Asymmetric design for compound elliptical concentrators (CEC) and its geometric flux implications

Lun Jiang[a] and Roland Winston[a*]
[a]UC Solar, University of California Merced, 5200 N Lake Rd, Merced,CA, 95343

ABSTRACT

The asymmetric compound elliptical concentrator (CEC) has been a less discussed subject in the nonimaging optics society. The conventional way of understanding an ideal concentrator is based on maximizing the concentration ratio based on a uniformed acceptance angle. Although such an angle does not exist in the case of CEC, the thermodynamic laws still hold and we can produce concentrators with the maximum concentration ratio allowed by them. Here we restate the problem and use the string method to solve this general problem. Built on the solution, we can discover groups of such ideal concentrators using geometric flux field, or flowline method.

Keywords: CEC, flowline method, asymmetric, string method, geometric flux

1. INTRODUCTION

The asymmetric ideal concentrators have been first accomplished by[1] using two parabolas. Other people built on it with a non-flat absorber and produced similar ideal concentrators with infinitely far away source. [2]. Such designs are generally referred to as the asymmetric compound parabolic concentrators (CPC), however, the parabola is not necessarily the accurate name to describe the shape when the absorber is non-flat. For concentrators with finite source, compound elliptical concentrator (CEC) was proposed, and extended to non-flat absorbers[3]. Parallel to this effort, the alternative method for building the CPC or CEC from a geometric flux field (i.e. flowline) perspective, have been showcased in several papers [4][5][6]. More recently, a discussion of using pharosage[7], or effectively the flowline method have been proposed for 3D ideal concentrators[8][9]. A generalized string method has been proposed for double stage symmetric concentrators[10]. Built on it, the asymmetric case was also developed for double stage concentrators[11]. Both the asymmetric and symmetric setups allow the second stage concentrators to degenerate into a small concentrator. This can be accomplished by keeping the absorber below the crossing over edge rays (A'B and AB'). It is also revealed in these discussions that the shape of the absorber can be any convex shape. Such a prior art, with both symmetric and asymmetric setups, will inevitably produce a CEC-type concentrators as the second stage, which is the main topic of discussion for this paper. Within this paper, instead of discussing about double stage concentrators, we give the same solution from the perspective of directly generating the general ideal concentrator. We discuss about the CEC with asymmetric finite sources and propose the method for building the ideal concentrator using the string method. This eliminate the complex setup of both first and second concentrators and directly arrive at the degenerated case. Built on it, we will also include a short derivation of generating a group of flowline concentrators with the asymmetric setup. The concept can be quickly extended to non-flat absorbers due to the flexibility of the string method. At the end of the paper we will predict several other designs that can be extended from the current concentrators. In the end, we will also discuss the potential of this method for angular transformers.

Ideal concentrators

Ideal concentrators obeying both the first and second law of thermodynamics can be defined using the understanding of the radiative heat transfer between a source and sink at equilibrium temperature T, assuming both bodies are ideal black bodies[12] [13]. Here we give a more robust argument to reframe the arguments made in [13].

*rwinston@ucmerced.edu; phone 1 209 201-2863; fax 1 209 228-2914; ucsolar.org

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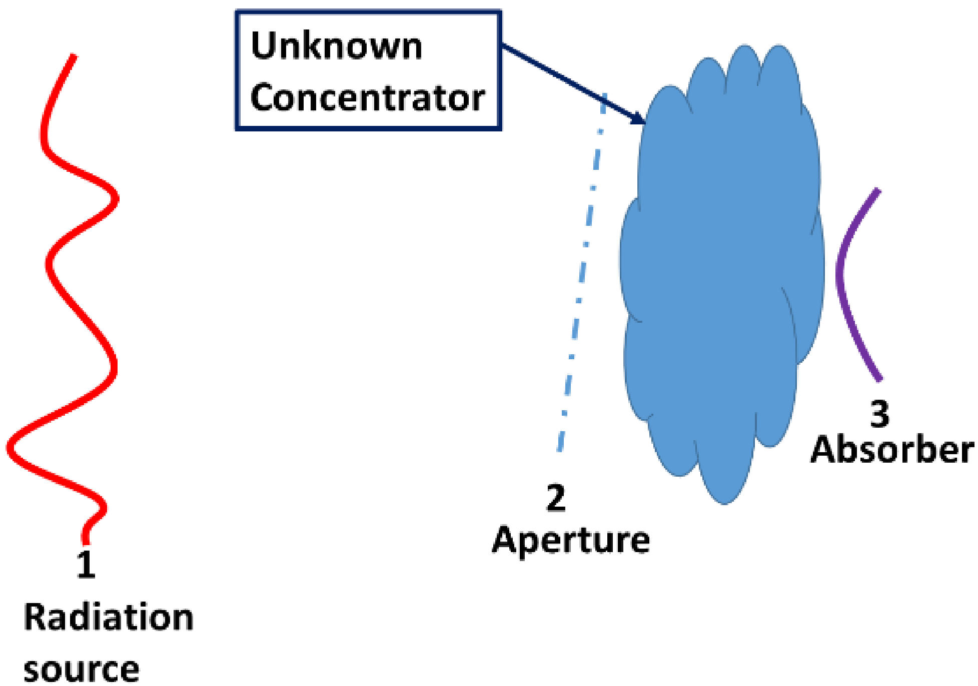

Figure 1. General problem of a concentrator design.

Here, we call the radiation source as 1, absorber as 3, the goal is to construct a concentrator that performs the highest geometrical from aperture 2 to absorber 3.

Define geometric concentration ratio $C = A_2/A_3$.

Define the probability of radiation power transferring from any blackbody a to b as P_{ab}. For example, the probability of any ray from 1 to 2 is defined as $P_{12} = Q_{12}/Q_1$, here, Q_{12} is the power(watts) going from 1 to 2, and Q_1 is the total power emitted by 1.

According to the first law of thermodynamics, energy would be conserved, assuming a zero loss system, concentrating light from 2 to 3:

$$Q_{12} = Q_{13}$$

$$Q_1 P_{12} = Q_1 P_{13}$$

Using Stefan-Boltzmann law:

$$A_1 \sigma T^4 P_{12} = A_1 \sigma T^4 P_{13}$$

$$A_1 P_{12} = A_1 P_{13} \tag{1}$$

According to the second law of thermodynamics, at equilibrium temperature, the radiative heat exchanged between the two black bodies must be the same, otherwise a colder body will transfer heat to a hotter body, considering only radiative heat transfer.

$$Q_{12} = Q_{21}$$

$$Q_{13} = Q_{31}$$

Or, similar to how we reached EQ (1),

$$A_1 P_{12} = A_2 P_{21} \tag{2}$$

$$A_1 P_{13} = A_3 P_{31} \tag{3}$$

Combining (1), (2) and (3), we conclude:

$$C = \frac{A_2}{A_3} = \frac{P_{31}}{P_{21}} \leq \frac{1}{P_{21}} \tag{4}$$

The maximum concentration ratio C_{max} will be reached if and only if $P_{31} = 1$. The conventional nonimaging optics is a group of such concentrators, where any ray from the absorber will reach the source only, as long as the absorber is treated as a black body. Here we define them as the thermodynamically ideal concentrators. These concentrators allow the absorber to "see" only the source. In other word, the usable etendue (or phase space) of the absorber can be traced to the source and the source only.

2. THERMODYNAMICALLY IDEAL CONCENTRATORS

String method

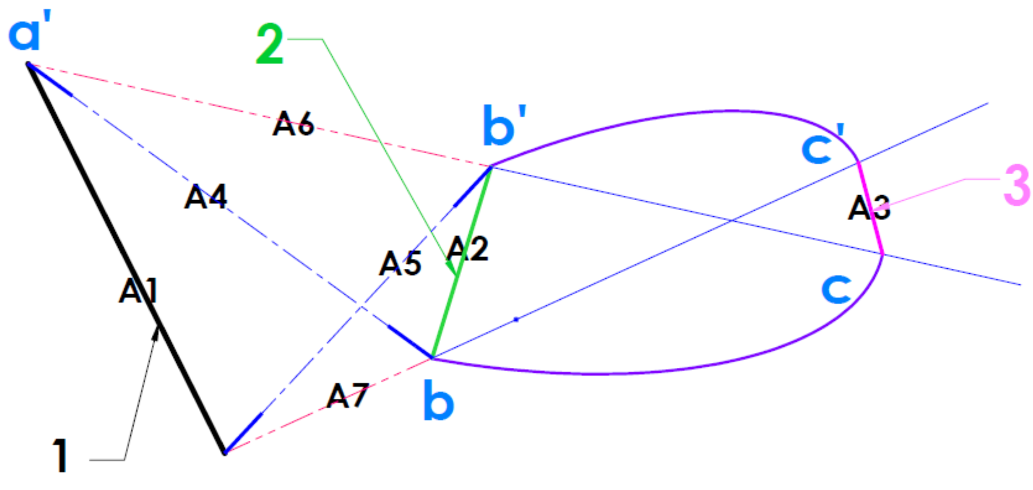

Figure 2. The asymmetric setup for ideal concentrators.

Now consider the asymmetric setup of the source and the absorber as shown in Fig.2. Due to the property of the elliptical curve b'c', ab'+b'c = ac'+c'c. Similarly a'b+bc'=a'c+cc'. Combine the above two equations and we get 2cc'=a'b+ab'-ab-a'b'. Imaginary strings can be used to form the curves, especially for the cc' part, where we can allow the string to be bent over a convex shape. This string method guarantees that $C = \frac{bb'}{cc'} = \frac{A_2}{A_3} = \frac{2A_2}{A_7+A_6-A_4-A_5} = \frac{1}{P_{21}} = C_{max}$ according to Hottel string[14]. As long as the string is taut and tied on points a and c for the b'c' curve, and tied on a'c' for the bc curve, the curves will be ideal concentrators. Two deductive conclusions can be made. First, cc' does not need to be flat, it can be any convex shape as shown in[3]. Second, due to the string's capability of dealing with asymmetry, the absorber shape does not need to be symmetric either. Third, the source aa' can be infinitely far away, giving ideal concentrators for asymmetric acceptance angle setup as mentioned in [2]. Now we propose the asymmetric, any convex shape of absorber, ideal concentrator solution as the following:

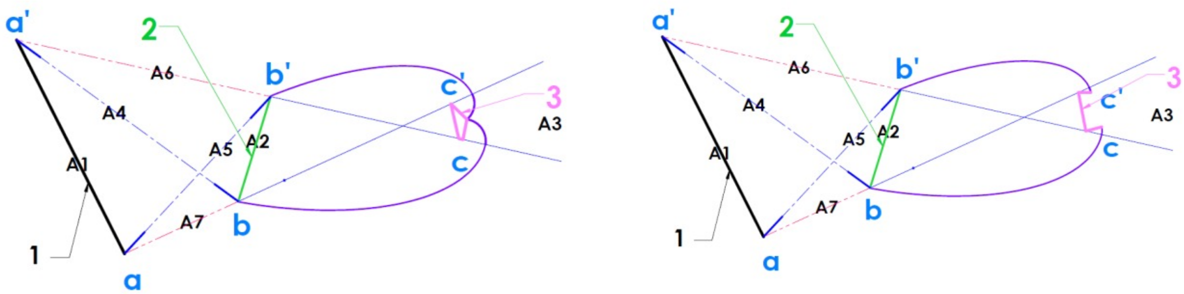

Figure 3. The ideal concentrator for any convex, asymmetric scenario.(a)The absorber is closed shape.(b)The absorber is open.(Note that these curves are not exactly traced out here.)

- Choose the source and absorber shape and position.
- Draw the crossover lines a’c and ac’, which are tangent to the upper and lower edge of the absorber.
- Choose one point on the absorber for the closed shape absorber Fig.3.(a), or the two ends of the open shape absorber Fig.3.(b), to start with the curve.
- Tie the corresponding strings to the chosen points and the source edges. For example, $ac'c$ are tied to a and c, with c' as the initial movable point for tracing out the curve $\widehat{c'b}$.
- Keep the string tight and trace out the upper and lower curve of the concentrator.
- Stop as the curves cross the crossover lines.

Notice that the source and absorber must be chosen first, once they are determined, the aperture will be fully determined too. This process cannot be reversed. This is also similar to the Figure.1. of [10], where the absorber is in region 3, and exactly tangential to AB and AB’. The difference between our method and [10] is that we predetermine the source and the absorber as the only configuration of the setup. The double stage concentrator, which has two foci potentially mismatching the absorber’s position, need additional curves to maintain the ideal concentration to the aperture of the second stage. Although exactly as described in [10], such a requirement degenerates by eliminating these curves once the absorber falls below the crossover of a’c and ac’.

Geometric flux implications of the ideal concentrators

From the ideal concentrators that do not disturb the geometric flux field, we can trace the flowline inside the ideal concentrator described in Fig.2. There are four different regions to generate the field, as shown in Fig.4.

- Region I, any point within this region sees one of the extreme rays being from c’ and the other from the reflection of a’, so flux is simply elliptical shape with a’and c’ as foci.
- Region II, any point within this region sees one of the extreme rays being from c and the other from the reflection of a, so flux is simply elliptical shape with ac as foci.
- Region III, here a point sees only the absorber cc’, and therefore are hyperbolas
- Region IV, similarly, a point sees the source aa’, and the flux line is hyperbola.

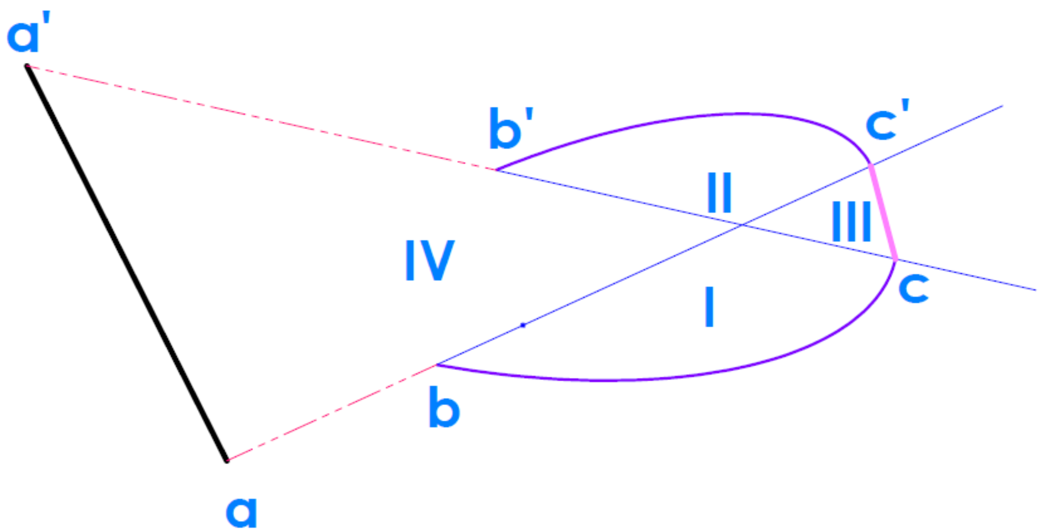

Figure.4. The regions of geometric flux field within an ideal asymmetric concentrator.

Using a simple Matlab® code, we can show case the flowline within the ideal concentrator as in Fig.5. The interesting fact is, when plotted over different regions as shown in Fig.4, the crossing over of the flowline from one region to the other is smooth (with the same tangent).

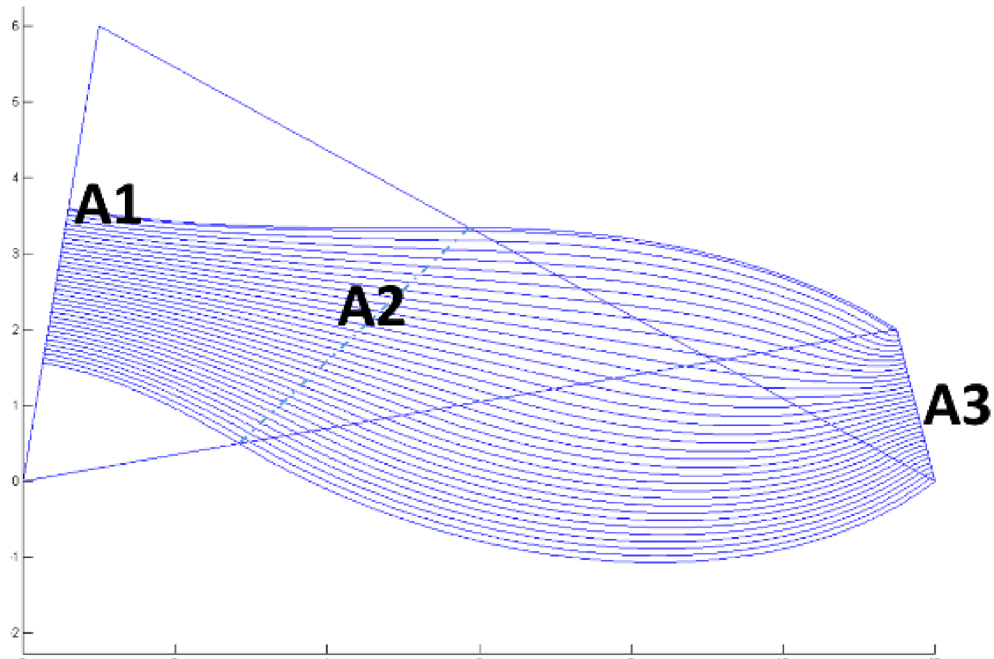

Figure. 5. The geometric flux within an ideal concentrator

By observing the geometric flux inside, we can reach the following conclusions.

- Because there is no sink or source for the flowline $\vec{J}$, $\nabla \cdot \vec{J} = 0$
- We can take any pair of the flowline in between and form an ideal concentrator, just as shown in [8] for the symmetric case, we have asymmetric ones here.
- Such a method can be utilized for any convex absorber as shown in Fig.3, both closed and non-closed.

- If we trace the flowline from the absorber A3 to source A1, for any pair of them, the absorber is always effectively put at the source. This is also why it achieves the highest concentration possible.

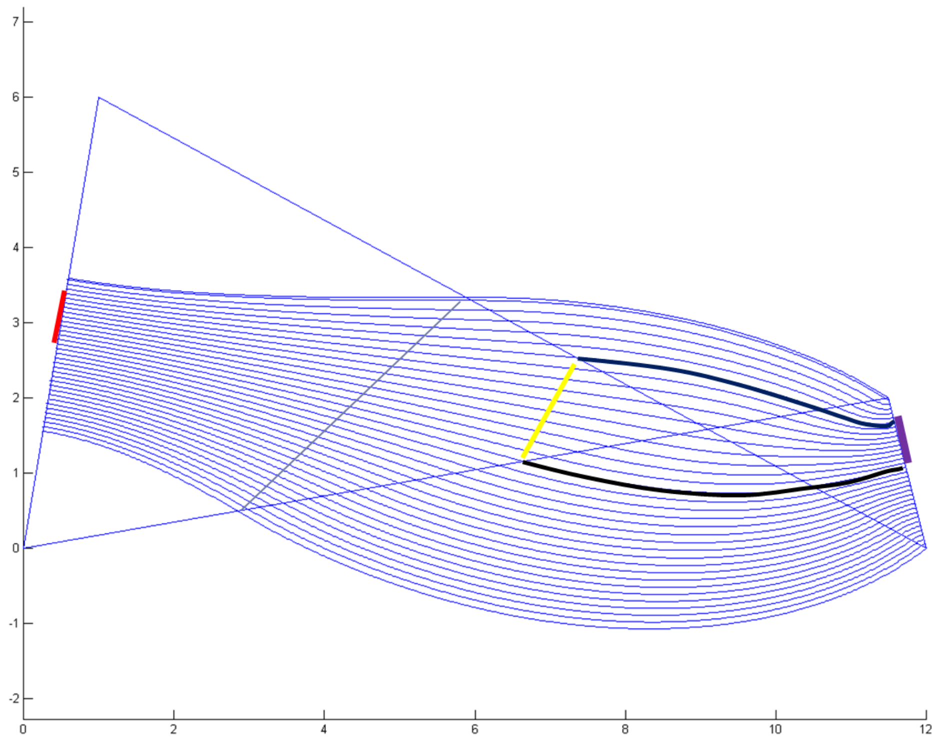

Figure.6. The flowline pair that can form the ideal concentrators. Here the aperture of the concentrator is yellow, the absorber is the purple, the concentrator shape is the pair of flowline connecting their edge. and if we trace the flowline all the way back to red. Due to $\nabla \cdot \vec{J} = 0$, the purple and the red will be the same.

Discussion

The idea of ideal concentrators formed using string method can be easily extended also to angular transformers, as long as the wave front can be represented by strings.[15] How to form a flowline method for these ideal concentrators have only been vaguely discussed[16]. It is obvious that a simple "mountain top" shape in 2D cannot produce rotational symmetric ideal results.[4] Therefore the question remains as in what way the geometric flux field in include both the source and sink information in a 3D setup. Even if such a field exists, is it possible to produce the ideal concentrators following the flowlines as reflectors?

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