independent of which half of the full 22-year solar cycle is examined, since the orientation of both the overlaying background poloidal field and the toroidal field providing the axial field of the structure flip when switching after 11 years. This mechanism therefore appears to be a reasonable candidate for the origin of the SHHR since it agrees with this major component of the rules.

Interestingly, the mechanism that we have discovered provides explanations for many of the further details of the SHHR. In particular, our mechanism can also (i) explain the fact that the SHHR is only a weak rule, in the sense that only 60-80% of active regions obey the rule, and therefore 20-40% are in violation, and, (ii) provide a plausible reason for the observed increased disparity in the rule around the period between cycles. Indeed, expanding on the latter, our model provides a prediction for the dependence of the adherence to the main SHHR over the cycle. These further affirmations of SHHR characteristics are gleaned from a deeper examination of the mechanism, which was the focus of much of the rest of the chapter.

To facilitate this understanding, we explored the reason behind these dynamics in detail via an analytical model of the forces acting on the magnetic structure. The success of an attempted rise depends on the internal magnetic buoyancy and tension forces of the structure as modified by the existence of the background field, as it competes with the induced tension force in the overlying large-scale back-

ground field created by the rise through that field. The effect of the introduction of large-scale background field on the flux tube is to adjust the overall buoyancy and internal tension forces in such a way that it can either facilitate or hinder its rise. Our mathematical model confirms that the predominant adjustment comes from the internal tension force, which we analytically show to be dependent on the product of q and B_s , thereby revealing again the dependence on the alignment (i.e. sign) of these two entities: having the same sign provides a positive (upward) tension force that encourages rise, whereas opposite signs create a negative tension force that counters the rise. Setting the net force balance of these three forces to zero provides a quadratic equation (2.29), whose roots define a regime where the selection mechanism operates. We call this region in parameter space the Selective Rise Regime (SRR). The region depends on the parameters q , B_s and H_B (the twist, the strength of the background field and the scale height of the background field), which we consider to be the main parameters of our studies; there are other parameters that we have fixed at canonical values. It turns out that the dependence on the configuration of the background field via H_B is weak, and therefore the SRR can be considered as mainly a region of $q - B_s$ parameter space, defined by the modulus of the roots of the characteristic quadratic equation (2.29). Between the (modulus of the) two roots is the SRR, where the selection mechanism operates, and one sign of twist rises preferentially over the other. Above the largest root, both signs of twist will rise successfully. Below the smallest root, both twists can rise or neither twist may rise depending on the nature of the roots, but structures of both signs behave similarly.

The analytical model SRR exhibits the general characteristics of the $q - B_s$ dependence of the simulation data, and clarifies the existence of the selection mechanism as a direct result of the asymmetry of the internal tension force in

the magnetic structure. The quantitative agreement of the analytical SRR and the selection regime found from the simulations is not perfect, mainly because estimation of the retarding effect of the tension from the overlying field is difficult, and we do not account for other retarding effects, such as drainage of the axial flux driving the buoyancy and the destruction of the trailing vortices that aid the rise. However, the analytic model and simulation results do, in tandem, provide an explanation for why violations of the SHHR that induce “scatter” in the observations, might be expected. The main reason for this is that structures containing twist of either sign whose strength is above the upper bound of the SRR (for a given B_s) will rise successfully. If it is expected that the twist of the magnetic structures arises randomly in the structure generation process, then some distribution of twist might be also be expected that includes strong values of either sign whose moduli lie outside the SRR. These values would not have the bias of the SHHR and would induce violations in the observed data.

In order to test these ideas, we carried out a MC study with the aim of synthetically reproducing the nearest equivalent of SHHR observations from our simulations. For a given B_s , we computed a significant number of simulations containing multiple magnetic flux structures with randomly assigned twists (and spacing) from a reasonably wide uniform distribution. From these, we calculated the sign and strength of the net helicity that “emerged” near the top of our simulation box. The relative fractional distribution of these signs (and strengths) mimic the observations that lead to the SHHR at that particular B_s . We associated the chosen B_s with a time in the solar cycle, with small B_s representing solar minimum and large B_s being related to solar maximum. We find that for low B_s , the fractional distribution of the signs of helicity at emergence is statistically close to 50%-50%. This fractional distribution increases as B_s increases, up to around 85%-15% in

favour of the sign expected from the SHHR for the largest B_S computed. Overall, this is in good agreement with the SHHR observations, where concurrence with the rule occurs for 60-70% of the total active regions over the whole cycle. The violations in our simulations can be checked to confirm that they mainly come from values of the twist that lie outside the SRR, although other odd effects from structure interactions can also occur in these multiple tube simulations.

A nice quality of these MC simulations is that they provide a prediction for the temporal evolution of the agreement with the SHHR, via the fractional distribution of the signs. The distribution depends on B_s (which we associate with epoch in the cycle) since the extent of the SRR (in $|q|$) depends on B_s , widening as B_s increases. This means that it is increasingly difficult to violate the SHHR (i.e. possess twist that is both below any physical cap and large enough to fall above the upper SRR boundary) as B_s increases, and hence the rule is obeyed more completely at higher B_s (note that this assumes that other effects at very small twist are less important). With SHHR observational data collected over more solar cycles, this prediction could be tested.

Various other models [75, 29] have attempted to address the issue of the origin of SHHR, as described in the introduction, and here it is pertinent to describe the main differences in the results from our model and the others, and what each can contribute to our understanding. A major distinction between our model and most others is that the magnetic flux structures in most other models are initially untwisted and then acquire the appropriate twist by some mechanism during transit, whereas in our model, all flux structures possess some twist initially (of random size and sign) and then the structures with the “correct” twist are selected for rise by a filtering mechanism. The models acquiring twist either do it directly, by rising into and then merging with other field (e.g. [29]), or indirectly, by

acquiring writhe from dynamics influenced by the Coriolis force which then, by helicity conservation, induces a compensatory twist of the “correct” sign (e.g. LFP). A big difference between the models is then the degree to which they explain the scatter in the adherence to the SHHR. For example, the Choudhuri model, as presented, predicts 100% agreement with the SHHR, except perhaps at solar minimum. The Σ mechanism of LFP, however, does produce a scatter since the twist is generated (indirectly) from interaction with the convective turbulence, and therefore a distribution of twist will result. It is worth noting that most of these previous models assume that the magnetic flux structures have zero twist initially, which seems unlikely. Allowing an initial distribution of twists (as in our model) would actually provide another source of scatter in the resultant twist observations in these models too.

The different models also provide different results for the solar cycle dependence of the SHHR, in particular, with respect to the cycle dependence of the scatter of the observations and the anti-SHHR behaviour observed at solar minimum [102, 98, 2, 10, 8, 9, 54, 56]. For example, our model predicts that agreement with the SHHR increases in the rise to solar maximum and decreases towards solar minimum, whereas the model of LFP predicts no variation in the agreement over the solar cycle. Our model and Choudhuri’s model both predict anti-SHHR behaviour in the transition between cycles if the switch of polarity of the poloidal and toroidal components of the dynamo field are slightly out of phase. The model of LFP does not know about this phase information and depends only on the influence of rotation on convection which is cycle independent.

All these mechanisms are complementary in some sense, and could all be contributing to the ultimate helicity content of rising structures and therefore the SHHR observations. One can imagine a scenario where our model is dominant

near the base of the convection zone, LFP’s model operates in the bulk of the convection zone, and Choudhuri’s model contributes near the surface. Our model is a very complete one, in the sense that it individually not only explains the preferred helicity of active regions, as required by the SHHR, but also the scatter in the observations, and possibly other elements, such as the anti-SHHR behaviour at the beginning of the cycle. We have already begun full 3D simulations to confirm our mechanism in that context. In future work, we will expand these to simulations that include a rotationally-influenced convective zone through which the structure must transit. In this manner, we will be able to examine the contributions from the various mechanisms, whilst moving away from the concept of isolated (and thin) flux tubes, as clearly the dynamics of magnetic flux concentrations are undeniably different.

One extra highly conjectural but fascinating possibility from our model is that it could potentially offer an indirect understanding of the elusive magnetic nature of the solar interior. Since the degree of agreement with the SHHR (the scatter fraction) is significantly dependent on the strength of the interior field (B_s) in our model (but only weakly dependent on the configuration, H_B), the exact observed fractional degree of scatter might provide a probe into the interior field strength at least, if not the vertical variation of the field. We therefore look forward to improved multi-cycle observations of the SHHR.

Chapter 3

Magnetic Flux Concentrations in a Background Field: Convective simulations

The following chapter contains work and text from Manek, Pontin & Brummell (2021), a paper in preparation to be submitted to The Astrophysical Journal. This work was initiated as a three-dimensional simulation project for this thesis, but faster progress was made with a two-dimensional version with Christina Pontin as part of a project for the Kavli Summer Program in Astrophysics (2021). Hence this is the more complete version and, therefore, the one described here. The status of the three-dimensional work is described briefly in an Appendix.

3.1 Introduction

The previous chapter dealt with the rise of flux tubes through an artificially imposed background field in a quiescent background. While this gave interesting insights into the rise dynamics of the flux tubes and the consequent selection

mechanism arising out of it, the initial state is highly idealized. In this Chapter, we add another level of realism to our simulations. In a realistic solar scenario, and based on our current understanding of the solar dynamo theory, we expect the flux tube to rise from the deep solar interior (radiative zone or tachocline) through the turbulent convection zone to finally emerge at the solar photosphere and corona. A comprehensive flux tube evolution model from the deep interior of the Sun to the solar surface is challenging to achieve, owing to the broad length and time scales involved in the dynamics. This chapter focuses on simulating the dynamics in the upper radiative zone and the lower to middle convection zone. To achieve this, in this chapter, we discuss simulations of rising flux tubes in a two-layer system - a convectively unstable layer sitting atop a stable region - with a self-consistent background field formed due to the action of convection on large-scale magnetic fields.

The premise of the dynamo theory on which Chapter 2 was based was that the spatial location of the toroidal magnetic flux formation was widely separated from where the observations were made. This necessitated a transport mechanism, magnetic buoyancy, that can transfer magnetic flux from the deep interior towards the surface. The introduction of turbulent overshooting convection to our simulations, as is the case in this chapter, leads to another transport mechanism that can counter buoyant transport by driving magnetic flux from the convection zone to the stably stratified layer below. The non-diffusive part of this transport, buried in the inductive term of the induction equation, is understood intuitively, mathematically and numerically by a number of different phenomenologies, and a number of different names, for example, flux expulsion, turbulent pumping, topological pumping, the mean-field γ -effect, diamagnetic pumping, etc.

Much two- and three-dimensional work has been done in the past studying

these turbulent pumping mechanisms. Drobyshevski and Yuferev (1974) [35] first invoked the idea of “topological pumping” in three-dimensional convection with a specific flow field pattern. They studied laminar flow patterns such that individual convective cells have upflows at the center and downflows at the edges (as observed in solar surface convection). They argued that downflows at the edges of the convective cells form an interconnected network that can transfer large-scale fields. In contrast, the isolated upflows at the center of the convective cell are only good to move small-scale magnetic fields. Arter, Proctor, and Galloway (1982) [6] and Arter (1983) [5] carried out three-dimensional calculations, which show that the pumping mechanism operates at higher magnetic Reynolds numbers, even though the exact working of the mechanism differs from Drobyshevski and Yuferev (1974). While these models give an insight into the pumping mechanism, the more simplistic flow profiles do not lead to very efficient transport of magnetic fields. Separate from the imposed flow fields with simplistic profiles, Tao, Proctor, and Weiss [106] carried out two-dimensional simulations to study the temporal expulsion of magnetic fields under a flow field driven by random forcing. This was motivated by the original work of Weiss [116], which for the first time explored the time-dependent nature of flux expulsion. Hurlburt et al. (1994) [63] carried out two-dimensional, fully compressible convection simulations to study the pumping process and gave scaling relations for the overshooting depth and the relative stability of the stable region, S .

A series of three-dimensional simulations compressible convection simulations have been studied by Tobias, Brummell, Clune, and Toomre [108, 110, 111] that addresses a more efficient, turbulent pumping mechanism. The authors considered a two-layer system, with turbulent overshooting convection sitting on top of a stable region, and studied the transport of a large-scale magnetic layer via com-

peting mechanisms of magnetic buoyancy and turbulent pumping. They found that, in these compressible simulations that have substantial asymmetry between upflows and downflows imposed by the stratification, a magnetic layer imposed in the convection zone gets transported by coherent downflow plumes, and eventually forms a large-scale layer with increasing magnetic energy. These authors investigated the dependence of this pumping mechanism on the many parameters of the problem in detail, noting that this turbulent mechanism was very robust.

Beyond simulations, these anisotropic turbulent pumping mechanisms have also been described in mean-field models by parameterizing the effect of small-scale velocity and field correlations in the electromotive force (EMF) on the mean-field. The isotropic part of the leads to the well-known α -effect, but the off-diagonal parts lead to terms that look like a transport velocity down the gradient of turbulent intensity [67, 66, 93, 16].

Regardless of characterization, here we want to examine the competition between the two transport effects, magnetic buoyancy and turbulent pumping, and any possible consequences on the SRR. To achieve this, we carry out a set of preparatory studies involving convection and magnetic flux tubes, as described in the following sections. The chapter is organized as follows: Section 3.2 describes the basic equations and how the two-layer formulation is implemented. Section 3.3.1 describes simulations of turbulent convection in the two-layer system alone in a purely hydrodynamic calculation. Section 3.3.2 explores the conditions under which *isolated* flux tubes can rise successfully through this turbulent convection (without any background field) in the MHD system. Section 3.3.3 considers different initial conditions in the MHD system in order to examine magnetic pumping in the two-layer system,. We carry out pumping calculations along the lines of [111], albeit in two dimensions, and show the self-consistent formation of a large-scale

magnetic layer. Section 3.3.4 combines all the previous sections and explores the rise of magnetic flux tubes through a self-consistent background magnetic layer in the presence of turbulent convection for a set of canonical parameters. Section 3.3.5 focuses on varying the strengths of this background magnetic layer and its consequences on the rise dynamics of the flux tube, essentially exploring the existence of a SRR in this setup. The presence of turbulent convection serves to introduce some inherent fluctuations in our model setup that can affect the initial conditions through which the flux tube is initiated. Therefore, Section 3.3.6 explores the robustness of all the results in the previous sections of this chapter, taking into account the random initial conditions that flux tubes may encounter via a suite of MC simulations. Section 3.3.7 examines the reasons behind the results of this chapter and compares with the ideas of the previous (non-convective) Chapter. Section 3.3.8 focuses on the effect of varying the main parameter (S) that controls the overshooting present in the two-layer system. Finally, we summarize the results in Section 3.4 and discuss the salient features and caveats of the work presented in this chapter.

3.2 Model and Methods

This study extends the work of Chapter 2. As in those earlier works, this chapter considers the evolution of a flux concentration in a large-scale background field. However, this work is substantially different in two main ways. Firstly, the concentration (often referred to as a “flux tube” merely for convenience) evolves by buoyancy through two vertically-stacked regions. The flux concentration begins in a deeper layer that is initially convectively-stable (representing a radiative zone, or a tachocline), and then, as it buoyantly rises, it transits through a convection zone. This is in contrast to the previous work where the buoyant rise only transited

through a single quiescent adiabatic layer. Secondly, the presence of this two-layer system of overshooting convection serves to generate a self-consistent distribution of the large-scale background field by magnetic pumping. In the previous works of Chapter 2, the profile of the background field was merely imposed artificially.

We use the formulation used in [111] to establish two layer overshooting convection with a large-scale background field. In our initial study of the problem here, we consider a 2-D Cartesian domain (x, z) , that contains a fully compressible ideal gas confined between two horizontal, impenetrable, stress-free boundaries. We keep all three components of the velocity and magnetic vector fields, but all components are independent of the missing third direction (i.e. $\partial/\partial y = 0$). This type of setup is often referred to as 2.5D. The Cartesian box is $x_{\text{m}}d$ wide and $z_{\text{m}}d$ deep, where d is the depth of the convection zone in the two layer system. We non-dimensionalize our system using d , T_0 (the temperature at the upper boundary), ρ_0 (the density at the upper boundary) and B_0 (some measure of the initial field strength) as our units of length, temperature, density and magnetic field strength. The thermal sound crossing time at the top of the domain, $(d^2/((c_p - c_v)T_0))^{1/2}$, is our unit of time, where c_p and c_v are the specific heats of the fluid at constant pressure and constant volume respectively (and their ratio $\gamma = c_p/c_v$ will be used later). With these units, the governing non-dimensional equations (the conservation of mass, momentum and energy, the equation of state for a perfect gas, the induction equation and the divergence-free condition for magnetic fields) are [111]

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{U}) = 0, \quad (3.1)$$

$$\partial_t (\rho \mathbf{U}) + \nabla \cdot (\rho \mathbf{U} \mathbf{U} - \alpha \mathbf{B} \mathbf{B}) = -\nabla p_t + \sigma C_k \left[\nabla^2 \mathbf{U} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{U}) \right] + \rho g \hat{z}, \quad (3.2)$$

$$\partial_t T + \nabla \cdot (\mathbf{U}T) + (\gamma - 2)T \nabla \cdot \mathbf{U} = \frac{\gamma C_k}{\rho} \nabla \cdot (\kappa_z \nabla T) + \frac{\zeta C_k \alpha (\gamma - 1)}{\rho} |\nabla \times \mathbf{B}|^2 + V_\mu, \quad (3.3)$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{U} \times \mathbf{B}) + C_k \zeta \nabla^2 \mathbf{B}, \quad (3.4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3.5)$$

$$p_t = p_g + p_m = \rho T + \alpha \frac{|\mathbf{B}|^2}{2}. \quad (3.6)$$

Here, in nondimensional form, $\mathbf{U} = (u, v, w)$ is the velocity, $\mathbf{B} = (B_x, B_y, B_z)$ is the magnetic field, T is the temperature and ρ is the density. Note again that all quantities are only functions of (x, z) (where z varies downwards from the top). The total pressure, p_t , is the sum of the gas pressure, p_g ($= \rho T$ for an ideal gas), and the magnetic pressure, $p_m = \alpha |\mathbf{B}|^2/2$, where $\alpha = \sigma \zeta Q C_k^2$ (where the latter parameters will be explained shortly). The rate of viscous heating is $V_\mu = (\gamma - 1) C_k \rho \sigma \partial_i u_j (\partial_i u_j + \partial_j u_i - (2/3) \nabla \cdot \mathbf{U} \delta_{ij})$.

Our two layer system consists of a convective layer (layer 1) overlying a convectively-stable layer (layer 2). This layering is enforced by a piecewise constant thermal conductivity (with a narrow smoothed junction between the two layers) that defines the two layers as being piecewise continuous polytropes when in a hydrostatic state. Since the total hydrostatic heat flux through the domain must remain the same at any depth, the thermal conductivities, κ_i , in the two layers can be described by the parameter $S = (m_2 - m_{\text{ad}})/(m_{\text{ad}} - m_1)$, related to the polytropic indices in the two layers, m_i , and the adiabatic index $m_{\text{ad}} = 1/(\gamma - 1)$

(= 1.5 for an ideal monatomic gas where $\gamma = 5/3$):

$$\frac{\kappa_2}{\kappa_1} = \frac{m_2 + 1}{m_1 + 1} = \frac{S(m_{\text{ad}} - m_1) + m_{\text{ad}} + 1}{m_1 + 1}. \quad (3.7)$$

We choose $m_1 = 1$ always, and specify the temperature gradient in the hydrostatic upper layer as θ . The relative stability of the two domains is then measured by S , often referred to as the stiffness parameter ([63, 19]). Increasing S increases the relative stability (“stiffness”) of the lower layer. Figure 3.1 shows the polytropic temperature and density profiles versus depth in the two layers for the three values of S that we use in these simulations, $S = 3, 7$ and 15 .

The other dimensionless parameters that govern the system are as follows. The Rayleigh number,

$$Ra(z) = \frac{\theta^2(m_i + 1)}{\sigma C_{k_z}^2} \left(1 - \frac{(m_i + 1)(\gamma - 1)}{\gamma} \right) (1 + \theta z)^{2m_i - 1}, \quad (3.8)$$

is a derived measure that evaluates the competition between buoyancy driving (given in terms of the stratification inputs θ and m_i) and diffusive effects. This is therefore a measure of the the supercriticality and vigor of the convection. The Ra involves the non-dimensional thermal conductivity $C_{k_z} = C_k \kappa_z$, where $\kappa_z = \kappa_i / \kappa_1$ and $C_k = \kappa_1 / \{d\rho_0 c_p [(c_p - c_v)T_0]^{1/2}\}$. which is different in the two layers. The Ra is a function of depth, and, in this study, the quoted value of Ra is evaluated at the middle ($z = 0.5$) of the unstable upper layer under the conditions of the initial polytrope.

The Prandtl number at a given depth is

$$\sigma_z = \frac{\mu c_p}{\kappa_z}, \quad (3.9)$$

where μ is the (constant) dynamic viscosity. The Prandtl number is depth-dependent, but $C_{k_z}\sigma_z = C_k\sigma$ is independent of κ_z and therefore is independent of depth. Any quoted Prandtl number is σ (the value in the upper layer). Similarly, the non-dimensional magnetic resistivity is controlled by a depth-dependent Schmidt number

$$\zeta_z = \frac{\eta c_p}{\kappa_z}, \quad (3.10)$$

but $C_{k_z}\zeta_z = C_k\zeta$ is independent of depth, and any quoted value is ζ .

The Chandrasekhar number,

$$Q = \frac{B_0^2 d^2}{\mu_0 \mu \eta}, \quad (3.11)$$

where μ_0 is the magnetic permeability, is a measure of the strength of the imposed magnetic field B_0 compared to diffusive effects. Note that the parameter truly governing magnetic effects is $\alpha = \sigma \zeta Q C_k^2$. This parameter determines the dynamic ratio of the gas pressure to the magnetic pressure, and is often referred to as the plasma β . Increasing Q (for fixed diffusivities) increases the dynamical back-reaction of any magnetic field on the flow field.

At the upper and lower boundaries of our domain, we apply impenetrable and stress-free boundary conditions

$$w = \partial_z u = \partial_z v = 0 \text{ at } z = 0, z_m, \quad (3.12)$$

which ensure that the mass flux and mechanical energy flux vanish on the boundaries conserving the total mass. The boundary conditions on temperature are

$$T = 1 \text{ at } z = 0, \quad \partial_z T = \frac{\kappa_2}{\kappa_1} \theta \text{ at } z = z_m, \quad (3.13)$$

therefore the imposed heat flux is the only flux of energy into and out of the system. The magnetic boundary conditions are specified as

$$B_x = B_y = 0 \text{ at } z = 0, z_m. \quad (3.14)$$

Note, that it is sufficient to impose boundary conditions only on the horizontal components of the magnetic field due to the solenoidality of $\mathbf{B}$. We note that non-zero vertical gradients of the horizontal field can be present at the boundaries so that the magnetic energy can decrease with time, i.e. these can be “run-down” systems.

The domain is periodic in the horizontal in all variables.

This system is solved numerically using the same code and same pseudo-spectral methods as in [109]. The resolution typically used is 512×600 for the 6×2.5 aspect ratio 2D domain.

3.3 Results

In order to achieve our goal of examining the effect of a self-consistent large-scale background field on the rise of a magnetic concentration through a convection zone, we must first complete some preparatory steps. First of all, we need to establish overshooting convection in our two layer system representing the base of the convection zone. Secondly, we must examine what strength of a flux tube is required for it to have enough buoyancy to rise through this convection. Thirdly, we must establish a vertical profile of pumped large-scale (mean) horizontal field with this overshooting convection (without the flux tube) to act as our self-consistent background field. Finally, we can then evolve the flux tube as a concentration amongst the volume-filling pumped background field in the convective simulation

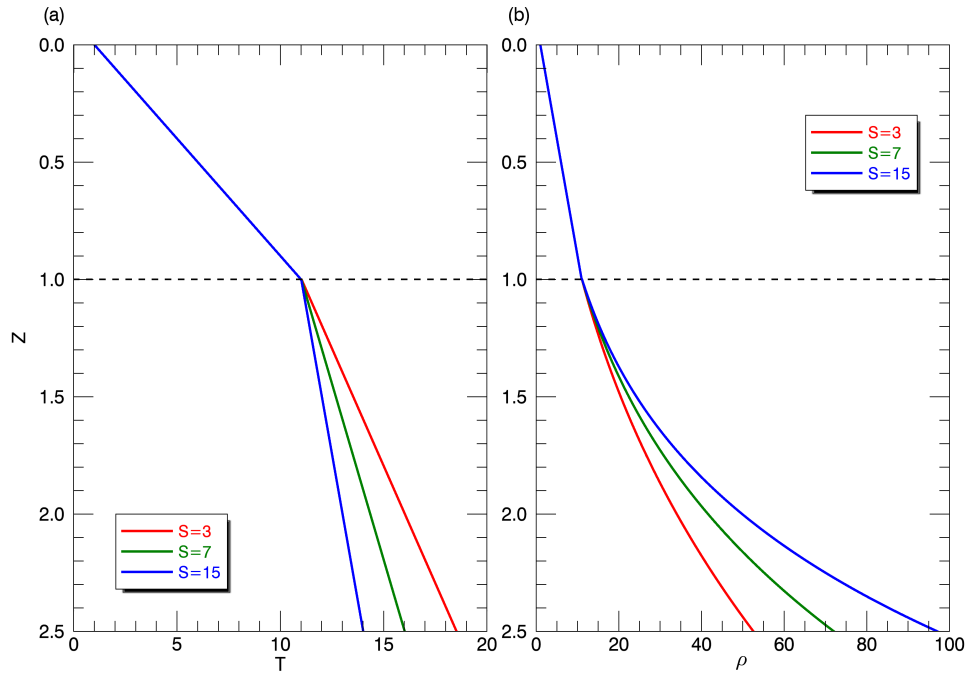


Figure 3.1: Profiles of polytropic thermodynamic initial conditions, a) T , and b) ρ , as a function of depth z at $x = 0$ for $S = 3, 7$ and 15 . The dashed line marks the transition between the convection zone and the radiative zone.

and examine the effect of that background field on the different signs of twist in the tube/concentration. We now describe each of these steps in detail.

3.3.1 Convection in 2D

We choose some canonical parameters for our study as follows. Our two-dimensional Cartesian box is of size $x_m = 6$, $z_m = 2.5$ so that our convection zone (of depth unity) has an aspect ratio of 6, allowing plenty of room for convective cells, and so that the domain has plenty of room for overshooting. For purely hydrodynamic convection, we only solve equations 3.1, 3.2, 3.3 (under the ideal gas equation of state) as an initial value problem. Throughout this study, we canonically use $Ra = 4 \times 10^4$ and $Pr = 0.1$, in order to achieve reasonably supercritical dynamics at a Prandtl number less than unity. These values are in the correct regime for astrophysical purposes, but are orders of magnitude away from their true astrophysical values due to numerical limitations. We set the stratification using $m_1 = 1$ and $\theta = 10$ and choose S from the values $S = 3, 7, 15$. These choices of S provide a range of overshooting dynamics from deep to fairly confined, as we shall see shortly. For much of this chapter, we use $S = 7$ as the canonical value, but all S are investigated and described in detail.

The time evolution of these equations from initial conditions consisting of a small amount of thermal noise in the convectively unstable layer sets up convection in the layer which can then overshoot into the lower convectively-stable layer. As an example, the time evolution of total kinetic energy in the simulation domain for the case with $S = 7$ is shown in Figure 3.2. An initial increase in the kinetic energy characterizes the formation of convection in the unstable layer, and, eventually, the kinetic energy settles to a statistical-steady (stationary) state.

Figure 3.3 shows snapshots of the vertical velocity (w) in the simulation do-

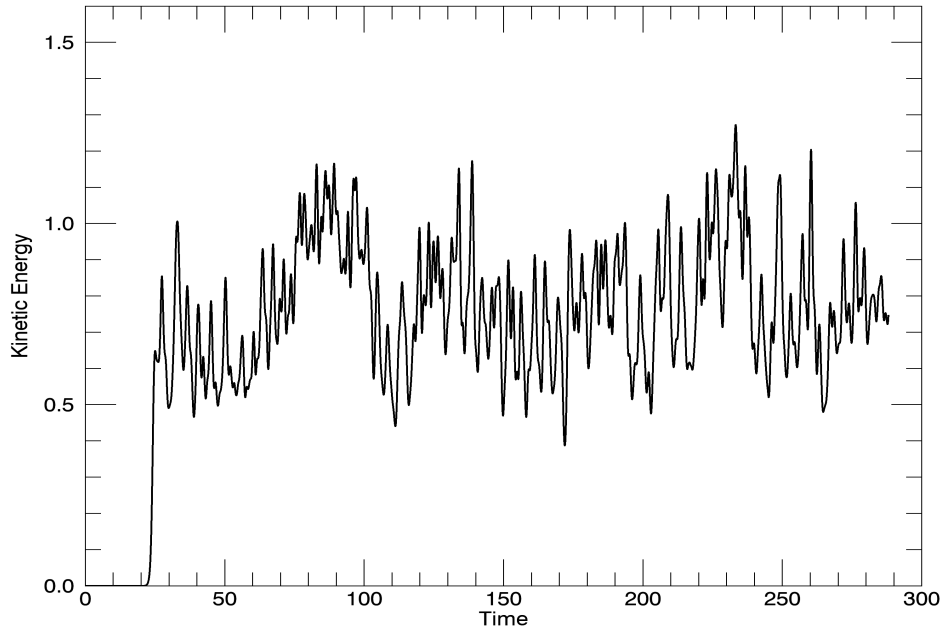


Figure 3.2: Kinetic energy as a function of time for the case with $S = 7$, $Ra = 4 \times 10^4$ and $Pr = 0.1$.

main at four different times during this evolution ($t \sim 24, 26, 71, 288$). We can see in Figure 3.3a that the initial evolution is marked by the formation of a cellular pattern of roughly five cells of convective motions, characterized by narrow down-flows (blue) and broader upflows (red). As the convection ramps up, these more regular cellular patterns (probably related to the linear eigenfunctions) quickly evolve into more turbulent nonlinear dynamics, consisting of more time-dependent plumes that begin to overshoot into the stable layer below, as seen in Figure 3.3b. During the later, more characteristic, evolution, as shown in Figures 3.3c and d, plumes can form, migrate, split or merge, and the flows have established a strong identity in the stable layer, connected to the convective layer, as if there were no boundary between the two regions, which is what we term “overshooting”.

The depth to which this overshooting occurs is a topic of great interest in astrophysics and has been studied in detail in both 2D and 3D (e.g. [62, 63,

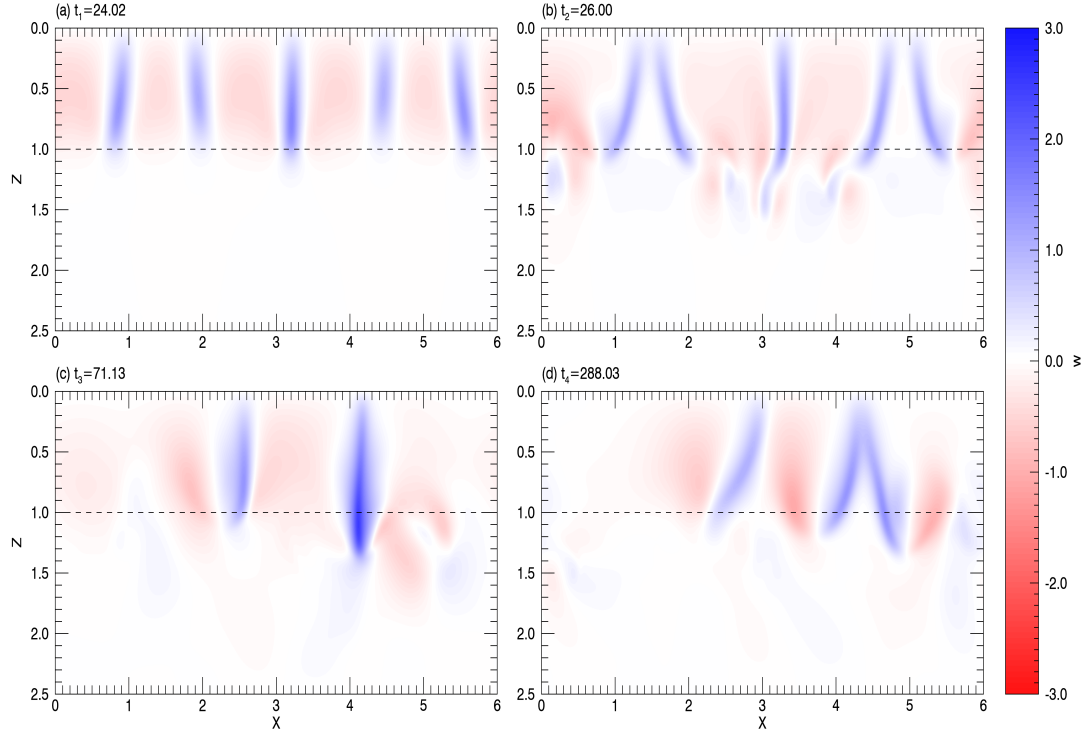


Figure 3.3: Vertical velocity (w) snapshots plotted at 4 different times for the case with $S = 7$, $Ra = 4 \times 10^4$ and $Pr = 0.1$. Blue color indicates downflows whereas upflows are red. The dashed line marks the transition between the convection zone and the radiative zone.

19]). This overshooting depth depends on many of the parameters, Ra , Pr , S and its scaling with these parameters is of great interest to astrophysics, but here we concentrate on the effect of the the relative convective stability of the two layers, quantified by the parameter S (introduced by [63], defined in Section 3.2). Increasing S increases the relative stiffness of the stable layer since we have fixed the stratification of the upper layer. By “stiffness” here, we mean the resistance to convective motions entering the layer, in the sense of the buoyant deceleration. For greater S , due to the more rapid increase in density in the stable region (as can be seen in Figure 3.1), a plume entering this region feels a larger (negative) density perturbation and therefore decelerates more rapidly than it would at lower S .

The level to which the convection penetrates the stable region can be quantified by examining the time- and horizontally-averaged kinetic energy flux (over a representative time in the stationary state of the convection), as shown in Figure 3.4. The overshoot depth is often taken to be the depth at which the kinetic energy reaches a fraction of its maximum value, typically 1% (e.g. [63, 19]). This value reflects where, on average (both in time and space), the convective motions die out. Figure 3.4 shows this average kinetic energy profile for the three different values of S , ($S = 3$, $S = 7$, $S = 15$). We can clearly see that the overshoot depth decreases as S increases, as has been found many times before. The overshoot depths are $z_o = 2.08, 1.63, 1.55$ for $S = 3, 7, 15$ respectively. This result is an average over many instantaneous realizations, such as those shown as examples in Figure 3.5, which shows the vertical velocity (as for Figure 3.3) at a representative time in each of the $S = 3$ and $S = 15$ cases. In this Figure, it can clearly be seen that the dynamics are active much deeper in the stable layer for $S = 3$ (down to at least $z = 2$) and less deep (down to $z = 1.5$ or so) for $S = 15$, when compared

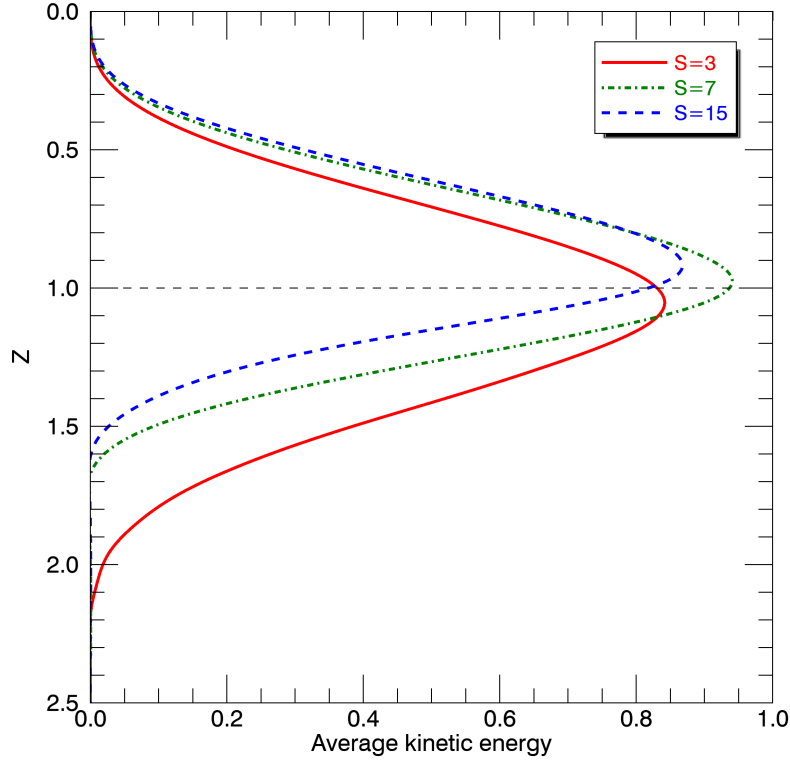


Figure 3.4: Time- and horizontally-averaged kinetic energy flux as a function of z for three different cases, $S = 3, 7$, and 15 . The horizontal dashed line at $z = 1$ marks the transition between the convection zone and the radiative zone. Overshooting depth is the z -location at which the curves meet the y -axis in the stable region.

to the case at $S = 7$ in the earlier figure. The scaling of the overshoot depth with S is of interest, but here we are more concerned with creating initial conditions for the later parts of our study.

We add a quick note regarding “overshooting” convection versus “penetrative” convection. If the dynamics in the overshoot layer below the convection zone are sufficiently energetic to mix the stable region strongly, driving it towards a well-mixed adiabatic system with a constant entropy, then the “overshooting convection” becomes known as “penetrative convection” [118]. All our cases here are overshooting convection and not penetrative convection. Figure 3.6 shows the

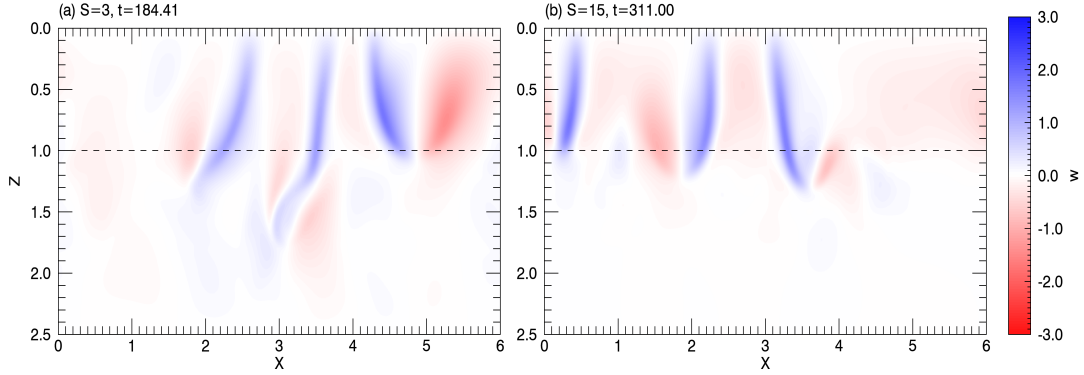


Figure 3.5: Vertical velocity (w) snapshots for (a) $S = 3$ and (b) $S = 15$.

time-averaged entropy for the three cases. We can see that significant mixing has not occurred below the convection zone, since there is still a strong entropy gradient below $z = 1$.

3.3.2 Rise of Flux tubes in Pure Convection

Having established a purely convective background state, we now wish to understand under what conditions a flux tube is able to rise through such a state in the absence of any large-scale volume-filling field. We need to do this since we are interested in effects on the flux tubes that would otherwise emerge buoyantly at the top of the convection zone. We therefore now turn to MHD simulations where we add an *isolated* flux tube to established convection. Again, similar simulations have been performed before, for example, in 3D for fully compressible and anelastic convection [34, 1]. A discovery of this past work has been that magnetic structures need to possess magnetic energy greater than the peak kinetic energy of the flow to survive transit of the convection zone. Here, we need to establish exact values for our particular setup to inform later, more complete and complex simulations including a large-scale background field.

The initial setup for these cases consists of an instance in time from the sta-

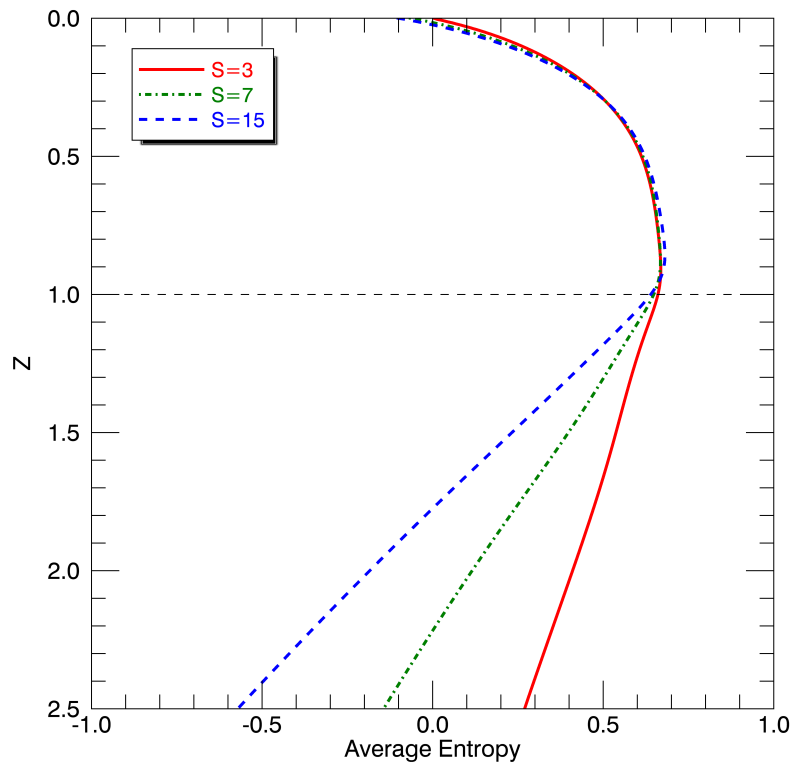


Figure 3.6: Time- and horizontally-averaged entropy as a function of height for $S = 3$, 7, and 15.

tionary state of one of the overshooting convection simulations, such as those in Figures 3.3c,d or 3.5. To this state, we add a twisted magnetic flux tube embedded at a chosen location in the stable zone (see, for example, Figure 3.7a or 3.8a). The total magnetic field of the flux tube is given by

$$\mathbf{B} = (B_x, B_y, B_z) = \mathbf{B}_{\text{tube}} = \left(-2q \frac{(z_c - z)}{r_o}, 1 - \frac{r^2}{r_o^2}, -2q \frac{(x - x_c)}{r_o} \right) \text{ for } r \leq r_o \quad (3.15)$$

where q is the initial twist of the field, x_c and z_c are the horizontal and vertical locations of the center of the flux tube, r_o is the outer radius of the structure, and $r = ((x - x_c)^2 + (z - z_c)^2)^{1/2}$ is the cylindrical radius measured from the center of the tube. Note that the field is purely axial at the center of the flux tube and the amplitude there is unity. With $|q| = 0.5$, the azimuthal field in the tube is unity at $r = r_o$. We only examine $|q| = 0.5$ in this study, and in this Section of the work, the sign of q does not matter, since there is no background field against which its orientation can be judged. We choose $x_c = 3, z_c = 2$ as the standard initial location, and we adopt $\zeta = 0.001$ from now. The latter value is small so that tubes, which can have strong gradients at the edges, do not diffuse too quickly, i.e. other dynamical times of interest, like the buoyant rise time, are quicker than the time to diffuse the flux tube magnetic perturbation. This value renders the magnetic Prandtl number $Pm = \sigma/\zeta = 100$ which is not representative of astrophysical values that are typically less than one. This is a statement that our simulations are too viscous and therefore not as turbulent as they should be, but this is a numerical limitation. At this point, we only examine $S = 7$ and leave the study of the effects of varying S later (see Section 3.3.8).

The canonical set of governing parameters is therefore now $S = 7, Ra = 4 \times 10^4, Pr = 0.1, \zeta = 0.001$. We are left to vary the Chandrasekhar number, Q ,

as the key parameter of interest. With unit maximum amplitude of both the axial and azimuthal fields of the flux tube, the Chandrasekhar number entirely determines the initial strength of the magnetic field (in this case, the flux tube) and determines the dynamical influence (via α) of the Lorentz force (see equation 3.2). When we add the flux tube to the existing fields, we do so assuming that the total pressure and the temperature equilibrate quickly, so that the increase in magnetic pressure is entirely compensated for by a drop in density. This means that the initial condition is not in equilibrium and the tube experiences an initial upwards buoyancy force.

Figure 3.7 shows the evolution of the full set of Equations 3.1-3.6 for such an initial state. This Figure shows intensity plots of the normalized axial field, B_y , which is a good indicator of the location of the tube, overplotted with the normalized vertical velocity, w , representing the convective flows. We use the absolute maximum values of B_y and w as the normalizing factors for the axial field and vertical velocity, respectively. The case shown is at the canonical parameters, for a positively-twisted flux tube with $Q = 2 \times 10^8$, and the Figure displays four different times. Figure 3.7a shows the initial location of the cylindrical flux tube at $(x_c, z_c) = (3.0, 2.0)$ deep in the stable region, along with the overshooting convective motions. Here, red and blue colors indicate upflows and downflows respectively. As the flux tube starts rising due to its magnetic buoyancy, the initial rise is reasonably symmetric, and quickly becomes accompanied by the formation of vortices, a characteristic reminiscent of non-convective flux tube rise (Fig. 3.7b). The impact of overshooting convection on the dynamics of the flux tube is minimal at this stage. As the flux tube enters the overshooting region ($1.0 \leq z \leq 1.55$) and gets closer to the transition between the stable and convective layer, the effect of convective motions becomes evident on the flux tube. Figure 3.7c shows that the

flux tube is located beneath a downflow. The symmetric rise of the flux tube is broken, as buoyant and advective forces act on the tube, and the driving vortices are stretched. However, due to sufficient initial twist ($|q| = 0.5$) and sufficient buoyancy ($\propto Q$), the flux tube maintains coherency and navigates its way into an upflow and thereby to the top of the convective simulation domain (Fig. 3.7d). We consider these dynamics as a successful rise of the flux tube (but will develop a more quantitative measure later).

Figure 3.8 represents a case similar to Figure 3.7 but for a lower Q . The simulation starts from the same initial steady-state convection and initial magnetic profile, albeit now with $Q = 5 \times 10^6$ (Figure 3.8a). With a weak buoyancy perturbation due to lower Q , the initial rise of the flux tube is very slow. Figure 3.8b shows that the flux tube barely rises in the stable zone with no distinct vortices forming. The slow rise brings the tube to the edge of the overshooting region ($z \sim 1.5$ where even these weak flows distort the cylindrical flux structure (Fig. 3.8c). Eventually, other forces dominate over buoyancy and the flux tube gets stretched, and its rise is halted (Figure 3.8d). We consider this as an example of an unsuccessful rise of the flux tube. Even with minimal interaction with the overshooting, the buoyancy was insufficient to enable a rise.

It is reasonable then, in this study, to conclude that the rise or non-rise of the flux tube is determined by the amount of buoyancy (controlled by Q) imparted to the flux tube (for fixed other parameters). We now explore a wider range of Q to understand this dependence in more detail. To determine the ultimate fate of a flux tube more concisely, we turn to a more quantitative and explicit measure of the rise characteristics, denoted by $z_{ft}(t)$, that tracks the z -location of the maximum of the axial field, B_y , as a function of time. From Figures 3.7 and 3.8, it is clear that this quantity tracks the progress of a tube reasonably well.

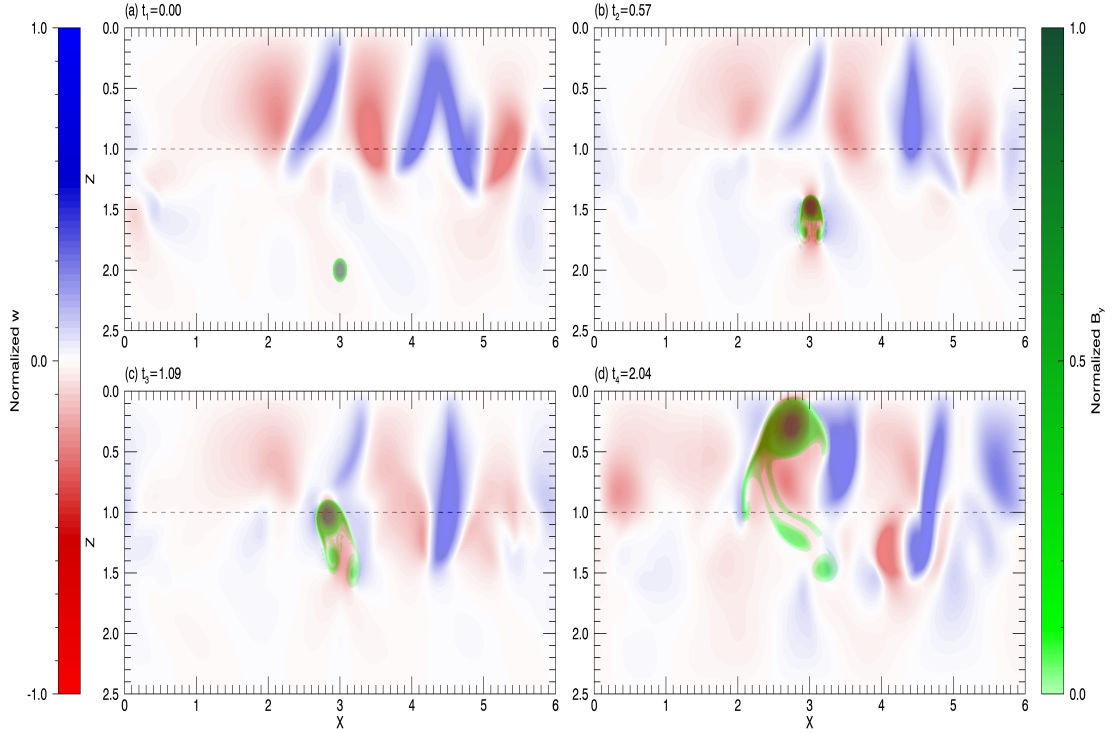


Figure 3.7: Intensity plots of normalized B_y , overplotted with the normalized vertical velocity, w , as a function of time for the case with $S = 7$, $Ra = 4 \times 10^4$, $Q = 2 \times 10^8$, $Pr = 0.1$ and $Pm = 100$. The initial location of the flux tube is $(x_c, z_c) = (3.0, 2.0)$. The dashed horizontal line indicates the transition between the convection zone and the radiative zone.

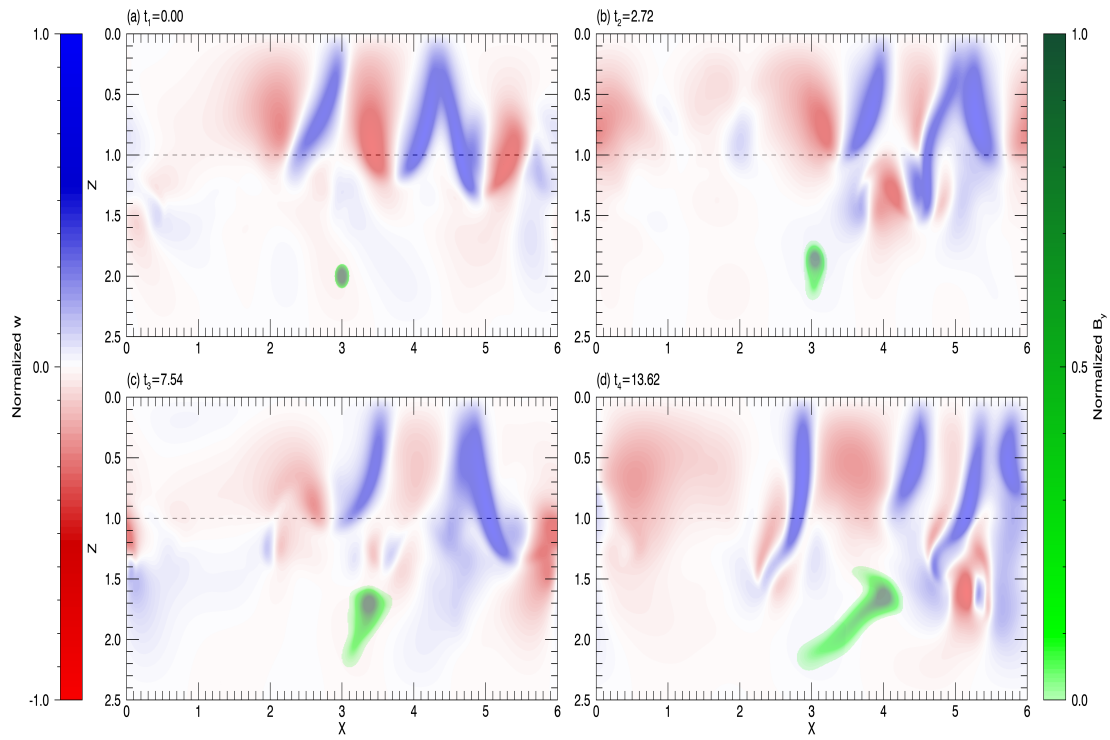


Figure 3.8: Same as Figure 3.7 but for $Q = 5 \times 10^6$.

Figure 3.9 shows $z_{ft}(t)$ for a range of simulations that were initiated identically but for varying Q . A relatively lower Q leads to an almost neutrally buoyant flux tube that does not rise and remains embedded in the stable zone ($Q = 5 \times 10^6, 1 \times 10^7$; red and green markers). A relatively middling value of Q , for example, $Q = 4 \times 10^7$ (blue markers) leads to a slow rise of the flux tube through the stable zone and into the convection zone, where it experiences some substantial variations due to interactions with upflows and downflows but nevertheless rises. With higher Q values, the flux tube rises quickly through both the stable zone and the convection zone to the top of the domain (e.g. $Q = 8 \times 10^7, 2 \times 10^8$; magenta and black markers). In this plot, the black marker corresponds to the $Q = 2 \times 10^8$ case discussed in Figure 3.7 and the red marker corresponds to the $Q = 5 \times 10^6$ in Figure 3.8. Note that we curtail such plots, stopping following the tube either after it reaches the top of the simulation domain or when it has become clear that a tube has stopped rising.

From this information, we choose to use $Q = 2 \times 10^8$ as the value of Q for our canonical set of parameters used in the next sections. Our aim is to evaluate the effect of the sign of twist in conjunction with a large-scale background field on emergent tubes. This case provides a clear and definite emergence in the absence of such effects. It can be interesting to examine other cases too, and this will be done later in this chapter when examining different S .

At this point, it is important to realize that here the sign of the twist of the flux tube does not make a difference to the dynamics. The evolution and fate of a positively-twisted flux tube and a negatively-twisted flux tube inserted into convection without any background field is the same. This is illustrated by Figure 3.10 which shows z_{ft} for two tubes with opposite signs (panel a and c), and a snapshot of the simulation (panels b and d) at a late stage for a negatively-

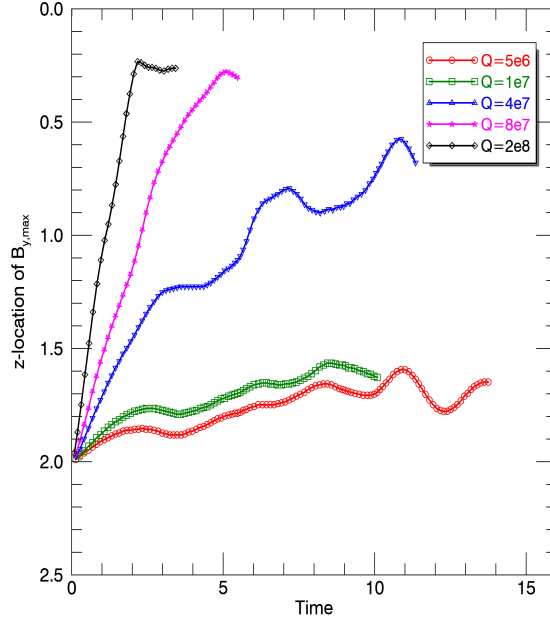


Figure 3.9: The maximum of B_y as a function of time z_{ft} for different Q . Other parameters are $Ra = 4 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, $q = 0.5$, and $S = 7$.

twisted tube which can be compared with that shown for a positively-twisted tube in Figure 3.7d and 3.8d respectively. Note that Figure 3.10a and b are for $Q = 2 \times 10^8$ and c and d for $Q = 5 \times 10^6$. Clearly, the dynamics are very similar, and the twist serves mainly to maintain the coherence of the tube. We will show that this symmetry is broken by the inclusion of a background field in later sections of this chapter.

3.3.3 Dynamical formation of Background Magnetic Field

Chapter 2 explored the effect of a background magnetic field on the rise of flux tubes by artificially imposing the background field as a function that decreased exponentially with height. We argued that the turbulent convective pumping of large-scale magnetic fields (e.g. [111]) would likely achieve some profile of the background field where the majority of the field was confined below the convection

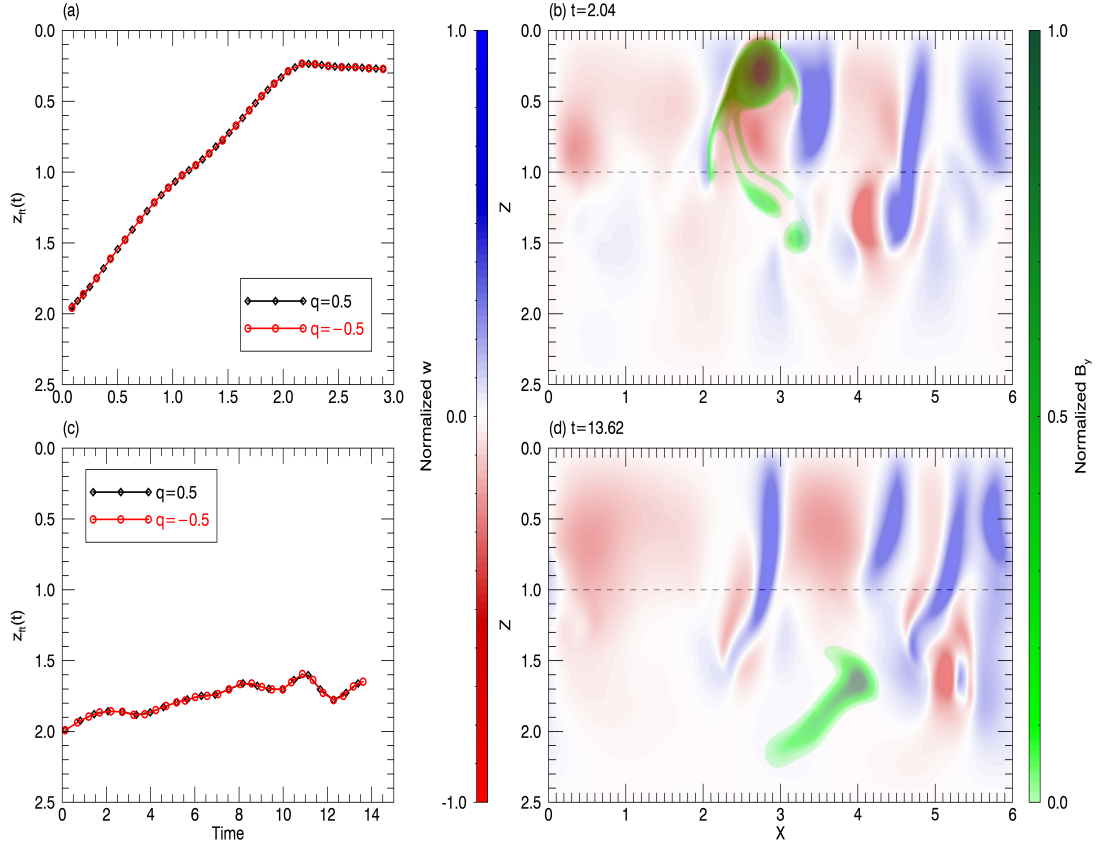


Figure 3.10: Comparison of the dynamics of positively- and negatively-twisted tubes for $Q = 2 \times 10^8$ (a and b) and $Q = 5 \times 10^6$ (c and d): (a) z_{ft} plotted as a function of time for both $q = 0.5$ and $q = -0.5$, (b) The equivalent of Figure 3.7d but for $q = -0.5$, (c) z_{ft} plotted as a function of time for both $q = 0.5$ and $q = -0.5$, (d) The equivalent of Figure 3.8d but for $q = -0.5$.

zone. The exponential profile was their attempt at a simple model of part of this profile. In this section of this chapter, as part of the setup of our initial state for the ultimate simulations, we explore the self-consistent formation of a large-scale background magnetic layer by magnetic pumping due to the presence of the overshooting turbulent convection, instead of assuming its existence, as in the previous work. Again, simulations of this type have been performed before (e.g. [111]) and transport of magnetic field out of convection zone into a stable layer down a gradient of turbulent intensity is fairly well-understood. Our aim here is simply to create self-consistent initial conditions for the large-scale background field, rather than artificial ones. Since the selection mechanism discovered in Chapter 2 depends on the relative strengths of the tube and the background field, our goal here is to be able to control the amplitude of the profile of the pumped field to explore this relationship.

We start with a steady-state pure convection solution from Section 3.3.1 and impose a thin horizontal magnetic layer concentrated in the convection zone. Note that, in this Section, no flux tube is imposed. The magnetic layer is given by

$$\mathbf{B} = (B_x, B_y, B_z) = \mathbf{B}_{\text{layer}} = \left(\tanh\left(\frac{z - z_{\text{bot}}}{\delta}\right) \tanh\left(\frac{z_{\text{top}} - z}{\delta}\right), 0, 0 \right) \text{ for } z_{\text{bot}} \leq z \leq z_{\text{top}} \quad (3.16)$$

where z_{top} and z_{bot} are the top and bottom location of the layer respectively, and δ is the smoothing width of the edges of the layer. For a standard initial condition, we use a layer where $z_{\text{bot}} = 0.80$, $z_{\text{top}} = 0.75$ and $\delta = 0.01$. The other parameters remain at the canonical values ($Ra = 4 \times 10^4$, $Pr = 0.1$, and $\zeta = 0.001$) and we can again choose Q to determine the initial strength of the magnetic layer. This, in turn, affects the initial magnetic buoyancy of the layer, since we again adjust the background thermodynamic state to maintain total pressure and temperature equilibrium, as in the previous section. We have experimented with omitting the

thermodynamic adjustment to avoid certain numerical issues, and found that this makes little difference, since the code can adjust pressure balance very quickly.

The choice of Q is a little complicated for the following reasons. In Section 3.3.2, we established that a flux tube with $Q = 2 \times 10^8$ rises through the convection and therefore we will need to run our ultimate simulations (involving both flux tubes and background field) at this Q value. The work of Manek & Brummell (2021) [80] showed that the key factor in the selection mechanism was the relative values (and orientation) of the azimuthal field in the tube and the background field strength. The former is initially dictated by q and can be different from the axial field strength of unity. However, our choice of $|q| = 0.5$ makes the peak azimuthal field also conveniently equal to unity, and therefore, both of these quantities are governed by the value of Q . However, we wish to be able to control the relative strengths of the azimuthal field in the tube and the background field in order to explore the regimes of [80]. From that previous work, we expect the background field strength to be roughly between 5% – 20% of the tube strength for the selection mechanism of Chapter 2 to manifest. We therefore need to be able change either the resultant azimuthal field strength or the resultant background field strength at a fixed Q to be able to realize this ratio. We could achieve this by varying $|q|$, but instead we choose to control the resultant strength of the evolved background field (in order to avoid having to repeat the simulations of the previous section many times). This is not particularly artificial, since any pumping calculation with the magnetic boundary conditions specified (Equation 3.12) runs down anyway, so that the magnetic field amplitude decreases from unity to some significantly lower value. Changing the initial layer amplitude allows any chosen final amplitude to be realized. In essence, there is an effective Q of this run-down state that we can control, given by the square of the amplitude of the state times the original Q :

$$Q_{\text{eff}} = Q|\mathbf{B}|^2.$$

With this in mind, we purposely start our pumping simulations with a lower Q ($= 4 \times 10^6$) in this setup. This allows a two-fold flexibility. Firstly, at large Q , strong field can quickly accumulate at the upper boundary causing numerical problems; lower initial Q can avert this unphysical issue. Secondly, after evolving the system at lower Q , we can evaluate the peak amplitude of the final horizontally-averaged profile, and then scale this state to whatever peak amplitude we desire, essentially adjusting the Q_{eff} . We can then take this as an initial condition, and run at the higher Q (the canonical Q that we must use when a tube is eventually present in the next Section) until it relaxes into the pumped state for that value. This process is equivalent to running at different amplitudes of the initial layer at fixed Q (and therefore different Q_{eff}) but allows us to have fast access to a pumped state with a controllable peak amplitude of the profile.

We illustrate this whole process with Figures 3.11 and 3.12. Figure 3.11 gives a good impression of the magnetic pumping process by showing snapshots of the horizontal magnetic field B_x (normalized by its maximum) as a function of time in the simulation at the canonical parameters. The initial magnetic layer can clearly be seen in Figure 3.11a. Panel b exhibits the early evolution of the layer, showing that these are dominated by the competing effects of magnetic buoyancy and downward transport of flux caused by advection in the convective plumes. Some sections of the layer rise by the combined effects of the magnetic buoyancy of the layer and upflow advection, whereas some sections get dragged towards and into the stable layer, in strong downflows where advection overcomes the buoyancy. After this initial adjustment, the advective churning of the field dominates, as can be seen in panels c-d. At this stage, the small-scale turbulent interactions with the magnetic field dominate, leading to the turbulent transport

mechanism of magnetic pumping. The overshooting convective motions therefore transport magnetic flux to the stable region, as seen in Figure 3.11e-f. The pumped fields slowly accumulate at the edge of the overshoot region, where their residual magnetic buoyancy is balanced by the pumping mechanism.

In panels g-h of Figure 3.11, we show an example of the adjustment of the peak amplitude of pumped field. We evaluate the peak strength of the current pumped layer by calculating a time average of the horizontal field. Due to the influence of magnetic diffusion and vertical boundary conditions, the peak strength of the pumped layer has diminished considerably from the initial value of unity. We re-scale the magnetic fields in the whole simulation domain by a chosen scaling factor. If required, we can also adjust the Chandrasekhar number of the simulation at this point to the canonical value of $Q = 2 \times 10^8$. We then further evolve the new system, treating the scaled magnetic field as a new initial condition, over many convective turnover times. This allows the system to relax to a new dynamically-consistent pumped magnetic layer, as shown in Figure 3.11g-h.

It should be noted that even though the pumped magnetic layer in Figure 3.11h (for example) looks fairly smooth, thanks to the considerably higher value now of the peak in the pumped layer, there are considerable local variations if we check the vertical profiles of the layer at different horizontal locations. In Chapter 2, the artificially-imposed background field was uniform in the horizontal direction, whereas here, the self-consistently generated pumped field state is not. In order to make contact between the two different works, we here calculate (and use for comparison) the time- and horizontal-average of the horizontal magnetic field B_x . First, we show as Figure 3.11 the time evolution of the horizontally-averaged field for various stages in the calculation shown in the previous Figure. For each time portion, the initial and final state is shown as a thicker line, and

the thinner lines show regular time intervals between these end points. Panel a again shows the initial dynamics where the profile evolves from the initial layer located between $z = [0.7, 0.8]$. Initially, it can be seen that the profile relaxes in shape and amplitude somewhat, and moves upwards due to magnetic buoyancy and starts to accumulate at the top. At the same time, some of the average field gets moved towards the lower stable layer, creating a second accumulation peaked at around $z = 1.4$. In panel b, later in time, we see that on average, the field starts to be evacuated from the upper boundary and the interior of the convection zone. The peak around $z = 1.4$ becomes (relatively) more prominent, but also a second peak around $z = 1.8$ starts to form, just below the overshoot region. This latter peak is the progenitor of the pumped layer, as can be seen in panel c, where, as time progresses, this peak becomes the dominant dynamical feature in the average field. Panel d shows how we control the amplitude of the pumped field. Here, we have taken the end result of the earlier calculations, and rescaled the pumped field by a chosen factor (a factor of 20 in this case) and continued the evolution. It can be seen that this achieves a stable profile of pumped field on average, with a peak amplitude of about 0.2. This type of state is what we desire as an initial condition for later calculations also containing a flux tube with peak field amplitude of unity. Such a profile provides the ratio of amplitudes where the background field is at about peak amplitude of 20% of the tube peak amplitude. It should be noted that pumped profile can still evolve and run down due to the magnetic boundary conditions. However, this evolution is in general very slow compared to other dynamics that we are interested in (such as the rise time of a flux tube) and therefore can be considered essentially steady. As can be seen in panel d, this is because the profile can evolve such that there are no gradients of the (average) field at the boundary (especially at higher S than is shown in this

case: see later) and/or the profile becomes essentially piecewise linear with very little diffusion acting (except at the junctions of the piecewise profile).

Figure 3.13 shows the final time- and horizontally-average state of the pumped field as a function of height at the canonical parameters with $Q = 2 \times 10^8$. This is the state that we will use for the next Section. The peak strength of this pumped layer is about 0.21, or 21%, compared to a unit strength flux tube. The filled-diamond markers are placed to indicate the locations where the average field strength is 1, 2.5, 5, 10, 15, and 20% on the upslope section of the pumped layer. We will use these locations from this layer in the next Section.

3.3.4 Rise of Flux tubes in the presence of a Background Field

We now come to the main point of this chapter and examine the dynamics of the rise of magnetic flux tubes in the presence of convection and a background magnetic field. We take the setup we arrived at in Section 3.3.3, where we have statistically steady-state convection with a self-consistent magnetic field in the form of a layer in the stable zone. To this we introduce a flux tube. The total magnetic field is then given by

$$\mathbf{B} = (B_x, B_y, B_z) = \mathbf{B}_{\text{tube}} + \mathbf{B}_{\text{pumped}} \quad (3.17)$$

where $\mathbf{B}_{\text{tube}}$ is the field of a tube as in Equation 3.15 and $\mathbf{B}_{\text{pumped}}$ is the field resultant from the end of the simulations in the previous Section 3.3.3. In this Section, we consider flux tubes with both positive (anticlockwise) and negative (clockwise) twist, q , with a fixed magnitude $|q| = 0.5$. The other parameters are the canonical set: $Ra = 4 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, $S = 7$ and $Q = 2 \times 10^8$.

There are a few important points to note at this stage. Firstly, introducing a

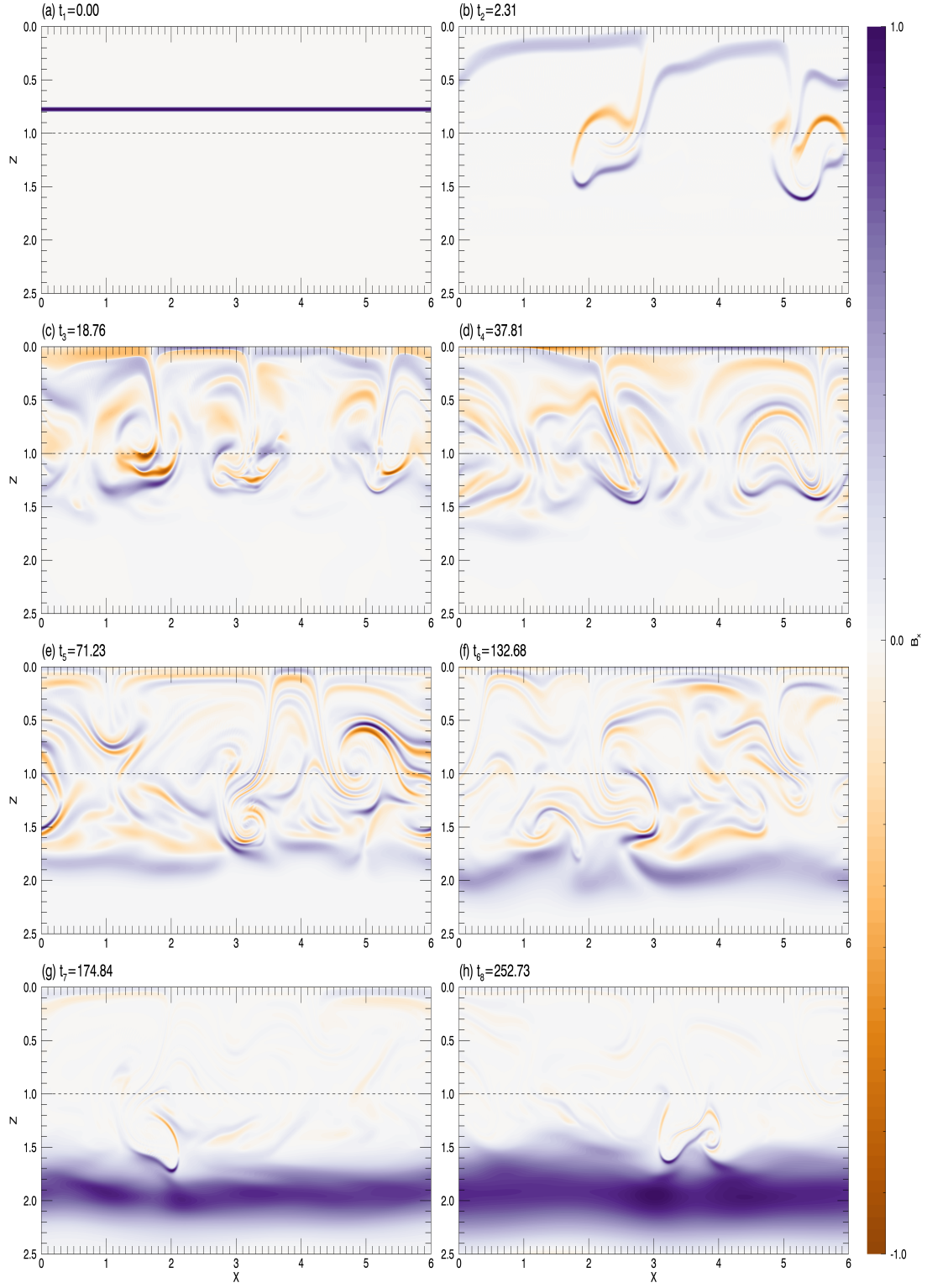


Figure 3.11: Snapshots of normalized horizontal magnetic field, B_x , as a function of time. (a)-(f) have $Q = 4 \times 10^6$ whereas (g)-(h) have $Q = 2 \times 10^8$.

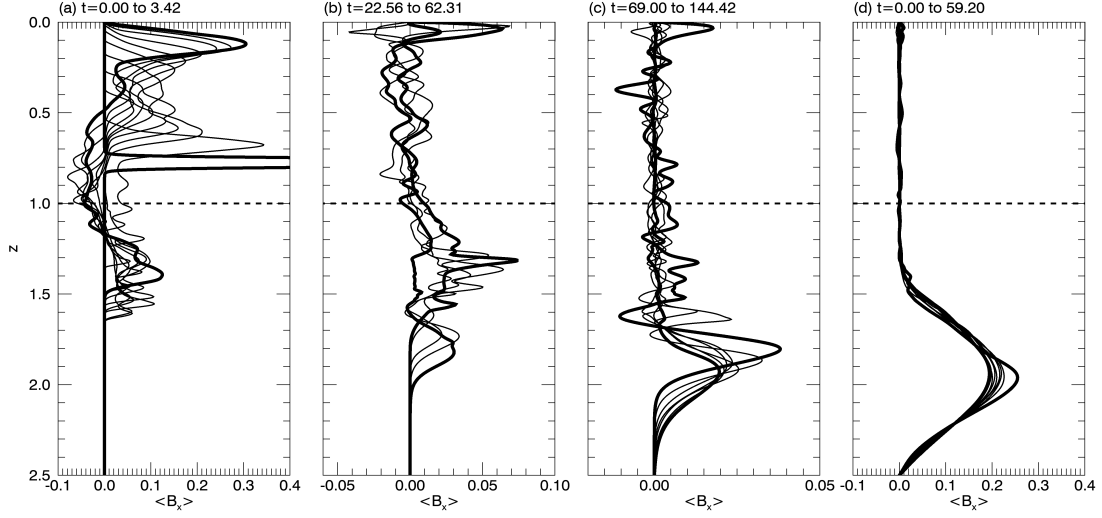


Figure 3.12: Time- and horizontally-averaged pumped horizontal field as a function of height (z) for the case with $S = 7$. Initial and final profiles are plotted with a thicker line in each of the subplots. (a) shows the initial pumping of the imposed magnetic layer. The final profile of the pumped field is multiplied by a factor of 20 and its evolution is shown in (d).

flux tube, $\mathbf{B}_{\text{tube}}$, again requires a thermodynamic perturbation in the background stratification for consistency, as in the case when there was no large-scale background magnetic field already present. As before, we choose to adjust the density whilst maintaining the current temperature in the tube. This imparts an initial magnetic buoyancy to the tube, as was the case in the absence of a background magnetic field. The pumped layer, $\mathbf{B}_{\text{pumped}}$, is already dynamically consistent with the background stratification (see Section 3.3.3) and the thermodynamics do not need to be adjusted for this component.

Secondly, as determined in detail in Chapter 2, the relative strengths of the twist of the tube and the background field are the key determining factors in the differential evolution of differently twisted flux tubes. Here, we have fixed the strength of the tube (via its unit maximum axial field strength and fixed twist value, q) and are left with two controls over the relative strength of the background

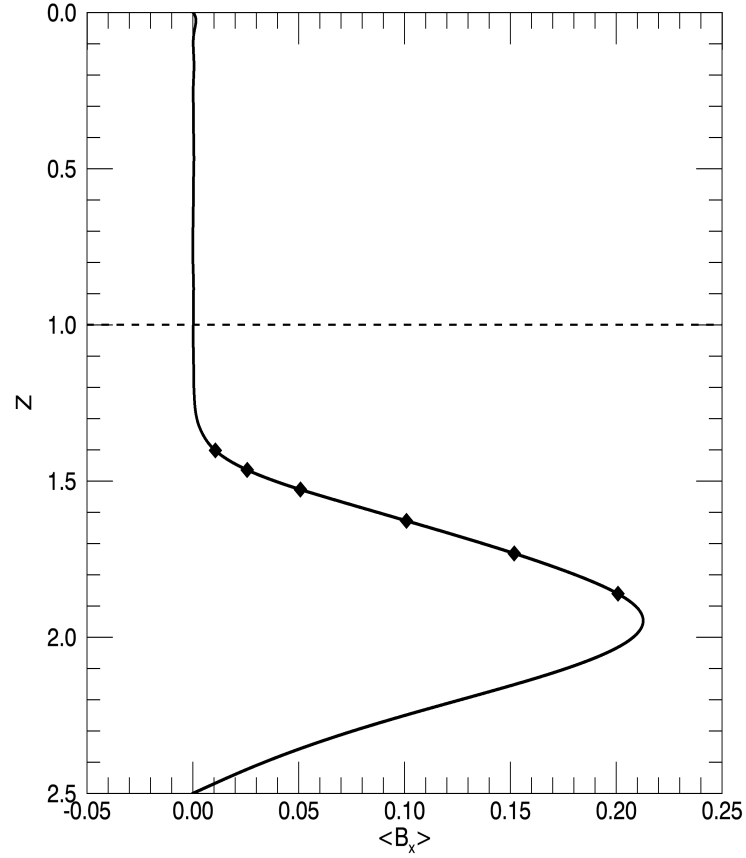


Figure 3.13: Time averaged pumped horizontal field as a function of height (z) for the case with $S = 7$. Black diamond markers indicate the different z -coordinates centred at which flux tubes are introduced. These z -coordinates correspond to 1, 2.5, 5, 10, 15 and 20% strength of the pumped horizontal field as compared to the flux tube strength. The dashed line marks the transition between the convection zone and the radiative zone.

field. One control is that we can adjust the peak amplitude of the pumped layer as described in the previous section. The other control is that, even for a given peak amplitude in the initial condition for $\mathbf{B}_{\text{pumped}}$, we can adjust the depth at which the flux tube is introduced, thereby determining both the background field value that the tube experiences initially and the amount of pumped background field the tube experiences during transit upwards.

Our strategy here initially is to select a case from the previous section where the peak amplitude of $\mathbf{B}_{\text{pumped}}$ lies in realm where Chapter 2 predicted that a selection mechanism may occur. We then select particular depths at which the flux tube is introduced into the pumped magnetic layer in order to vary the precise ratio of the twist field in the tube to the averaged pumped field strength (see diamond markers in Fig. 3.13). We investigate the details of varying the peak amplitude and related local effects later. We initially choose all the locations of the introduction of the flux tube to be on the upper side of the pumped layer. We do this because we assume that flux tubes are likely to be formed by magnetic buoyancy instabilities, which require that the magnetic field increases sufficiently strongly downwards, therefore the most likely initiation of tubes is on the upper side of the pumped layer. We argue also that, while flux tubes may perhaps form below the peak of the pumped magnetic layer, the eventual rise of such tubes is more likely to fail completely due to the significant extent of the strong magnetic field that it has to rise through, and that failed cases are less interesting to us initially. We adopt the notation that, for example, $B_s = 0.10$ corresponds to a time-averaged pumped background field strength (at the depth of the center of the tube) that is 10% of the unit peak strength flux tube (in both axial and, more importantly, azimuthal field). Locating the flux tube at the vertical position corresponding to this strength in the average pumped field is now one of our key

control parameters.

Figure 3.14 shows intensity plots of normalized B_y (green color scale), overplotted with normalized vertical velocity, w (red-blue color scale), for both a positively-twisted (panel a) and negatively-twisted (panel b) flux tube for four different times in the evolution of the system. Here the red and the blue color represents upwards and downwards vertical velocity respectively. For this case, the flux tube in both cases is introduced at a depth such that $B_s = 0.10$ as explained above. Figures 3.14a and b therefore look identical, with the flux tube embedded between two convective downflow plumes. However, the dynamics subsequently evolve substantially differently in the two cases.

The flux tube with a positive twist rises initially due to its imposed magnetic buoyancy, as in the case with no background field. However, the rise this time is not entirely coherent as the flux tube is rising through the pumped horizontal background field. Instead, as the tube rise, it experiences some axial flux loss along the background field B_x accompanied by substantial disruption of the vortices that form in the isolated tube case. This can be seen qualitatively but clearly in Figure 3.14a at $t_2 = 0.81$. These effects, due to the presence of the background field, were also seen and described in Chapter 2. Furthermore, the flux tube in this case is located between two merging downflows at these early stages in the plots. However, even with all these relatively unfavorable circumstances for its rise, the flux tube successfully enters the convectively unstable layer (Fig. 3.14a at $t_3 = 1.35$) and eventually rises to the top in a weak upflow next to the strong merged downflow (Fig. 3.14a at $t_4 = 2.29$). Note that the effect of convective motions on the rise is immediately evident as the flux tube does not rise symmetrically (about the line $x = 3$) and so these simulations are clearly distinct from those of Chapter 2.

In stark contrast, the dynamics of the flux tube with a negative twist are entirely different. Figure 3.14b at $t_2 = 0.24$ shows that, in the presence of the background field, the flux tube does not even begin to rise through the convective background. The flux tube structure simply gets distorted quickly and breaks apart, with axial flux rapidly drained along the B_x fieldlines (see Fig. 3.14b at t_3 and t_4).

At this point, we have already arrived at an important conclusion that parallels those of Chapter 2, i.e. that the flux tube dynamics in the presence of a background field are very different compared to cases where there was no background field present. With no background field, tubes of both twists rose similarly, with the dynamics dominated by their identical magnetic buoyancy (see Section 3.3.2), whereas, here, in the presence of background fields, positively- and negatively-twisted tubes evolve very differently. In these new simulations, these conclusions, that are similar to those of Chapter 2, are reached despite the presence of convection and a more self-consistent background field. Put in a slightly different way, we have shown that a positively-twisted flux tube is more likely to rise than a negatively-twisted tube in the presence of convection and a self-consistently pumped background field of strength, $B_s = 0.10$. Since the magnetic buoyancy forces exerted by both signs of twist are nearly identical, we attribute these effects to magnetic tension effects, as described in Chapter 2, even though this system is more complex. We examine this quickly with the Figure 3.15 and investigate this in more detail later. Figure 3.15 shows the vertical profile of the vertical tension force $F_{z,tens} = (B_x \partial_x + B_z \partial_z) B_z$ on the $x = 3$ line through the center of the tube at $t = 0$, corresponding to the state in panels a and b in Figure 3.14. These line plots confirm that, compared to the case where no background field is present (dashed line), the positively-twisted tube has enhanced negative (upward) tension

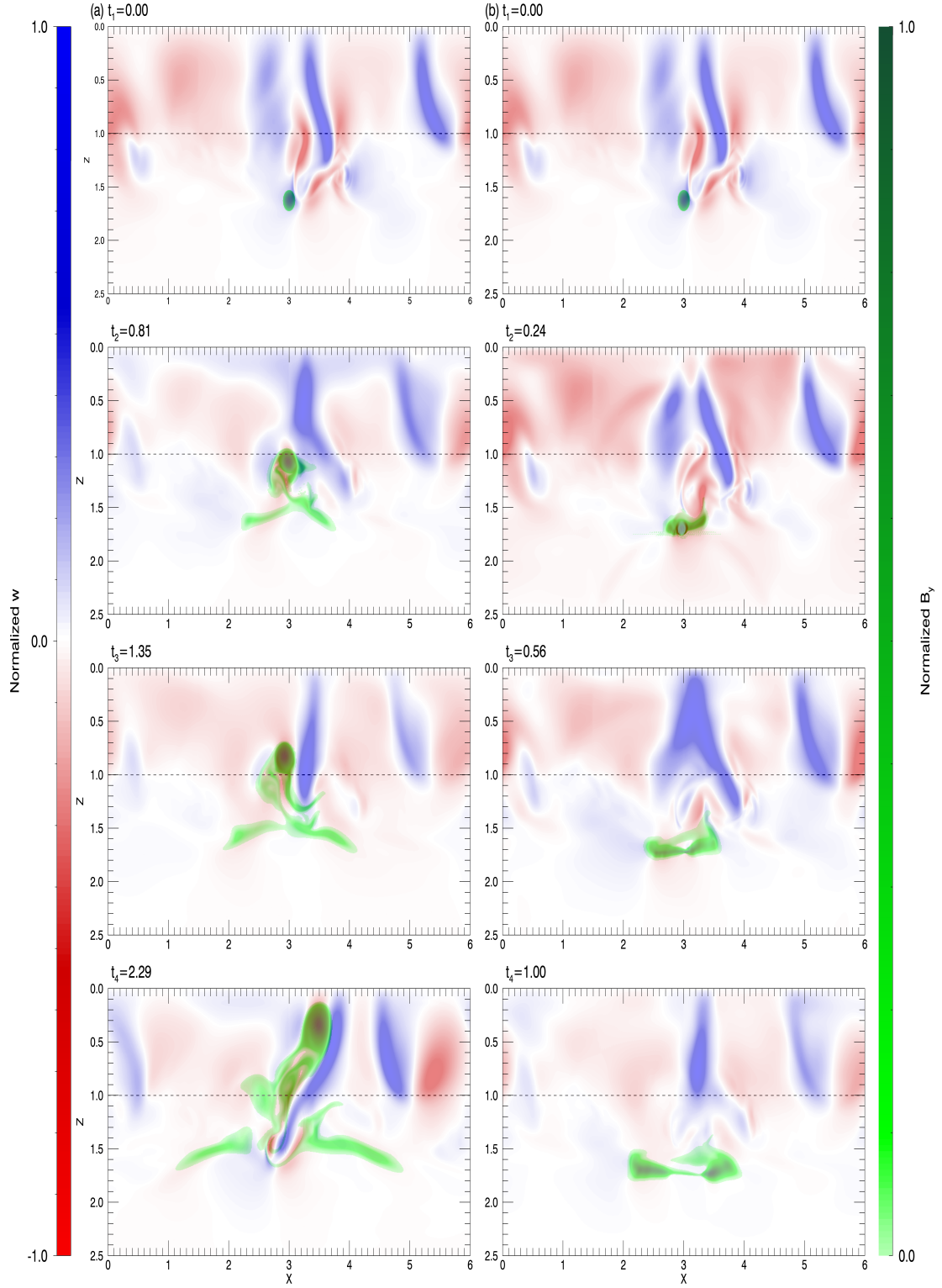


Figure 3.14: Intensity plots of normalized B_y , overplotted with normalized vertical velocity, w , for (a) $q = 0.5$ and (b) $q = -0.5$ at four different times. The time-averaged pumped background field strength, B_s , at the center of flux tube in both cases is 0.10, i.e., 10% of the initial strength of the flux tube.

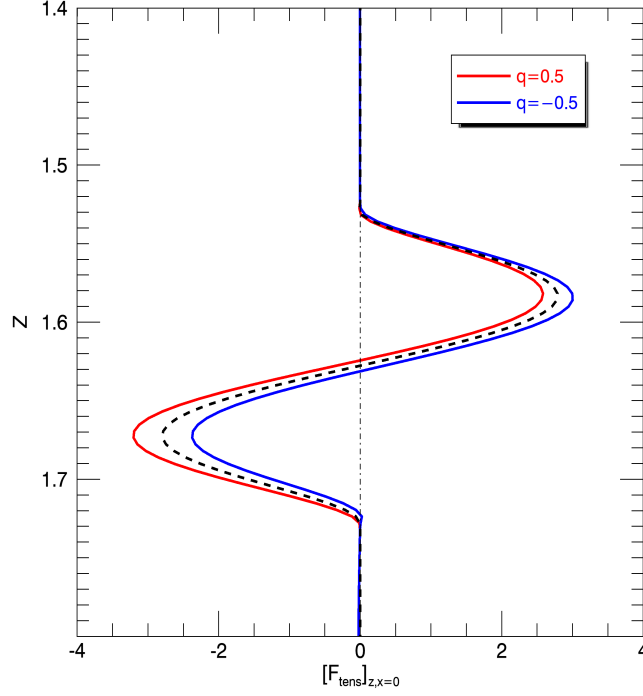


Figure 3.15: Plots from the canonical case at $S = 7$ (from Section 3.3.4) showing the tension force acting in the z -direction, $F_{z,tens} = (B_x \partial_x + B_z \partial_z) B_z$, evaluated at $x = 3.0$ and $t = 0.0$ for both positive ($q = 0.5$) and negative ($q = -0.5$) twisted tube. The dashed line shows the profile from the symmetrical tube from Section 3.3.2 where no pumped background field is present.

in the lower half of the tube, whereas the negatively-twisted tube has enhanced positive (downward) tension in the upper half of the structure. In the former case, the tension acts in concert with the buoyancy force, helping the rise of the tube, whereas in the latter case, tension acts against buoyancy, decreasing the chances of rise. This is the case despite the fact that the background pumped field may have significant variations away from the mean profile at this particular location.

3.3.5 Dependence on B_s or vertical location

In order to explore the dependence of this selection mechanism exhibited above on the relative strength of the background field, as suggested by Chapter 2, we

vary the initial vertical location of the flux tube and examine the subsequent rise dynamics. We explore a broad range of B_s , ranging from relatively very weak ($B_s = 0.01$) to very strong ($B_s = 0.20$) in the canonical case. The vertical locations corresponding to these pumped background field strengths are marked in Figure 3.13. The parameters for all of these simulations remain the same at the canonical values: $Ra = 4 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, $Q = 2 \times 10^8$, and $S = 7$.

Rather than qualitatively analyzing intensity plots to determine a successful/unsuccessful rise, we return to plots of z_{ft} that track the location of the peak field and therefore (usually) the tube as a function of time for the various cases. These time traces are shown in Figure 3.16.

Figure 3.16 shows z_{ft} as a function of time for a positively-twisted flux tube (panel a) and a negatively-twisted flux tube (panel b) for otherwise identical canonical parameters. Both panels show line plots for each of the six different locations corresponding to the six different relative background field strengths at those locations, namely $B_s = 0.01, 0.025, 0.05, 0.10, 0.15, 0.20$. One can confirm that the initial z_{ft} location, $z_{ft}(t = 0)$, for each of these values is different and corresponds to the appropriate point in Figure 3.13.

Figure 3.16a shows that the positively-twisted flux tube rises through the stable layer into the convectively unstable layer and reaches the top of the domain for all cases surveyed except $B_s = 0.20$. It appears that background field strengths $B_s > 0.15$ are sufficient to prevent the rise of a positively-twisted tube. On the other hand, Figure 3.16b shows that a negatively-twisted flux tube rises for relatively weak background field strengths ($B_s = 0.01, 0.025, 0.05$) but does not rise for stronger background field strengths ($B_s = 0.10, 0.15, 0.20$).

These simple plots verify the conclusions of Chapter 2 in these more complex simulations. That is, at low background field strengths ($B_s \leq 0.05$ for this q),

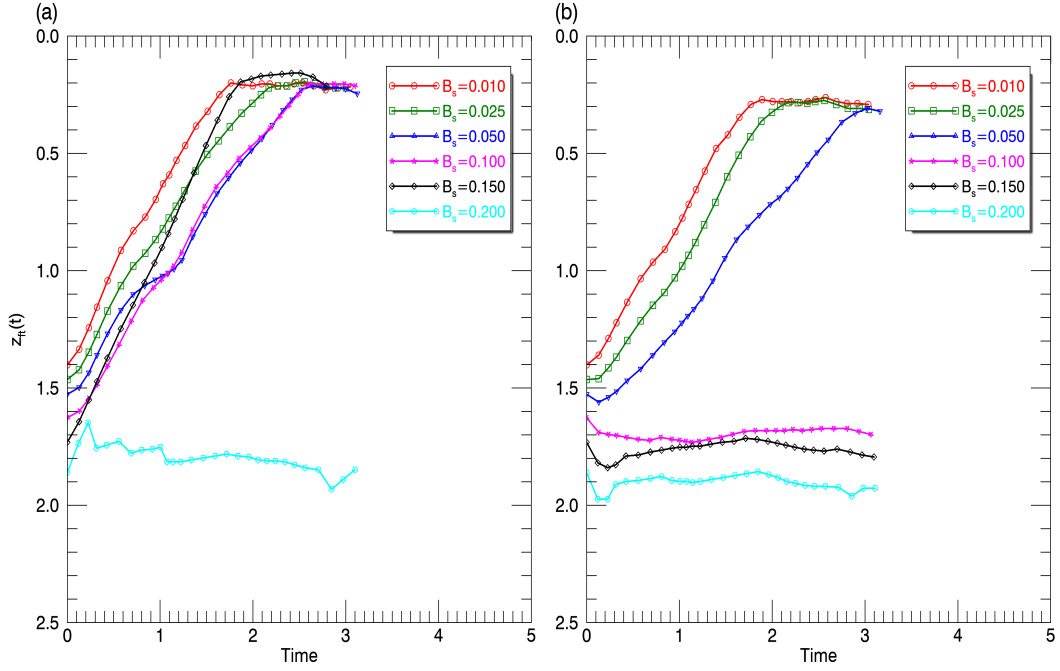


Figure 3.16: Plot of z_{ft} as a function of time for (a) $q = 0.5$ and (b) $q = -0.5$. Each subplot shows five cases with different B_s (namely, $B_s = 1, 2.5, 5, 10, 15, 20\%$ of the initial flux tube strength) at the center of the flux tube.

tubes with both signs of twist can rise uninhibited. At relatively high background field strengths ($B_s \geq 0.2$ for this q), the background field completely quenches the rise of tubes with either sign of twist. For (approximately) $0.05 < B_s < 0.2$, there exists a Selective Rise Regime (SRR) of parameter space, where positively-twisted tubes rise, and negatively-twisted tubes do not.

3.3.6 Dependence on horizontal location and initial timestep

With the role of time-averaged pumped background field in creating an asymmetry between the rise of differently twisted flux tubes firmly established in a canonical case above, it is essential to note that the pumped magnetic layer can have significant local temporal and spatial variations, due to turbulent nature of

the overshooting convection. These variations can potentially affect the initial background state that the flux tube experiences quite substantially. One could argue that the results that we presented earlier were not necessarily representative since they may be a result of a favorable (or unfavorable) fluctuation in the initial conditions. Furthermore, as explained and explored in Chapter 2, we might expect substantial variations in behavior, including violations to any “rules” governing the dynamics. This is not a bad thing, since the Solar Hemispherical Helicity Rule that we are trying to understand is itself actually only a statistical rule that is relatively weakly obeyed, in the sense that there are substantial violations to the rule.

With this in mind, and to better understand the implications of initial conditions, we carry out a Monte Carlo-type (MC) study that examines two sources of local variations that can affect the initial background conditions and hence the rise dynamics of the flux tube. Firstly, while the time-averaged pumped field reaches a reasonably well-established statistically-steady profile, the overshooting convection is highly time-dependent, and the associated local thermodynamic and magnetic state are similarly so. Therefore, the local state in which we embed a flux tube may deviate substantially from the time average. Selecting the background state at different times from the magnetic layer pumping simulations (from Section 3.3.3) can change the conditions the flux tube experiences initially. Secondly, even at a particular time and a particular vertical location, there may be substantial variations in the dynamics in the horizontal. Again, local conditions at the point of insertion of the tube can be significantly different from the horizontal mean, deeming the latter unrepresentative. These two types may be essentially equivalent in creating random variations, but we explore both sources of variation here.

The simulations in Section 3.3.2 all started with the horizontal flux tube location at $x_c = 3.0$ (the middle of the horizontal domain) and used a particular timestep, $t = 62.01$ from the simulation described in Figure 3.12d. We now explore the consequences of varying these. At our canonical parameters, for each nominal background field strength B_s (gleaned from the time and horizontally-averaged field), we consider three randomly-chosen different timesteps from the simulations of Section 3.3.3. For each of these three pumped field initial conditions, we run a series of simulations where we add single flux tubes placed at one of five different horizontal locations. Each of these simulations is run with both positively-twisted and negatively-twisted tubes. Overall, this amounts to 15 separate cases of flux tube dynamics for each type of twisted tube that somewhat randomly sample different background dynamics.

Figure 3.17 shows z_{ft} as a function of time utilising the initial tube insertion level nominally corresponding to $B_s = 0.10$ for all the MC cases at the canonical parameters. Panels a, c, and e are for positively-twisted flux tubes at the three chosen times, $t = t_1$, t_2 , and t_3 , where $t_1 = 35.62$, $t_2 = 48.78$, and $t_3 = 62.01$, chosen from the canonical pumping simulation exhibited in Figure 3.12d. Panels b, d, and f are the same times for a negatively-twisted flux tube. Each panel in Figure 3.17 contains five traces of z_{ft} corresponding to the five different horizontal locations of the inserted flux tube chosen (non-randomly) as $x = 1, 2, 3, 4$, and 5 .

Some general conclusions can quickly be reached from this Figure. Studying any individual panel shows that there can be significant variations in the rise characteristics due to the horizontal positioning of the flux tube. For example, Figure 3.17a, using the $t = t_1$ timestep as an initial condition, shows that embedding a flux tube at different horizontal locations makes some difference to the subsequent dynamics. The flux tube embedded at $x_c = 1.0$ takes a relatively long time to

reach the top of the convective layer, whereas the flux tube at $x_c = 2.0$ reaches the top much faster. Other horizontal locations lead to very similar rise times to each other, with this rise time intermediate between the fastest and slowest. Looking at the other timesteps, we see very similar rise behaviour (to both each other and to most locations in timestep $t = t_1$) from most locations in Figure 3.17c, although now the tube initiated from position $x_c = 5$ rises significantly more slowly. Picking yet another initial timestep $t = t_3$ in 3.17e leads to similar overall behaviour except that in this case, the flux tube initiated at $x_c = 2.0$ initially rises but then is halted completely and moved back to the overshoot layer. If the total dynamics from this simulation are examined in detail, we find that, in this unusual case, a strong convective plume disrupts the pumped layer locally such that the rising flux tube experiences an unusually thick layer of the pumped field during its rise, and therefore fails to rise. This may need to be framed slightly differently. The case being discussed is $S = 7$ and the unusually thick layer is only convincingly visible in $S = 3$ case. It's just a downward convective plume that messes up the rise for this particular case. Fluctuations in the convective magnetohydrodynamics are responsible for most of these fluctuations. While the dynamics in individual cases may somewhat differ due to such fluctuations, the broader conclusion is that the vast majority of the cases initiated with a positively-twisted flux tube lead to a successful rise of the flux tube. Similarly, from Figures 3.17b, d and f, we can quickly see that, for the initial vertical location associated nominally with $B_s = 0.10$, all of the negatively-twisted flux tube cases even more consistently failed to rise. Regardless of timestep or horizontal location of initiation, all of these flux tubes remained in the overshoot layer.

Overall then, the MC set of simulations represented by Figure 3.17 does indeed reinforce the idea discussed in Section 3.3.4 that there are strongly different

dynamics depending on the sign of the twist of the flux tube. One sign of the twist (positive in this case, with the orientation of the pumped background field fixed as the positive direction) is statistically far likelier to rise than the other. While there are variations in rise times and even violations to this rule due to the precise location of the origination of the flux tube in space and time and therefore the exact magnetic-convective conditions the tube experiences, statistically the result is clear.

Thus far, we only examined one vertical location, corresponding to one nominal average pumped background field strength, $B_s = 0.1$, in this more statistical MC manner. Figures 3.18 and 3.19 are the same as Figure 3.17 but for $B_s = 0.01$ and 0.20 , respectively. From our discussion in Section 3.3.4, specifically from Figure 3.16, we know that a relatively weak background field strength ($B_s = 0.01, 0.025, 0.05$) at the flux tube initiation site can lead to a successful rise for both positively- and negatively-twisted tubes. Similarly, a strong background field strength ($B_s = 0.20$) at the flux tube injection location can lead to an unsuccessful rise in both cases. We confirm that these results are statistically representative with these new Figures.

Figure 3.18 shows that most of the flux tubes initiated in the presence of a $B_s = 0.01$ background field successfully rise through the stable layer into the convective layer and reach its top. Of course, some variations in the background state do affect the rise of individual flux tubes. For example, the tube initiated at $x = 5$ in panel a rises more slowly than average. However, in general, all the positively-twisted tubes in panels a, c, and e rise relatively consistently. In this weak background field case, this is also relatively true for negatively-twisted tubes in panels b, d and f. However, there is greater variation for negatively-twisted tubes. For example, the tube initiated at $x = 1$ in panel b (red marker) and the

tubes initiated at $x = 2, 5$ in panel d (green and black markers) show negatively-twisted tubes at different horizontal locations that rise much more slowly. There is even one case where there is no rise at all (see green marker in Figure 3.18f). That the variations for the negatively-twisted flux tubes are more obvious might be expected, since, according to Chapter 2, these tubes add a tension effect that reduces the likelihood of rise, and possibly even quench the rise if the state is close to marginal buoyancy, whereas the existing rise is only enhanced (to varying degrees) in positively-twisted tubes.

Figure 3.19 exhibiting the MC runs for $B_s = 0.20$, simply shows that all the cases studied in the presence of a strong background field fail to rise, as was predicted in Chapter 2 (for dynamics in the absence of convection). Fluctuations in these cases are minimal since, with the immediate defeat of the rise by the strong background field, the flux tubes do not interact with overshooting convection much at all.

Section 2.3.7 of Chapter 2 discussed in detail the possible mechanisms that can lead, in the absence of convection, to variations and violations in the selection mechanism that might contribute to the observational scatter in the SHHR. In the presence of convection and a dynamically-formed pumped field, variations in background state can clearly be a significant additional source of scatter.

3.3.7 Tension forces

That these simulations seem to demonstrate a selection mechanism akin to the one suggested in Chapter 2 now seems well-established, at least at the canonical parameters. The quick check of the tension force exhibited as Figure 3.15 showed that the explanation in terms of tension forces from Chapter 2 may indeed carry over to the more complicated dynamics here. We now confirm this by plotting

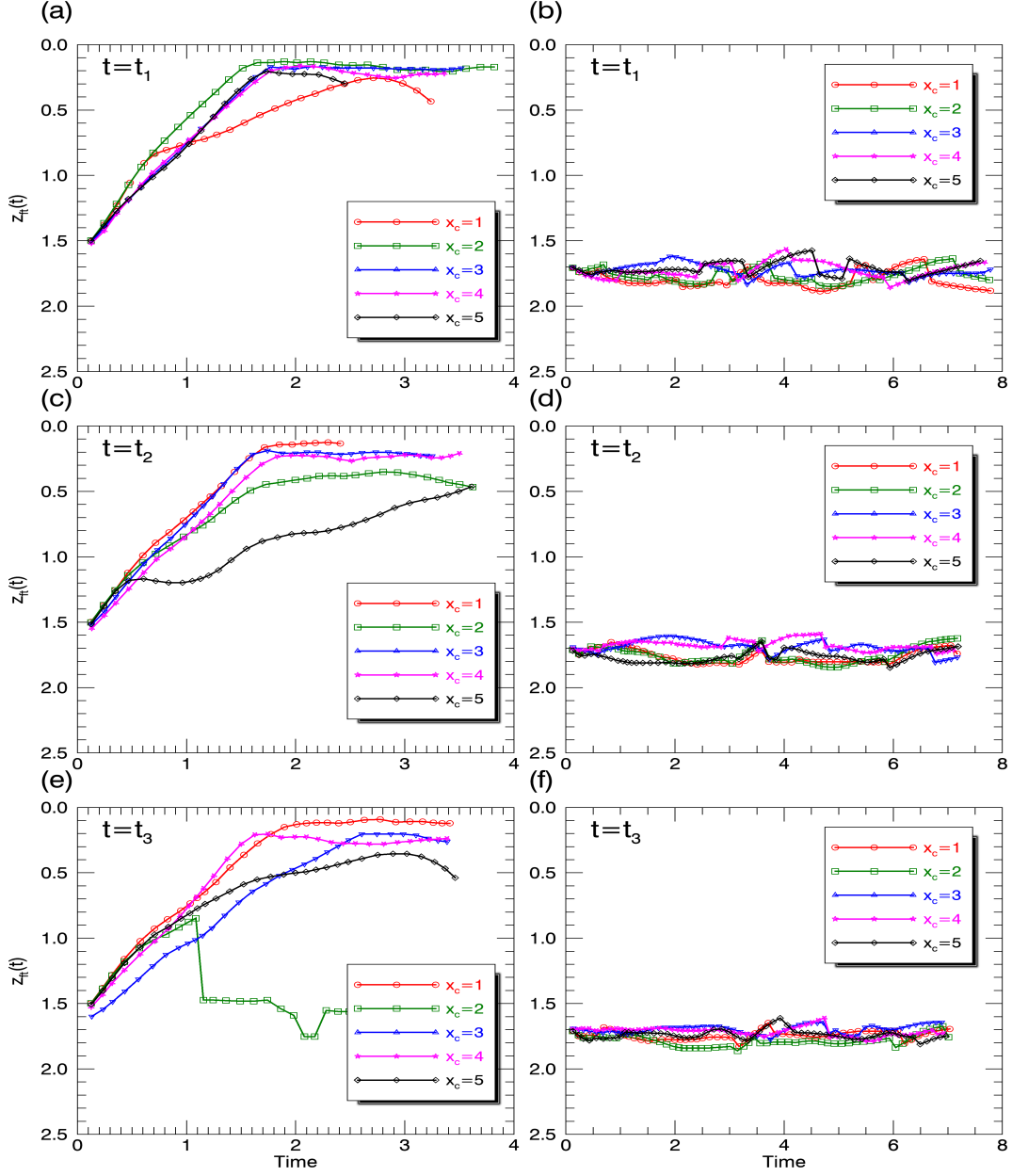


Figure 3.17: Plots of z_{ft} as a function of time showing the successful/unsuccessful rise of the flux tube in the presence of a pumped background field of strength, $B_s = 0.10$. The first column (subplots a, c and e) shows the cases with $q = 0.5$ whereas the second column (subplots b, d and f) has $q = -0.5$. Each of the three rows represent different initial conditions chosen from the results of Section 3.3.3 at times, $t_1 < t_2 < t_3$. Each subplot shows five different horizontal locations of the flux tube, x_c .

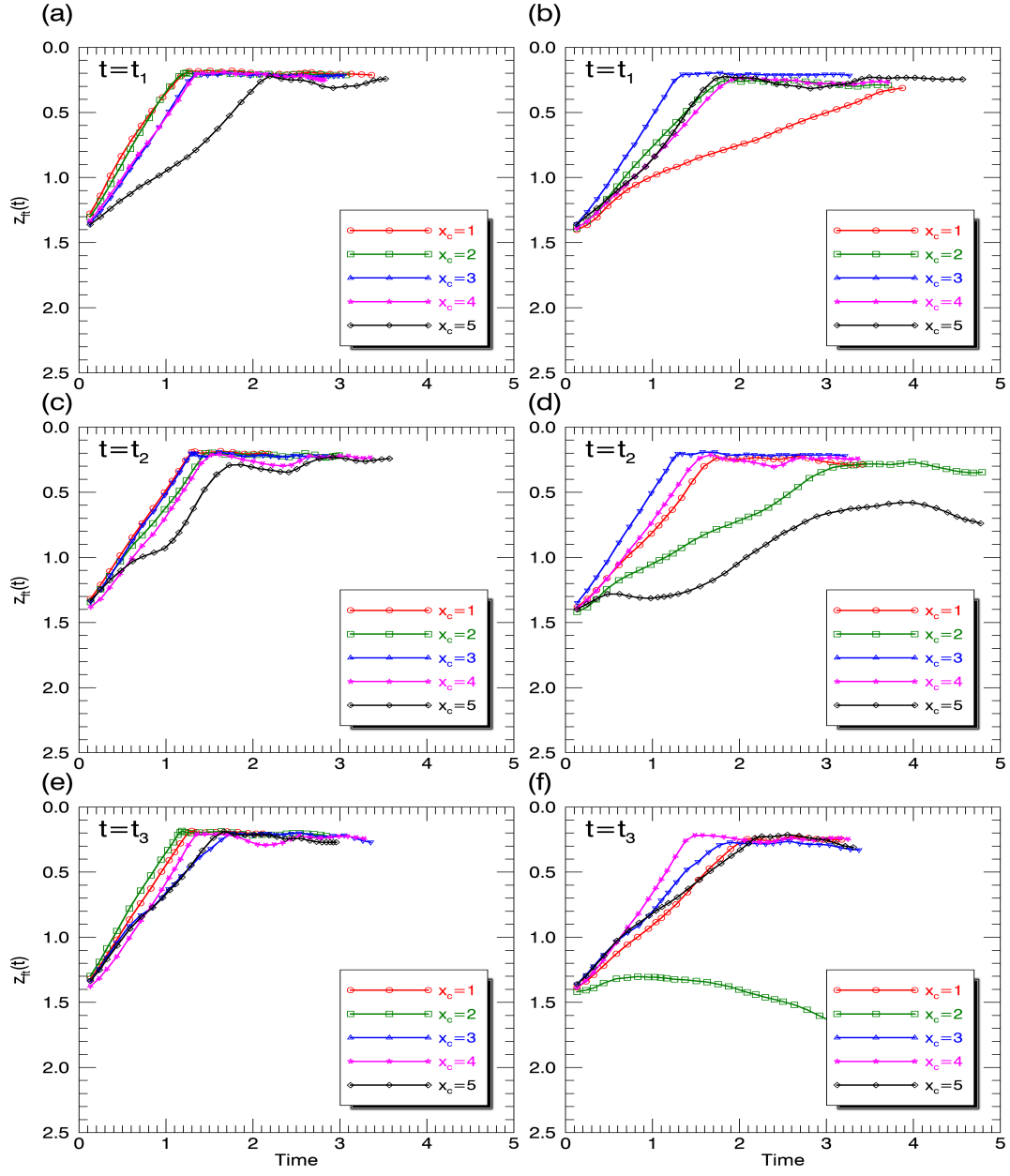


Figure 3.18: Same as Figure 3.17 with $B_s = 0.01$.

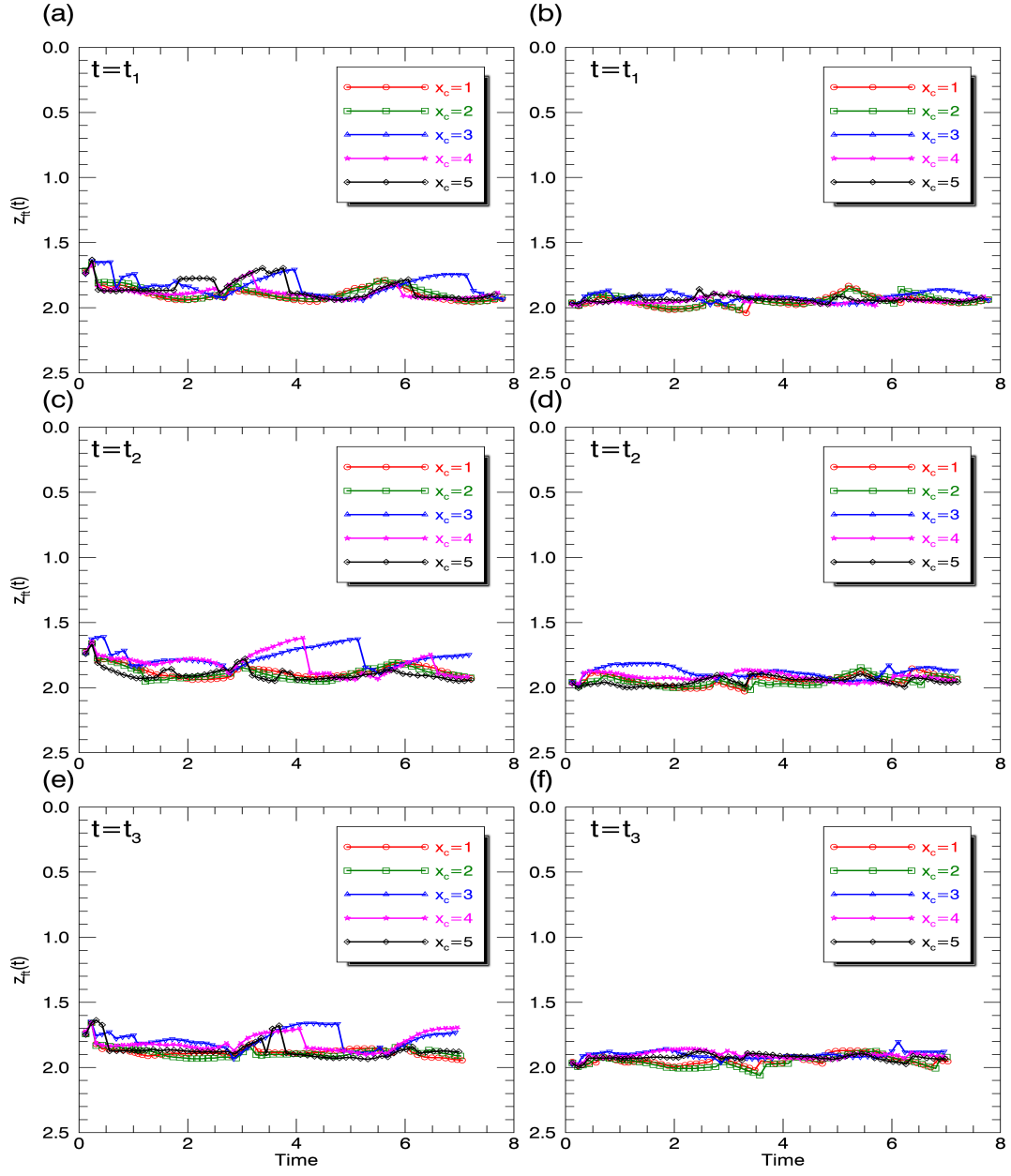


Figure 3.19: Same as Figure 3.17 with $B_s = 0.20$.

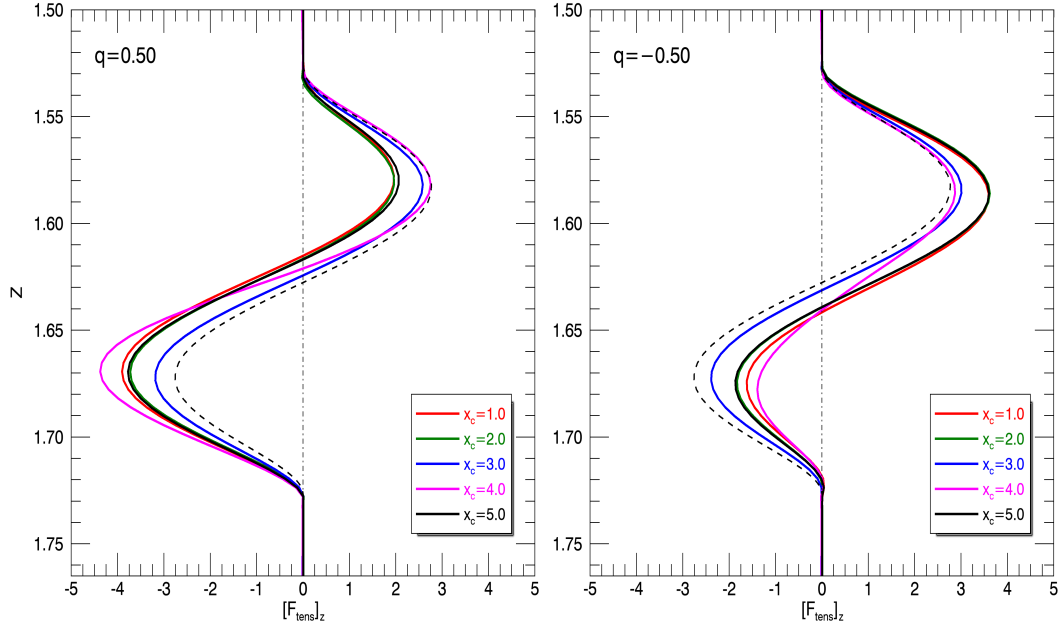


Figure 3.20: Plots of the tension force acting in the z -direction, $F_{z,tens} = (B_x \partial_x + B_z \partial_z) B_z$, from the MC data in Section 3.3.6). The data in this Figure corresponds to that in Figure 3.17e and f. The line cuts of tension force are evaluated at $t = 0.0$ and the center of each tube, for both positively-twisted ($q = 0.5$) and negatively-twisted ($q = -0.5$) tubes. The dashed line shows the profile from the symmetric tube of Section 3.3.2 where no pumped background field is present.

line cuts of the initial tension forces for some of the MC simulations to show that this effect is universal. Figure 3.20 shows line cuts of the vertical tension force for the MC cases previously exhibited in Figure 3.17. These cases are at the canonical parameters, using timestep $t = t_3$ as an initial condition, and the two panels (of both figures) show the results of positively-twisted and negatively-twisted initial tube conditions. Figure 3.20 clearly shows that, for all the cases with a positive twist, there is a net shift of the profile towards more negative (upwards) force values (when compared with the dashed line, which represents a tube with no background field present from Section 3.3.2), and that for all cases with a negative twist, the shift is towards more positive (downwards) values. This MC set of simulations however shows that, despite this net effect, there are significant variations, which can help explain the variations in Figure 3.17. For example, of the positively-twisted tubes, the one placed at location $x = 3.0$ (blue lines) has the slowest rise and can be seen to have the weakest deviation from the symmetric flux tube. These plots also reveal that some of the drastic variations in Figure 3.17 that occur later in the rise are clearly due to advective effects of the convection. For example, the positively-twisted tube placed at $x = 2.0$ has a substantial bias in the tension force, and initially rises strongly due to this, but meets a convective downflow on entering the convection zone that overcomes the buoyancy and tension effects and drives it back downwards. It now seems convincingly proven that the selection mechanism does indeed operate and certainly lends a bias to the rise statistics, but the convection provides an additional source of significant variation.

3.3.8 Variation with S

The previous simulations have all been carried out at a canonical set of parameters including a stiffness parameter of $S = 7$. We now examine the implications of varying the stiffness, S . Changing S changes the polytropic initial stratification of the lower stable layer, with increasing S leading to increased buoyancy deceleration in the stable layer. This leads to a reduction of the extent of overshooting convection in the stable region, as was mentioned earlier and was shown in Figure 3.4, and as was investigated in detail in the past in [19], for example. If a pumping simulation is performed by adding a magnetic layer as an initial condition as in Section 3.3.3, then the depth to which the magnetic layer is pumped is also reduced at higher S for layer strengths that are relatively passive (high plasma β) since the pumping depth is then strongly related to the extent of overshooting. This was investigated in detail in [109]. As the form of the pumped background field forms a key part of our initial conditions in which we embed the flux tube, and plays a crucial role in the subsequent evolution of a flux tubes as found above, it is essential to ascertain the role of S .

In this section, we carry out a suite of Monte Carlo-like simulations, similar to those in Section 3.3.6, but for $S = 3$ and 15. We keep all the other parameters at the canonical values: $Ra = 4 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$. A number of preliminary simulations must be completed to create a meaningful initial condition, as for the $S = 7$ canonical case. We first take the simulations of purely hydrodynamic overshooting convection from Section 3.3.1 that were performed at $S = 3$ and 15. As in Section 3.3.2, we then perform a number of simulations at each S where we add an *isolated* flux tube to the convection at various different Q values to determine the necessary value of Q that dictates that a flux tube will rise through the convection. Figure 3.21 shows the characteristic rise plots for the new S values

(equivalent to Figure 3.9 for the $S = 7$ case). From these plots, we choose to use $Q = 4 \times 10^7$ for both cases. For both $S = 3$ and $S = 7$, the tube clearly rises quickly at this Q . For $S = 15$ the tube rises, but is slower. When choosing Q for $S = 7$ before, we chose the case at $Q = 2 \times 10^8$ that exhibited a very clear rise. Here, we choose a value of $Q (= 4 \times 10^7)$ that is a definite rise for $S = 3$ but is closer to a marginal rise for $S = 15$. Being close to the marginal state is not necessarily a bad thing, as then the effects of tension can be highlighted, if they exist, by clearly creating distinctive rise or no-rise realisations.

The next step in preparing the ultimate initial conditions is to create a pumped layer. We take the statistically-stationary state of the purely hydrodynamic turbulent convection calculations (without a flux tube) again and introduce a magnetic layer for the two new S values, as was done for $S = 7$ in Section 3.3.3. In each of these cases, we evolve the layer for a significant amount of time and, again, the overshooting convection pumps the magnetic layer into the stable region, albeit with significant differences. Figure 3.22 shows a snapshot of the final pumped state for each S (equivalent to Fig. 3.11 and Figure 3.23 shows the time- and horizontal-average of this state (equivalent to Fig. 3.13). It can be seen that the depth at which pumped layer is formed is different, as is its width. For the lower $S = 3$ case, the extent of overshooting is larger than that for the earlier $S = 7$ case, leading to a thicker and deeper pumped magnetic layer. For the $S = 15$ case, a more compact pumped layer is generated, located higher up in the stable region due to the reduced overshooting in this stiffer case.

We now have all the knowledge and initial conditions necessary to run simulations for $S = 3$ and $S = 15$ that involve turbulent overshooting convection, its associated pumped field, plus a flux tube as an initial condition. We choose to run simulations only for an intermediate value of the relative background field

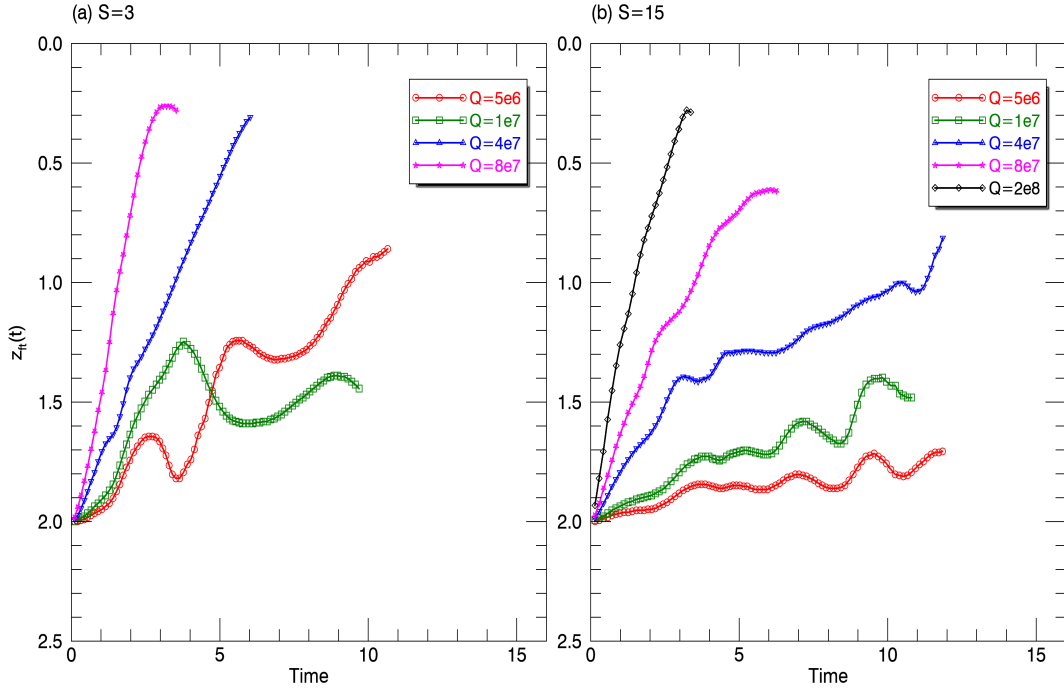


Figure 3.21: Same as Figure 3.9 but for (a) $S = 3$ and (b) $S = 15$.

strength, $B_s = 0.1$ (relative to the twisted azimuthal field amplitude), where selective dynamics are most likely. This strength ($B_s = 0.10$) corresponds to the vertical locations, marked on Figure 3.23 for each case. We run a suite of new simulations for each S starting from a random late timestep of the appropriate pumping simulations (which we denote arbitrarily by $t = t_1$), initiating flux tubes at the desired vertical location and at one of the five different positions for each, as in Section 3.3.6.

Figure 3.24 is similar to Figure 3.17 but for $S = 3$. Figure 3.24a shows the traces of z_{ft} for the five different horizontal locations of a positively-twisted flux tube. We notice that the dynamics of each case are markedly different and reveal some interesting characteristics of the impact of overshooting convection on the rise of the tube. These can be explained as follows.

Tubes located at $x = 1$ and 2 (red and green markers) rise through the stable

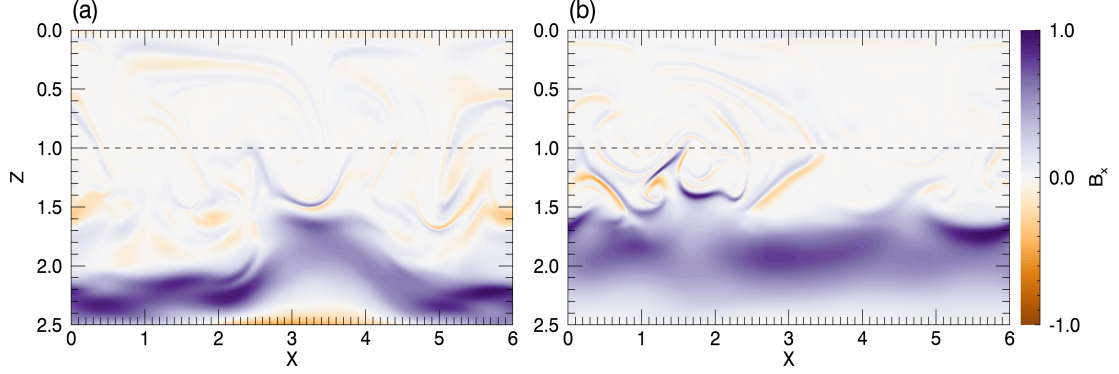


Figure 3.22: Snapshots of normalized horizontal magnetic field, B_x , for (a) $S = 3$ and (b) $S = 15$, at time $t = t_1$. Note that t_1 represents different times in both the subplots. Here, we have $Q = 4 \times 10^7$.

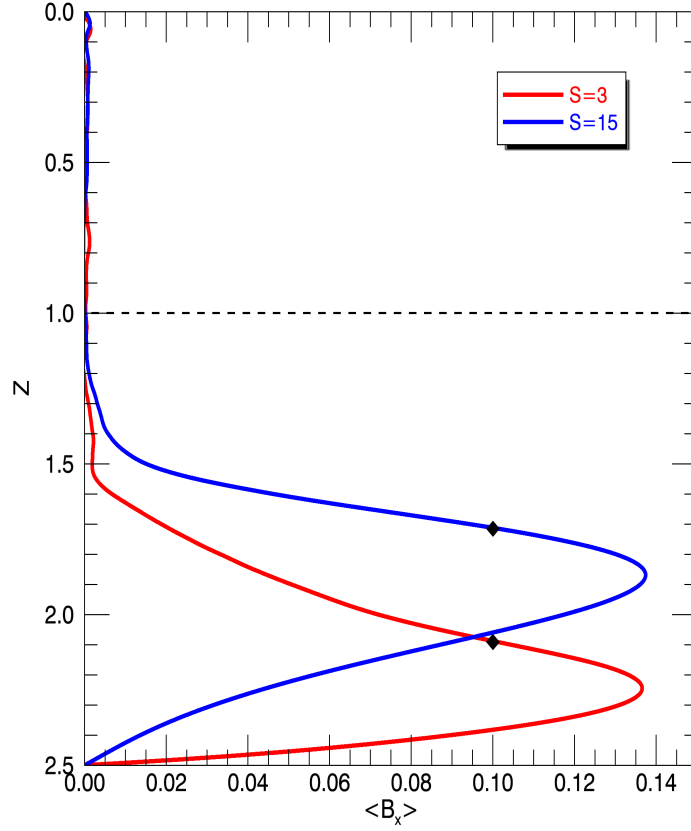


Figure 3.23: Time and horizontally averaged pumped field as a function of z for $S = 3$ and 15. Diamond markers indicate the vertical location corresponding to $\langle B_x \rangle = B_s = 0.10$.

and convective layer and reach its top, but the rise is slow compared to any of the cases studied in the previous section for $S = 7$. These flux tubes take about twice as much time to reach the top of the convective layer compared to any of the previously discussed successful rise cases. There are significant reasons for this. First, the initial Q for this case is weaker than any of the previous simulations, which results in lower initial buoyancy imparted to the tube. Second, the deep overshooting convective motions in this case influence the directness of the rise of the tube, making its path to the top of the convective layer more convoluted, thus taking more time. Additionally, the flux tubes are rising from a vertical location deeper in the stable layer, so the distance they have to cover during their rise is comparatively longer.

Flux tubes located near the center of the horizontal domain ($x = 3$ and 4) have a similar z_{ft} profile, each of them showing that the rise is unsuccessful. The influence of deep overshooting convection and its impact on the pumped layer is stark here. Figure 3.22a shows the background pumped field that forms the initial condition (taken at $t = t_1$) in which the flux tube has been embedded. Due to the deep overshooting motions, the pumped layer gets lifted at horizontal locations roughly between $x = 2$ and 4.5 . This increases the effective thickness of the pumped layer through which the flux tube has to rise when it is inserted at the $B_s = 0.1$ vertical location deduced from the average. The local vertical profile of B_x at this horizontal location is substantially different from that average. This hampers the rise from these two locations sufficiently to make them actually fail.

The tube located at $x = 5$ also has an interesting trajectory. In this case, we see that the flux tube rises quickly through the stable layer and enters the convective layer. However, it soon interacts with a downward convective plume and is advected downwards back into the stable layer, resulting ultimately in an

unsuccessful rise.

From the limited set of cases explored here, we have two examples of the successful rise of flux tube, another where the tube starts to rise but eventually fails, and the other two that fail dramatically. These are intriguing results that show how local fluctuations can impact the overall dynamics of the flux tube, and in the solar context, possibly contribute to the SHHR scatter. The fluctuations in the overshoot layer in this $S = 3$ case are substantial.

In contrast, Figure 3.24b shows identical simulations to those discussed for panel a but where negatively-twisted flux tubes are added instead of positively-twisted tubes. Here, very clearly none of the tubes rise. The retarding effect of the negative twist is strong enough that no fluctuations in the overshooting or pumped layer are sufficient to allow rise. There is clearly a significant difference between positively- and negatively-twisted tubes in this $S = 3$ case, commensurate with what was observed in the $S = 7$ case, and the ideas of Chapter 2.

Examining the case with a relatively stiffer stable region at $S = 15$, the results are far more consistent between realizations and still commensurate with the selection mechanism ideas. Figure 3.25 shows the traces of z_{ft} for positively-twisted (panel a) and negatively-twisted flux tubes (panel b). Each panel again shows the five cases corresponding to the different horizontal locations of the flux tube. Figure 3.25a shows that each individual case is slightly different due to the local background variations that the tube experiences. However, in this case, all the flux tubes regardless of horizontal location successfully rise through the stable layer into the convective layer and eventually reach the upper boundary. In contrast, once again, Figure 3.25b shows that none of the negatively-twisted tubes rise when $B_s = 0.10$. The greater regularity here can be attributed to the closer correlation between the time-average field and the local conditions that the

tubes experience. Figure 3.22 shows that the overshooting convection motions only weakly influence the pumped field layer in the stiffer $S = 15$ case.

Again, we can check that the ideas of Chapter 2 regarding tension forces are indeed responsible for these effects. Figures 3.26 and 3.27 show line cuts of the tension forces through the MC sets of simulation initial conditions for $S = 3$ and $S = 15$ respectively. In all these cases, the expected tension biases do indeed occur, with positively-twisted tubes contributing a rise-enhancing force, and negatively-twisted tubes creating a net tension that opposes buoyancy. Again, the MC sets show variations which can account for the individual variations in rise times. It should be noted that the variation for $S = 3$ is far greater than that for $S = 15$, leading to some of the less common effects after the initial rise seen in Figure 3.24a, for example.

In this section, we have shown how changes in the stiffness of the stable region impact the overall rise dynamics of flux tubes. There exist some significant differences, such as the variation of the overshoot depth, and therefore the depth and form of the pumped field profile, which were previously known. Here, we have found that these can have a fairly substantial impact on the rise dynamics of individual tubes, but overall, the selection mechanism still operates to enhance the statistical likelihood of rise of tubes with one sign of twist (here positive) over the other (here negative). The more significant variations in the overshooting and pumped layer in less stiff layers (e.g. $S = 3$) lead to greater variability in the agreement with this average result for lower S cases. Higher S cases conform to the selection rules more uniformly.

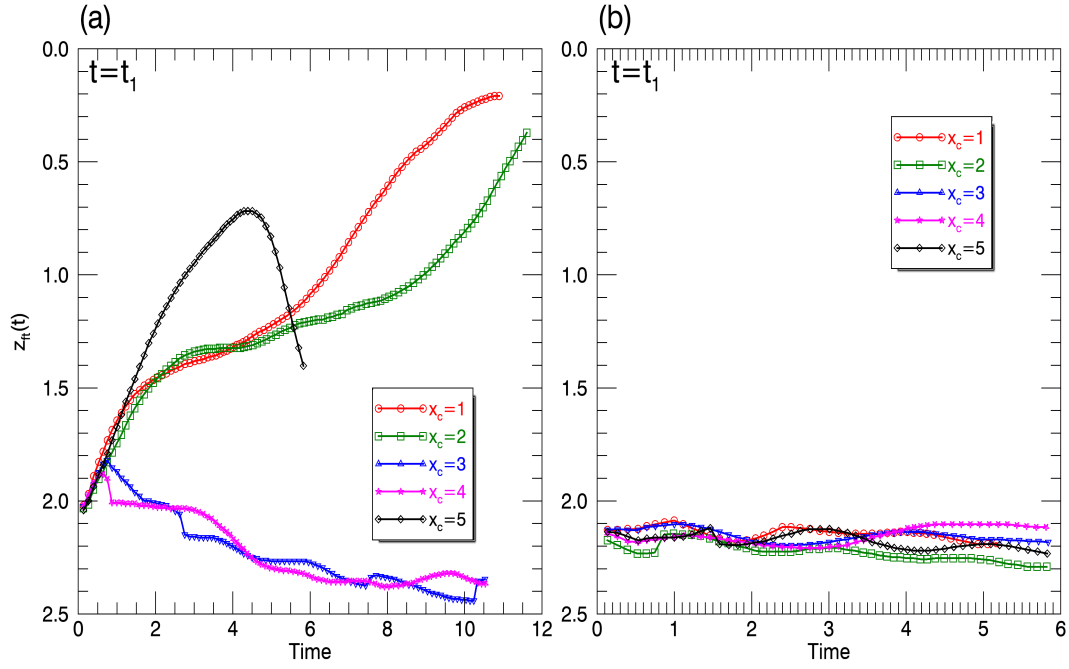


Figure 3.24: z_{ft} plotted as a function of time, at different x_c , for twists, (a) $q = 0.5$, and (b) $q = -0.5$. Here, $t = t_1$ indicated the time at which the pumped background field was inserted with a flux tube. Note that $S = 3$ for this case. Here, $Q = 4 \times 10^7$.

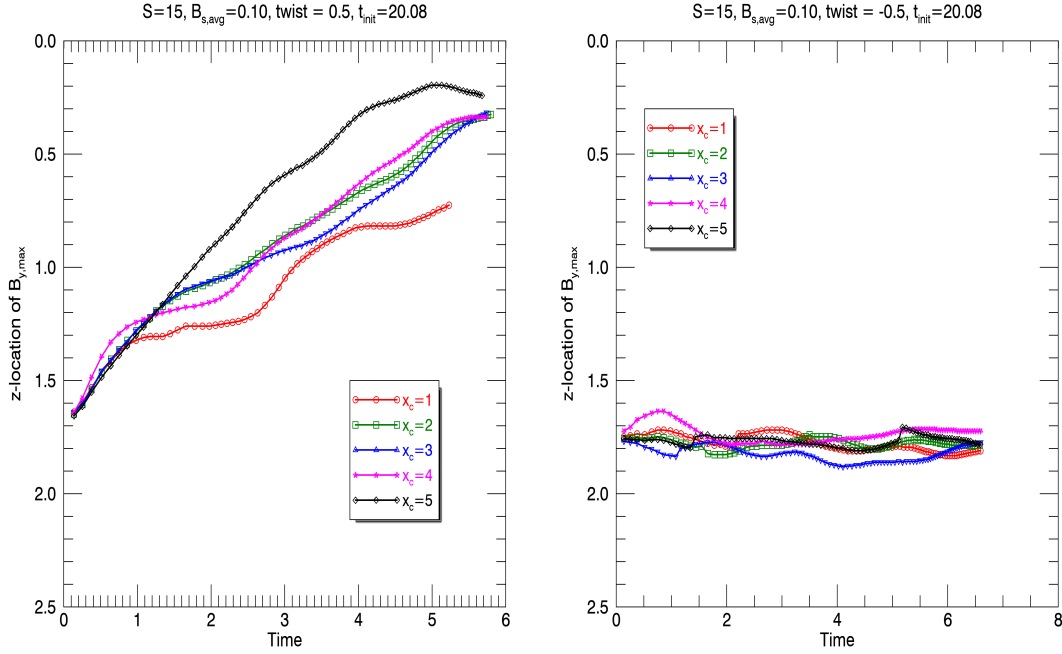


Figure 3.25: Same as Figure 3.24 but for $S = 15$.

3.4 Discussion

In this chapter, via a series of numerical simulations, we have clearly established an asymmetry between the buoyant rise of oppositely twisted flux tubes when in the presence of overshooting convection and a dynamically-formed large-scale background field. Specifically, we find that a positively-twisted (anticlockwise) flux tube is more likely to rise than a negatively-twisted (clockwise) tube when the rise is in the presence of a pumped field that is, on average, oriented horizontally in the positive direction. This result affirms the existence of a selection mechanism based on tension effects akin to the non-convective case described in Chapter 2. This mechanism operates because of the relative orientations of the azimuthal field of the tube and the horizontal background field. For a positively-oriented background field, the azimuthal field of a positively-twisted flux tube is enhanced on the lower side and reduced on the upper side, leading to a net

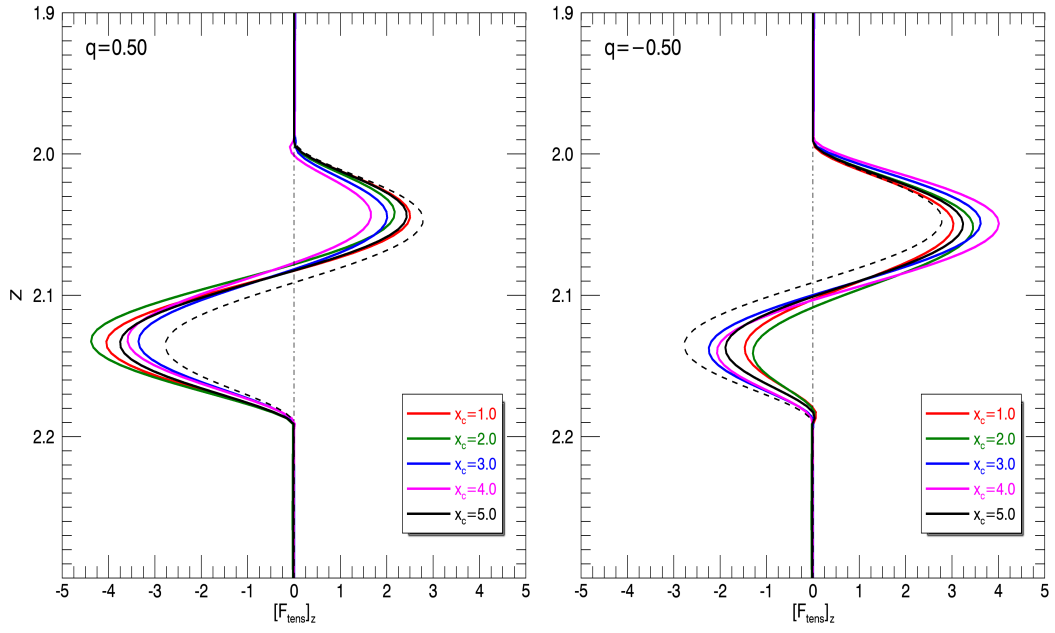


Figure 3.26: Plots of the tension force acting in the z-direction, $[F_{tens}]_z$ on line cuts through the middle of the tubes at $t = 0$ for various different x_c and both signs of q : $q = 0.5, -0.5$. This case corresponds to the $S = 3$ MC cases of Figure 3.24a and b.

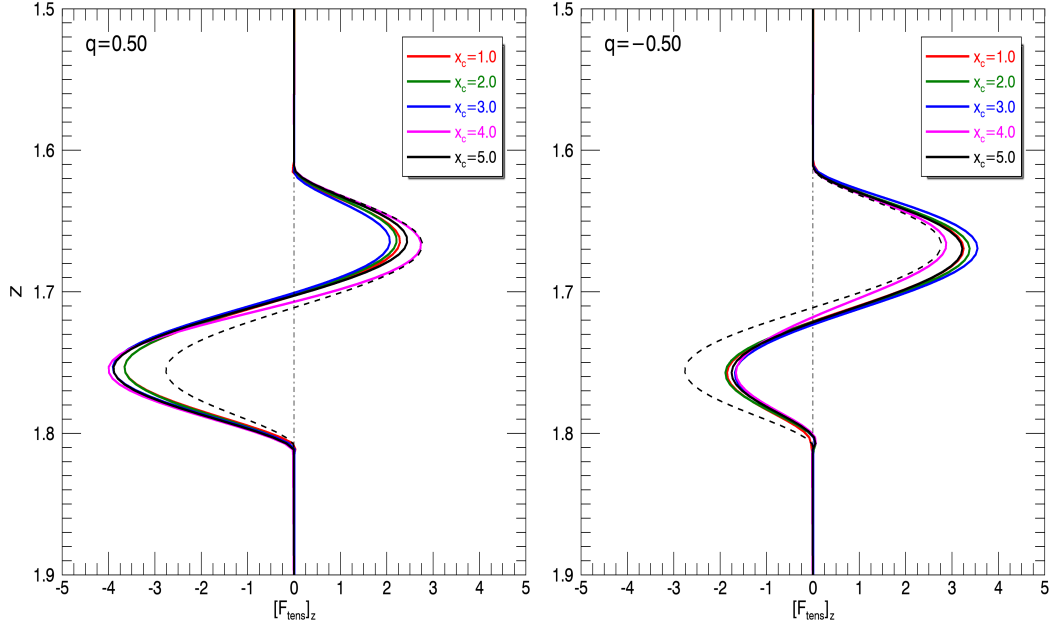


Figure 3.27: Same as Figure 3.26 but for MC cases at $S = 15$ that correspond to those in Figure 3.25.

tension force that acts upwards in concert with buoyancy, encouraging rise. For negatively-twisted flux tubes, the resultant net tension force is downwards, acting against buoyant rise. As explained in Papes 1 and 2, this selection mechanism, when related to the solar context, is in agreement with the solar hemispherical helicity rule(s).

We find that the effect of convection on the *initial* rise dynamics of the flux tubes is weak. The reason for this is mainly because the initial rise characteristics are set by tension effects generated by the combination of the flux tube field with the field of the pumped layer, as described above, and the fact that the pumped layer peaks where the overshooting convective power drops towards zero. Therefore the selection mechanism only happens in a region of weak convection. Because of this, examining the tension forces of the initial conditions is a strong predictor of at least the initial rise (or lack of rise).

Convective effects are not necessarily insignificant overall though. In order to assess this issue, we performed out a detailed set of MC simulations to establish the statistical relevance of the selection mechanism described. We studied the effect of temporal and spatial variations in the convection and its associated pumped background field. The overshooting process is quite intermittent, with some strong events that can lead to significant perturbations in the local environment that influences the tube’s rise dynamics. This can be strong deviations of the local pumped field away from the average, or strong advective effects. These can lead to less predictable outcomes. Furthermore, even if the tube initially rises, as it enters more vigorous convection in the overshoot zone and the actual convection zone, it can be subject to advective forces that can significantly affect the rise. Overall, the presence of convection is a significant source of variation in the *overall* rise dynamics of the tube. However, the ultimate conclusions are the same: that there is a definite difference in the dynamics depending on the twist of the flux tube (for fixed orientation of the average background field), and that the rise of one (positive, in this case) is preferred.

This work also confirms other elements of the predictions of Chapter 2 under the more complex dynamics examined here. For example, Chapter 2 predict that there is a lower threshold in the strength of the background field below which both types of twisted tubes will rise, and another upper threshold in above which both types of tubes will fail to rise, thereby confining the selective dynamics to a region in parameter space that Chapter 2 deem the Selective Rise Regime (SRR). Here, we statistically confirm the presence of this region in the convective dynamics.

The role of the stiffness parameter S was also investigated. We performed a MC suite of simulations and a statistical analysis of the presence of the selection mechanism for $S = 3, 7$ and 15 , covering the range from a weakly-stable lower

region to a fairly strongly-stable lower region. For lower S , the motions overshoot a substantial fraction of the convection zone depth into the stable lower layer, thereby creating a much deeper and somewhat wider pumped layer, whereas for higher S the overshoot and pumped layer are shallower and more compact. Regardless, the selection mechanism appears to operate consistently at all S . At lower S , due to the greater degree of fluctuating dynamics that the tube must encounter in any attempted ride, the variations in rise dynamics are greater than at higher S , where agreement with the selection rules is more uniform. It is an interesting question as to what is the correct S for the solar context. Early (and more recent) works [84, 38] on fitting polytropic models to solar data put the adiabatic index of the stable region in the range $m_2 = 3 - 4$. This corresponds to $S = 3 - 5$ here. However, many researchers quote an S that is much higher, usually based in the fact that, in the turbulent stationary state, convection relaxes the upper layer to an adiabatic state away from any boundaries. This would imply $m_1 = 1.5$ and therefore an effective “turbulent” S of infinity in our definition. We think this logic is a bit misleading, but have nonetheless erred on the side of including simulations at higher S to accommodate researchers who think this way. Regardless, the $S = 15$ case is very similar to the $S = 7$ in all respects. Perhaps the more important distinction is that our simulations are not at high enough Ra to be fully representative of the solar context. Note that increasing S also increases the plasma β of the dynamics at fixed Q . We have considered flux tubes of fixed strength, both in the amount of twist and the axial field strength, and thereby the flux tube has a fixed (initial) magnetic pressure. For larger S , this implies a large plasma β since the gas pressure is increased at fixed magnetic pressure. This can be compensated for via a larger Q . However, note that the choice of Q cannot be arbitrarily large as the thermodynamic quantities need to

be adjusted accordingly to preserve total pressure balance and this sets a limit.

Of course, there are many caveats to our results. Perhaps the foremost issue is that the flux tubes we examine are not self-consistently generated from the background magnetic field. Our aim has been to examine volume-filling fields where flux tubes are concentrations in the field. The work presented here is a very crude attempt at this. A better approach would be to incorporate a depth-dependent shear below the convection zone, as in the solar tachocline, and thereby to create a layer of B_y from the stretching of the pumped B_x . The hope would be that this layer would go unstable by magnetic buoyancy instabilities and create rising self-consistent magnetic structures that are truly magnetic concentrations in volume-filling fields. Such simulations in the absence of convection have been performed before (e.g. [112]) and doing this for the current problem is future work.

In terms of parameters, our work could be critiqued in a number of ways. The Rayleigh number is orders of magnitude below stellar values and the Prandtl number is orders of magnitude too large, and therefore our simulations do not operate at anything like astrophysically realistic levels of turbulence. These limitations are numerical. We could have done better values for these parameters using higher resolution in these 2D studies if we had possessed greater patience. These simulations, however, if nothing else, serve as a first insight into mildly turbulent convective effects. The magnetic Prandtl number, $Pm = Pr/\zeta$, which in our case is $Pm = 100$ is also not astrophysical, since, for astrophysical objects, the viscosity should be smaller than the resistivity, leading to $Pm < 1$. We worked with low resistivity but higher viscosity to avoid issues of the magnetic flux tubes diffusing too quickly, but this leads to $Pm > 1$. While having a low P_m would be ideal, the role of P_m is more critical in dynamo simulations.

It is interesting to question what might happen at higher levels of turbulence, and this brings up the complex dependence of the necessary Q on the other non-dimensional parameters. To overcome the increased convective kinetic energy in a high Ra and low Pr scenario, a high Q value is required for the flux tube to rise successfully. Moreover, we expect vigorous convection to transport (pump) magnetic fields from the convection zone toward its base and into the stable region more efficiently. Efficient pumping of magnetic fields is expected to create a relatively strong pumped background magnetic field layer, also requiring a high Q for a flux tube to rise through it successfully. This dependence, where very strong tubes might be required for transit through highly turbulent convection zones, is somewhat unnerving. However, some of this discomfort is caused by the fact that our flux tubes are not self-consistently generated from the background field. If they were, we might expect shear to create toroidal field (B_y) that is substantially stronger than any B_x that is present by stretching, providing a causal link between the convective energy and the energies bound up in the two components of the magnetic field (pumped layer and tubes) below the convection zone. It is essential to note that none of this directly influences the significant result of the chapter, the selection mechanism, since this merely depends on the relative strengths of the azimuthal field in the tube compared to the background field. If background field strengths change, a different range of q may define in the Selection Rise Region. There are some limitations on this, as described in Chapter 2.

The non-astrophysical Pm that we used brings up the more general question of timescales. With the magnetic boundary conditions that we have chosen, horizontal magnetic flux is not conserved in the domain and the pumping simulations can lose the initial B_x and run down. When we achieve a pumped state, the majority of the horizontal field becomes isolated from the boundaries (in most

cases), and almost piecewise linear, and therefore only diffuses and runs down very slowly, in the sense that the rundown is a lot slower than the timescales of the other dynamics that we are interested in. For example, the timescale of the rise is set via Q and we make sure that this is relatively quick compared to the rundown timescale. Our simulations are once again not perfectly astrophysical but we believe that the ordering of timescales is the relevant one, with the rise being slow compared to convective overturning, but fast compared to any significant evolution of the large-scale background field. If we use other boundary conditions, then magnetic flux can be conserved, but field then accumulates temporarily at the upper boundary during the initial evolution, causing unphysical numerical issues.

Similarly, we attempt to get the ordering of length scales correct. The mildness of the turbulence means that an extended inertial cascade of dynamic scales is not realized. However, the most influential issue is probably the relative size of the flux tube compared to the deeper convective dominant scales. Unfortunately there is no direct solar data on either of these scales. We might expect the scales of buoyancy instabilities to be of the order of the depth of the shear (the tachocline), and the strong stratification suggests that the scale of the flux tubes in the deep interior might be far smaller than those observed to emerge at the solar surface. Therefore, it seems likely that flux tubes at initiation are small compared to some characteristic overturning scale of the convection. The flux tubes in our simulations obey this ordering. We suspect, if anything, our flux tubes are likely far larger than they should be, since we choose a scale smaller than the main turnover scale, but large enough to be well-resolved by our numerical grid resolution.

Finally, it is important to acknowledge that the work described in this chapter

has been carried out in 2D as an initial investigation, and, of course, 3D simulations would be more realistic. The 3D simulations are already underway and will be reported soon, but it is worth noting here some differences that might be anticipated. [19] and [108, 111] carried out 3D numerical simulations investigating penetrative and overshooting convection, and the transport (pumping) of magnetic fields and there are some notable differences between 2D and 3D versions of these processes. In particular, the overshooting depth of convection in 2D is significantly larger than in 3D. This is due to the more coherent and space-filling nature of 2D convection. Convection in 2D even at high Reynolds numbers is characterized by large overturning motions. In 3D, the downwards motions are characterized by much more local plumes. The sparse nature of 3D convection leads to reduced mixing below the convection zone, and therefore reduced overshoot. This further means that the pumping mechanism in 2D calculations is far more efficient than in 3D. We find in 2D that the average pumped field is virtually absent in the convective layer; this is not the case in three-dimensional calculations where the overshooting convection transfers the majority of magnetic fields from the convective layer to its base but does not entirely evacuate it. As a corollary, the pumped layer in 2D seems to sit firmly below the overshoot region, whereas in 3D the two seem to overlap more substantially. In terms of the rise of magnetic structures then, we might expect that tubes initiate from a slightly different location in 3D and might experience more interaction with the background field in the convective layer during transit. With regards to the selection mechanism, this suggests that the relative strengths of the background field that prevent a positive and a negative twisted flux tube from rising may be some what different than found here, with lower values likely required. While this is an important detail and deems further investigation, it is not likely that this will impact the

broader conclusion of this chapter, i.e., that there is an asymmetry in the rise of oppositely-twisted flux tubes.

Overall, we have clearly established a case for the selection mechanism of Chapter 2 being relevant to the rise of magnetic structures in the more complex solar situation involving a convection zone. The earlier Chapter established the connection between this mechanism and the SHHR, and it appears that this mechanism could be a contributor to this bias. There are other elements that could also contribute, such as the Σ mechanism suggested by [75], and it remains to be seen what the contribution of each mechanism is to the total helicity budget. The mechanism presented here has some sound theoretical arguments for the statistical error in the SHHR. Indeed, the observed error could even be used to perhaps infer the some of the likely magnetic conditions at the base of the convection if this mechanism were the primary contributor, providing a very useful probe into the deep solar interior.

Chapter 4

Conclusion

This doctoral work’s primary goal was to better understand the dynamics of transport processes that lead to the rise of magnetic fields from the deep interior of the Sun. Our work was motivated by various sets of previous computational and observational studies of the Sun. We were inspired by observations of the magnetic fields at the solar surface, in particular, the nature of sunspots embedded in active regions and their helicity as they emerge into the photosphere. These observations motivate modeling the structure of such emerging flux as helical tubes. Computationally, we were further inspired by previous sets of studies ([112, 113]) that focused on the formation of such tubular structures by instabilities. As well as showing the formation of “tube-like” magnetic structures, these studies revealed another vital aspect of magnetic fields in the solar interior that has typically been ignored in the past for convenience. That is, the magnetic flux tubes formed in these simulations, and probably also the solar interior, are *not isolated* but are instead magnetic flux concentrations embedded in a large-scale background field. We have built the work presented in this thesis on these ideas.

The very simple first models of non-isolated magnetic concentrations studied in this thesis, consisting of a flux concentration rising through a large-scale back-

ground field under various circumstances, have already revealed various exciting new facets of flux tube dynamics, creating a better understanding of transport processes, and even informing us on potential reasons for the surprisingly ordered solar surface observations of helicity.

This work started with that presented in Chapter 2. This work used 2.5-dimensional simulations that examined magnetic flux concentrations rising in the absence of a convective background but in the presence of an artificially-imposed large-scale, volume-filling background field. This work showed that changing the orientation of the large-scale background field changed the rise dynamics of a flux tube of a particular twist. These results have also been confirmed with three-dimensional simulations that are reported in Appendix 5.3. Even with the complicated dynamics of the third dimension taken into account, we show, through a broad survey of parameters, that the relative orientation of the background field and twist plays an essential determining factor in the rise dynamics of 2.5D and 3D flux tubes. This bias in the dynamics is shown to be driven by tension force effects due to the interaction of the background field with the internal tension of the concentration, which depends on the relative orientation of the background field and the twist of the concentration.

This selective nature in the rise of flux tubes led to a deep dive into the dependence of the effect on the parameters and the various observational consequences of these results. Ultimately, our simulations in Chapter 2 now show the existence of a “Selective Rise Regime (SRR)”, which, in conjunction with a set of Monte Carlo (MC) simulations, explains many of the statistical features of the Solar Hemispherical Helicity Rule(s) (SHHR). In particular, we have been able to show that our numerical results reproduce:

1. The correct sign of helicity in both the solar hemispheres.

2. The cycle-independence of helicity rules.
3. The significant scatter in the adherence to the general helicity rule.

Additionally, using the MC simulations, we have explicitly listed scenarios that can lead to SHHR violations, which have the potential to be verified observationally.

Another important feature of observations that is not very well documented but that our simulations reveal, is the *temporal variation of the scatter* inherent in this rule. Our model predicts that, as the solar cycle progresses, the number of violations to the SHHR also changes. In our simulations, the temporal variation of the violations is primarily linked to the changing strength of the background field (assuming that flux tubes are produced with the same random distribution of twist and strength). It could be possible that, using this temporal signature of the violations from the models, constraints could be placed on the strengths and structure of the large-scale interior field, something that is normally frustratingly elusive. A collaborative effort between detailed observations and such numerical simulations could be fruitful here.

These results revive the discussion on the origin of SHHR. The most promising theoretical explanation of the origin of SHHR previously was that proposed by Longcope et al. [75] (LFP). This theory states that the action of rotationally-influenced convective turbulence on a rising flux tube can impart the correct sign of twist to emerging structures, and that the turbulent nature of convection can account for the large scatter in such helicity observations. The authors term this mechanism as Σ effect. While this mechanism may well be very important, a couple of caveats need attention. As the scatter is controlled by convective turbulence, it is unclear what happens to the amount of scatter when the intensity/nature of turbulence in the numerical simulations changes. Additionally, in the Σ mechanism there is no basis for the temporal variations in helicity as the

solar cycle progresses. Observationally, a few studies have focused on the temporal adherence to SHHR [8, 9, 54, 56], but a comprehensive concerted effort is still missing. However, there seems to be agreement that the helicity violations are more prominent towards the transition between solar cycles [83]. This leads us to suggest that the SRR discovered by our flux tube simulations in this thesis likely at least acts in concert with the Σ effect to reveal the overall SHHR observations.

Chapter 3 added another level of realism to our simulations by studying the rise of flux tubes in the presence of a more realistic background state incorporating overshooting convection. This addition also brings into the picture another competing transport mechanism, turbulent magnetic pumping, that serves to self-consistently create the large-scale background field through which the flux tube rises, but also has the potential to disrupt the rise of field. We show that even with this added level of complexity, the SRR persists. Furthermore, the presence of convection adds fluctuations that eventually contribute to the overall scatter observed in SHHR.

A considerable part of this work has already been done in 3D, and the current results are reported in Appendix 5.4. However, more needs to be done in this regard and hence this is still a work in progress. The current status of this work is that, apart from simply establishing overshooting convection in three dimensions at reasonable parameters, we have successfully determined the necessary conditions required for an *isolated* 3D flux tube to rise through this turbulent overshooting convection. These conditions form a pivotal component in the broader work that needs to be completed in 3D. The three-dimensional pumping to self-consistently create a large-scale background magnetic layer is currently in the works and will form the final piece tying the two-dimensional work with the more realistic three-dimensional studies. A preliminary 3D pumping study is

also reported in Appendix 5.4. This preliminary study exhibits the challenges of three-dimensional pumping, in contrast to the 2D studies reported in Chapter 3. Firstly, the overshooting depth in 3D is much smaller than the equivalent 2D simulations (see Figure 3.4 and 5.11 for a comparison). Secondly, the sparse nature of 3D convection, as opposed to the space-filling nature of its 2D counterpart, leads to a significant average horizontal magnetic flux in the convection zone even after evolving the system over multiple convective turnover times. This weaker pumping action in 3D creates numerical complications in achieving an eventual background field profile that is of sufficient strength compared to the flux tube.

Certainly, there are many shortcomings of the simulations presented in this thesis. For instance, a major one is that our simulations have assumed the existence of twisted flux tubes rather than studying their formation. Some studies have shown that flux tubes formed in the deep interior by flux sheet instabilities are naturally twisted. But the distribution of the amount of twist has not been explicitly quantified, and the resulting relative strengths of tubes and the background field are not clear. This offers an opportunity for future studies to examine the self-consistent formation of flux tubes by flux sheet instabilities and their subsequent evolution through the background field and convection.

Our simulations have also not accounted for rotation. As discussed above, the Σ (and other) effects can impart helicity to rising flux tubes. This demands a comprehensive three-dimensional study involving rotation, which may help us explicitly delineate contributions to the total helicity budget of emerging structures from the various different mechanisms proposed. It is important to note, though, that we believe that our mechanism could likely be a significant contributor, since in essence it acts as a filter that operates before the other mechanisms can act. This comprehensive work is in progress and we hope to be able to unravel the

total helicity budget in the near future.

Chapter 5

Appendix

5.1 Robustness of flux tube dynamics

Initial conditions used in Chapter 2 can be set up in multiple ways. Here, we explore the robustness of the flux tube dynamics, as discussed in Chapter 2, to different initial thermodynamic conditions and different initial axial magnetic profiles.

Different initial thermodynamic conditions

The effect of the introduction of a magnetic flux structure into a stratified gas is to add an extra magnetic pressure to the total pressure. However, it is generally assumed that the total pressure equilibrates quickly under the circumstances of interest, thereby adjusting the gas pressure in the magnetic concentration. This change in gas pressure can lead to a change in density, temperature, or both. There is no unique way of specifying the initial thermodynamic conditions inside the magnetic flux tube. In order to check robustness of our results to these different formulations, we have tested two commonly-used sets of initial thermal

conditions. The crucial result to highlight here is that changing these makes essentially no difference to our major results and only very minor differences to any of the dynamics of the problem. We supply some description here to verify this fact, and to provide details of the formulations that we use in the main text (Case 2). Incidentally, we did also examine a case that did not initial obey the condition of total pressure equilibration, which would be considered unphysical by most. Since this case again produced very similar results, it is not included here.

When a flux tube is introduced into the stratification, the assumption of total pressure balance introduced above dictates that

$$p_{gas,in} + p_{B,in} = p_{gas,out} + p_{B,out} \quad (5.1)$$

throughout the interior or at least at the edge of the tube. Here, $p_{gas,in}$ and $p_{B,in}$ are the gas and magnetic pressure inside the flux tube respectively, and, similarly, $p_{gas,out}$ and $p_{B,out}$ are the gas and magnetic pressure outside the flux tube. The latter two are known, given respectively by Equation 2.2 and

$$p_{B,out} = \frac{|\mathbf{B}_{out}|^2}{2} = B_s^2 \exp\left(\frac{y_c - y}{H_B}\right). \quad (5.2)$$

These therefore depend solely on height y . If $B_s \neq 0$, the magnetic pressure inside the flux tube, $p_{B,in}$, depends on the interior magnetic field $\mathbf{B}_{in}$ which includes contributions from both the magnetic field of the flux tube ($\mathbf{B}_{tube}$) and the background field ($\mathbf{B}_{back}$), and so, from Equation (2.6)

$$p_{B,in} = \frac{|\mathbf{B}_{in}|^2}{2} = \frac{1 + 4q^2r^2 - 4q(y - y_c)B_{back} + B_{back}^2}{2} \quad (5.3)$$

where we have used $x^2 + (y - y_c)^2 = r^2$ for simplicity. This is a function of x and

y .

Equations (5.1, 5.3 and 5.2) together dictate an adjusted gas pressure inside the concentration, $p_{gas,in}$. In forming a complete set of initial conditions, it therefore remains to determine how to compensate for this adjustment with the temperature and the density inside the flux structure. We consider two cases.

Case 1: The simplest setup is to assume that both the temperature also equilibrates quickly to match the external thermodynamics. The assumptions of total pressure and temperature equilibration are perhaps the most commonly used ones to explain the existence of magnetic buoyancy in magnetic field concentrations. Under these assumptions, we have

$$p_{gas,in}(x, y) = p_{gas,out}(y) + p_{B,out}(y) - p_{B,in}(x, y), \quad (5.4a)$$

$$T_{gas,in}(y) = T_{gas,out}(y), \quad (5.4b)$$

$$\rho_{gas,in}(x, y) = \frac{p_{gas,in}(x, y)}{T_{gas,in}(y)}, \quad (5.4c)$$

Above, $T_{gas,in}$, the gas temperature inside the concentration, gets set to the polytropic gas temperature outside, $T_{gas,out}$, by the assumption, and then the density inside the concentration is determined by the ideal gas equation from this temperature and the adjusted gas pressure $p_{gas,in}$. Figure (5.1i) shows the properties of these initial conditions (zoomed in to focus on the flux tube and its near exterior).

Case 2: A somewhat less restrictive interpretation of the internal thermodynamics was used by some authors, (e.g. [60]). Here, the temperature is considered merely continuous at the edge of the tube and allowed to vary such that the density and total pressure inside the tube are solely a function of height. To achieve this, the pressure and temperature at the edge of the concentration ($r = \sqrt{x^2 + (y - y_c)^2} = R$) are used to set the adjusted density at the edge:

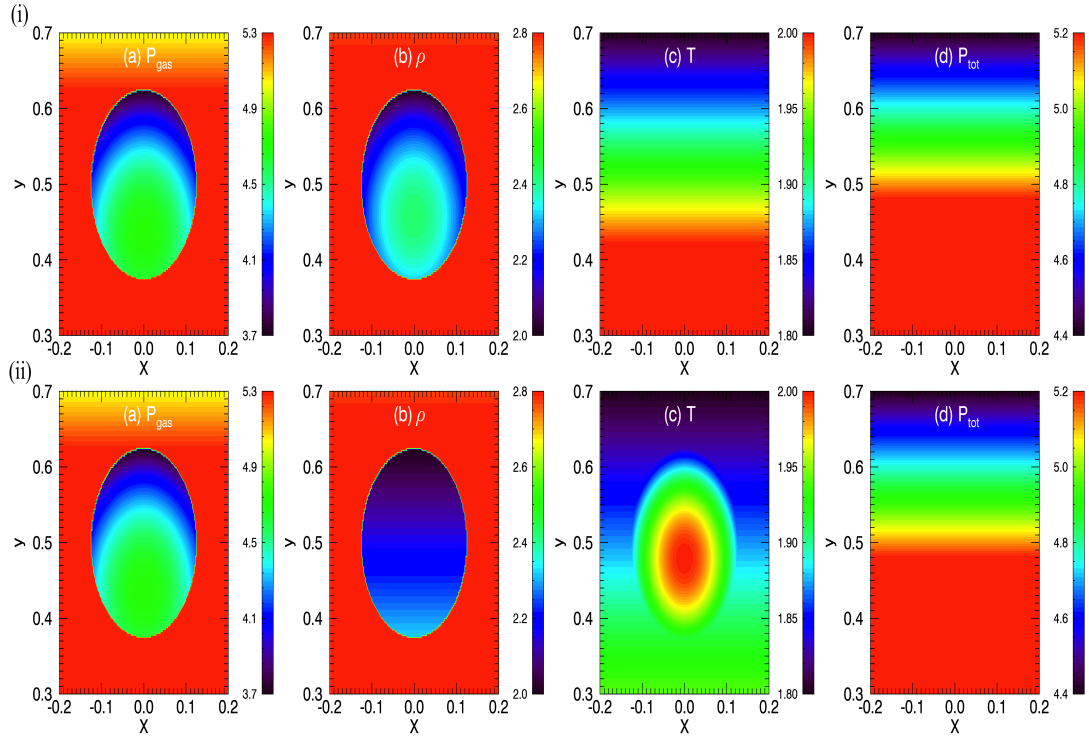


Figure 5.1: Thermodynamic initial conditions for (i) Case 1 and (ii) Case 2: (a) gas pressure, (b) density, (c) temperature, and (d) total pressure. A subset of the domain is shown for clarity.

$$p_{gas,edge}(y) = p_{gas,out}(y) + p_{B,out}(y) - p_{B,in}(x, y)|_{r=R}, \quad (5.5a)$$

$$T_{gas,edge}(y) = T_{gas,out}(y), \quad (5.5b)$$

$$\rho_{gas,edge}(y) = \frac{p_{gas,edge}(y)}{T_{gas,edge}(y)}, \quad (5.5c)$$

This density is then propagated over all x , and, with the adjusted internal pressure from Equation (5.1), used to create the internal temperature:

$$p_{gas,in}(x, y) = p_{gas,out}(y) + p_{B,out}(y) - p_{B,in}(x, y) \quad (5.6a)$$

$$\rho_{gas,in}(y) = \rho_{gas,edge}(y) \quad (5.6b)$$

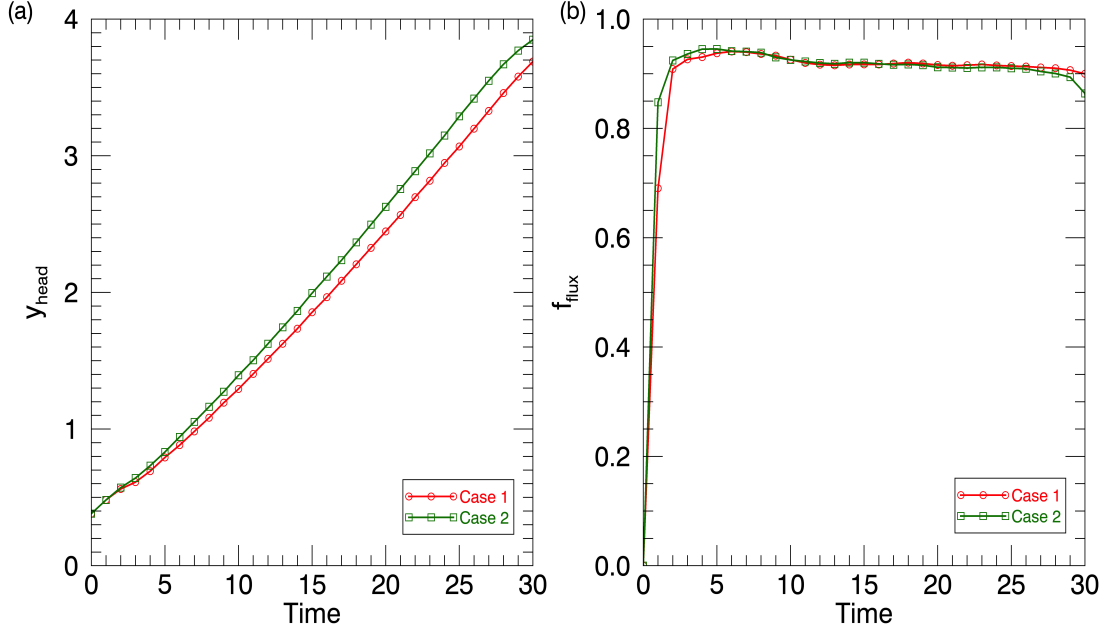


Figure 5.2: Comparison of (a) vertical position of the head of the rising flux tube and (b) rising flux fractions for two different initial thermodynamic conditions for $B_s = 0$. Flux fraction (calculated with $v_{y,\text{threshold}} = 0.03$) in (b) has been curtailed at the time when the flux tube structure reaches the top of the simulation box.

$$T_{\text{gas},in}(x, y) = \frac{p_{\text{gas},in}(x, y)}{\rho_{\text{gas},in}(y)} \quad (5.6c)$$

Figure (5.1ii) shows the properties for Case 2.

Figure 5.2 compares the vertical position of the head of the magnetic flux structure and rising flux fraction from Equation (5.9) for both cases for simulations in the absence of background field ($B_s = 0$). We find the location of the head of the flux tube by measuring the vertical distance to the maximum of B_z on the central plane, $x = 0$, as a function of time. We curtail this plot at the time (denoted by the rise time) when the flux tube structure has reached the top of the simulation box. The two cases have quite different internal density functions inside the initial magnetic structure, since, from above, in Case 1, is a function of x and y whereas for Case 2 it is solely a function of y . Despite these differences in the initial thermodynamic conditions, the time it takes for the flux tube to reach the top

of the simulation box is very similar (see figure 5.2a), with Case 1 being slightly slower since its integrated density perturbation is slightly smaller. The rising flux fractions, as shown in figure (5.2b), are also very similar, showing the usual characteristics of successful rise. Hence, it appears that the simulation results are robust to different interpretations of the initial thermodynamic conditions. We have used Case 2 as our canonical case for all the results of this thesis.

5.2 Gaussian flux tube dynamics

While we explored the dynamics of a “top-hat” flux tube profile (in the axial flux, B_z) in Section 2.3.1, here we check the robustness of our results with respect to a different initial magnetic profile for both $B_s = 0$ and $B_s \neq 0$. In this section, rather than using a top-hat profile, we use a Gaussian profile in radius given by Equation 5.8, as also used in, for example, [27]. The top-hat profile is given simply by

$$B_z(r, \theta) = \begin{cases} 1 & \text{if } r \leq R_o \\ 0 & \text{if } r > R_o, \end{cases} \quad (5.7)$$

whereas a Gaussian profile is given by

$$B_z(r, \theta) = \begin{cases} B_o e^{-r^2/R^2} & \text{if } r \leq 2R_o \\ 0 & \text{if } r > 2R_o. \end{cases} \quad (5.8)$$

We choose B_o such that the integral of B_z^2 over the concentration is the same in both cases, and therefore the buoyancy forces induced by each are also the same.

Fig. 5.3 shows line cuts of these two initial magnetic field profiles through the center of the flux tube at a height of $y = 0.5$. A top hat profile has unrealistically

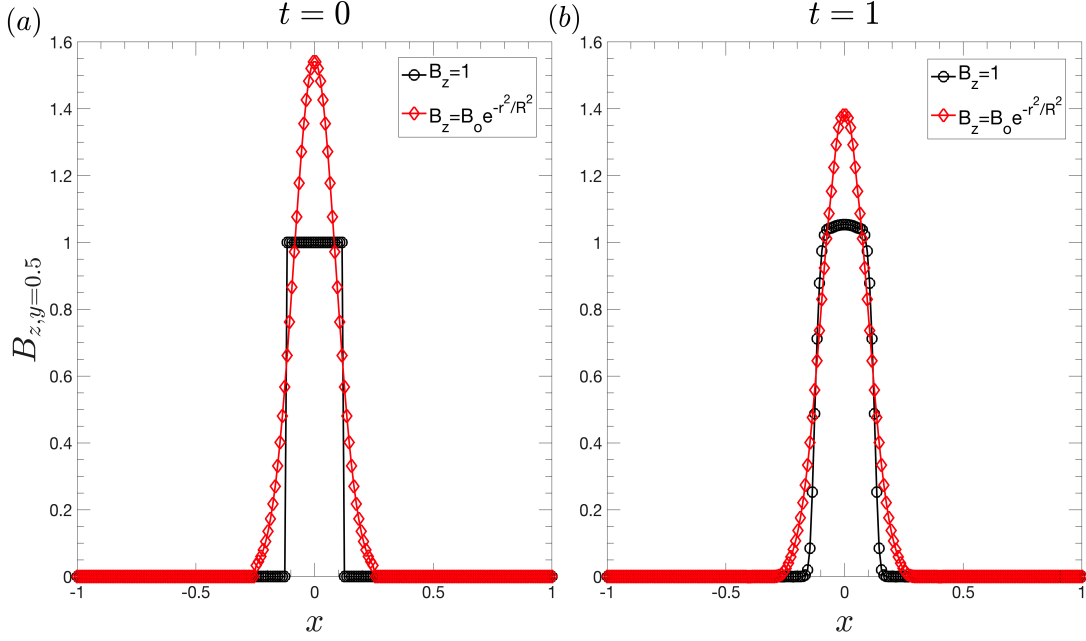


Figure 5.3: Line cuts of B_z against x at the height of the center of the magnetic concentration, $y = 0.5$, for simulations started with both top hat and Gaussian profiles inside the magnetic structure. (a) The initial condition at $t = 0$. (b) A short time later, at $t = 1$

strong magnetic field gradients at the edge of the flux tube, although this appears not to be an issue, since, at the parameters of the simulations here, diffusion acts quickly to smooth these gradients. This can be seen in Fig. 5.3b, which shows the profiles at $t = 1$. Even at this very early stage in the rise, the top hat profile has been smoothed to a more physical profiles and the two profiles have become very similar.

Ultimately, we again found that the choice of profile had little consequences for the dynamics of interest. Fig. 5.4 shows the rise time and flux fraction as a function of time for both these case (when $B_s = 0$) and it can be seen that the dynamics are very similar. A more stringent test is the comparison of, for example, Fig. 5.5 in this Appendix with Fig. 2.3 from Chapter 2. From these, it is clear that the results are not substantially influenced by the choice of profile.

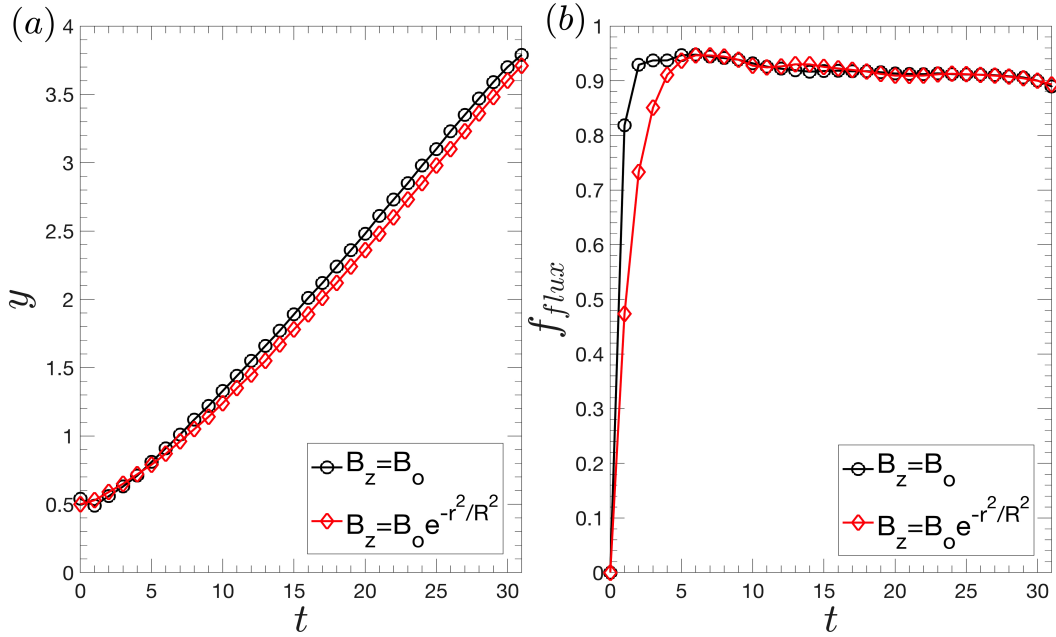


Figure 5.4: (a) Rise time and (b) flux fraction for top-hat and Gaussian magnetic field profiles at $B_s = 0$

Figure 5.5 shows intensity plots of B_z as a function of time for a number of simulations at varying background field strength and orientation, but otherwise fixed parameters (equivalent to Figure 2.3 of Section 2.3). The crucial results, which we now describe, are essentially the same, regardless of the form of the magnetic initial condition. Figure 5.5a shows a canonical case exhibiting the characteristic rise of an isolated twisted magnetic flux tube in a field-free environment by the action of magnetic buoyancy. The flux tube rises coherently, forming a “mushroom-like” structure, without any non-diffusive flux loss, a result found by many (e.g. HFJ).

Section 2.3.1 found that the introduction of background field significantly affects these rise dynamics, and this is also seen here in these new results in the subsequent panels of Fig. 5.5. In the presence of a background field of strength, $B_s = 0.1$ (Fig. 5.5b), the rising flux structure experiences some initial flux loss due to transport down the overlaying background fieldlines, but rises in a qual-

itatively similar manner to the canonical case. Here, the rising flux structure is still reasonably coherent and is able to transport upwards a major portion of the initial flux in the flux tube. With a stronger background field strength, $B_s = 0.2$ (Fig. 5.5c), the coherency of the initial flux structure is completely disrupted and no significant upwards flux transport is observed. A more fine-grained parameter study of varying background field strengths reveals that an overlying background field with a typical strength of $B_s \geq 0.16$ can completely halt the rise of an initially-buoyant twisted magnetic flux tube of the canonical type studied. The first surprising result of Section 2.3, and repeated here in a slightly different configuration, is therefore that a relatively weak background field (only 16% of the initial tube strength) can totally suppress the buoyant rise of the structures.

The second, perhaps more surprising, result of Section 2.3.1, is that the quenching threshold depends on the relative orientation of the twist of the magnetic concentration (i.e. its local azimuthal field) and the background field. Section 2.3.1 investigated this by keeping fixed twist, reversing the background field and varying the background field strength to find the value where rise was suppressed again, which we repeat here with our new Gaussian tube. Figure 5.5d shows the rise of a magnetic flux tube in the presence of an overlying large-scale background field of strength, $B_s = -0.02$ (2% of the initial axial field at tube center), oriented in the negative horizontal (x) direction, showing that this very weak strength does not affect the dynamics in any significant way. However, by slightly increasing the overlying field strength to $B_s = -0.06$ (Fig. 5.5e), the rise of the buoyant tube is completely suppressed and the coherent initial structure completely disintegrates. This surprising behavior sets another, even lower, background field strength threshold for the quenching of the flux tube rise with this field orientation.

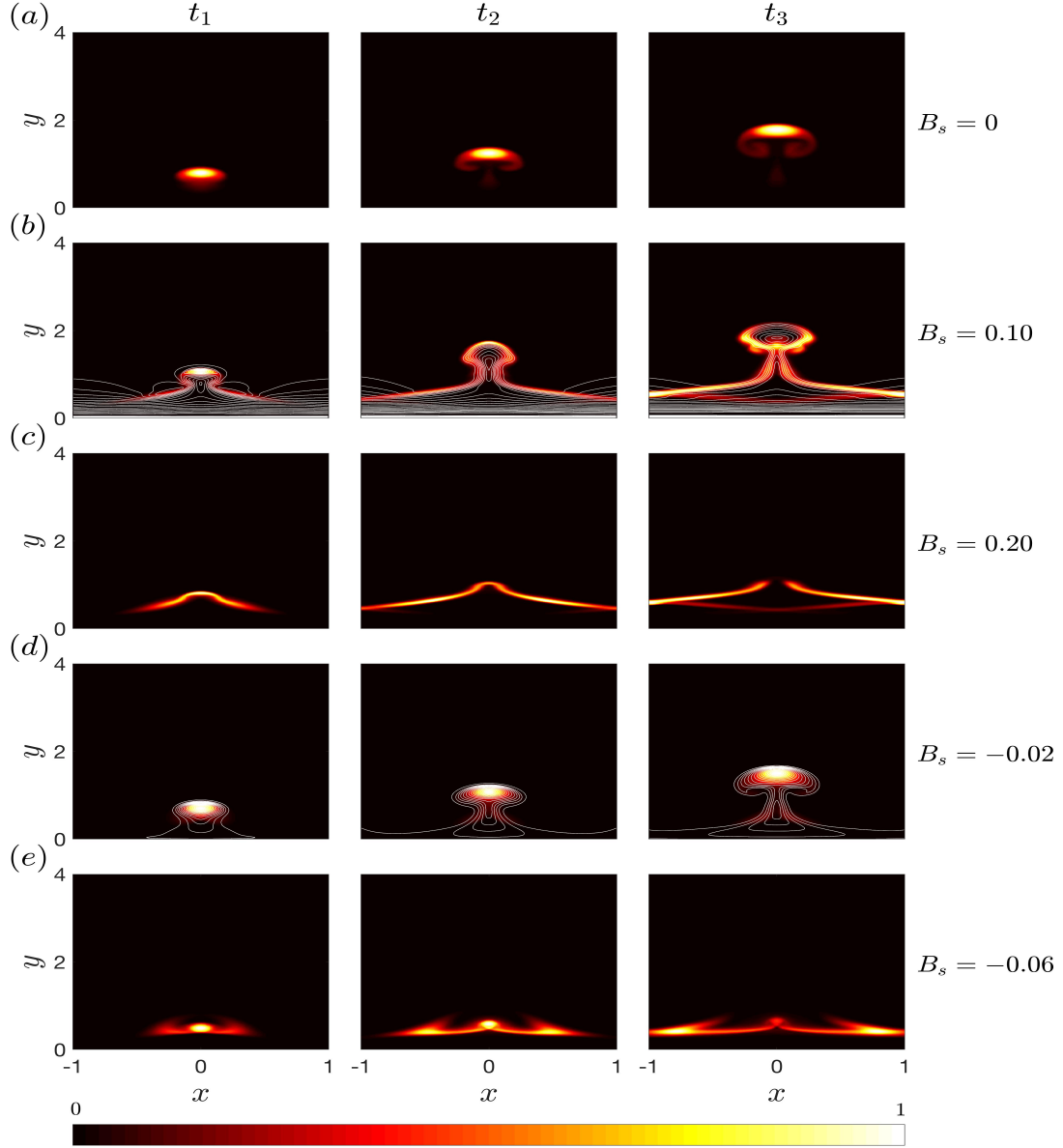


Figure 5.5: Intensity plots showing the evolution of (normalized) $B_z(x, y, t)$ for various background field strengths. Shown in rows (a, b, c, d and e respectively) are $B_s = 0, 0.1, 0.2, -0.02, -0.06$. For all cases, $H_B = 0.125$. Three times are shown for each case. In (a, b, d, e) the times are $(t_1, t_2, t_3) = (5, 10, 15)$ but for (c) the times are $(t_1, t_2, t_3) = (2, 5, 8)$. Contours of A_z have been added to panels (b) and (d). Panels (a, b and d) demonstrate cases of successful rise, whereas in panels (c and e) the rise is suppressed. The initial axial magnetic field profile in each of these cases is given by Equation (5.8).

The qualitative picture of the rise and suppression of tubes in the presence of a background field as described by Fig. 5.5 can be corroborated more quantitatively by calculation of the rising flux fraction, $f_{flux}(t)$ (as was done in Section 2.3.1 for the top hat initial condition rather than the Gaussian one here). The rising flux fraction is given by

$$f_{flux}(t) = \frac{\int \int B_{z,t}^* dx dy}{\int \int B_{z,0} dx dy} \quad (5.9)$$

where $B_{z,0} = B_z(x, y, t = 0)$ and $B_{z,t}^* = B_z((x, y, t)$ where $v_y(x, y, t) > v_{y,threshold}$). Here, $v_{y,threshold}$ is a judiciously chosen velocity that tracks any flux rising upwards at or above this threshold velocity value. A large flux fraction therefore indicates that a significant portion of the original axial flux in the flux tube is still rising, and lower flux fractions indicate that normal buoyant rise is being impeded. Figure 5.6 shows the rising flux fraction as a function of time for various background field strengths and orientations with $H_B = 0.125$. Panel (a) of Fig. 5.6 corroborates the first surprising result that even a relatively weak background field strength ($\sim 20\%$) can suppress the rise of the flux tube. In this panel, the canonical result with $B_s = 0$ can be seen to maintain the rise of almost all the flux (with only a slight diffusive loss) whereas $B_s = 0.1$ reduces the rising fraction to less than 0.5. Panel (b) corroborates the second result regarding the effect of changing the orientation of the background field for a similarly twisted flux tube. Here, for a negatively-oriented background field (retaining a positive twist in the structure), the substantially more dramatic reduction of rising flux fraction by the same background field strengths is readily apparent. Note that both the panels show the rise only until the time that the flux tube either reaches the top of the simulation box or is completely stopped.

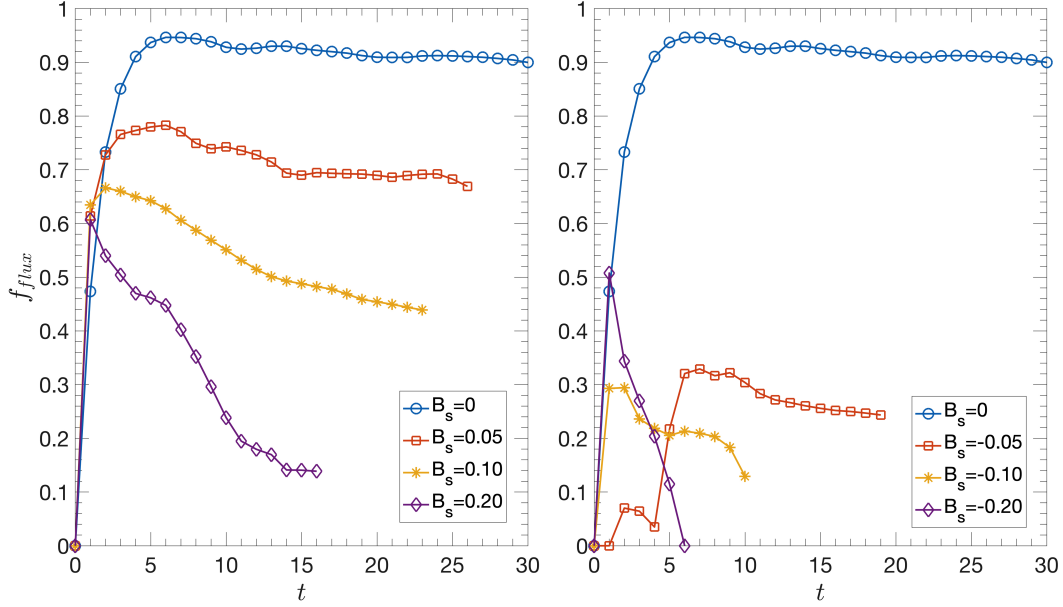


Figure 5.6: Rising flux fraction, $f_{flux}(t)$ for the initial Gaussian magnetic axial field profile. (a) $B_s \geq 0$, (b) $B_s \leq 0$ with $H_B = 0.125$ and $v_{y,threshold} = 0.02$.

5.3 Three-dimensional flux tube dynamics: Non-convective simulations

Chapter 2 studied the rise of two-dimensional flux tubes through an artificially imposed large-scale background magnetic field. We discovered the presence of a selection mechanism that preferentially selects flux tubes of a particular twist for a given background field orientation to rise over others. We verify the existence of this selection mechanism with three-dimensional calculations.

The initial conditions for the three-dimensional calculations are set up in a very similar fashion to the model described in Section 2.2 but with significant differences being the inclusion of the third dimension and the nature of the initial density perturbation imparted to the flux tube in this direction. In the 3D case, apart from adjustment due to magnetic pressure, we also introduce an additional density perturbation such that the central portion of the flux tube is more buoyant

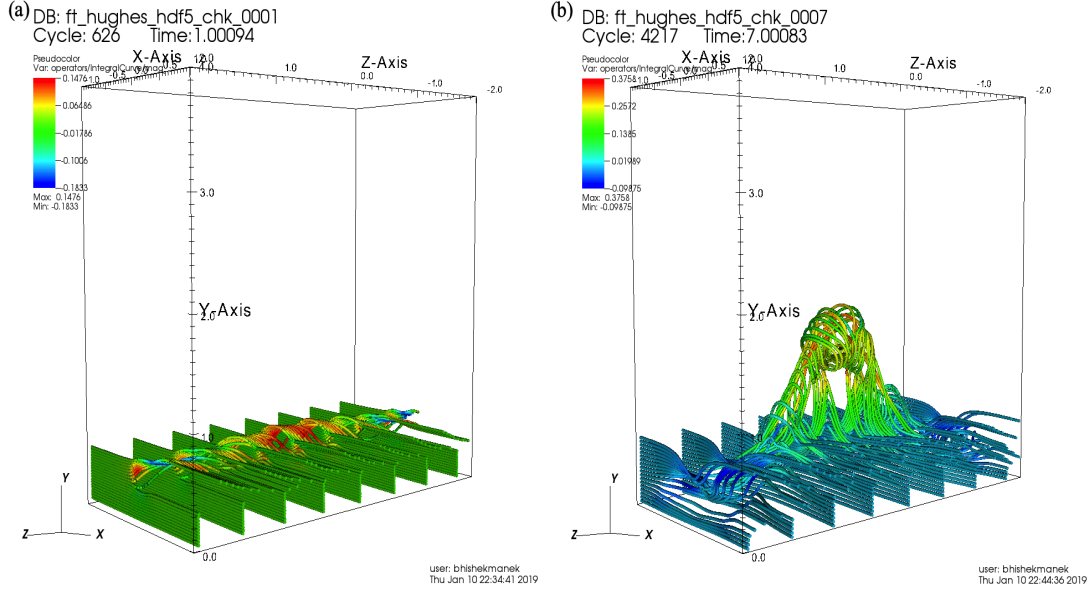


Figure 5.7: Side-on perspective of 3D magnetic field lines for the case with $B_s = 0.10$ at two different times.

than its ‘legs.’ That is,

$$\rho_{in} = \rho_{tube} - \rho_{pert} \exp(-z^2/\sigma^2) \quad (5.10)$$

where ρ_{in} is the final gas density inside the flux tube, ρ_{tube} is the gas density inside the flux tube after adjusting for the magnetic pressure (assuming thermal equilibrium, refer Case I in Appendix 5.1), and $\rho_{pert} = 0.1 \rho_{mean}$ and $\sigma = 0.375$. Here, we considered ρ_{mean} as the mean density at the center of the flux tube.

Figure 5.7 shows a side-on perspective of the three-dimensional magnetic field lines during the evolution of a flux tube with $B_s = 0.10$ at two different times. Figure 5.7a is plotted early in the evolution and closely demonstrates the initial magnetic conditions. A twisted flux tube, aligned along the (horizontal) z axis, is embedded in a horizontal background field (in the positive x direction in this case). Note that the background field has an exponential dependence in vertical direction

y , as in the two-dimensional cases, and the field lines higher up in the simulation box are not plotted for clarity. Figure 5.7b shows the later stage of evolution with the central portion of the flux tube rising due to the imposed buoyancy. The field line connectivity is complex, and there is substantial interaction of the rising tube with the background field.

Now, similar to Section 2.3.1, we systematically explore the effect of varying the background field strength and orientation to corroborate the presence of the SRR. Figure 5.8 shows the volume rendering of B^2 at four different times for cases, (a) $B_s = 0.0$, (b) $B_s = 0.05$, and (c) $B_s = -0.05$. Note that we keep the sign of the twist of the flux tube the same in all three cases and vary the strength and orientation of the background field. Figure 5.8a shows the characteristic rise of a flux tube in the absence of large-scale background magnetic field, i.e., $B_s = 0.0$. The rising flux tube quickly develops vortices, as seen in Figure 5.8a at timestep=50, a feature very similar to what we observed in the two-dimensional calculations (see Section 2.3.1). Figure 5.8a at timestep=60 shows that the flux tube reaches the top of the simulation domain in a reasonably coherent fashion.

The introduction of the background field leads to markedly different dynamics, something we expect based on our findings of Chapter 2. Figure 5.8b shows the rise of flux tube in the presence of a background field of strength $B_s = 0.05$. In this case, the formation of vortices in the rising flux tube is inefficient due to the connectivity with the background field (see Figure 5.8b at timestep=20 and 30). Even with this complexity, we find that the flux tube successfully rises through the background field and reaches the top of the simulation domain. There is possibly some flux loss to the background field during the transit, but we are not presenting a quantitative estimate of the rising flux fraction here.

On changing the orientation of the background field, that is, using $B_s = -0.05$,

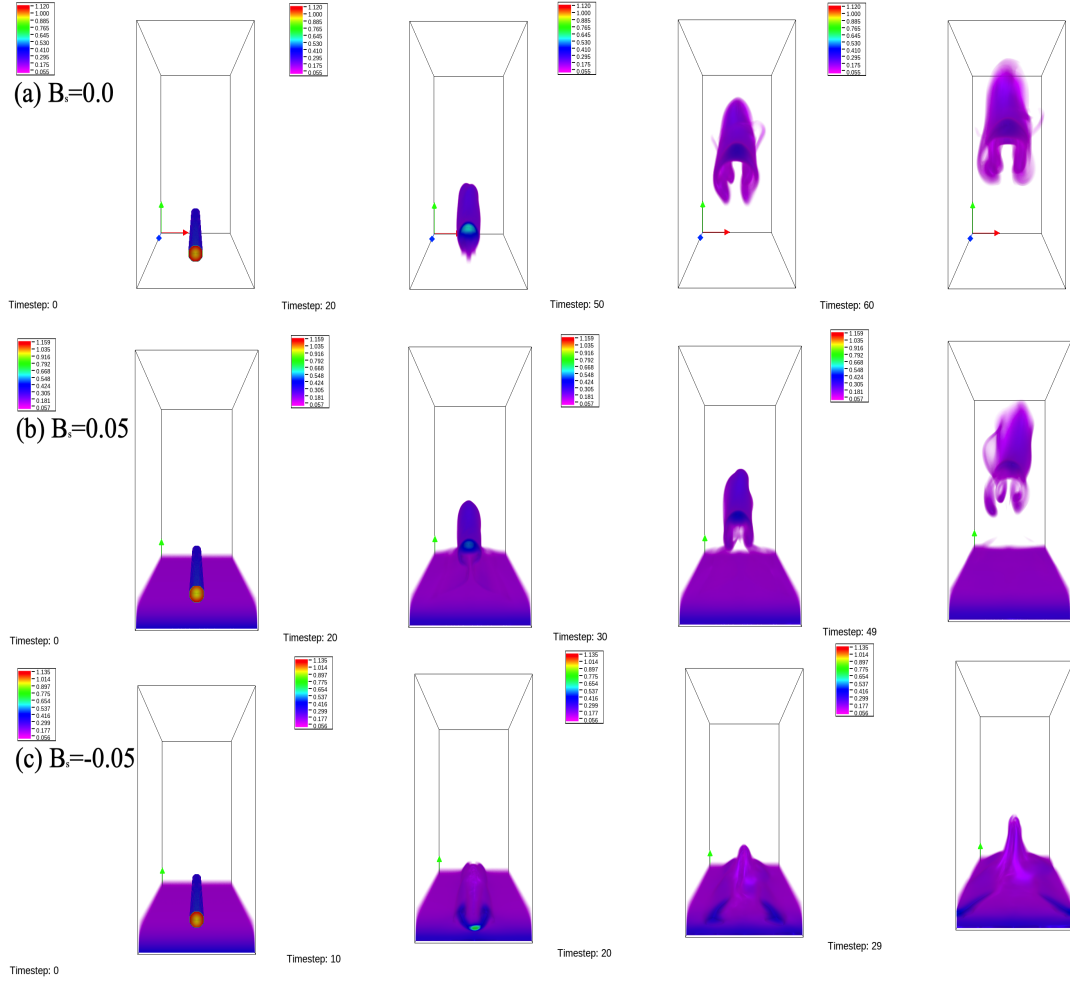


Figure 5.8: Volume rendering of B^2 : (a) $B_s = 0.0$, (b) $B_s = 0.05$, and (c) $B_s = -0.05$. The twist of the flux tube is fixed at $q = 2.5$. Red, green, and blue arrows at the edge of the box (clearly visible in (a)) correspond to the x, y, and z axes.

we find that the rise of the flux tube is completely halted, a result that we have established in substantial detail in Chapter 2. The flux tube gets stretched and breaks apart, becoming a part of the large-scale background field (see Figure 5.8c at timestep=10, 20, and 29). This confirms the presence of a selection mechanism similar to that which we discovered with two-dimensional simulations in Chapter 2.

5.4 Three-dimensional flux tube dynamics: Convective simulations

Chapter 3 studied the rise of two-dimensional flux tubes in the presence of convection and a self-consistent background magnetic layer. The eventual goal of the work initiated in Chapter 3 is to simulate similar dynamics in more realistic three-dimensional simulations. There are some significant differences that we expect in three-dimensional studies. Firstly, as convective turbulence influences the rise of the flux tube, it may no longer rise as a single coherent entity, but rather have differential rise along the newly-introduced horizontal direction. Moreover, previous numerical simulation studies of overshooting convection have shown that overshooting, and therefore also the pumping efficiency, is reduced in 3D case when compared to the 2D, due to the localized nature of plumes in three dimensions. This could lead to a scenario where the rising flux tube experiences more background field during its transit through the convection zone in a 3D setup than in a similar 2D setting, since the convection zone interior may not get so completely evacuated of field by the pumping process. This can have a significant effect on the rise dynamics.

We have begun this work, and a considerable part has already been completed.

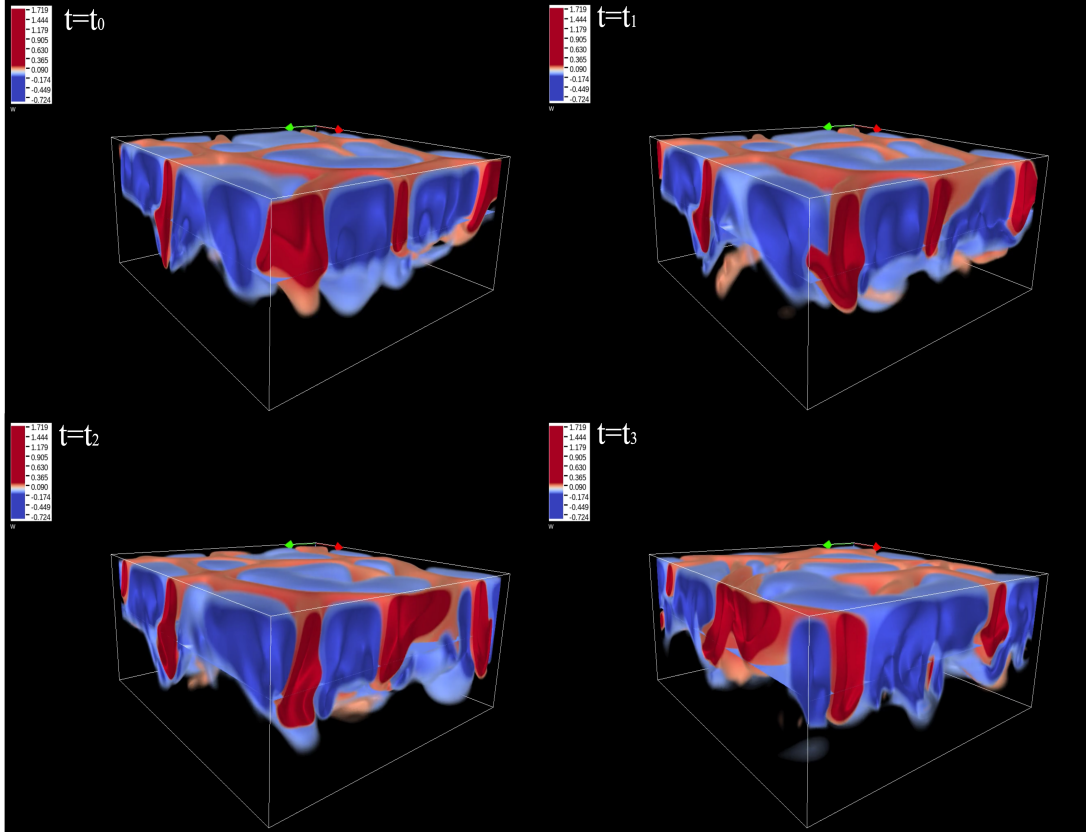


Figure 5.9: Volume renderings of vertical velocity (w) at four different times for $Ra = 1 \times 10^4$ and $Pr = 0.1$. A faintly marked surface separates the convective layer on the top with the stable layer at the bottom. Red and green arrows indicate the x and y directions.

In particular, we have set up a turbulent convective simulation (see Figure 5.9) and studied the conditions under which flux tubes can rise successfully through it (see Figure 5.10). Furthermore, we have begun the next step of examining turbulent pumping in these three-dimensional, two-layer simulations (see Figure 5.11). We now provide brief descriptions of this work in progress,

We start with a model setup that is the same as discussed in Section 3.2, except for the inclusion of the dynamics in the third dimension. Figure 5.9 shows volume renderings of vertical velocity (w) for $Ra = 1 \times 10^4$, $Pr = 0.1$, and $S = 3$ at four different times in the later stages of the evolution. Convective motions

overshooting into the stable region below the convective layer are visible.

The next step is to determine the conditions under which a twisted flux tube can successfully rise through this fairly turbulent convection. The introduction of a magnetic flux tube leads to an adjustment in the density inside it such that the tube is buoyant. Figure 5.10 shows the successful rise of a flux tube with volume renderings of B^2 in the presence of convection at four different times (same as Figure 5.9). The parameters in this case are $Ra = 1 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, and $Q = 4 \times 10^6$. Initially, the whole tube is buoyant and starts rising and reaches the boundary separating the convective layer from the stable region (see Figure 5.10 at $t = t_1$). At this point, the effect of convection becomes pretty evident in the subsequent dynamics. Parts of the flux tube that encounter downflow plumes are advected back towards the stable region, whereas the portions of the tube that are in the upflows keep rising owing to their initial buoyancy as well as advective motions (see Figure 5.10 at $t = t_2$). This leads to a naturally convoluted three-dimensional flux tube which is otherwise usually artificially imposed in various studies (including ours discussed above in Appendix 5.3). Figure 5.10 at $t = t_3$ shows that the flux tube successfully reaches the top of the convective layer but in a complex form. Note that one could consider some sections of the tube as arching on arrival at the surface, but the scale of this arching is the convective scale, which is possibly not the one associated with active region sunspot pairs.

Figure 5.11 shows a result from a first-cut at three-dimensional pumping simulations that we have carried out as part of this work. This Figure shows the time- and horizontally-averaged pumped horizontal magnetic field plotted as a function of z . In this case, we had started with an initial condition of a thin magnetic layer of amplitude $B_{x,0} = 0.1$ in the convection zone, located in $0.7 < z < 0.8$ (where z is now the vertical direction). The parameters in this case are $Ra = 1 \times 10^4$,

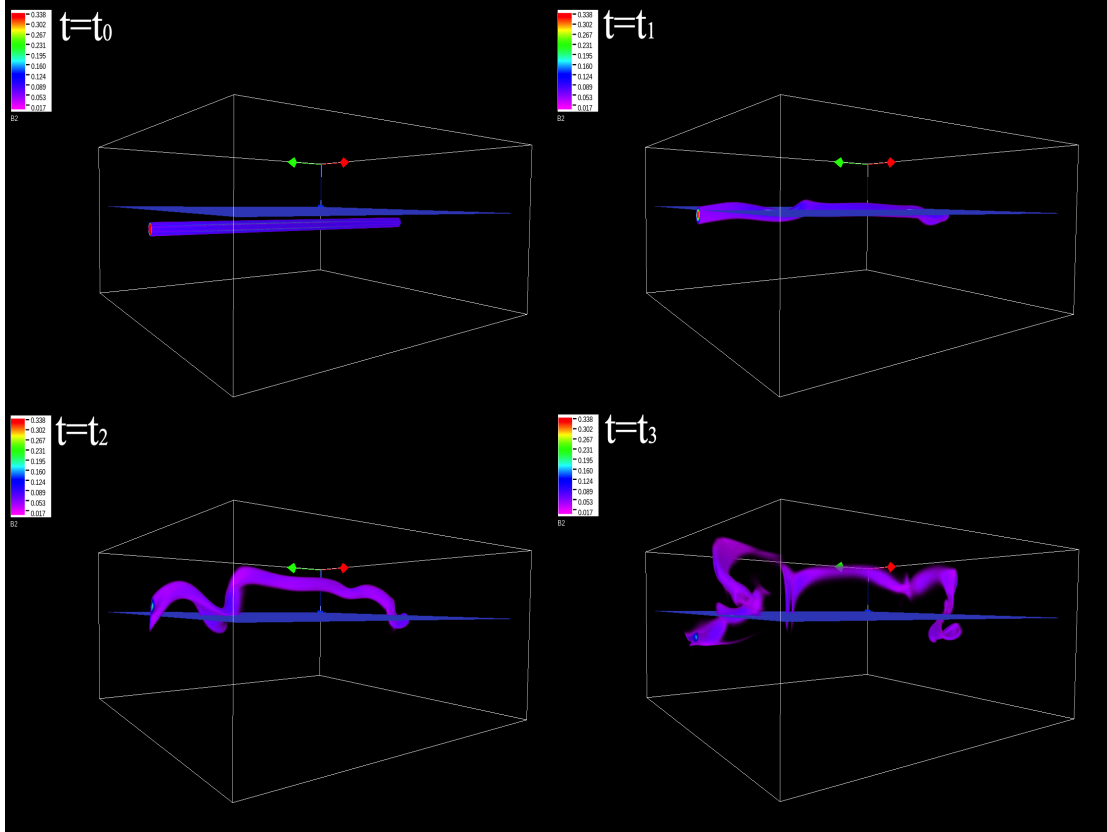


Figure 5.10: Volume renderings of B^2 at four different times for $Ra = 1 \times 10^4$, $Pr = 0.1$, $\zeta = 0.001$, and $Q = 4 \times 10^6$. The plane layer separates the convective layer at the top from the stable layer at the bottom. Red and green arrows indicate the x and y directions.

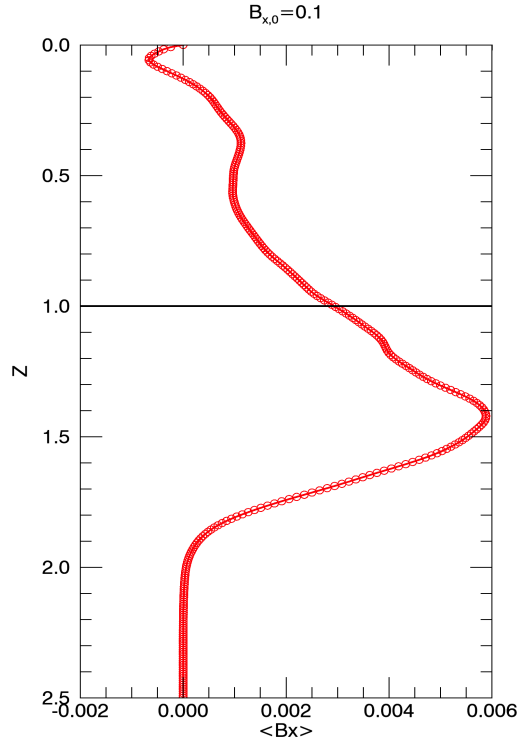


Figure 5.11: Time- and horizontally-averaged pumped horizontal field as a function of height (z) for the case with $S = 3$.

$Pr = 0.1$, $\zeta = 0.001$, and $Q = 4 \times 10^6$. The pumping mechanism is successfully demonstrated by the accumulation of magnetic field in the stable layer. While this is encouraging, there are many challenges in these simulations related to control of the strength of the final pumped layer. As can be seen in Figure 5.11, the maximum average field in the pumped layer is around 0.006. In other words, this corresponds to a background layer of strength, $B_s = 0.006$, which is about 0.6% of a unit strength magnetic flux tube. Our two-dimensional simulations have given us insights into the typical threshold values of the B_s that can dramatically affect the flux tube dynamics; we expect to need $B_s \sim 5-15\%$ for evidence of a selection bias. Furthermore, we know from Tobias et al. [111] that three-dimensional turbulent pumping does not entirely evacuate the magnetic fields from the convective layer. A rising flux tube is therefore expected to experience considerable back-

ground field throughout its rise. For this reason, we do not expect the threshold values of B_s for complete quenching of the rise of tubes to be as large in this 3D setup. Still, the value of 0.6% seems considerably lower than we expect to need to demonstrate a bias, and thus the pumped layer may need further amplification to achieve the effects we expect to see. Unfortunately, in three-dimensions, the “run-down” nature of the boundary conditions and more substantial flux pileup (than 2D) at the top boundary lead to numerical challenges, and then scaling of the pumped layer is not straightforward. These issues deem further investigation and are currently in progress and will be reported soon.

5.5 Magnetic Flux Emergence

The following appendix contains work and text from Manek, Fang & Fan (2022), a paper in preparation to be submitted to The Astrophysical Journal. This work was initiated as part of the Newkirk Fellowship with Dr. Yuhong Fan. Newkirk fellowship is administered by the High Altitude Observatory (HAO), National Center for Atmospheric Research (NCAR). NCAR is funded by the National Science Foundation. This work was financially supported in part by the NASA LWS grant 80NSSC19K0070 to NCAR.

The goals of this project were broad, and we have carried out three-dimensional MHD simulations of the formation of a rotating blowout jet, from the buoyant emergence of a twisted magnetic flux tube from the upper convection zone, through the photosphere, and into the corona in the presence of an inclined open field. Figure 5.12 shows the setup of the simulation. We use a Cartesian domain as shown in Figure 5.12(a) with the horizontal sizes: $x \in [-26.5 \text{ Mm}, 26.5 \text{ Mm}]$, $y \in [-26.5 \text{ Mm}, 26.5 \text{ Mm}]$, and the vertical size: $z \in [-6.1 \text{ Mm}, 49.8 \text{ Mm}]$, where $z = 0$ corresponds to the photosphere (the gray horizontal plane in Figure 5.12a).

We first initialize a plane-parallel, vertically stratified atmosphere, as shown in Figure 5.12(b), which contains an adiabatically stratified polytropic layer representing the top of the convection zone, an approximately isothermal layer representing the photosphere and chromosphere, transitioning to a high-temperature corona layer, permeated with a uniform slanted open magnetic field of 6 G (illustrated by the field lines shown in the vertical cross-section in Figure 5.12(a)). A horizontal twisted magnetic flux tube oriented along the x -direction is inserted into the convection zone layer below the photosphere (see Figure 5.12(a)), with the magnetic field for the flux tube given as follows:

$$\mathbf{B} = B_x(r) \hat{\mathbf{x}} + B_\theta(r) \hat{\theta}, \quad (5.11)$$

with

$$B_x(r) = B_0 \exp(-r^2/a^2), \quad (5.12)$$

$$B_\theta(r) = qr B_x(r), \quad (5.13)$$

where $\hat{\mathbf{x}}$ denotes the tube axial direction, $\hat{\theta}$ denotes the azimuthal direction in the tube cross-section, q is the twist of the flux tube, and r denotes the radial distance to the tube axis. The details of the numerical code and the MHD equations solved are available in detail in Fan (2017) [42].

Over here, we presented a limited set of results from this work. In Chapters 2 and 3, we discussed the dynamics of magnetic flux tubes in the presence of a large-scale background field in the deep interior of the Sun, simulating the upper tachocline and lower convection zone. We can envisage a similar setup in these magnetic flux emergence simulations, where the flux tube rises through the upper convection zone, photosphere, and the corona. The significant difference in the initial magnetic conditions is that the background field, in this case, is

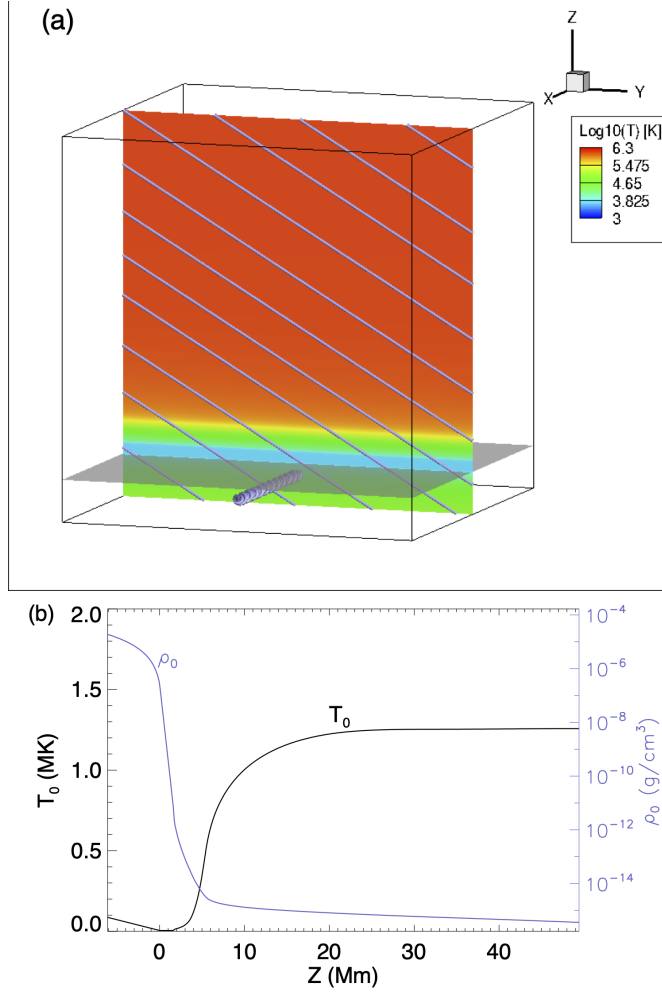


Figure 5.12: (a) The initial set up of the simulation domain. The gray plane corresponds to the photosphere. The color image in the vertical cross-section shows the temperature of the initial background equilibrium plane-parallel atmosphere permeated with a uniform open magnetic field represented by the field lines in the vertical cross-section. A twisted magnetic flux tube, as represented by the twisted field lines, is inserted into the interior layer below the photosphere. (b) The temperature $T_0(z)$ (black curve) and density $\rho_0(z)$ (blue curve) profiles of the initial background equilibrium plane-parallel atmosphere. See text for detailed description.

represented by the slanted open magnetic fields, which mimic the polar coronal fields. We follow a similar approach as we did in Chapters 2 and 3; the sign (and the magnitude) of the twist of the flux tube is kept fixed while the orientation of the background field is changed. Another unique aspect of this work is that

with the extensive availability of observational magnetic field data at the solar photosphere and corona, the initial strength of the flux tube, $B_0 = 5677$ G, and the strength of the slanted open magnetic fields, $B_s = 6$ G, are better constrained.

Figure 5.13 shows the intensity plots of (a) density and (b) temperature, overplotted with velocity and magnetic field vectors, in the central plane of the simulation domain ($x = 0$), for four different times. Note that the color bar in this plot is on a logarithmic scale. The initial background stratification along with the non-axial magnetic field vectors are shown in Figure 5.13a and b at $t = 0$. The flux tube’s twist is clockwise, and it is located at $z_c = -1.7$ Mm below the photosphere. In this case, the background field is negatively oriented ($B_s < 0$). Figure 5.14 shows the same setup as Figure 5.13 but with $B_s > 0$. In both these cases, we find that the flux tube emerges through the background field into the photosphere and successfully evolves into the corona. The evolution of dynamics (magnetic reconnection, jet formation) further on will be reported in a forthcoming paper. However, the important result is that the background field’s presence doesn’t seem to enforce a selection mechanism on the rising flux tube. This is not unexpected as the observationally constrained magnetic field strengths are such that the background magnetic field is roughly 0.1% of the initial flux tube strength. We know from Chapters 2 and 3 that the threshold background field strengths for the selection mechanism to operate can be relatively higher. This suggests that the selection mechanism can primarily work in the deeper interior of the Sun, and the role of the near-surface layer in selecting flux tubes of particular helicity, in accordance with SHHR, is minimal.

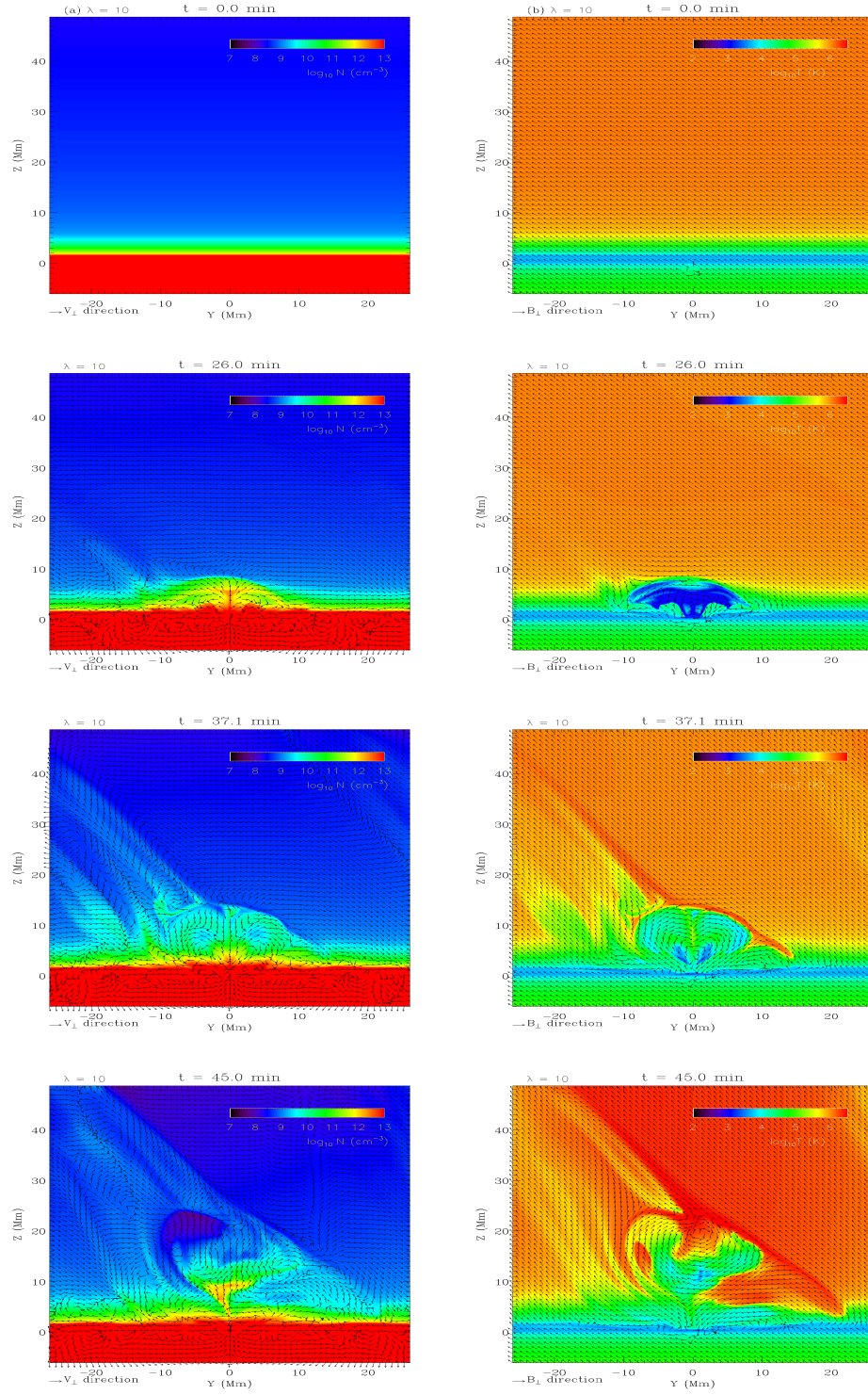


Figure 5.13: Intensity plots of (a) density and (b) temperature (in log scale) overplotted with velocity and magnetic field vectors respectively. For this case, $B_s > 0$.

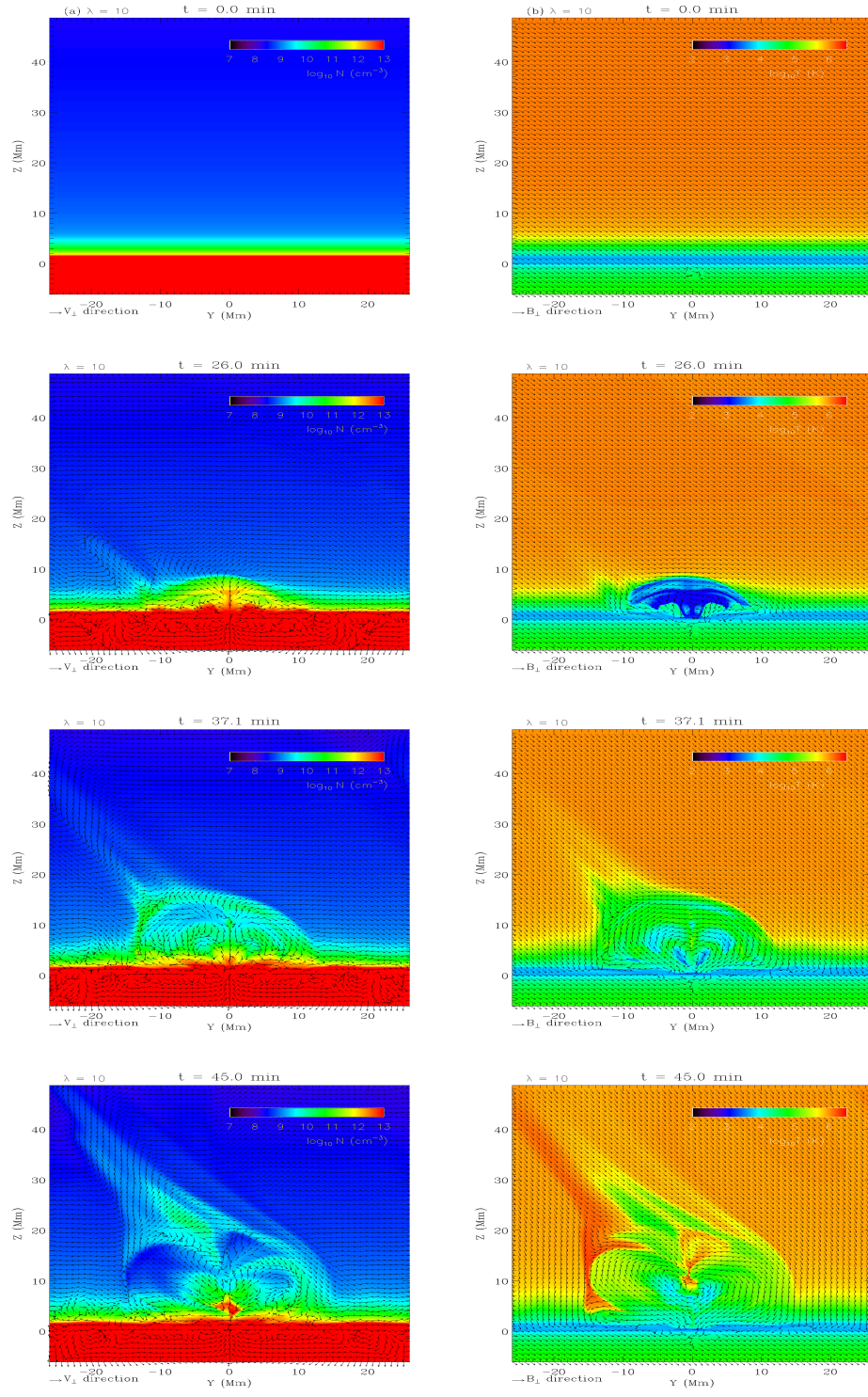


Figure 5.14: Same as 5.13 but for $B_s < 0$.

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