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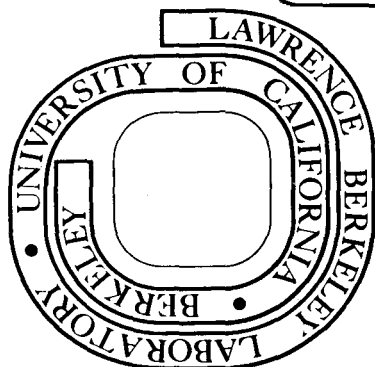
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CLASSICAL-LIMIT DESCRIPTION OF ROTATION-VIBRATIONAL  
BAND EXCITATION IN DEFORMED EVEN-EVEN NUCLEI

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## ABSTRACT

Classical-limit S-matrix (CLSM) theory, previously formulated for Coulomb excitation of the ground band in even-even nuclei, is extended to rotation-vibrational bands. A perturbation approximation is introduced for which the results are conceptually simple, and lend themselves to an illuminating classical description of rotational-vibrational excitation. Numerical calculations performed with this formalism for the  $K=0$  octupole band in  $^{238}\text{U}$  are in good agreement with calculations based on the semi-classical Alder-Winther theory. It is suggested that methods analogous to those described here could be used to describe nucleon and cluster transfer in deformed systems.

## 1. INTRODUCTION

Classical-limit S-matrix theory (CLSM) for rotational excitation of deformed nuclei by heavy-ion projectiles<sup>1-5</sup> may be extended to investigate other processes in heavy-ion scattering having classical analogs. In this paper we extend the CLSM to the case where both rotational and vibrational modes of the target nucleus are excited by the heavy-ion projectile. We concentrate specifically on the rotational signature of the vibrational bands in deformed even-even nuclei. In Section 2 we study the excitation of a permanently deformed even-even target nucleus with axial symmetry, and in Section 3 we consider the case of shape vibrations. Finally, in Section 4 we give a less rigorous but more illuminating derivation of the S-matrix elements and discuss its implications.

Before closing this brief introduction let us mention that the simpler case of vibrational excitation in spherical nuclei has been successfully treated by an approach closely related to the one described here.<sup>6</sup>

## 2. PERMANENT DEFORMATION

Let us consider, as in Ref. 3, an even-even target nucleus with an axially symmetric shape, and a projectile nucleus, incident with zero impact parameter on the target nucleus. We assume the projectile to be spherical and disregard any projectile excitation during the collision process.

The classical Hamiltonian for this system may be written

$$\begin{aligned}
 H(r, \chi, p_r, p_\chi) = & \frac{p_r^2}{2m} + \frac{p_\chi^2}{2} \left( \frac{1}{\mathcal{J}} + \frac{1}{mr^2} \right) + \frac{Z_p Z_T e^2}{r} \\
 & + \frac{Z_p Q_0^{(2)} e^2 P_2(\cos \chi)}{2r^3} + V_{\ell > 2}^{\text{Coul}}(r, \chi) + V_{\text{nuc}} \quad (1)
 \end{aligned}$$

where  $V_{\ell > 2}^{\text{Coul}}(r, \chi)$  represents multipole-monopole Coulomb interactions of higher order than quadrupole, and  $V_{\text{nuc}}$  represents a nuclear potential (possibly complex). The reason for writing the Hamiltonian in this particular form will become apparent shortly. The angle  $\chi$  is defined by the symmetry axis of the target and the line joining the centers of target and projectile,  $r$  is the distance between these centers,  $p_\chi$  and  $p_r$  are the quantities canonically conjugate to  $\chi$  and  $r$ , defining the rotational angular momentum of the target and the relative linear momentum between target and projectile, respectively. See Fig. 1 for an illustration of the coordinate system. The other quantities appearing in Eq. (1) are the charges of projectile and target,  $Z_p e$  and  $Z_t e$  respectively, the reduced mass of the system  $m$ , the moment of inertia  $\mathcal{J}$  of the target, and the electric quadrupole moment of the target  $Q_0^{(2)}$ . The usual Legendre polynomial is denoted by  $P_2(\cos \chi)$ . The term  $V_{\ell > 2}^{\text{Coul}}(r, \chi)$  is given explicitly by

$$V_{\ell > 2}^{\text{Coul}}(r, \chi) = \sum_{\ell > 2} \frac{Z_p Q_0^{(\ell)} e^2 P_\ell(\cos \chi)}{2r^{\ell+1}} \quad (2)$$

where

$$Q_0^{(\ell)} e \equiv 2 \int r^\ell P_\ell(\cos \theta) \rho(r, \theta) d^3 r \quad (2a)$$

and  $\rho(r, \theta)$  is the nuclear charge density.

We remark that so far we are considering only permanent deformations. Shape oscillations will be introduced in the next section.

Since the nuclear potential  $V_{\text{nuc}}$  enters the CLSM formalism on the same footing as the Coulomb interaction it is easily and accurately included in CLSM calculations (see ref. 4). However, we will neglect its influence here for simplicity, and consider only Coulomb excitation in the discussion to follow. The classical limit of the quantum-mechanical S-matrix is given in section 2.2 of ref. 5 as

$$S_{0 \rightarrow I}^{J=0} = \frac{1}{2} \sqrt{2I+1} \int_0^\pi P_I(\cos \bar{\chi}) \sqrt{\sin \chi_0 \sin \bar{\chi} \frac{d\bar{\chi}}{d\chi_0}} e^{i\phi'} d\chi_0 \quad (3)$$

where  $\phi'$  is given by

$$\phi' = -\frac{1}{\hbar} \int [r(t) dp_r(t) + \chi(t) dp_\chi(t)] + \frac{1}{\hbar} \int_{\tilde{r}_T}^r \tilde{p}_r d\tilde{r} + \sigma_0(\eta_0) + \sigma_I(\eta_I) \quad (4)$$

The quantities appearing in Eqs. (3) and (4) are those used in Ref. 5. The bar and tilde quantities are defined in the Appendix. They basically represent the previously defined dynamical variables transformed to an interaction representation.<sup>3,5</sup>

As demonstrated in refs. 3 and 5, this expression may be integrated numerically, or evaluated by saddle-point methods, to yield a highly accurate approximation to the quantum-mechanical S-matrix. However, we may simplify evaluation of this expression in the limit that the higher-order multipole terms of  $V_{\ell>2}^{\text{Coul}}(r, X)$  are small compared to the quadrupole term in Eq. (1). In that case the S-matrix elements [Eq. (3)] may be calculated by considering  $V_{\ell>2}^{\text{Coul}}(r, X)$  as a perturbation on the phase  $\phi'$ ,



neglecting its effect on the classical orbit itself. The contribution to the phase of  $V_{\ell>2}^{\text{Coul}}(r, \chi)$  in this limit is given by

$$\Delta\phi'_V = \phi'_{V \neq 0} - \phi'_{V=0} = \frac{1}{\hbar} \int_{-\infty}^{\infty} V(r(t), \chi(t)) dt \quad (5)$$

where  $r(t)$ ,  $\chi(t)$  in Eq. (5) are evaluated with the unperturbed Hamiltonian, i.e., setting  $V_{\ell>2}^{\text{Coul}} = 0$  in Eq. (1).

This approximation is related in spirit to the Alder-Winther semi-classical method<sup>13,14</sup> which treats the quadrupole excitation of nuclear states using quantum mechanics, but neglects the effect of the quadrupole potential on projectile dynamics. However, we note that in this approximation we accurately include the effect of the quadrupole potential on the orbit, and only ignore the (generally smaller) higher-order terms. This approximation may also be compared to Broglia, et al.<sup>15</sup> who neglect the effect of an imaginary nuclear potential on the projectile orbit, including only its contribution to an imaginary phase which gives rise to a damping of amplitudes.

For a single term of  $V_{\ell>2}(r, \chi)$  in Eq. (2) we have

$$\Delta\phi'_{V_\ell} = \frac{1}{\hbar} \int_{-\infty}^{\infty} \frac{Z_p Q_0^{(\ell)} e^2 P_\ell(\cos\chi(t))}{2r(t)^{\ell+1}} dt \quad (6)$$

Since  $\ell > 2$  and  $\chi(t)$  varies slowly for excitation of a heavy target, most of the contribution to the integral in (6) is around the point of closest approach (CA) of the trajectory. Therefore we replace  $\chi(t)$  in Eq. (6) by its value at this point  $\chi_{\text{CA}}$ ;  $\Delta\phi'_{V_\ell}$  is now given by

$$\Delta\phi'_{V_\ell} \approx \frac{1}{2\hbar} Z_p Q_0^{(\ell)} e^2 P_\ell(\cos\chi_{\text{CA}}) \int_{-\infty}^{\infty} \frac{dt}{r(t)^{\ell+1}} \quad (6a)$$

Replacing  $\phi'$  in Eq. (3) by

$$\phi'_{V \neq 0} = \phi'_{V=0} + \sum_{\ell > 2} \phi'_{V_\ell} \quad (7)$$

then

$$S_{0 \rightarrow I}^{J=0} = \frac{1}{2} \sqrt{2I+1} \int_0^\pi P_I(\cos \bar{\chi}) \sqrt{\sin \chi_0 \sin \bar{\chi} \frac{d\bar{\chi}}{d\chi_0}} e^{i(\phi'_{V=0} + \sum_{\ell > 2} \phi'_{V_\ell})} d\chi_0 \quad (8)$$

Under the present assumption that  $V_{\ell > 2}^{\text{Coul}}(r, \chi)$  constitutes a perturbation, we can expand

$$e^{i \sum_{\ell > 2} \phi'_{V_\ell}} \approx 1 + i \sum_{\ell > 2} \phi'_{V_\ell} = 1 + i \sum_{\ell > 2} C_\ell P_\ell(\cos \chi_{CA}) \quad (9)$$

where

$$C_\ell = \frac{1}{2} Z_p Q_0^{(\ell)} e^2 \int_{-\infty}^{\infty} \frac{dt}{r(t)^{\ell+1}} \quad (10)$$

The S-matrix element is finally written as

$$S_{0 \rightarrow I}^{J=0} = \frac{1}{2} \sqrt{2I+1} \int_0^\pi P_I(\cos \bar{\chi}) \sqrt{\sin \chi_0 \sin \bar{\chi} \frac{d\bar{\chi}}{d\chi_0}} e^{i\phi'_{V=0}} \left\{ 1 + \sum_{\ell > 2} C_\ell P_\ell(\cos \chi_{CA}) \right\} d\chi_0 \quad (11)$$

In this form the contribution from each multipole deformation ( $\ell > 2$ ) appears explicitly as a form factor  $P_\ell(\cos \chi_{CA})$  multiplied by a strength coefficient  $C_\ell$ .

We reiterate that in this approximation all quantities appearing in Eq. (11) are evaluated considering the Hamiltonian [Eq. (1)] with  $V_{\ell > 2}^{\text{Coul}}(r, \chi)$  set equal to zero.

In the preceding analysis the quadrupole potential was formally separated from the higher multipole interaction terms because of its normal dominance in rotational excitation. Obviously this is not a restriction, and one could group the terms in a different way. For example, if the target nucleus has a very large hexadecapole electric moment, we could pull the hexadecapole interaction term from  $V_{\ell>2}^{\text{Coul}}(r, \chi)$  and include it in the unperturbed part of the Hamiltonian. The modification of Eq. (11) in such a case is obvious. We stress that the expression for the CLSM given by Eq. (3) is accurate for large values of  $V_{\ell>2}^{\text{Coul}}(r, \chi)$  but Eq. (11) is valid only as long as those terms ( $\ell > 2$ ) may be considered as a perturbation relative to the monopole and quadrupole terms.

### 3. SHAPE VIBRATIONS

The formalism developed in the previous section is especially useful for nuclear vibrations, since they will affect the classical motion of the system less than a permanent deformation, due to their oscillatory character. Therefore, if these vibrations are not too large in amplitude, they can also be considered as a perturbation.

To fix ideas let us assume we have a deformed target nucleus with just a quadrupole deformation, and we are interested in studying a particular harmonic monopole vibration of order  $2^{\ell}$ . From the previous section it is straightforward to add other permanent deformations or vibrations.

The Hamiltonian for this system is now

$$\begin{aligned}
 H(r, \chi, q, p_r, p_\chi, n) &= \frac{p_r^2}{2m} + \frac{p_\chi^2}{2} \left( \frac{1}{\mathcal{J}} + \frac{1}{mr^2} \right) + \hbar \omega_\ell (n + \frac{1}{2}) + \frac{z_p z_T e^2}{r} \\
 &+ \frac{z_p Q_0^{(2)} e^2 P_2(\cos \chi)}{2r^3} + V_{\ell > 2}^{\text{Coul}}(r, \chi, q, n) + V_{\text{nuc}} \quad (12)
 \end{aligned}$$

where

$$V_{\ell > 2}^{\text{Coul}}(r, \chi, q, n) = \frac{z_p Q_0^{(\ell)}(q, n) e^2 P_\ell(\cos \chi)}{2r^{\ell+1}} \quad (13)$$

In Eqs. (12,13),  $q$  is the phase of the vibration and  $n$ , the classical analog of the vibrational quantum number, is canonically conjugated to  $q$ . As before, we will neglect for simplicity the nuclear potential  $V_{\text{nuc}}$  and consider only Coulomb excitation in the following.

The oscillating electric multipole moment  $Q_0^{(\ell)}$  is related to  $q$  and  $n$  by

$$Q_0^{(\ell)}(q, n) = \kappa_\ell \sqrt{n + \frac{1}{2}} \cos q \quad (14)$$

where  $\kappa_\ell$  is a proportionality factor depending only on the charge distribution and radius of the nucleus.

The expression for the S-matrix to be used is that of ref. 10 and 11, with the considerations made in ref. 3 for the case of Coulomb excitation, i.e., a generalization of the previous expression for the CLSM [Eq. (3)] to include the new degree of freedom, the oscillator phase.

It is given by

$$S_{0,0 \rightarrow I,n}^{J=0} = \frac{\sqrt{2I+1}}{4\pi} \int_0^\pi d\chi_0 \int_0^{2\pi} d\bar{q}_0 P_I(\cos \bar{\chi}) e^{i\bar{q}n} \sqrt{\sin \chi_0 \sin \bar{\chi} \frac{\partial(\bar{q}, \bar{\chi})}{\partial(\bar{q}_0, \chi_0)}} e^{i\phi'} \quad (15)$$

The definitions of  $\bar{q}_0$  and  $\bar{q}$  are similar to that of  $\bar{\chi}$ , and are given in the Appendix. Equation (15) may be integrated numerically in a tractable but time-consuming double integration. However, considerable simplification and insight results if it is possible to treat the  $2^\ell$ -pole vibration as a perturbation in the sense previously discussed. Assuming that the vibration affects only the phase and not the other quantities appearing in the integrand, the Jacobian may be factored as

$$\frac{\partial(\bar{q}, \bar{\chi})}{\partial(\bar{q}_0, \chi_0)} = \frac{\partial \bar{q}}{\partial \bar{q}_0} \cdot \frac{\partial \bar{\chi}}{\partial \chi_0} \quad (16)$$

since  $\partial \bar{\chi} / \partial \bar{q}_0 \cong 0$ .

This factorization and the separation of  $\phi'$  into two terms, corresponding to permanent quadrupole and oscillating  $2^\ell$ -pole contributions, allows separation of the double integral in Eq. (15) as follows:

$$\begin{aligned} S_{0,0 \rightarrow I,n}^{J=0} &= \frac{\sqrt{2I+1}}{2} \int_0^\pi d\chi_0 P_I(\cos \bar{\chi}) \sqrt{\sin \chi_0 \sin \bar{\chi} \frac{\partial \bar{\chi}}{\partial \chi_0}} e^{i\phi'_{V=0}} \\ &\times \frac{1}{2\pi} \int_0^{2\pi} d\bar{q}_0 \sqrt{\frac{\partial \bar{q}}{\partial \bar{q}_0}} e^{i(\phi'_{V_\ell} + \bar{q}n)} \end{aligned} \quad (17)$$

In Eq. (17),  $\phi'_{V_\ell}$  is defined as

$$\begin{aligned} \phi'_{V_\ell} &\equiv \int_{-\infty}^{\infty} \frac{Z_p Q_0^{(\ell)}(q(t)) e^2 P_\ell(\cos \chi(t))}{2 r(t)^{\ell+1}} dt \\ &\approx \frac{Z_p e^2 P_\ell(\cos \chi_{CA})}{2} \int_{-\infty}^{\infty} \frac{Q_0^{(\ell)}(q(t))}{r(t)^{\ell+1}} dt = C_\ell P_\ell(\cos \chi_{CA}) \end{aligned} \quad (18)$$

By expanding  $\exp(i\phi'_{V_\ell})$  as before [Eqs. (9,10)] we find that

$$S_{0,0 \rightarrow I,n}^{J=0} = \frac{\sqrt{2I+1}}{2} \int_0^\pi d\chi_0 P_I(\cos\bar{\chi}) \sqrt{\sin\chi_0 \sin\bar{\chi} \frac{\partial\bar{\chi}}{\partial\chi_0}} F_n e^{i\phi'_{V=0}} \times (1 + C_\ell P_\ell(\cos\chi_{CA})) \quad (19)$$

where

$$F_n \equiv \frac{1}{2\pi} \int_0^{2\pi} d\bar{q}_0 \sqrt{\frac{\partial\bar{q}}{\partial\bar{q}_0}} e^{i\bar{q}n} \quad (20)$$

We again see the perturbation appearing as a form factor multiplying the integrand of the unperturbed CLSM expression [Eq. (3)]. The form factor has the same functional form as in the case of small permanent deformations, which is to be expected if these are considered as vibrations with a very large period compared to the collision time.

As an application of this formalism we will consider excitation of the lowest octupole vibrational band of  $^{238}\text{U}$ , which has been strongly excited by heavy-ion beams.<sup>12</sup> This band is primarily  $K=0$  for the low spin members, with  $K$ -mixing evident for higher spins.

Since only the lowest vibrational state is excited in this case, then  $n=1$ . Further, the band levels have spins  $I=1,3,5,\text{etc.}$  due to the octupole symmetries. Therefore for the sum  $(1 + C_3 P_3(\cos\chi_{CA}))$  appearing in the integrand of Eq. (19), only the second term has a non-zero contribution. (We note also that for the ground band,  $n=0$ ,  $I=0,2,4,\text{etc.}$ , and only the first term (1) in that sum would contribute.)

The final CLSM expression for the  $\kappa = 0$  octupole band is

$$S_{0 \rightarrow I}^{J=0} = \frac{\sqrt{2I+1}}{2} F_1 C_3 \int_0^\pi dX_0 P_I(\cos \bar{X}) P_3(\cos X_{CA}) \sqrt{\sin X_0 \sin \bar{X} \frac{\partial \bar{X}}{\partial X_0}} e^{i\phi'}_{V=0} \quad (20)$$

where  $I = 1, 3, 5, \dots$

Apart from the constant factor  $F_1 C_3$ , the only difference between Eq. (20) and Eq. (3) (which is valid for the ground band) is in the form factor  $P_3(\cos X_{CA})$ .

Comparisons of the  $K=0$  octupole band excitation signature found by means of Eq. (20) and by the standard Winther-deBoer computer code,<sup>13</sup> which is based on the semiclassical Alder-Winther (A-W) theory<sup>14</sup> for Coulomb excitation is shown for two cases in Figs. 2 and 3. The agreement found is seen to be quite reasonable.

We remark that just as in the case of the ground state rotational band, in the limit where the Sommerfeld parameter  $\eta$  becomes infinity and the adiabaticity parameter  $\xi$  approaches zero, the expression (20) for the classical-limit S-matrix becomes identical to the corresponding one in the Alder-Winther theory<sup>3,5</sup> (see Eq. 5.7 in Ref. 14). Therefore, the considerations of Ref. 5 concerning the nature and accuracy of semiclassical and classical-limit scattering theories are applicable here also.

#### 4. ALTERNATIVE DESCRIPTION AND ITS IMPLICATIONS

Expression (20) for the S-matrix differs from the one for the case of pure rotational excitation of the ground band only in the factor  $P_3(\cos X_{CA})$  appearing in the integrand. From its derivation it is apparent that this factor is due to the octupole vibration, and it may be explained

in a less rigorous but more illuminating way. The  $K=0$  octupole vibration appears as a standing-wave shape vibration of the nuclear surface with a  $P_3(\cos\theta)$  dependence. The excitation of this vibration by a projectile will be strongly dependent on the particular trajectory followed by the projectile, especially at the point of closest approach where the interaction is at its maximum.

In particular we expect that trajectories, such as the one labeled (1) in Fig. 4, that approach the target along a node of the octupole vibration will excite that mode only slightly, while those like (2) will excite it much more since their point of closest approach will be near the region where the vibrational amplitude is maximum. If the excitation intensity is small enough (as is actually the case) the excitation amplitude will be linear in the vibration amplitude at the point of closest approach, i.e., in  $P_3(\cos X_{CA})$ .

The amplitude for the excitation of the rotation and the vibration is then found by integrating the product of the rotation amplitude times this vibration amplitude over all initial orientations. (This implies that the coupling between the two motions is neglected.) The rotational excitation amplitude for a given trajectory is given by

$$\frac{1}{2} \sqrt{(2I+1) \frac{d(\cos \bar{X})}{d(\cos X_0)}} P_I(\cos X) \exp(i\Delta_1/\hbar) \quad (21)$$

Thus we have rederived Eq. (20) and the term  $P_3(\cos X_{CA})$  is now interpreted as a form factor for the excitation of the octupole vibration.

This interpretation allows us to extend this particular approximation to the CLSM method to other situations where conditions similar to those encountered in the case of the  $K=0$  octupole vibrations are met; more



precisely a small interaction strength, little coupling to the rotational motion, and the possibility of describing the process in classical terms. We are presently considering the extension of this formalism to cases that satisfy these conditions, in particular, sub-barrier nucleon and cluster transfer processes. In both cases it is expected that the amplitude (form factor) for transfer will be peaked in certain regions of the deformed nuclear surface, due to the angular orientation of the high-lying nucleon orbits available for transfer. Such an angle-dependent form factor for transfer is formally analogous to the angle-dependent form factor for vibrational excitation which we have just discussed, and should in like manner give rise to a characteristic signature in the ground rotational band of the deformed transfer product.

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APPENDIX

As noted in the text, the bar and tilde quantities appearing in Eq. (4) and thereafter arise from a unitary transformation of the original variables appearing in the Hamiltonian, Eq. (1). The purpose of the transformation is to remove an oscillatory asymptotic time dependence in the integral representation for the S-matrix element. Therefore it is closely related to the unitary transformation from the Schrödinger to interaction representation in quantum mechanics. We sketch the basic equations here. A further discussion may be found in Refs. 3, 5, and 11.

The quantities  $\bar{\chi}$ ,  $\bar{q}_0$  and  $\bar{q}$  are defined in a similar way as in Ref. 11, i.e., by considering the tangential elastic trajectory in the asymptotic regions defined by

$$\bar{E} = \frac{\tilde{p}_r^2}{2m} + \frac{p_\chi^2}{2} \left( \frac{1}{\mathcal{J}} + \frac{1}{\tilde{m}r^2} \right) + \hbar\omega_0 \left( n + \frac{1}{2} \right) + \frac{z_p z_T e^2}{\tilde{r}} \quad (\text{A.1})$$

In this expression  $p_\chi$  and  $n$  are taken to be constant and equal to their value at some point in the asymptotic region for the collision;  $\tilde{r}$  and  $\tilde{p}_r$  are the radial coordinate and momentum for an elastic collision trajectory that coincides with the actual trajectory at the point mentioned above. By defining  $\tilde{r}_T$  as the turning point of the radial motion in this elastic trajectory, i.e.

$$p_r(\tilde{r}_T, p_\chi, n, \bar{E}) = 0 \quad ,$$

$\bar{\chi}$ ,  $\bar{q}_0$  and  $\bar{q}$  are given by

$$\bar{\chi} = \chi + \int_{\tilde{r}_{T\text{out}}}^r \frac{\partial \tilde{p}_r}{\partial \tilde{p}_\chi} d\tilde{r} \quad (\text{A.2})$$

$$q_0 = q_0 + \int_{\tilde{r}_{T\text{inc}}}^r \frac{\partial \tilde{p}_r}{\partial n} d\tilde{r} \quad (\text{A.3})$$

$$\bar{q} = q + \int_{\tilde{r}_{T\text{out}}}^r \frac{\partial \tilde{p}_r}{\partial n} d\tilde{r} \quad (\text{A.4})$$

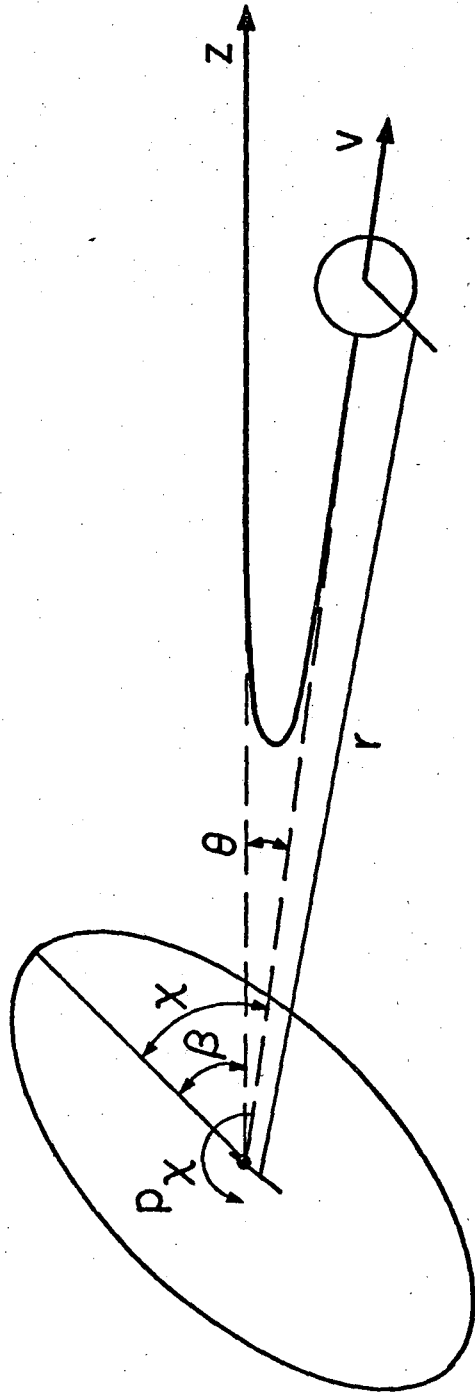
As we see in Eqs. (A.2 - A.4),  $\bar{q}_0$  is defined on the incoming branch of the trajectory and  $\bar{q}$ ,  $\bar{\chi}$ , are defined on the outgoing branch.

REFERENCES

1. S. Levit, U. Smilansky, and D. Pelte, Phys. Lett. 53B (1974) 39.
2. H. Massmann and J.O.Rasmussen, Nucl. Phys. A243 (1975) 155.
3. R. Donangelo, M.W.Guidry, J.P.Boisson, and J.O.Rasmussen, Phys. Lett. 64B (1976) 377.
4. M.W.Guidry, H. Massmann, R. Donangelo, and J.O.Rasmussen, Nucl. Phys. A274 (1976) 183.
5. M.W.Guidry, R. Donangelo, J.P.Boisson, and J.O.Rasmussen, Nucl. Phys. A295 (1978) 482; and R. Donangelo, LBL-5825, Ph.D. Thesis, (1977) unpublished.
6. R.A.Malfliet, Symposium on Classical and Quantum Mechanical Aspects of Heavy-Ion Collisions, Heidelberg (1974).
7. T. Koeling and R.A.Malfliet, Phys. Reports 22C (1975) 182.
8. J. Knoll and R. Schaeffer, Phys. Lett. 52B (1974) 131.
9. J. Knoll and R. Schaeffer, Ann. Phys. 97 (1976) 307.
10. W.H.Miller, J. Chem. Phys. 53 (1970) 3578.
11. R.A.Marcus, J. Chem. Phys. 56 (1972) 3548.
12. E. Grosse, J. de Boer, R.M.Diamond, F.S.Stephens, and P. Tjøm, Phys. Rev. Letters 35 (1975) 565.
13. A. Winther and J. de Boer, California Institute of Technology Technical Report (November 18, 1965); reprinted in "Coulomb Excitation", by K. Alder and A. Winther (Academic Press, New York, New York, 1966).
14. K. Alder and A. Winther, Mat. Fys. Medd. Dan. Vid. Selsk 32 (1960) 8.
15. R.A.Brogia, S. Landowne, R.A.Malfliet, V.Rostokin, and A. Winther, Phys. Reports 11C (1974) 1.

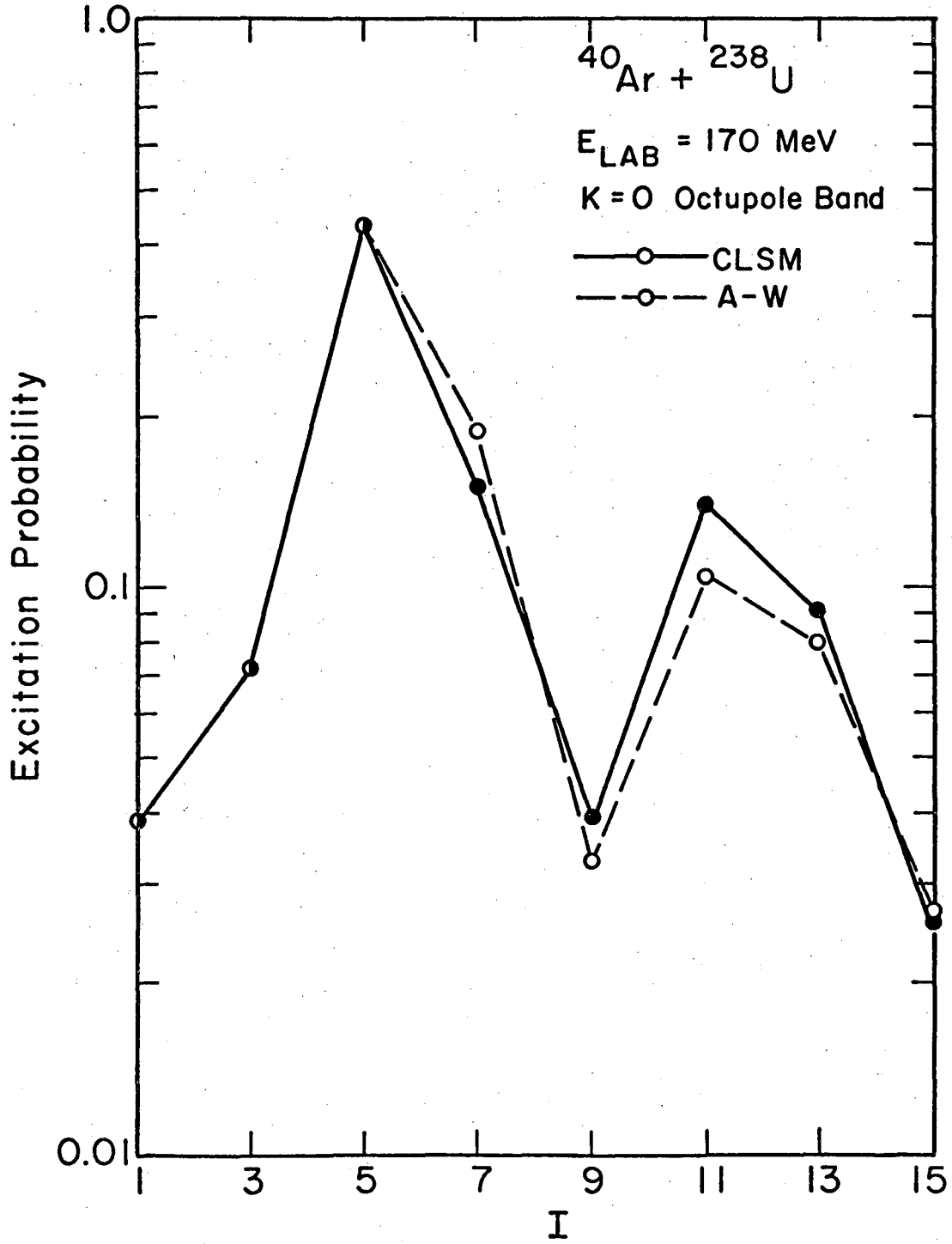
FIGURE CAPTIONS

- Fig. 1. Geometrical illustration of the coordinate system used in the calculations.
- Fig. 2. Signature of the  $K=0$  octupole band excitation in  $^{238}\text{U}$  by 170 MeV  $^{40}\text{Ar}$  ions scattered at backward angles. The energies are taken from the rotational model with  $E_{1-} = 0.7313$  MeV for the octupole band and  $E_{2+} = 0.0449$  MeV for the ground band. The quadrupole moment of  $^{238}\text{U}$  is taken to be 11.12b for both bands. The solid line represents the calculation described here, and the dashed line that of the Alder-Winther method.<sup>14</sup> The probabilities for octupole band excitation are normalized to unity.
- Fig. 3. Same as Fig. 2 using 400 MeV  $^{86}\text{Kr}$  ions as projectiles.
- Fig. 4. The  $K=0$  octupole vibration is represented as a standing wave on the nuclear surface. The trajectory labeled (2), which has its point of closest approach near a maximum in the vibrational amplitude, excites the octupole vibration much more than trajectory (1), which has its point of closest approach near a node in the vibration.



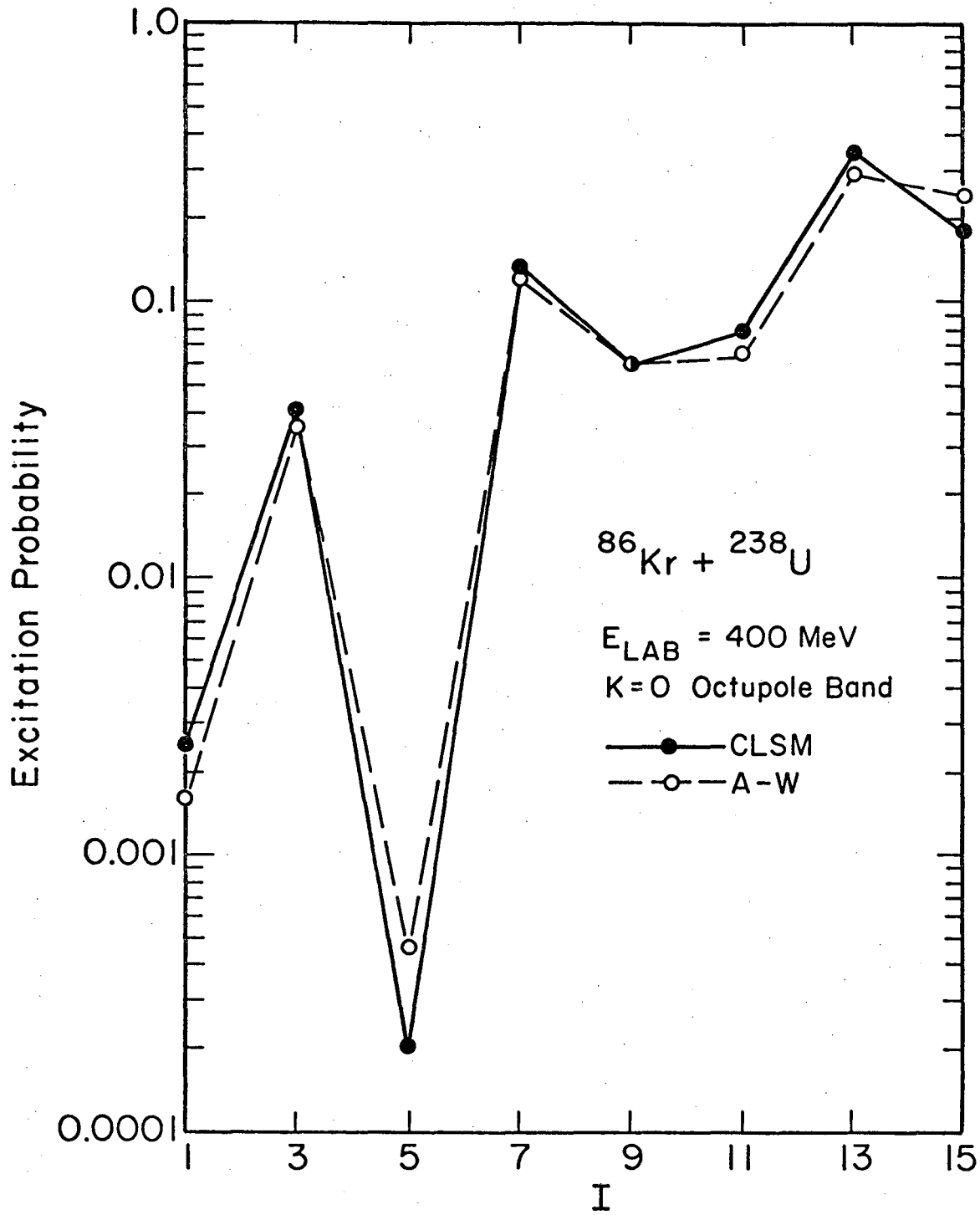
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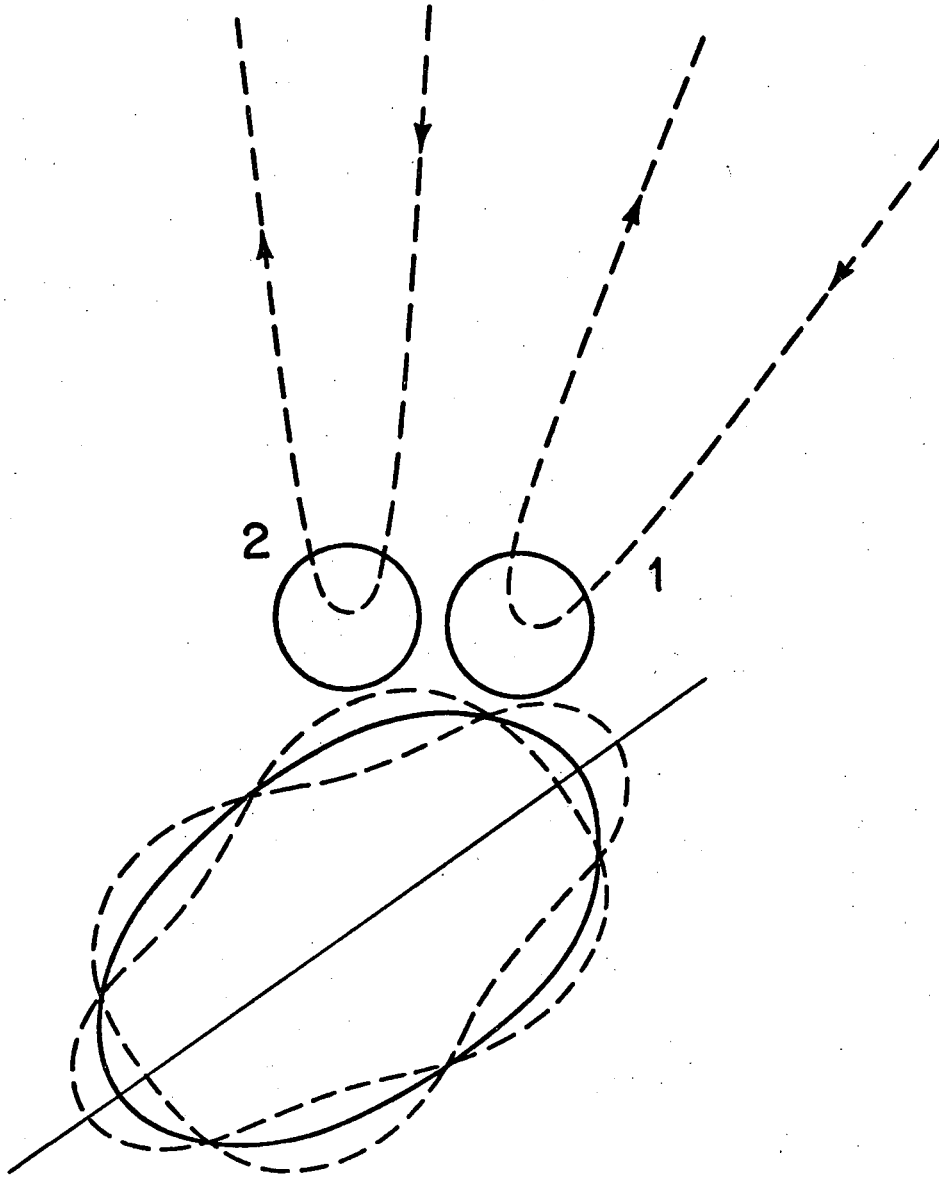
Figure 2  
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Figure 3  
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