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### **Turbulence elasticity—A new mechanism for transport barrier dynamics**

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We present a new, unified model of transport barrier formation in "elastic" drift wave-zonal flow (DW-ZF) turbulence. A new physical quantity—the delay time (i.e., the mixing time for the DW turbulence)—is demonstrated to parameterize each stage of the transport barrier formation. Quantitative predictions for the onset of limit-cycle-oscillation (LCO) among DW and ZF intensities (also denoted as I-mode) and I-mode to high-confinement mode (H-mode) transition are also given. The LCO occurs when the ZF shearing rate ( $|\langle v \rangle'_{ZF}|$ ) enters the regime  $\Delta \omega_k < |\langle V \rangle'_{ZF}| < \tau_{cr}^{-1}$ , where  $\Delta \omega_k$  is the local turbulence decorrelation rate and  $\tau_{cr}$  is the threshold delay time. In the basic predator-prey feedback system,  $\tau_{cr}$  is also derived. The I-H transition occurs when  $|\langle V \rangle'_{E\times B}| > \tau_{cr}^{-1}$ , where the mean  $E \times B$  shear flow driven by ion pressure "locks" the DW-ZF system to the H-mode by reducing the delay time below the threshold value. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4894695]

Cyclic phenomena appear in many nonlinear dynamical systems, e.g., biological populations, and ecologies.<sup>1</sup> In biological processes, gestation and maturation times (also called delay times) play a key role in inducing LCO phenomena.<sup>2</sup> In confined plasmas, there are also many experimental observations of LCO phenomena (also denoted as I-mode) in spontaneous transport barrier formation, especially the L-H transition.<sup>3</sup> These feature a stable phase lag between DWand ZF-intensities (or radial electric field). Besides theoretical interest, a physical understanding of the LCO is also practically important to understanding the transition mechanism and to achieving a scaling of L-H transition thresholds with firm physical basis. Recently, experiments<sup>4</sup> with high spatial and temporal resolution suggest that the floating radial electric field (associated with the ZF), rather than the mean electric field (associated with ion pressure gradient), is central to triggering the transport barrier formation.<sup>5</sup> LCOs often occur in a "intermediate" regime, where the ZF shear is stronger than the turbulence decorrelation rate, but is not sufficiently strong to fully quench the DW turbulence. Though the LCO-like phenomena in DW-ZF system were noted in the extended predator-prey systems, such as the 2 predator + 1 prey system composed of ZF ("predator"), mean shear flow ("predator"), and DW ("prey"),<sup>6</sup> the existing studies are limited to qualitative descriptions, and do not present predictions, e.g., state under what circumstance the LCO onset or stop? While zonal shear decorrelation of the large eddies is widely invoked as the physical mechanism underpinning the dynamics of the DW-ZF system, most reduced models of transport treat zonal flows in a rather desultory fashion. As the L-H transitions occur in a strong flowshear regime with persistent turbulence, any further unified understanding of the transition mechanism requires a precise treating of the wave-flow interaction, e.g., the history dependent DW-ZF coupling, especially in regimes of stronger shear. The ubiquity of zonal flow LCO phenomena suggest that a robust mechanism is at work setting the required time delay.

An important, but not yet appreciated, property of the DW turbulence is its elasticity, which appears as a finite delay time in the response of the DW turbulence to the zonal shear. The delay time is a new time scale, which is set by transport of polarization charge (potential vorticity mixing), reflects the history of the DW-ZF coupling, and hence is an essential element in DW-ZF dynamics. It is a ubiquitous and robust mechanism for a time delay. In contrast to turbulent viscosity, turbulent elasticity introduces wave-like behavior to the evolution equation of the ZF, which fundamentally changes the dynamical structure of the DW-ZF system. The effect of the turbulent elasticity becomes prominent as the system enters the strong shear regime (e.g., the Dimits shift regime<sup>8</sup>), where the diffusive turbulent momentum flux ansatz fails and the ZF evolution equation changes from a diffusion equation to a telegraph equation.<sup>7</sup> As turbulent elasticity features delayed response of DW turbulence to ZF shear, a straightforward consequence is that it can induce a phase lag between DW and ZF. If the phase lag is stabilized, a steady LCO state will form. Motivated by this observation, here we propose a generic and simple DW-ZF LCO model, which is relevant to determining the onset of I-phase<sup>9,10</sup> as a step toward the transport barrier formation. An essential ingredient of our 2-fields elastic predator-prey (PP) model is the history dependent DW-ZF coupling, which reflects the time delay effect. In the new predator-prey feedback system, we predict and calculate a critical delay time (which also corresponds to a critical zonal shear). When this time is exceeded, the DW-ZF system will evolve into a steady LCO state. The mechanism for entering the LCO state is that once the delay time exceeds a critical value, both fixed points of

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the DW-ZF system will become unstable, and a stable phase lag between the ZF and the DW develops. According to the Poincaré-Bendixson theorem, the DW-ZF system will then be "attracted" to a LCO state. This mechanism is analogous to that for onset of LCOs in ecological systems, where the gestation time plays a crucial role. We also discuss the impact of the mean electrostatic field driven by ion pressure gradient on the dynamics of the DW-ZF system. We argue that the mean  $E \times B$  shear can "lock" the DW-ZF system to the H-mode-like fixed point by reducing the delay time below its threshold value. This provides a new robust and unified viewpoint for understanding the physical mechanism of transport barrier formation via a cyclic state.

To facilitate the following discussion, we briefly summarize the main results of the conventional 2-fields PP model,<sup>11</sup> which is composed of two first order differential equations:

$$\frac{\partial}{\partial t}\varepsilon_D(t) = \gamma_l \varepsilon_D(t) - \gamma_{nl} \varepsilon_D^2(t) - \alpha \varepsilon_Z(t) \varepsilon_D(t), \qquad (1)$$

$$\frac{\partial}{\partial t}\varepsilon_Z(t) = -\gamma_d \varepsilon_Z(t) + \alpha \varepsilon_D(t) \varepsilon_Z(t).$$
(2)

Equations (1) and (2) are the simplest, nontrivial version of PP model.  $\varepsilon_D(\varepsilon_Z)$  is the energy intensity of the DW (ZF),  $\gamma_l$  is the linear growth rate of the DW,  $\gamma_{nl}$  describes the local coupling between DWs,  $\gamma_d$  is the ZF frictional damping, and  $\alpha$ describes the nonlocal coupling between DW and ZF. The sign of the DW-ZF coupling in Eq. (2) is opposite to that in Eq. (1), so that energy conservation is guaranteed during DW-ZF interaction. Here "ZF" refers to shear flow driven by the DW turbulence. The exact forms of these coefficients are not crucial to the conclusion of this letter, so we simply take them as given parameters. The two fixed points of Eqs. (1) and (2) are  $(\varepsilon_D, \varepsilon_Z) = \left(\frac{\gamma_I}{\gamma_{nI}}, 0\right)$  and  $(\varepsilon_D, \varepsilon_Z) = \left(\frac{\gamma_d}{\alpha}, \frac{\gamma_I}{\alpha} - \frac{\gamma_{nI}\gamma_d}{\alpha^2}\right)$ , which correspond to the L-mode and the H-mode, respectively, and hereafter will be labelled as L-solution and Hsolution. Existence of H-solution requires exceeding a lower limit on the linear growth rate,  $\gamma_l > \gamma_{nl} \gamma_d / \alpha$ . Once this condition is satisfied, the L-solution will always be unstable.<sup>12</sup> To analysis the stability of the H-solution, linearizing Eqs. (1) and (2) near the H-solution, one yields the trace of the corresponding Jacobian matrix

$$tr(J_H) = -\frac{\gamma_{nl}\gamma_d}{\alpha}.$$
 (3)

As  $tr(J_H) < 0$ , the H-solution is always an "attractor." Therefore, one obtains a condition of the L-H transition in the conventional 2-fields PP model,<sup>11</sup> which is just  $\gamma_l > \gamma_{nl}\gamma_d/\alpha$ . A remarkable feature of Eqs. (1) and (2) is that system tends to be "attracted" to the H-solution in the presence of a nonzero seed ZF. The reason is that the ZF is modulationally unstable in the weak shear regime ( $\Delta \omega_k > |\langle V \rangle'_{ZF}|$ ), so that a seed ZF can be continually amplified until the H-solution is achieved. In other words, the basin of the attraction of Hsolution is  $\varepsilon_Z \in (0, \infty)$  and  $\varepsilon_D \in (0, \infty)$ , while the basin of the attraction of the L-solution is  $\varepsilon_Z = 0$  and  $\varepsilon_D \in (0, \infty)$ . As the H-solution in this model is always an "attractor," a sustained I-phase does not exist.

In the conventional Predator-Prey model, the DW-ZF couplings are dependent on the product of  $\varepsilon_D$  and  $\varepsilon_Z$  at the same time, i.e., the model assumes an instantaneous response of the DW turbulence to the ZF.<sup>11</sup> However, it has been demonstrated<sup>7</sup> that the response/delay time is constrained by the turbulent relaxation time of the DW-ZF system. In the wave turbulence picture, turbulent relaxation (or potential vorticity mixing) is driven by the "collisions" between the DW packets, which are a function of the local DW-DW scattering (on the order of the turbulence decorrelation time,  $\Delta \omega_k^{-1}$ ) and the DW-DW scattering mediated by the ZFs (on the order of the ZF shearing time,  $|\langle V \rangle'_{ZF}|^{-1}$ ). Prior to the L-H transition, the DW-ZF system enters the Dimits shift regime, when the ZF shearing rate exceeds the turbulence decorrelation rate

$$|\langle V \rangle'_{ZF}| > \Delta \omega_k. \tag{4}$$

In this marginally stable regime, the DW fluctuations are nearly quenched by the ZFs, so turbulent relaxation is mediated mainly by the DW-ZF scattering, i.e., the delay time  $\tau$  is set by the ZF shear time,<sup>7</sup>  $\tau \simeq |\langle V \rangle'_{ZF}|^{-1}$ , which is also a necessary condition for having a sustained I-phase. In this scenario of dominance of nonlocal interaction, the dynamical structure of the conventional 2-fields PP model will be qualitatively changed.

Taking account of the delayed response of the DW turbulence to the ZF, the evolution of  $\varepsilon_D$  at time *t* is actually influenced by the "distortion" field ( $\varepsilon_Z$ ) at  $(t - \tau)$ , i.e., the nonlinear coupling between *DW* and *ZF* is history dependent. Correspondingly, the strength of the "elastic force" (here means the back-reaction of the DW on the ZF) "felt" by the ZF at time *t* depends on the deformation of the DW induced by the ZF at time  $(t - \tau)$ . A standard way to model this time delay effect<sup>2</sup> is to replace  $\varepsilon_D(t)\varepsilon_Z(t)$  by  $\varepsilon_D(t)\varepsilon_Z(t - \tau)$ . As the delay time (i.e.,  $|\langle V \rangle'_{ZF}|^{-1}$ ) is much shorter than the period  $(\omega_{LCO}^{-1})$  of the LCO,<sup>9,10</sup> the history dependent nonlinear coupling can then be approximated as

$$\varepsilon_D(t)\varepsilon_Z(t-\tau)\simeq \varepsilon_D(t)\varepsilon_Z(t)-\tau\varepsilon_D\frac{\partial}{\partial t}\varepsilon_Z,$$
 (5)

where  $\partial_t \ln \varepsilon_D = \omega_{LCO}$ . Replacing  $\varepsilon_D(t)\varepsilon_Z(t)$  in Eqs. (1) and (2) by Eq. (5), one obtains the elastic 2-fields PP model with history dependent DW-ZF coupling

$$\frac{\partial}{\partial t}\varepsilon_D = \gamma_l \varepsilon_D - \gamma_{nl} \varepsilon_D^2 - \alpha \varepsilon_Z \varepsilon_D - \frac{\alpha \tau \gamma_d - \alpha^2 \tau \varepsilon_D}{1 + \alpha \tau \varepsilon_D} \varepsilon_Z \varepsilon_D, \quad (6)$$

$$\frac{\partial}{\partial t}\varepsilon_Z = \frac{-\gamma_d \varepsilon_Z + \alpha \varepsilon_D \varepsilon_Z}{1 + \alpha \tau \varepsilon_D}.$$
(7)

Equations (6) and (7) are the simplest nonlinear system that incorporates the effect of turbulent elasticity. They are equivalent to a "projection" of a more realistic system, such as the 3-fields system composed of the evolution of  $\varepsilon_D$ ,  $\varepsilon_Z$ , and the turbulent momentum flux. Here, the effect of dynamical evolution of the turbulent momentum flux is "modeled" by a history dependent DW-ZF coupling. However, the reduced model captures the essence (i.e., history dependent DW-ZF coupling) of the time delay effect, and also is more analytically tractable. Here, we use the elastic 2-fields PP model as a paradigm to illustrate the critical role of the delay time in introducing new dynamical structures to the DW-ZF system.

In contrast to Eqs. (1) and (2), the time delay effect in Eqs. (6) and (7) can be interpreted as follows: for the evolution of  $\varepsilon_D$ , the nonlocal coupling coefficient  $\alpha$  (constant in time) is renormalized to  $\alpha' = \alpha + (\alpha \tau \gamma_d - \alpha^2 \tau \varepsilon_D)/(1 + \alpha \tau \varepsilon_D)$ , which varies with time. For the evolution of  $\varepsilon_Z$ , the inertia of the ZF is renormalized to  $(1 + \alpha \tau \varepsilon_D)$ , which is a "shielding effect" and will induce a phase lag between  $\varepsilon_Z$  and  $\varepsilon_D$ . As in Eqs. (1) and (2), the L-solution in Eqs. (6) and (7) is always unstable in the presence of the H-solution. The positions of the fixed points of the elastic 2-fields PP model are the same as those of Eqs. (1) and (2), but the stability of the H-solution can change. The trajectory of  $J_H$  of Eqs. (6) and (7) now gives

$$tr(J_H) = -\frac{\gamma_{nl}\gamma_d}{\alpha} + \frac{\alpha\tau\gamma_d}{1+\tau\gamma_d} \left(\frac{\gamma_l}{\alpha} - \frac{\gamma_{nl}\gamma_d}{\alpha^2}\right).$$
 (8)

In Eq. (8), the delay time makes a positive contribution, and hence tends to destabilize the H-solution. The delay time for transition is

$$tr(J_H) = 0 \Rightarrow \tau_{cr} = \frac{\gamma_{nl}}{\alpha \gamma_l - \gamma_{nl} \gamma_d}.$$
 (9)

Once  $\tau > \tau_{cr}$ , both the fixed points in Eqs. (6) and (7) will become "repellers." As the delay time  $\tau \simeq \langle V \rangle_{ZF}^{\prime-1}$ , the corresponding critical strength of zonal shear is

$$|\langle V \rangle_{cr}'| = \tau_{cr}^{-1} = \frac{\alpha \gamma_l}{\gamma_{nl}} - \gamma_d.$$
(10)

To have an unstable H-solution requires that the ZF shearing rate not be "too" strong, i.e.,  $|\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|$ . Combining with the nonlocality dominance condition (Eq. (4)), one obtains the condition for the DW-ZF system to enter the LCO state as  $\Delta \omega_k < |\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|$ . When this criterion is met, the Poincaré-Bendixson theorem implies that the trajectory of the phase point in the phase space of  $\varepsilon_Z$  and  $\varepsilon_D$  will then be "attracted" to a closed orbit, i.e., a limit cycle, and the system will enter a new steady LCO state. In this state, the DW turbulence is not quenched, but oscillates. The physical process evolution is sketched in Figure 1. The direction of the LCO is determined by the causality between  $\varepsilon_Z$  and  $\varepsilon_D$ . Here, the ZF gains energy from the DW turbulence, and hence the system executes a clockwise LCO (Fig. 1). The nonlinear dynamics of the elastic 2-fields PP model is numerically illustrated with the following parameters:  $\gamma_l = 0.8$ ,  $\gamma_{nl} = 1$ ,  $\alpha = 2$ ,  $\gamma_d = 0.3$ , and initial phase point ( $\varepsilon_{D,0}$ ,  $\varepsilon_{Z,0}$  = (0.8, 0.3). With these parameters, both L- and Hsolutions exist and they are the H-solution  $(\varepsilon_Z, \varepsilon_D) = (0.15, \varepsilon_D)$ 0.325) and the L-solution,  $(\varepsilon_Z, \varepsilon_D) = (0.8, 0)$ . The corresponding critical delay time is  $\tau_{cr} = 0.77$ . In Fig. 2, the delay time is  $\tau = 1.8$ , which exceeds the critical value, and the system evolves to a LCO state. The ratio of the delay time to the period of the LCO is  $\tau/\omega_{CLO}^{-1} \simeq 1.8/50 = 0.036 \ll 1$ ,



FIG. 1. Sketch of physical mechanism of LCO.

and hence the use of the expansion in Eq. (5) is also seen to be valid. This ratio is also consistent with experimental observations, e.g.,  $\tau/\omega_{CLO}^{-1} \simeq 0.01$  in Ref. 10. From Fig. 2, the local turbulence decorrelation rate is estimated as  $\Delta \omega_k \simeq \gamma_{nl} \varepsilon_D \simeq 0.3 \le |\langle V \rangle'_{ZF}| \simeq 1.8^{-1}$ , and hence the precondition  $\Delta \omega_k < |\langle V \rangle'_{ZF}|$  is satisfied. A constant phase mismatch ( $\pi/2$ ) between  $\varepsilon_D$  and  $\varepsilon_Z$  also appears in Fig. 2.

It is widely recognized that the mean electric field driven by ion pressure gradient plays an important role in the L-H transition. In the preceding section, we showed the delay time to be a new parameter, controlling the dynamical structure of the DW-ZF system. A natural question is how the delay time is modulated by the mean  $E \times B$  shear. The ramping injected power can enhance the turbulent Reynolds stress, which then drives stronger ZFs. During this process, the ZF continuously extracts energy from the DW turbulence, and eventually drives the DW-ZF system to a socalled Dimits shift state and initiates the LCO (I-phase). Besides driving the DW-ZF system into the strong shear regime, the increased edge ion pressure can also drive a large mean  $E \times B$  shear flow. The newly generated mean  $E \times B$ shear flow will increase the DW scattering to ZF shear via the same mechanism. This extra scattering will enhance the turbulence decorrelation rate and then reduce the delay time. If the ion pressure profile is steepened sufficiently, the mean  $E \times B$  shearing rate will then exceed the ZF shearing rate, so that the mean  $E \times B$  shear becomes the dominant turbulence decorrelation mechanism and the delay time is then determined by the mean  $E \times B$  shearing rate,  $\tau \sim |\langle V \rangle_{E \times B}^{\prime - 1}|$ . In other words,  $\langle V \rangle_{E \times B}$  becomes the main "controller" in the later phase of the L-H transition. Once  $|\langle V \rangle'_{E \times B}| > \tau_{cr}^{-1}$ , the H-solution becomes an "attractor," so that the DW-ZF system will transit from the LCO state to H-mode (e.g., Figure 3). In the process of the L-H transition, the evolution of the ion pressure (or the mean electrostatic field) is determined by

$$\frac{\partial}{\partial t}P_i = (\chi_i(\varepsilon_D) + \chi_{neo})\frac{\partial^2}{\partial x^2}P_i + S, \qquad (11)$$

where  $\chi_{neo}$  is the neoclassical heat conductivity, which is smaller than the turbulent heat conductivity  $\chi_i$ , and *S* stands for the injected external power. The turbulent heat



FIG. 2. Delay time  $\tau = 1.8 > \tau_{cr}$ .

conductivity scales as  $\chi_i(\epsilon_D) \sim \epsilon_D$ . Because  $\epsilon_D$  is participating in a LCO,  $\chi_i$  is oscillating, too. In Eq. (11), the turbulence intensity  $\epsilon_D$  plays the role of reducing the pressure amplitude, which is opposite to effect on the ZF amplitude. For a constant power injection rate, the fastest rise rate of  $P_i$  corresponds to the minimal value of  $\epsilon_D$ . Thus, the turbulence intensity lags behind the ion pressure by a phase of  $\pi/2$ . Compared to the LCO among  $\epsilon_Z$  and  $\epsilon_D$ , it is then no surprise that a counter-rotating LCO among  $P_i$  and  $\epsilon_D$ , would occur.<sup>5</sup> Taking  $P_i$  as passive, which is adequate for the I-phase, we sketched the counter-rotated limit-cycle in Fig. 4. This scenario is also consistent with the bifurcation model of a LCO, where, by employing a model S-curve of the effective diffusivity of particle density, one can obtain a LCO solution for the mean radial electric field (or ion pressure).<sup>13</sup> Here, the turbulent elasticity induced LCO among DW and ZF can naturally provide a multi-valued turbulent diffusivity/conductivity and so cause the oscillation of ion pressure profile. Finally, one arrives at a unified paradigm for the spontaneous transport barrier formation as sketched in Fig. 5. At the beginning, the DW-ZF system is near an unstable L-mode, and then the ZF starts to grow. When the



FIG. 3. Delay time  $\tau = 0.5 < \tau_{cr}$ .

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FIG. 4. The LCO among  $\epsilon_D$  and  $P_i$  with s = 0.2 and  $\chi_{neo} \ll \chi_i$ . The cycling direction is opposite to that in Figure 2.

ZF shearing exceeds local turbulence decorrelation (i.e.,  $|\langle V \rangle'_{ZF}| > \Delta \omega_k$ ), but is below the critical shearing rate  $(|\langle V \rangle'_{ZF}| < |\langle V \rangle'_{cr}|)$ , the system will be "attracted" a limit-cycle and enters the I-phase. During the above process, the mean  $E \times B$  shear driven by the ion pressure gradient is continuously increasing, and finally becomes the dominant turbulence decorrelation mechanism  $(|\langle V \rangle'_{E \times B}| > |\langle V \rangle'_{ZF}|)$ . Once  $|\langle V \rangle'_{E \times B}| > |\langle V \rangle'_{cr}|$  (i.e.,  $\tau < \tau_{cr}$ ), the DW-ZF system will evolve into the H-mode state via reverse Hopf bifurcation. Note that the delay time parameterizes each stage of the transition.

In summary, we propose a new mechanism for the transport barrier formation via a cyclic state. This new mechanism follows from the time history of the DW-ZF coupling, which originates from the turbulent mixing of the DW turbulence, and hence is general and robust. In the elastic 2-fields predator-prey feed back system, we predict a critical delay time  $\tau_{cr}$  and show that the LCO occurs for an "intermediate" strong ZF shear regime,  $\Delta \omega_k < |\langle V \rangle'_{ZF}| < \tau^{-1}_{cr}$ , which is in agreement with recent experiments<sup>9,10,14</sup> and is different from the Waltz rule for L-H transition,<sup>15</sup>  $|\langle V \rangle'_{E\times B}| > \gamma_l$ . It is also found that the oscillating turbulence intensity can result in a LCO among DW intensity and ion pressure, with opposite rotation direction, in the later phase of the I-mode. Since the flip of LCO happens during the I-H transition, it is a signature of the formation of H-mode.<sup>5</sup> It is argued that if the



FIG. 5. A sketch of the unified paradigm of L-I-H transition (s-injected power).

delay time is reduced below a critical value by the mean shear flow, the DW-ZF system will be "locked" to the Hsolution. Therefore, turbulent elasticity is a critical "controller" of transport barrier dynamics.

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