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Time-Geography in Four Dimensions: Potential Path Volumes around 3D Trajectories

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Abstract

An upcoming increase in availability and accuracy of 3D positioning requires development of new analytical approaches that will incorporate the third positional dimension, the elevation and model space and time as a 4D concept. In this paper we propose the extension of time geography into four dimensions. We generalise the time geography concept of a Potential Path Area into a Potential Path Volume around a 3D trajectory and present its mathematical definition. The algorithm for calculating PPVs around 3D trajectories is currently being implemented and will be tested on simulated data and real 3D data from movement ecology.

1. Introduction

Movement data are collected in the form of trajectories, which are sequences of locations collected at certain times. These data are collected using trackers, such as GPS devices, which are capable of recording location in three dimensions. However, typically only the two geographical dimensions (either longitude/latitude or easting/northing) are used for analysis and the third dimension, elevation, is neglected (Belant *et al*. 2012). This is first because the accuracy of GPS elevation measurements is poor, but also because including the third dimension into any kind of geometrical calculations necessary for correct mathematical analysis introduces higher complexity. The upcoming deployment of the European Galileo and the Chinese COMPASS/Beidou systems are expected to improve elevation accuracy (Li *et al*. 2015). Therefore an upcoming critical gap is for analytical methods for 3D movement.

In this paper we propose a generalisation of a well-established movement analysis framework, time geography (Hägerstrand 1970) to consider location in three dimensions. This framework originally operates in the conceptual space of a Space-Time Cube (STC), which consists of a 2D geographic plane and time as the third axis. We propose to extend this concept into a Space-Time HyperCube (STHC), which we define as a 4D space, consisting of a 3D geographic space and fourth dimension - time. By applying the conceptual extension from STC to STHC, we can mathematically generalise time geography into four dimensions.

We focus on one particular time geography concept: the Potential Path Area (PPA), a popular accessibility measure in transport (Patterson and Farber 2015). A PPA is the projection of a Space-Time Prism (STP) onto the geographical plane (Miller 2005). The STP is an accessibility volume within the STC, which represents all the paths which the object could have traversed between two observed positions, P_i and P_{i+1} (Figure 1a). In the 2D case, the PPA is an ellipse. If this ellipse is calculated around each segment of a trajectory, their union can be used to delineate the range of a moving object (Long and Nelson 2012).

To extend this principle into four dimensions, we propose to generalise the PPA ellipse into an ellipsoid located within the three dimensional geographic space. This ellipsoid is the projection of the four-dimensional Space-Time Prism (the 4D accessibility volume between the two observed positions) onto the 3D base space of the STHC. We call this ellipsoid the Potential Path Volume (PPV, Figure 1b) and propose that it can be used in the same way as the Potential Path Area for trajectories where location is measured in three dimensions.

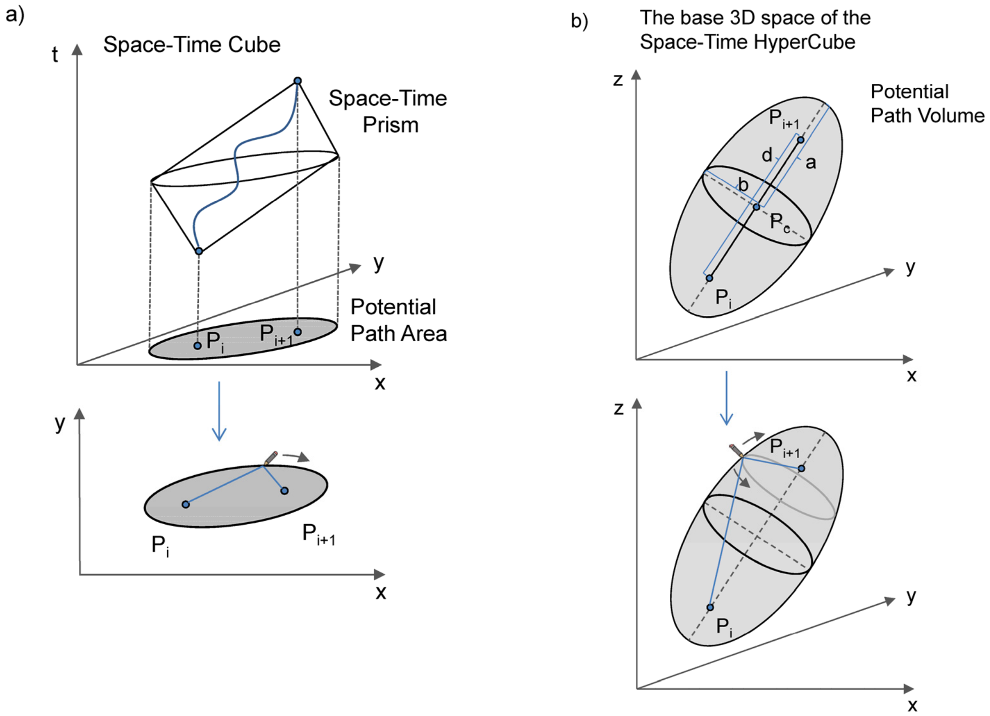

Figure 1: a) The definition of the Potential Path Area as the projection of the Space-Time Prism in an STC and b) its 4D generalisation the Potential Path Volume.

In the remainder of this paper we lay out the mathematical definition of the Potential Path Volume and describe the algorithm for calculation of PPVs for a set of given trajectories. As this is work in progress, implementation and testing of the algorithm are currently on-going.

2. Mathematical definition of the Potential Path Volume

The PPA is an ellipse around a movement segment, so that the two locations (start and end points of a segment, P_i and P_{i+1}) are placed in the foci of the ellipse. Given the maximum possible velocity and time difference between P_i and P_{i+1}, the maximum length of the path between P_i and P_{i+1} can be imagined as a string of this length, fixed in P_i and P_{i+1}. By placing a pen into this string and tracing as far out as possible in all directions, an ellipse is generated that covers all possible paths that the moving object could have passed (Figure 1a).

The same principle can be applied in 3D (Figure 1b). P_i and P_{i+1} are now placed into foci of an oblique ellipsoid. The longest possible length of the path between P_i and P_{i+1} is calculated based on maximum possible velocity and time difference. Then the ellipsoid is generated by tracing the ellipse first in one plane (a plane parallel to movement direction) and then this ellipse is rotated around the axis represented by the direction of movement between P_i and P_{i+1}. This construction results in a special type of an ellipsoid, a prolate spheroid (e.g. the form of a rugby ball), where the two minor axes are identical (Figure 1b), that is, the ellipsoid major axis is a and the two minor axes are b and b. This follows an assumption that movement in the two directions perpendicular to line P_i to P_{i+1} is equally possible. In a more general case, a scalene ellipsoid, where all three axes are different (i.e. major axis a and two minor axes b and c), would be more appropriate – this will be considered in our future work for the types of movements where velocities differ based on direction of movement.

The PPV ellipsoid is defined by the following quantities (Figure 1b): the distance between the two foci (d), the length of the major axis (a), the length of the two minor axes (b), the origin point of the ellipsoid (P_c) and the two rotation angles (α, β) that transform the original coordinate system into a coordinate system defining the ellipsoid (Figure 2a). For each segment on the trajectory, with start and end points $P_i(x_i, y_i, z_i)$ and $P_{i+1}(x_{i+1}, y_{i+1}, z_{i+1})$, the origin point P_c is given as the central point:

$$P_c = (x_c, y_c, z_c) = (\frac{x_i + x_{i+1}}{2}, \frac{y_i + y_{i+1}}{2}, \frac{z_i + z_{i+1}}{2}) \quad (1)$$

The distance d between P_i and P_{i+1} is Euclidean distance between two 3D points:

$$d = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2} \quad (2)$$

The major axis of the ellipsoid, a, can be calculated knowing the time difference Δt between P_i and P_{i+1} and the maximum possible velocity v_{max}. For this velocity we could take the maximum observed velocity, however, that would create a degenerate ellipsoid and we therefore follow a more robust calculation of v_{max} (Long and Nelson 2012):

$$v_{max} = 2 \cdot v_m - v_{m-1} \quad (3)$$

where v_m is the maximum observed velocity and v_{m-1} is the next largest observed velocity. Then, a and b are given as this:

$$a = \frac{v_{max} \cdot \Delta t}{2} \qquad b = \sqrt{a^2 - \frac{d^2}{4}} \quad (4)$$

Finally, we also need to find the transformation of the original axes (x, y, z) onto the ellipsoid axes (x', y', z') (Figure 2b). These are created by a combination of the translation of the coordinate origin into the central point P_c and then two rotations, the first one for angle α around the z axis and the second one for angle β around the rotated y-axis (Figure 2a). These two angles are the Tait–Bryan nautical angles of pitch and yaw (because of the symmetry of movement around the axis P_i to P_{i+1}, the roll is not important) and can be calculated as:

$$\alpha = \arctan(\frac{y_{i+1} - y_i}{x_{i+1} - x_i}), \quad \beta = \arcsin(\frac{z_{i+1} - z_i}{d}) \quad (5)$$

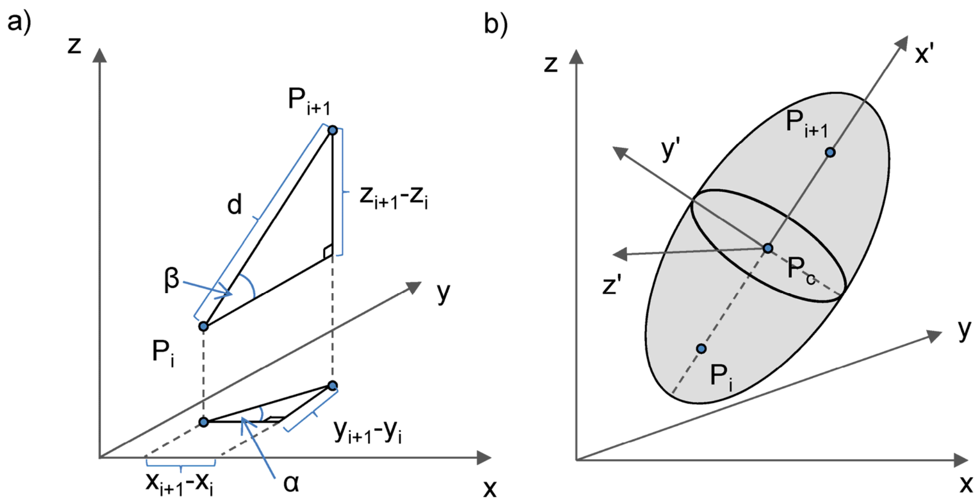

Figure 2. a) Definition of angles α and β and b) transformation of the original coordinate system (x, y, z) into the coordinate system of the ellipsoid (x',y',z').

New coordinates are then calculated as per (6), where $R(\alpha)$ and $R(\beta)$ are rotation matrices (7):

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R(\beta) \cdot R(\alpha) \cdot \begin{pmatrix} x - x_c \\ y - y_c \\ z - z_c \end{pmatrix} \quad (6)$$

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R(\beta) = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \tag{7}$$

The order of rotations in eq. (6) corresponds to the right to left order in the matrix product, i.e. α first, then β. Given any point *(x,y,z)* in the original coordinate system, we can then determine if the point is within the ellipsoid: this is the case when the following is satisfied:

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{b^2} \leq 1 \tag{8}$$

We can use this mathematical formulation to define an algorithm that calculates PPVs for a set of 3D trajectories. The algorithm runs through all trajectories and first identifies the largest possible velocity on each trajectory. Then, for each segment of each trajectory it builds the respective PPV as a volumetric representation. That is, it builds a volume where all voxels which, given the respective maximum possible velocity are within the PPV for this trajectory segment (based on (8)), are given value 1 and all others value 0. In the last step, the total PPV volume (for all trajectories and segments) is built as the union of all individual PPV volumes. That is, each voxel of the total PPV volume is assigned value 1 if it belongs to at least one of the individual PPV volumes. The resulting volume delineates the 3D accessibility space that the moving object could have reached. Our algorithm is currently being implemented in R and will be tested on simulated data and real data from movement ecology.

3. Conclusions

A limitation of our algorithm is that the computational complexity is $\mathcal{O}(n \times p \times v)$, where *n* is the number of trajectories, *p* the length of the longest trajectory and *v* the number of voxels. The complexity could be decreased by at each step considering only voxels from the box that bounds the ellipsoid around every segment instead of the entire volume. This has been done previously for space-time densities (Demšar and Virrantaus 2010).

Analogously to the use of PPAs for wildlife trajectories (Long and Nelson 2012), the union of PPVs for all segments could be used as a delineation of a home range for an animal that moves freely in 3D, such as birds or marine animals. We plan to test our approach on a set of real 3D bird trajectories (Tarroux *et al.* 2016). Since the union of PPVs for a set of trajectories defines the volume which an object can reach given its maximum possible velocity, this means that the object could not have been located outside this volume at any time. This could also be used to improve other alternative measures of three dimensional home ranges in ecology, such as the 3D Brownian bridges density (Tracey *et al.* 2014).

References

Belant JL, Millspaugh JJ, Martin JA and Gitzen RA, 2012. Multi-dimensional space use: the final frontier. *Frontiers in Ecology*, doi:10.1890/12.WB.003.

Demšar U and Virrantaus K, 2010. Space-time density of trajectories: exploring spatio-temporal patterns in movement data. *International Journal of Geographical Information Science*, 24(10):1527–1542.

Hägerstrand T, 1970, What about people in regional science? *Papers of the Regional Science Assoc.*, 24:7-21.

Li X, Zhang X, Ren X, Fritsche M, Wickert J and Schuh H, 2015, Precise positioning with current multi-constellation GNSS: GPS, GLONASS, Galileo and BeiDou. *Nature Scientific Reports* 5:8328.

Long JA and Nelson TA, 2012. Time Geography and Wildlife Home Range Delineation. *The Journal of Wildlife Management*, 76(2):407-413.

Miller HJ, 2005, A Measurement Theory for Time Geography. *Geographical Analysis*, 37:17-45.

Patterson Z and Farber S, 2015. Potential Path Areas and Activity Spaces in Application: A Review. *Transport Reviews*, 35(6):679-700.

Tarroux A, *et al.*, 2016. Data from: Flexible flight response to challenging wind conditions in a commuting Antarctic seabird: do you catch the drift? *Movebank Data Repository*. doi:10.5441/001/1.q206rm6b

Tracey JA, Sheppard J, Zhu J, Wei F, Swaisgood RR and Fisher RN, 2014. Movement-Based Estimation and Visualization of Space Use in 3D for Wildlife Ecology and Conservation. *PloS One*, 9(7):e101205.