Title
Loan Loss Severity and Optimal Mortgage Default

Authors
Lekkas, Vassilis
Quigley, John
Van Order, Robert

Publication Date
1993

Peer reviewed

iber

Institute of
Business and
Economic Research

University of
California at
Berkeley

CENTER FOR REAL ESTATE AND URBAN ECONOMICS

WORKING PAPER SERIES

WORKING PAPER NO. 93-216

LOAN LOSS SEVERITY AND OPTIMAL MORTGAGE DEFAULT

By

VASSILIS LEKKAS
JOHN QUIGLEY
ROBERT VAN ORDER

WALTER A. HAAS SCHOOL OF BUSINESS

Loan Loss Severity and Optimal Mortgage Default

by

Vassilis Lekkas*

John Quigley**

Robert Van Order*

*Federal Home Loan Mortgage Corporation

**University of California, Berkeley

Working Paper No. 93-216
May 1993

I. Introduction

Over the past few years there has been increased attention to the pricing of credit risk in the mortgage market, (e.g., Cunningham and Hendershott [1984], Kau et al [1986, 1991] and Hendershott and Van Order [1987]), and to empirical models of default (e.g., Campbell and Dietrich [1983], Foster and Van Order [1984], Cooperstein, Redburn, and Myers [1991], and Quigley and Van Order [1992]). For the most part, these models focus on the contingent claims approach to default, which treats default as the exercise of a put option.

The importance of homeowner equity, which is a measure of the extent to which the option is "in the money," in virtually all studies of default suggests that the options approach is a fruitful way to analyze default. Kau et al [1992] simulate default frequencies in a frictionless model (i.e., one with no transaction costs) and argue that these frequencies are not much different from those actually observed. They also argue that the introduction of transactions costs into their simulation implies implausibly low default rates. This leads them to conclude that the frictionless model does a good job of explaining default behavior. These are, however, weak tests of qualitative properties. One can introduce different types of transaction costs that are fully consistent with the data on observed default propensities (see Quigley and Van Order [1992]).

In this paper we analyze this issue using a completely different body of data: loan loss severities on defaulted mortgages. The severity of loan losses can be interpreted as a measure of the extent to which the option is in the money when it is exercised. The frictionless options model has well-defined predictions about loss severities, which we test in some detail. The frictionless model does not do very well in these tests.

The data for this exercise include individual loans originated during the period 1975 through 1990 and purchased by Freddie Mac. We analyze a random sample of mortgage loans, investigating the probability of mortgage default. We use these data and Freddie Mac's data on loan losses to analyze the loss severities for some 15,000 mortgages which defaulted during the period.

II. Optimal Exercise

Rational borrowers in a perfectly competitive market will exercise options when they can thereby increase their wealth. Absent either transactions costs or reputation costs (which reduce credit ratings), wealth can be increased by defaulting when the market value of the mortgage equals or exceeds the value of the house. Similarly, by prepaying when market value exceeds par, borrowers can increase wealth by refinancing. Note that the value of the mortgage exceeds the present value of the remaining payment stream because the mortgage claim includes both the options to prepay and also to default at some subsequent date. Thus, even if the market value of the house is less than the present value of future mortgage payments (*i.e.*, the default option is "in the money"), it may not be optimal to exercise the default option.

The problem of determining when to exercise an option requires specifying the underlying state variables and parameters that determine the price of any contingent claim and then deducing the rule for exercise that maximizes borrower wealth. For residential mortgages, the key state variables are interest rates and house values. The value of a mortgage, M(c, i, t, V, T), depends upon the coupon rate, c, a vector of relevant interest rates, i, property value, V, the age of the mortgage, t, the remaining time to maturity, T, and various parameters. A standard arbitrage argument is sufficient to derive an equilibrium condition for M (a second order partial differential equation), specifying that the expected return plus capital gains must equal the risk-free rate of return plus a risk adjustment. This condition applies to any claim that is contingent on the underlying state variables. It implies that the value of the mortgage equals the risk-adjusted expected present value of its net cash flows.[1]

To simplify matters and to isolate the default option, assume that interest rates are non-stochastic (so that the only source of risk is house price volatility) and there is no prepayment option. Assume that house price changes are continuous, with an instantaneous mean, μ, (which need not be constant) and a standard deviation, σ. Let ρ be the imputed rent payout ("dividend") rate. The arbitrage model implies that the value of the mortgage M satisfies

$$(1) \qquad (\tfrac{1}{2}M^2\sigma^2)(\partial^2 M/\partial V^2) + M(i-\rho)(\partial M/\partial V) + (\partial M/\partial t) + C = iM,$$

where i is the interest rate and C is the coupon payment on the mortgage (which depends on the coupon rate, c).

This follows almost directly from the analysis of Black and Scholes [1973]. Stochastic house prices are assumed to follow

$$(2) \qquad dV = \mu(V,t)dt + \delta(V,t)\,dz$$

$$= \mu V dt + \sigma V\,dz,$$

where z is a normally distributed error term. Arbitrage-free equilibrium requires that the expected return, θM, on holding any security, M, whose value is contingent on V, equal the risk-free return plus a risk adjustment based on the market price of house price risk

[1]See Cox, Ingersol and Ross [1985], lemma 4.

(3) $\theta M = i M + \lambda\delta(\partial M/\partial V),$

where λ is the market price of house-price risk[2]. Equation (3) applies to the house itself, so that in equilibrium the expected return, θV, on holding a house must equal the risk-free return plus the risk adjustment

(4) $\theta V = iV + \lambda\delta$.

This expected return is, however, nothing other than the flow of services from the asset, at rate ρ, and the expected capital gain

(5) $\theta V = \rho V + \mu V$

Finally, the expected yield on the mortgage, from Ito's lemma, is given by

(6) $\phi M = C + (\partial M/\partial t) + \mu(\partial M/\partial V) + (\frac{1}{2})\delta^2(\partial^2 M/\partial V^2)$

Equating (4) and (5), substituting into (3) and equating (3) and (6) yields equation (1). Equation (1) simply states that the expected return (coupon plus expected, risk-adjusted capital gains) must equal the risk-free rate.

Note that the expected appreciation rate μ of traded assets (in this case, houses) does not appear on equation (1) nor does the price of risk, λ, for holding the asset.[3] If the underlying state variables are traded assets, then arbitrage leads to a risk-neutral interpretation of the price of a contingent claim on an asset relative to the price of that asset. The value of the option is the expected present value of the outcome, where prices are projected to grow at a mean rate of $i-\rho$ (and variance $\sigma^2 T$) and are discounted at the risk-free rate. This is equivalent to

---

[2]For a general deviation of (3) and in particular of λ see Brennan and Schwartz (1985).

[3]This is because (4) and (5) implies that $(i-\rho)V = \mu V - \lambda\sigma$.

assuming risk neutrality (See Smith [1976] for a discussion and Cox, Ingersoll and Ross [1985] for a proof).

An infinite number of functions satisfy (1), which reflect the infinite number of ways that coupon plus capital gain can equal the required expected return. By incorporating the optimal call and/or put strategies, the function appropriate for a particular mortgage can be determined. Optimal exercise strategies are determined by wealth maximization.

If there are no costs to default other than losing the house, the optimal default "strategy" is characterized simply by the house value at time t, V_t*, at which default takes place. The optimal V_t* minimizes the value of the mortgage (this maximizes the borrower's net worth), subject to the condition that V_t* equal the value of the remaining balance when the option is exercised.

Figure 1, adapted from Quigley and Van Order [1992], illustrates the optimal strategy. This strategy is represented by the lowest curve (i.e., the one that minimizes M) which satisfies equation (1) and is not above the 45 degree line (where the remaining balance equals the value of the house). If the solution is an interior one, it is represented by the tangency depicted in the figure. The curve must also be below the horizontal line M, which gives the value of a riskless mortgage. The curve approaches M asymptotically as V increases. The tangency determines V_t*, the default "strategy." The entire curve gives the market relationship between mortgage values and house prices. The distance X is the value of the default option, the premium for insurance that a competitive mortgage insurer would charge. At V_t* the distance S (=X) represents the extent to which the option must be in the money before default. Note, however, that this distance is also the severity of the loss by the lender or mortgage insurer (absent transactions costs) from selling the house immediately after foreclosure.

The virtue of the contingent claim model is its simplicity. The default option is exercised at V_t^*, which depends only on the variables in (1) and on the boundary and tangency conditions, which in turn depend on the variables in (1). The equilibrium condition has the property that the mean price change of any traded asset as well as the risk premium are irrelevant in pricing the option or in exercising it. Thus, circumstances under which default occurs depend only on ρ,σ, c, T and M relative to V; they are independent of the original house price, expected price appreciation, the original loan-to-value ratio (LTV), the historic path of prices and the market price of risk.

The logic of the model is relatively straightforward. The "cost" of exercising the option now, even if the option is in the money, is the inability to exercise it later on. Things that cause the option value to be high will cause the borrower to delay exercising it until it is further in the money, causing severity (S in Figure 1) to be higher. In particular, the longer the time to maturity, the lower the coupon rate relative to current interest rates, the higher the rental rate and the higher its volatility, the more valuable is the option. We cannot measure the last two with our data set. However, from the data that we have we can test the following four propositions about loss severities:

1. *Ceteris paribus*, severity should be independent of initial LTV. However, high LTV loans almost always have insurance if they are purchased by Freddie Mac. The cost of insurance increases the effective coupon rate to the borrower for high LTV loans, but not

the mortgage coupon rate measured in financial data. Thus for this data set, the frictionless model predicts that severity should fall as initial LTV increases.

2. *Ceteris paribus*, severity should be the same in regions with high default frequencies as in regions with low frequencies and should be the same for loans originated in "good" years (e.g., those followed by house price inflation) as in "bad" years.

3. Severity should decrease with the age of the mortgage. This is because older loans have a shorter time to maturity, and therefore there is less value to keeping the option alive. The option will be exercised when it is less far into the money.

4. Severity should decrease as coupon-rate-minus-the-current-interest-rate increases. The higher the coupon, the less value there is in keeping the option alive, and thus the sooner it will be exercised.

III. Data and Results

Table 1 tabulates loss severity for all Freddie Mac defaults on single-family, owner-occupied, non-condominium loans purchased within one year of origination from 1975-1990.[4] Loss severity, gross of any insurance payments, is measured by the difference between the mortgage balance and the value of the house for defaulted loans. It excludes all transactions costs and foregone interest. House values are measured in two ways. The first is based on an appraisal at the time a defaulted property is acquired by Freddie Mac. The second is the actual sale price when (about a year later) the house is sold from the Freddie Mac inventory. Neither is a perfect measure of the extent to which the option was in the money when the borrower

[4]Seasoned loans acquired by Freddie Mac were eliminated from these comparisons.

chose to default. Nonetheless, there is no reason to believe there is a systematic bias by LTV, coupon rate, interest rate, or age.

Panel A presents loss severity as a fraction of loan balance for the loans described above that defaulted from 1975-1990. The first column indicates the losses based on appraisal data at acquisition ("Loss I"), while the second column uses eventual selling prices ("Loss II"). On average, actual losses consistently exceeded appraised losses, by about 8 to 10 percent. In both columns, however, there is a strong effect of LTV. High LTV loans have much higher severity rates. This is not consistent with the first prediction of the ruthless model.

Panel B of the table presents similar calculations for Texas defaults during the same time period. The LTV effect remains, though it appears to be much smaller. However, the Texas losses are substantially higher. This is not consistent with the second prediction of the frictionless model.[5]

Of course, these static comparisons do not hold other things constant. For instance, the table does not control for the age of the loan. *Ceteris paribus*, high LTV loans will have negative equity at younger ages than low LTV loans. Because younger loans have larger option values than older ones, severity should be lower. To control for this and for other factors, we report regressions relating individual severity rates on defaulted loans to LTV categories, a dummy variable for Texas loans, dummy variables for origination years, the age of the mortgage at the time of default, and coupon-rate-minus current-mortgage-rates.

---

[5]Institutional differences, such as homestead provisions and state laws requiring delays in enforcing eviction, may cause average loss rates to vary among states. In general, however, Texas provides fewer protections against eviction than any other state. Thus, based on institutional differences alone, Texas loss rates should be lower than elsewhere. See Clauretie and Herzog [1989].

A. A Crude Test

Table 2 summarizes these regressions using loss rates estimated at the time of property acquisition by Freddie Mac. All four predictions are apparently rejected in the results reported in column 1. The effects of LTV and of Texas loans are the same as those reported in Table 1. Age has a positive effect, as does the interest rate differential.

These tests are not definitive. We cannot control for σ and ρ directly, and it is possible that they are correlated with the explanatory variables[6]. The simple linear model does not capture all the nonlinearities implicit in the options model. In column 2, we add quadratic terms for the age of the mortgage and for coupon-minus-current interest rate. Again, all four predictions are rejected. The result for coupon minus current interest rate is, however, fragile and disappears when the specification is altered slightly as in column 3. The coefficient estimated for the age of the mortgage is also fragile, and its sign changes in columns 3 and 4. In contrast, the effect of LTV appears to be quite robust to changes in specification.

Table 3 reports a second set of regressions relying upon loss severity rates computed using the actual sale prices of defaulted properties acquired by Freddie Mac. The specifications of the four regression equations are the same as those reported in Table 2. The effect of LTV is again highly significant as is the dummy variable for Texas loans. The effects of mortgage age and coupon interest rate are mixed.

---

[6]For instance σ might be high in Texas, causing severity to be higher--though whether this could explain a 7 percentage point difference in average losses is unclear.

In both Tables 2 and 3, the dummy variables for origination year (not presented) are highly significant. Losses were much lower on mortgages originated in the 1970s when inflation was high than they were later on when inflation was low.

B. A More Precise Test

Of course, the 14,867 observations on loss severities analyzed in Tables 1, 2, and 3 are hardly a random sample of the millions of mortgages purchased by Freddie Mac During the 1975-1990 time period. In particular, the "rule" for selecting observations for the analysis of loss severities is the fact of default by the borrower. The contingent claims model predicts that the probability of default is itself a function of homeowner equity. Homeowner equity is a function of initial LTV and the subsequent course of house prices, which vary by geographical region and time period. Thus, the selectivity of the sample implies that the coefficients will be biased measures of the unconditional loss severities in the population.

Consistent estimates of the behaviorial parameters can be obtained by including the Mills ratio of the selection rule as an additional variable in the regression (See Heckman [1976]). The Mills ratio (f/[1-F], where f is the probability density function and F is the the cumulative density function) can be computed directly from the hazard of default for each observation in the analysis sample.

Appendix A presents estimates of the default hazard for a random sample of 277,199 observations on mortgages purchased by Freddie Mac during the 1976-1990 period (2,036 of these mortgages actually defaulted). The coefficients of this hazard model, together with the underlying baseline hazard, permit us to compute the Mills ratio for each of the 14,867 observations in the analysis sample.

FIGURE 1: Optimal Default

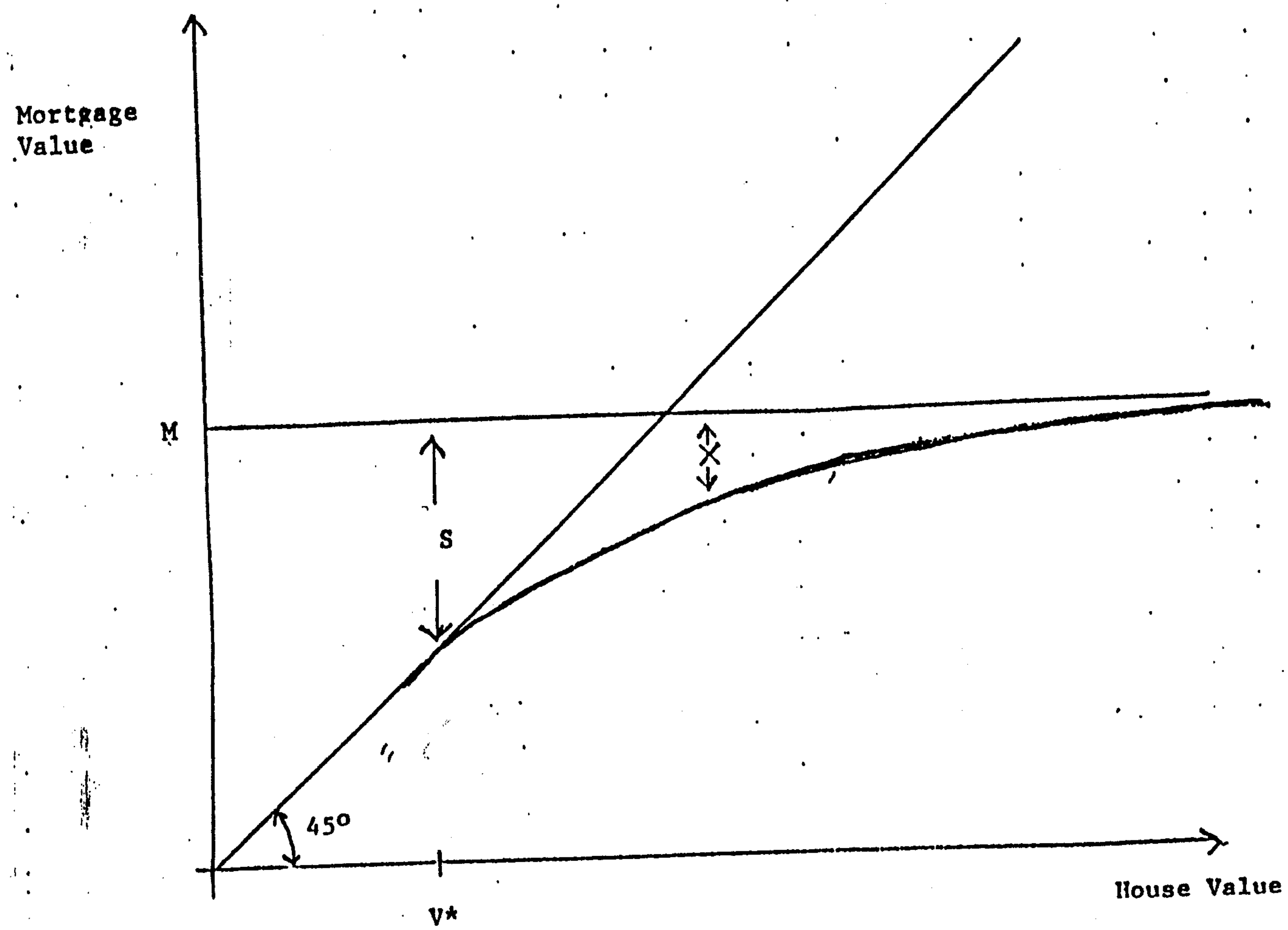

Table 4 presents the same specification as reported previously, this time including the Mills ratio as an additional explanatory variable. The dependent variable is the loss rate calculated by using the appraised value of the property at acquisition.

Table 5 presents similar results using the alternative measure of the loss rate.

In each of the regressions reported, the Mills ratio is significant and is quite large in magnitude. For example, a coefficient of about 500 in the regressions suggests that an observation with a predicted probability of default of one half percent will have a loss rate, on average, about two percentage points higher than an observation with a predicted probability of default of one tenth of one percent[7]. The historical average for Freddie Mac is around 0.2 percent.

Despite this, however, the signs and levels of significance of the other variables are unchanged, and the magnitudes are similar. Once again, the dummy variables for origination year are highly significant. The pattern of coefficients does not arise from the selectivity of observations into the sample of loan losses.

The central finding remains. Each of the four hypotheses is rejected by the data.

III. Conclusions

The frictionless model predicts that, holding term and coupon constant, there is an optimal amount of "in-the-moneyness" that will induce borrowers to default, and that amount is independent of initial LTV. Our results show a strong relationship to LTV, and even after

---

[7]i.e., $500 \left(\frac{.005}{.995} - \frac{.001}{.999}\right) \approx 2$

econometric adjustments, the results by LTV are not qualitatively different from those given in Table 1, which depicts the raw data by LTV.

Indeed, the data appear to be consistent with the hypothesis that people wait until the values of their houses have dropped by twenty percent (for Loss I) or thirty percent (for Loss II), and then they default. This rule of thumb is hardly consistent with the frictionless or ruthless model of wealth maximizing behavior.

An alternative explanation adds transaction costs and liquidity to the model. Suppose that transactions or reputation costs are such that (as in Kau et al [1992]) people seldom exercise their options "ruthlessly." Assume instead that people "get into trouble," e.g., by losing a job, and have only enough liquidity to last a specified period, e.g., a year at the end of that period they must either sell the house or default. Under these circumstances they may default if the option is just barely in the money.

At the end of a the period, all those in trouble will have experienced housing price declines of approximately the same percent, which means that loans with higher initial LTVs, will have bigger severities. This model is consistent with our data and our empirical results. In particular, it is consistent with the pronounced effects of LTV noted above and also with the ambiguity of mortgage age and coupon rate. The model is consistent with the importance of the dummy variables for origination year and for loans made in Texas during this period.

Table 1
Loss Severity by LTV as a Percent of Mortgage Balance*
1975-1990

Original LTV	Loss I	Loss II
A. All Loans		
51-60	-7.0 %	2.0 %
61-70	-0.3	8.7
71-75	0.9	10.0
76-80	2.1	11.7
81-90	5.2	15.1
91-95	14.76	24.3
B. Texas Loans		
51-60	-1.5 %	6.9 %
61-70	13.1	21.6
71-75	13.8	21.7
76-80	16.7	24.5
81-90	18.2	26.8
91-95	23.0	32.0

Notes:

*Average losses on all defaulted loans, 1975-1990, excluding defaults on seasoned loans purchased by Freddie Mac.

Loss I is computed as mortgage balance minus appraised value at acquisition.

Loss II is computed as mortgage balance minus actual sales price at time of sale.

Source: Freddie Mac.

Table 2
Regression Models of Actual Loss Severity in Percent
(Loss Severity Measured by Appraised Value Minus Mortgage Balance
14,867 observations

Variable	1*	2*	3	4
LTV 51-60 (dummy)	11.06 (1.46)	11.11 (1.47)	9.44 (1.17)	9.64 (1.20)
LTV 61-70 (dummy)	18.09 (2.63)	18.15 (2.64)	14.92 (2.04)	15.14 (2.07)
LTV 71-75 (dummy)	23.68 (3.47)	23.74 (3.48)	19.98 (2.75)	20.22 (2.78)
LTV 76-80 (dummy)	30.49 (4.58)	30.71 (4.61)	25.36 (3.58)	25.59 (3.61)
LTV 81-90 (dummy)	36.80 (5.53)	37.01 (5.57)	30.75 (4.35)	30.95 (4.37)
LTV >90 (dummy)	41.93 (6.31)	42.09 (6.33)	38.95 (5.51)	39.26 (5.55)
Age of mortgage (thousand days)	8.27 (17.27)	12.03 (8.95)	-2.64 (8.37)	-2.72 (8.64)
Age of mortgage squared (x 10^7)		-7.77 (2.93)		
Coupon minus current rate	169.05 (4.93)	184.97 (4.91)	-78.21 (2.39)	
Coupon minus current rate squared (x10^{-2})		5.41 (0.38)		
Texas (dummy)	6.64 (7.83)	6.74 (7.95)		
Intercept	-20.85 (0.91)	-22.00 (0.96)	-25.48 (3.60)	-25.47 (3.60)
R^2	0.16	0.16	0.05	0.05

Notes: * Regression also includes dummy variables for each year of origination, 1975-1989.

t-ratios are reported in parentheses.

Table 3
Regression Models of Actual Loss Severity in Percent
(Loss Severity Measured by Value at Time of Sale Minus Mortgage Balance)
(14,867 observations)

Variable	1*	2*	3	4
LTV 51-60 (dummy)	10.84 (2.33)	10.93 (2.36)	10.93 (2.26)	10.91 (2.26)
LTV 61-70 (dummy)	18.73 (4.52)	18.91 (4.57)	17.46 (4.05)	17.46 (4.05)
LTV 71-75 (dummy)	22.94 (5.60)	23.16 (5.67)	21.69 (5.10)	21.70 (5.10)
LTV 76-80 (dummy)	23.61 (5.92)	24.03 (6.04)	21.12 (5.09)	21.09 (5.09)
LTV 81-90 (dummy)	28.75 (7.22)	29.19 (7.35)	25.38 (6.13)	25.36 (6.13)
LTV >90 (dummy)	34.34 (8.62)	34.73 (8.74)	32.73 (7.91)	32.68 (7.90)
Age of mortgage (thousand days)	4.20 (11.90)	11.04 (11.29)	-2.62 (10.19)	-2.59 (10.12)
Age of mortgage squared (x 10^7)		-1.55 (7.53)		
Coupon minus current rate	224.06 (8.06)	271.99 (9.20)	36.48 (1.41)	
Coupon minus current rate squared (x10^{-2})		39.11 (3.34)		
Texas (dummy)	6.985 (11.46)	7.32 (12.01)		
Intercept	-21.59 (4.73)	-25.63 (5.59)	-15.75 (3.81)	-15.81 (3.82)
R^2	0.11	0.11	0.04	0.04

Notes: * Regression also includes dummy variables for each year of origination, 1975-1989. t-ratios are reported in parentheses.

Table 4
Regression Models of Actual Loss Severity in Percent
(Loss Severity Measured by Appraisal Value minus Mortgage Balance)
(14,867 observations)

Variable	1*	2*	3	4
LTV 51-60 (dummy)	11.08 (1.46)	11.14 (1.47)	9.70 (1.21)	9.70 (1.21)
LTV 61-70 (dummy)	17.84 (2.60)	17.93 (2.61)	14.94 (2.05)	14.94 (2.05)
LTV 71-75 (dummy)	23.19 (3.40)	23.31 (3.42)	19.62 (2.71)	19.61 (2.71)
LTV 76-80 (dummy)	29.86 (4.48)	30.13 (4.52)	24.90 (3.53)	24.90 (3.53)
LTV 81-90 (dummy)	35.72 (5.37)	36.04 (5.42)	29.20 (4.14)	29.19 (4.14)
LTV >90 (dummy)	40.00 (6.00)	40.38 (6.06)	35.13 (4.97)	35.13 (4.97)
Age of mortgage (thousand days)	8.46 (17.58)	11.40 (8.21)	-1.64 (4.89)	-1.64 (4.89)
Age of mortgage squared (x 10^7)		-6.12 (2.60)		
Coupon minus current rate	175.27 (5.01)	188.82 (5.01)	0.85 (0.03)	
Coupon minus current rate squared (x10^{-2})		610.60 (0.43)		
Texas (dummy)	5.67 (6.38)	5.88 (6.58)		
Mills Ratio	234.72 (3.61)	205.22 (3.09)	484.11 (8.61)	483.72 (8.94)
Intercept	-20.44 (0.89)	-21.41 (0.93)	-27.53 (3.91)	
R^2	0.16			

Notes: * Regression also includes dummy variables for each year of origination, 1975-1989. t-ratios are reported in parentheses.

Table 5
Regression Models of Loss Severity in Percent
(Loss Severity Measured by Value Minus Mortgage Balance at time of sale)
(14,867 observations)

Variable	1*	2*	3	4
LTV 51-60 (dummy)	10.84 (2.34)	10.94 (2.36)	10.90 (2.27)	10.86 (2.26)
LTV 61-70 (dummy)	18.45 (4.46)	18.69 (4.52)	17.24 (4.02)	17.26 (4.02)
LTV 71-75 (dummy)	22.34 (5.47)	22.66 (5.56)	21.16 (5.00)	21.25 (5.02)
LTV 76-80 (dummy)	22.94 (5.26)	23.44 (5.89)	20.39 (4.97)	20.39 (4.93)
LTV 81-90 (dummy)	27.45 (6.90)	28.08 (7.07)	23.56 (5.72)	23.69 (5.75)
LTV >90 (dummy)	31.74 (7.94)	32.54 (8.16)	28.51 (6.90)	28.75 (6.95)
Age of mortgage (thousand days)	4.5 (12.70)	10.0 (10.36)	-2.0 (-6.49)	-2.0 (6.53)
Age of mortgage squared (x 10^7)		-13.1 (6.25)		
Coupon minus current rate	236.08 (8.41)	279.98 (9.47)	118.99 (4.48)	
Coupon minus current rate squared (x10^{-2})		3946.14 (3.37)		
Texas (dummy)	5.62 (8.82)	6.15 (9.59)		
Mills Ratio	381.87 (7.17)	313.92 (5.79)	453.76 (12.19)	492.81 (11.42)
Intercept	-21.31 (-4.67)	-24.82 (5.42)	-17.23 (4.12)	-17.28 (4.20)
R^2	0.11	0.12	0.05	0.04

Notes: * Regression also includes dummy variables for each year of origination, 1975-1989. t-ratios are reported in parentheses.

Appendix A

As discussed in Quigley and Van Order [1992], the contingent claims model implies that the hazard of default is a function of homeowner equity. In particular, some level of negative equity is associated with a much higher probability of exercising the put option. Table A1 presents parameter estimates of a proportional hazard model estimated from a random sample of 277,199 mortgages purchased by Freddie Mac during the 1976-1990 period. The model includes the determinants of individual equity: the initial LTV, region, and the origination year. The parameters of hazard model are highly significant, and the parameters are broadly consistent with Quigley and Van Order [1992]. Table A2 presents the baseline hazards estimated by the method of Kaplan and Meier (see Kalbfleisch and Pretince [1980]). These parameters are used to estimate the Mills ratio for each of the 14,867 observations in the sample analyzed in the tent.

Table A1
Hazard Model

Variable	Coefficient/(t ratio)
YR76	-0.11 (-2.6)
YR77	0.25 (0.7)
YR78	1.31 (4.1)
YR79	2.24 (7.0)
YR80	3.22 (3.8)
YR81	3.70 (11.6)
YR82	3.50 (10.3)
YR83	3.30 (10.0)
YR84	3.13 (9.9)
YR85	2.76 (8.4)
YR86	1.74 (5.3)
YR87	1.83 (3.9)
YR88	1.83 (5.4)
YR89	2.29 (6.7)
YR90	2.64 (7.8)
NC	-0.12 (-1.7)
NE	-0.31 (-4.0)
SE	-0.03 (-0.4)
SW	0.91 (1.5)
LTV 6170	1.17 (6.5)
LTV 7175	1.78 (10.2)

Variable	Coefficient/(t ratio)
LTV 7680	1.90 (12.7)
LTV 8190	2.57 (16.7)
LTV 9195	3.16 (20.4)

Notes:

1. YR76-YR90: Dummies for origination year (1975 ommitted)

2. NC-SW: Dummies for region (W (western region) omitted)

3. LTV 6170-LTV 9195: Dummies for original loan-to-value ratio. LTV 6170 is loan-to-value ratio from 6170 through 70%. LTV 60% and below omitted.

Table A-2

Baseline Hazard (Dummy Variables for Age of Mortgage)

Age	Coefficient (in tenth of a basis point per year)
1	0.1
2	5.3
3	12.5
4	18.8
5	19.6
6	19.0
7	13.6
8	2.6
9	2.3
10	2.0
11	1.8
12	1.3
13	0.7
14	0.6
15	0.6
16	0.2
17	0.0
18	0.0

References

Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," **Journal of Political Economy**, Vol. 81, 1973, pp. 637-659.

Brennan, M. and E.S. Schwartz, "Determinants of GNMA Mortgage Prices," **Journal of AREUEA,** Vol. 13, 1985, pp. 209-228 (1985).

Campbell T., and J. Dietrich, "The Determinants of Default on Insured Conventional Residential Mortgage Loans," **Journal of Finance,** December 1983, pp. 1569-1580.

Clauretie, T., and T. N. Herzog, "How State Laws Affect Foreclosure Costs," **Secondary Mortgage Markets,** Vol. 6, Spring 1989, pp. 25-28.

Cooperstein, R., F.S. Redburn and H.G. Myers, "Modelling Mortgage Terminations in Turbulent Times," **AREUEA Journal**, 19 (4) (1991) pp. 473-494.

Cox, J., J. Ingersol, and S. Ross, "An Intertemporal General Equilibrium Model of Asset Prices," **Econometrica**, Vol. 53, 1985, pp. 363-364.

Cunningham, D. and Patric Hendershott, "Pricing FHA Default Insurance," **Housing Finance Review**, Vol. 3, #4, 1984, p. 342-373.

Foster, Chester and Robert Van Order, "FHA Terminations: A Prelude to Rational Mortgage Pricing," **Journal of AREUEA,** Vol. 13, 1985, pp. 373-291.

Heckman, James, "Sample Selectivity Problems as a Specification Error," Econometrica, 1976.

Hendershott, P., and R. Van Order, "Pricing Mortgages: An Interpretation of the Models and Results," **Journal of Financial Services Research**, 1, pp. 77-111, (1987).

Kalbfleisch, J.D. and R.L. Prentice, **The Statistical Analysis of Failure Time Data**, New York: John Wiley and Sons, 1980.

Kau J., D. Keenan, W. Muller III, and J. Eppensor, "Option Theory and Fixed-Rate Mortgages," University of Georgia, 1986, mimeo.

Kau, James B., Donald C. Keenan and Taewon Kim, "Default Probabilities for Mortgages," University of Georgia, 1992, mimeo.

Quigley, John M. and Robert Van Order, "More on the Efficiency of the Market for Single Family Homes: Default," Center for Real Estate and Urban Economics, Working Paper, University of California, Berkeley, 1992

Smith, Clifford W., "Option Pricing," **Journal of Financial Economics**, Vol. 3, Jan-Mar 1976, pp. 616-621.