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Comments on "Acoustic Velocity Formulation for Sources in Arbitrary Motion"

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Ghorbaniasl et al. [1] have published time-domain analytical formulations of the acoustic velocity for sources in arbitrary motion by extending Farassat's formulations [2]. They claimed that their formulations have an advantage over pressure gradient formulations [3] in that their formulations are simpler and provide faster computations. This comparison is based on the assumption that the pressure gradient formulations were developed and used to obtain the acoustic velocity components indirectly. However, this assumption is incorrect. This note will clarify the confusion about the relationship between the acoustic velocity formulations and the acoustic pressure gradient formulations.

The first point is that pressure gradient formulations were developed to solve acoustic scattering problems [4, 5, 6, 7, 8] that require the pressure gradient as the boundary condition in acoustic scattering problems rather than an indirect step in computing the acoustic velocity. Thus pressure gradient formulations are required in their own right. Therefore, there is no need to compare two independent formulations and to assess the superiority of one formulation over the other.

Secondly, the acoustic velocity formulations can be considered as an extension of the pressure gradient formulations. The relationship between acoustic pressure gradient and acoustic velocity is shown in the following derivation.

First, the acoustic pressure is related to the acoustic velocity potential

$$p'(\mathbf{x},t) = -\rho_0 \frac{\partial \phi(\mathbf{x},t)}{\partial t}, \tag{1}$$

or

$$\phi(\mathbf{x},t) = -\frac{1}{\rho_0}\int_{t_0}^{t} p'(\mathbf{x},t^*)dt^*, \tag{2}$$

where $\mathbf{x}$ and t are the observer position vector and observer time. t^* is a dummy variable of integration and t_0 is the initial observer time. The variables p' and ϕ are the acoustic pressure and acoustic velocity potential, respectively, and ρ_0 is the ambient density.

The gradient of the acoustic velocity potential, which is the acoustic velocity, is given as

$$\nabla\phi(\mathbf{x},t) = -\frac{1}{\rho_0}\int_{t_0}^{t} \nabla p'(\mathbf{x},t^*)dt^*. \tag{3}$$

Now that the acoustic velocity is written in terms of the acoustic pressure gradient, it is straightforward to see that the acoustic velocity can be derived in terms of the acoustic pressure gradient.

The acoustic pressure gradient formulation G1 [3] is written as

$$4\pi\nabla p'_T(\mathbf{x},t) = -\frac{\partial}{\partial t}\left\{\frac{1}{c}\int\limits_{f=0}[\hat{\mathbf{r}}E_T]_{\mathrm{ret}}\,dS + \int\limits_{f=0}\left[\frac{(\mathbf{r}-\mathbf{M})\rho_0 U_n}{r^2(1-M_r)^2}\right]_{\mathrm{ret}} dS\right\}. \tag{4}$$

where

$$E_T = \left[\frac{\rho_0(\dot{U}_n + U_{\dot{n}})}{r(1-M_r)^2}\right]_{\text{ret}} + \left[\frac{\rho_0 U_n(r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}\right]_{\text{ret}}, \tag{5}$$

for the thickness noise and

$$\begin{aligned} 4\pi\nabla p'_L(\mathbf{x},t) =& \frac{1}{c}\frac{\partial}{\partial t}\left\{-\int\limits_{f=0}[\hat{\mathbf{r}}E_L]_{\text{ret}}\,dS + \int\limits_{f=0}\left[\frac{\mathbf{L} - L_r\hat{\mathbf{r}}}{r^2(1-M_r)}\right]_{\text{ret}} dS \right.\\ &\left. - \int\limits_{f=0}\left[\frac{L_r\hat{\mathbf{r}} - L_r\mathbf{M}}{r^2(1-M_r)^2}\right]_{\text{ret}} dS\right\} + \int\limits_{f=0}\left[\frac{\mathbf{L} - 3L_r\hat{\mathbf{r}}}{r^3(1-M_r)}\right]_{\text{ret}} dS, \end{aligned} \tag{6}$$

where

$$\begin{aligned} E_L =& \frac{1}{c}\left[\frac{\dot{L}_r}{r(1-M_r)^2}\right]_{\text{ret}} + \left[\frac{L_r - L_M}{r^2(1-M_r)^2}\right]_{\text{ret}} \\ &+ \frac{1}{c}\left[\frac{L_r\,(r\dot{M}_r + c(M_r - M^2))}{r^2(1-M_r)^3}\right]_{\text{ret}}. \end{aligned} \tag{7}$$

for the loading noise where $r = |\mathbf{x} - \mathbf{y}|$ and c is the speed of sound in the undisturbed medium. The subscripts r, n and M imply the dot product of the vector with either the unit vector in the radiation direction $\hat{\mathbf{r}}$, outward normal vector $\hat{\mathbf{n}}$ to the surface $f = 0$, or the surface Mach number $\mathbf{M}$, respectively. The dot over a variable indicates source time differentiation. The variables U_i and L_i are defined by

$$U_i = [1 - (\rho/\rho_0)]v_i + (\rho u_i/\rho_0), \tag{8}$$

$$L_i = P_{ij}\hat{n}_j + \rho u_i(u_n - v_n), \tag{9}$$

where u_i are the components of the local flow velocity vector and v_i are the components of the local blade surface velocity vector and P_{ij} is the compressive stress tensor. Eqs. (8) and (9) are the form used for a permeable surface, which is useful if the flow field around surfaces is used especially for including nonlinear sources. For an impermeable surface, such as the actual blade surface, $U_i = v_i$ and $L_i = P_{ij}\hat{n}_j$.

Substituting Eqs. (4)–(7) into Eq. (3) yields

$$4\pi\rho_0 \nabla\phi_T(\mathbf{x}, t) = \frac{1}{c}\int\limits_{f=0} [\hat{\mathbf{r}}E_T]_{\text{ret}}\, dS + \int\limits_{f=0} \left[\frac{(\mathbf{r} - \mathbf{M})\rho_0 U_n}{r^2(1 - M_r)^2}\right]_{\text{ret}} dS, \tag{10}$$

for the thickness noise and

$$\begin{aligned} 4\pi\rho_0 \nabla\phi_L(\mathbf{x}, t) =& \frac{1}{c}\int\limits_{f=0} [\hat{\mathbf{r}}E_L]_{\text{ret}}\, dS - \frac{1}{c}\int\limits_{f=0} \left[\frac{\mathbf{L} - L_r\hat{\mathbf{r}}}{r^2(1 - M_r)}\right]_{\text{ret}} dS \\ &+ \frac{1}{c}\int\limits_{f=0} \left[\frac{L_r\hat{\mathbf{r}} - L_r\mathbf{M}}{r^2(1 - M_r)^2}\right]_{\text{ret}} dS - \int_{t_0}^{t} \left\{ \int\limits_{f=0} \left[\frac{\mathbf{L} - 3L_r\hat{\mathbf{r}}}{r^3(1 - M_r)}\right]_{\text{ret}} dS \right\} dt^*, \end{aligned} \tag{11}$$

for the loading noise.

Note that Eqs (10) and (11) were referred to as formulation V1A in Ghorbaniasl et al. [1]. They derived the formulations directly taking the spatial derivative of the noise source terms which involved heavy algebraic manipulations while derivation just presented shows that the acoustic velocity formulation is obtained directly from the acoustic pressure gradient formulation. The formulation V1 in Ghorbaniasl et al. [1] can also be derived from intermediate steps to formulation G1. The detailed derivations are not provided in this note because it is not relevant to the context.

In sum, this note clarifies the confusion between the acoustic velocity formulations and the acoustic pressure gradient formulations. They both have distinctive purposes and are used independently, yet the acoustic velocity formulations can be directly obtained from the acoustic pressure gradient formulations and the former can be considered as an extension of the latter.

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